노트 제목 2010-05-25

- oblique shocks

$$V_{t_1} = V_1 \cos\sigma \qquad V_{t_2} = V_2 \cos(\sigma-\delta)$$
$$V_{n_1} = V_1 \sin\sigma \qquad V_{n_2} = V_2 \sin(\sigma-\delta)$$
$$Ma_{n_1} = Ma_1 \sin\sigma \qquad Ma_{n_2} = Ma_2 \sin(\sigma-\delta)$$

continuity $\quad \rho_1 V_{n_1} = \rho_2 V_{n_2}$ — ①

mtm in t-direction : $(\rho_1 V_{n_1}) V_{t_1} = (\rho_2 V_{n_2}) V_{t_2}$

$\rightarrow V_{t_1} = V_{t_2} = V_t$ — ② $\quad \cancel{\rho_1 \ \rho_2}$

mtm in n-direction :

$$p_1 - p_2 = (\rho_2 V_{n_2}) V_{n_2} - (\rho_1 V_{n_1}) V_{n_1} \quad — ③$$

energy eq : $c_p T_1 + \frac{1}{2} V_1^2 = c_p T_2 + \frac{1}{2} V_2^2$ — ④

$$V_2^2 - V_1^2 = V_{n_2}^2 + \cancel{V_{t_2}^2} - (V_{n_1}^2 + \cancel{V_{t_1}^2}) = V_{n_2}^2 - V_{n_1}^2$$

perfect gas relation : $c_p T = c_p \frac{p}{\rho R} = \frac{k}{k-1} \frac{p}{\rho}$

④ $\rightarrow \frac{k}{k-1}\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right) = \frac{V_{n_1}^2 - V_{n_2}^2}{2}$ ——— ⑤

p_1, ρ_1, V_{n_1}, V_{t_1} are specified → ① ② ③ ⑤ determine p_2, ρ_2, V_{n_2}, V_{t_2}

Similar relations bet V_{n_2} and V_{n_1} ~ ft. of Ma_{n_1}

$$\frac{p_2}{p_1} = \frac{1}{k+1}\left[2k\,Ma_1^2 \sin^2\sigma - (k-1)\right]$$

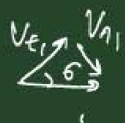
$$\frac{\rho_2}{\rho_1} = \frac{(k+1)\,Ma_1^2 \sin^2\sigma}{(k-1)\,Ma_1^2 \sin^2\sigma + 2} = \frac{V_{n_1}}{V_{n_2}}$$

$$\frac{T_2}{T_1} = \left[2 + (k-1)\,Ma_1^2 \sin^2\sigma\right]\frac{2k\,Ma_1^2 \sin^2\sigma - (k-1)}{(k+1)^2\,Ma_1^2 \sin^2\sigma}$$

$$T_{0_2} = T_{0_1}$$

$$\frac{P_{0_2}}{P_{0_1}} = \left[\frac{(k+1)\,Ma_1^2 \sin^2\sigma}{2+(k-1)\,Ma_1^2 \sin^2\sigma}\right]^{\frac{k}{k-1}}\left[\frac{k+1}{2k\,Ma_1^2 \sin^2\sigma - (k-1)}\right]^{\frac{1}{k-1}}$$

$$Ma_{n_2}^2 = \frac{(k-1)\,Ma_{n_1}^2 + 2}{2k\,Ma_{n_1}^2 - (k-1)}$$



Deflection angle $\delta = \sigma - (\sigma - \delta)$

$$= \tan^{-1}\frac{V_{n_1}}{V_{t_1}} - \tan^{-1}\frac{V_{n_2}}{V_{t_2}}$$

max δ? → $\dfrac{\partial \delta}{\partial V_t} = 0$

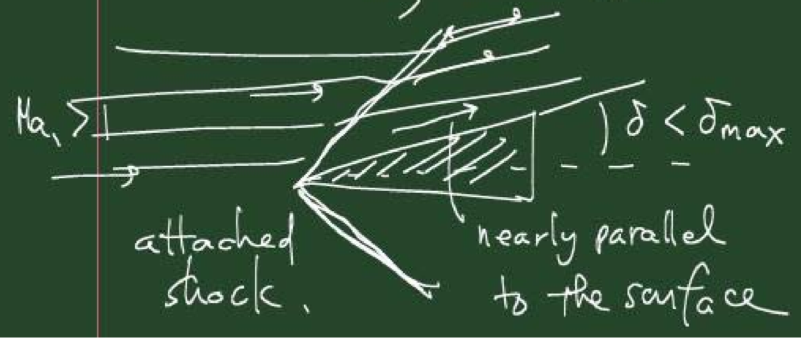
$$\rightarrow \frac{V_{n_1}}{V_{t_1}} = \left(\frac{V_{n_1}}{V_{n_2}}\right)^{\frac{1}{2}}, \quad \frac{V_{n_2}}{V_{t_2}} = \frac{V_{n_2}}{V_{n_1}}\frac{V_{n_1}}{V_{t_2}} = \left(\frac{V_{n_2}}{V_{n_1}}\right)^{\frac{1}{2}}$$

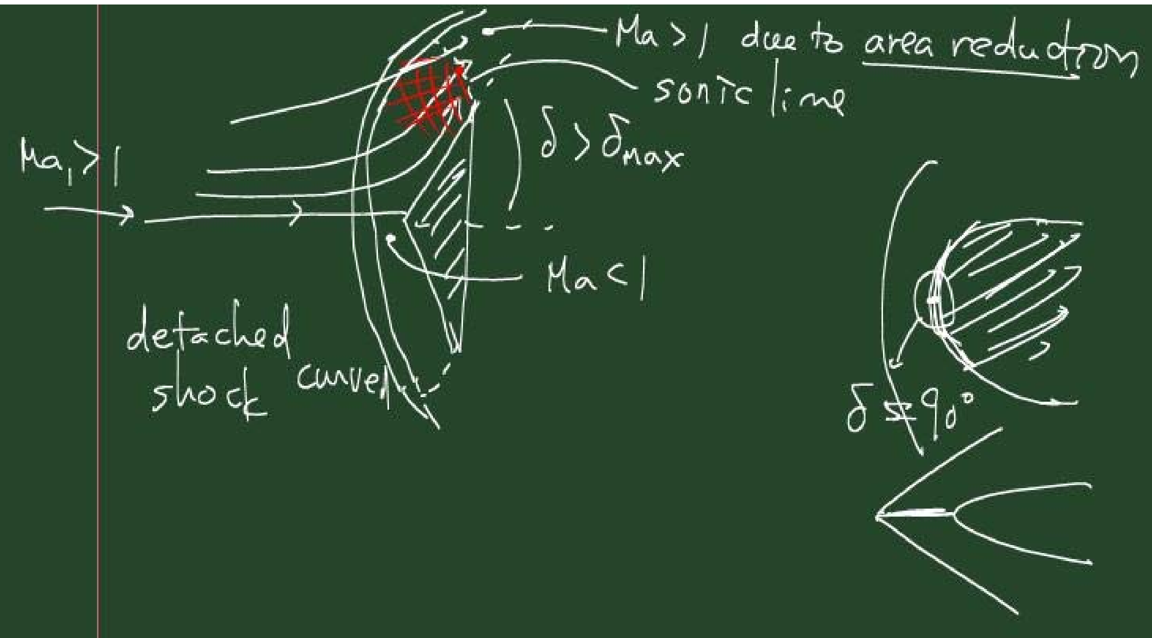
$$\rightarrow \delta_{max} = \tan^{-1}\left(\frac{V_{n1}}{V_{n2}}\right)^{\frac{1}{2}} - \tan^{-1}\left(\frac{V_{n1}}{V_{n2}}\right)^{-\frac{1}{2}}$$

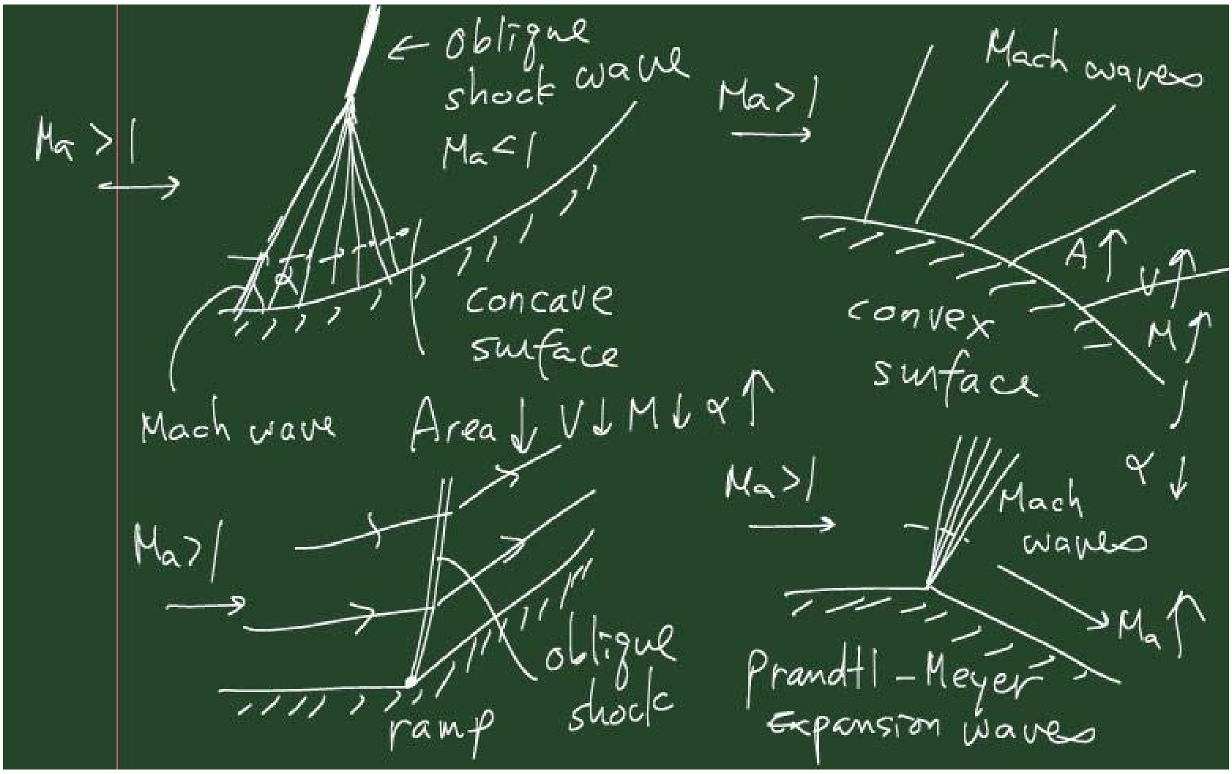
ft(Ma_{n1})

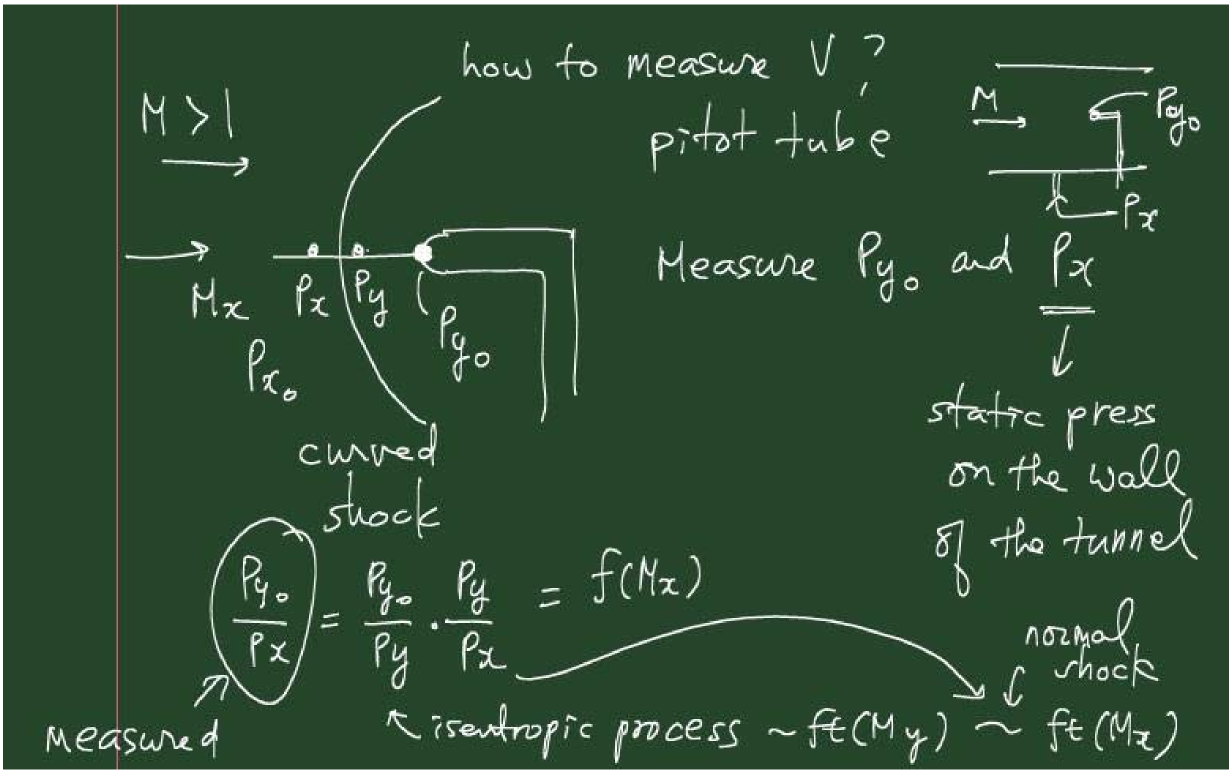
if $Ma_{n1} = 3$, $\frac{V_{n1}}{V_{n2}} = 3.8571 \rightarrow \delta_{max} = 36.03^\circ$

" $\rightarrow \infty$, " $= 6.0$ $= \underline{\underline{45.58^\circ}}$






oblique shock wave
Ma>1
Ma<1
α
concave surface
Mach wave
Area↓ V↓ M↓ α↑
Ma>1
Mach waves
A↑ V↑
convex surface
M↑
α↓
Ma>1
ramp
oblique shock
Ma>1
Mach waves
Ma↑
Prandtl-Meyer Expansion waves


how to measure V ?
M>1
pitot tube
M
Py0
Px
Mx Px Py Py0
Px0
curved shock
Measure Py0 and Px
static press on the wall of the tunnel
Py0/Px = Py0/Py · Py/Px = f(Mx)
measured
normal shock
isentropic process ~ ft(My) ~ ft(Mx)

→ obtain M_x. supersonic pitot tube

$M_x < 1$

isentropic relation ~ $f(M_x)$

measured compute

⑤ Flow in constant-area ducts with friction

When the ducts are reasonably short,

the flow is approximately adiabatic.

when " " extremely long,

the flow is non-adiabatic and

approximately isothermal

two limiting cases

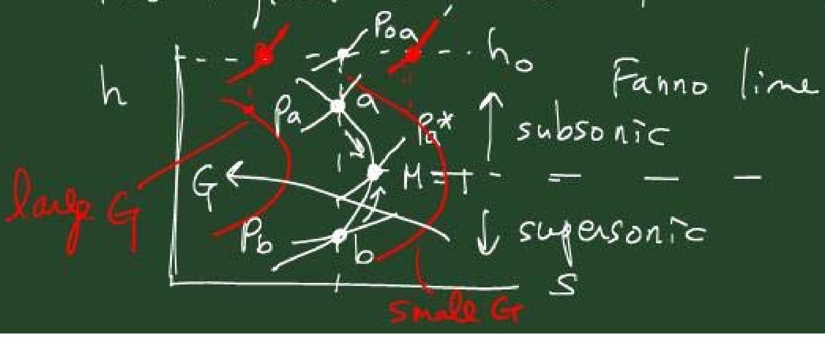
① Adiabatic flow

energy $\quad h + \frac{1}{2}V^2 = h_o$

cont. $\quad \frac{w}{A} = \rho V \equiv G$ (mass velocity)

$\longrightarrow \quad h = h_o - \frac{G^2}{2\rho^2}$ → relation between h and ρ.

For a given flow, h_o & G are const.



adiabatic → $dq = 0$ → $ds \geq 0$

If the flow is subsonic (pt. a), due to friction,

$V\uparrow, \; M\uparrow, \; p\downarrow, \; h\downarrow$

(never exceed $M=1$)

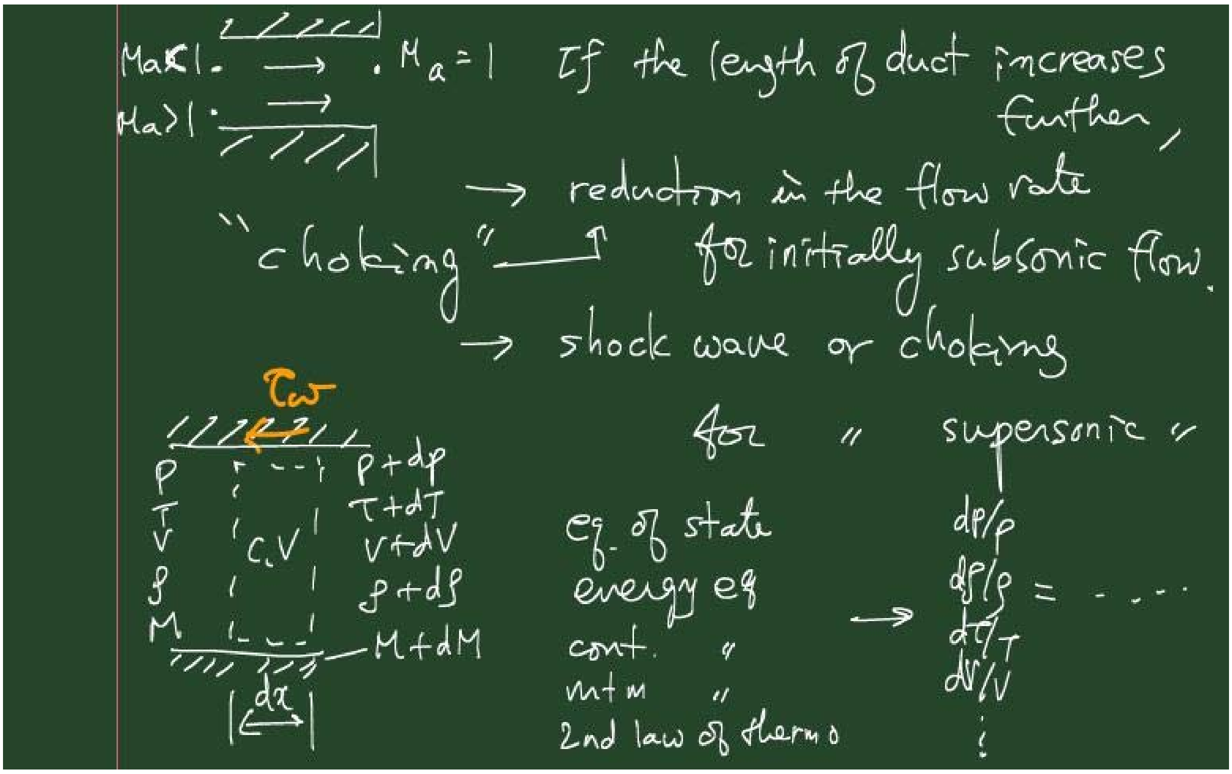
If the flow is supersonic (pt. b),

$V\downarrow, \; M\downarrow, \; p\uparrow, \; h\uparrow$.

(never below $M=1$)

limiting press → p^* @ $M=1$

choking occurs.


Ma<1. ⟶ . Ma = 1
Ma>1.
If the length of duct increases further,
→ reduction in the flow rate
"choking"
for initially subsonic flow.
→ shock wave or choking
for " supersonic "
τw
p p+dp
T T+dT
V V+dV
ρ ρ+dρ
M M+dM
C.V
dx
eq. of state
energy eq
cont. "
mtm "
2nd law of thermo
dp/p
dρ/ρ = ...
dT/T
dV/V
⋮