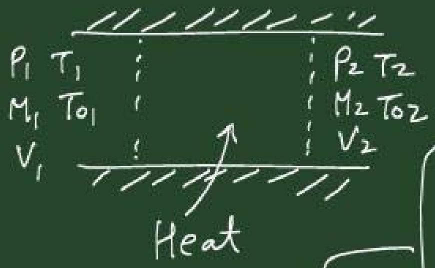
노트 제목 2010-06-01

- Flow in ducts with heating or cooling



cont: $\rho_1 V_1 = \rho_2 V_2 \rightarrow \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2}$ — ①

mtm: $P_1 - P_2 = \frac{w}{A}(V_2 - V_1)$

perfect gas: $p = \rho R T \quad c = \sqrt{kRT}$

$\rightarrow \rho V^2 = k p M^2$

$$\frac{P_2}{P_1} = \frac{1 + kM_1^2}{1 + kM_2^2} \quad \text{— ②}$$

perfect gas law: $\frac{P_2}{P_1} = \frac{\rho_2}{\rho_1}\frac{T_2}{T_1}$ — ③

Mach number: $\frac{M_2}{M_1} = \frac{V_2}{V_1}\frac{c_1}{c_2} = \frac{V_2}{V_1}\sqrt{\frac{T_1}{T_2}}$ — ④

Impulse ft $\quad F \equiv P(1 + kM^2)$

$$\frac{F_2}{F_1} = \frac{P_2}{P_1} \cdot \frac{1 + kM_2^2}{1 + kM_1^2} \overset{②}{=} 1$$

Isentropic stag. press

$$\frac{P_{o2}}{P_{o1}} = \frac{P_2}{P_1} \frac{\left(1 + \frac{k-1}{2} M_2^2\right)^{\frac{k}{k-1}}}{\left(1 + \frac{k-1}{2} M_1^2\right)^{\frac{k}{k-1}}}$$

Entropy change $\quad \frac{S_2 - S_1}{c_p} = \ln \frac{T_2/T_1}{(P_2/P_1)^{\frac{k-1}{k}}}$

stag. temp $T_0 = T + \frac{V^2}{2c_p} = T\left(1 + \frac{V^2}{2c_pT}\right)$

$\left(h_0 = h + \frac{V^2}{2}\right)$, $h = c_pT$ $\quad = T\left(1 + \frac{k-1}{2}M^2\right)$

energy eq. $\frac{T_{02}}{T_{01}} = \frac{T_2}{T_1}\,\frac{1+\frac{k-1}{2}M_2^2}{1+\frac{k-1}{2}M_1^2}$

$Q = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} = c_p(T_{02} - T_{01})$

① & ② → ③ → ④

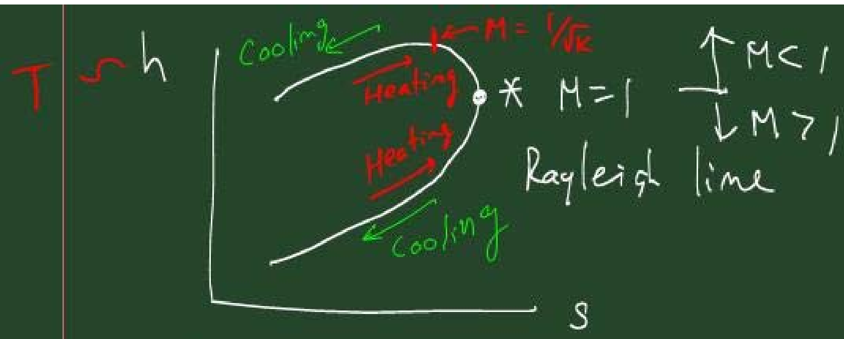
$\frac{T_2}{T_1} = \frac{M_2^2}{M_1^2}\,\frac{(1+kM_1^2)^2}{(1+kM_2^2)^2}$

$T_{02} = T_{01} + Q/c_p$

→ $\frac{T_{02}}{T_{01}} = 1 + Q/c_pT_{01}$

$$\frac{T_{02}}{T_{01}} = \frac{M_2^2}{M_1^2}\,\frac{(1+kM_1^2)^2}{(1+kM_2^2)^2}\cdot\frac{1+\frac{k-1}{2}M_2^2}{1+\frac{k-1}{2}M_1^2} = 1 + \frac{Q}{c_pT_{01}}$$

	Heating $M_1<1$	Heating $M_1>1$	Cooling $M_1<1$	Cooling $M_1>1$
T_0	↑	↑	↓	↓
M	↑	↓	↓	↑
T	↑ for $M<\frac{1}{\sqrt{k}}$ ↓ " >	↑	↓ for $M<\frac{1}{\sqrt{k}}$ ↑ " " > "	↓
P	↓	↑	↑	↓
P_0	↓	↓	↑	↑
V	↑	↓	↓	↑



choking effect

Heating ($T_0 \uparrow$) : $M \uparrow$ for $M_1 < 1$ ⟹ $\boxed{M \to 1}$

Heating $M \downarrow$ $M_1 > 1$

$M<1$ . . . . • $M=1$ ⇐ Max. allowable M_1 exist

$M>1$ - - - - - - • $M=1$ ⇐ Min. allowable M_1

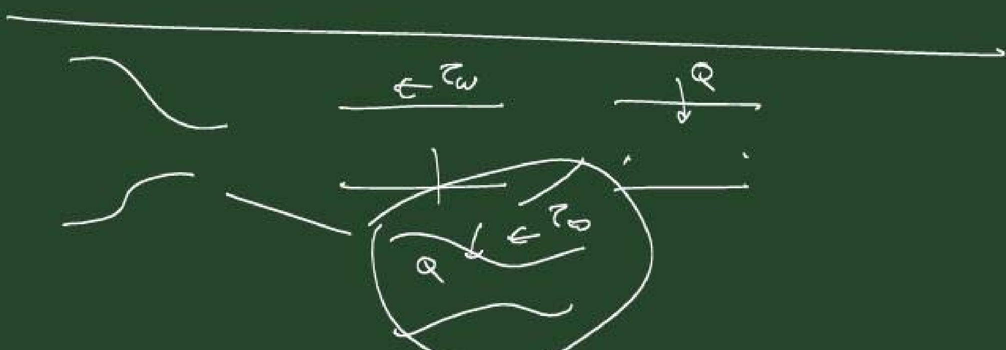
when T_{0_1} and $\Delta T_0 (\sim Q)$ are given.

⟹ Maximum possible ratio of T_{0_2}/T_{0_1}

Cooling ($T_0 \downarrow$) : $M \downarrow$ for $M_1 < 1$

$M \uparrow$ 〃 $M_1 > 1$

no limitation in T_{0_1}/T_{0_2} for $M_1 < 1$.



- ✓ tensor
- ✓ governing eqs. – principal laws
- ✓ potential flows
- ✓ surface waves
- ✓ compress. flow

spring – inviscid flow

fall – viscous flow