Mid-term exam : up to Chapter 5. April 28 (Wed)
18:30 –
(2 hrs)

노트 제목 2010-04-13

⊙ Plane wake

U_c : char. convection vel.

$U_s = U_c - \langle U(x, y{=}0) \rangle$: char. vel. difference

$y_{\frac{1}{2}}$: $\langle U(x, y = y_{\frac{1}{2}}) \rangle = U_c - \frac{1}{2} U_s(x)$

$\xi = y / y_{\frac{1}{2}}$

$f(\xi) = \dfrac{U_c - \langle U(x,y) \rangle}{U_s(x)}$: self-similar vel. defect.

$f(0) = 1$, $f(\pm 1) = \frac{1}{2}$

mtm-deficit flow rate

$$\dot{M}(x) = \int_{-\infty}^{\infty} \rho \langle U \rangle (U_c - \langle U \rangle) dy$$

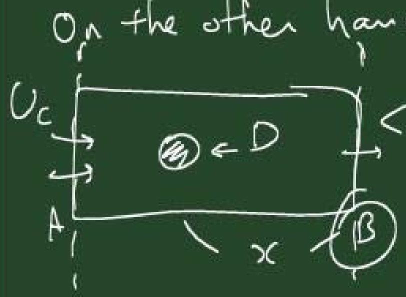
$$= \rho U_c U_s(x) y_{\frac{1}{2}}(x) \int_{-\infty}^{\infty} \Big(1 - \underbrace{\frac{U_s}{U_c}}_{\text{decays as } x \uparrow} f(\xi)\Big) f(\xi) d\xi$$

ft. (x)

∴ it is not exactly self-similar but

asymptotically self-similar

On the other hand,



$$\Sigma F_x = -D + \int p_A\, dy - \int p_B\, dy$$

$$= \int_{-\infty}^{\infty} \rho \langle U\rangle (\langle U\rangle - U_c)\, dy$$

$$\Rightarrow D = \int_{-\infty}^{\infty} \rho \langle U\rangle (U_c - \langle U\rangle)\, dy$$

$$\therefore \dot{M}(x) = D = \text{const.}$$

$\therefore$ In the far wake $\left(U_s/U_c \to 0\right)$,

$U_s(x)\, y_{\frac{1}{2}}(x)$ is indep. of x.

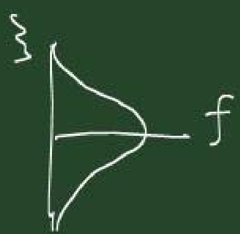
Self-similarity and boundary layer eq.

$$\Rightarrow S = \frac{U_c}{U_s}\frac{dy_{\frac{1}{2}}}{dx}, \quad \frac{S(\xi f)'}{\xi} = \frac{-g'}{\xi} \quad \left(g(\xi) = \frac{\langle uv\rangle}{U_s^2}\right)$$

$$S = \text{const.}$$

$$U_s \sim x^{-\frac{1}{2}}, \quad y_{\frac{1}{2}} \sim x^{\frac{1}{2}}$$

$$f(\xi) = \exp(-\alpha \xi^2) \quad (\alpha = \ln 2)$$

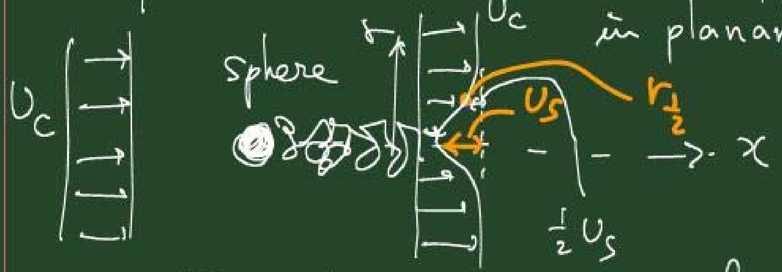


$$Re_0 = \frac{U_s y_{\frac{1}{2}}}{\nu} = \text{const.}$$

$$Re_T = \frac{U_s y_{\frac{1}{2}}}{\nu_T} = \frac{1}{\hat{\nu}_T} = \frac{2\ln 2}{S} \simeq 16.7 \qquad S_{cylinder} = 0.083$$

$\langle u^2 \rangle^{\frac{1}{2}}_{max} / U_s = 0.32$

- Axisymmetric wake – similar approach as in planar wake



self-similarity is possible only as $\frac{U_s}{U_c} \to 0$

$\to S = \frac{U_c}{U_s} \frac{dr_{\frac{1}{2}}}{dx} = \text{const.}$ $\quad$ ↳ $50 < x/d < 150$.

mtm deficit flow rate = Drag $\sim \rho U_c U_s r_{\frac{1}{2}}^2$

$\to U_s \sim x^{-\frac{2}{3}}, \quad r_{\frac{1}{2}} \sim x^{\frac{1}{3}}$

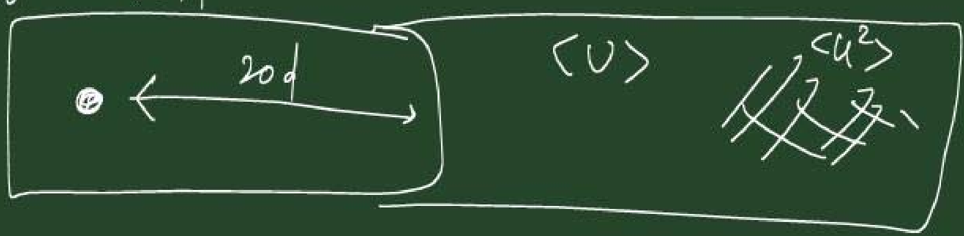
$Re_0 = \frac{U_s r_{\frac{1}{2}}}{\nu} \sim x^{-\frac{1}{3}}$ decreasing in x !

$\langle u^2 \rangle^{\frac{1}{2}}_{max} / U_s \sim 0.9$!

↓ flow can be assumed to relaminarize.

laminar flow $\to U_s \sim x^{-1}, \quad r_{\frac{1}{2}} \sim x^{\frac{1}{2}}$

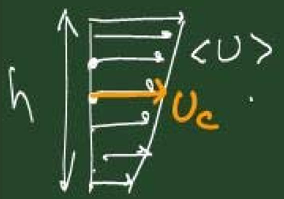
$Re_d = 10^5 \to 20M$



- Homogeneous shear flow

  So far, $\frac{d\langle U\rangle}{dy} \neq$ const.

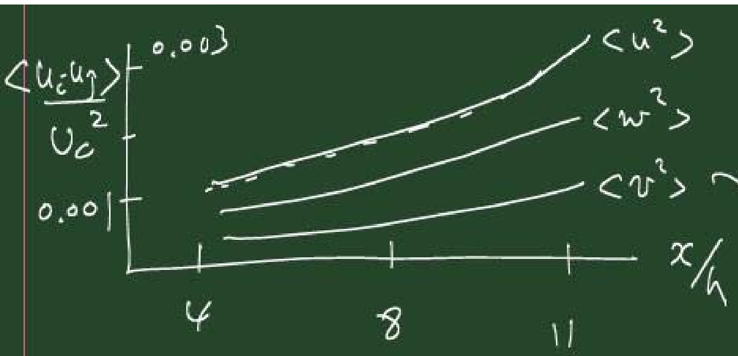
  $\frac{d\langle U\rangle}{dy} =$ const.



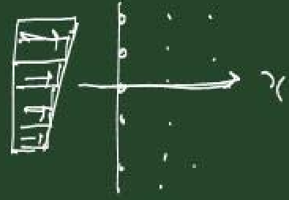
$\langle V\rangle = \langle W\rangle = 0$

$u_i(\underline{x},t)$ and $p'(\underline{x},t)$ are statistically homogeneous.

→ imposed mean vel. grad $\frac{\partial\langle U_i\rangle}{\partial x_j}$ must be uniform (i.e. not ft of $\underline{x}$) but may vary in time. (ex. 5.40 & 5.41)



axial variation



In a frame moving w/ U_c, turb. is approx. homo.

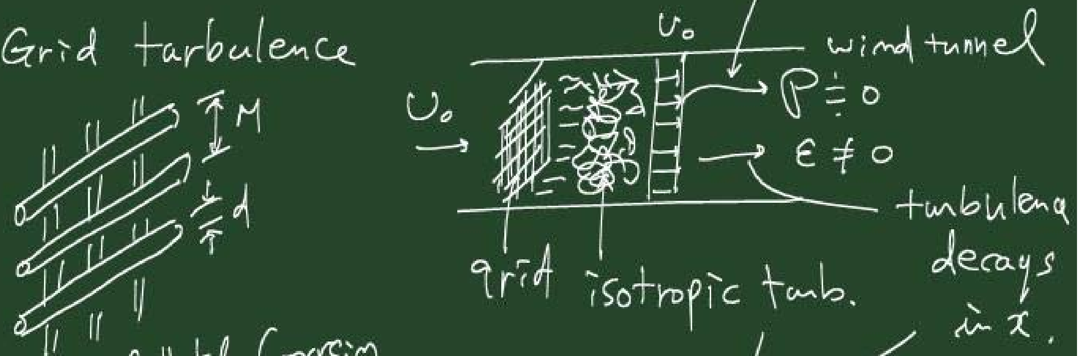
Direct numerical simulations :

Rogallo (1981), Rogers & Moin (1987)

→ homo. shear flow becomes self-similar after some time.

→ statistics normalized by $S = \frac{\partial\langle U\rangle}{\partial y}$ and k become indep. of time. ∵ no mean. vel. grad

- Grid turbulence

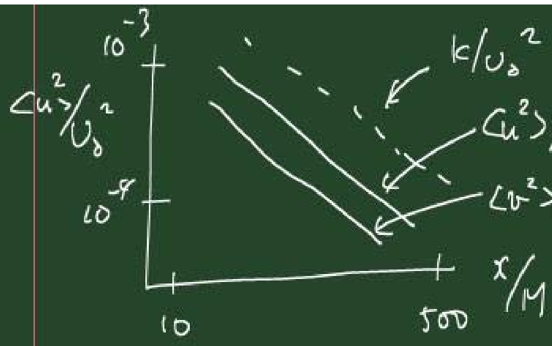


decaying isotropic turbulence.

Kang, Meneaveu (JFM)

Comte-Bellot & Corrsin (19__, JFM)

In the frame moving with U_0, turbulence is homo. and evolves in time $(t = x/U_0)$.



$\langle u^2\rangle/U_0^2 \simeq 1.1\langle v^2\rangle/U_0^2$, $\langle v^2\rangle/U_0^2 \simeq \langle w^2\rangle/U_0^2$ } isotropic turbulence

$\langle uv\rangle = \langle vw\rangle = \langle uw\rangle = 0$

grid turbulence is called as homo. isotropic turb.

$$\frac{k}{U_0^2} = A\left(\frac{x-x_0}{M}\right)^{-n}$$

x_0 : virtual origin

n : 1.15 ~ 1.45

moving frame → $k(t) = k_0\left(\frac{t}{t_0}\right)^{-n}$

t_0 : arbitrary ref. time

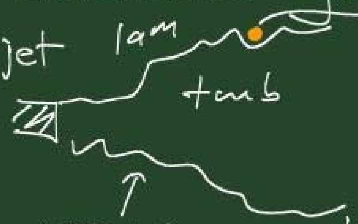
k_0 : $k = k_0$ at $t = t_0$

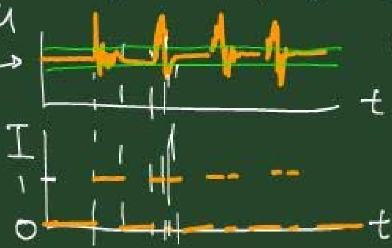
$$\frac{dk}{dt} = -\left(\frac{nk_0}{t_0}\right)\left(\frac{t}{t_0}\right)^{-n-1} = -\varepsilon$$

$$\therefore\ \varepsilon = \varepsilon_0 \left(\frac{t}{t_0}\right)^{-n-1}, \quad \varepsilon_0 = nk_0/t_0$$

5.5 Further observations

- Conserved scalar — skip
- Intermittency (I)





flow is intermittent.

viscous superlayer : highly contorted moving surface that separates regions of turb. & non-turb. flows.

- PDFs and higher moments — skip
- Large-scale turbulent motion

Fig. 5.51 <plane mixing layer>

Fig. 5.52 <axisym. wakes>

coherent structures — turb. production

⇒ turbulent processes are non-local in space & time due to large-scale structures.

turbulence — ~~random motion~~ ?

random motion
+ organized motion
(coherent)
(large-scale)