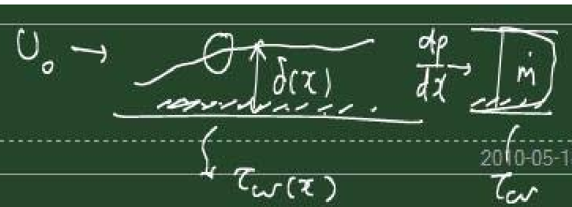
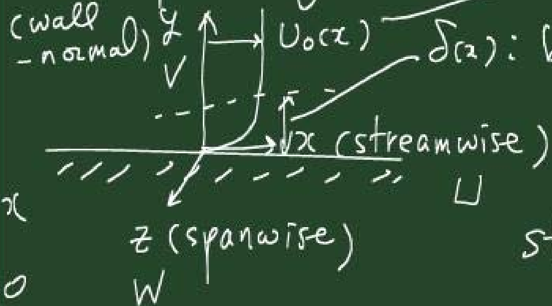
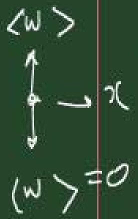


노트 제목 2010-05-18

7.3 Boundary layers

- Description of the flow — free-stream velocity





$\delta(x)$: boundary layer thickness

$\langle W \rangle = 0$, $W \neq 0$

statistics vary in y and x.

- free-stream pressure P_0

$$P_0(x) + \frac{1}{2}\rho U_0^2(x) = \text{const}$$

$$\rightarrow -\frac{dP_0}{dx} = \rho U_0 \frac{dU_0}{dx} \qquad \frac{dU_0}{dx} > 0 \text{ favorable press. grad.}$$

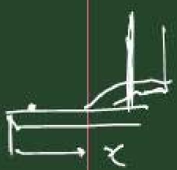
$$\qquad\qquad\qquad\qquad\qquad \text{〃} < 0 \text{ adverse 〃 〃}$$

- $\underline{\delta(x)}$ at which $\langle U \rangle = 99\%$ of $U_0 = 0.99U_0$

$0.99U_0 \rightarrow \underline{0.995U_0}$

$0.999U_0$

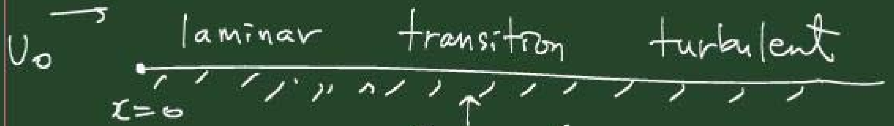
more reliable ones:



displacement thickness $\delta^*(x) = \int_0^\infty \left(1 - \frac{\langle U \rangle}{U_0}\right) dy$

momentum 〃 $\theta(x) = \int_0^\infty \frac{\langle U \rangle}{U_0}\left(1 - \frac{\langle U \rangle}{U_0}\right) dy$

- $Re_x = \frac{U_0 x}{\nu}$, $Re_\delta = \frac{U_0 \delta}{\nu}$, $Re_{\delta^*} = \frac{U_0 \delta^*}{\nu}$, $Re_\theta = \frac{U_0 \theta}{\nu}$

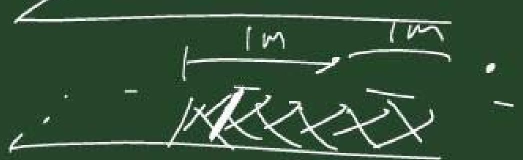


$U_0 =$ $\qquad Re_x \simeq 10^6$

Air $15\,m/s$ $\qquad Re_x = \frac{U_0 x}{\nu} = 10^6 \rightarrow x = \frac{10^6 \times 1.5\times10^{-5}}{15}$

$= 1\,m$

wind tunnel



boundary layer trip. wire $\rightarrow$ promote transition to turbulence

⊙ Mean-mtm eqs.

Boundary layer approx.

$$\frac{\partial}{\partial y}\langle\cdot\rangle \gg \frac{\partial}{\partial x}(\cdot)$$

y-mtm : $\langle p\rangle + \rho\langle v^2\rangle = P_0(x)$

@ wall, $\langle P_w\rangle = P_0(x)$

x-mtm : $\langle U\rangle \frac{\partial\langle U\rangle}{\partial x} + \langle V\rangle \frac{\partial\langle U\rangle}{\partial y}$

$$= \nu \frac{\partial^2\langle U\rangle}{\partial y^2} - \frac{\partial\langle uv\rangle}{\partial y} - \frac{1}{\rho}\frac{dP_0}{dx}$$

$$\equiv \frac{1}{\rho}\frac{\partial\tau}{\partial y} + U_0\frac{dU_0}{dx}$$

$$-\frac{\partial\langle uu\rangle}{\partial x} \neq 0$$

$$-\cancel{\frac{\partial\langle uw\rangle}{\partial z}} \; 0$$

$$\tau = \rho\nu\frac{\partial\langle U\rangle}{\partial y} - \rho\langle uv\rangle \; : \text{total shear stress}$$

@ wall, $\frac{1}{\rho}\frac{\partial \tau}{\partial y}\Big|_{y=0} = -U_0 \frac{dU_0}{dx}$

for $U_0 = \text{const.}$, $\frac{\partial \tau}{\partial y}\Big|_{y=0} = 0 = \mu \frac{\partial^2 \langle U \rangle}{\partial y^2}\Big|_{y=0} - \rho \frac{\partial \langle uv \rangle}{\partial y}\Big|_{y=0}$

$$\left(v \sim \underbrace{v_w}_{0} + y \underbrace{\frac{\partial v}{\partial y}\Big|_w}_{-\frac{\partial u}{\partial x}|_w - \frac{\partial w}{\partial z}|_w = 0} + \frac{1}{2} y^2 \frac{\partial^2 v}{\partial y^2}\Big|_w + \cdots \right) \quad \langle uv \rangle \sim y^3$$

$$\Downarrow$$

$$\frac{\partial \langle uv \rangle}{\partial y} \sim y^2 \to 0 \text{ @ } y=0$$

. integrate x-mtm eq. : $\int_0^\infty dy$

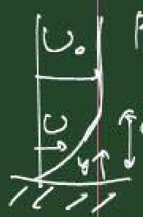
$$\tau_w = \frac{d}{dx}(\rho U_0^2 \theta) = \rho U_0^2 \frac{d\theta}{dx}$$ for zero press grad. boundary layer

von Karman's integral mtm eq.

$$C_f \equiv \frac{\tau_w}{\frac{1}{2}\rho U_0^2} = 2\frac{d\theta}{dx} \quad \text{for } \frac{dP_0}{dx} = 0$$

skin-friction coeff.

$$\hookrightarrow \frac{C_f}{2} = \frac{d\theta}{dx}.$$



For laminar flow w/ $\frac{dP_0}{dx} = 0$, Blasius (1908) similarity sol.

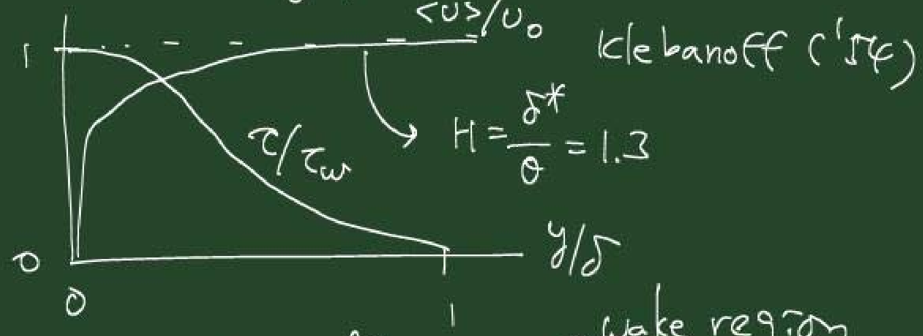
$$\frac{U(x,y)}{U_0} = f\left(\frac{y}{\delta_x}\right), \quad \delta_x = \sqrt{\frac{x\nu}{U_0}}$$

$$\rightarrow \frac{\delta}{x} \sim 5.0\, Re_x^{-\frac{1}{2}}, \quad \frac{\delta^*}{\delta} \sim 0.35, \quad \frac{\theta}{\delta} \sim 0.14$$

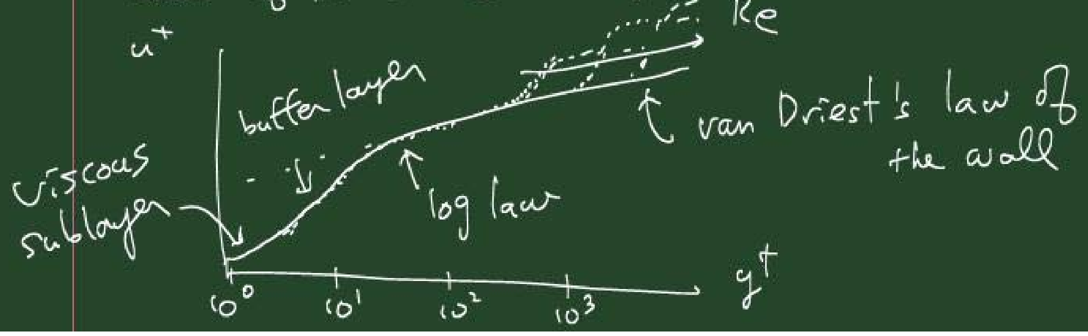
$$c_f \sim 0.664\, Re_x^{-\frac{1}{2}}, \quad H = \frac{\delta^*}{\theta} \sim 2.6$$

shape factor

⊙ Mean velocity profiles



• Law of the wall



From mixing-length hypothesis,

$$\frac{1}{\rho}\tau(y) = \nu\frac{\partial\langle U\rangle}{\partial y} - \langle uv\rangle \qquad \ell\cdot u$$

$$= \nu\frac{\partial\langle U\rangle}{\partial y} + \nu_T\frac{\partial\langle U\rangle}{\partial y} \qquad \nu_T = \ell_m\frac{\partial\langle U\rangle}{\partial y}$$

$$= \nu\frac{\partial\langle U\rangle}{\partial y} + \ell_m^2\left[\frac{\partial\langle U\rangle}{\partial y}\right]^2$$

$$\left(\ell_m^+ = \ell_m/\delta_\nu\right)$$

Normalize this eq. in wall units

$$\frac{\tau}{\tau_w} = \frac{\partial u^+}{\partial y^+} + \left(\ell_m^+\frac{\partial u^+}{\partial y^+}\right)^2$$

Solve for $\partial u^+ / \partial y^+$

$$\rightarrow \frac{\partial u^+}{\partial y^+} = \frac{2\tau/\tau_w}{1 + \left[1 + 4\,\tau/\tau_w \,(l_m^+)^2\right]^{\frac{1}{2}}}$$

In the inner layer, $\tau/\tau_w \simeq 1$.

van Driest's law of the wall

$$\rightarrow u^+ = f_w(y^+) = \int_0^{y^+} \frac{2}{1 + \left[1 + 4\left[l_m^+(y^+)\right]^2\right]^{\frac{1}{2}}}\, dy^+$$

In the log-law region, $l_m = \kappa y \rightarrow l_m^+ = \kappa y^+$

In the viscous sublayer, $\langle U \rangle \sim y$

$$\nu_T \frac{\partial \langle U \rangle}{\partial y} = l_m^2 \left(\frac{\partial \langle U \rangle}{\partial y}\right)^2 \sim y^2 \quad \text{if } l_m = \kappa y$$

but $\langle uv \rangle \sim y^3$ — wrong

$\downarrow$

correction is needed

or l_m should be damped.

Van Driest('56): $l_m^+ = \kappa y^+ \underbrace{\left[1 - \exp(-y^+/A^+)\right]}_{\text{van Driest damping ft.}}$

$(A^+ = 26, \quad \kappa = 0.41)$

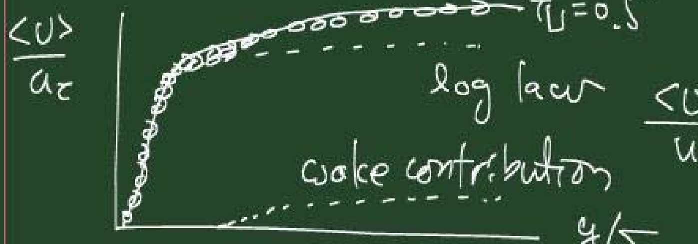
w/ $\frac{dp_0}{dx}$ $\rightarrow$ log law is observed w/ $\kappa = 0.41$

A^+ increases significantly

when $-\frac{\partial \bar{\tau}}{\partial y}\Big|_{y=0} > 2 \times 10^{-3} \frac{\tau_w}{\delta_\nu}$.

• Velocity-defect law

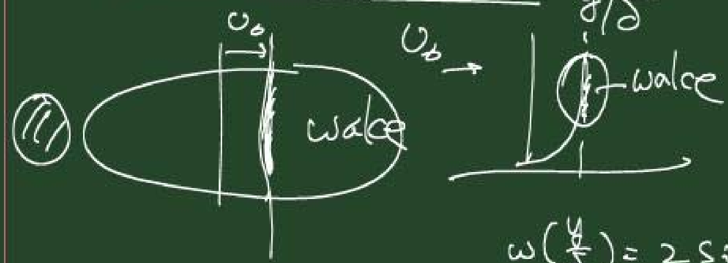
In the defect layer ($y/\delta > 0.2$, say), the mean vel. deviates from the log law.



Coles (1956)

$$\frac{\langle U \rangle}{u_\tau} = f_w\left(\frac{y}{\delta_\nu}\right) + \frac{\Pi}{\kappa}\, w\left(\frac{y}{\delta}\right)$$

law of the wall, law of the wake



$w(0) = 0$, $w(1) = 2$ normalization condition

$$w\left(\frac{y}{\delta}\right) = 2\sin^2\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

Π: wake strength parameter depending on the flow type.

from Coles formula,

$f_w \rightarrow$ log law, $\langle U \rangle_{y=\delta} = U_0$

$$\frac{U_0}{u_\tau} = \frac{1}{\kappa}\ln\frac{\delta}{\delta_\nu} + B + \frac{\Pi}{\kappa}\, w(1) \quad @\ y = \delta$$

$$-\ \frac{\langle U \rangle}{u_\tau} = \frac{1}{\kappa}\ln\frac{y}{\delta_\nu} + B + \frac{\Pi}{\kappa}\, w\left(\frac{y}{\delta}\right)$$

$$\frac{U_0 - \langle U \rangle}{u_\tau} = \frac{1}{\kappa}\left\{-\ln\frac{y}{\delta} + \Pi\left[2 - w\left(\frac{y}{\delta}\right)\right]\right\}$$

• friction law

Coles formula @ $y = \delta$

$$
\begin{cases}
\dfrac{U_0}{u_\tau} = \dfrac{1}{\kappa} \ln\left(\dfrac{\delta u_\tau}{\nu}\right) + B + \dfrac{2\Pi}{\kappa} \\[2ex]
\qquad = \dfrac{1}{\kappa} \ln\left(Re_\delta \dfrac{u_\tau}{U_0}\right) + B + \dfrac{2\Pi}{\kappa} \\[2ex]
C_f = 2\left(\dfrac{u_\tau}{U_0}\right)^2
\end{cases}
$$

Schultz-Grunow formula: $C_f = 0.370 \left[\log_{10} Re_x\right]^{-2.584}$