Week 3, 15 & 17 March

Mechanics in Energy Resources Engineering - Chapter 2. Axially Loaded Members

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1st exam

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- 31 March 09:30 – 10:45 (09:00 – 10:45)
- If you can solve the home assignment with confidence, you will do a good job.
- More than 50% from the home assignments.
- ~90% from the examples and the problems from the textbook.
- Try to interpret the problem in terms of physical behaviour. You will be required to explain your answer physically.

Review

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- Introduction to Mechanics of Materials (재료역학)
- Normal Stress and Normal Strain (수직응력과 수직변형율)
- Mechanical Properties of Materials (역학적 성질)
- Elasticity, Plasticity, and Creep (탄성, 소성 및 크리프)
- Linear Elasticity, Hooke's Law, and Poisson's Ratio (선형탄성, Hooke의 법칙, 포아송비)
- Shear Stress and Shear Strain (전단응력과 전단변형율)
- Allowable Stresses and Allowable Loads (허용응력과 허용하중)
- Design for Axial Loads (축하중의 설계)
- Review of Statics (정역학 복습)

Q & A
Example 1-3.

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- Increase of Δd_1 & Δd_2?

- I do understand that d_2 will increase but I don't understand why d_1 will also increase. Doesn't d_1 decrease?

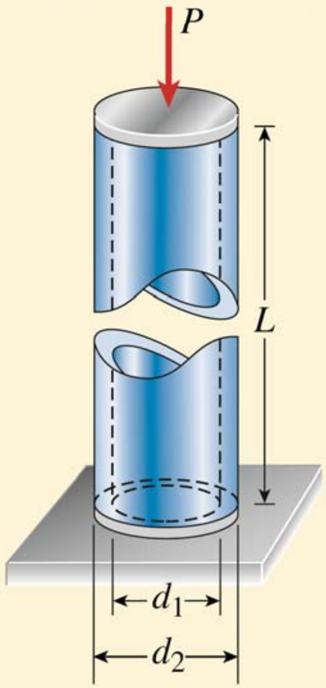
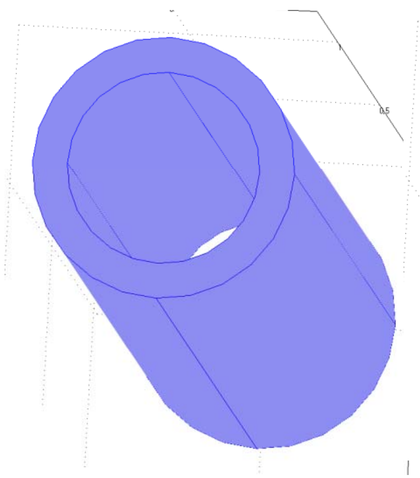


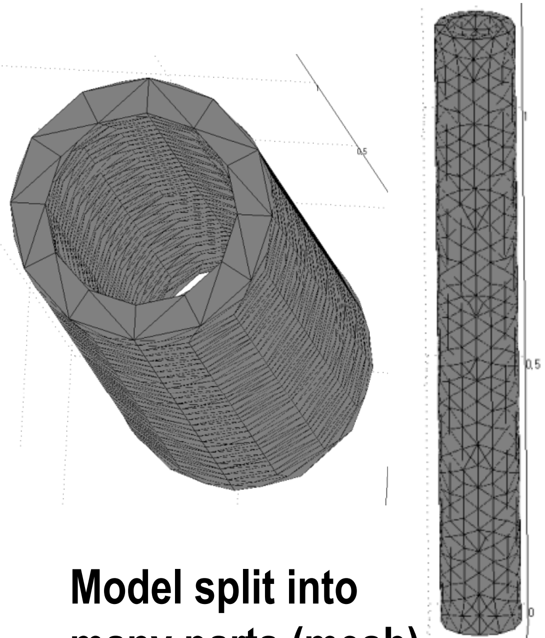
FIG. 1-23 Example 1-3. Steel pipe in compression

Q & A
Example 1-3.

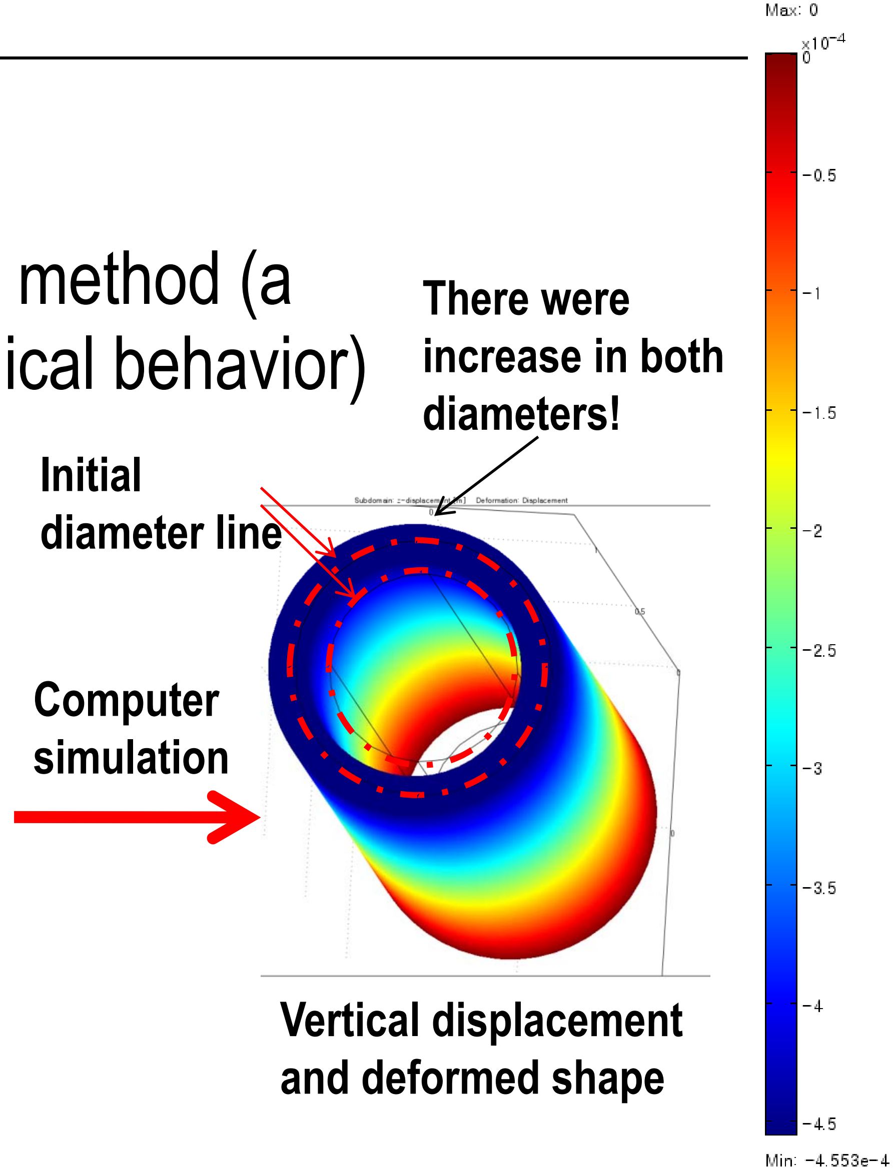
- Increase of Δd_1 & Δd_2?
- An investigation by a numerical method (a computer simulation of mechanical behavior)



model



Model split into many parts (mesh)



Vertical displacement and deformed shape

Preview

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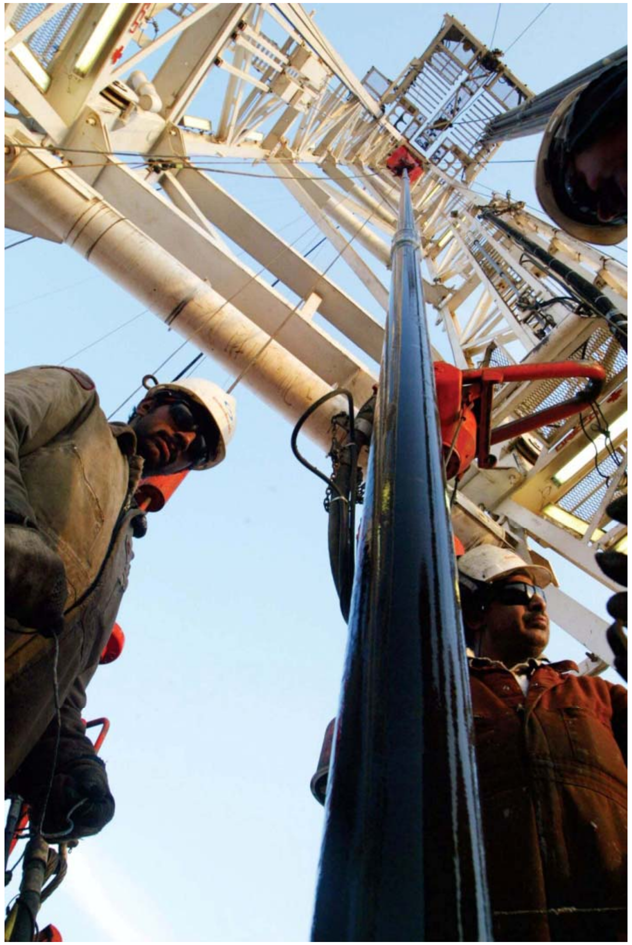
- Introduction
- Changes in Lengths of Axially Loaded Members (축하중을 받는 부재의 길이변화)
- Changes in Lengths Under Nonuniform Conditions (균일봉 길이변화)
- Statically Indeterminate Structures (부정정 구조물)
- Thermal Effects, Misfits, and Prestrains (열효과, 어긋남 및 사전변형)
- Stresses on Inclined Sections (경사면에서의 응력)
- Strain Energy (변형율 에너지)
- Impact Loading (충격하중)
- Stress Concentrations* (응력집중)

Introduction

- Axially loaded members
  - Structural components subjected only to tension or compression
  - Solid bars with straight longitudinal axes
  - Examples) truss members, connecting rods in engines, columns in buildings.
  - We already learned the 'stress-strain' behavior and normal stress and strain.



Drilling rig comprised of axially loaded members

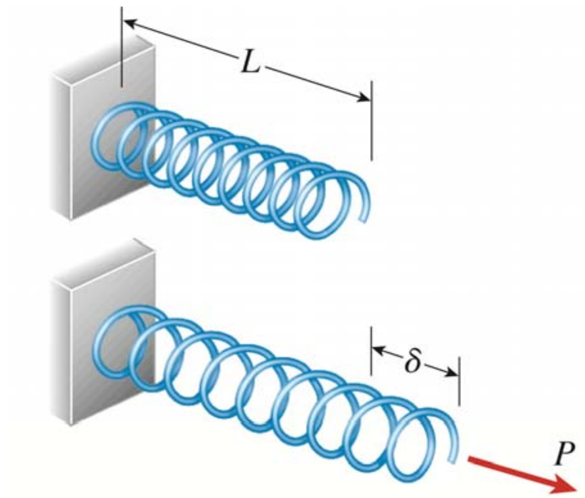
Changes in lengths of axially loaded members

- The relationship between the load and elongation

$$P = k\delta \qquad \delta = fP$$

P: load
δ: elongation
k: stiffness (강성도)
(spring constant)

f: flexibility
(compliance)



Changes in lengths of axially loaded members

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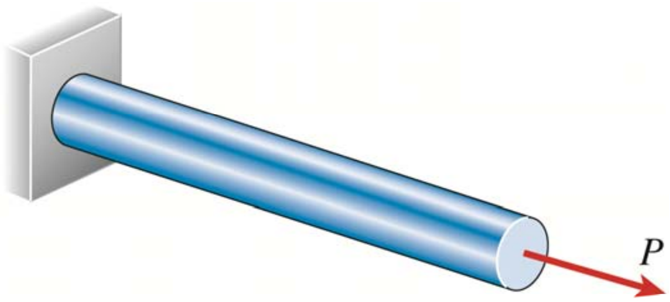
- Prismatic bar (균일단면봉)
  - A structural member having a straight longitudinal axis and constant cross section throughout its member

- Elongation of a bar

$$\delta = \frac{PL}{EA}$$

EA: axial rigidity



$$k = \frac{EA}{L}$$

stiffness

Example 2.2

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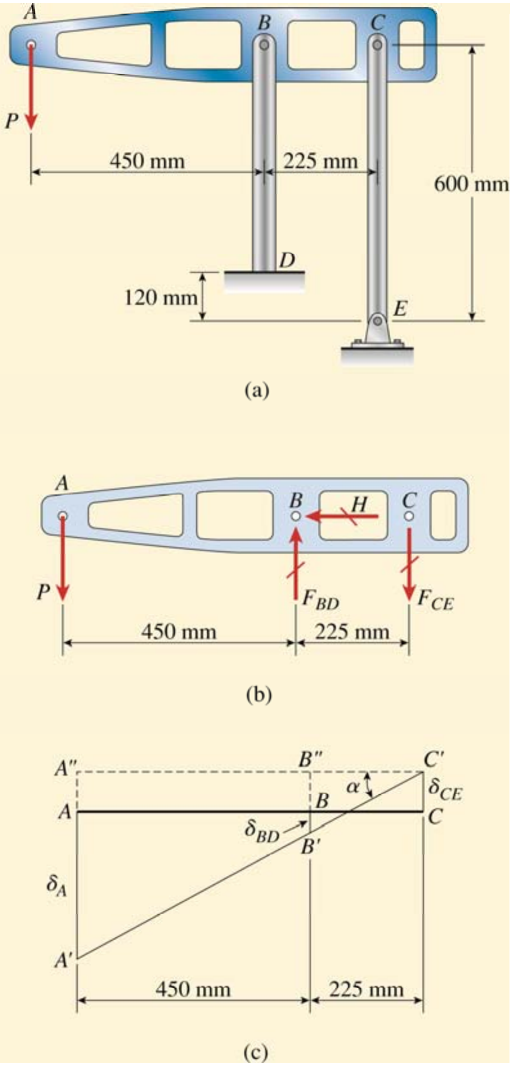

FIG. 2-8 Example 2-2. Horizontal beam *ABC* supported by two vertical bars

Nonuniform conditions

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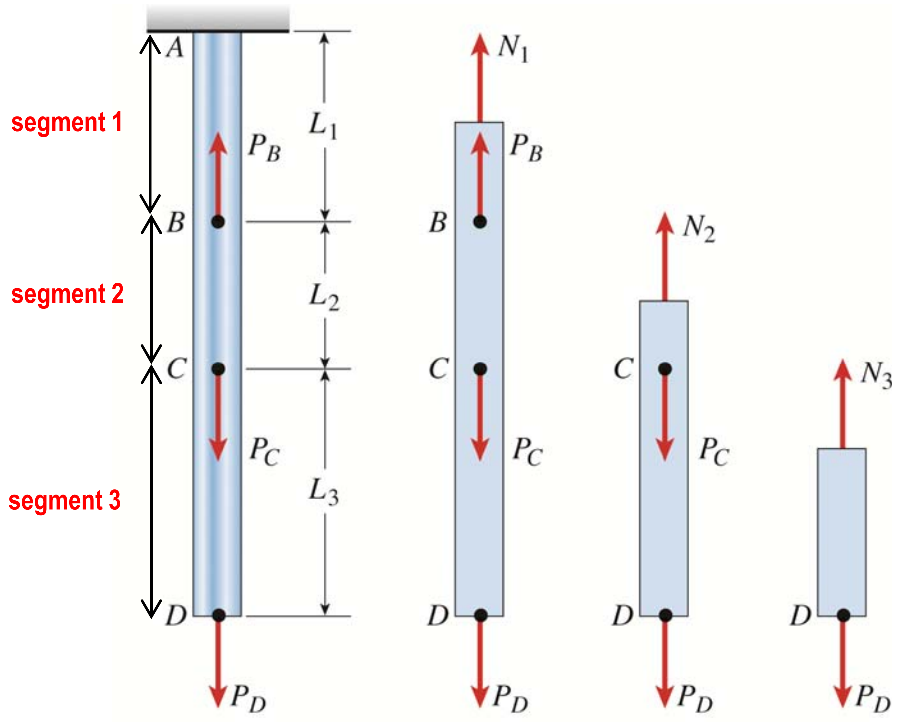
- More general situation with one or more axial loads



- Identify the segments of the bar
- Determine the *internal* axial forces N_1, N_2 and N_3. From equilibrium,

$$N_1 = -P_B + P_c + P_D$$

$$N_2 = P_c + P_D$$

$$N_3 = P_D$$

- Determine the length changes in segments

$$\delta_1 = \frac{N_1 L_1}{EA} \qquad \delta_2 = \frac{N_2 L_2}{EA} \qquad \delta_3 = \frac{N_3 L_3}{EA}$$

- Total changes (← be careful about the sign)

$$\delta = \delta_1 + \delta_2 + \delta_3$$

Nonuniform conditions

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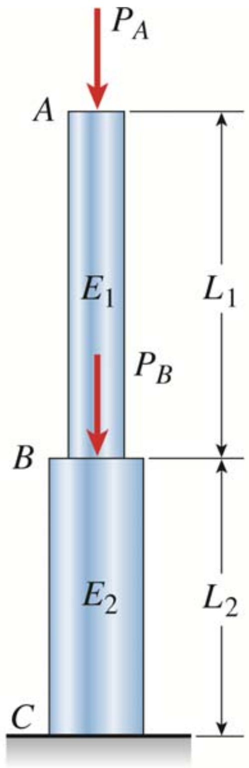

FIG. 2-10 Bar consisting of prismatic segments having different axial forces, different dimensions, and different materials

- Bars with (different loads + dimension + materials)

$$\delta = \sum_{i=1}^{n} \frac{N_i L_i}{E_i A_i}$$

- *N* is the total number of segments
- N_i is not an external load but is the internal axial force in segment *i*.

Nonuniform conditions

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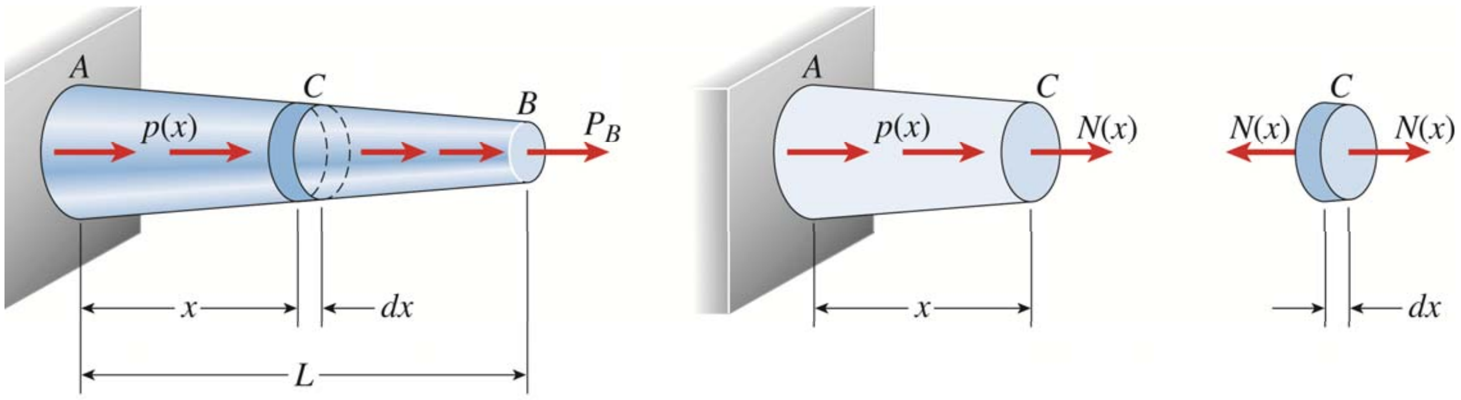
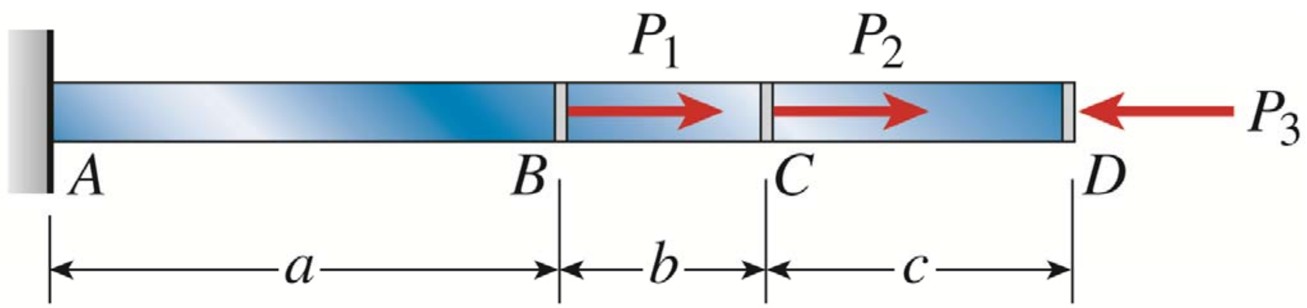
- Axial force *N* and cross-sectional area *A* vary continuously by the tapered bar.
  - Single force P_B & distributed forces *p(x)*
  - Define at differential element

$$d\delta = \frac{N(x)dx}{EA(x)} \longrightarrow \delta = \int_0^L d\delta = \int_0^L \frac{N(x)dx}{EA(x)}$$



Nonuniform conditions
problem 2.3-3

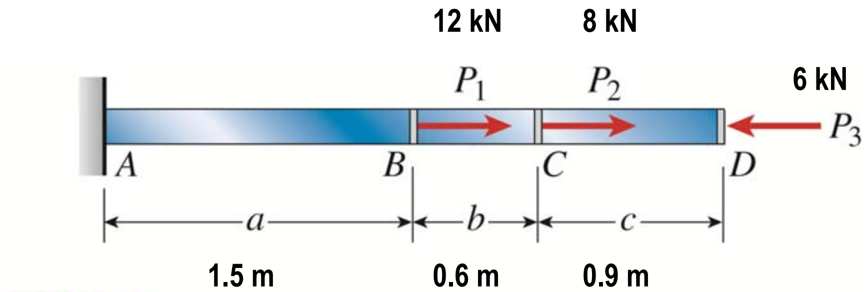
- A = 260 mm^2, P_1 = 12 kN, P_2 = 8 kN, P_3 = 6 kN
- A = 1.5 m, b = 0.6 m, c = 0.9 m
- E = 210 GPa, δ?



PROB. 2.3-3

Nonuniform conditions

- Prob: 2.3-3, (a) total elongation, (b) increase of P_3 for zero elongation



PROB. 2.3-3

- Identify the segments of the bar
- Determine the *internal* axial forces N_{AB}, N_{BC} and N_{CD}. From equilibrium,

$$N_{AB} = P_1 + P_2 - P_3$$

$$N_{BC} = P_2 - P_3$$

$$N_{CD} = -P_3$$

- Determine the length changes in segments

$$\delta_1 = \frac{N_{AB} L_1}{EA} \quad \delta_2 = \frac{N_{BC} L_2}{EA} \quad \delta_3 = \frac{N_{CD} L_3}{EA}$$

- Total changes (← be careful about the sign)

$$\delta = \delta_1 + \delta_2 + \delta_3$$

Statically indeterminate structure
example

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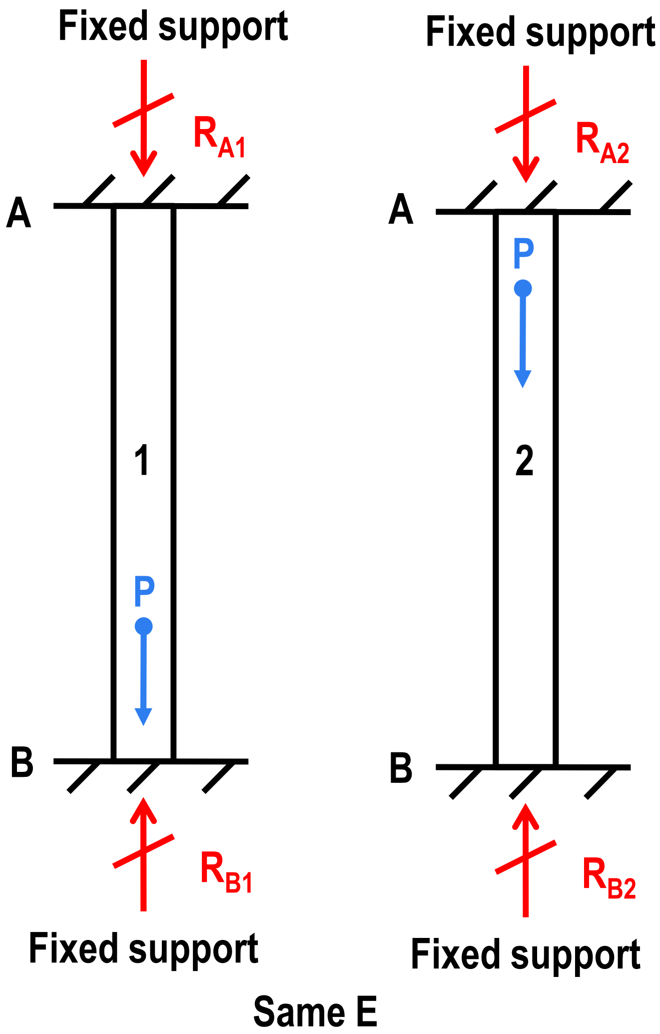

- Structures 1 &2 are fixed at both ends and are under a force P in different locations as shown in the left.
  - Can be determine the reactions R_A & R_B with equilibrium equations alone?
  - R_{B1} vs. R_{B2}. Which is bigger? What is the reason for your answer?

  → In addition to equilibrium equation, we need to consider that both ends are fixed (compatibility equations) and deformation characteristics (force-displacement relation) to determine the reactions.

Statically indeterminate structure
Definition

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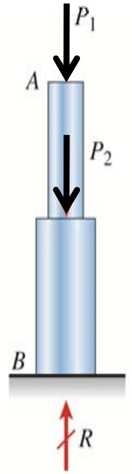
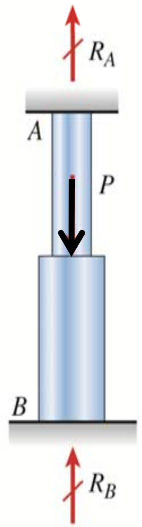


$$R = P_1 + P_2$$

Unknowns can be solved by Equil. Eq. →정정

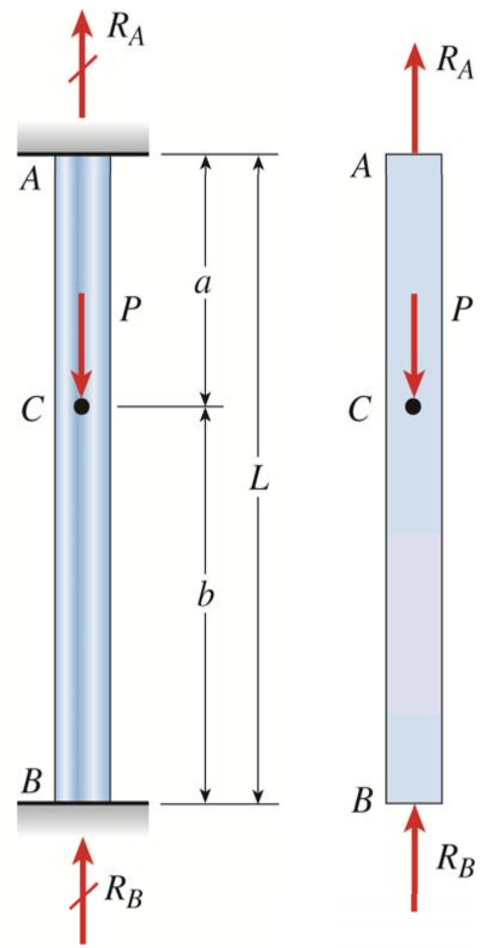


$$R_A + R_B = P$$

Equil. Eq. is not enough to solve the unknowns → 부정정

- Statically determinate (정정, 靜定)
  - reactions and internal forces can be obtained from equilibrium equations alone (via Free Body Diagram)
  - No need to know the properties (e.g., E, v, G) of the materials
- Statically indeterminate (부정정, 不靜定)
  - Equilibrium + additional equations related to the displacement
  - Need to know the properties (e.g., E, v, G) of the materials

Statically indeterminate structure



1) Equilibrium Equation $\sum F_{ver} = 0 \longrightarrow R_A - P + R_B = 0$

2) Clue for additional equation:

– A bar with both ends fixed does not change in length

$$\delta_{AB} = 0$$

←Compatibility equation: the change in length must be compatible with the conditions at the supports

3) Compat. Eq. in terms of Forces: force-displacement relations

$$\delta = \frac{PL}{EA} \longrightarrow \delta_{AC} = \frac{R_A a}{EA} \qquad \delta_{CB} = -\frac{R_B b}{EA}$$

Why minus here?

- By combining 2) and 3),

$$\delta_{AB} = \delta_{AC} + \delta_{CB} = \frac{R_A a}{EA} - \frac{R_B b}{EA} = 0$$

- Finally, combining above with 1) Equil. Eq.

$$R_A = \frac{Pb}{L} \qquad R_B = \frac{Pa}{L}$$

- We can also calculate the displacements

$$\delta_C = \delta_{AC} = \frac{R_A a}{EA} = \frac{Pab}{LEA}$$

Statically indeterminate structure

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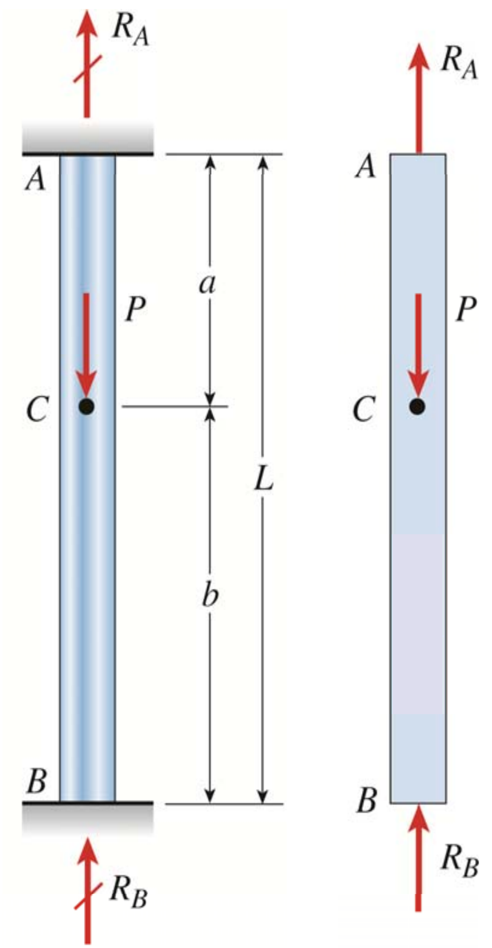

1) Equilibrium Equation

$$\sum \mathbf{F} = 0$$

2) Compatibility Equation

Conditions on the displacement of the structure

ex) $\delta_{AB} = 0$

3) Force-displacement Equation

$$\delta = \frac{PL}{EA}$$

<u>1) + 2) + 3) → Unknown forces & displacement</u>

Statically indeterminate structure
Example 2-5

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- Compressive force in the steel cylinder, P_s & in copper tube P_c?
- Stresses, σ_s & σ_c?

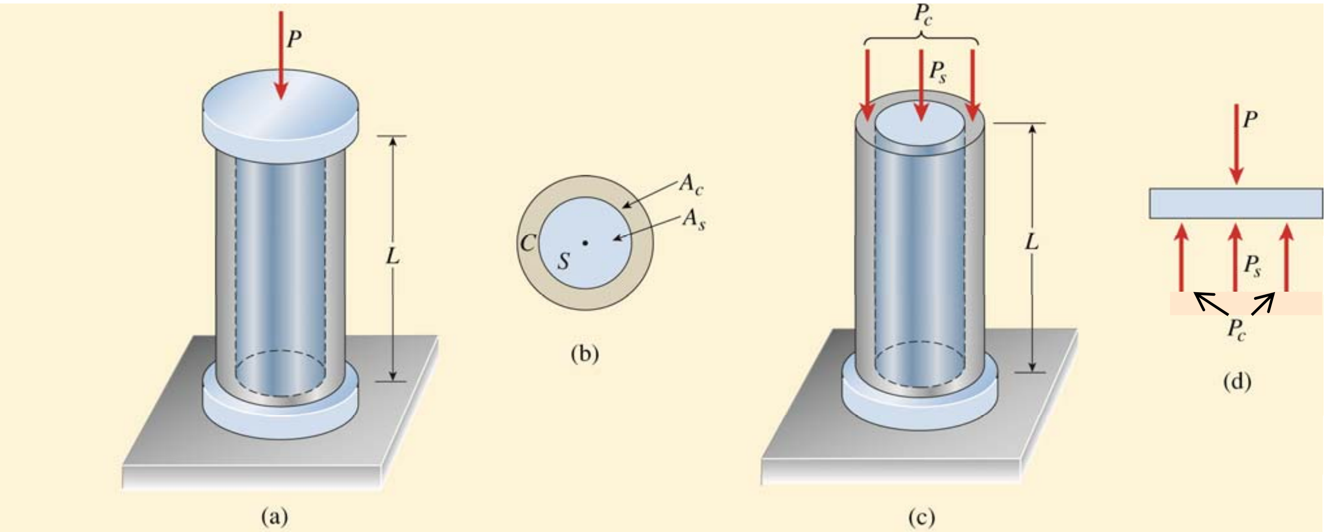

FIG. 2-17 Example 2-5. Analysis of a statically indeterminate structure

Preview

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Thermal Effects, misfits, and prestrains

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- Other sources of stresses and strains other than 'external loads'
  - Thermal effects: arises from temperature change
  - Misfits: results from imperfections in construction
  - Prestrains: produced by initial deformation

Thermal Effects

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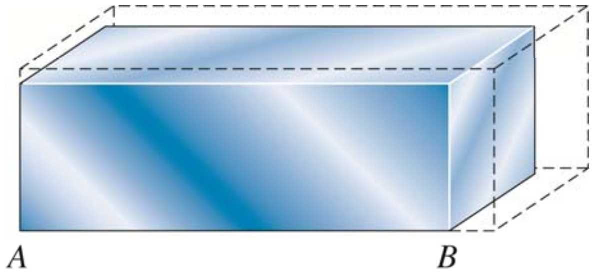
- Changes in temperature produce expansion or contraction → thermal strains



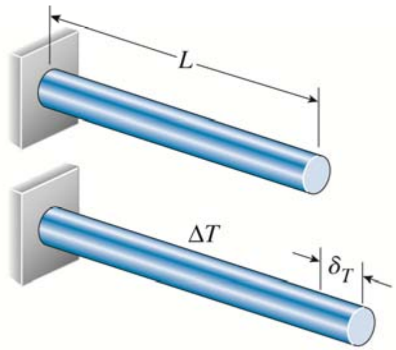
- Thermal strain, ε_T

$$\varepsilon_T = \alpha(\Delta T)$$

  - *α:* coefficient of thermal expansion (1/K or 1/°C). e.g., granitite: ~1.0 ×10^{-5} /°C
  - Heated → Expansion (+), Cooled → contraction (-)

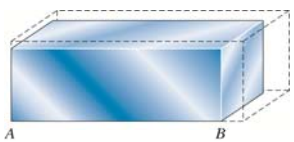
- Displacement by thermal expansion

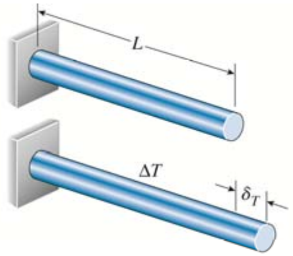
$$\delta_T = \varepsilon_T L = \alpha(\Delta T) L$$

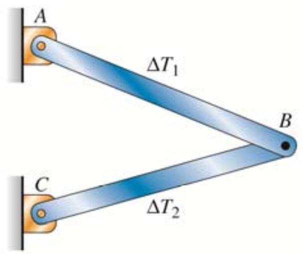


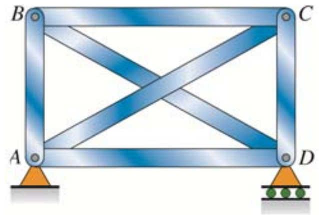
Thermal Effects

- No restraints → free expansion or contraction
  - Thermal strain is NOT followed by thermal stress
  - Generally, statically determinate structures do not produce thermal stress

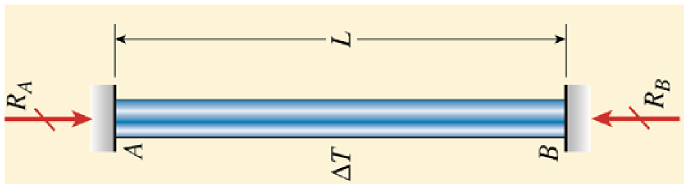








- With supports that prevent free expansion and contraction → <u>Thermal stress generated</u>
  - How much thermal stress?



Thermal Effects

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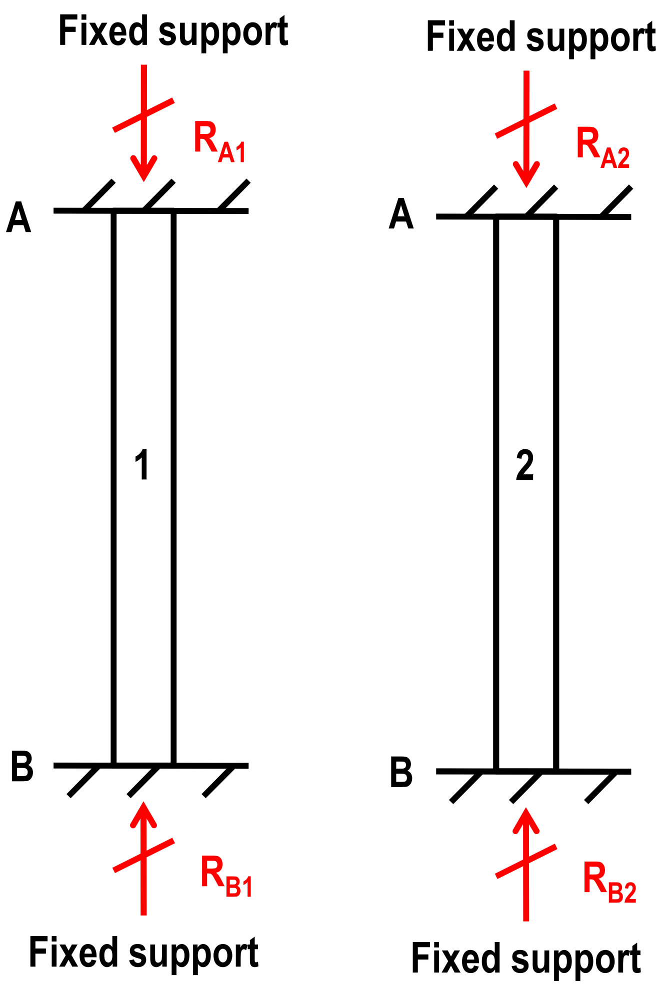


- Two bars in the left were under uniform temperature increase of ΔT.
  - E: Elastic Modulus
    α: Coefficient of Thermal Expansion
  - If $E_1=E_2$ and $\alpha_1 > \alpha_2$, which bar will generate the higher thermal stress?
  - If $\alpha_1 = \alpha_2$ and $E_1>E_2$, which bar will generate the higher thermal stress?
  - Will R_A and R_B the same?

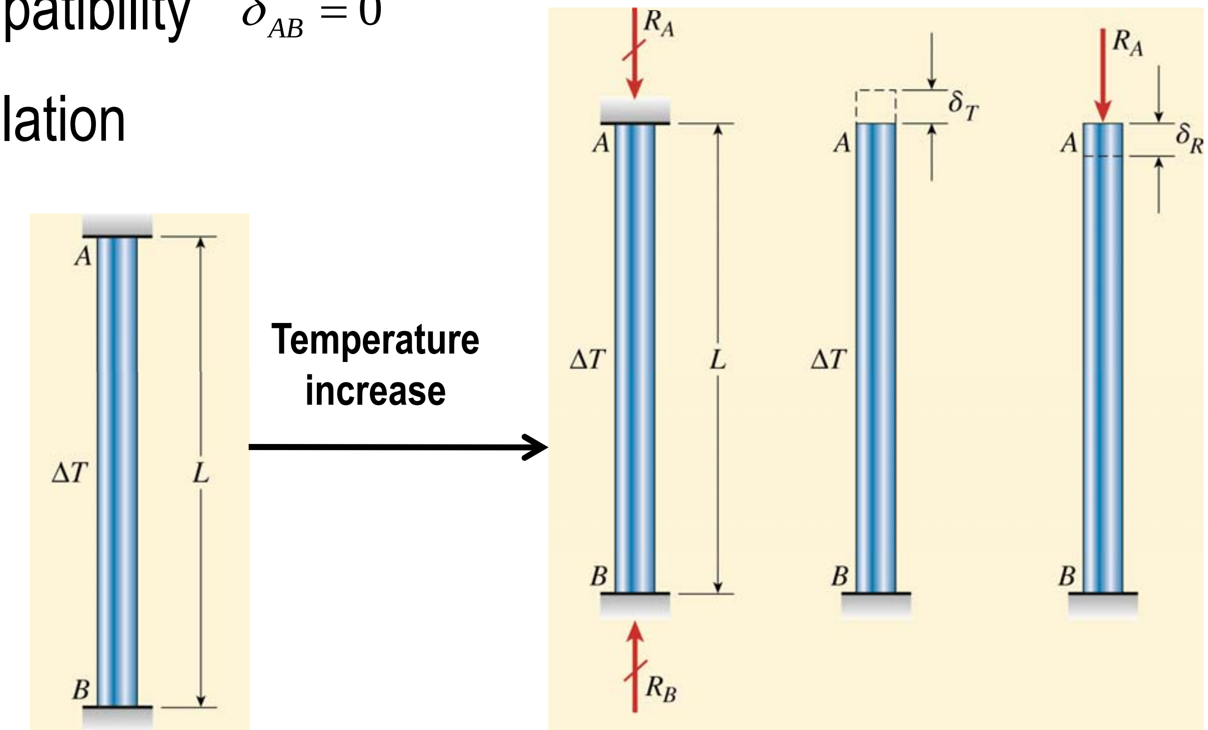
Thermal Effects

Calculation of Thermal stress (Example 2-7)

- Approach similar to the analysis of statically indeterminate structure
  - Equation of Equilibrium $\sum \mathbf{F} = 0$
  - Equation of compatibility $\delta_{AB} = 0$
  - Displacement relation

$$\delta_T = \alpha(\Delta T)L$$

$$\delta = \frac{PL}{EA}$$



Thermal Effects

Calculation of Thermal stress (Example 2-7)

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- Equilibrium Eq.

$$\sum \mathbf{F} = 0 \longrightarrow \sum F_{ver} = R_B - R_A = 0$$

- Compatibility Eq.

$$\delta_{AB} = \delta_T - \delta_R = 0$$

- Displacement Relations

$$\delta_T = \alpha(\Delta T)L \qquad \delta_R = \frac{R_A L}{EA}$$

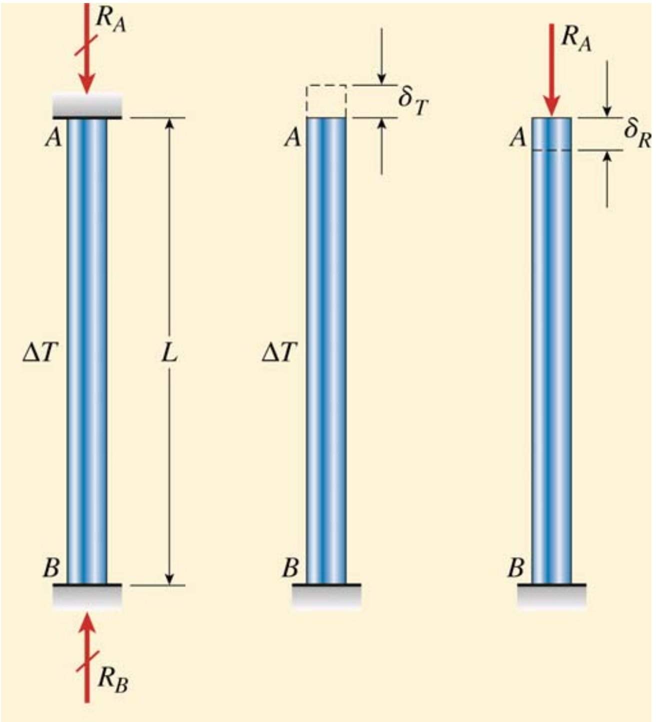
- Compat. Eq. ← Displ. Rel.

$$\delta_T - \delta_R = \alpha(\Delta T)L - \frac{R_A L}{EA} = 0$$

- Reactions

$$R_A = R_B = EA\alpha(\Delta T)$$

- Thermal Stress in the bar → $\sigma_T = \frac{R_A}{A} = \frac{R_B}{A} = E\alpha(\Delta T)$

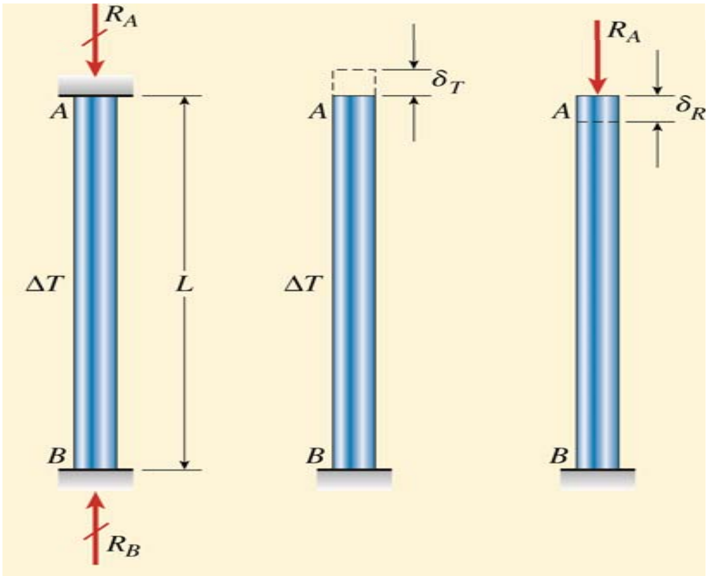


Thermal Effects
Calculation of Thermal stress

- Thermal Stress in the bar

$$\sigma_T = \frac{R_A}{A} = \frac{R_B}{A} = E\alpha\left(\Delta T\right)$$

  – Stress independent of the length (*L*) & cross-sectional area (*A*)
  – Assumptions: *ΔT* uniform, homogeneous, linearly elastic material
  – Lateral strain?



Thermal Effects
Example 2-8

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- Calculation of Thermal stress & elongation

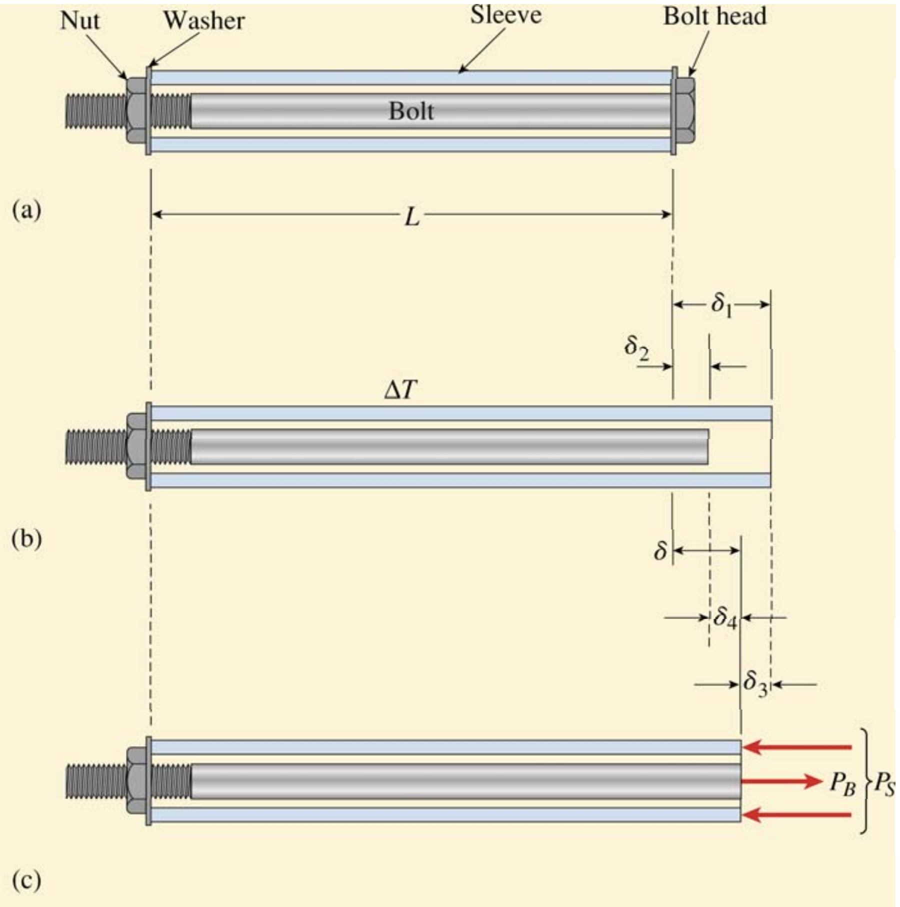

FIG. 2-24 Example 2-8. Sleeve and bolt assembly with uniform temperature increase ΔT

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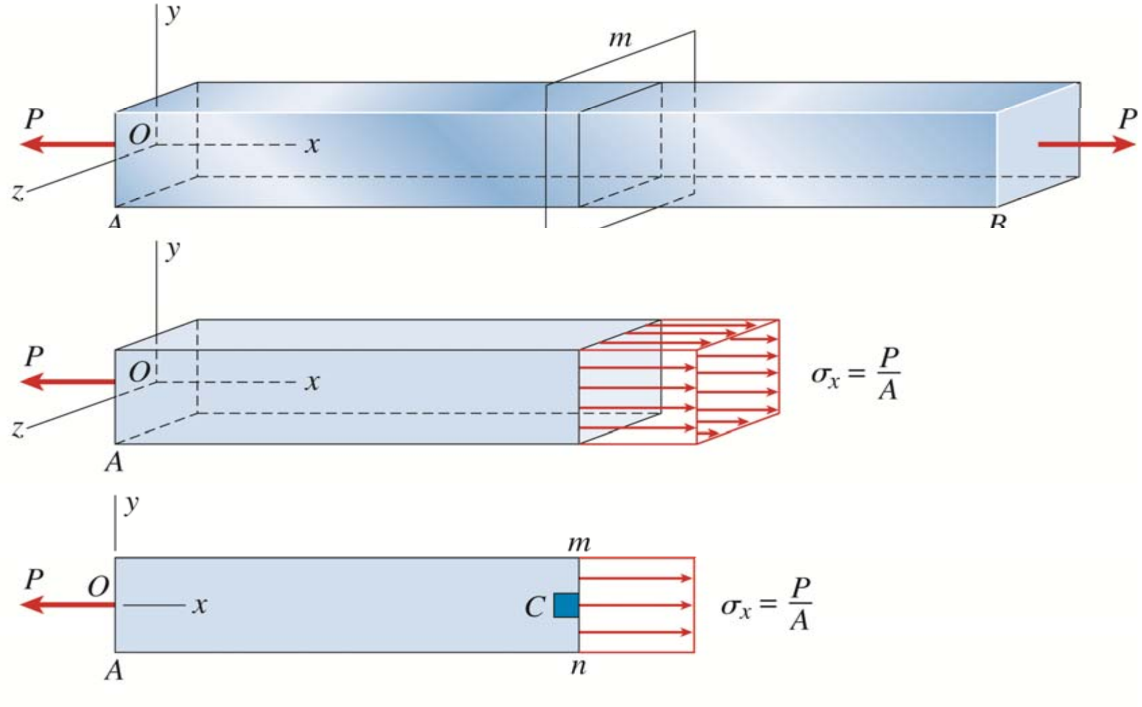
Stresses on inclined sections

- Normal stress $\sigma_x = P / A$
  - P act at the center (centroid)
  - Cross section n is away from localized stress concentration



Stresses on inclined sections

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- Stresses on inclined sections ←a more complete picture
  - Finding the stresses on section *pq*.
  - Resultant of stresses : still *P*
  - Normal Force (N) and Shear Force (V)

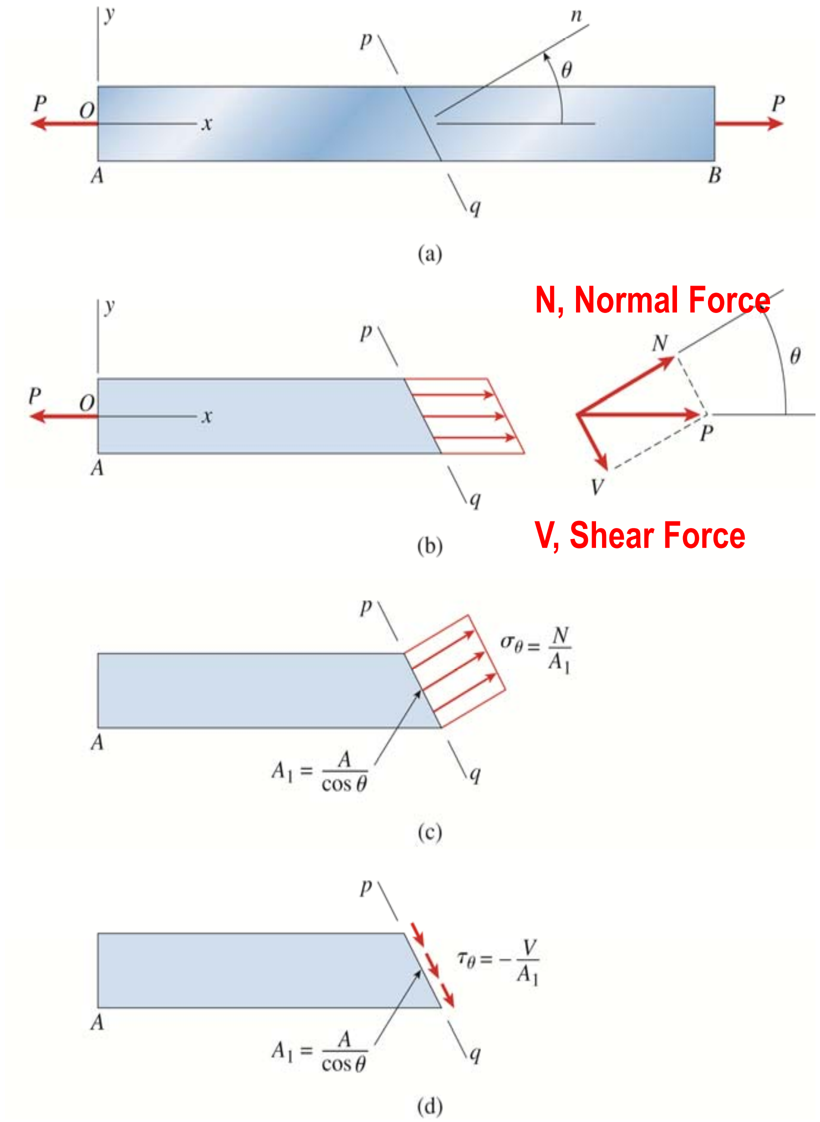
$$N = P\cos\theta \qquad V = P\sin\theta$$

  - Normal Stress (σ) and shear stress (τ)

$$\sigma = \frac{N}{A_1} \qquad \tau = \frac{V}{A_1}$$

A: area of cross-section
A1: area of inclined section

$$A_1 = \frac{A}{\cos\theta}$$



Stresses on inclined sections

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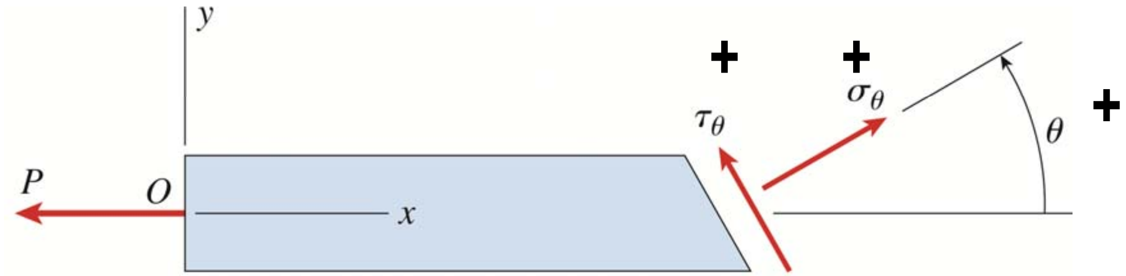
- Notation and sign convention



  - Notation: Subscript θ indicate that the stresses on a section inclined at an angle θ
  - Sign convention: norm (positive tension), shear (+, for tendency of counter clockwise rotation)

Stresses on inclined sections

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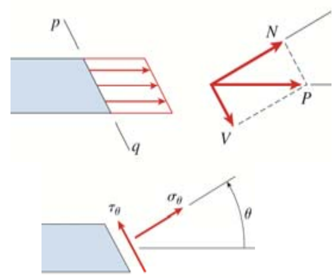
- Based on the sign convention (note minus shear stress),

$$\sigma_\theta = \frac{N}{A_1} = \frac{P}{A}\cos^2\theta \qquad \tau_\theta = -\frac{V}{A_1} = -\frac{P}{A}\sin\theta\cos\theta$$



$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta) \qquad \sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$$

$$\sigma_\theta = \sigma_x\cos^2\theta = \frac{1}{2}\sigma_x(1+\cos 2\theta) \qquad \tau_\theta = -\sigma_x\sin\theta\cos\theta = -\frac{\sigma_x}{2}\sin 2\theta$$

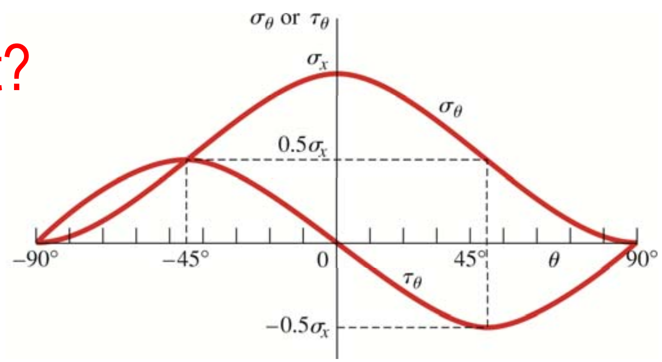
- Above equation are independent of material (property and elastic…).
- Maximum stresses…why is this important?

$$\sigma_{max} = \sigma_x$$

When $\theta = 0$

When $\theta = -45°$

$$\tau_{max} = \frac{\sigma_x}{2}$$



Stresses on inclined sections

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- Element A:
  - maximum normal stress
  - no shear $\sigma_{max} = \sigma_x$
- Element B:
  - The stresses at θ = 135°, -45° and -135° can be obtained from previous equations.
  - Maximum shear stresses $\tau_{max} = \dfrac{\sigma_x}{2}$
  - One-half the maximum normal stress

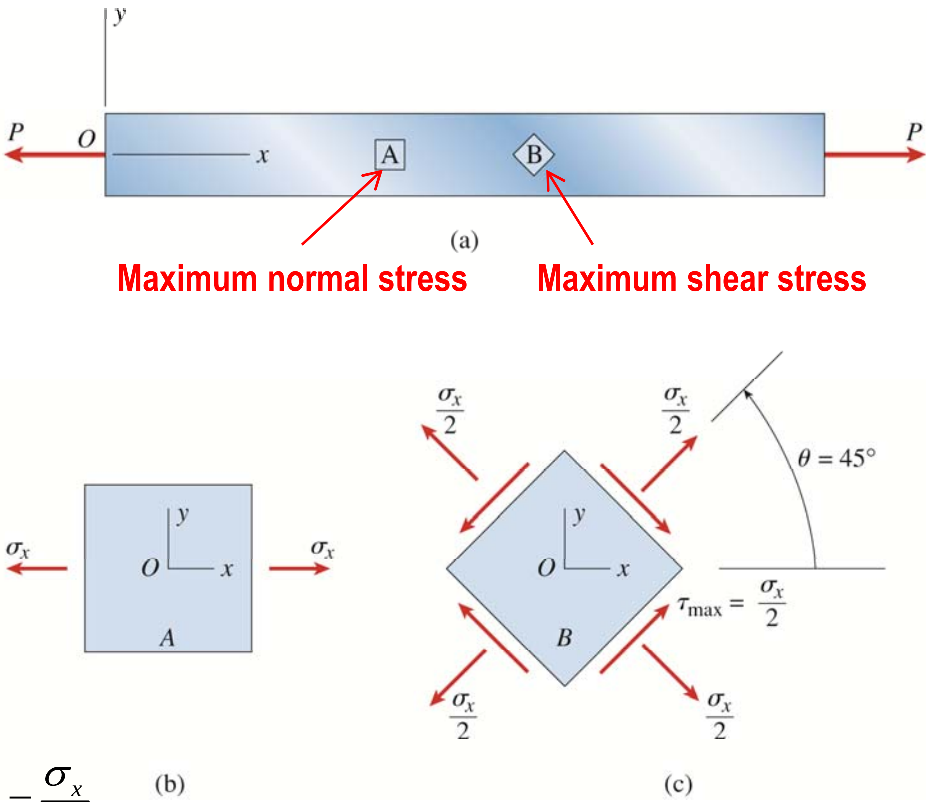

FIG. 2-36 Normal and shear stresses acting on stress elements oriented at $\theta = 0°$ and $\theta = 45°$ for a bar in tension

Stresses on inclined sections

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- Same equations can be used for uniaxial <u>compression</u>
- What will happen if material is much weaker in shear than in compression (or tension)
  - Shear stress may cause failure

FIG. 2-37 Shear failure along a 45° plane of a wood block loaded in compression

Stresses on inclined sections
Example 2-10

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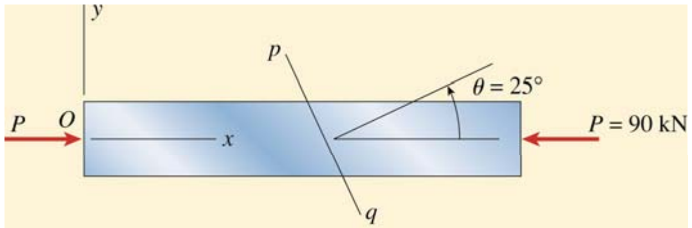
1) Determine the stresses acting on an inclined section *pq* cut through the bar at an angle θ=25°.

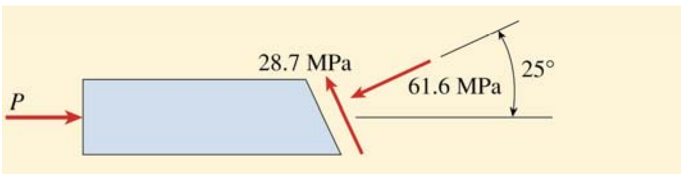


A=1200 mm^2

$$\sigma_x = -\frac{P}{A} = \frac{90\,kN}{1200mm^2} = -75MPa$$

$$\sigma_\theta = \sigma_x \cos^2\theta = (-75MPa)(\cos 25°)^2 = -61.6MPa$$

$$\tau_\theta = -\sigma_x \sin\theta\cos\theta = (75MPa)(\sin 25°)(\cos 25°) = 28.7MPa$$



Stresses on inclined sections
Example 2-10

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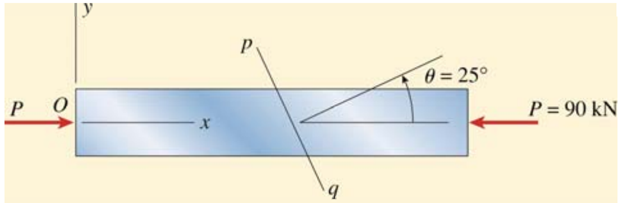
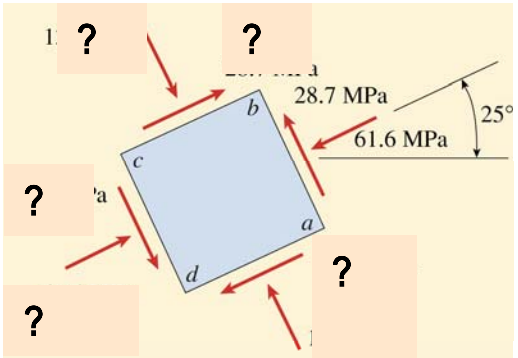
2) Determine the complete state of stress for θ=25° and show the stresses on a properly oriented stress element.



A=1200 mm^2

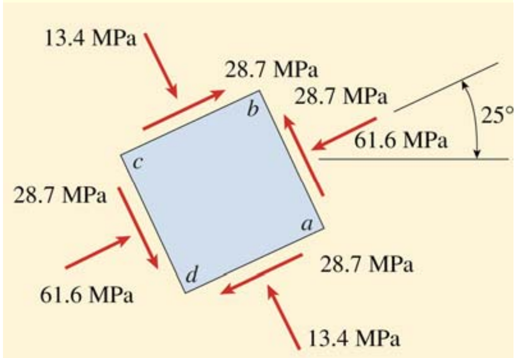


- For face ab, normal and shear stresses are just obtained
- For face ad, we substitute θ=25°-90° = -65°

$$\sigma_\theta = \sigma_x \cos^2\theta = (-75MPa)(\cos -65°)^2 = -13.4MPa$$

$$\tau_\theta = -\sigma_x \sin\theta\cos\theta = (75MPa)(\sin -65°)(\cos -65°) = -28.7MPa$$

- The same applies to the faces *bc* and *cd* by putting θ=115° & 205° .



Summary

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- Introduction
- Changes in Lengths of Axially Loaded Members (축하중을 받는 부재의 길이변화)
- Changes in Lengths Under Nonuniform Conditions (균일봉 길이변화)
- Statically Indeterminate Structures (부정정 구조물)
- Thermal Effects, Misfits, and Prestrains (열효과, 어긋남 및 사전변형)
- Stresses on Inclined Sections (경사면에서의 응력)
- Strain Energy (변형율 에너지)
- Impact Loading (충격하중)
- Stress Concentrations* (응력집중)