Week 6 & 7, 5 & 12 March

Mechanics in Energy Resources Engineering - Chapter 4 Shear Forces and Bending Moments

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1st exam

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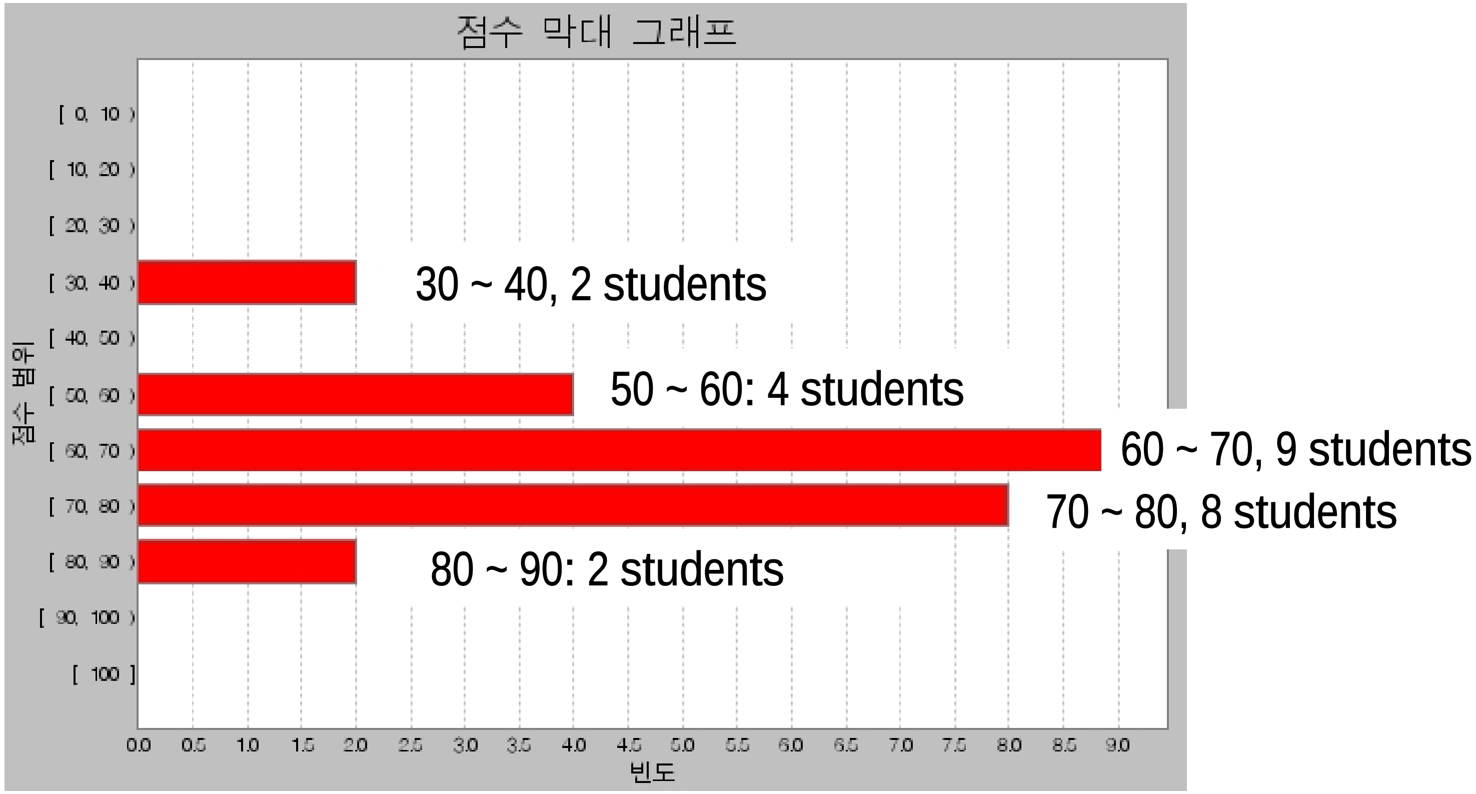
- Mean: 65.3, standard deviation: 12.9
- Max: 86.0, Min: 30.0



1st Exam

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- In general, you demonstrated your understanding to a reasonable extent and you are in good positions to study further.
- Try to thoroughly understand the home assignments. I encourage discussion with your peers.
- This time only, partial point was around 10% - 70%. However, it will be minimized next time. Max partial point will be 30%.
- Level of difficulty will be similar in the 2nd and 3rd exam.
- 2nd exam: Ch. 4, 5 & 12
- 3rd exam: entire chapters.

Review

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- Introduction
- Torsional Deformations of a circular bar (원형봉의 비틀림 변형)
- Circular bars of linearly elastic materials (선형탄성 원형봉)
- Nonuniform torsion (불균질 비틀림)
- Stresses and Strains in Pure Shear (순수전단에서의 응력과 변형율)
- Relationship Between Moduli of Elasticity E and G (탄성계수 E와 G의 관계)
- Statically Indeterminate Torsional Members (부정정 비틀림 부재)
- Strain Energy in Torsion and Pure Shear (비틀림과 순수전단에서의 변형에너지)

Change of schedule

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- 5 April (Ch.4) 7 April (Ch.12) by Jae-Won Lee
- 12 April (Ch.4) 14 April (Ch.5), hw#4 due
- 19 April (Ch.5), hw#5 due 21 April (Ch.5)
- 26 April (Review), 28 April (2nd Exam), hw#6 due

Shear Forces and Bending Moments
Preview

- Introduction
- Types of Beams, Loads, and Reactions
- Shear Forces and Bending Moments
- Relationships Between Loads, Shear Forces and Bending Moments
- Shear-Force and Bending-Moment Diagrams

Introduction

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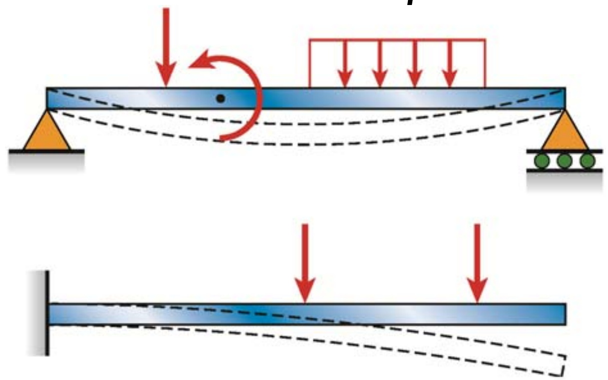
- Structural members (구조용 부재)
  - Axially loaded bar (봉): forces along the axis
  - A bar in torsion: torques along the axis (moment vectors)
  - Beam (보): lateral loads
- *Planar structure* (평면구조물) lie in a single plane
  - Loads and deflections occurs in the *plane of bending*



Types of Beams, Loads, and Reactions
beams

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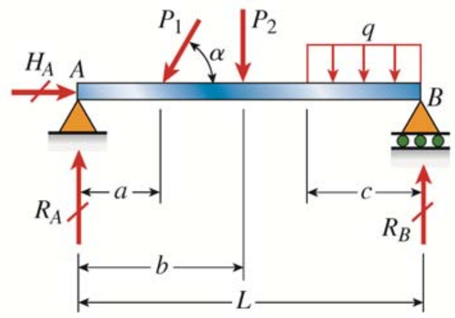
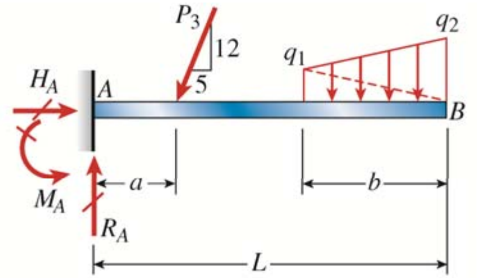
- Assumptions
  - Loads act in the plane of the figure: force vectors in the plane of figure & bending moments have their moments vectors perpendicular to the plane of the figure
  - Beam is symmetric about that plane → deflect only in the plane of bending

Types of Beams, Loads, and Reactions
Types of Beams

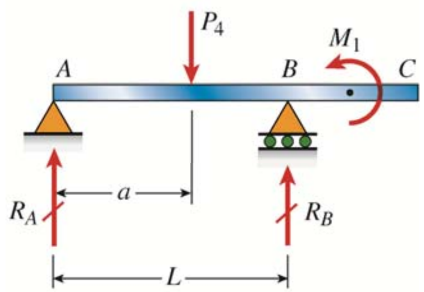
- Simple beam (단순보)



- Cantilever beam (캔틸레버 보)



- Beam with an overhang (돌출보)



Types of Beams, Loads, and Reactions

Types of supports

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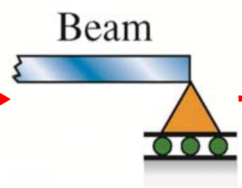
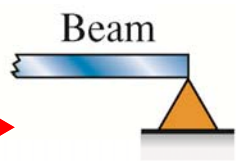
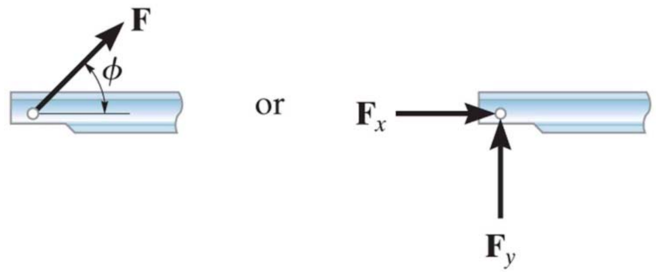


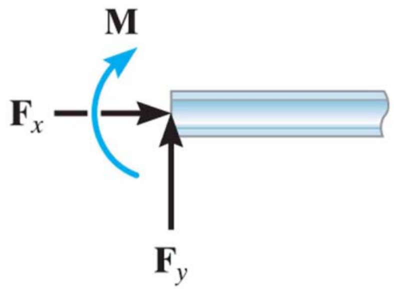
Free body Diagram



Free body Diagram



Free body Diagram



Types of Beams, Loads, and Reactions
Actual Examples

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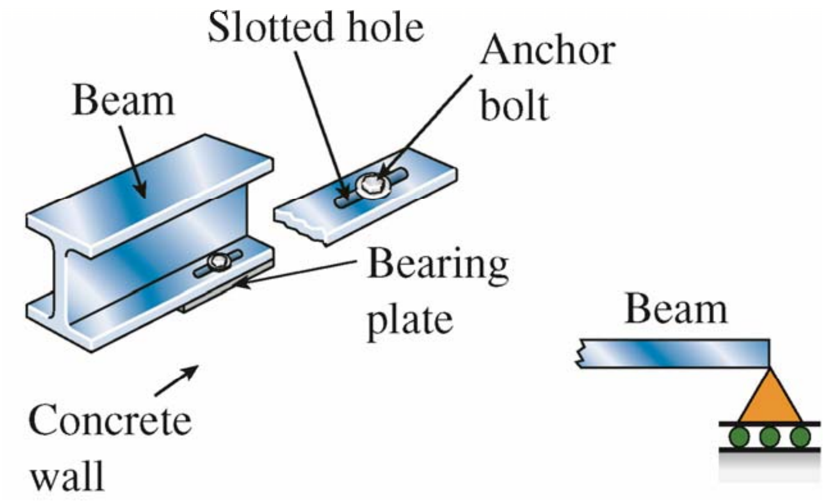
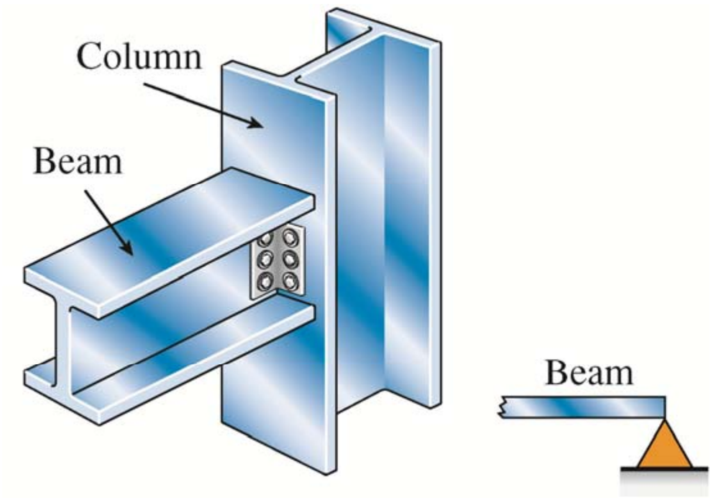
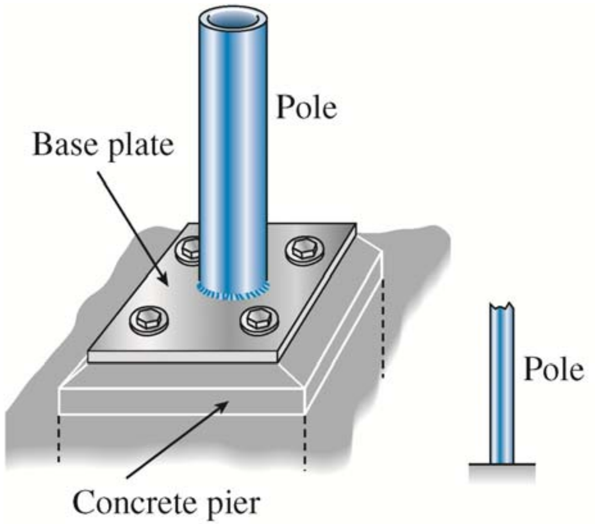
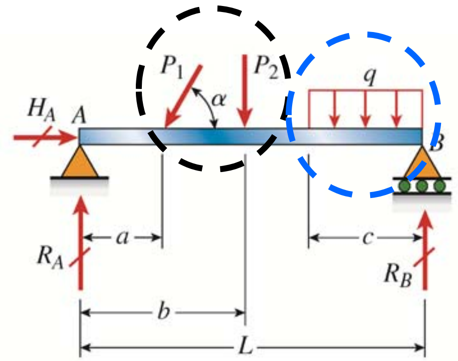


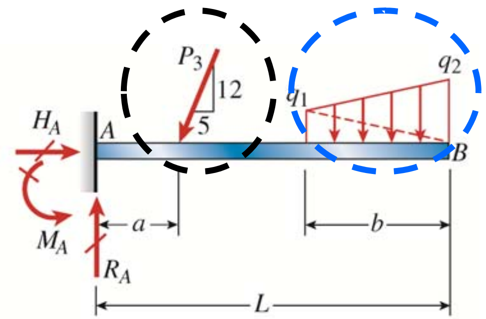


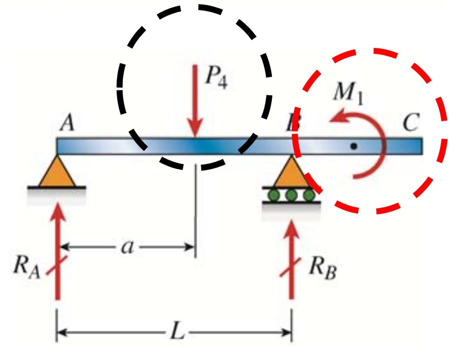


Types of Beams, Loads, and Reactions
Loads

- Concentrated load
  - applied over a very small area
- Distributed load
  - Spread along the axis of a beam
  - Measured by their intensity (Force/unit distance)
  - Uniformly distributed & linearly varying load
- Couple
  - The couple of moment M_1 (bending moment) acting on the overhang







Types of Beams, Loads, and Reactions
Reactions

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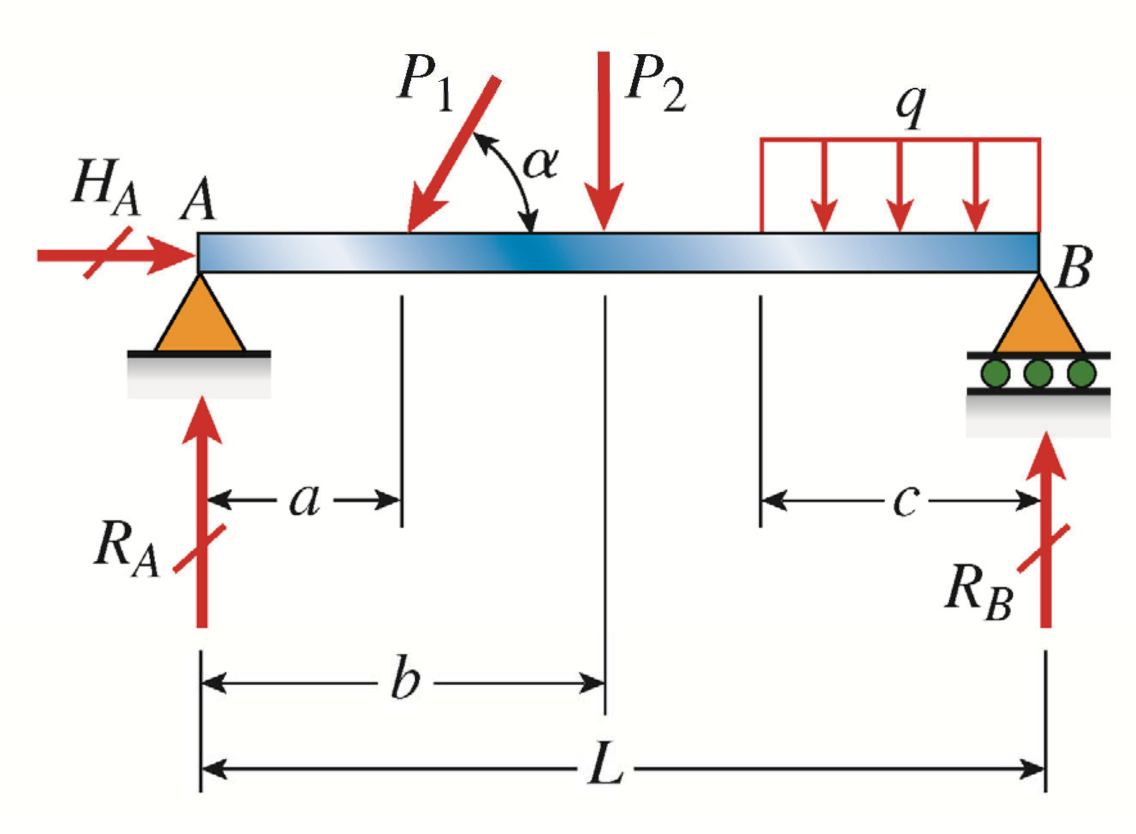
- Simple beam



Types of Beams, Loads, and Reactions
Reactions

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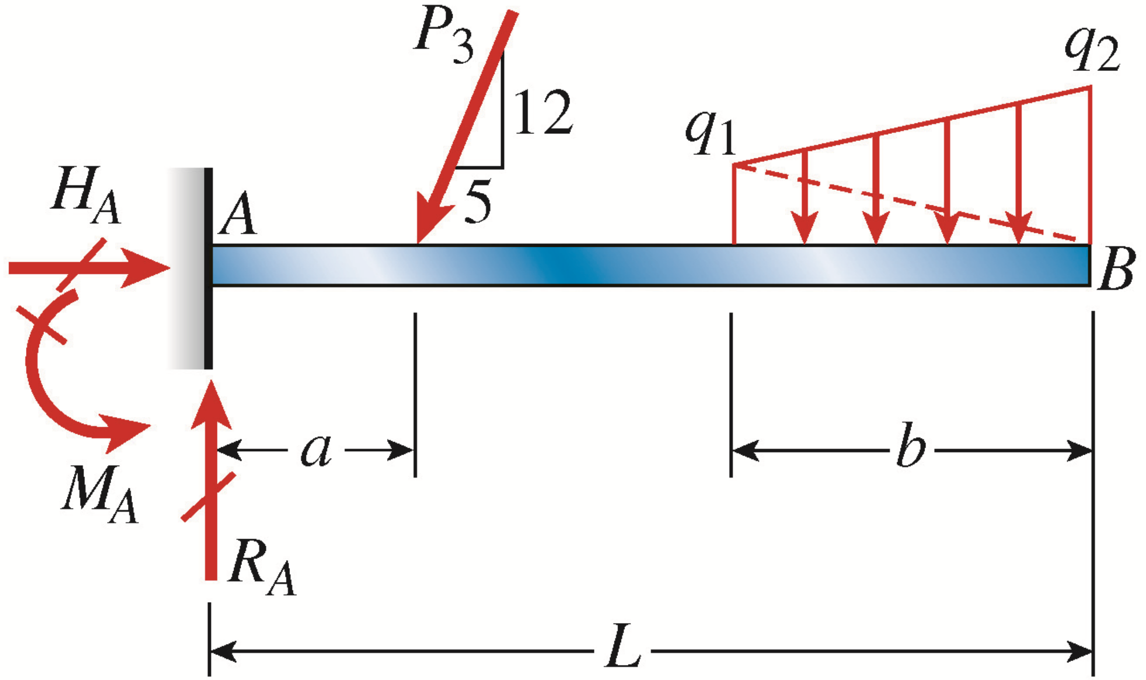
- Cantilever beam



Types of Beams, Loads, and Reactions
Reactions

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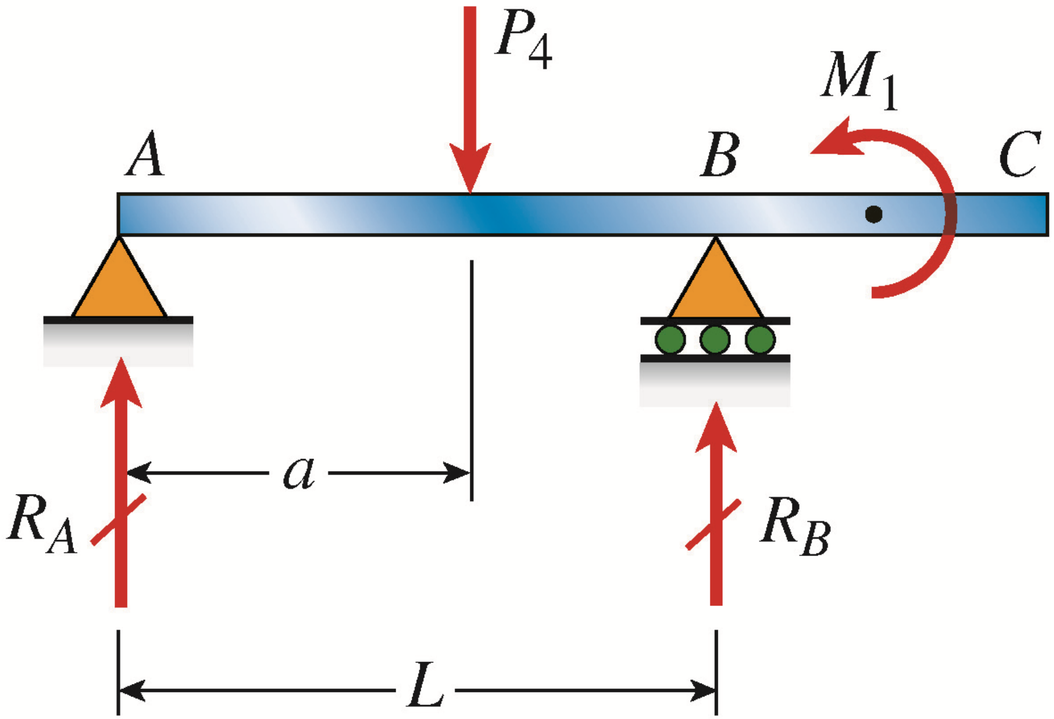
- Beam with an overhang



Shear Forces and Bending Moments
basic concepts

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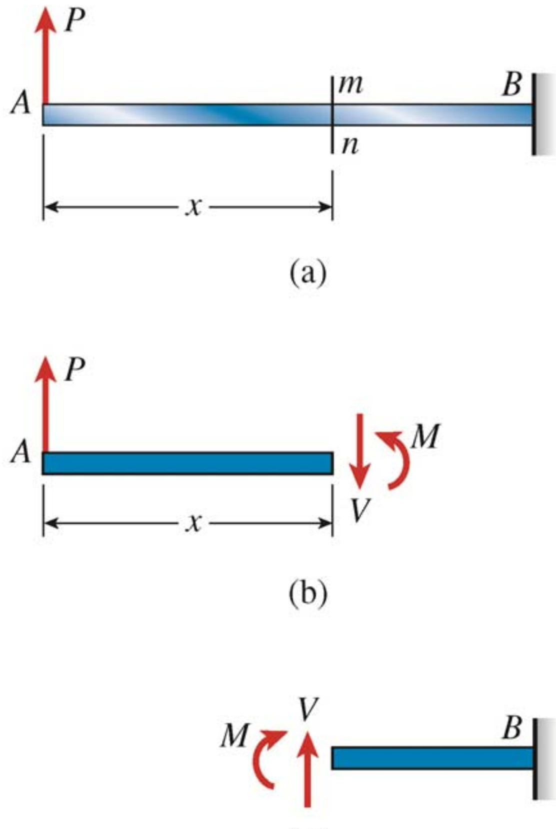
- Beams under forces or moment → stresses and strains are created throughout the interior of the beam.

- We first find the internal forces and couple (bending moment) on the cross section.
  - Stress resultant (합응력): resultants of stresses distributed over the cross section.

- Free Body Diagram – isolate left or right hand part.



Shear Forces and Bending Moments
methodology

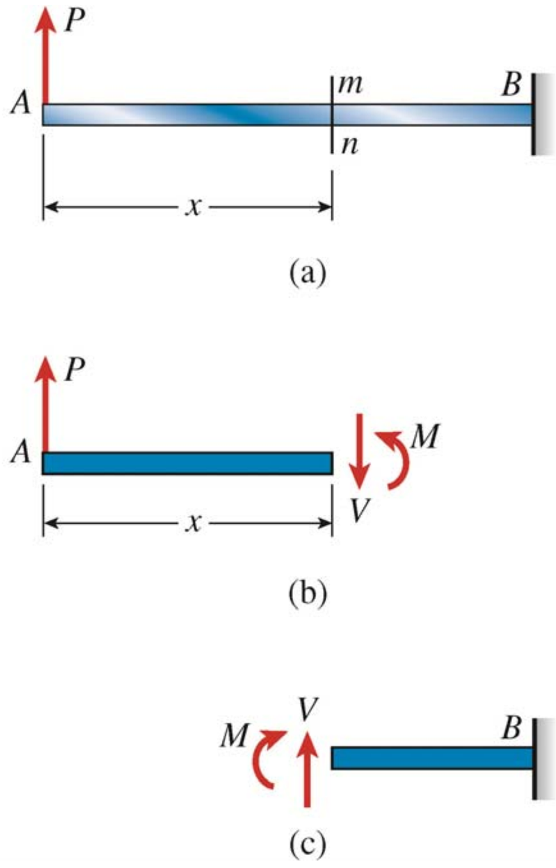
- Equilibrium Equation

$$\sum F_{ver} = 0 \qquad P - V = 0$$

$$V = P$$

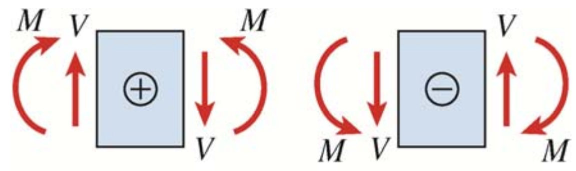
$$\sum M = 0 \qquad M - Px = 0$$

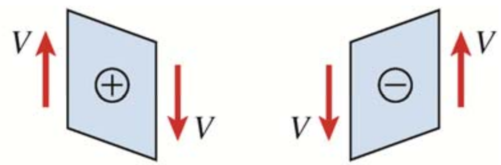
$$M = Px$$



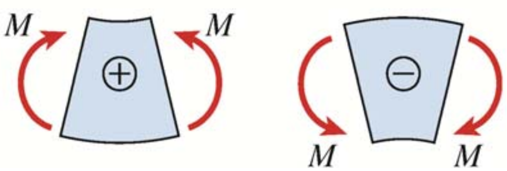
Shear Forces and Bending Moments
sign conventions for stress resultants

- 'deformation sign convention'
  - Based on how the material is deformed.
  - (+) shear force: acts clockwise
  - (-) shear force: ….. counter-clockwise
  - (+) bending moment: compress upper part
  - (-) bending moment: ………. lower part
- ' static sign convention'
  - Forces/moments are (+) or (-) according to their directions
  - Sign convention for Equilibrium Equation.





(a)



Summary

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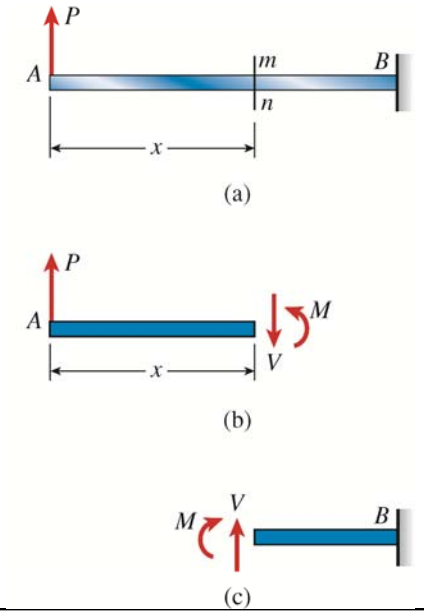
- Introduction
- Types of Beams, Loads, and Reactions
- Shear Forces and Bending Moments



- Relationships Between Loads, Shear Forces and Bending Moments
- Shear-Force and Bending-Moment Diagrams

Next Monday

Shear Forces and Bending Moments
Example 4-1

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- Shear force V & bending moment M at the right and left of mid point?
  - R_A & R_B?
  - Free Body Diagram.

$$\sum M_A = 0 \longrightarrow \quad R_B = \frac{P}{4} + \frac{M_0}{L}$$

$$\sum M_B = 0 \longrightarrow \quad R_A = \frac{3}{4}P - \frac{M_0}{L}$$

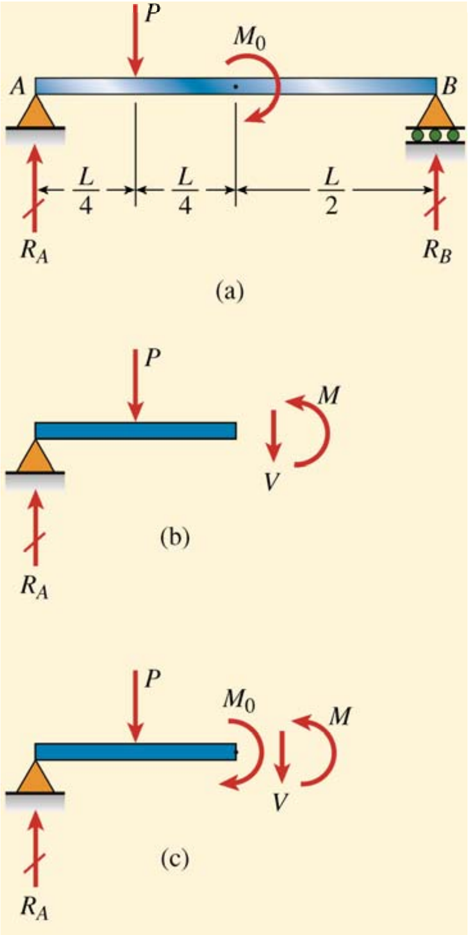
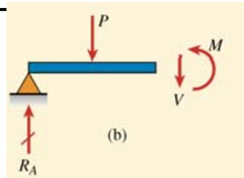


FIG. 4-11 Example 4-1. Shear forces and bending moment in a simple beam (parts (a) and (b) repeated)

Shear Forces and Bending Moments
Example 4-1

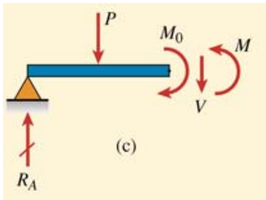
- To the left



$$\sum F_V = 0 \longrightarrow V = -\frac{P}{4} - \frac{M_0}{L}$$

$$\sum M_A = 0 \longrightarrow M = \frac{PL}{8} - \frac{M_0}{2}$$

- To the right



$$\sum F_V = 0 \longrightarrow V = -\frac{P}{4} - \frac{M_0}{L}$$

No change in shear force

$$\sum M_{cut} = 0 \longrightarrow M = \frac{PL}{8} + \frac{M_0}{2}$$

Increase by M_0

Shear Forces and Bending Moments
Example 4-2

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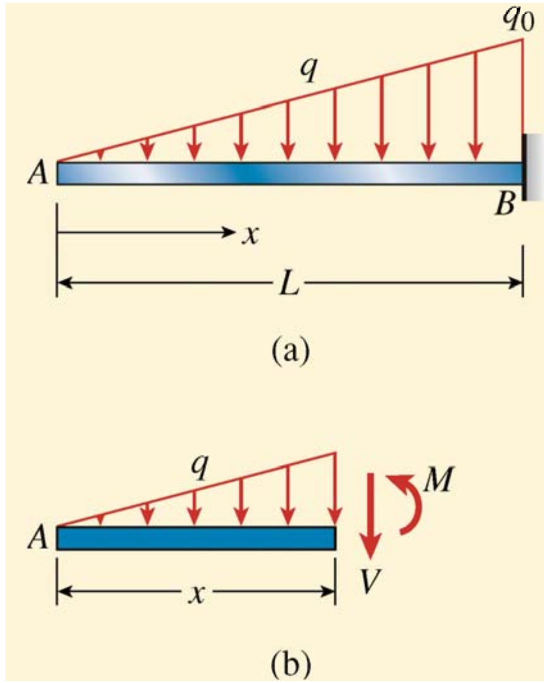
- Shear force V & Bending moment M?
  - Intensity of the distributed load at x

$$q = \frac{q_0}{L} x$$

  - Shear Force $V = -\frac{q_0 x^2}{2L}$ $\qquad V_{max} = -\frac{q_0 L}{2}$
  - Bending moment

$$\sum M = 0 \longrightarrow M + \frac{1}{2}\left(\frac{q_0 x}{L}\right)(x)\left(\frac{x}{3}\right) = 0$$

$$M = -\frac{q_0 x^3}{6L} \qquad M_{max} = -\frac{q_0 L^2}{6}$$



$$\frac{dV}{dx} = -q \qquad \frac{dM}{dx} = V$$

Shear Forces and Bending Moments
Example 4-3

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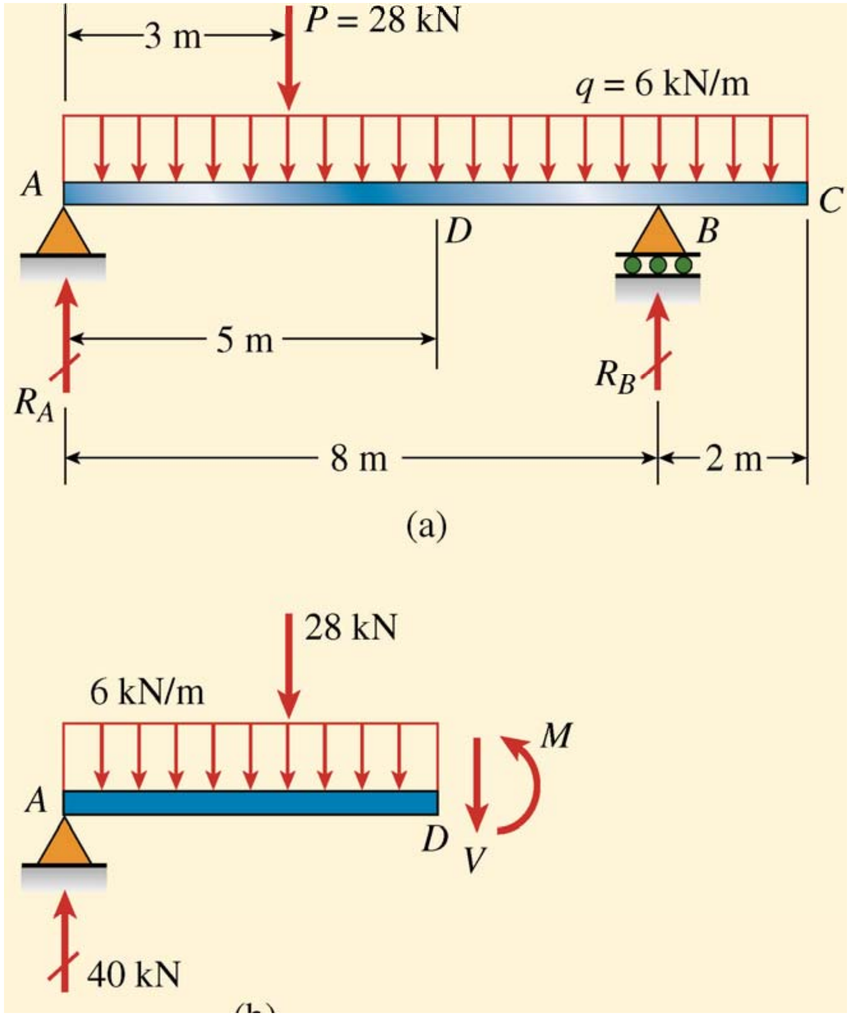
- Shear force V & bending moment M at D?



Relationships Between Loads, Shear Forces and Bending Moments

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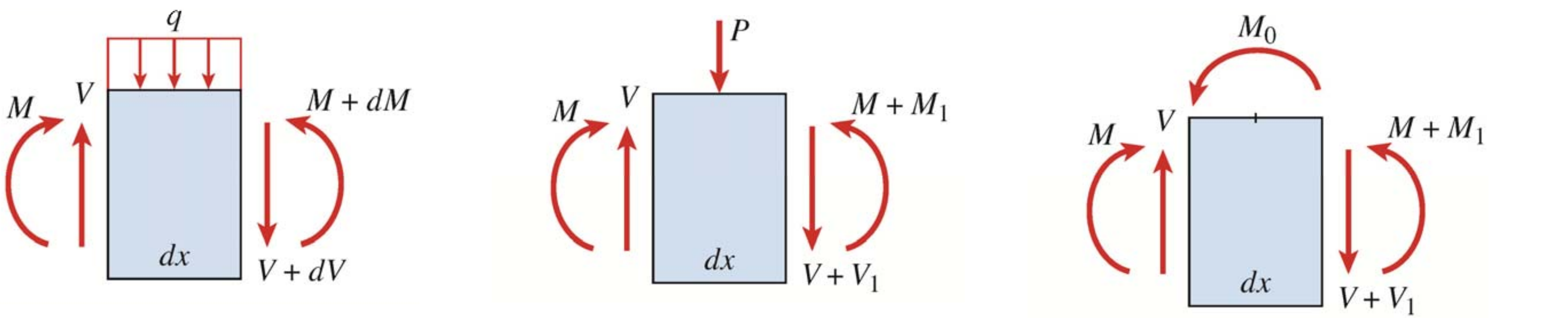
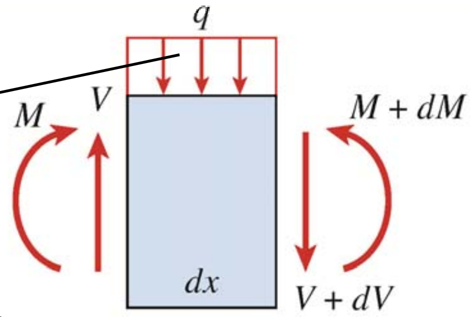
- Relationships between loads, shear forces, and bending moments in beams.
  - Useful for investigating the shear forces and bending moments throughout the entire length of a beam
  - Helpful when constructing shear-force and bending-moment diagrams
  - In general, the V & M varies along the axis of the beam.

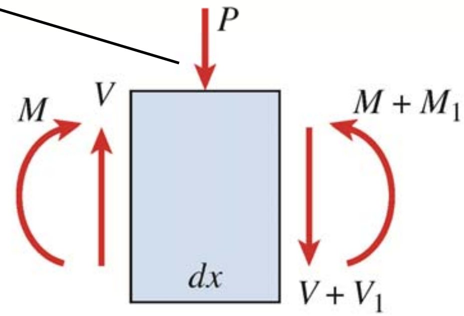


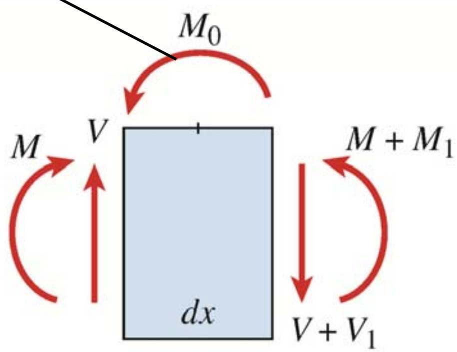
Relationships Between Loads, Shear Forces and Bending Moments

sign convention

- Sign convention for loads
  - (+) downward on the beam
  - (-) upward on the beam
- Sign convention for a couple (moment)
  - (+) counterclockwise
  - (-) clockwise







Distributed Loads

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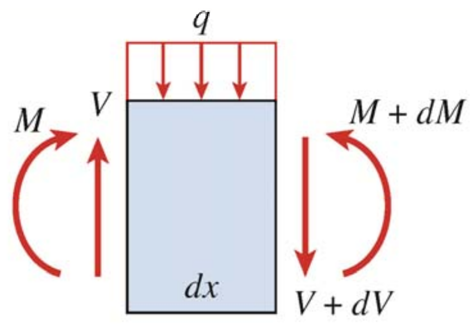
- From Equilibrium of forces

$$\sum F_V = 0 \qquad V - q dx - (V + dV) = 0$$

$$\frac{dV}{dx} = -q$$



  - Rate of change of the shear force at any point on the axis of the beam = negative of the intensity of the distributed load
  - (-) sign change to (+) for positive upward distributed load
  - If q = 0, dV/dx = 0 and V is constant in that part of the beam
  - If q = constant, dV/dx is also constant and V varies linearly in that part of the beam

Relationships Between Loads, Shear Forces and Bending Moments

Distributed Loads

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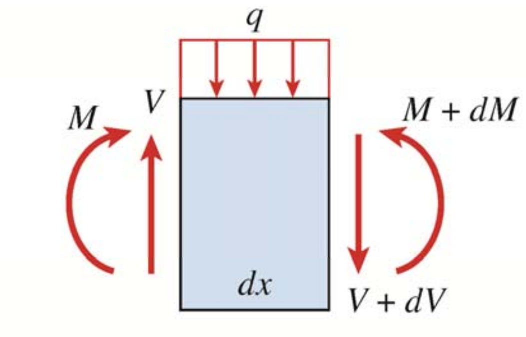
$$\frac{dV}{dx} = -q$$



- Integrate above equation after multiplying with dx

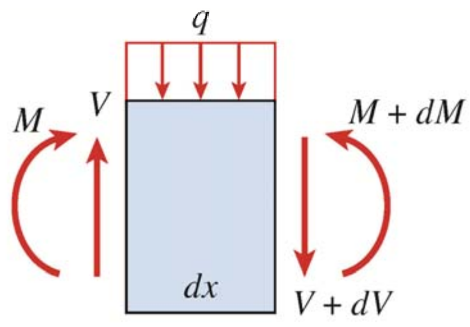
$$\int_A^B dV = -\int_A^B q dx$$

$$V_B - V_A = -\int_A^B q dx = -(\text{area of the loading diagram between } A \text{ and } B)$$

Change in shear force between two points along the axis of the beam = Negative of the total downward load between those points

Distributed Loads

- From Moment Equilibrium



$$\sum M_{left} = 0 \quad -M - q\,dx\frac{dx}{2} - (V + dV)dx + M + dM = 0$$

$$\frac{dM}{dx} = V$$

- Rate of change of the bending moment at any point on the axis of the beam = shear force at that same point

$$\int_A^B dM = \int_A^B V dx$$

$$M_B - M_A = -\int_A^B V dx = -(area\ of\ the\ shear - force\ diagram\ between\ A\ and\ B)$$

Relationships Between Loads, Shear Forces and Bending Moments

Concentrated Loads

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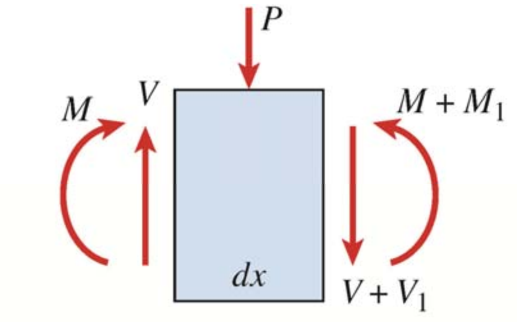

- From Force Equilibrium,

$$\sum F_V = 0 \qquad V - P - (V + V_1) = 0 \qquad V_1 = -P$$

  - Shear force decreases by P

- From Moment Equilibrium,

$$\sum M_{left} = 0 \qquad -M - P\frac{dx}{2} - (V + V_1)dx + M + M_1 = 0$$

$$M_1 = P\frac{dx}{2} + Vdx + V_1dx$$

- dx is infinitesimally small → M_1 is also very small
- Bending moment does notchange as we pass through the point of application of a concentrated load

Loads in the form of couples

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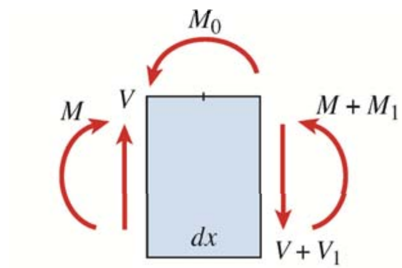

- From Force Equilibrium,

$$V_1 = 0$$

  - Shear force does not change at the point of application of a couple

- From Moment Equilibrium,

$$\sum M_{left} = 0 \qquad -M + M_0 - (V + V_1)dx + M + M_1 = 0$$

$$M_1 = -M_0$$

- Bending moment decreases by M_0 as we move from left to right through the point of load application → bending moment change abruptly.

Shear-Force and Bending-Moment Diagrams

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- Graph in which shear force and bending moment are plotted with respect to distance x along the axis of the beam.
- How shear forces and bending moments vary throughout the length of the beam? Maximum?
- Shear Force Diagram (SFD, 전단력선도)
- Bending Moment Diagram (BMD, 굽힘모멘트선도)

Shear-Force and Bending-Moment Diagrams
Concentrated Load

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- Determine Reactions from moment equilibrium,

$$R_A = \frac{Pb}{L} \qquad R_B = \frac{Pa}{L}$$

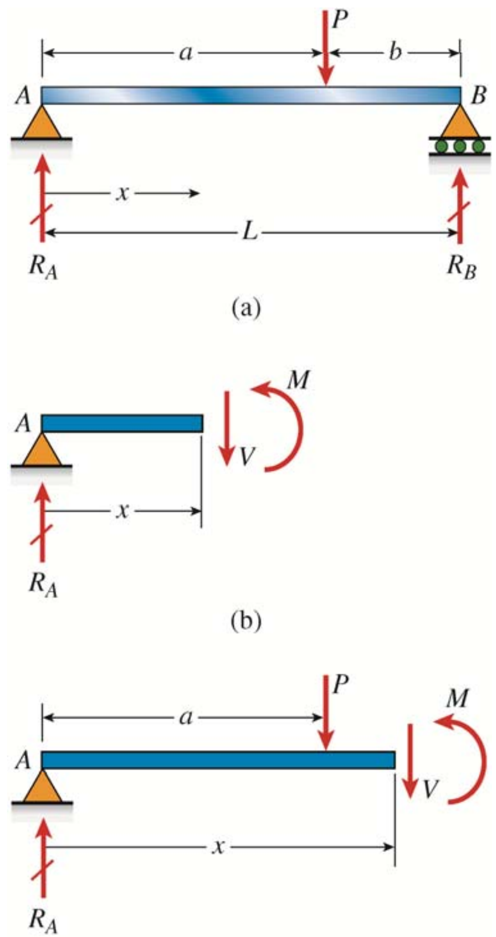
- V & M at the left part (0 < x < a),

$$V = R_A = \frac{Pb}{L} \qquad M = R_A x = \frac{Pbx}{L}$$

- V & M at the right part (a < x < L),

$$V = R_A - P = \frac{Pb}{L} - P = -\frac{Pa}{L}$$

$$M = R_A x - P(x-a) = \frac{Pbx}{L} - P(x-a) = \frac{Pa}{L}(L-x)$$



Shear-Force and Bending-Moment Diagrams
Concentrated Load

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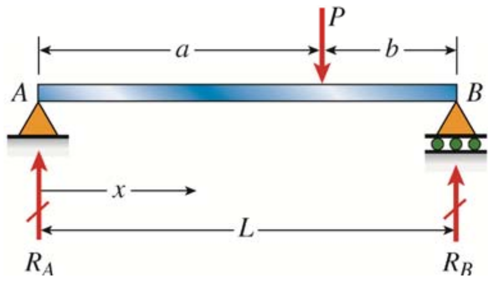
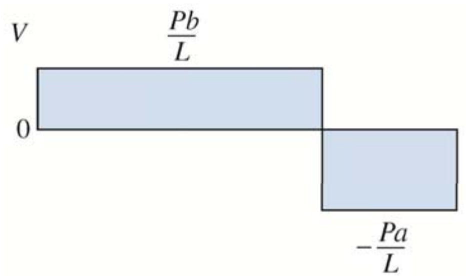


- Shear Force Diagram,

$$V = \frac{Pb}{L} \qquad (0 < x < a)$$

$$V = -\frac{Pa}{L} \qquad (0 < x < a)$$

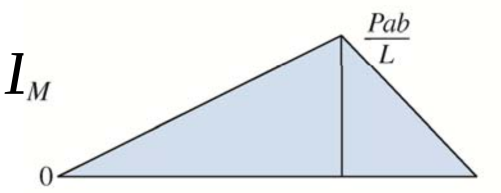


Slope dV/dx =0
→ q=0
Area for x < a
→ increase in M
Area for a<x<b
→ decrease in M

- Bending Moment Diagram

$$M = \frac{Pbx}{L} \qquad (a < x < L)$$

$$M = \frac{Pa}{L}(L - x) \qquad (a < x < L$$



Slope dM/dx = V

Shear-Force and Bending-Moment Diagrams
Uniform Load

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- From Moment Equilibrium,

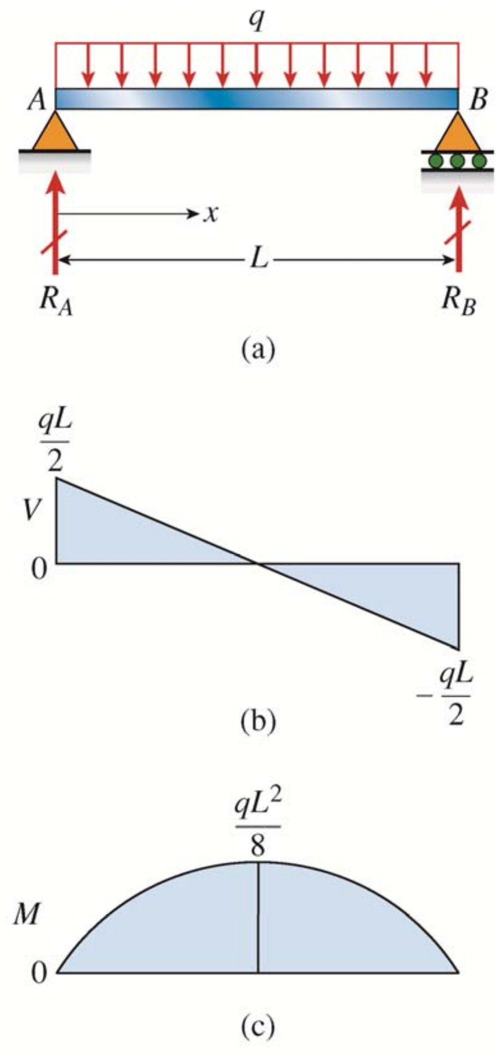
$$R_A = R_B = \frac{qL}{2}$$

- From Free Body Diagram,

$$V = R_A - qx = \frac{qL}{2} - qx$$

$$M = R_A x - qx\left(\frac{x}{2}\right) = \frac{qLx}{2} - \frac{qx^2}{2}$$

- Slope of V?
- Slope of M?



Shear-Force and Bending-Moment Diagrams
Several Concentrated Loads

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- From Moment Equilibrium,

$$R_A + R_B = P_1 + P_2 + P_3$$

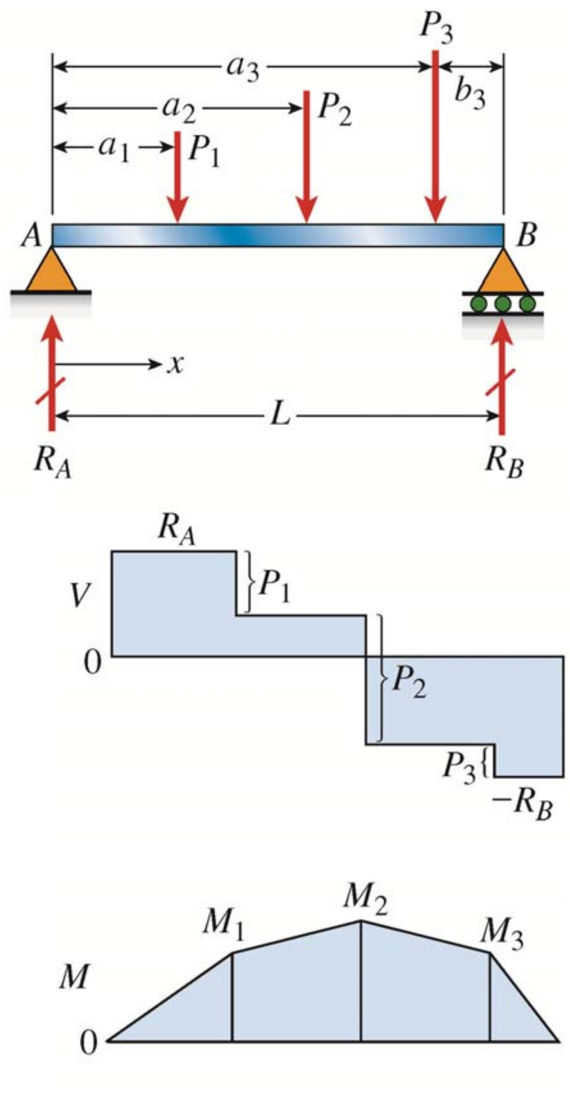
- From Free Body Diagram,

$$V = R_A \quad M = R_A x \qquad (0 < x < a_1)$$

$$V = R_A - P_1 \quad M = R_A x - P_1(x - a_1) \qquad (a_1 < x < a_2)$$

$$V = -R_B + P_3$$
$$M = R_B(L - x) - P_3(L - b_3 - x) \qquad (a_2 < x < a_3)$$

$$V = -R_B$$
$$M = R_B(L - x) \qquad (a_3 < x < L)$$



Shear-Force and Bending-Moment Diagrams
Several Concentrated Loads

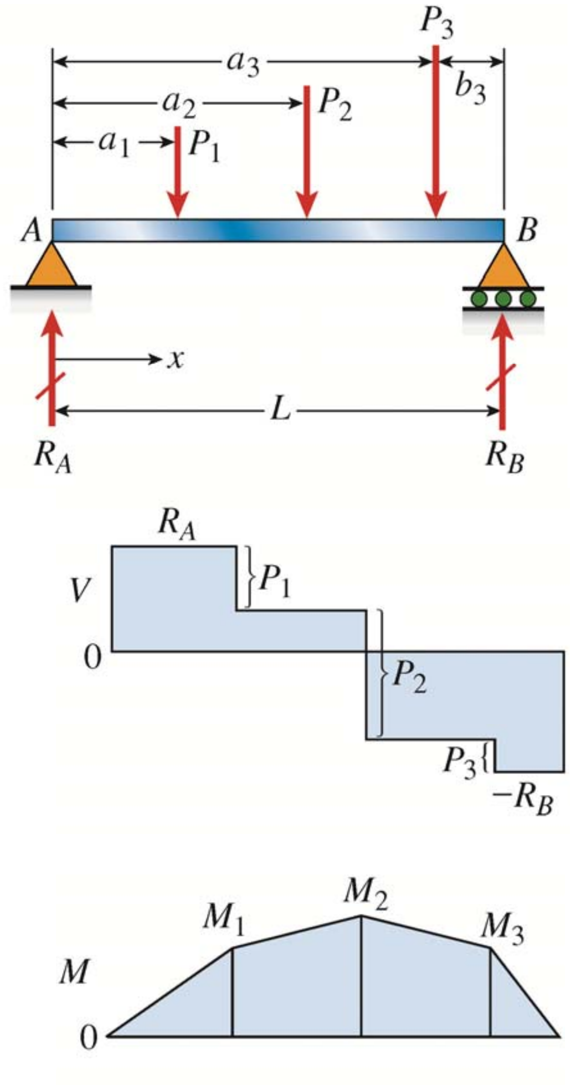
- Bending moment,

$$M_1 = R_A a_1$$

$$M_2 = R_A a_2 - P_1(a_2 - a_1)$$

$$M_3 = R_B b_3$$

- Maximum positive moment
- Maximum negative moment → numerically largest negative moment



Shear-Force and Bending-Moment Diagrams
Several Concentrated Loads

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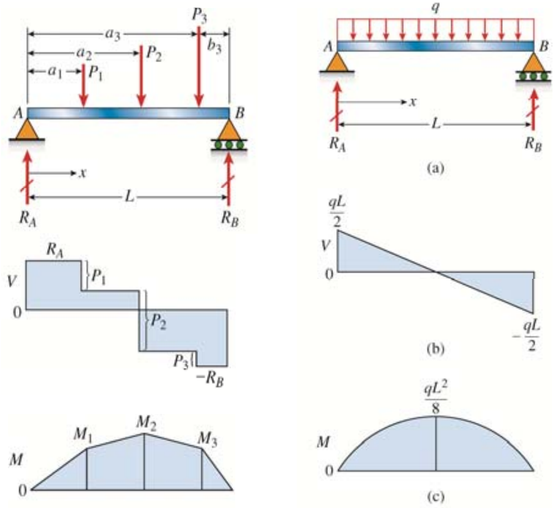
- Maximum positive and negative bending moments
  - A cross section where a concentrated load is applied and shear force changes sign
  - A cross section where the shear force =0
  - A point of support where a vertical reaction is present
  - A cross section where a couple is applied



Summary

- Introduction
- Types of Beams, Loads, and Reactions
- Shear Forces and Bending Moments
- Relationships Between Loads, Shear Forces and Bending Moments
- Shear-Force and Bending-Moment Diagrams

Next three lectures

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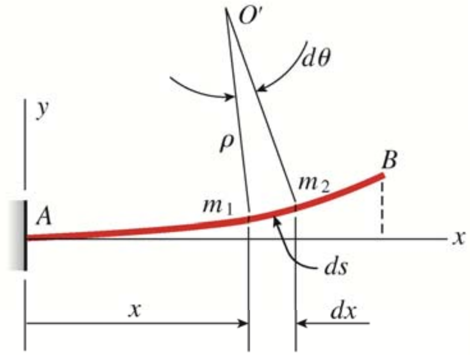
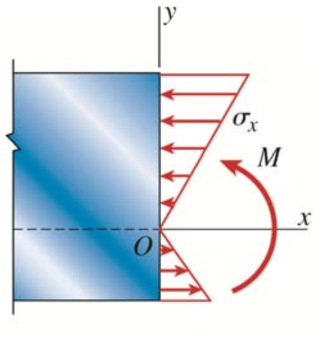
- Introduction
- Pure Bending and Nonuniform Bending
- Curvature of Beam
- Longitudinal Strains in Beams
- Normal Stress in Beams
- Design of Beams for Bending Stresses
- Nonprismatic Beams
- Shear Stresses in Beams of Rectangular Cross Section
- Shear Stresses in Beams of Circular Cross Section
- Shear Stresses in the Webs of Beams with Flanges

(a)





(a)