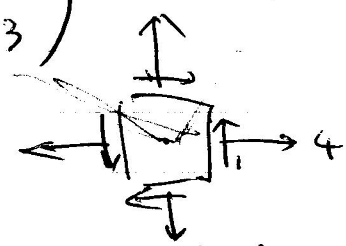
" we make a "

Before∨start Date 9. March 2009 No.

* homework check.
- invariant
- (n_x, n_y) calculation.
- more interaction.
- in English

$T_i = T_{ji} \cdot n_j$

$$\begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix} . \qquad \begin{pmatrix} 5 & 0 \\ 0 & 3 \end{pmatrix}$$



* $$\begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{5}{\sqrt{2}} \\ \frac{5}{\sqrt{2}} \end{pmatrix} \quad \left(\frac{5}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$$

magnitude → 5.

$$\begin{pmatrix} 5 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{5}{\sqrt{2}} \\ -\frac{3}{\sqrt{2}} \end{pmatrix} \quad \left(\frac{5}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\right)$$

$$\sqrt{\frac{25}{2} + \frac{9}{2}} = \underline{\underline{\sqrt{17}}}.$$

* Stress more important than strain
* importance of generating knowledge.
* 1st order 2nd order tensor ...
* Graphical representation of stress

K: Bok) - gradient
- body moment
- Cosserat model
- linear algebra (symmetric part)
- rotation KTH continuum.

Homework.

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prove that $\begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{pmatrix}$ is a second order tensor

similar to σ tensor.

p46 - p47.

check 2-7 Chou & Pagano.

check KTH continuum mechanics

* Graphical representations of stress

$$\sigma = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$$

$$\tau = -\frac{(\sigma_1 - \sigma_2)}{2}\sin 2\theta$$

$r^2 = x^2 + y^2$

$\left\langle \begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array} \right\rangle$

when expressed in terms of σ_1 & σ_2

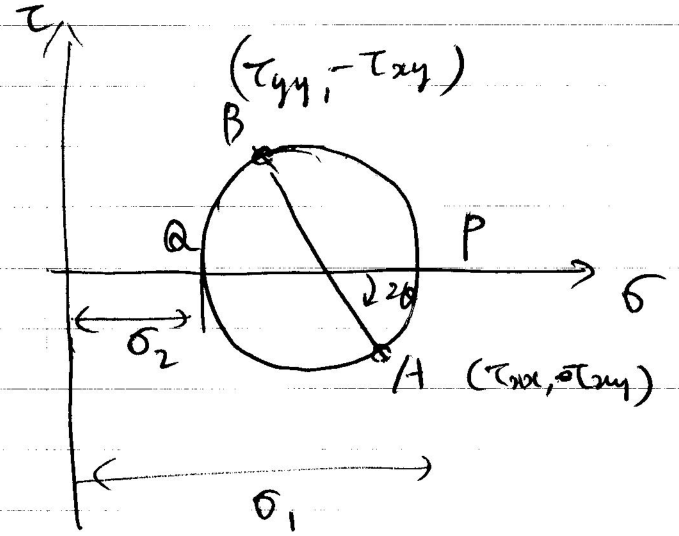
These are the equations of a circle in the (σ, τ) plane, with its center $\left(\frac{\sigma_1+\sigma_2}{2}, 0\right)$ and radius $\frac{(\sigma_1-\sigma_2)}{2}$

the circle is parameterized in the clockwise direction with angle 2θ.

$\left\langle \begin{array}{l} \cos 2\theta \rightarrow \cos 2\theta \\ \sin 2\theta \rightarrow -\sin 2\theta \end{array} \right.$

- σ_2 is $90°$ ($2 \times 90°$ in circle) ~~ap~~ apart from σ_1
- τ_{max} = radius of circle.

Fig 2.6



A & B.

$$\frac{\tau_{xx} + \tau_{yy}}{2} = \frac{\sigma_1 + \sigma_2}{2}$$

another proof of the fact that the value of the mean normal stress is independent of coordinate system

& θ

Finding σ_1, σ_2 from Mohr Circle Date No.
from a state of stress.

ex) $\begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}$

- two ways of plotting Mohr Circle.

Fig 2.6 (b).
MC can also be used to graphically find the orientation of the plane on which certain tractions act.

* Tensor

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For any vector quantity $\underline{F}$, it can be shown that its components transform to a primed coordinate system according to the rule.

$$\underline{F}' = \underline{\underline{R}}\,\underline{F} \quad \text{or} \quad F_i' = \beta_{ij} F_j$$

This equation may be taken as the definition of a vector, or tensor of the first order.
" $\underline{F}$ referred to a coordinate system x_i and transformed to another system x_i' by above equation is defined as a vector.

$$\underline{\underline{T}}' = \underline{\underline{R}}\,\underline{\underline{T}}\,\underline{\underline{R}}^T \quad \text{or} \quad \tau_{ij}' = \beta_{im}\beta_{jn}\tau_{mn}$$

$$\beta_{ij} = \begin{pmatrix} \cos(x',x) & \cos(x',y) & \cos(x',z) \\ \cos(y',x) & \cos(y',y) & \cos(y',z) \\ \cos(z',x) & \cos(z',y) & \cos(z',z) \end{pmatrix}.$$

2nd order tensor — stress, k_{ij}
<u>permeability</u>

Topic 2: Displacement and Strain ~~and Dis~~

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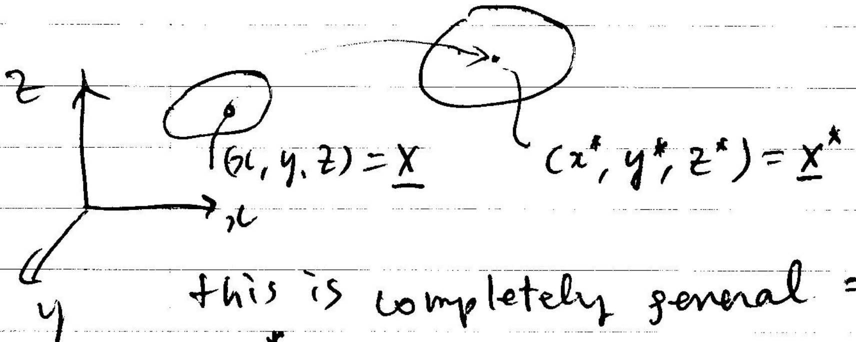
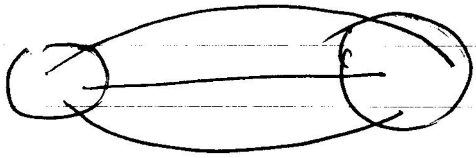


$\underline{x}^* = f(\underline{x})$

this is completely general = deformation.

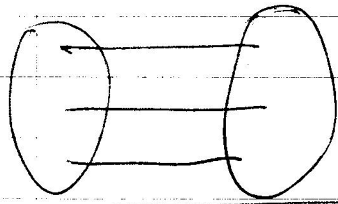
$\underline{x}^* = \underline{x} + \underline{u}$, $\underline{u}$: displacement.

$(x^*, y^*, z^*) = (x, y, z) + (u, v, w)$



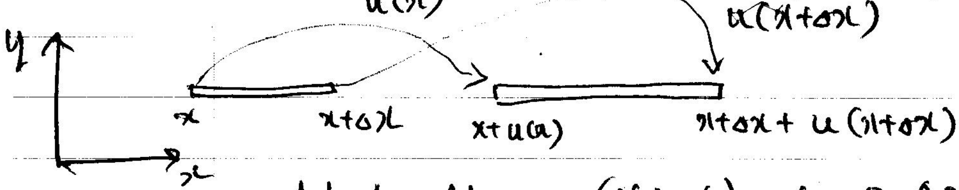
We think of the case in which different points moves to different points.

ex) if $\underline{u}$ is same each point (x, y, z)



⇒ rigid body displacement.
⟶ no stress involved.

Let's start with 1D problem



old length $= (x+\Delta x) - x = \Delta x$

new length $= x+\Delta x + u(x+\Delta x) - \{x + u(x)\} = \Delta x + u(x+\Delta x) - u(x)$

change of length, $\Delta L = u(x+\Delta x) - u(x)$

fractional change in length : $\dfrac{\Delta L}{L^{old}} = \dfrac{u(x+\Delta x) - u(x)}{\Delta x}$

Let $\Delta x \to 0$ (in order to define at a point instead of between two points)

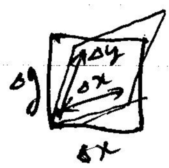
$$\frac{\Delta L}{L_{old}} = \frac{\partial u}{\partial x} = \varepsilon_{xx}, \quad \frac{du}{dx} \text{ in 1D}$$

" fractional elongation in X direction ".
positive = elongation.

Likewise $\varepsilon_{yy} = \dfrac{\partial v}{\partial y}$, $\varepsilon_{zz} = \dfrac{\partial w}{\partial z}$

However, ε_{xx}, ε_{yy}, ε_{zz} may not be enough to define deformation.

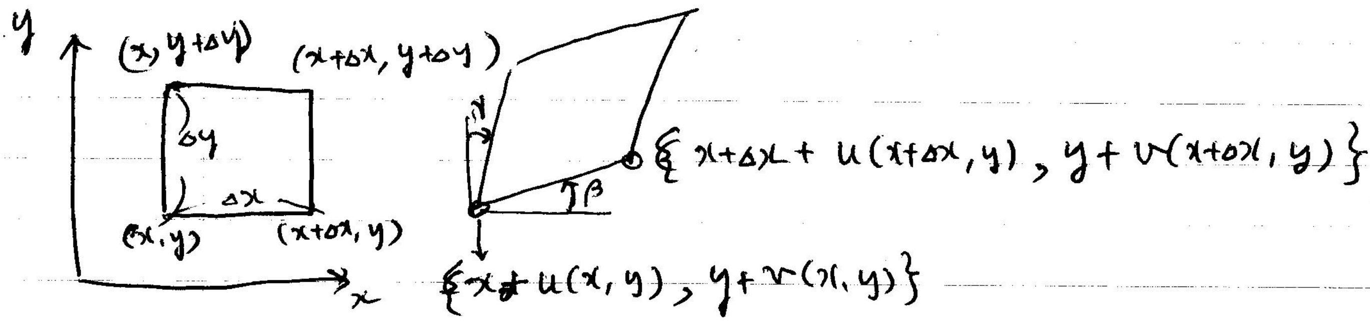
ex)



deformation without volume change

$\downarrow$

We need some other form of strain.



$$\tan\beta = \frac{\{y+v(x+\Delta x,y)\} - \{y+v(x,y)\}}{\{x+\Delta x+u(x+\Delta x,y)\} - \{x+u(x,y)\}} = \frac{v(x+\Delta x,y)-v(x,y)}{\Delta x+u(x+\Delta x,y)-u(x,y)}$$

$$= \frac{v(x+\Delta x,y) - v(x,y)}{\Delta x\cdot\left\{1 + \dfrac{u(x+\Delta x,y)-u(x,y)}{\Delta x}\right\}}$$

let $\Delta x \to 0$, $\tan\beta = \dfrac{\frac{\partial v}{\partial x}}{1+\frac{\partial u}{\partial x}}$ → very small compared to 1.

$\dfrac{\partial u}{\partial x} \ll 1$.

$$= \frac{\partial v}{\partial x}.$$

ex) $\dfrac{\partial v}{\partial x} = 0.01$, $\dfrac{\partial u}{\partial x} = 0.01$ we can neglect $\dfrac{\partial u}{\partial x}$ in this equation.

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if $\left|\frac{\partial v}{\partial x}\right| << 1$, $\tan\beta = \beta = \frac{\partial v}{\partial x}$.

likewise, $\gamma = \frac{\partial u}{\partial y}$, derivative of u with respect to y

old angle $= 90^\circ$
new angle $= 90^\circ - \beta - \gamma$
– change of angle = decrease in angle $= \beta + \gamma$
$= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = \gamma_{xy}$: engineering shear strain.

used before tensor was invented.

shear strain, $\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$, $\varepsilon_{yx} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$.

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) = \varepsilon_{zx}$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) = \varepsilon_{zy}.$$

\# Stress was symmetric only when body moment was zero.
But strain is symmetric "By definition".

Let's fit everything. Everything follows the same pattern.

$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x}\right)$ normal strain.

$$\underline{\underline{\varepsilon}} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$$

Strain matrix
→ symmetric by definition.

$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ we will talk about this later

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With u, v, w & x, y, z there are nine derivatives but there are only 6 components.
What happened to 3 others?

$$\underline{\underline{\varepsilon}} = \frac{1}{2}\begin{pmatrix} \frac{\partial u}{\partial x}+\frac{\partial u}{\partial x} & \frac{\partial v}{\partial x}+\frac{\partial u}{\partial y} & \frac{\partial w}{\partial x}+\frac{\partial u}{\partial z} \\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}+\frac{\partial v}{\partial y} & \frac{\partial w}{\partial y}+\frac{\partial v}{\partial z} \\ \frac{\partial u}{\partial z}+\frac{\partial w}{\partial x} & \frac{\partial v}{\partial z}+\frac{\partial w}{\partial y} & \frac{\partial w}{\partial z}+\frac{\partial w}{\partial z} \end{pmatrix}$$

$$= \frac{1}{2}\left\{ \underbrace{\begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{pmatrix}}_{\text{Displacement gradient } \nabla u} + \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z} \end{pmatrix} \right.$$

$$\underline{\underline{\varepsilon}} = \frac{1}{2}\left(\nabla \underline{u} + (\nabla \underline{u})^T \right)$$

strain matrix = symmetric part of displacement gradient

Def. $\frac{1}{2}(\underline{\underline{A}}+\underline{\underline{A}}^T)$ = symmetric part of A.

proof. $\underline{\underline{A}}_{sym}^T = \frac{1}{2}(\underline{\underline{A}}^T+\underline{\underline{A}}) = \frac{1}{2}(\underline{\underline{A}}+\underline{\underline{A}}^T)$

Def. $\frac{1}{2}(\underline{\underline{A}}-\underline{\underline{A}}^T)$ = antisymmetric part of A.

proof. $\left(\frac{1}{2}(A-A^T)\right)^T = \frac{1}{2}(A^T-A) = -\frac{1}{2}(A-A^T)$

$$A_{anti}^T = -A_{anti}^T$$

$$A_{anti} + A_{sym} = \frac{1}{2}(\underline{\underline{A}}+\underline{\underline{A}}^T) + \frac{1}{2}(\underline{\underline{A}}-\underline{\underline{A}}^T) = \underline{\underline{A}}$$

$\underline{\underline{W}}$ = antisymmetric part of displacement gradient

$$\underset{\sim}{W} = \frac{1}{2}\left(\nabla u - (\nabla u)^T\right), \leftarrow \underline{\underline{\varepsilon}} + \underline{\underline{W}} = \nabla \underline{\underline{u}}$$

$$W = \frac{1}{2}\begin{pmatrix} 0 & \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} & \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} & 0 & \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} & \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} & 0 \end{pmatrix}$$

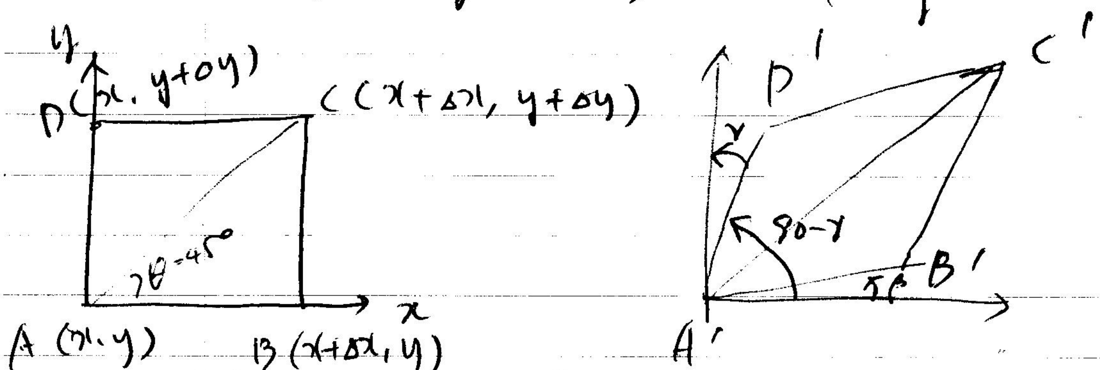
$\longrightarrow$ 3 independent component.

Q: What does W represent?

A: rigid body rotation.

$$\frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = \omega_{xy} \quad \text{rotation around z axis.}$$

$$\underset{\sim}{W} = \begin{pmatrix} 0 & \omega_{xy} & \omega_{xz} \\ -\omega_{xy} & 0 & \omega_{yz} \\ -\omega_{xz} & -\omega_{yz} & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix}.$$



$\overline{AC}$ is at 45° to the x-axis

$A'B'$ is now at angle $\beta = \frac{\partial v}{\partial x}$

$A'D'$ is now at angle $\gamma = \frac{\partial u}{\partial y}$

new angle $A'D'$ is $90° - \gamma = 90 - \frac{\partial u}{\partial y}$

Bisector line is now at angle $= \frac{1}{2}\left(\frac{\partial v}{\partial x} + 90° - \frac{\partial u}{\partial y}\right)$

$$= 45° + \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$= 45° + \frac{1}{2}(\beta - \gamma)$$

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change in angle $= \frac{1}{2}(\beta - \gamma) = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = w_{yx}$

$\therefore$ $\underline{\underline{W}}$ represent a rigid body rotation.

$\underline{\underline{\varepsilon}}$ " changes in shape, size ~~but no~~ rotation.

finitesimal deformation three additive components of

i) rigid body translation — no strain or rotation

ii) stretching and/or distorting deformation — non zero strain tensor

iii) infinitesimal rotations — nonzero rotation tensor.

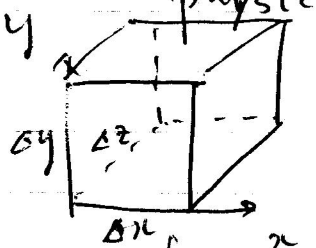
$\underline{\underline{\varepsilon}}$ can be also expressed $\underline{\underline{\varepsilon}}' = \underline{\underline{R}}\,\underline{\underline{\varepsilon}}\,\underline{\underline{R}}^T$,

3 principal directions, $\perp$ each other,

$$\begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix} \longrightarrow \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}$$

Like the case in stress, there are always direction where $\varepsilon_{xy}, \varepsilon_{xz}, \varepsilon_{yz} = 0$.

physical explanation???

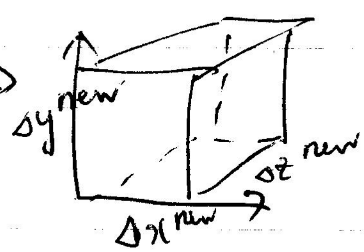


→ imagine that x-y-z are principal axes for strain tensor.

$$V_{old} = \Delta x \Delta y \Delta z$$

$$\varepsilon_{xx} = \frac{L^{new} - L^{old}}{L^{old}}$$

$$= \frac{\Delta x^{new} - \Delta x^{old}}{\Delta x^{old}}$$



$$\varepsilon_{xx}\Delta x^{old} = \Delta x^{new} - \Delta x^{old}$$

$$\Delta x^{new} = \Delta x^{old}(1 + \varepsilon_{xx})$$

$$\Delta y^{new} = \Delta y^{old}(1 + \varepsilon_{yy})$$

$$\Delta z^{new} = \Delta z^{old}(1 + \varepsilon_{zz})$$

$$V^{new} = \Delta x^{new}\Delta y^{new}\Delta z^{new} = \Delta x^{old}\Delta y^{old}\Delta z^{old}(1+\varepsilon_{xx})(1+\varepsilon_{yy})(1+\varepsilon_{zz})$$

Volumetric Strain

$$= \frac{V^{new} - V^{old}}{V^{old}} = \frac{V^{old}\cdot\{(1+\varepsilon_{xx})(1+\varepsilon_{yy})(1+\varepsilon_{zz}) - 1\}}{V^{old}}$$

$$= (1+\varepsilon_{xx})(1+\varepsilon_{yy})(1+\varepsilon_{zz}) - 1$$

small strain.

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$= 1 + \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} + \varepsilon_{xx}\varepsilon_{yy} + \varepsilon_{yy}\varepsilon_{zz} + \varepsilon_{xx}\varepsilon_{zz} + \varepsilon_{xx}\varepsilon_{yy}\varepsilon_{zz} - 1$

$\approx \varepsilon_x + \varepsilon_y + \varepsilon_z$ = volumetric strain.

$= \mathrm{Trace}(\underline{\underline{\varepsilon}})$

$\hookrightarrow$ change of shear strain does not affect volume!

*) For the symmetric tensor $T_{ij} = \begin{pmatrix} 7 & 3 & 0 \\ 3 & 7 & 4 \\ 0 & 4 & 7 \end{pmatrix}$

determine the principal values and the directions of the principal axes.

A) $\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} = \begin{pmatrix} 12 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

direction cosine, R

$$\begin{pmatrix} \cos(x^*, x) & \cos(x^*, y) & \cos(x^*, z) \\ \cos(y^*, x) & \cos(y^*, y) & \cos(y^*, z) \\ \cos(z^*, x) & \cos(z^*, y) & \cos(z^*, z) \end{pmatrix}$$

$$= \begin{pmatrix} 0.4243 & 0.7 & 0.56 \\ 0.8 & 0 & -0.6 \\ -0.4243 & 0.7 & -0.56 \end{pmatrix}.$$

confirmation

$$\begin{pmatrix} 12 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 2 \end{pmatrix} = R\, T_{ij}\, R^T.$$

— Dong Keun.

— oral exam.