

16.222 Mechanics of Composite Materials

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will have { 10 Problem Sets (40 %)
{ 3 Quiz (60 %)

• Academic Honesty

- problem sets : discussion with others O.K.
but should be referenced
- Quiz : open book, take home
No discussion or collaboration

• Ref :

1. R.M. Jones, "Mechanics of Composite Mat'l"
McGraw-Hill, 1975
--- Main Text
2. Tsai & Hahn, "Intro to Composite Mat'l"
Technomic Publication, 1980
3. Take Good Notes

• Purpose :

To introduce composites,
basic analysis methods,
behavior of laminated materials

• Course supplemented by

16.202 (Manufacturing Lab classes)

- outline

1. Introduction
2. Micromechanics
3. Ply Elasticity
4. Laminate Theory
5. Failure
6. Bending and Coupling
7. Thermal Stresses
8. Advanced Topics

• builds on basic solid mechanics and structures
 goes into anisotropic properties, matrix manipulation of stress & strain, laminated materials

1. Introduction

• what is a composite material?

Generally, Materials with 2 or more constituents combined by a physical process on a macroscopic scale

{ fibers or particles in a matrix
 Layers of dissimilar materials

• why composite?

- Can achieve properties not possible with a monolithic (homogeneous) material

- Constituents have two different functions

Reinforcement : provides most of properties

Matrix : binds together reinforcement

• Some Examples

Reinforcement

straw

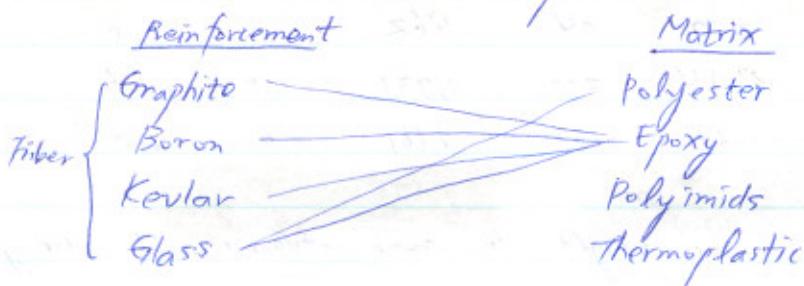
Matrix

clay

Sand, rock
cellulose

Cement, Water
Lignin (wood)

- we care about advanced (aerospace) use



Particles

- will limit our attention to

- ① continuous fibers
- ② cuttimated fibers



often comes in "prepreg" (preimpregnated) sheets

- Why use composites?

Better specific properties

specific stiffness, E/ρ

specific strength, σ_{ult}/ρ

for minimum weight structures

composites are light, low ρ

Mat'l	ρ (SG)	E (GPa)	σ_{ult} (MPa)	E/ρ^* (normalized on Al)	σ_{ult}/ρ^* (normalized on Al)
Al 2024-T3	2.77	74	462	1.0	1.0
Steel 300M	7.84	200	1,931	1.0	1.5
Gr/E _f	1.61	138	1,661	3.0	6.2

- composites --- less weight for same stiffness and strength

Importance in Aerospace

$$\text{Range of A/C, } R = \frac{L}{D} \frac{V}{(SFC)} \ln \frac{w_0}{w_f} \quad (\text{Breguet Eq.})$$

$$\text{Speed of Rocket, } V = V_E \ln \frac{w_0}{w_f} \quad (\text{Rocket Eq.})$$

exhaust velocity

where $\begin{cases} w_0 : \text{Gross Weight} \\ w_f : \text{Final weight} \end{cases}$

Less structural weight $\rightarrow \begin{cases} \text{more range / speed} \\ \text{more payload} \end{cases}$

A/C $\sim \$300/\text{lb payload}$

Satellite $\sim \$10,000/\text{lb}$

Round world flight (Voyager)

High Speed Civil Transport } we composites

single-stage - orbit

- Other advantages

- Tailoring --- Pick fiber, matrix, arrangement to get properties you want
(include zero coefficient of Tension/Extension, Bending/Torsion coupling, strength)

- Good fatigue life

- Good corrosion resistance

- Manufacture can be cheaper (molding, automatic process)

- stealth (low radar response)

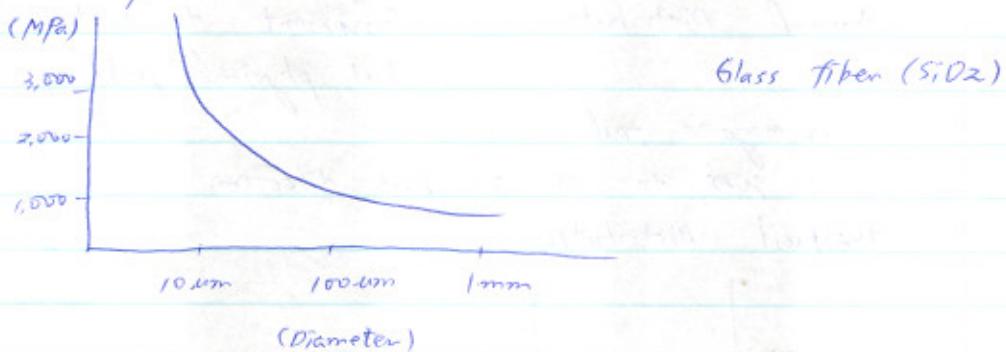
- Disadvantages
 - Cost (materials, more steps now)
 - Environmental Effects (H_2O , Impact, heating, space)
 - Technological Risk
 - Harder to Design

2. Micromechanics of Composites

Look at fibers, matrix and interactions in a polymer matrix composite.

Fibers --- very small diameter fibers of glass are much stronger than bulk properties of glass

Griffith Experiment, 1921



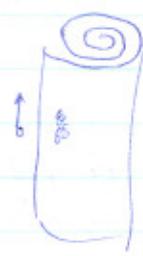
For brittle materials, strength $\propto \frac{1}{\text{fibre size}}$ flaws size
 small fibers \rightarrow smaller flaws, fewer flaws
 \rightarrow much higher strengths
 than large fibers, bulk properties

similarly for graphite fibers, etc.

Fibers for composites - graphite

Pitch

PAN (Polyacrylonitrile)

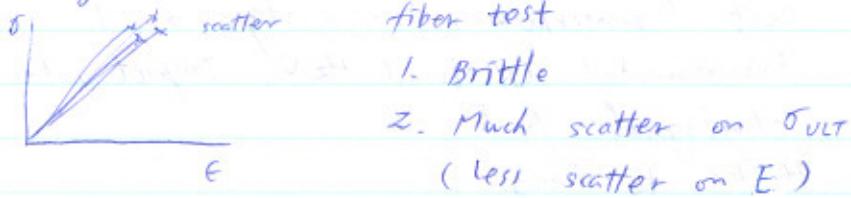


more organized

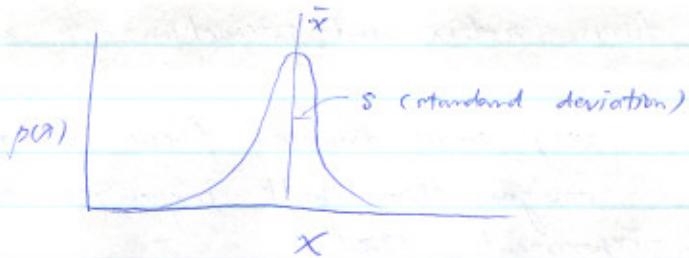


3-10 cm dia.

strong along fiber direction, weak bonds \perp to fiber dir.



• statistics of Failure

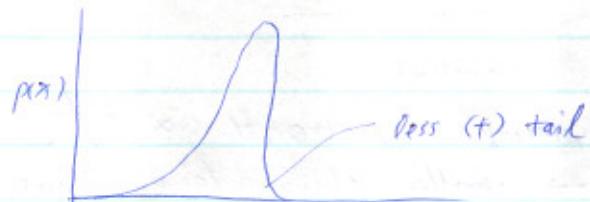


Normal Distribution --- Convenient for statistics
but physical problem

i) negative tail

ii) goes to ∞ in both directions

Weibull Distribution



Weibull $p(x) = \frac{\alpha}{\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} e^{-(x/\beta)^\alpha}$ Better for fits here for $x \geq 0$

β : scale factor (analogous to mean)

$$\text{Mean: } \bar{x} = \beta \underbrace{\Gamma\left(\frac{1}{\alpha} + 1\right)}_{*}$$

α	\bar{x}
5	0.92
25	0.98

$$\bar{x} \approx \beta$$

α : shape factor

$$S \text{ (standard dev.)} = \beta \sqrt{\Gamma\left(\frac{2}{\alpha} + 1\right) - \Gamma^2\left(\frac{1}{\alpha} + 1\right)}$$

$$\approx \beta/\alpha$$

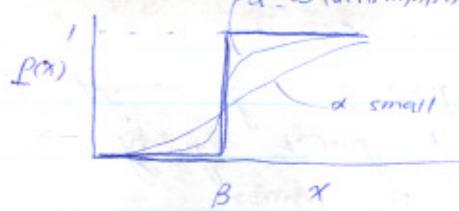
α	$\frac{\sigma}{\alpha}$
5	1.05
25	1.23

$$\text{Coeff. of Variation} = \frac{\sigma}{\alpha} \approx \frac{1}{\alpha}$$

Cumulative Distribution

$$P(x) = \int_0^x p(x) dx = 1 - e^{-\left(\frac{x}{\alpha}\right)^{\alpha}}$$

$\alpha = \infty$ (deterministic)



$P(x)$ is probability that failure will occur before load x is reached.

where, Mean $\approx \beta$

$$\text{S.D.} \approx \beta (\text{approx}) \approx \beta/\alpha$$

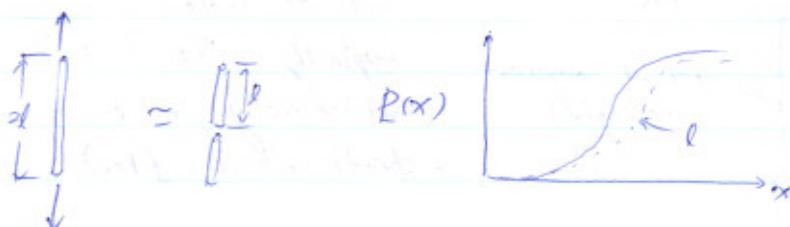
$$\text{C.O.V.} \approx 1/\alpha$$

Typical Values

Fiber	β	α	C.O.V.
Kevlar	$\sim 600 \text{ KSI}$	6	17 %
Graphite	$\sim 450 \text{ KSI}$	4	25 %
Steel	$\sim 200 \text{ KSI}$ (100 KSI = 690 MPa)	25-50	$\sim 4 \%$

→ a lot of scatter

Consider longer fiber, zl



d_l : weaker, more scatter

Consider a bundle of fibers,



When one fiber breaks, others carry load
stress goes up since net area is down.
Generally, greater less scatter, not more strength

Fiber bundles called tows

12K tow \rightarrow 12,000 fibers

20K " \rightarrow 20,000 fibers

Gr/Ep \rightarrow fiber sum

tow 100 μm

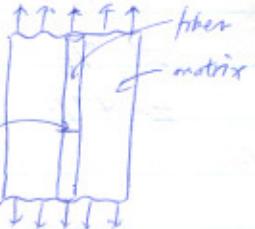
Use of fiber bundles good -- high strength, less scatter,
but need greater rigidity \rightarrow compression
as well as tension

Use matrix to enclose fibers.

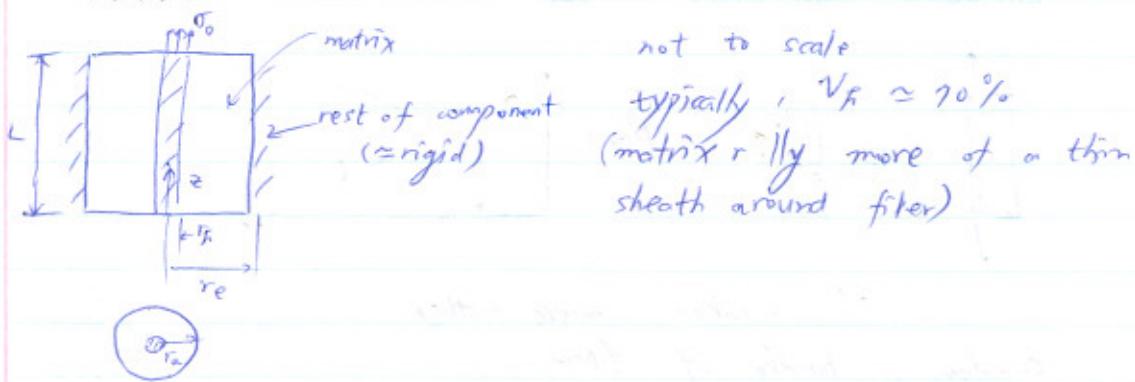
- Key role of matrix

1. Protect fibers
2. Provide rigidity for fibers
3. Stress transfer about fiber fracture
4. Reduce stress concentration @ break

- Role of Matrix in Stress Transfer


if matrix and fiber are well-bonded,
what happens?

First consider simple pull-out problem



B.C.'s

$$@ z = 0, \sigma_f = 0$$

$$@ z = L, \sigma_f = \sigma_0$$

@ $r = r_a$, Displacement = 0 (rigid)

Assume uniform σ_f

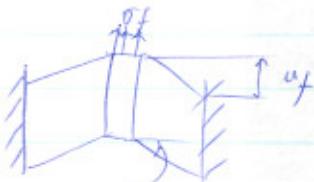
$$\text{zero} \cdot \sigma_m \quad (E_m \approx 3.5 \text{ GPa}, E_f \approx 210 \text{ GPa})$$

Matrix acts only in shear (adhesive yes)

Unknowns u_f - displacement

ϵ_f, γ_m - strain,

σ_f, τ_m - stress.



$\alpha \approx -\gamma_m$ strain - Displacement Egn.

$$\epsilon_f = \frac{\partial u_f}{\partial z} = u_f' \quad \dots \quad \textcircled{1}$$

$$\gamma_m = \frac{u_f}{r_a - r_f} \quad \dots \quad \textcircled{2}$$

Equilibrium Egn.

$$\begin{aligned} & (\sigma_f + \frac{\partial \sigma_f}{\partial z} dz) \pi r_f^2 - \sigma_f \pi r_f^2 \\ & + T_m z \pi r_f dz = 0 \\ & \frac{\partial \sigma_f}{\partial z} + \frac{z \gamma_m}{r_f} = 0 \\ & \sigma_f' + \frac{z \gamma_m}{r_f} = 0 \quad \dots \quad \textcircled{3} \end{aligned}$$

Stress - strain Egn.

$$\sigma_f = E_f \epsilon_f \quad \dots \quad \textcircled{4}$$

$$T_m = G_m \gamma_m \quad \dots \quad \textcircled{5}$$

5 Egn.s \rightarrow 5 unknowns

$$\textcircled{1} \rightarrow \textcircled{3} \quad \sigma_f' + \frac{z G_m \gamma_m}{r_f} = 0 \quad \dots \quad \textcircled{6}$$

$$\textcircled{2} \rightarrow \textcircled{6} \quad \sigma_f' + \frac{z G_m}{r_f} \left(-\frac{u_f}{r_a - r_f} \right) = 0 \quad \dots \quad \textcircled{7}$$

Taking derivative

$$\sigma_f'' - \frac{z G_m}{r_f(r_a - r_f)} u_f' = 0 \quad \dots \quad \textcircled{8}$$

$$\textcircled{1} \rightarrow \textcircled{1} \quad \sigma_f'' - \frac{zG_m}{r_f(r_a-r_f)E_f} \epsilon_f = 0 \quad \dots \textcircled{9}$$

$$\textcircled{4} \rightarrow \textcircled{9} \quad \sigma_f'' - \frac{zG_m}{r_f(r_a-r_f)E_f} \epsilon_f = 0 \quad \dots \textcircled{10}$$

$$\boxed{\sigma_f'' - \lambda^2 \sigma_f = 0} \quad \dots \textcircled{10}$$

where $\lambda^2 = \frac{z}{r_f(r_a-r_f)} \frac{G_m}{E_f}$

Geom. Mat'l

Solving, $\sigma_f = A \sinh \lambda z + B \cosh \lambda z$

B.C. @ $z=0$, $\sigma_f = B = 0$

@ $z=L$, $\sigma_f = A \sin \lambda L = \sigma_0$

Final solution, $\sigma_f = \sigma_0 \frac{\sinh \lambda z}{\sinh \lambda L}$

Useful to non-dimensionalize problem,

Define, $\eta = \frac{z}{r_f}$, $\eta_{max} = \frac{L}{r_f}$.

$\lambda z = \lambda r_f \eta = 5\eta$

then $\lambda^2 = \lambda^2 r_f^{-2} = \frac{z r_f^{-2}}{r_f(r_a-r_f)} \frac{G_m}{E_f}$

or $\lambda^2 = \frac{z (r_f/r_a)}{1 - (r_f/r_a)} \frac{G_m}{E_f}$

Define fiber volume fraction

$$V_f = \frac{\text{Volume of fibers}}{\text{Total volume}}$$



$$V_f = \frac{\pi r_f^2 L}{\pi r_a^2 L} = \frac{r_f^2}{r_a^2}$$

$$\therefore \lambda^2 = \frac{z \sqrt{V_f}}{1 - \sqrt{V_f}} \frac{G_m}{E_f}$$

$\sigma_0 = \sqrt{\frac{z \sqrt{V_f}}{1 - \sqrt{V_f}} \frac{G_m}{E_f}}$ Non-dim. parameter in terms of measurable composite properties

$$\sigma_f = \sigma_0 \frac{\sinh 5\eta}{\sinh 5\eta_{max}}$$

For shear stress in matrix, recall

$$\sigma_f' + \frac{z t_m}{r_f} = 0$$

$$t_m = -\frac{r_f}{z} \sigma_f'$$

$$t_m = -\sigma_0 \frac{5}{2} \frac{\cosh 5\eta}{\sinh 5L}$$

Also, from $\frac{v_f}{r_f} = - (r_a - r_f) \gamma_m$, can show

$$\frac{v_f}{r_f} = - \left(1 - \frac{r_f}{r_a}\right) \frac{\gamma_m}{G_m}$$

Consider magnitude of ζ (will scale problem)

$$\zeta = \sqrt{\frac{2\sqrt{V_F}}{1-V_F}} \sqrt{\frac{G_m}{E_F}}$$

Typical $G_F/E_F \dots G_m = 1.33 \text{ GPa}, E_F = 193 \text{ GPa}$

$$\sqrt{\frac{G_m}{E_F}} = 0.83$$

$$\frac{r_a}{r_f} \quad V_F \quad \sqrt{\frac{2\sqrt{V_F}}{1-V_F}} \quad \zeta$$

$$0.16 \quad 0.4 \quad 1.86 \quad 0.154$$

$$0.25 \quad 0.5 \quad 2.20 \quad 0.102$$

$$0.36 \quad 0.6 \quad 2.26 \quad 0.218$$

$$0.49 \quad 0.7 \quad \text{practical value} \quad 3.20 \quad 0.226$$

\uparrow typically $\zeta < 1$

Look at stress distribution in fiber of

Transform word to η'

$$\eta = \eta_{max} - \eta'$$

$$\frac{\sigma_f}{\sigma_0} = \frac{\sinh \zeta \eta}{\sinh \zeta \eta_{max}} = \frac{\frac{1}{2} (e^{5\eta} - e^{-5\eta})}{\frac{1}{2} (e^{5\eta_{max}} - e^{-5\eta_{max}})}$$

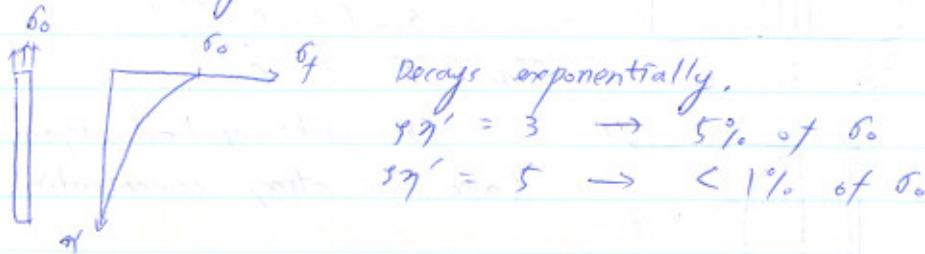
$$= \frac{e^{5\eta_{max}} - e^{-5\eta'}}{e^{5\eta_{max}} - e^{-5\eta_{max}}}$$

$$L \gg r_f \rightarrow \eta_{max} \gg 1$$

$$e^{5\eta_{max}} \gg e^{-5\eta_{max}}$$

$$\frac{\sigma_f}{\sigma_0} \approx e^{-5\eta'}$$

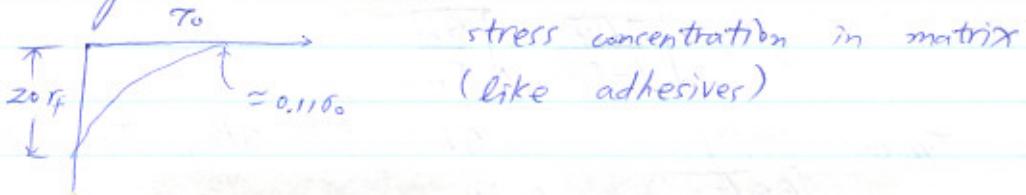
$$(Also similarly, \frac{\tau_m}{\sigma_0} \approx -\frac{\zeta}{2} e^{-5\eta'})$$



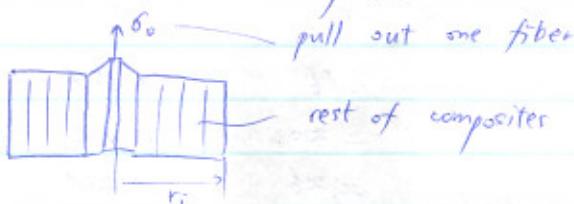
$$\eta' = \frac{5}{3} \rightarrow \frac{\epsilon'}{r_f} = \frac{5}{0.25\pi} = 23$$

\uparrow
 $V_f = 0.6$

By $\sim 20 r_f$ (10 diam.), fiber all gone
Similarly for shear stress



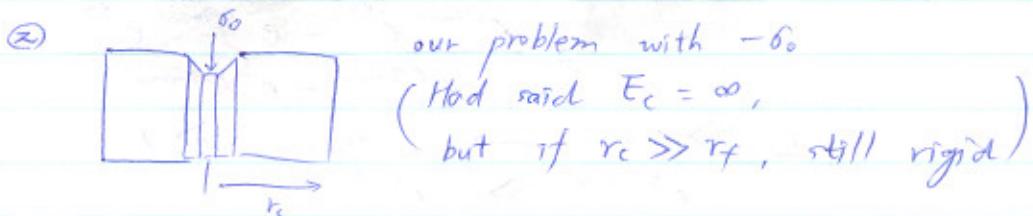
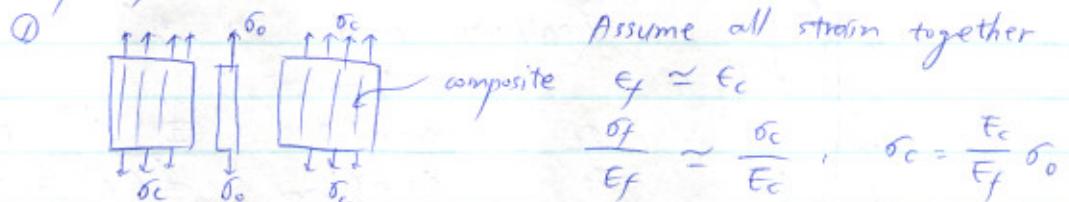
Have solved this problem



Formed load transfer quickly to matrix.

To examine fiber-break problem.

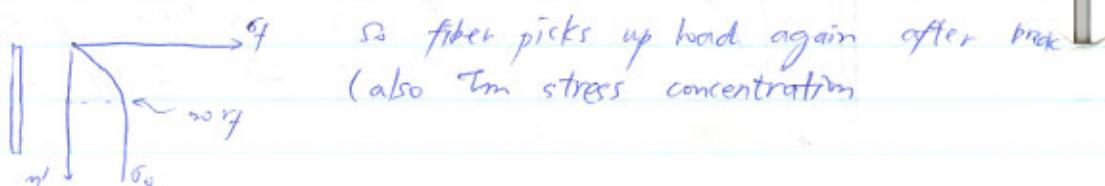
superimpose 2 solutions



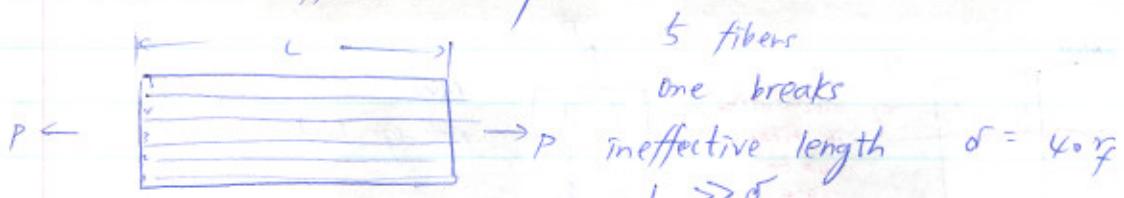
Adding ① & ② gives

$\sigma_f = \sigma_0 - \sigma_0 e^{-5\eta'}$
 $= \sigma_0 (1 - e^{-5\eta'})$

$T_m = \sigma_0 \frac{5}{2} e^{-5\eta'}$

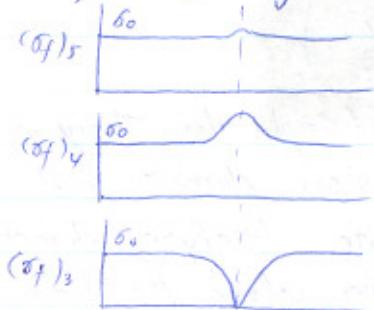


- Less than 10 fiber diameters from break, stress in fiber reaches $\sim \sigma_0$
- This region called "ineffective zone"
 - Total ineffective length for one break $\approx 20 \text{ df}$ (one zone each side)
- Real fibers a little worse
 - matrix deform plastically
 - debonding, sliding
- How this affects a composite



No. of breaks	No Matrix		With Matrix	
	# fibers	Ave. load	Ave. # fibers	Ave. load
0	5	$P/5$	5	$P/5$
1	4	$P/4$	$5 - \delta_L$	$\frac{P}{5 - \delta_L} = \frac{P}{5}$ still good

- Locally, neighboring fibers pick up load.



See Sastri & Phoenix

"shielding et Magnification of Loads
in Composites"

SAMPE Journal

Vol. 30, #4, July-Aug. 1997 p. 61

- Locally have load $> \frac{P}{5}$, but it is over small length - less chance of break
- Chance of break goes up for larger specimens (more flaws) but damage is localized.

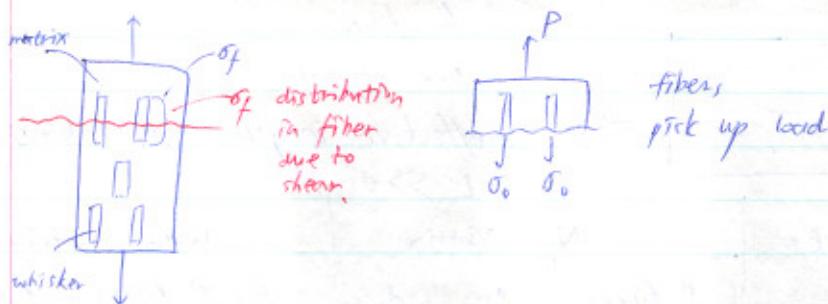


more length,
more breaks,
but localized

- So, Matrix transfers load, - only local effect when fiber breaks
 - distribution shifts and tightens
 - length scaling goes down (fewer flaws)

Fiber Kerfing	without matrix	with matrix
Ave bundle strength	350 Ksi	550 Ksi
C.V.	20~25 %	4~5 %
α	4~5	20~25

- Also have "Whisker Problem"



- Effective Properties of a Composite

See Tones, Chap. 3

Would like to predict effects of composite constituents and fiber volume fraction on macro-properties of a laminate (modulus, Poisson's ratio, strength, thermal expansion, conductivity, etc.)

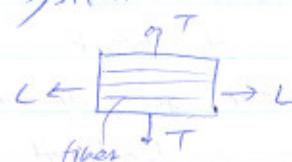
- Use mechanics of Materials Approach (simpler than Theory of Elasticity)

Basic idea - choose representative volume element and repeat to form composite. Analyze element

Importance of fiber volume fraction

Define L-T coordinate system

length transverse



Assumptions - i) Composite (Lamina) is

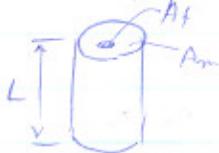
- macroscopically homogeneous
- linear elastic (but orthotropic)
- initially stress free

- b) Fibers are
- homogeneous
 - linear elastic
 - isotropic ?
 - regularly spaced, aligned
- c) Matrix is
- homogeneous
 - linear elastic ?
 - isotropic
 - void free ?

Matrix and fibers assumed perfectly bonded.

- Measuring Volume Fractions
 - i) cross-section, polish and count fibers in microscope
(area fraction \rightarrow volume fraction)
 - ii) Dissolve matrix, weigh fibers \rightarrow get mass fraction
From densities \rightarrow vol. fraction

First property to model, $\rho_c \rightarrow$ Density



$$M_f = \rho_f A_f L, \quad M_m = \rho_m A_m L$$

$$\rho_c = \frac{M_f + M_m}{\text{Vol}} = \frac{\rho_f A_f L + \rho_m A_m L}{(A_f + A_m)L}$$

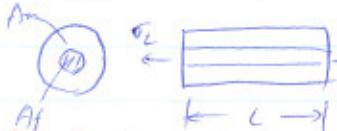
$$\boxed{\rho_c = V_f \rho_f + V_m \rho_m} \quad \text{Rule of Mixtures}$$

where $V_f = \frac{A_f}{A_f + A_m} = \frac{\pi r^2}{\pi r_a^2}, \quad V_m = 1 - V_f$

fiber vol. fraction

if no voids

- Look @ E_L - Longitudinal Modulus



$$E_L = \frac{\sigma_L}{\epsilon_L} = ?$$

Assume : Perfect bond $\Rightarrow \epsilon_L = \epsilon_{Lf} = \epsilon_{Lm}$

No lateral constraint, $\sigma_T = 0$

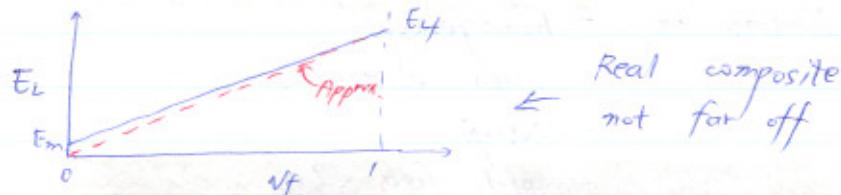
$$\sigma_L = \frac{A_f \sigma_f + A_m \sigma_m}{A_f + A_m} = V_f \sigma_f + V_m \sigma_m$$

$$\sigma_f = E_f \epsilon_f, \quad \sigma_m = E_m \frac{\epsilon_{Lm}}{\epsilon_L}$$

$$\frac{\sigma_L}{E_L} = \boxed{E_L = V_f E_{f,f} + V_m E_m}$$

R.O.M.
again

Note, if $E_{f,f} \gg E_m$ $\rightarrow E_L \approx V_f E_{f,f}$
 $\sim 270 \text{ GPa}$ $\sim 3 \text{ GPa}$ for reasonable V_f



Look @ E_T - Transverse Modulus



This looks messy.

Simplify as lumped series model.

$$\begin{array}{c} \uparrow \uparrow \uparrow \uparrow \uparrow \\ \sigma_T \\ \hline \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \downarrow \downarrow \downarrow \end{array} \quad \uparrow \uparrow \uparrow \uparrow \uparrow \sigma_T \quad \text{Assume: } \sigma_f = \sigma_m$$

Note $E_f = \frac{\sigma_f}{E_{f,f}}$, $E_m = \frac{\sigma_m}{E_m}$

$$\sigma_f = E_f \alpha_f, \quad \sigma_m = E_m \alpha_m$$

$$\begin{aligned} \text{Consider displacement, } u_T &= u_f + u_m \\ &= E_f \alpha_f + E_m \alpha_m \end{aligned}$$

$$E_T = \frac{u_T}{\alpha_m + \alpha_f} = \frac{E_f \alpha_f + E_m \alpha_m}{\alpha_m + \alpha_f}$$

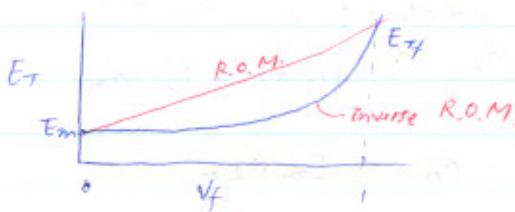
$$\text{For the same width } \frac{\alpha_f}{\alpha_m + \alpha_f} = V_f, \quad \frac{\alpha_m}{\alpha_m + \alpha_f} = V_m$$

$$E_T = E_f V_f + E_m V_m$$

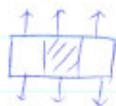
Divide by stress σ_T ($= \sigma_f = \sigma_m$)

$$\frac{E_T}{\sigma_T} = V_f \frac{\sigma_f}{\sigma_f} + V_m \frac{\sigma_m}{\sigma_m}$$

$$\boxed{\frac{1}{E_T} = \frac{V_f}{E_{f,f}} + \frac{V_m}{E_m}} \quad \text{Inverse R.O.M.}$$



But if we picked parallel model



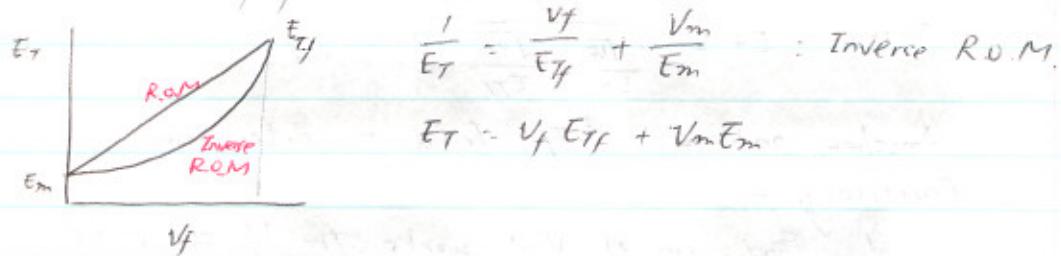
$$\epsilon_T = \epsilon_f = \epsilon_m$$

Get R.O.M.

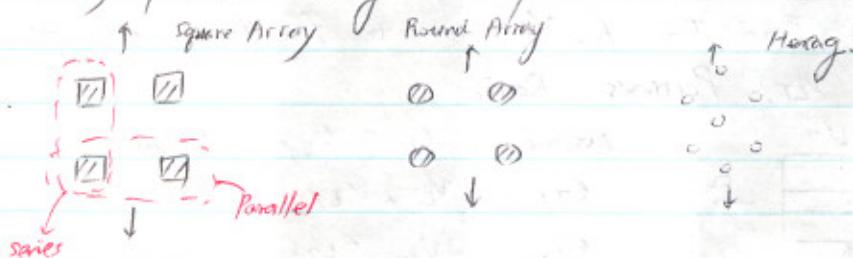
$$E_T' = E_{Tf} V_f + E_m V_m$$

These 2 cases represent bounds on E_T

For transverse properties, wide bounds from R.O.M.



Many possible Theory - depends on model used



More complex to analyze

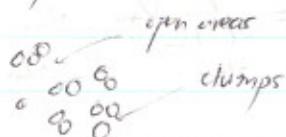
Elasticity Theorem

F.E.M.

Rayleigh-Ritz

But still might be approx.

Real composite - statistical distribution of fibers

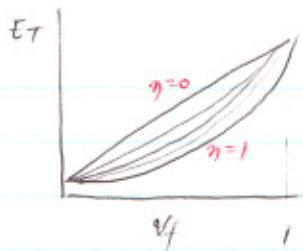


Mixed Models (empirical)

Simplest idea

$$E_T = E_T \underset{R.O.M.}{\text{(inverse)}} * \eta + E_T \text{ (R.O.M.)} * (1-\eta)$$

Fit η to data



Family of curves,
try fit

Much work along these lines

$$\text{Hahn} \quad E_T = \frac{1 - V^*}{\frac{1}{E_f} + \frac{V^*}{E_m}}, \quad V^* = \eta' \frac{V_m}{V_f}$$

$$\text{Chancus} \quad E_T = \frac{1}{\frac{1 - V_f}{E_m} + \frac{\sqrt{V_f}}{E_f}}, \quad \text{etc.}$$

Another problem, E_{Tf} hard to determine

Practically -

1. Find $\alpha_m \eta$ that works for $V_f \approx 0.60$
2. Get E_{Tf} as best problem
3. Find E_T for V_f not too far from 0.60 $\rightarrow 0.55 \sim 0.70$

• Look @ ν_{LT} , Poisson's Ratio

$$\begin{array}{c} \downarrow E_T \\ \left[\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right] \end{array} \quad \begin{array}{l} \text{Assume } \epsilon_L = \epsilon_{Lf} = \epsilon_{Lm} \\ \epsilon_{Tf} = -\nu_{LTf} \epsilon_{Lf} \\ \epsilon_{Tm} = -\nu_{m} \epsilon_{Lm} \end{array}$$

$$\begin{array}{l} u = u_m + u_f \\ = \epsilon_{Lm} \alpha_m + \epsilon_{Lf} \alpha_f \\ -u = \alpha_m \nu_m \epsilon_{Lm} + \alpha_f \nu_{LTf} \epsilon_{Lf} \end{array}$$

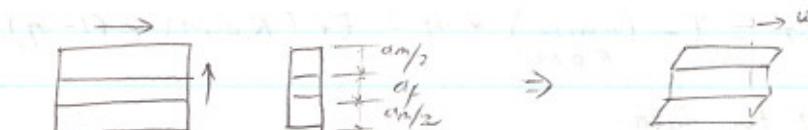
Divides by $\alpha_m + \alpha_f$,

$$\begin{aligned} -\frac{u}{\alpha_m + \alpha_f} &= \left(\frac{\alpha_m}{\alpha_m + \alpha_f} \nu_m + \frac{\alpha_f}{\alpha_m + \alpha_f} \nu_{LTf} \right) \epsilon_L \\ &= E_T = (\nu_m \nu_m + \nu_f \nu_{LTf}) \epsilon_L \\ -\frac{E_T}{\epsilon_L} &= \nu_{LT} = \nu_m \nu_m + \nu_f \nu_{LTf} : \text{R.O.M.} \end{aligned}$$

Model seems to work.

Anyway ν_m & ν_{LTf} both ~ 0.3 , so anything works.

• Look at Shear Modulus, G_{LT}



$$T_m = T_f = T$$

$$\gamma_m = T_m/G_m$$

$$u = \gamma_m a_m + \gamma_f a_f$$

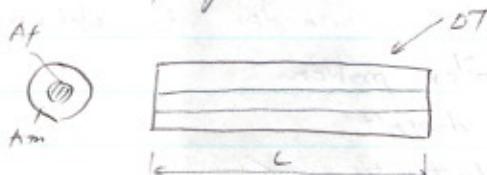
$$\frac{u}{a_m + a_f} = \gamma_m \frac{a_m}{a_m + a_f} + \gamma_f \frac{a_f}{a_m + a_f}$$

$$\frac{\gamma}{\tau} = \gamma_m V_m + \gamma_f V_f$$

$$\frac{\gamma}{\tau} = \frac{\gamma_m}{T_m} V_m + \frac{\gamma_f}{T_f} V_f$$

$$\frac{1}{G} = \frac{V_m}{G_m} + \frac{V_f}{G_{Lf}} \quad : \text{Inverse R.O.M.}$$

- Could also do a parallel model, get R.O.M. rule
 - Use some Mixed model or some experimental fit of γ
 - Hard to measure G_{Lf} Coef. Thermal Expansion
- Another Property, Look @ Thermal Expansion, CTE \propto



If matrix and fiber were independent,

$$\begin{array}{c} \xrightarrow{\Delta T} \\ m \end{array} \quad \epsilon_m = \alpha_m \Delta T$$

$$\begin{array}{c} \xrightarrow{\Delta T} \\ f \end{array} \quad \epsilon_f = \alpha_f \Delta T$$

$\left. \begin{array}{c} \epsilon_m = \alpha_m \Delta T \\ \epsilon_f = \alpha_f \Delta T \end{array} \right\} \text{not same}$

Assume bonded,

$$\begin{array}{c} \xrightarrow{\Delta T} \\ m \end{array} \quad \delta_m$$

$$\begin{array}{c} \xrightarrow{\Delta T} \\ f \end{array} \quad \delta_f$$

$$\delta_m = E_m (\epsilon_m - \alpha_m \Delta T)$$

$$\delta_f = E_f (\epsilon_f - \alpha_f \Delta T)$$

No total load over end, so

$$\delta_m A_m + \delta_f A_f = 0 \quad (3)$$

$$\text{Note } \epsilon_m = \epsilon_f = \epsilon_c = \alpha_c \Delta T \quad (4)$$

composite

Placing (1), (2), (4) into (3) gives

$$E_m (\epsilon_c - \alpha_m \Delta T) A_m + E_f (\epsilon_c - \alpha_f \Delta T) A_f = 0$$

Multiply by L and divide by volume

and recall $V_m = \frac{A_m L}{\text{vol.}}$, etc. $V_f = \text{etc.}$

$$E_m (\epsilon_c - \alpha_m \Delta T) V_m + E_f (\epsilon_c - \alpha_f \Delta T) V_f = 0$$

This yields

$$\epsilon_c = \frac{\alpha_m E_m V_m + \alpha_f E_f V_f}{E_m V_m + E_f V_f} \Delta T = \alpha_{LC}$$

Modulus weighted R.O.M.

Deal with $\frac{E_f V_f}{E_m V_m + E_f V_f}$ instead of $\frac{V_f}{V_m + V_f}$

Note: composite stress-free at cure temperature when cools down ($\Delta T = -$), residual stresses σ_m, σ_f form.

- Transverse C.T.E., α_{TC} , harder to obtain.
Moisture causes a similar problem.
Matrix swells, fiber doesn't.
- Look @ Thermal Conductivity, K_L



$$q_x = -K_f \frac{\partial T}{\partial x} \quad q_m = -K_m \frac{\partial T}{\partial x}$$

$$\text{Assume } \left(\frac{\partial T}{\partial x}\right)_f = \left(\frac{\partial T}{\partial x}\right)_m = \frac{\partial T}{\partial x} \quad (\text{long geometry})$$

$$q_c A_T = q_f A_f + q_m A_m$$

$$q_c = -(K_f V_f + K_m V_m) \frac{\partial T}{\partial x}$$

Note $K_f \gg K_m$

$$\text{so } K_L \approx K_f V_f \quad \text{like stiffness}$$

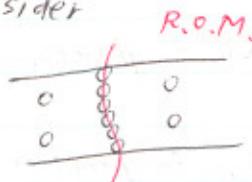
Also K_f very high for fibers
specific conductivity K/ρ can be greater than Al or Cu

- Transverse thermal conductivity - K_T

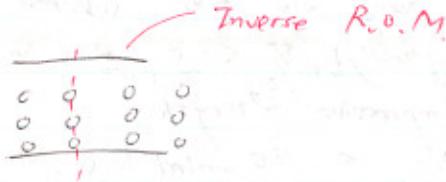
$$K_{Tf} \ggg K_m, K_{Lf} > K_{Mf}$$

good conductor

consider



R.O.M.



Inverse R.O.M.

V_f 's same, but K_T 's much different

K_T very dependent on micro structure
can't make good simplified model

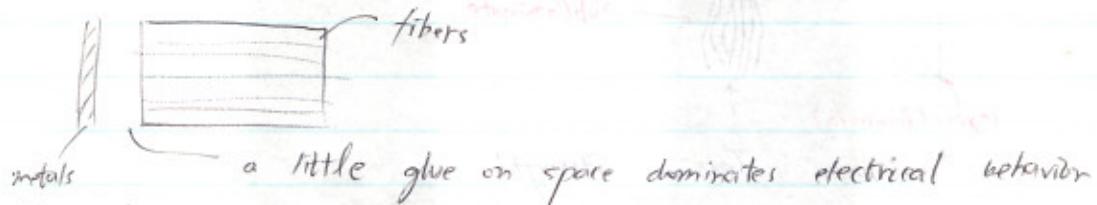
- Electrical conductivity - C_e somewhat similar

$$C_f \ggg C_m$$

$$\text{so } C_{eC} \approx R.O.M. \approx C_f V_f$$

$C_e \rightarrow$ difficult to establish (paths & microstructure)

Also, electrical behavior is dominate by contact.



- Strength

Difficult to predict

Look @ Tension



Tempting to use R.O.M

$$X_T = X_{fT} V_f + X_{mT} V_m$$

Let's try this

$$X_{fT} = 1990 \text{ MPa} \quad (\text{length?})$$

$$\text{AS1/2576 } X_{OT} = 1660 \text{ MPa} \quad (\text{typical Gr/Ep}) \quad V_f = 0.60$$

$$X_{mT} = 70 \text{ MPa}$$

$$X_{CT} = 1990 (.6) + 70 (.4) = 1290 \text{ MPa} \quad \text{No!}$$

R.O.M. should have been Upper Bound

Effective fiber strength is increased by load sharing through matrix

$$X_{fT}^{\text{eff}} \approx \frac{X_T}{V_f} = \frac{1660}{0.6} = 2770 \text{ MPa}$$

For very low fiber volume fractions.

actually, $V_f < 1.0$ is reasonable in practice.

- Compressive Strength



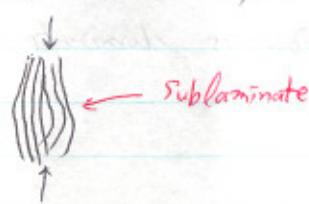
Dominated by fiber buckling

Controlled by fiber & matrix stiffness,
fiber geometry

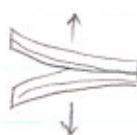
For most composites, material behavior gives

$$X_c = X_T$$

But this is not true for Kevlar, Pitch Fiber Gr/Epoxy, and
in structures, careful of delamination, buckling



- Transverse Tension strength



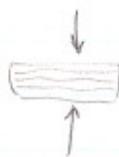
Matrix dominated

γ_t : very low

crack runs along fibers

impede formation of a plastic zone

- Transverse Compression



Matrix dominated

but less of problem

$$Y_c \approx (4 \text{ to } 7) \times Y_T$$

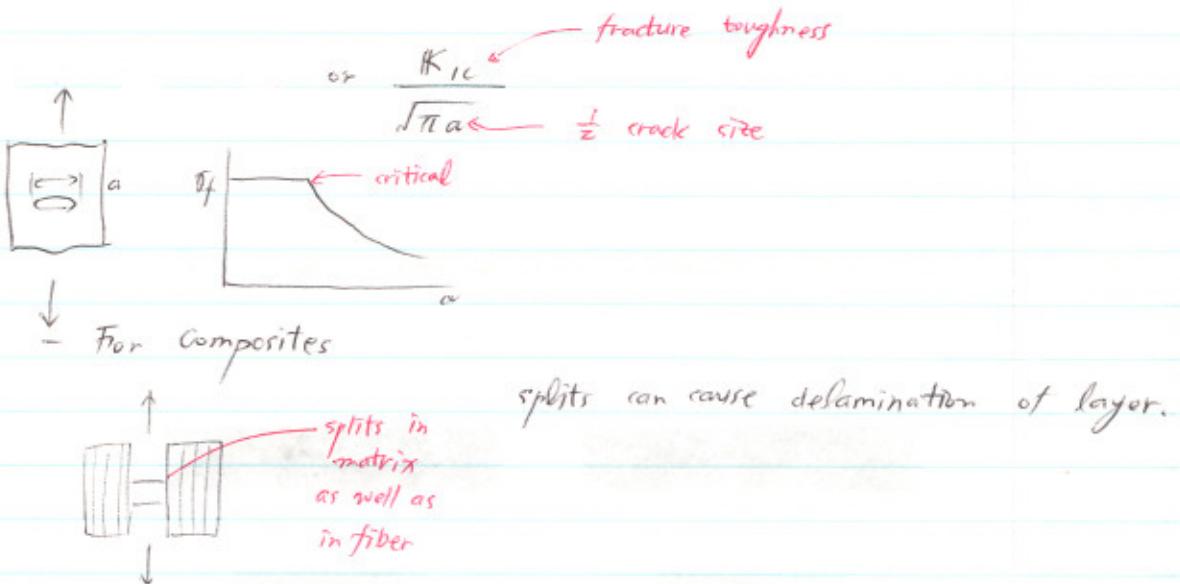
- Shear → matrix dominated

$$Y_c > S > Y_T \text{ typical}$$

- Other strength Related Properties

- For fracture

$$\text{For metals } \sigma_f = \sigma_y$$



- Fatigue

- For metals \rightarrow crack growth under cyclic loads a major problem
- For carbon fibers are very good in fatigue
0° dominated structures fatigue resistant (in tension)
Careful of delaminations in compression and off axis plies

- Impact

For Gr/Ep \rightarrow generally low strain to failure,
impact a major concern



- Environmental Resistance

Moisture intake \rightarrow changes matrix properties
Temperature \rightarrow can change matrix properties
People can concern with Hot, Wet, Post Impact,
Compression test.

- Talked about Micromechanics

(How composites work, trends, and usefulness ...)
Actually will use Experimental Data in design of
Structures from Composites

Will now talk about Macromechanics
using composites to design structure