

Digital Logic Design

4190.201.001

2010 Spring Semester

4. Working with combinational logic

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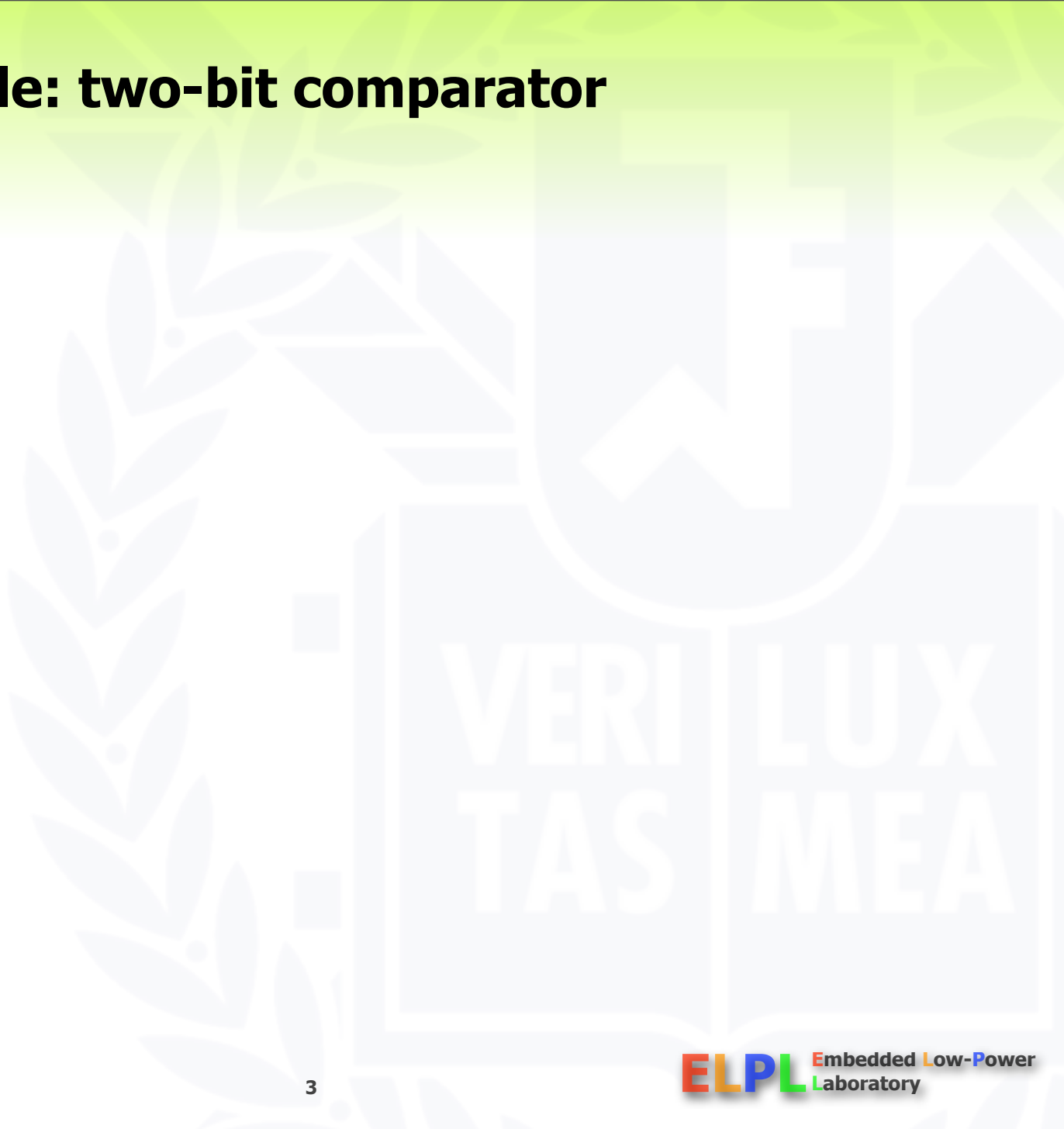


What to cover

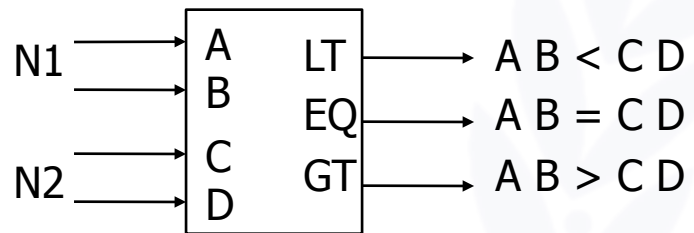
- Simplification
 - Two-level simplification
 - Exploiting don't cares
 - Algorithm for simplification
- Logic realization
 - Two-level logic and canonical forms realized with NANDs and NORs
 - Multi-level logic, converting between ANDs and ORs
- Time behavior
- Hardware description languages



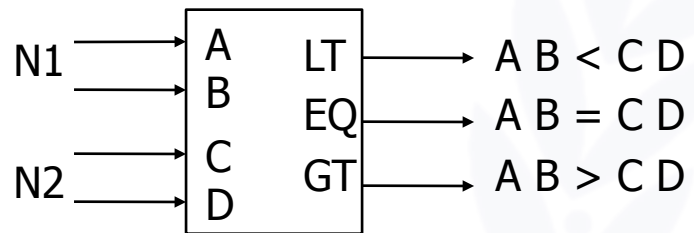
Design example: two-bit comparator



Design example: two-bit comparator



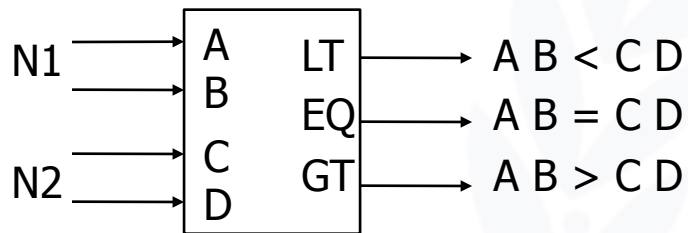
Design example: two-bit comparator



Block diagram



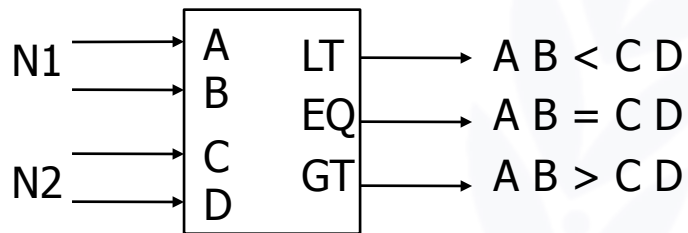
Design example: two-bit comparator



Block diagram

A	B	C	D	LT	EQ	GT
0	0	0	0	0	1	0
		0	1	1	0	0
		1	0	1	0	0
		1	1	1	0	0
0	1	0	0	0	0	1
		0	1	0	1	0
		1	0	1	0	0
		1	1	1	0	0
1	0	0	0	0	0	1
		0	1	0	0	1
		1	0	0	1	0
		1	1	1	0	0
1	1	0	0	0	0	1
		0	1	0	0	1
		1	0	0	0	1
		1	1	0	1	0

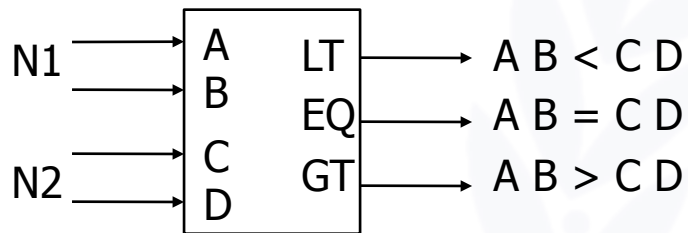
Design example: two-bit comparator



Block diagram
and
truth table

A	B	C	D	LT	EQ	GT
0	0	0	0	0	1	0
		0	1	1	0	0
		1	0	1	0	0
		1	1	1	0	0
0	1	0	0	0	0	1
		0	1	0	1	0
		1	0	1	0	0
		1	1	1	0	0
1	0	0	0	0	0	1
		0	1	0	0	1
		1	0	0	1	0
		1	1	1	0	0
1	1	0	0	0	0	1
		0	1	0	0	1
		1	0	0	0	1
		1	1	0	1	0

Design example: two-bit comparator



Block diagram
and
truth table

A	B	C	D	LT	EQ	GT
0	0	0	0	0	1	0
		0	1	1	0	0
		1	0	1	0	0
		1	1	1	0	0
0	1	0	0	0	0	1
		0	1	0	1	0
		1	0	1	0	0
		1	1	1	0	0
1	0	0	0	0	0	1
		0	1	0	0	1
		1	0	0	1	0
		1	1	1	0	0
1	1	0	0	0	0	1
		0	1	0	0	1
		1	0	0	0	1
		1	1	0	1	0

We'll need a 4-variable Karnaugh map
for each of the 3 output functions



Design example: two-bit comparator

		A		
	0	0	0	0
	1	0	0	0
C	1	1	0	1
	1	1	0	0
		B		

K-map for LT

		A		
	1	0	0	0
	0	1	0	0
	0	0	1	0
C	0	0	0	1
			B	

K-map for EQ

		A			
C	0	1	1	1	D
	0	0	1	1	
	0	0	0	0	
	0	0	1	0	
		B			

K-map for GT

LT =

EQ =

GT =



Design example: two-bit comparator

		A		
	0	0	0	0
	1	0	0	0
C	1	1	0	1
	1	1	0	0
	B			D

K-map for LT

		A		
	1	0	0	0
	0	1	0	0
C	0	0	1	0
	0	0	0	1
	B			D

K-map for EQ

		A		
	0	1	1	1
	0	0	1	1
C	0	0	0	0
	0	0	1	0
	B			D

K-map for GT

$$LT = A' B' D + A' C + B' C D$$

$$EQ =$$

$$GT =$$



Design example: two-bit comparator

		A		
	0	0	0	0
	1	0	0	0
C	1	1	0	1
	1	1	0	0
	B			D

K-map for LT

		A		
	1	0	0	0
	0	1	0	0
C	0	0	1	0
	0	0	0	1
	B			D

K-map for EQ

		A		
	0	1	1	1
	0	0	1	1
C	0	0	0	0
	0	0	1	0
	B			D

K-map for GT

$$LT = A' B' D + A' C + B' C D$$

$$EQ = A' B' C' D' + A' B C' D + A B C D + A B' C D'$$

$$GT =$$



Design example: two-bit comparator

		A		
	0	0	0	0
	1	0	0	0
C	1	1	0	1
	1	1	0	0
	B			D

K-map for LT

		A		
	1	0	0	0
	0	1	0	0
C	0	0	1	0
	0	0	0	1
	B			D

K-map for EQ

		A		
	0	1	1	1
	0	0	1	1
C	0	0	0	0
	0	0	1	0
	B			D

K-map for GT

$$LT = A' B' D + A' C + B' C D$$

$$EQ = A' B' C' D' + A' B C' D + A B C D + A B' C D' = (A \text{ xnor } C) \cdot (B \text{ xnor } D)$$

$$GT =$$



Design example: two-bit comparator

		A		
	0	0	0	0
	1	0	0	0
C	1	1	0	1
	1	1	0	0
	B			D

K-map for LT

		A		
	1	0	0	0
	0	1	0	0
C	0	0	1	0
	0	0	0	1
	B			D

K-map for EQ

		A		
	0	1	1	1
	0	0	1	1
C	0	0	0	0
	0	0	1	0
	B			D

K-map for GT

$$LT = A' B' D + A' C + B' C D$$

$$EQ = A' B' C' D' + A' B C' D + A B C D + A B' C D' = (A \text{ xnor } C) \cdot (B \text{ xnor } D)$$

$$GT = B C' D' + A C' + A B D'$$



Design example: two-bit comparator

		A		
	0	0	0	0
	1	0	0	0
C	1	1	0	1
	1	1	0	0
	B			D

K-map for LT

		A		
	1	0	0	0
	0	1	0	0
C	0	0	1	0
	0	0	0	1
	B			D

K-map for EQ

		A		
	0	1	1	1
	0	0	1	1
C	0	0	0	0
	0	0	1	0
	B			D

K-map for GT

$$LT = A' B' D + A' C + B' C D$$

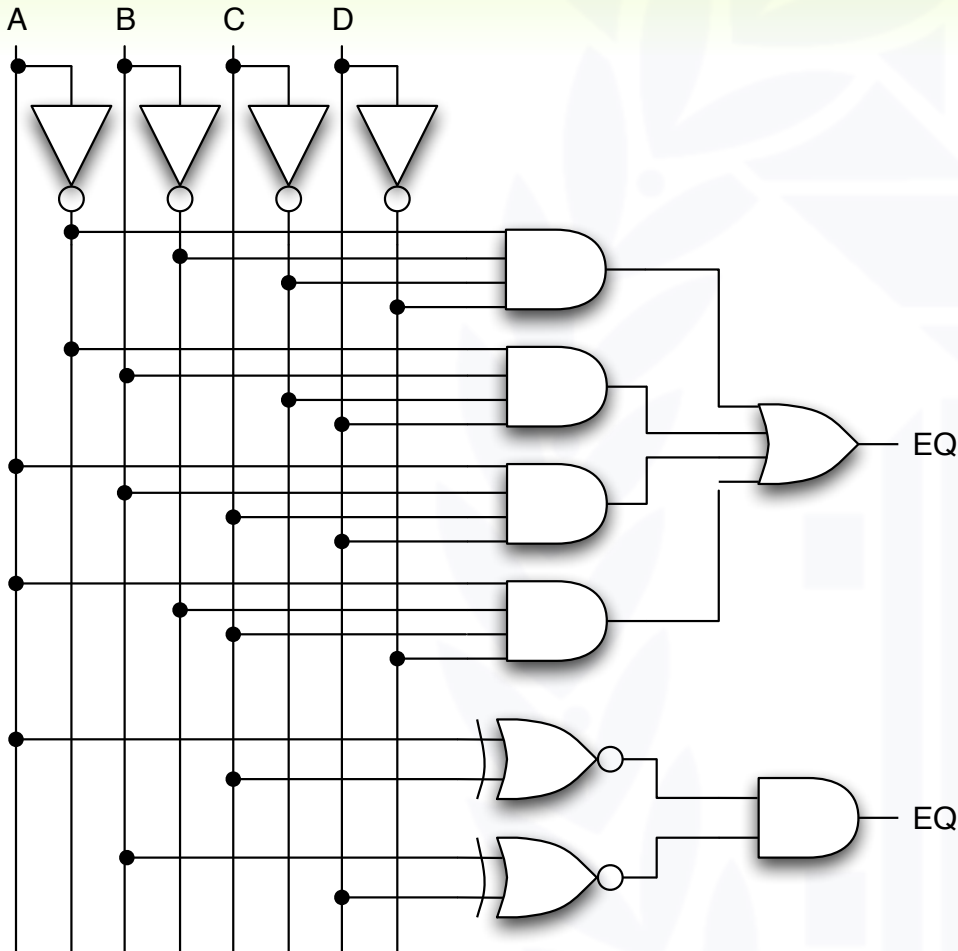
$$EQ = A' B' C' D' + A' B C' D + A B C D + A B' C D' = (A \text{ xnor } C) \cdot (B \text{ xnor } D)$$

$$GT = B C' D' + A C' + A B D'$$

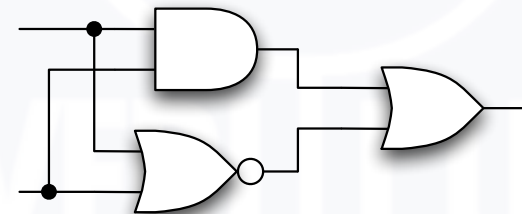
LT and GT are similar (flip A/C and B/D)



Design example: two-bit comparator

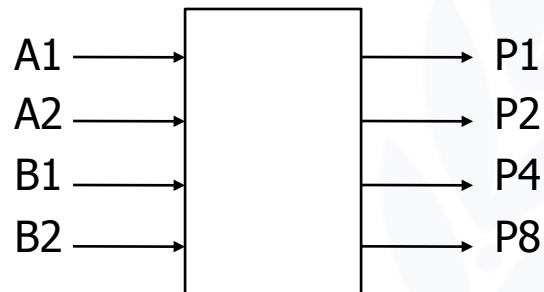


Two alternative implementations of EQ with and without XOR



XNOR is implemented with at least 3 simple gates

Design example: 2x2-bit multiplier

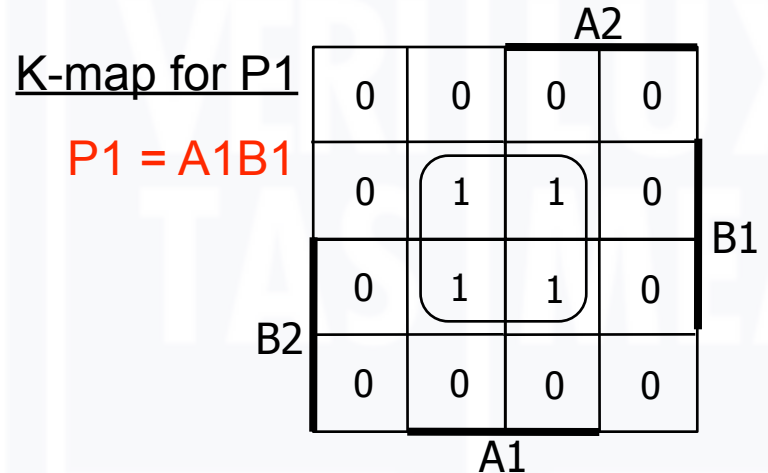
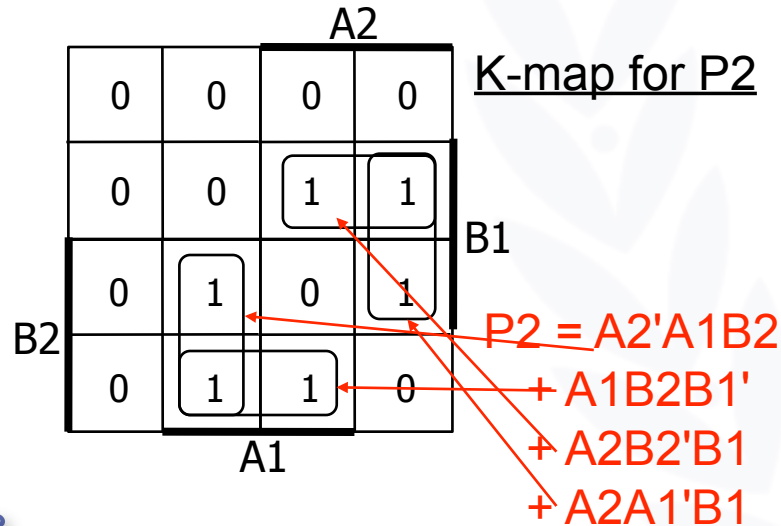
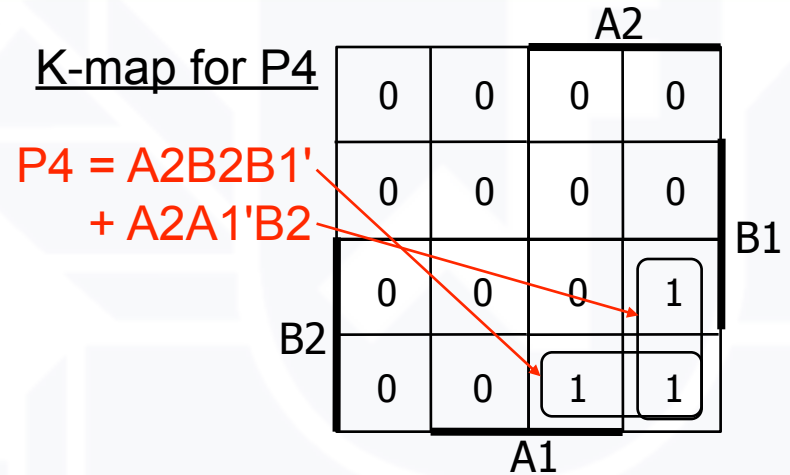
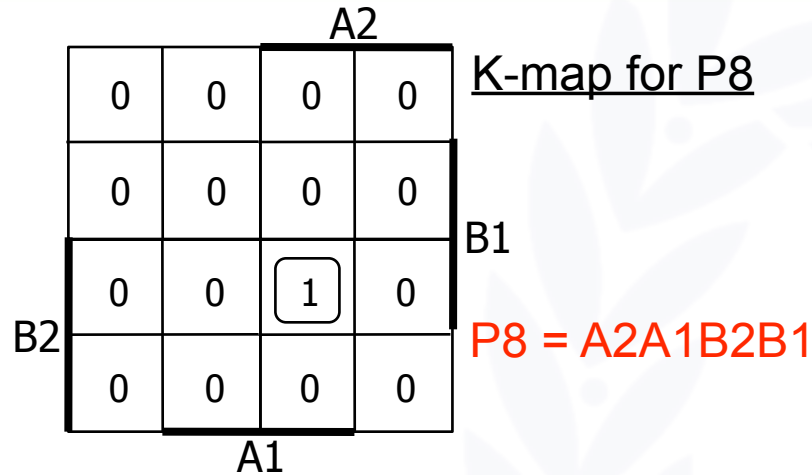


Block diagram
and
truth table

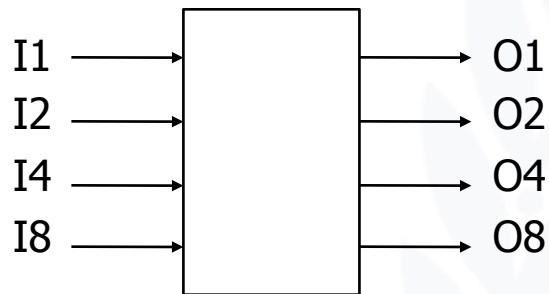
A2	A1	B2	B1	P8	P4	P2	P1
0	0	0	0	0	0	0	0
		0	1	0	0	0	0
		1	0	0	0	0	0
		1	1	0	0	0	0
0	1	0	0	0	0	0	0
		0	1	0	0	0	1
		1	0	0	0	1	0
		1	1	0	0	1	1
1	0	0	0	0	0	0	0
		0	1	0	0	1	0
		1	0	0	1	0	0
		1	1	0	1	1	0
1	1	0	0	0	0	0	0
		0	1	0	0	1	1
		1	0	0	1	1	0
		1	1	1	0	0	1

4-variable K-map
for each of the 4
output functions

Design example: 2x2-bit multiplier



Design example: BCD increment by 1



Block diagram
and
truth table

I8	I4	I2	I1	O8	O4	O2	O1
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
1	0	1	0	X	X	X	X
1	0	1	1	X	X	X	X
1	1	0	0	X	X	X	X
1	1	0	1	X	X	X	X
1	1	1	0	X	X	X	X
1	1	1	1	X	X	X	X

4-variable K-map for each of
the 4 output functions

Design example: BCD increment by 1

				I8	
I2	0	0	X	1	I1
	0	0	X	0	
	0	1	X	X	
	0	0	X	X	
				I4	

O8

$$O8 = I4 I2 I1 + I8 I1'$$

$$O4 = I4 I2' + I4 I1' + I4' I2 I1$$

$$O2 = I8' I2' I1 + I2 I1'$$

$$O1 = I1'$$

				I8	
I2	0	0	X	0	I1
	1	1	X	0	
	0	0	X	X	
	1	1	X	X	
				I4	

O2

				I8	
I2	0	1	X	0	I1
	0	1	X	0	
	1	0	X	X	
	0	1	X	X	
				I4	

O4

				I8	
I2	1	1	X	1	I1
	0	0	X	0	
	0	0	X	X	
	1	1	X	X	
				I4	

O1



Definition of terms for two-level simplification

- Implicant
 - Single element of ON-set or DC-set or any group of these elements that can be combined to form a subcube
- Prime implicant
 - Implicant that can't be combined with another to form a larger subcube
- Essential prime implicant
 - Prime implicant is essential if it alone covers an element of ON-set
 - Will participate in ALL possible covers of the ON-set
 - DC-set used to form prime implicants but not to make implicant essential
- Redundant prime implicant
- Objective:
 - Grow implicant into prime implicants (minimize literals per term)
 - Cover the ON-set with as few prime implicants as possible (minimize number of product terms)



Examples to illustrate terms

		A		
	0	X	1	0
	1	1	1	0
C	1	0	1	1
	0	0	1	1
	B			
				D

Examples to illustrate terms

	A			
	0	X	1	0
	1	1	1	0
C	1	0	1	1
	0	0	1	1
	B			

6 prime implicants:

$A'B'D$, BC' , AC , $A'C'D$, AB , $B'CD$



Examples to illustrate terms

	A			
	0	X	1	0
	1	1	1	0
C	1	0	1	1
	0	0	1	1
	B			

6 prime implicants:

$A'B'D$, BC' , AC , $A'C'D$, AB , $B'CD$

essential



Examples to illustrate terms

		A			
	0	X	1	0	
	1	1	1	0	
C	1	0	1	1	D
	0	0	1	1	
		B			

6 prime implicants:

$A'B'D$, BC' , AC , $A'C'D$, AB , $B'CD$

essential

Minimum cover: $AC + BC' + A'B'D$

Examples to illustrate terms

	A			
	0	X	1	0
	1	1	1	0
C	1	0	1	1
	0	0	1	1
	B			

6 prime implicants:

$A'B'D$, BC' , AC , $A'C'D$, AB , $B'CD$

essential

Minimum cover: $AC + BC' + A'B'D$

	A			
	0	0	1	0
	1	1	1	0
C	0	1	1	1
	0	1	0	0
	B			



Examples to illustrate terms

		A		
	0	X	1	0
	1	1	1	0
	1	0	1	1
C	0	0	1	1
		B		

6 prime implicants:

$A'B'D$, BC' , AC , $A'C'D$, AB , $B'CD$

essential

Minimum cover: $AC + BC' + A'B'D$

				A
	0	0	1	0
	1	1	1	0
	0	1	1	1
C	0	1	0	0
				B



Examples to illustrate terms

				A
	0	X	1	0
	1	1	1	0
	1	0	1	1
C	0	0	1	1
				B

6 prime implicants:

$A'B'D$, BC' , AC , $A'C'D$, AB , $B'CD$

essential

Minimum cover: $AC + BC' + A'B'D$

5 prime implicants:

BD , ABC' , ACD , $A'BC$, $A'C'D$

				A
	0	0	1	0
	1	1	1	0
	0	1	1	1
C	0	1	0	0
				B
				D



Examples to illustrate terms

				A
	0	X	1	0
	1	1	1	0
	1	0	1	1
C	0	0	1	1
				B

6 prime implicants:

$A'B'D$, BC' , AC , $A'C'D$, AB , $B'CD$

essential

Minimum cover: $AC + BC' + A'B'D$

5 prime implicants:

BD , ABC' , ACD , $A'BC$, $A'C'D$

essential

				A
	0	0	1	0
	1	1	1	0
	0	1	1	1
C	0	1	0	0
				B



Examples to illustrate terms

		A		
	0	X	1	0
	1	1	1	0
C	1	0	1	1
	0	0	1	1
	B			

6 prime implicants:

$A'B'D$, BC' , AC , $A'C'D$, AB , $B'CD$

essential

Minimum cover: $AC + BC' + A'B'D$

5 prime implicants:

BD , ABC' , ACD , $A'BC$, $A'C'D$

essential

Minimum cover: 4 essential implicants

				A
	0	0	1	0
	1	1	1	0
	0	1	1	1
C	0	1	0	0
				B



Algorithm for two-level simplification

- Algorithm: Minimum sum-of-products expression from a Karnaugh map
 - Step 1: Choose an element of the ON-set
 - Step 2: Find "maximal" groupings of 1s and Xs adjacent to that element
 - Consider top/bottom row, left/right column, and corner adjacencies
 - This forms prime implicants (number of elements always a power of 2)
 - Repeat Steps 1 and 2 to find all prime implicants
 - Step 3: Revisit the 1s in the K-map
 - If covered by single prime implicant, it is essential, and participates in final cover
 - 1s covered by essential prime implicant do not need to be revisited
 - Step 4: If there remain 1s not covered by essential prime implicants
 - Select the smallest number of prime implicants that cover the remaining 1s



Algorithm for two-level simplification (example)



Algorithm for two-level simplification (example)

		A		
	X	1	0	1
	0	1	1	1
	0	X	X	0
C	0	1	0	1
		B		
				D



Algorithm for two-level simplification (example)

A			
X	1	0	1
0	1	1	1
0	X	X	0
0	1	0	1
B			
C			
D			

A			
X	1	0	1
0	1	1	1
0	X	X	0
0	1	0	1
B			
C			
D			

2 primes around $A'BC'D'$



Algorithm for two-level simplification (example)

A				D
X	1	0	1	
0	1	1	1	
0	X	X	0	
C	0	1	0	B
	0	1	1	

A				D
X	1	0	1	
0	1	1	1	
0	X	X	0	
C	0	1	0	B
	0	1	1	

2 primes around $A'BC'D'$



Algorithm for two-level simplification (example)

A			
X	1	0	1
0	1	1	1
0	X	X	0
0	1	0	1
B			
C			
D			

A			
X	1	0	1
0	1	1	1
0	X	X	0
0	1	0	1
B			
C			
D			

2 primes around $A'BC'D'$

A			
X	1	0	1
0	1	1	1
0	X	X	0
0	1	0	1
B			
C			
D			

2 primes around $ABC'D$

Algorithm for two-level simplification (example)

				A	
C	X	1	0	1	
	0	1	1	1	
	0	X	X	0	D
	0	1	0	1	
				B	

				A	
C	X	1	0	1	
	0	1	1	1	
	0	X	X	0	D
	0	1	0	1	
				B	

2 primes around $A'BC'D'$

				A	
C	X	1	0	1	
	0	1	1	1	
	0	X	X	0	D
	0	1	0	1	
				B	

2 primes around $ABC'D$

Algorithm for two-level simplification (example)

		A				
		X	1	0	1	
		0	1	1	1	
		0	X	X	0	D
C		0	1	0	1	
		B				

		A				
		X	1	0	1	
		0	1	1	1	
		0	X	X	0	D
C		0	1	0	1	
		B				

2 primes around $A'BC'D'$

		A				
		X	1	0	1	
		0	1	1	1	
		0	X	X	0	D
C		0	1	0	1	
		B				

2 primes around $ABC'D$

		A				
		X	1	0	1	
		0	1	1	1	
		0	X	X	0	D
C		0	1	0	1	
		B				

3 primes around $AB'C'D'$



Algorithm for two-level simplification (example)

		A				
		X	1	0	1	
		0	1	1	1	
		0	X	X	0	D
C		0	1	0	1	
		B				

		A				
		X	1	0	1	
		0	1	1	1	
		0	X	X	0	D
C		0	1	0	1	
		B				

2 primes around $A'BC'D'$

		A				
		X	1	0	1	
		0	1	1	1	
		0	X	X	0	D
C		0	1	0	1	
		B				

2 primes around $ABC'D$

		A				
		X	1	0	1	
		0	1	1	1	
		0	X	X	0	D
C		0	1	0	1	
		B				

3 primes around $AB'C'D'$



Algorithm for two-level simplification (example)

	A				
	X	1	0	1	
	0	1	1	1	
C	0	X	X	0	D
	0	1	0	1	
	B				

	A				
	X	1	0	1	
	0	1	1	1	
C	0	X	X	0	D
	0	1	0	1	
	B				

2 primes around $A'BC'D'$

	A				
	X	1	0	1	
	0	1	1	1	
C	0	X	X	0	D
	0	1	0	1	
	B				

2 primes around $ABC'D$

	A				
	X	1	0	1	
	0	1	1	1	
C	0	X	X	0	D
	0	1	0	1	
	B				

3 primes around $AB'C'D'$

	A				
	X	1	0	1	
	0	1	1	1	
C	0	X	X	0	D
	0	1	0	1	
	B				

2 essential primes



Algorithm for two-level simplification (example)

	A				
	X	1	0	1	
	0	1	1	1	
	0	X	X	0	D
C	0	1	0	1	
	B				

	A				
	X	1	0	1	
	0	1	1	1	
	0	X	X	0	D
C	0	1	0	1	
	B				

2 primes around $A'BC'D'$

	A				
	X	1	0	1	
	0	1	1	1	
	0	X	X	0	D
C	0	1	0	1	
	B				

2 primes around $ABC'D$

	A				
	X	1	0	1	
	0	1	1	1	
	0	X	X	0	D
C	0	1	0	1	
	B				

3 primes around $AB'C'D'$

	A				
	X	1	0	1	
	0	1	1	1	
	0	X	X	0	D
C	0	1	0	1	
	B				

2 essential primes

	A				
	X	1	0	1	
	0	1	1	1	
	0	X	X	0	D
C	0	1	0	1	
	B				

Minimum cover (3 primes)



Algorithm for two-level simplification (example)

	A				
	X	1	0	1	
	0	1	1	1	
	0	X	X	0	D
C	0	1	0	1	
	B				

	A				
	X	1	0	1	
	0	1	1	1	
	0	X	X	0	D
C	0	1	0	1	
	B				

2 primes around $A'BC'D'$

	A				
	X	1	0	1	
	0	1	1	1	
	0	X	X	0	D
C	0	1	0	1	
	B				

2 primes around $ABC'D$

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	0	1	1	1	
	0	X	X	0	D
C	0	1	0	1	
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3 primes around $AB'C'D'$

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	0	1	1	1	
	0	X	X	0	D
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	B				

2 essential primes

	A				
	X	1	0	1	
	0	1	1	1	
	0	X	X	0	D
C	0	1	0	1	
	B				

Minimum cover (3 primes)



Activity

- List all prime implicants for the following K-map:

A				D
X	0	X	0	
0	1	X	1	
0	X	X	0	
C	X	1	1	B
	1	1	1	

- Which are essential prime implicants?
- What is the minimum cover?



Activity

- List all prime implicants for the following K-map:

				A	
		X	0	X	0
		0	1	X	1
C	0	X	X	0	
	X	1	1	1	
		B			

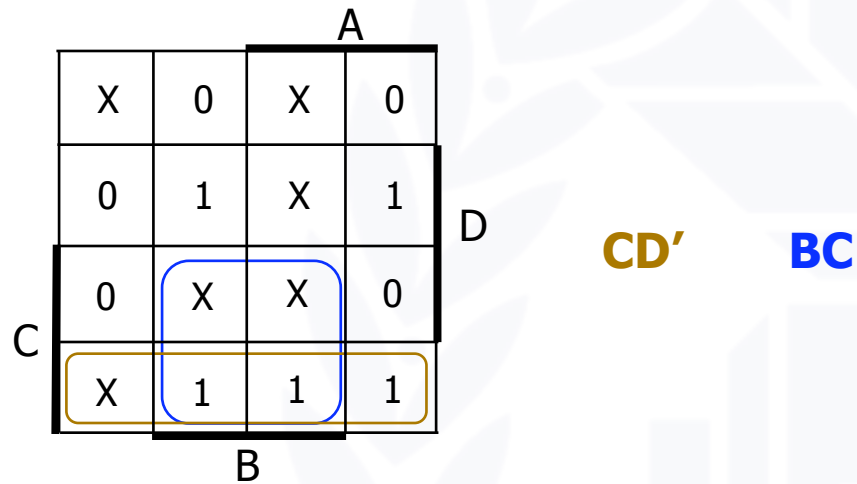
CD'

- Which are essential prime implicants?
- What is the minimum cover?



Activity

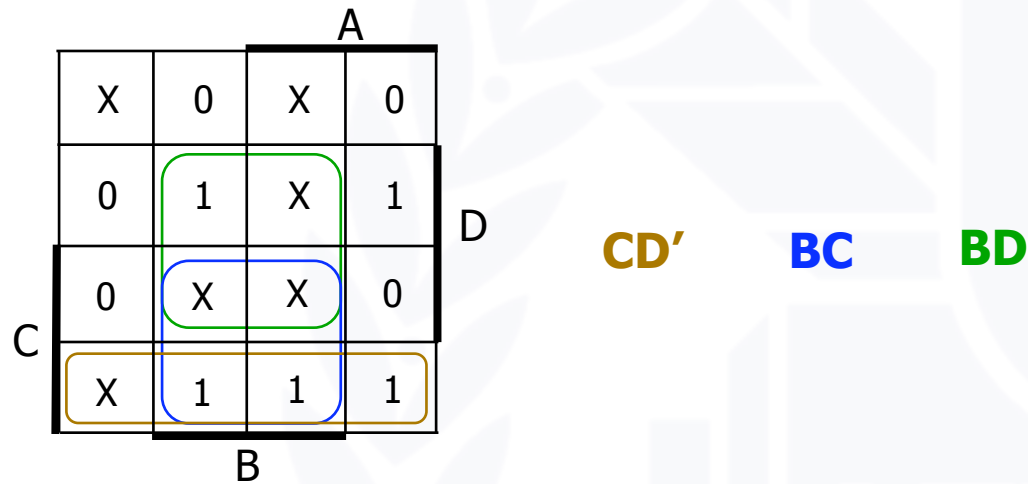
- List all prime implicants for the following K-map:



- Which are essential prime implicants?
- What is the minimum cover?

Activity

- List all prime implicants for the following K-map:



- Which are essential prime implicants?
- What is the minimum cover?



Activity

- List all prime implicants for the following K-map:

		A		
	X	0	X	0
	0	1	X	1
C	0	X	X	0
	X	1	1	1
		B		

CD'

BC

BD

AB

- Which are essential prime implicants?
- What is the minimum cover?



Activity

- List all prime implicants for the following K-map:

		A		
	X	0	X	0
	0	1	X	1
C	0	X	X	0
	X	1	1	1
		B		

CD'

BC

BD

AB

$AC'D$

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Activity

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AB

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	X	1	1	1
		B		

CD'

BC

BD

AB

$AC'D$

- Which are essential prime implicants?

CD'

BD

- What is the minimum cover?



Activity

- List all prime implicants for the following K-map:

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	0	1	X	1
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		B		

CD'

BC

BD

AB

$AC'D$

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- What is the minimum cover?

CD'

BD

$AC'D$



Activity

- List all prime implicants for the following K-map:

		A		
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	0	1	X	1
C	0	X	X	0
	X	1	1	1
		B		

CD'

BC

BD

AB

$AC'D$

- Which are essential prime implicants?

- What is the minimum cover?

CD'

BD

$AC'D$

CD'



Activity

- List all prime implicants for the following K-map:

		A		
	X	0	X	0
	0	1	X	1
C	0	X	X	0
	X	1	1	1
		B		

CD'

BC

BD

AB

$AC'D$

- Which are essential prime implicants?

- What is the minimum cover?

CD'

BD

$AC'D$

CD'

BD



Activity

- List all prime implicants for the following K-map:

		A		
	X	0	X	0
	0	1	X	1
C	0	X	X	0
	X	1	1	1
		B		

CD'

BC

BD

AB

$AC'D$

- Which are essential prime implicants?

CD'

BD

$AC'D$

- What is the minimum cover?

CD'

BD

$AC'D$



Activity

- List all prime implicants for the following K-map:

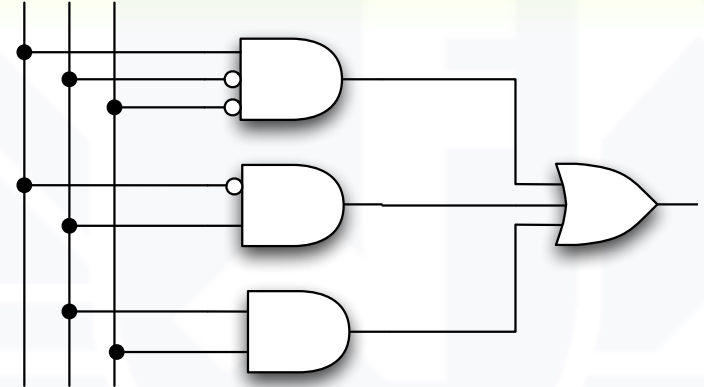
		A		
	X	0	X	0
	0	1	X	1
C	0	X	X	0
	X	1	1	1
		B		

- Which are essential prime implicants?
- What is the minimum cover?

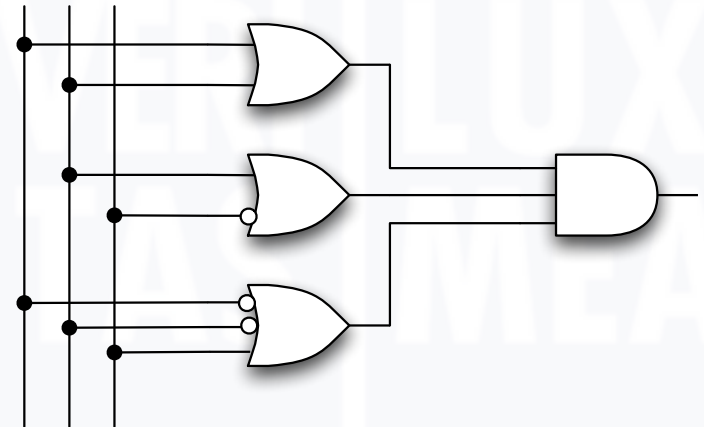


Implementations of two-level logic

- Sum-of-products
 - AND gates to form product terms (minterms)
 - OR gate to form sum

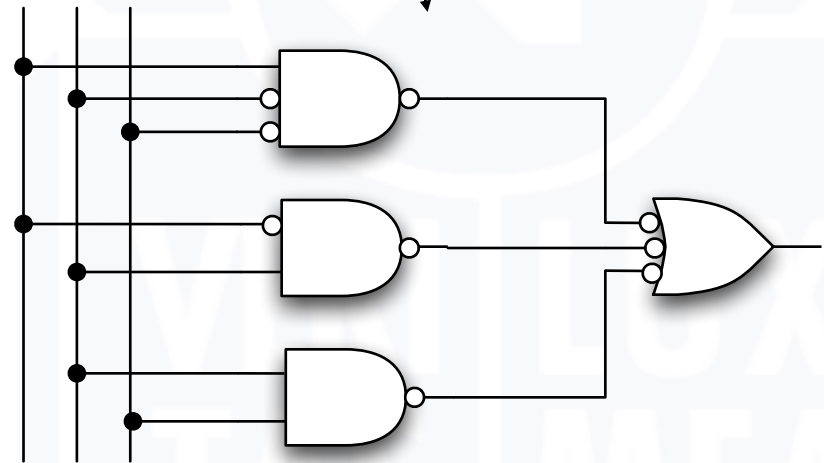
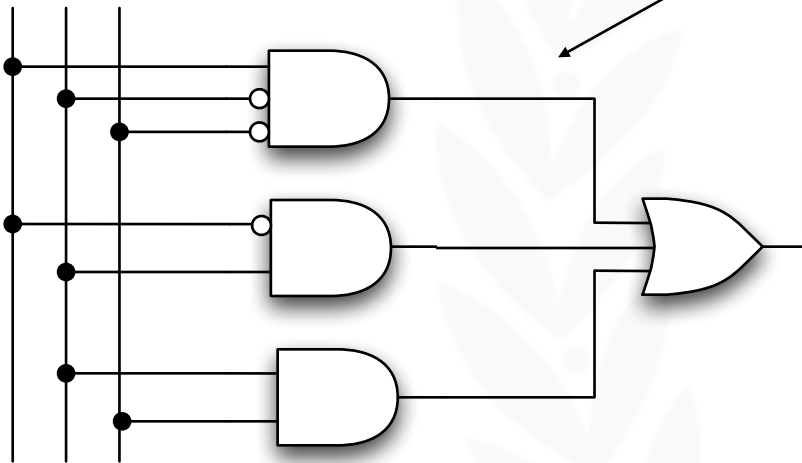


- Product-of-sums
 - OR gates to form sum terms (maxterms)
 - AND gates to form product



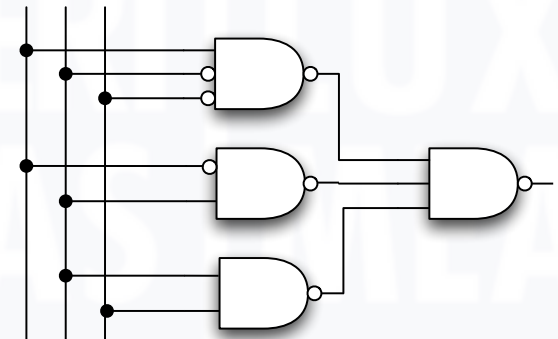
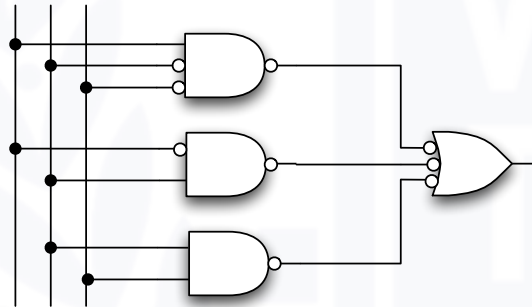
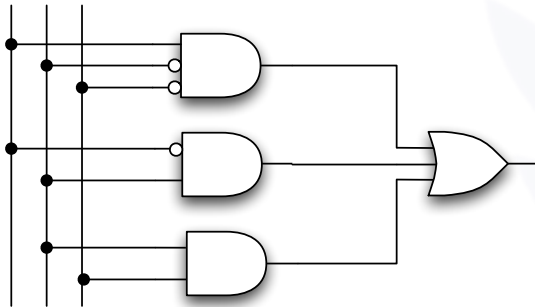
Two-level logic using NAND gates

- Replace minterm AND gates with NAND gates
- Place compensating inversion at inputs of OR gate



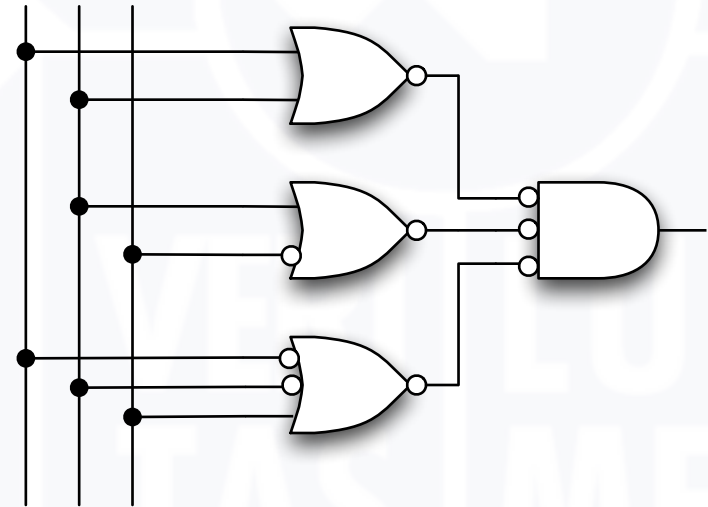
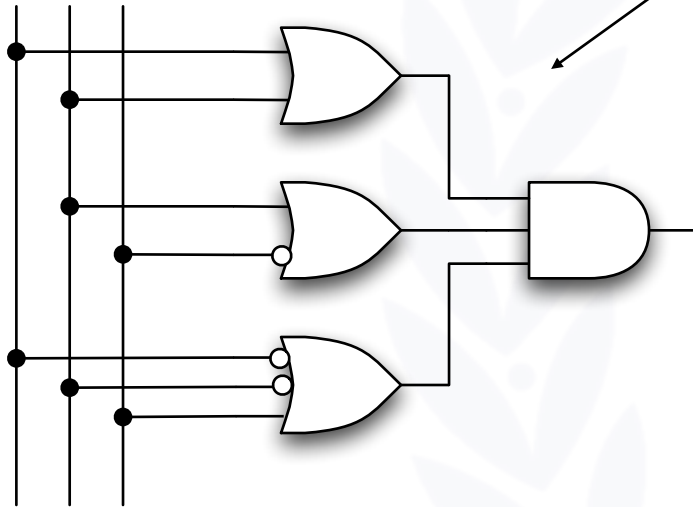
Two-level logic using NAND gates

- OR gate with inverted inputs is a NAND gate
 - de Morgan's: $A' + B' = (A \cdot B)'$
- Two-level NAND-NAND network
 - Inverted inputs are not counted
 - In a typical circuit, inversion is done once and signal distributed



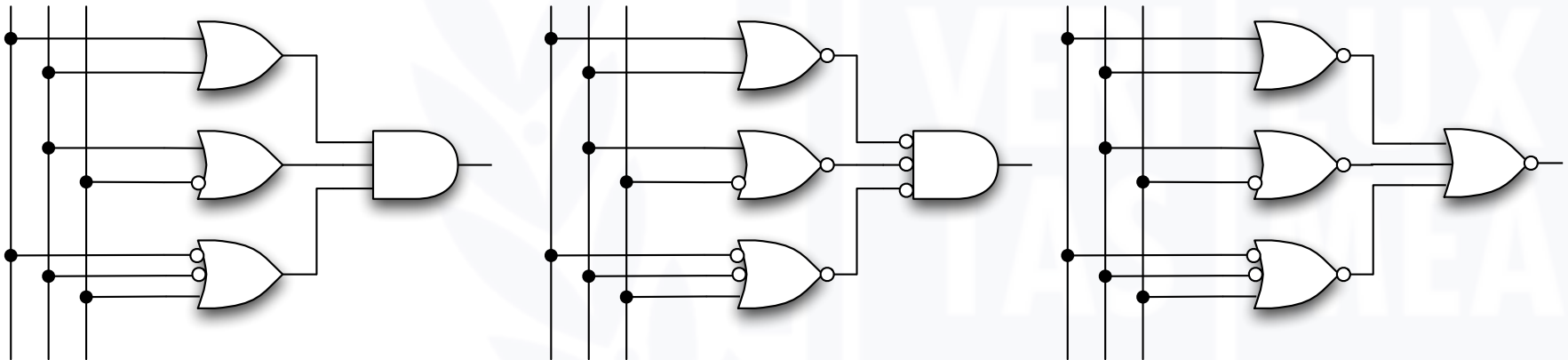
Two-level logic using NOR gates

- Replace maxterm OR gates with NOR gates
- Place compensating inversion at inputs of AND gate



Two-level logic using NOR gates (cont'd)

- AND gate with inverted inputs is a NOR gate
 - De Morgan's: $A' \bullet B' = (A + B)'$
- Two-level NOR-NOR network
 - Inverted inputs are not counted
 - In a typical circuit, inversion is done once and signal distributed



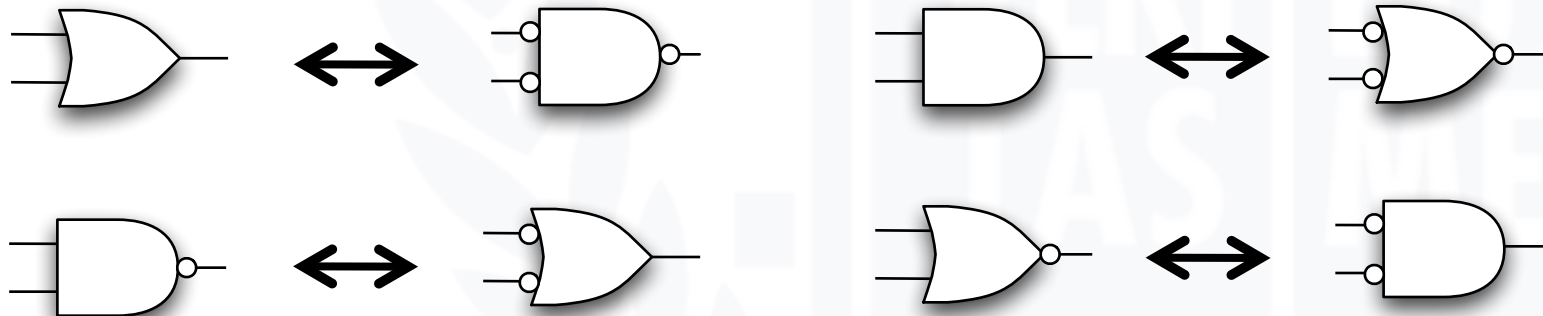
Two-level logic using NAND and NOR gates

- NAND-NAND and NOR-NOR networks

- de Morgan's law: $(A + B)' = A' \cdot B'$ $(A \cdot B)' = A' + B'$
- Written differently: $A + B = (A' \cdot B')'$ $A \cdot B = (A' + B')'$

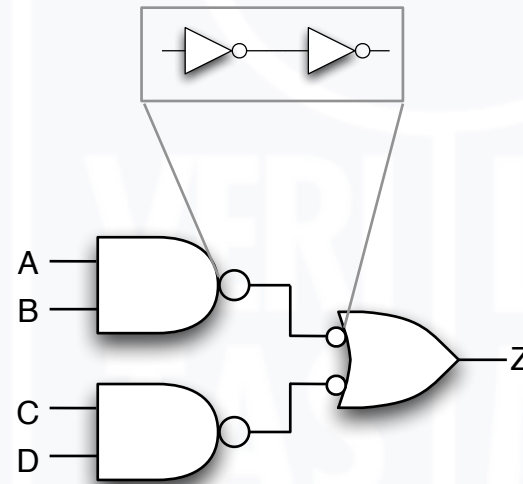
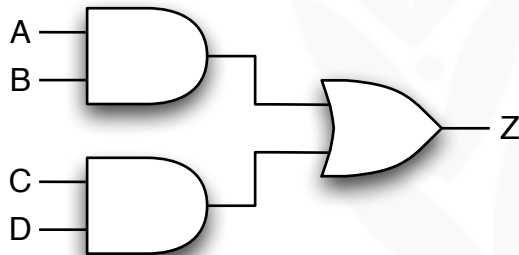
- In other words —

- OR is the same as NAND with complemented inputs
- AND is the same as NOR with complemented inputs
- NAND is the same as OR with complemented inputs
- NOR is the same as AND with complemented inputs



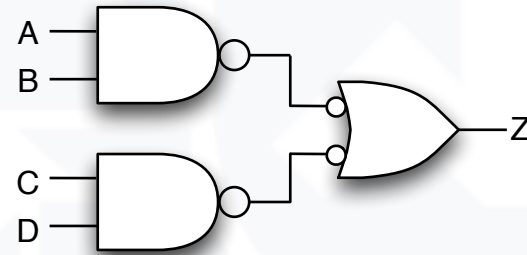
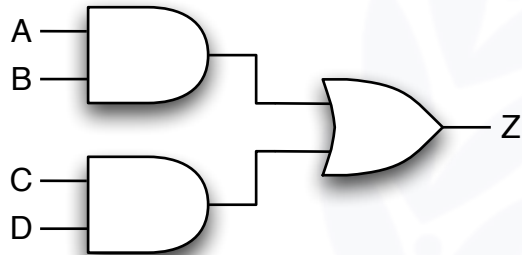
Conversion between forms

- Convert from networks of ANDs and ORs to networks of NANDs and NORs
 - Introduce appropriate inversions ("bubbles")
- Each introduced "bubble" must be matched by a corresponding "bubble"
 - Conservation of inversions
 - Do not alter logic function
- Example: AND/OR to NAND/NAND



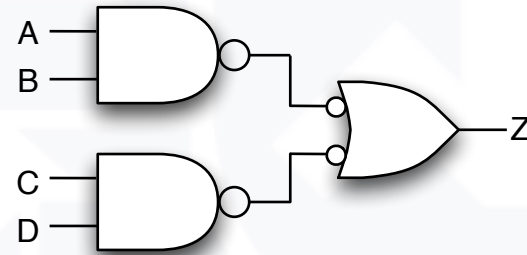
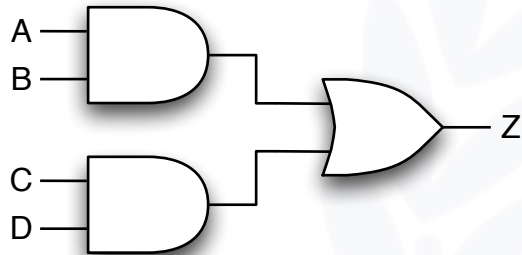
Conversion between forms (cont'd)

- Example: Verify equivalence of two forms



Conversion between forms (cont'd)

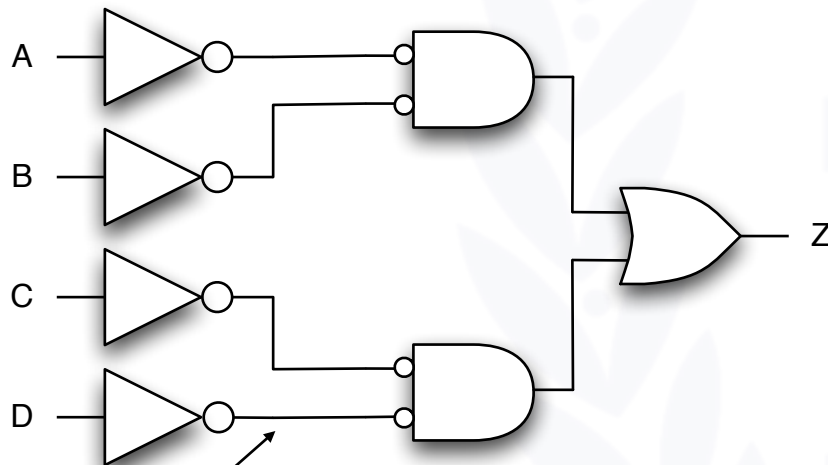
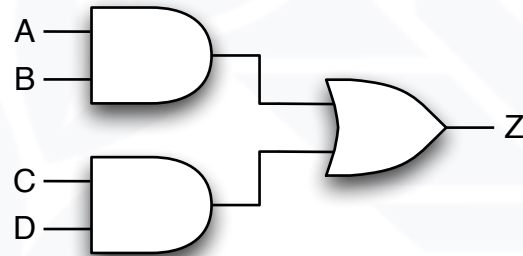
- Example: Verify equivalence of two forms



$$\begin{aligned} Z &= [(A \cdot B)' \cdot (C \cdot D)']' \\ &= [(A' + B') \cdot (C' + D')]' \\ &= [(A' + B')' + (C' + D')'] \\ &= (A \cdot B) + (C \cdot D) \end{aligned}$$

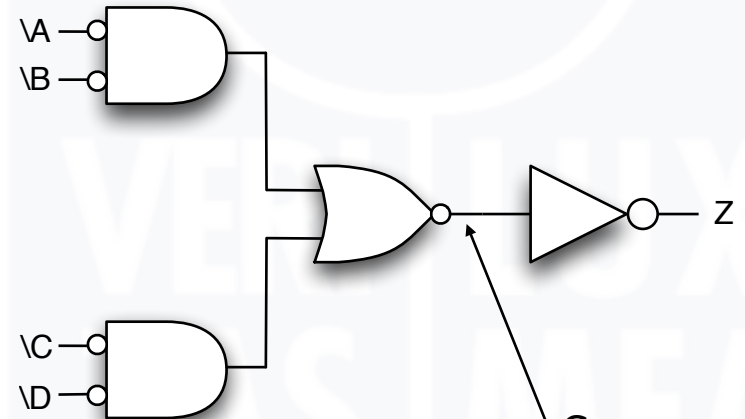
Conversion between forms (cont'd)

- Example: map AND/OR network to NOR/NOR network



Conserve
"bubbles"

Step 1



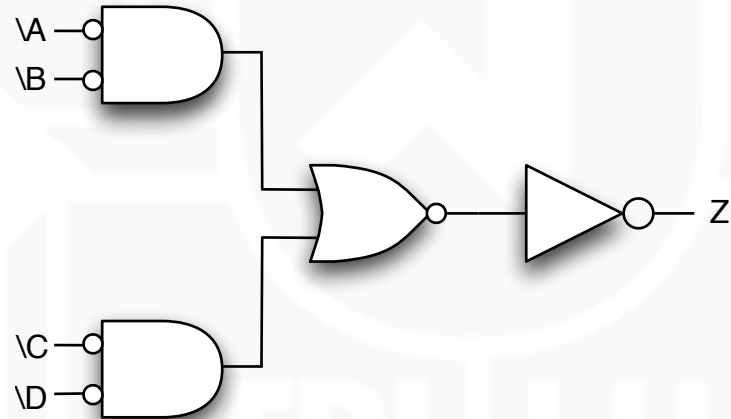
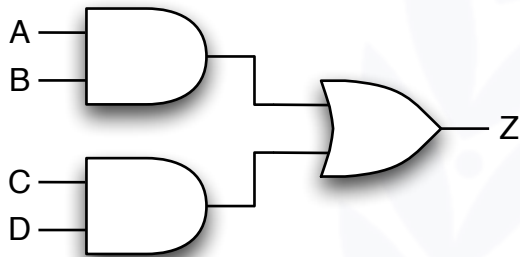
Conserve
"bubbles"

Step 2



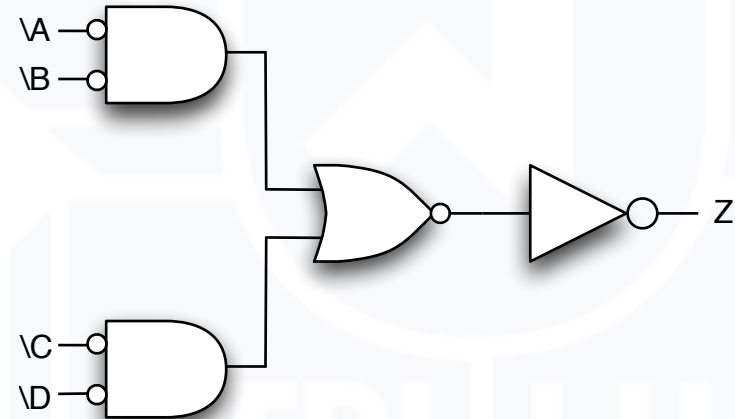
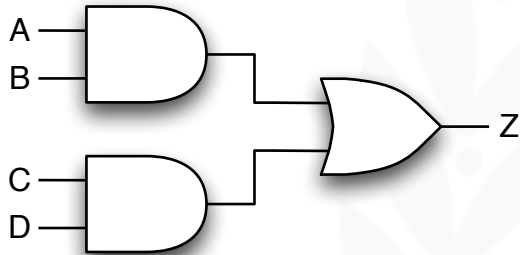
Conversion between forms

- Example: Verify equivalence of two forms



Conversion between forms

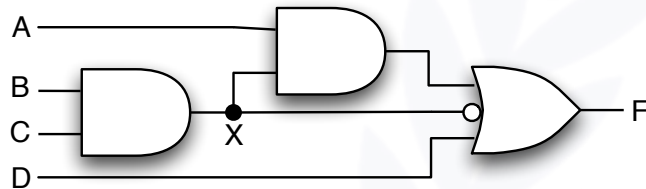
- Example: Verify equivalence of two forms



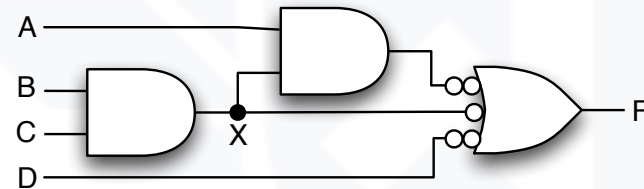
$$\begin{aligned} Z &= \{ [(A' + B')' + (C' + D')']' \}' \\ &= \{ (A' + B') \cdot (C' + D') \}' \\ &= (A' + B')' + (C' + D')' \\ &= (A \cdot B) + (C \cdot D) \end{aligned}$$

Conversion between forms

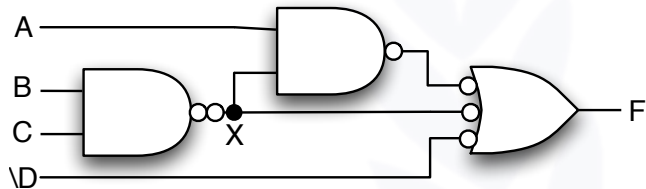
Example



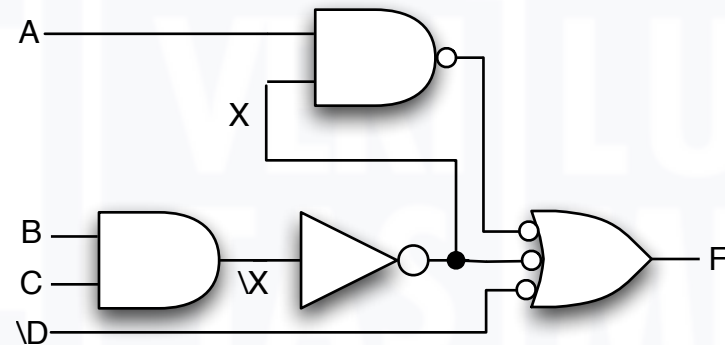
Original circuit



Add double bubbles to invert all inputs of OR gate



Add double bubble to invert output of AND gate

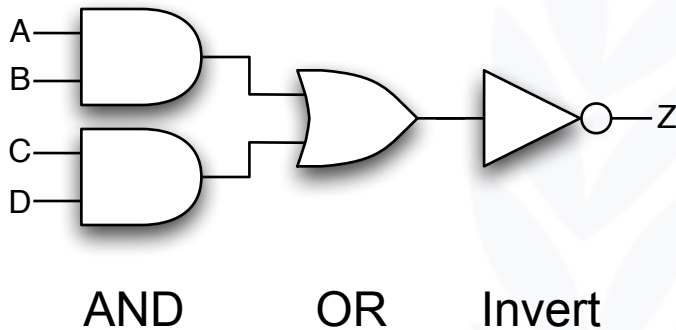


Insert inverters to eliminate double bubbles on a wire

AND-OR-invert gates

- AOI function: three stages of logic — AND, OR, Invert
 - Multiple gates "packaged" as a single circuit block

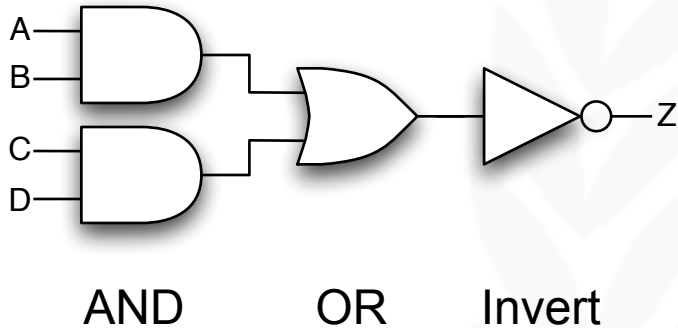
Logical concept



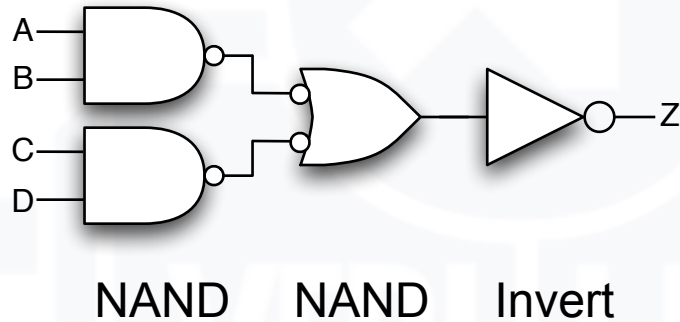
AND-OR-invert gates

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Logical concept



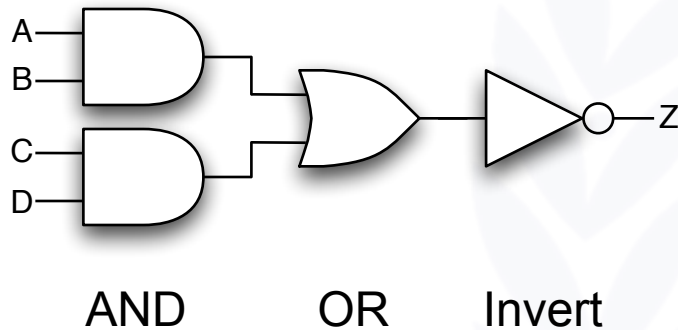
Possible implementation



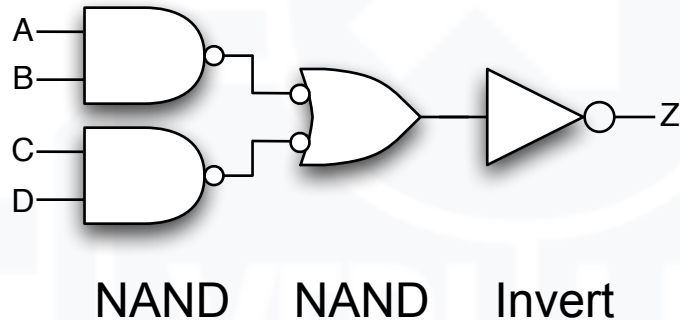
AND-OR-invert gates

- AOI function: three stages of logic — AND, OR, Invert
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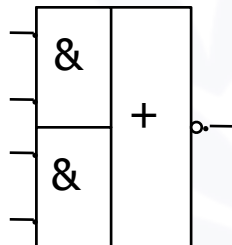
Logical concept



Possible implementation



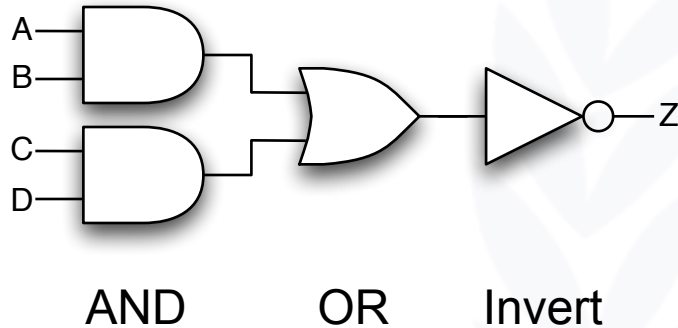
2x2 AOI gate symbol



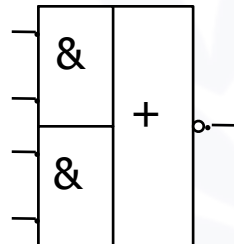
AND-OR-invert gates

- AOI function: three stages of logic — AND, OR, Invert
 - Multiple gates "packaged" as a single circuit block

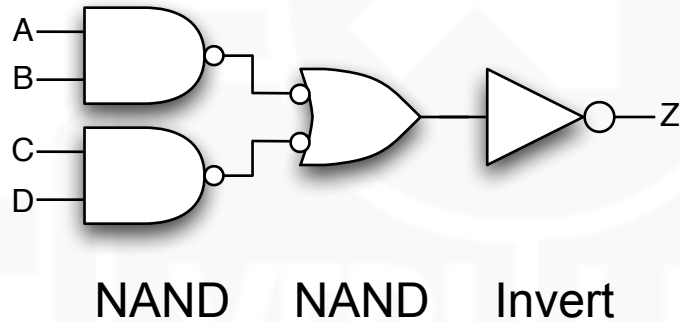
Logical concept



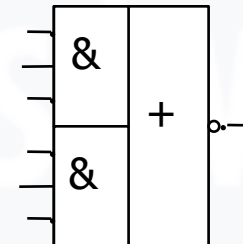
2x2 AOI gate symbol



Possible implementation

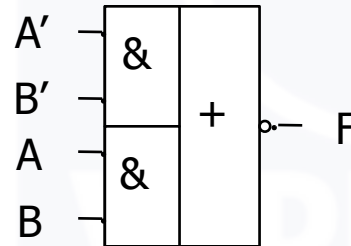


3x2 AOI gate symbol



Conversion to AOI forms

- General procedure to place in AOI form
 - Compute the complement of the function in sum-of-products form
 - By grouping the 0s in the Karnaugh map
- Example: XOR implementation
 - $A \text{ xor } B = A' B + A B'$
 - AOI form:
 - $F = (A' B' + A B)'$



Summary for multi-level logic

- Advantages
 - Circuits may be smaller
 - Gates have smaller fan-in
 - Circuits may be faster
- Disadvantages
 - More difficult to design
 - Tools for optimization are not as good as for two-level
 - Analysis is more complex



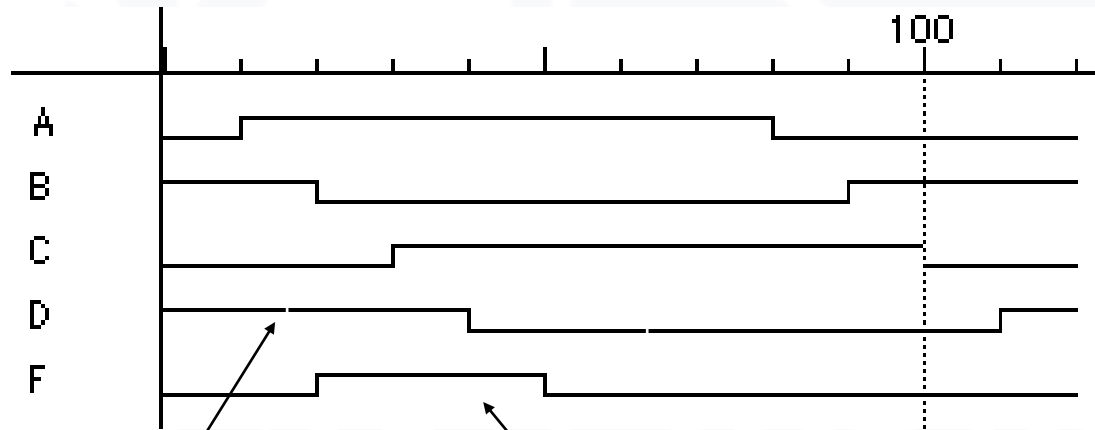
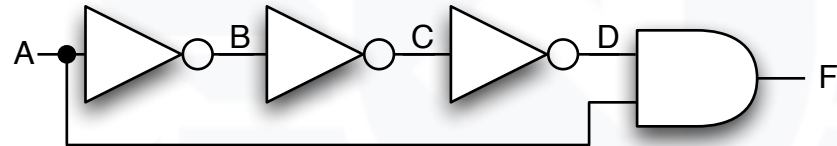
Time behavior of combinational networks

- Waveforms
 - Visualization of values carried on signal wires over time
 - Useful in explaining sequences of events (changes in value)
- Simulation tools are used to create these waveforms
 - Input to the simulator includes gates and their connections
 - Input stimulus, that is, input signal waveforms
- Some terms
 - Gate delay — time for change at input to cause change at output
 - Min delay – typical/nominal delay – max delay
 - Careful designers design for the worst case
 - Rise time — time for output to transition from low to high voltage
 - Fall time — time for output to transition from high to low voltage
 - Pulse width — time that an output stays high or stays low between changes



Momentary changes in outputs

- Can be useful — pulse shaping circuits
- Can be a problem — incorrect circuit operation (glitches/hazards)
- Example: pulse shaping circuit
 - $A' \cdot A = 0$
 - Delays matter



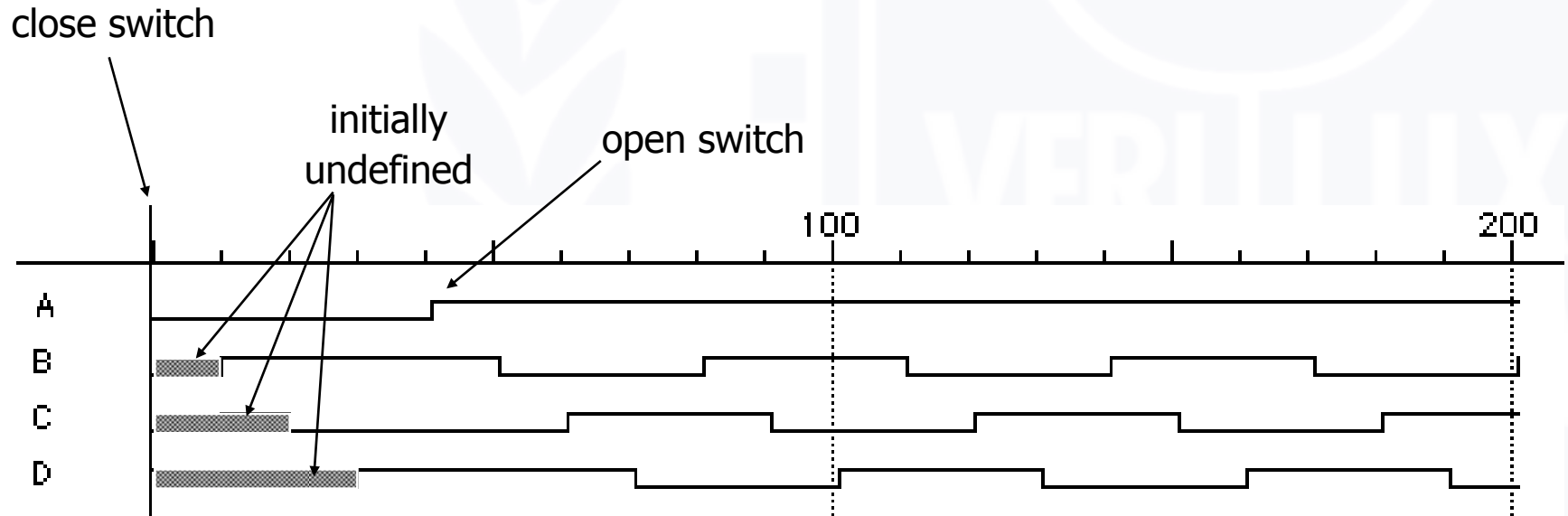
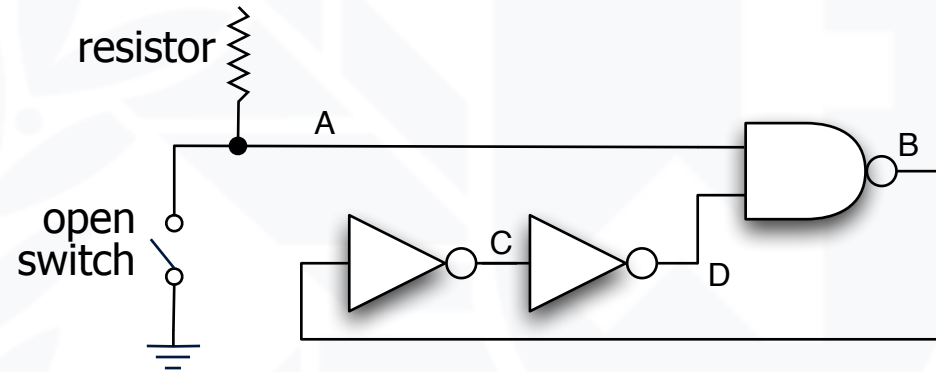
D remains high for three gate delays after A changes from low to high

F is not always 0 pulse 3 gate-delays wide



Oscillatory behavior

- Another pulse shaping circuit



Hazards and glitches

- Glitch
 - Unwanted pulse of a combinational logic network
- Hazard
 - A circuit has a potential to generate glitch
 - Intrinsic characteristic
 - A circuit with a hazard may or may not generate glitch depending on the input pattern
- Static hazard
 - Static 0-hazard: momentarily 1 while the output is 0
 - Static 1-hazard: momentarily 0 while the output is 1
- Dynamic hazard
 - Generate glitch more than once for a single transition 0 to 1 or 1 to 0
- Hazard-free circuit generation
 - Can avoid hazard if there is only single input change
 - Hazard caused by simultaneous multiple input changes is unavoidable



Hazard-free circuit

- No hazard
 - When the initial and final inputs are covered by the same prime implicant
- Hazard
 - When the input change spans prime implicants
- Generalized static hazard-free circuits
 - Add a redundant prime implicants so that all the single-input transitions are covered by one prime implicant
- Dynamic hazard-free circuits
 - Extension of the static hazard-free method
 - Beyond the scope of this class



Hardware description languages

- Describe hardware at varying levels of abstraction
- Structural description
 - Textual replacement for schematic
 - Hierarchical composition of modules from primitives
- Behavioral/functional description
 - Describe what module does, not how
 - Synthesis generates circuit for module
- Simulation semantics



HDLs

- Abel (circa 1983) - developed by Data-I/O
 - Targeted to programmable logic devices
 - Not good for much more than state machines
- ISP (circa 1977) - research project at CMU
 - Simulation, but no synthesis
- Verilog (circa 1985) - developed by Gateway (absorbed by Cadence)
 - Similar to Pascal and C
 - Delays is only interaction with simulator
 - Fairly efficient and easy to write
 - IEEE standard
- VHDL (circa 1987) - DoD sponsored standard
 - Similar to Ada (emphasis on re-use and maintainability)
 - Simulation semantics visible
 - Very general but verbose
 - IEEE standard



PALASM

```
TITLE      <Design title>
PATTERN    <Identification such as file name>
REVISION   <Version or other ID>
AUTHOR     <Name of designer>
COMPANY    <Organization name>
DATE       <Relevant date>

CHIP  <Description>  <Device name>
; <Pin numbers, eg 1   2   3   4   5   6   7   8>
  <pin names,      eg Clk Clr Pre I1 I2 I3 I4 GND>
; <Pin numbers, eg 9   10  11  12  13  14  15  16>
  <pin names,      eg NC  NC  Q1  Q2  Q3  Q4  NC  Vcc>

STRING  <Name>  '<Characters to substitute>'
        <more string definitions>

EQUATIONS
  <combinatorial equations of the form
    OutName = Name1 Op1 Name2 ..... OpN NameM>

  <registered equations of the form
    OutName := Name1 Op1 Name2 ..... OpN NameM>
```



PALASM

- Operators
 - / NOT or active-low
 - * AND
 - + OR
 - :+: XOR
 - = Combinational output
 - *= Latched output
 - := Registered output
- Simulation feature



ABEL

- Extended version of PALASM for PLD design
- Logic is expressed by
 - Equations
 - Truth Tables
 - State Diagrams
 - Fuses
 - XOR Factors
- Operators
 - Logical: similar to PALASM
 - ! !A NOT: one's complement
 - & A & B AND
 - # A # B OR
 - \$ A \$ B XOR: exclusive OR
 - !\$ A !& B XNOR exclusive NOR



ABEL

Operators

Very limited arithmetic

- -A Twos complement (negation)

- A - B Subtraction

- + A + B Addition

- The following operators are not valid for sets:

- * A*B Multiplication

- / A/B Unsigned integer division

- % A%B Modulus remainder from division

- << A<<B Shift A left by B bits

- >> A>>B Shift A right by B bits

- ABC = 3 * 17;

Relational

- == A == B Equal

- != A != B Not equal

- < A < B Less than

- <= A <= B Less than or equal

- > A > B Greater than

- >= A >= B Greater than or equal



ABEL

- Statements
 - IF THEN ELSE
 - STATE MACHINE
- Architecture independent extensions:
 - .CLK: Clock input to an edge triggered flip-flop
 - .OE: Output enable
 - .PIN: Pin feedback
 - .FB: Register feedback
- Architecture specific dot extensions:
 - .J: J input to an JK-type flip-flop
 - .K: K input to an JK-type flip-flop
 - .R: R input to an SR-type flip-flop
 - And many more



Verilog

- Supports structural and behavioral descriptions
- Structural
 - Explicit structure of the circuit
 - Net list
 - e.g., each logic gate instantiated and connected to others
- Behavioral
 - Program describes input/output behavior of circuit
 - Many structural implementations could have same behavior
 - e.g., different implementation of one Boolean function



Structural model

```
module xor_gate (out, a, b);  
    input      a, b;  
    output     out;  
    wire       abar, bbar, t1, t2;  
  
    inverter invA (abar, a);  
    inverter invB (bbar, b);  
    and_gate and1 (t1, a, bbar);  
    and_gate and2 (t2, b, abar);  
    or_gate  or1 (out, t1, t2);  
  
endmodule
```



Simple behavioral model

Continuous assignment

```
module xor_gate (out, a, b);  
  input          a, b;  
  output         out;  
  reg            out;  
  
  assign #6 out = a ^ b;  
  
endmodule
```

Simulation register
keeps track of
value of signal

Delay from input change
to output change



Simple behavioral model

- Always block

```
module xor_gate (out, a, b);  
  input          a, b;  
  output         out;  
  reg            out;  
  
  always @(a or b) begin  
    #6 out = a ^ b;  
  end  
  
endmodule
```

Specifies when block is executed
ie. triggered by which signals



Driving a simulation through a “testbench”

```
module testbench (x, y);  
    output        x, y;  
    reg [1:0]     cnt;
```

2-bit vector

```
    initial begin  
        cnt = 0;  
        repeat (4) begin  
            #10 cnt = cnt + 1;  
            $display ("@ time=%d, x=%b, y=%b, cnt=%b",  
                $time, x, y, cnt); end  
            #10 $finish;  
        end
```

Initial block executed
only once at start
of simulation

Print to a console

```
        assign x = cnt[1];  
        assign y = cnt[0];  
    endmodule
```

Directive to stop
simulation



Hardware description languages vs. programming languages

- Program structure
 - Instantiation of multiple components of the same type
 - Specify interconnections between modules via schematic
 - Hierarchy of modules (only leaves can be HDL in Aldec ActiveHDL)
- Assignment
 - Continuous assignment (logic always computes)
 - Propagation delay (computation takes time)
 - Timing of signals is important (when does computation have its effect)
- Data structures
 - Size explicitly spelled out - no dynamic structures
 - No pointers
- Parallelism
 - Hardware is naturally parallel (must support multiple threads)
 - Assignments can occur in parallel (not just sequentially)



Hardware description languages and combinational logic

- Modules - specification of inputs, outputs, bidirectional, and internal signals
- Continuous assignment - a gate's output is a function of its inputs at all times (doesn't need to wait to be "called")
- Propagation delay- concept of time and delay in input affecting gate output
- Composition - connecting modules together with wires
- Hierarchy - modules encapsulate functional blocks



Working with combinational logic summary

- Design problems
 - Filling in truth tables
 - Incompletely specified functions
 - Simplifying two-level logic
- Realizing two-level logic
 - NAND and NOR networks
 - Networks of Boolean functions and their time behavior
- Time behavior
- Hardware description languages
- Later
 - Combinational logic technologies
 - More design case studies



Not included in the lecture

- Advanced Boolean optimization
 - Quine_McCluskey method
 - Espresso method

