## Digital Logic Design

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## 10. Sequential Logic Technology

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## Sequential logic implementation

Q Implementation

- Random logic gates and FFs
- Programmable logic devices (PAL with FFs)

Q Design procedure

- State diagrams
- Design
- Verification (branch condition)
- Reduction (implicant chart or raw matching)

Q State transition table

- State assignment
- Tight encoding for random logic
- One-hot for FPGA
- Output-based for PLD
- Next state functions
- Input synchronization


## Median filter FSM

Q Remove single 0s between two 1 s (output = NS3)


|  | PS1 |  | PS2 | PS3 | NS1 | NS2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | NS3

## Median filter FSM (cont'd)

Q Realized using the standard procedure and individual FFs and gates

|  | PS1 |  | PS2 | PS3 | NS1 | NS2 | NS3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |  |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 |  |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 |  |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 |  |
| 0 | 1 | 0 | 1 | $X$ | X | X |  |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 |  |
| 0 | 1 | 1 | 1 | 0 | 1 | 1 |  |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 |  |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 |  |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 |  |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 |  |
| 1 | 1 | 0 | 0 | 1 | 1 | 0 |  |
| 1 | 1 | 0 | 1 | $X$ | $X$ | $X$ |  |
| 1 | 1 | 1 | 0 | 1 | 1 | 1 |  |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |  |

NS1 = Reset' (I)
NS2 = Reset' (PS1 + PS2 I )
NS3 = Reset' PS2
$\mathrm{O}=\mathrm{PS} 3$

## Median filter FSM (cont'd)

Q But it looks like a shift register if you look at it right


## Median filter FSM (cont'd)

Q An alternate implementation with S/R FFs

- Personally I do not recommend this!


Q The set input (S) does the median filter function by making the next state 111 whenever the input is 1 and PS2 is 1 (1 input to state $x 1 x$ )

## FSM implementation with a shift register

Q String recognizer
Q Good candidate for a shift register implementation


## FSM implementation with a counter

Q Three functions of a counter

- Count
- Reset
- Jump
- State machine implementation with a counter
- Next state function
- Count (CNT), Reset (R) and Load (LD)
- Sequencer



## FSM implementation with a ROM

Q PS + FSM input

- Address input
- NS
- Data output

Q Input synchronization is applied here
Q Both Moore and Mealy machines can be implemented
Q Advantage and disadvantages?

- Same to the combinational logic implementation with a ROM



## FSM implementation using PALs

Q Programmable logic building block for sequential logic

- Aacro-cell: FF + logic
- D-FF
- Two-level logic capability like PAL (e.g., 8 product terms)



## Vending machine example (Moore PLD mapping)

$$
\begin{array}{ll}
\text { D0 } & =\operatorname{reset}^{\prime}\left(\mathrm{QO} 0^{\prime} \mathrm{N}+\mathrm{Q} 0 \mathrm{~N}^{\prime}+\mathrm{Q} 1 \mathrm{~N}+\mathrm{Q} 1 \mathrm{D}\right) \\
\mathrm{D} 1 & =\operatorname{reset}^{\prime}(\mathrm{Q} 1+\mathrm{D}+\mathrm{Q} 0 \mathrm{~N}) \\
\text { OPEN } & =\text { Q1Q0 }
\end{array}
$$



## Vending machine (synch. Mealy PLD mapping)

OPEN = reset'(Q1Q0N' + Q1N + Q1D + Q0'ND + QON'D)


## 22V10 PAL

Q Combinational logic elements (SoP)
Q Sequential logic elements (D-FFs)

- Up to 10 outputs
- Up to 10 FFs

Q Up to 22 inputs

Functional Logic Diagram for PALC22V10D


## 22V10 PAL macro cell

Q Sequential logic element + output/input selection


## FSM implementation with an FPGA

9 Altera MAX 3000 CPLD architecture


## FSM implementation with an FPGA

Q Altera MAX 3000 CPLD macrocell


## FSM implementation with an FPGA

Q Xilinx Vertex-5 slice


## Example: traffic light controller

Q A busy highway is intersected by a little used farmroad
Q Detectors C sense the presence of cars waiting on the farmroad

- With no car on farmroad, light remain green in highway direction
- If vehicle on farmroad, highway lights go from Green to Yellow to Red, allowing the farmroad lights to become green
Q These stay green only as long as a farmroad car is detected but never longer than a set interval
- When these are met, farm lights transition from Green to Yellow to Red, allowing highway to return to green
Q Even if farmroad vehicles are waiting, highway gets at least a set interval as green
Q Assume you have an interval timer that generates:
- A short time pulse (TS) and
- A long time pulse (TL),
- In response to a set (ST) signal.

Q TS is to be used for timing yellow lights and TL for green lights

## Example: traffic light controller (cont')

Q Highway/farm road intersection

highway

## Example: traffic light controller (cont')

Q Tabulation of inputs and outputs

| inputs | description | outputs | description |
| :--- | :---: | :---: | :---: |
| reset | place FSM in initial state | HG, HY, HR | assert green/yellow/red highway lights |
| C | detect vehicle on the farm road | FG, FY, FR | assert green/yellow/red highway lights |
| TS | short time interval expired | ST | start timing a short or long interval |
| TL | long time interval expired |  |  |

Q Tabulation of unique states - some light configurations imply others
state description
HG highway green (farm road red)
HY highway yellow (farm road red)
FG farm road green (highway red)
FY farm road yellow (highway red)

## Example: traffic light controller (cont')

- State diagram



## Example: traffic light controller (cont')

Q Generate state table with symbolic states

- Consider state assignments

Output encoding - similar problem to state assignment
(Green $=00$, Yellow $=01$, Red $=10$ )

| Inputs |  |  | Present State |  | Next State | Outputs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | TL | TS |  |  |  | ST | H | F |
| 0 | - | - |  | HG | HG | 0 | Green | Red |
| - | 0 | - |  | HG | HG | 0 | Green | Red |
| 1 | 1 | - |  | HG | HY | 1 | Green | Red |
| - | - | 0 |  | HY | HY | 0 | Yellow | Red |
| - | - | 1 |  | HY | FG | 1 | Yellow | Red |
| 1 | 0 | - |  | FG | FG | 0 | Red | Green |
| 0 | - | - |  | FG | FY | 1 | Red | Green |
| - | 1 | - |  | FG | FY | 1 | Red | Green |
| - | - | 0 |  | FY | FY | 0 | Red | Yellow |
| - | - | 1 |  | FY | HG | 1 | Red | Yellow |
|  | SA1: |  | $\mathrm{HG}=00$ |  | $\mathrm{FG}=11$ | $\mathrm{FY}=10$ |  |  |
|  | SA2: |  | $\mathrm{HG}=00$ |  | $\mathrm{FG}=01$ | $\mathrm{FY}=11$ |  |  |
|  | SA3: |  | $\mathrm{HG}=000$ |  | $\mathrm{FG}=0100$ | $\mathrm{FY}=1000$ | (0) | hot) |

## Logic for different state assignments

- SA1

$$
\begin{aligned}
& \text { NS0 }=\text { C•TL•PS1'•PSO' }+\mathrm{C} \cdot \text { TL'•PS1•PS0 }+ \text { PS1'•PS0 } \\
& \text { ST }=\text { C•TL•PS1' } \bullet \text { PSO' }+ \text { TS•PS1'•PS0 }+ \text { TS•PS1•PSO' }+C^{\prime} \cdot P S 1 \bullet P S 0 ~+~ T L \cdot P S 1 \cdot P S 0 ~ \\
& \mathrm{H} 1=\mathrm{PS} 1 \quad \mathrm{H} 0=\mathrm{PS} 1 \text { '•PS0 } \\
& \text { F1 = PS1' } \quad \text { F0 }=\text { PS1•PSO }
\end{aligned}
$$

Q SA2

$$
\begin{aligned}
& \text { NS1 }=\text { C•TL•PS1' }+ \text { TS'•PS1 + C'•PS1'•PS0 } \\
& \text { NSO }=\text { TS•PS1•PS0' }+ \text { PS1'•PS0 }+ \text { TS'•PS1•PS0 } \\
& \text { ST }=\mathrm{C} \cdot T L \bullet P S 1 \text { ' }+\mathrm{C}^{\prime} \cdot P S 1 \text { '•PS0 }+\mathrm{TS} \cdot P S 1 \\
& \text { H1 = PS0 } \\
& \text { H0 = PS1•PS0' } \\
& \text { F1 = PSO' } \\
& \text { F0 = PS1•PS0 }
\end{aligned}
$$

Q SA 3

$$
\begin{aligned}
& \text { NS3 }=\text { C'•PS2 + TL•PS2 + TS'•PS3 } \\
& \text { NS2 }=\text { TS•PS1 + C•TL'•PS2 } \\
& \text { NS1 }=\text { C•TL•PS0 }+ \text { TS'•PS1 } \\
& \text { NSO }=\mathrm{C} \cdot \bullet \text { PSO }+\mathrm{TL} \cdot \bullet \text { PSO }+\mathrm{TS} \bullet \text { PS3 } \\
& \text { ST }=\mathrm{C} \cdot \mathrm{TL} \bullet \text { PSO }+\mathrm{TS} \bullet \text { PS } 1+\mathrm{C} \cdot \bullet P S 2+\mathrm{TL} \bullet P S 2+\mathrm{TS} \bullet P S 3 \\
& \text { H1 = PS3 + PS2 } \\
& \mathrm{H} 0=\mathrm{PS} 1 \\
& F 1=P S 1+P S 0 \\
& \mathrm{FO}=\mathrm{PS} 3
\end{aligned}
$$

## Sequential logic implementation summary

Q Models for representing sequential circuits
Q Finite state machines and their state diagrams

- Mealy, Moore, and synchronous Mealy machines

Q Finite state machine design procedure

- Deriving state diagram
- Deriving state transition table
- Assigning codes to states

Q Determining next state and output functions

- Implementing combinational logic
- Implementation technologies
- Random logic + FFs

Q PAL with FFs (programmable logic devices - PLDs)

