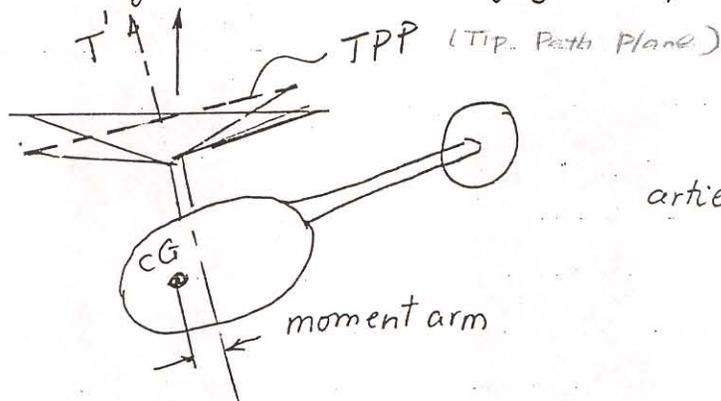


Simple Control Concepts for Helicopters

These concepts are useful when dealing with forward flight.
Pitch and Roll Control

Can be achieved by tilt or offset of thrust, this simple mechanism can be used for both pitch and roll control as illustrated by the schematic figures presented below.

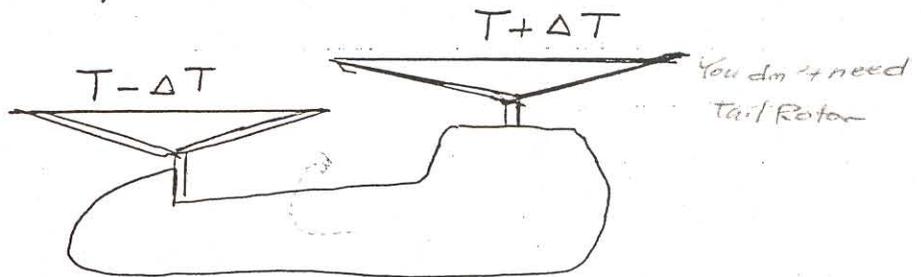


articulated rotor

$e \uparrow \alpha$
 better control
 (maneuvering)
 Flexible wing of control power
 control power

For an articulated rotor tip path tilted by controls, moment arm of T and C.G. offset provides pitching moment also hub moment, due to blade offset, tilts the plane of the rotor called the tip path plane

For tandem helicopter rotors



The thrust differential shown provides pitch control. Roll control is achieved by tilt of the tip path plane about a longitudinal axis

For hingeless blades, blade bending or flexibility

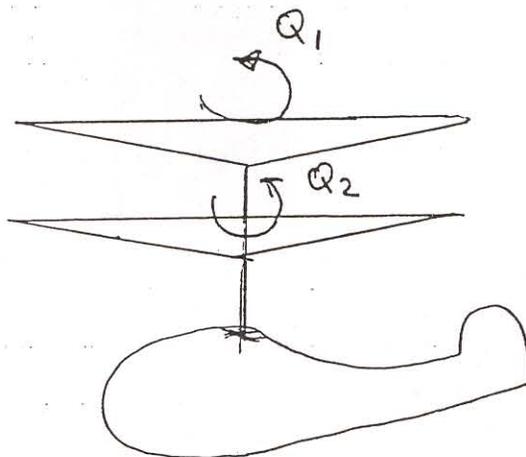
produces both a combination of tilt and offset using the hub moment generated at the blade root.

Yaw Control

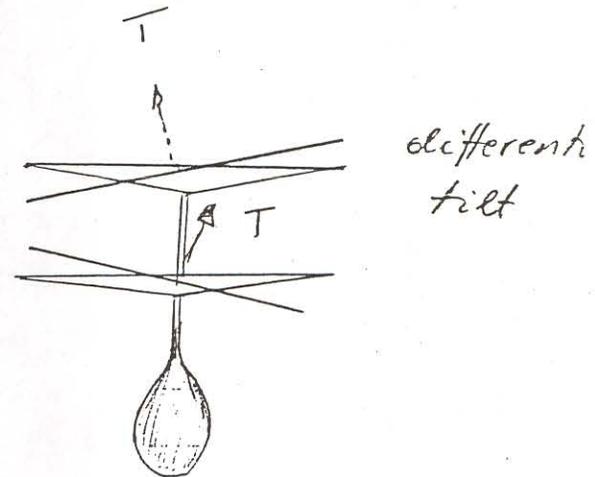
On a conventional helicopter is achieved by the tail rotor, operated by the pilots pedals.

On a tandem rotor helicopter it can be accomplished by a differential tilt of the two rotors.

On a coaxial helicopter differential torque on the two rotors can produce yaw



$$Q_1 - Q_2 = \Delta Q - \text{produces yaw}$$



Climb and Descent

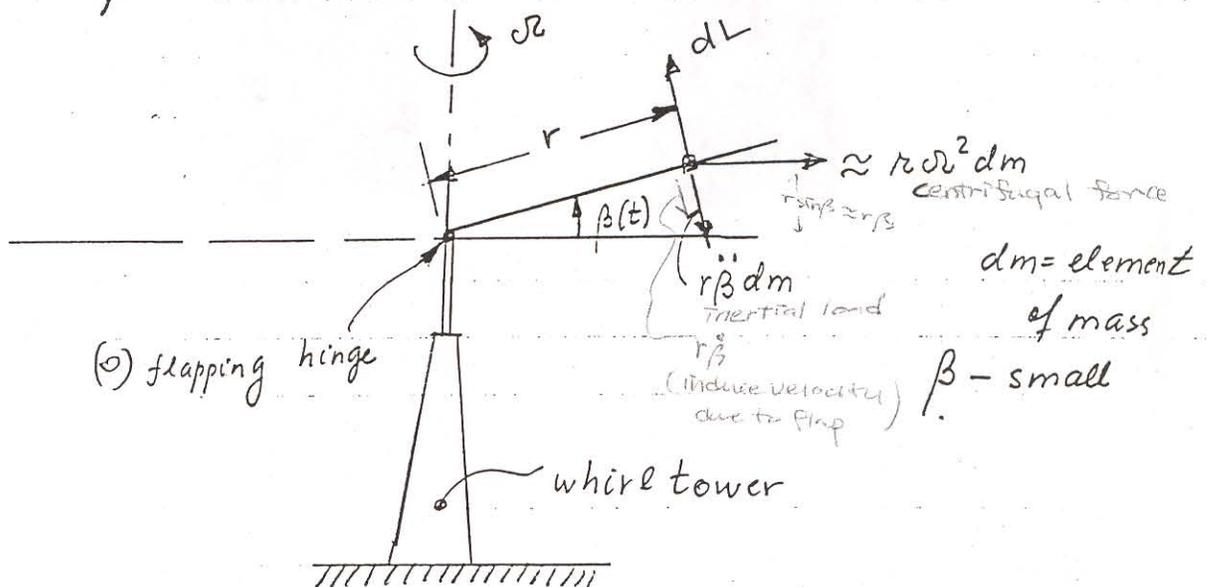
Are achieved by a combination of collective pitch and throttle. On some light, piston engine helicopters these two can be interconnected

Gas turbine helicopters have engine RPM governor.

Response of a Fully Articulated Rotor with Centrally Hinged Blades to Cyclic Pitch Control

Consider an articulated rotor with centrally hinged blades mounted on a whirl tower (a test stand used for testing rotors in hover.) The blade and swash plate mechanism are schematically shown on the next page

As has been mentioned cyclic pitch inputs are used to control the rotor in forward flight. However before dealing with the forward flight problem it is useful to consider the behavior of a blade having only the flapping d.o.f which is excited by a cyclic pitch input. The geometry of the problem is shown below



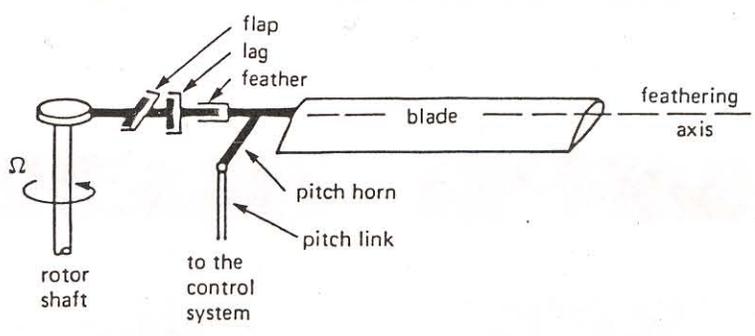
Writing moment equilibrium about the hinge point O , yields

$$\int_0^R r dL = \int_0^R (r^2 \ddot{\beta} dm + r^2 \Omega^2 \beta dm)$$

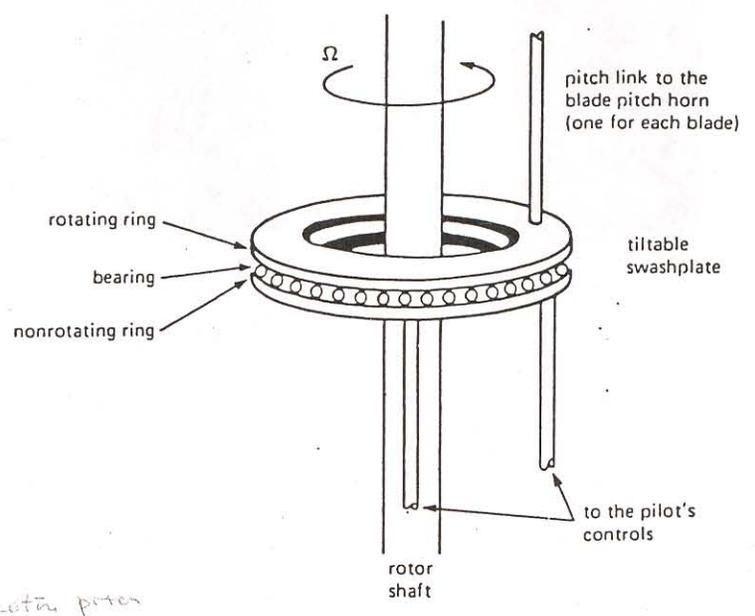
$$\text{or } I_1 (\ddot{\beta} + \Omega^2 \beta) = \int_0^R r dL \quad (1)$$

Define flapping moment of inertia as

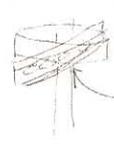
46



Schematic of the flap and lag hinges, and the pitch bearing at the hub of an articulated blade.



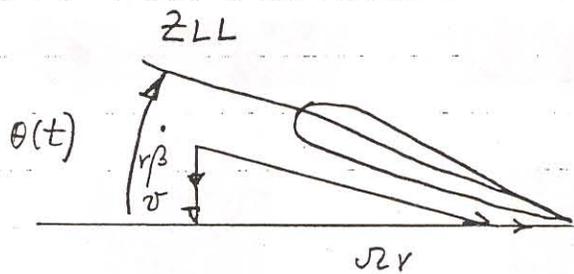
Schematic of the rotor swashplate



 collector pitch
 tilt - cyclic pitch

$$I_1 = \int_0^R r^2 dm = \int_0^R r^2 m(r) dr$$

Natural frequency of the hinged blade is Ω . Consider next the aerodynamic load and use blade element theory



$$dL = \frac{1}{2} \rho a c \Omega^2 r^2 \left[\theta(t) - \left(\frac{v + r\dot{\beta}}{\Omega r} \right) \right] dr$$

Assume $\theta(t)$ given by cyclic pitch
Excitation cyclic pitch

$$\theta(t) = \theta_0 + \theta_{1s} \sin \psi + \theta_{1c} \cos \psi \quad (a) \Rightarrow \text{response } \beta(\psi) = \beta_0 + \beta_{1c} \cos \psi + \beta_{1s} \sin \psi$$

$$\int_0^R r dL = \frac{1}{2} \rho a c \Omega^2 \int_0^R r^2 \left[\theta(\psi) - \frac{v}{\Omega r} - \frac{\dot{\beta}}{\Omega} \right] r dr$$

$$\int_0^L r dL = \frac{1}{2} \rho a c \Omega^2 \left[\frac{\theta(\psi) R^4}{4} - \frac{v R^3}{\Omega 3} - \frac{\dot{\beta} R^4}{4\Omega} \right] = I_1 \ddot{\beta} + I_1 \Omega^2 \beta$$

$$= \frac{1}{2} \rho a c \Omega^2 \left(-\frac{\dot{\beta} R^4}{4\Omega} \right) + \frac{1}{2} \rho a c \Omega^2 R^4 \left[\frac{\theta(\psi)}{4} - \frac{\lambda}{3} \right] \quad (2)$$

Combining Eq(1) and (2)

$$I_1 \ddot{\beta} + \frac{1}{8} \rho a c \Omega^2 R^4 \frac{\dot{\beta}}{\Omega} + I_1 \Omega^2 \beta = \frac{1}{2} \rho a c \Omega^2 R^4 \left[\frac{\theta(\psi)}{4} - \frac{\lambda}{3} \right]$$

divide through by $I_1 \Omega^2$ to get

$$\frac{\ddot{\beta}}{\Omega^2} + \frac{\gamma}{8} \frac{\dot{\beta}}{\Omega} + \beta = \frac{\gamma}{2} \left[\frac{\theta(\psi)}{4} - \frac{\lambda}{3} \right] \quad (3)$$

Aerodynamic Damping in flap Lock number $\gamma = \frac{\rho a c R^4}{I_1} = \frac{\text{aerodynamic moment}}{\text{inertial moment}}$

where $\gamma = \frac{\rho a c R^4}{I}$ = Lock number = $\frac{\text{aerodynamic moment}}{\text{inertia moment}}$
 (reasonable range $4 < \gamma < 10$)

Assume a steady state solution in form of

$$\beta(\psi) = \beta_0 + \beta_{1c} \cos \psi + \beta_{1s} \sin \psi \quad (4)$$

where $\psi = \Omega t$, thus

$$\frac{\dot{\beta}}{\Omega} = -\beta_{1c} \sin \psi + \beta_{1s} \cos \psi \quad (5)$$

$$\frac{\ddot{\beta}}{\Omega^2} = -\beta_{1c} \cos \psi - \beta_{1s} \sin \psi = \beta_0 - \beta \quad (6) \text{ (from Eq (4))}$$

Substituting Eqs (4), (5), (6) into Eq (3) yields

$$\begin{aligned} & \beta_0 - \beta + \frac{\gamma}{\rho} (-\beta_{1c} \sin \psi + \beta_{1s} \cos \psi) + \beta = \\ & = \frac{\gamma}{2} \left[\frac{1}{4} (\theta_0 + \theta_{1c} \cos \psi + \theta_{1s} \sin \psi) - \frac{\lambda}{3} \right] \quad (7) \end{aligned}$$

Steady state yields

$$\beta_0 = \frac{\gamma}{2} \left(\frac{\theta_0}{4} - \frac{\lambda}{3} \right) \quad (8)$$

and $\left. \begin{array}{l} \beta_{1c} = -\theta_{1s} \\ \beta_{1s} = \theta_{1c} \end{array} \right\} (9) \text{ Equivalence of flapping \& feathering}$
 response to actuation at 90° phase lag

Equations (9) are known as the equivalence between flapping and feathering.

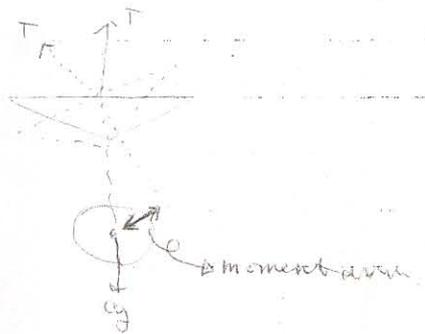
(57)

- β_0 - coning angle
- [β_{1c} - represents the pitching of the TPP
- β_{1s} - represents the rolling of the TPP

In resonance the response of the forced system lags by 90° after the input

- $\frac{d\theta}{dt}$ of Gravity Effect는 무시

- θ is small (ex. $\theta < 10^\circ$)



* HINGELESS BLADE 2차원
 moment arm of θ is $r \sin \theta$
 θ is small, $\sin \theta \approx \theta$
 θ is small, $\cos \theta \approx 1$

Blade Element Theory in Nonaxial Flight

(+ performance in forward flight & additional fixed flight effects)

Use no feathering plane (NFP) i.e. swashplate plane as a reference plane for velocity

• Articulated Rotor

• Inplane dynamics (lag motion)

is neglected

• Everything is in the

(NFP)

flapping or feathering

cyclic pitching neglected

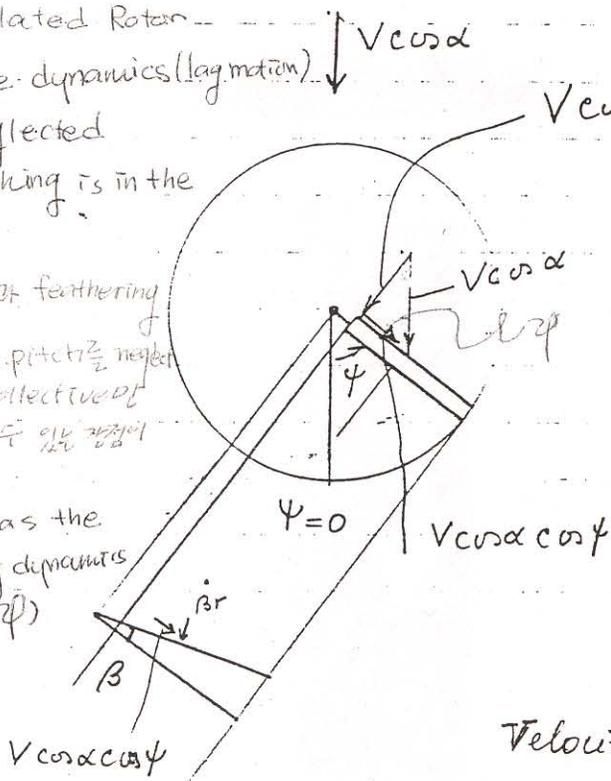
& collective

pitching due to inflow

is neglected

• Blade has the flapping dynamics

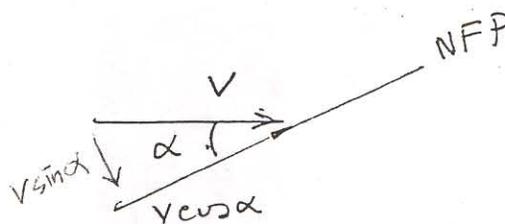
$\beta = \beta(\psi)$



INFLOW IS SMALL

$$(U_T^2 + U_P^2 + \Omega R^2)^{1/2} \approx U_T$$

$$\left(\frac{U_P}{U_T}\right), \left(\frac{\Omega R}{U_T}\right) \ll 1$$



Velocity components relative to the

blade element can be viewed tangential and normal,

furthermore consider only flapping motion and neglect

lead-lag motion, also β -small, $\sin \beta = \beta$, $\cos \beta = 1$
(assume uniform inflow over the disk)

Inside (tangential) inside the ref plane

$$U_T = \Omega r + V \cos \alpha \sin \psi = \Omega r + \mu \Omega R \sin \psi \quad (1)$$

perpendicular to a ref plane (+ disk positive down)

$$U_P = r \dot{\beta} + (V \cos \alpha \cos \psi) \sin \beta + (v + V \sin \alpha) =$$

$$U_P = r \dot{\beta} + \mu \Omega R \beta \cos \psi + \lambda \Omega R \quad (2)$$

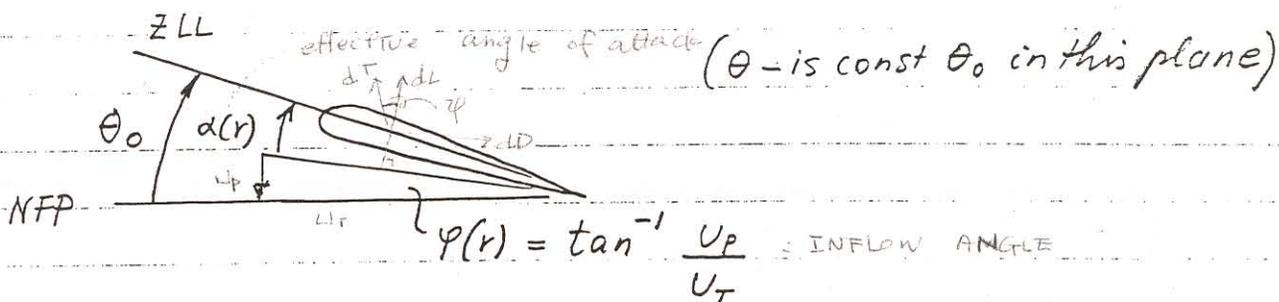
radial flow component

$$U_R = \text{radial component} = \mu \Omega R \cos \psi = V \cos \alpha \cos \psi \quad (3)$$

$$C_T = \frac{\sigma}{2} \left[\frac{C_p}{2} - \frac{\lambda}{2} \right] \Rightarrow C_T = \frac{\sigma}{2} \left[\frac{C_p}{2} \left(1 + \frac{3}{2} \left(\frac{U_T}{U_T} \right)^2 \right) - \frac{\lambda}{2} \right]$$

$$\mu = \frac{V \cos \alpha}{\Omega R}$$

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Also $U_T^2 + U_P^2 + U_R^2 = U_{RES}^2 \approx U_T^2$

$\alpha(r) = \theta_0 - \tan^{-1} \frac{U_P}{U_T} \approx \theta_0 - \frac{U_P}{U_T}$ (4)

The elemental thrust

$dL = \frac{1}{2} \rho U_T^2 c \left[\theta_0 - \frac{U_P}{U_T} \right] dr \cdot \alpha$

$dT = \frac{1}{2} \rho ac (U_T^2 + U_P^2 + U_R^2) \left[\theta_0 - \tan^{-1} \left(\frac{U_P}{U_T} \right) \right] dr$

$dT \approx \frac{1}{2} \rho ac U_T^2 \left[\theta_0 - \frac{U_P}{U_T} \right] dr$

Average thrust, based on the assumption of uniform inflow λ

$T = \frac{b}{2\pi} \int_0^{2\pi} \int_0^R \frac{dT}{dr} dr d\psi$

In forward flight the blade will undergo flapping, taking into account the first harmonic

$\beta = \beta_0 + \beta_{1c} \cos \psi + \beta_{1s} \sin \psi$ (a)

$\dot{\beta} = (-\beta_{1c} \sin \psi + \beta_{1s} \cos \psi) \Omega$ (b)

$T \approx \frac{\rho a b c}{4\pi} \int_0^{2\pi} \left[\theta_0 \left(\frac{\Omega^2 r^3}{3} + 2\mu \Omega^2 R \frac{r^2}{2} \sin \psi + \mu^2 \Omega^2 R^2 r \sin^2 \psi \right) - \left(\lambda \Omega^2 R \frac{r^2}{2} + \Omega \dot{\beta} \frac{r^3}{3} + \mu \Omega^2 R \beta \frac{r^2}{2} \cos \psi + \mu \lambda \Omega^2 R^2 r \sin \psi + \mu \Omega R \dot{\beta} \frac{r^2}{2} \sin \psi + \mu^2 \Omega^2 R^2 \beta r \sin \psi \cos \psi \right) \right] \int_0^R dr d\psi$

$$= \frac{\rho abc \Omega^2 R^3}{4\pi} \int_0^{2\pi} \left[\theta_0 \left(\frac{1}{3} + \mu \sin \psi + \mu^2 \sin^2 \psi \right) - \left(\frac{\lambda}{2} + \frac{1}{3} \frac{\beta}{\Omega} + \frac{1}{2} \mu \beta \cos \psi + \lambda \mu \sin \psi + \frac{1}{2} \mu \frac{\beta}{\Omega} \sin \psi + \frac{1}{2} \mu^2 \beta \sin 2\psi \right) \right] d\psi$$

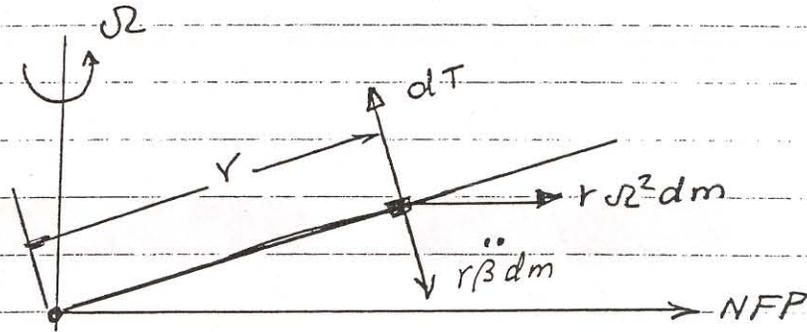
insert eqs (a) and (b) to get

$$\begin{aligned} T &= \frac{\rho abc \Omega^2 R^3}{4\pi} \int_0^{2\pi} \left\{ \theta_0 \left[\frac{1}{3} + \mu \sin \psi + \mu^2 \left(\frac{1}{2} - \frac{1}{2} \cos 2\psi \right) \right] - \left[\frac{\lambda}{2} - \frac{1}{3} \beta_{1c} \sin \psi + \frac{1}{3} \beta_{1s} \cos \psi + \frac{1}{2} \mu \beta_0 \cos \psi + \frac{1}{2} \mu \beta_{1c} \left(\frac{1}{2} + \frac{1}{2} \cos 2\psi \right) + \frac{1}{4} \mu \beta_{1s} \sin 2\psi + \lambda \mu \sin \psi - \frac{1}{2} \mu \beta_{1c} \left(\frac{1}{2} - \frac{1}{2} \cos 2\psi \right) + \frac{1}{4} \mu \beta_{1s} \sin 2\psi + \frac{1}{4} \mu^2 \beta_0 \sin 2\psi + \frac{1}{4} \mu^2 \beta_{1c} \left(\frac{1}{2} \sin \psi + \frac{1}{2} \sin 3\psi \right) + \frac{1}{2} \mu^2 \beta_{1s} \left(\frac{1}{2} \cos \psi - \frac{1}{2} \cos 3\psi \right) \right] \right\} d\psi \\ &= \frac{\rho abc \Omega^2 R^3}{4\pi} \left[\theta_0 \left(\frac{2\pi}{3} + \mu^2 \pi \right) - \lambda \pi \right] \\ &= \frac{\rho abc \Omega^2 R^3}{2} \left[\frac{\theta_0}{2} \left(1 + \frac{3}{2} \mu^2 \right) - \frac{\lambda}{2} \right] \\ \therefore C_T &= \frac{\sigma a}{2} \left[\frac{\theta_0}{3} \left(1 + \frac{3}{2} \mu^2 \right) - \frac{\lambda}{2} \right] \quad (5) \end{aligned}$$

$$\sin^2 \psi = \frac{1}{2} - \frac{1}{2} \cos 2\psi ; \quad 2 \sin \psi \cos \psi = \sin 2\psi$$

$$\cos^2 \psi = \frac{1}{2} + \frac{1}{2} \cos 2\psi$$

Flapping Motion and Flapping Angles



Taking moments about the flapping hinge

$$I_1 \beta'' + I_1 \Omega^2 \beta = \int_0^R r \frac{dT}{dr} dr \quad (6)$$

substituting and equating harmonics

$$\beta_0 = \frac{\gamma}{2} \left[\frac{\theta_0}{4} (1 + \mu^2) - \frac{\lambda}{3} \right] \quad (7)$$

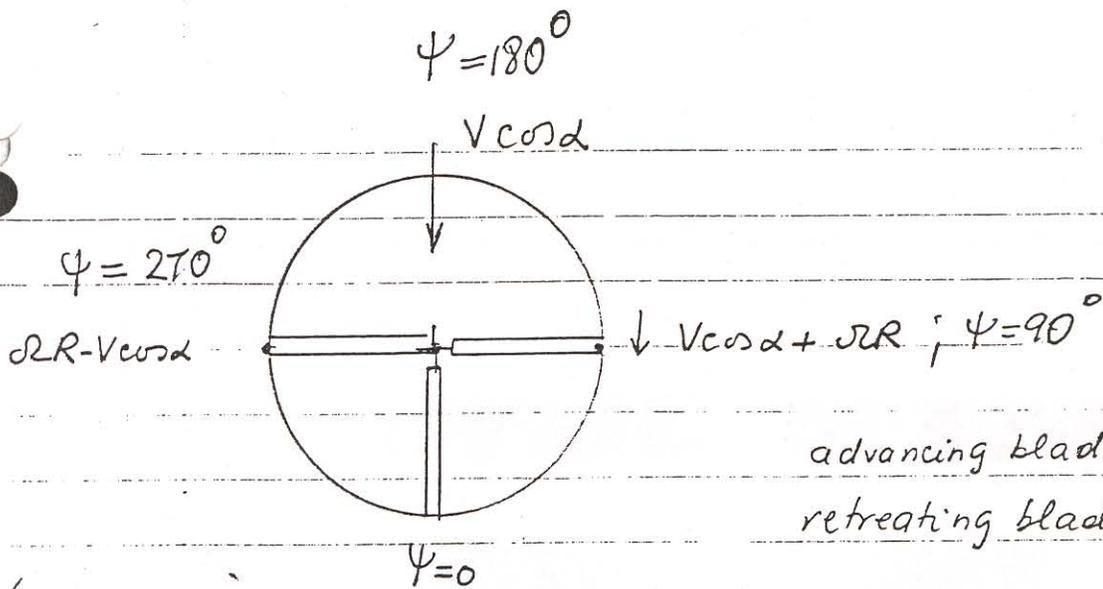
page 60
 $\beta_0 = \frac{\gamma}{2} \left[\frac{\theta_0}{4} - \frac{\lambda}{3} \right]$
 HOVER ~~where~~ $\mu = 0.0$

$$\gamma = \frac{\rho a c R^4}{I_1}$$

$$\beta_{1s} = - \frac{8/3 \mu \left[\theta_0 - \frac{3}{4} \lambda \right]}{1 - \frac{1}{2} \mu^2} \quad (8)$$

$$\beta_{1s} = \frac{-4/3 \mu \beta_0}{1 + \frac{1}{2} \mu^2} \quad (9)$$

β_{1s}, β_{1c} are perturbations of the disk from being parallel to the swashplate. The blade responds in resonance 90° after the aerodynamic forcing.



advancing blade $0 < \psi < 180^\circ$
retreating blade $180 < \psi < 360^\circ$

The disk is tilting to the right and back.

Reversed Flow Region and Stall

Consider Eq (1) (on p. 53), at some r on the retreating blade

$$U_T = \Omega r + \mu \Omega R \sin \psi = 0$$

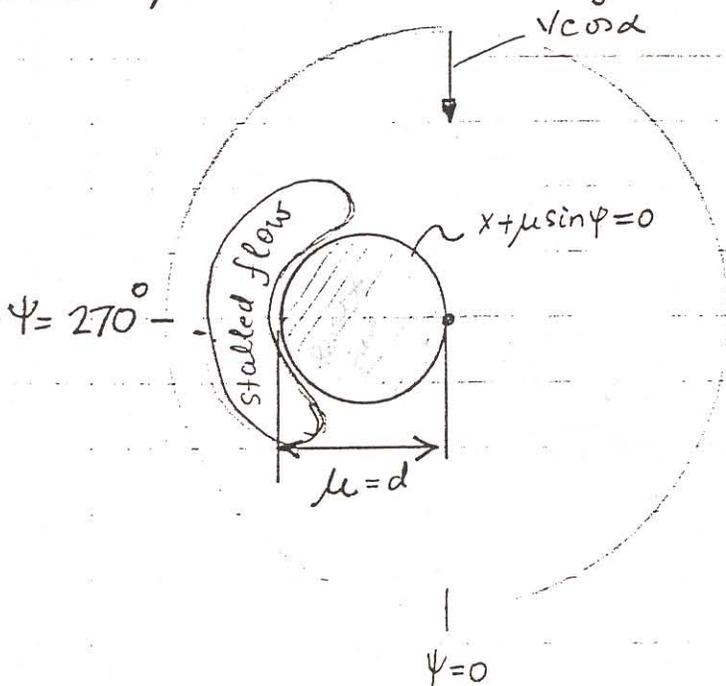
$$\left(\frac{r}{R}\right) + \mu \sin \psi = 0 \rightarrow x + \mu \sin \psi = 0 \quad (10)$$

Equation (10) describes the root locus of a circle which represents the reversed flow region. This region is a

circle as shown in the figure with a diameter (d)

$$d = \mu$$

In the reversed flow region the flow on the cross section is from the TE to the leading edge and in this region it is assumed that the blade produces no thrust.



Obviously in the neighborhood of the reversed flow region

$$\frac{U_p}{U_T} \text{ is not small anymore since } U_T \rightarrow 0$$

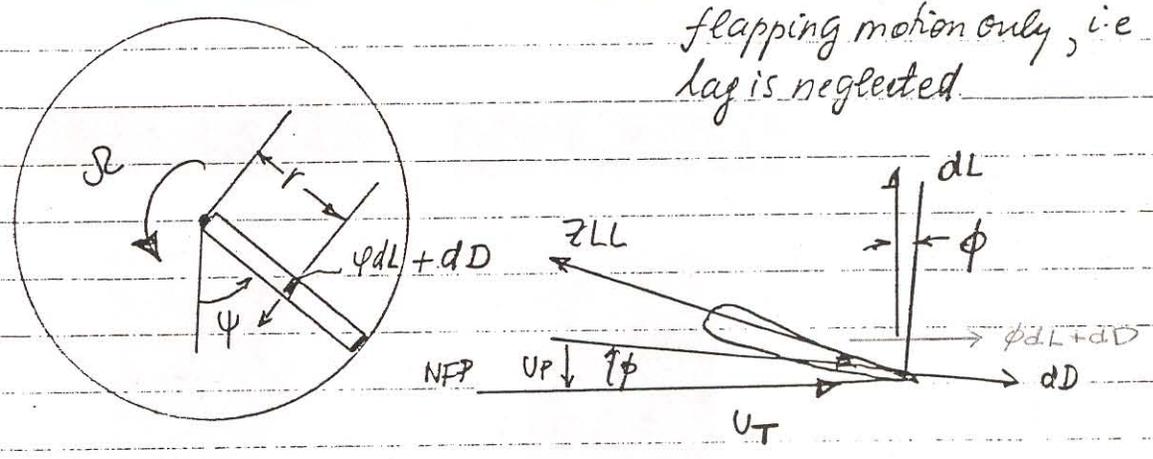
therefore this region is surrounded by a kidney shaped region in which we shall encounter stall on the blade

Obviously the expression for C_T as well as the flapping coefficients given by Eqs (5), (7), (8) and (9) are evaluated neglected both stall and reversed flow effects.

Torque Coefficients

$V \cos \alpha$

Note This analysis is based on flapping motion only, i.e. lag is neglected.



The elemental torque is given by

$dQ \approx r(\phi dL + dD)$ where $\phi \approx \frac{U_P}{U_T}$

drag to overcome to keep the blade turning

이걸로는 행기를 남기잖아! Horizontal force component를 더해주어야 + 다음 page!

$Q = \frac{1}{2\pi} \int_0^{2\pi} \int_0^R dQ dr d\phi$

and $dD \approx \frac{1}{2} \rho U_T^2 c c_{D0} dr \approx \frac{1}{2} \rho U_T^2 c [\delta_0 + \delta_1 \alpha_{eff} + \delta_2 \alpha_{eff}^2] dr$

$= \frac{1}{2} \rho U_T^2 c [\delta_0 + \delta_1 (\theta_0 - \frac{U_P}{U_T}) + \delta_2 (\theta_0 - \frac{U_P}{U_T})^2] dr$

$dL \approx \frac{1}{2} \rho U_T^2 a [\theta_0 - \frac{U_P}{U_T}] c dr$

$Q = \frac{b}{2\pi} \int_0^{2\pi} \int_0^R \left\{ \left[\frac{U_P}{U_T} \frac{1}{2} \rho U_T^2 a (\theta_0 - \frac{U_P}{U_T}) c \right] + \left[\frac{1}{2} \rho U_T^2 c [\delta_0 + \delta_1 (\theta_0 - \frac{U_P}{U_T}) + \delta_2 (\theta_0 - \frac{U_P}{U_T})^2] \right] \right\} r d\phi dr$

After a considerable amount of algebra, Eq(10) will yield

$C_Q = \frac{Q}{\rho (\pi R^2) (U_T)^2 R} = C_Q(\theta_0, \lambda, \mu)$ (11)

No FEATHERING PLANE $\theta_0, \theta_1, \theta_2$ 고정

2247 NFP는 articulated rot

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Horizontal Force

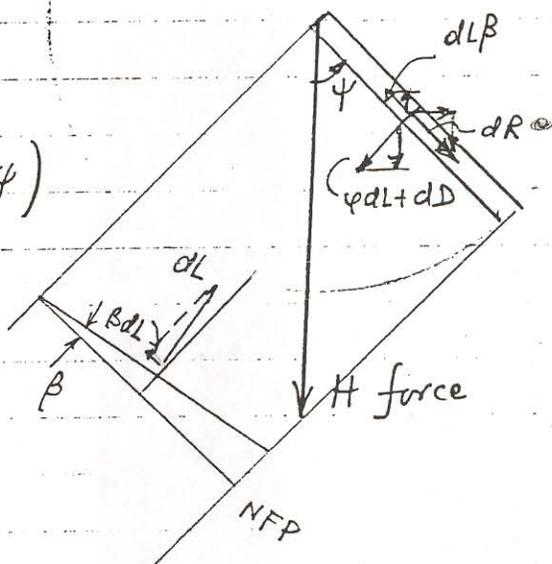
H-force is the force in the plane of the disk pointing \downarrow to rear $V \cos \alpha$

Free H is composed of components of forces shown in the figure. Again lag motion is neglected

$$dH = dR \cos \psi + (\varphi dL + dD) \sin \psi - \beta dL \cos \psi \quad (12)$$

(NASA literature neglects $dR \cos \psi$)

Note H force acts in no feathering plane and is positive in the aft direction



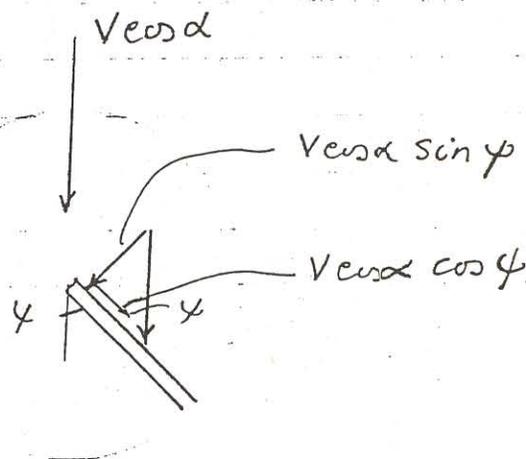
$$dR = \frac{1}{2} \rho U_R^2 C_{D0} c dr$$

$$U_R = V \cos \alpha \cos \psi$$

dR - friction drag

$$H = \frac{b}{2\pi} \int_0^{2\pi} \int_0^R \frac{dH}{dr} dr d\psi$$

$$C_H = \frac{H}{\rho (\pi R^2) (UR)^2} = C_H(\theta_0, \mu, \lambda) \quad (13)$$



(61)

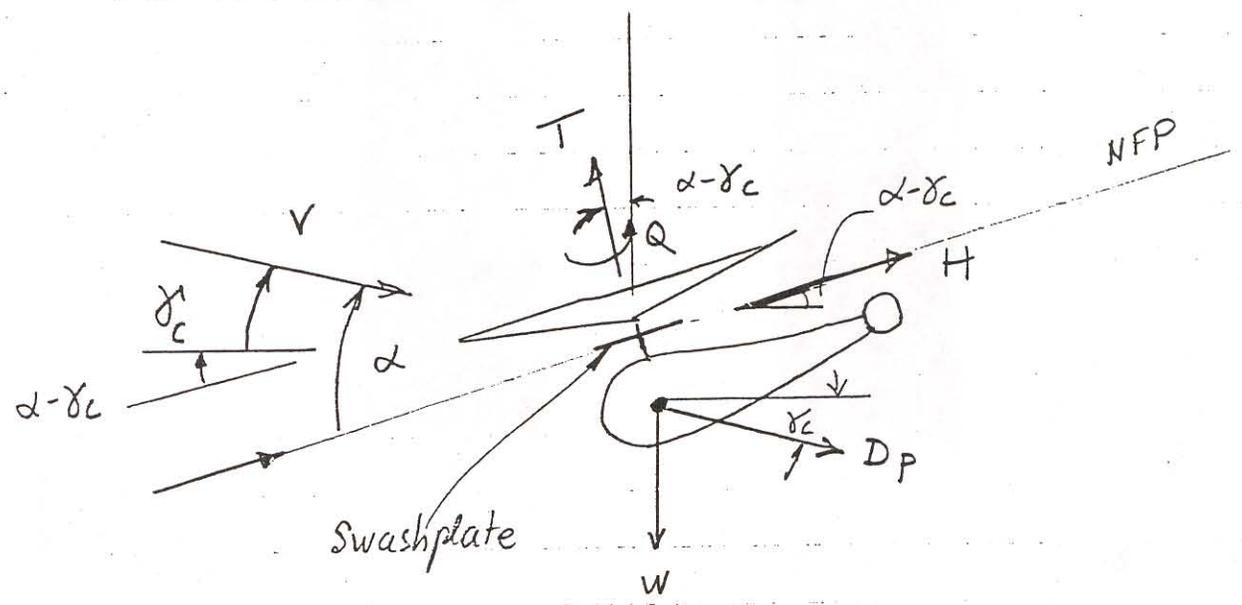
Lateral Force, Positive to the Right (Starboard)

$$dY = -(\varphi dL + dD) \cos\psi - \beta dL \sin\psi + dR \sin\psi \quad (14)$$

Again based on the assumption that lag motion is neglected

$$C_Y = \frac{Y}{\rho \pi R^2 (VR)^2} = C_Y(\theta_0, \lambda, \mu) \quad (15)$$

Forward Flight Performance Using Blade Element Theory



For equilibrium

$$\sum F_z = 0 \quad T \cos(\alpha - \gamma_c) + H \sin(\alpha - \gamma_c) - W - D_p \sin \gamma_c = 0$$

$$\sum F_x = 0 \quad T \sin(\alpha - \gamma_c) - H \cos(\alpha - \gamma_c) + D_p \cos \gamma_c = 0$$

A/c treated as a point mass -

T, H elaborate fens of (θ_0, λ, μ) , divide by $\rho \pi R^2 (VR)^2$ to nondimensionalize

$$\lambda = \mu \tan \alpha + \frac{C_T}{2 \sqrt{\mu^2 + \lambda^2}}$$

Problem: α unknown, therefore λ is unknown

Solution: for a given (μ, γ_c) ^{velocity & γ_c}

Assume $T=W \rightarrow C_T(\lambda, \theta_0)$

$C_T(\lambda, \theta_0, \mu)$
 $C_Q(\lambda, \theta_0, \mu)$ ^{known}

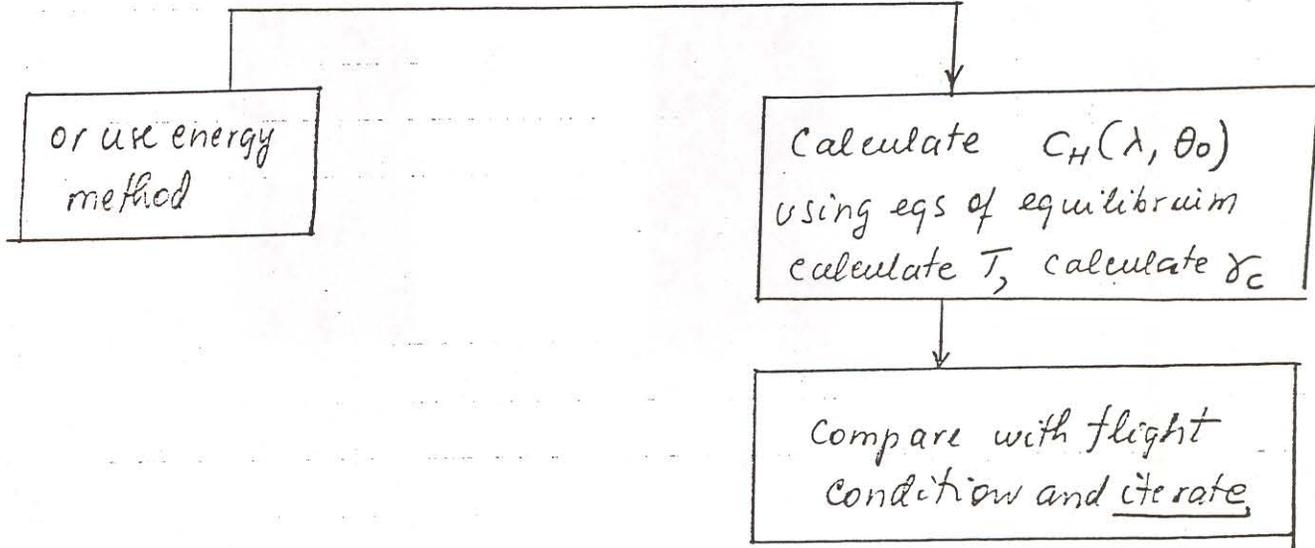
Assume a value of $C_Q(\lambda, \theta_0)$

$C_T = C_T(\lambda, \theta_0)$
 $C_Q = C_Q(\lambda, \theta_0)$ } Solve 2 eqs with 2 unknowns

$\mu \text{ and } \frac{C_L}{\mu^2 + \lambda^2}$

λ, θ_0

Balance of Force method



A good reference on balance of forces method

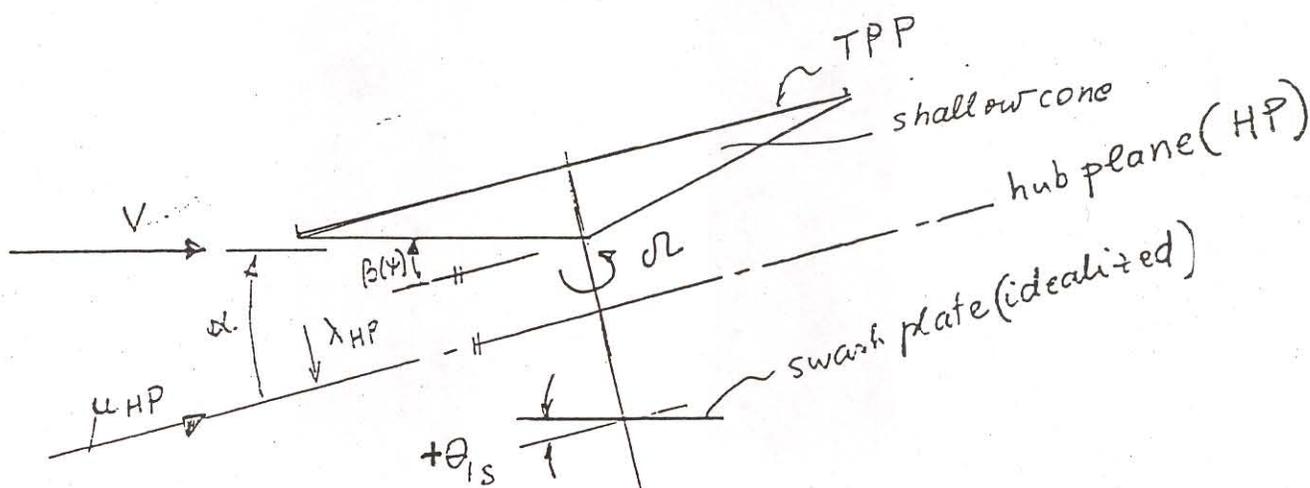
Tanner, NASA CR-114, 1964
Charts for Estimating Rotary Wing
Performance in Hover and Fwd Flight.

(63)

Reference Planes Used in Forward Flight Analyses

Hub Plane

- 1) Hub plane is a plane perpendicular to the rotor shaft. Both cyclic pitch and flapping are present. It is the best reference plane to use for hingeless or bearingless rotors, dynamics, aeroelasticity and vibrations. It is also suitable for articulated rotors with a large hinge offset.
- Airfoil $\theta(\psi)$ = measured from hub plane



Side view

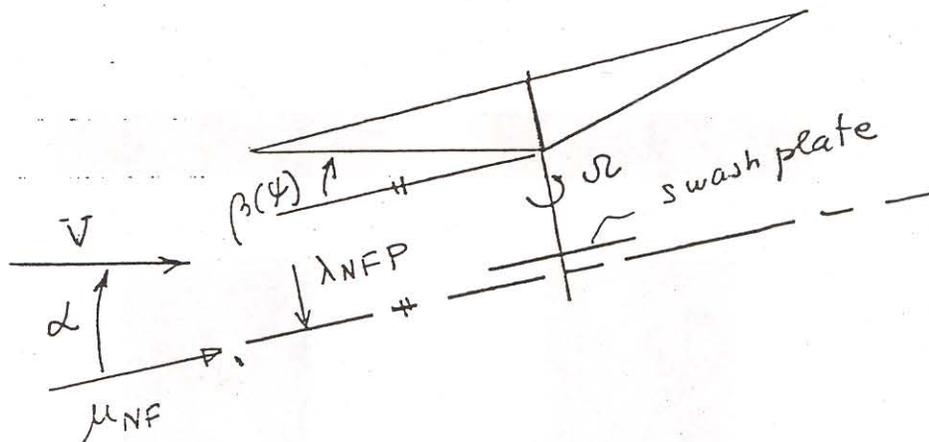
- 2) No feathering plane (NFP)

When using this plane there is no cyclic pitch. Good for performance calculations of articulated rotors.

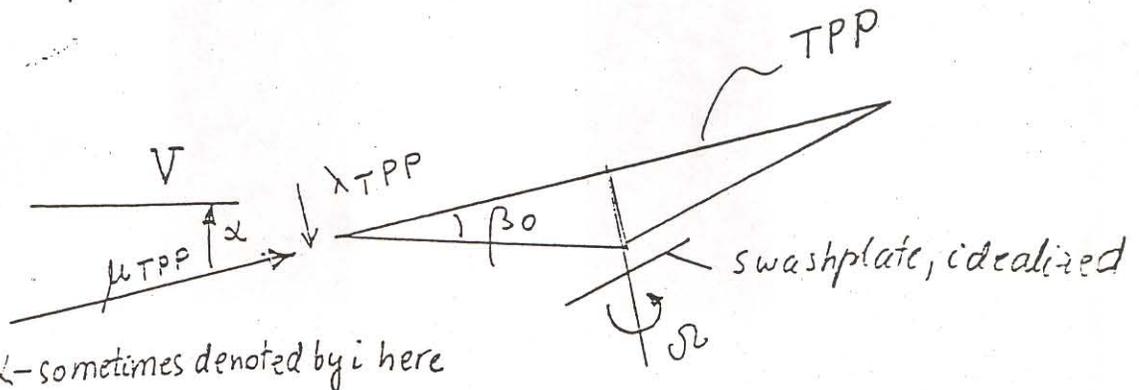
It is useless for hingeless and bearingless rotors. It is not in widespread use in modern helicopter analysis, thus it represents a sentimental link with the past. (see next page). This plane coincides with the swashplate plane.

Airfoil θ_0 (no cyclic) measured from NFP.

(64)



3) Tip Path Plane (TPP)



No flapping component. Airfoil angle $\theta(\psi)$. Good for inflow calculations, unsteady airloads on the rotor. Frequently, eventually one has to switch to the hub plane through an appropriate transformation.

4) Stability Plane (stability & control all $\frac{1}{2}$ plane)

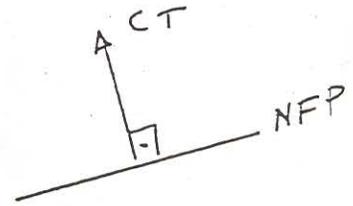
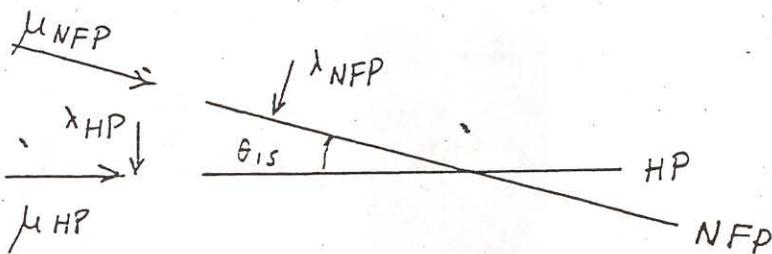
Is an inertial axis system, and is useful when motions are referred to inertial space.

(65)

Change in Reference Planes (Cyclic pitch \neq relative small) *HP ASSUME

When using the NFP we have derived the expression

$$C_T = \frac{\sigma_a}{2} \left[\frac{\theta_0}{3} \left(1 + \frac{3}{2} \mu_{NFP}^2 \right) - \frac{\lambda_{NFP}}{2} \right] \quad (1)$$



from Figure above

$$\mu_{NFP} \cos \theta_{15} - \lambda_{NFP} \sin \theta_{15} = \mu_{HP}$$

$$\lambda_{NFP} \cos \theta_{15} + \mu_{NFP} \sin \theta_{15} = \lambda_{HP}$$

Assume $\mu_{HP} \approx \mu_{NFP}$

$$\lambda_{HP} \approx \mu_{NFP} \theta_{15} + \lambda_{NFP}$$

for small angles

where $\sin \theta_{15} = \theta_{15}$

$\cos \theta_{15} = 1.0$

$$\lambda_{NFP} = \lambda_{HP} - \mu_{NFP} \theta_{15} = \lambda_{HP} - \mu_{HP} \theta_{15} \quad (2)$$

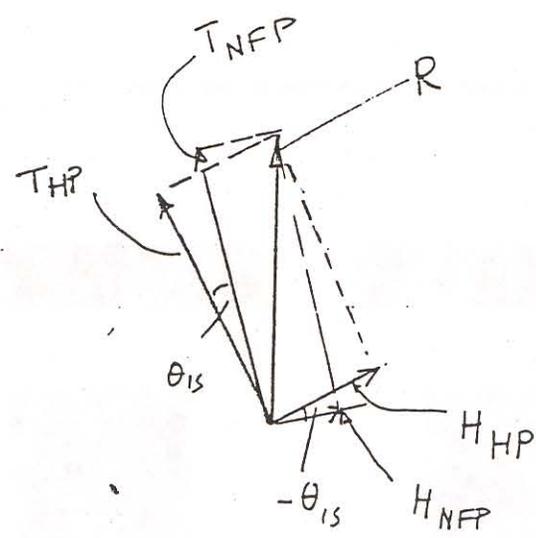
Combining Eqs (1) and (2)

$$C_T = \frac{\sigma_a}{2} \left[\frac{\theta_0}{3} \left(1 + \frac{3}{2} \mu_{HP}^2 \right) + \frac{\mu_{HP} \theta_{15}}{2} - \frac{\lambda_{HP}}{2} \right] \quad (3)$$

Next consider some other forces acting on the rotor as shown on the next page

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$$T_{NFP} = T_{HP} \cos \theta_{15} + H_{HP} \sin \theta_{15} \approx T_{HP} + H_{HP} \theta_{15}$$

H force is 2 orders of magnitude smaller than T , so $H_{HP} \theta_{15}$ is three orders of magnitude smaller than T

and is therefore negligible, thus

$$T_{HP} \approx T_{NFP}$$