

## Trim Analysis

The purpose of a trim analysis is to determine the rotor control settings, the rotor disk orientation and the overall helicopter orientation for the prescribed flight or test conditions. In general the two basic trim procedures that have been used are:

(a) Propulsive trim - in this trim procedure, which simulates actual forward flight conditions, the rotor is maintained at a fixed value of the thrust coefficient  $C_T$  with forward flight. Horizontal and vertical force equilibrium is maintained. Furthermore zero pitching and rolling moments are also enforced.

A more comprehensive version of this trim procedure accounts also for the effect of the tail rotor. In this case yaw moment and side force equilibrium are also maintained

(b) Wind tunnel trim or moment trim - this trim procedure, which simulates conditions under which a rotor would be tested in the wind tunnel, pitching and rolling moments on the rotor are maintained at zero. Force equilibrium is not required for this case because the rotor is mounted on a supporting structure in the wind-

tunnel (known as the test stand) and it provides the appropriate force reactions that are needed for equilibrium. Yaw moment equilibrium is also usually provided by the test stand.

### Propulsive Trim

This flight condition simulates straight and level flight at a fixed advance ratio  $\mu$ . The weight of the helicopter is a constant quantity  $W$ . The control parameters that can be used to achieve trim are provided below:

- ② • Collective pitch,  $\theta_0$ , controls the magnitude of the rotor thrust. The collective is changed by a lever operated by the pilot's left hand, with an upward pulling motion required for an increase in thrust.
- Cyclic pitch components,  $\theta_{ls}$  and  $\theta_{lc}$ , control the pitching and rolling moments on the rotor. Lateral cyclic pitch ( $\theta_{lc}$ ) produces rotor disk tilt left and right. This changes the orientation of the rotor thrust vector, producing both a side force and a rolling moment. Longitudinal cyclic ( $\theta_{ls}$ ) imparts a once-per-revolution cyclic pitch change to the blades, such that the rotor disk can be tilted fore and aft. Both lateral and longitudinal cyclic are controlled by the pilot using a cyclic stick

(similar to the conventional stick on a fixed-wing aircraft) which is held in the pilot's right hand.

- Yaw - is controlled by using tail rotor thrust. The pilot has a set of pedals, which are operated by the pilot's feet, just like the rudder of a fixed-wing aircraft. By pushing the pedals in the required direction, the collective pitch on the tail rotor is changed, producing a change in the tail rotor thrust, and inducing yaw to the right or left. The tail rotor has no cyclic pitch control and therefore it operates essentially in an untrimmed mode.

Trim is a complicated and to some extent nonlinear problem, particularly at high advance ratios. There are also many cross coupling effects which are unavoidable, and usually cross coupling effects are minimized by the appropriate design of the electronics (automatic pilot, control-feedback and mechanical or structural design of the rotor)

To discuss trim in the framework of a class, several approximations have to be made to yield a tractable problem from a computational (actually algebraic) point of view. The simplifying assumptions made are listed next.

- Use blade element theory for the calculation of the aerodynamic loads, with certain assumptions for the wake inflow to calculate the blade flapping and control angles.

The principal approximation here is the neglect of blade flexibility and unsteady aerodynamics.

- Neglect reverse flow and dynamic stall, these effect are neglected here for convenience (from a calculation-point of view) and should not be neglected when dealing with an actual problem.
- It is rare that a trim solution can be obtained in closed form, usually an iterative/numerical procedure is used to obtain trim.

Mathematically when the propulsive trim problem is considered the known (or given) quantities are:

$W$  - weight of the helicopter.

$\mu$  = the advance ratio in straight and level flight

together with the geometry of the vehicle.

The quantities that have to be obtained in the process of solution are :

$\theta_0, \theta_{IC}, \theta_{IS}, \alpha$

$\beta_0, \beta_{IS}, \beta_{IC}$  and the sideslip angle  $\gamma$   
and the pitch and roll angles  $\theta_F, \phi_F$

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The geometry of the problem is illustrated in Figs. 1 and 2.

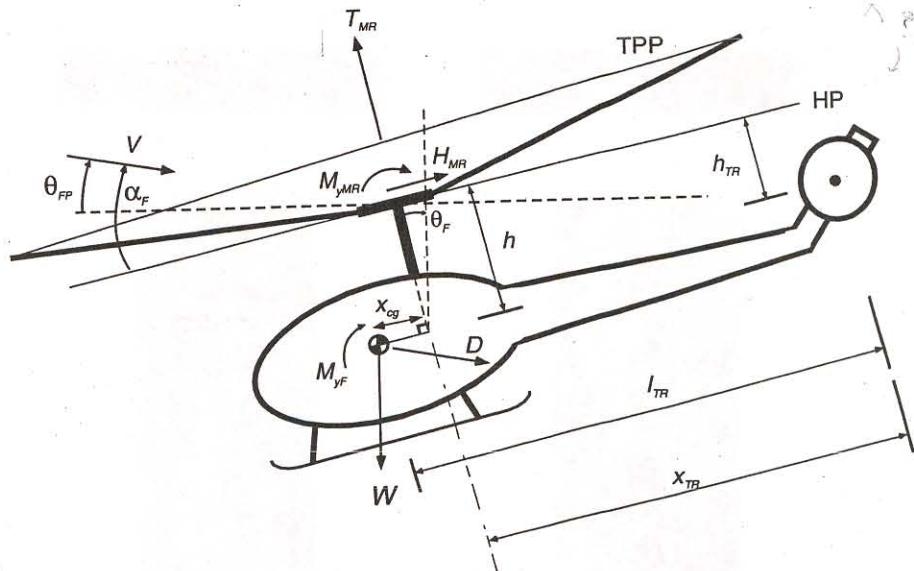


Fig. 1 Longitudinal forces and moments acting during propulsive trim

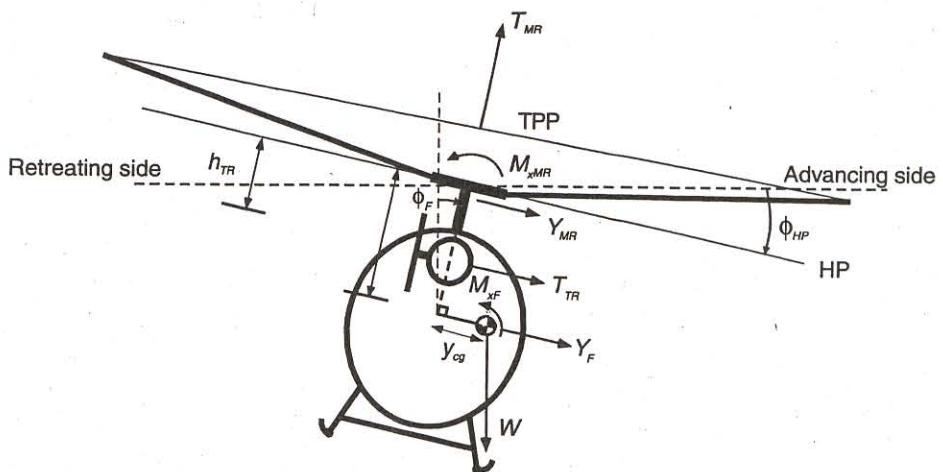


Fig. 2 Lateral forces and moments acting during propulsive trim.

## T-6

Figure 1 shows the forces and moment acting on the helicopter in the longitudinal plane and Fig. 2 shows these quantities for the lateral plane. It is convenient to decompose each contribution (force or moment) according to its source. Thus the various contributions are identified by subscripts, such as:

MR - main rotor; F - fuselage; HT - horizontal tail; VT - vertical tail, TR - tail rotor, and other sources are identified by - O . For example the pitching moment can be written as:

$$M = M_{MR} + M_F + M_{HT} + M_{VT} + M_{TR} + M_O$$

In the description of the trim procedure that follows the hub plane (HP) is used as the reference plane. The flight path (FP) angle is  $\theta_{FP}$ . It will be also assumed that there is no sideslip, thus the fuselage side force  $Y_F$  contribution can be assumed to be negligible. Also, it will be assumed for simplicity that there are no contributions from the horizontal and vertical tails.

Vertical force equilibrium yields:

$$W - T_{MR} \cos \alpha_F \cos \phi_F + D \sin \theta_{FP} - H_{MR} \sin \alpha_F + Y_{MR} \sin \phi_F + Y_{TR} \sin \phi_F = 0 \quad (1)$$

Longitudinal force equilibrium yields:

$$D \cos \Theta_{FP} + H_{MR} \cos \alpha_F - T_{MR} \sin \alpha_F \cos \phi_F = 0 \quad (2)$$

Lateral force equilibrium

$$Y_{MR} \cos \phi_F + T_{TR} \cos \phi_F + T_{MR} \cos \alpha_F \sin \phi_F = 0 \quad (3)$$

where  $T_{TR}$  - is the thrust of the tail rotor.

Pitching moment equilibrium about the hub yields :

$$M_{YMR} + M_{YF} - W(x_{cg} \cos \alpha_F - h \sin \alpha_F)$$

$$- D(h \cos \alpha_F + x_{cg} \sin \alpha_F) = 0 \quad (4)$$

Rolling moment equilibrium about the hub yields :

$$M_{XMR} + M_{XF} + T_{TR} h_{TR} + W(h \sin \phi_F - y_{cg} \cos \phi_F) = 0 \quad (5)$$

Torque equilibrium about the shaft yields:

$$Q_{MR} - Y_{TR} l_{TR} = 0 \quad (6)$$

Using small angle assumptions allows one to simplify equations (1) - (6), i.e. all angles used in the Eqs (1)-(6) are small their  $\sin \frac{\alpha}{\pi} \alpha_F$ ;  $\cos \alpha_F = 1$

$$\underline{W - T_{MR}} + \underline{D \Theta_{FP}} - \underline{H_{MR} \alpha_F} = 0$$

The underlined terms in the last equation are negligible compared to the first two, thus

$$W - T_{MR} = 0 \quad (7)$$

Carrying out similar operations on the other equations

yields

$$D + H_{MR} - T_{MR} \alpha_F = 0 \quad (8)$$

$$Y_{MR} + T_{TR} + T_{MR} \phi_F = 0 \quad (9)$$

$$M_{YMR} + M_{YF} - W(x_{cg} - h\alpha_F) - D(h + x_{cg}\alpha_F) = 0$$

assuming  $x_{cg}\alpha_F \ll h$  allows one to simplify  
the last equation

$$M_{YMR} + M_{YF} + W(h\alpha_F - x_{cg}) - hD = 0 \quad (10)$$

$$M_{XMR} + M_{XF} + T_{TR} h_{TR} + W(h\phi_F - y_{cg}) = 0 \quad (11)$$

$$Q_{MR} - T_{TR} l_{TR} = 0 \quad (12)$$

where  $Q_{MR}$  is the torque of the main rotor.

Recall that the thrust is simply the average of the blade lift during one revolution, multiplied by the number of blades  $N_b$ , thus

$$T_{MR} = \frac{N_b}{2\pi} \int_0^{2\pi} \int_0^R dF_z \, d\psi \quad (13)$$

where  $dF_z = dL \, dr$ .

The thrust coefficient can be written as

$$C_{TMR} = \frac{T_{MR}}{\rho (\pi R^2)(\Omega R)^2} = \frac{\alpha \sigma}{2} \frac{1}{2\pi} \int_0^{2\pi} \int_0^R \left[ \left( \frac{U_T}{\Omega R} \right)^2 - \left( \frac{U_p}{\Omega R} \right) \left( \frac{U_T}{\Omega R} \right) \right] dr \, d\psi \quad (14)$$

Because of the complexity of  $U_p, U_T$  and  $\theta$  this equation must be usually treated numerically.

However, by assuming uniform inflow, that is

$\pi(r, \psi) = \lambda = \text{const}$  over the disk, and

$C - \text{chord const}$ ;  $C_d = C_{d0}$ , and linear twist

given by  $\theta_{tw}$ , the thrust coefficient can be expressed as

$$C_{TMR} = \frac{C_a}{2} \left[ \frac{\theta_0}{3} \left( 1 + \frac{3}{2} \mu^2 \right) + \frac{\theta_{tw}}{4} \left( 1 + \mu^2 \right) + \mu \frac{\theta_{IS}}{2} - \frac{\lambda}{2} \right] \quad (15)$$

The rotor torque, side force, drag force, and moments about the appropriate axes can be computed by a similar manner.

The rotor drag force, also denoted often as the the H-force, described on page 69 of the class notes can be written as

$$H_{MR} = \frac{N_b}{2\pi} \int_0^{2\pi} \int_0^R [dF_x \sin \psi + dF_R \cos \psi] d\psi \quad (16)$$

or in nondimensional form

$$\begin{aligned} C_{HMR} = & \frac{C_a}{2} \frac{1}{2\pi} \int_0^{2\pi} \int_0^R \left\{ \sin \psi \left[ \left( \frac{U_p}{J\omega R} \right) \left( \frac{U_I}{J\omega R} \right) \theta - \right. \right. \\ & \left. \left. \left( \frac{U_p}{J\omega R} \right)^2 \right] - \beta \cos \psi \left[ \left( \frac{U_I}{J\omega R} \right)^2 \theta - \left( \frac{U_p}{J\omega R} \right) \left( \frac{U_I}{J\omega R} \right) \right] \right. \\ & \left. + \frac{Cd\theta}{a} \left( \frac{U_I}{J\omega R} \right)^2 \sin \psi \right\} . \end{aligned} \quad (17)$$

The rotor side force, also known as the Y-force, see page 70 of the class notes, is given by

$$Y_{MR} = \frac{N_b}{2\pi} \int_0^{2\pi} \int_0^R [-dF_x \cos \psi + dF_R \sin \psi] d\psi \quad (18)$$

The rotor torque is given by

$$Q_{MR} = \frac{N_b}{2\pi} \int_0^{2\pi} \int_0^R y dF_x d\psi \quad (19)$$

The rotor rolling moment is given by.

$$M_{xMR} = \frac{Nb}{2\pi} \int_0^{2\pi} \int_0^R y dF_2 \sin \psi d\psi \quad (20)$$

The rotor pitching moment is given by

$$M_{yMR} = \frac{Nb}{2\pi} \int_0^{2\pi} \int_0^R y dF_2 \cos \psi d\psi \quad (21)$$

(Clearly, eqs (18)-(21) can be manipulated further.

To complete the formulation of the trim problem one need appropriate relations for the inflow on the main rotor and the tail rotor. Assuming uniform inflow distribution

$$\lambda = \tan \alpha + \frac{C_T}{2(\mu^2 + \lambda^2)}$$

$$\lambda_{MR} = \mu_{MR} \tan \alpha_{MR} + \frac{C_T}{2\sqrt{\mu_{MR}^2 + \lambda_{MR}^2}} \quad (22)$$

$$\lambda_{TR} = \mu_{TR} \tan \alpha_{TR} + \frac{C_{TTR}}{2\sqrt{\mu_{TR}^2 + \lambda_{TR}^2}} \quad (23)$$

In Eq (22) it is common practice to set

$$\alpha_{MR} = \alpha \quad (24)$$

and then  $\mu_{MR} = \mu$ .

However, for the tail rotor

$\mu_{TR} = \frac{V \cos \alpha_{TR}}{\omega_{TR} R_{TR}}$  where  $\alpha_{TR}$  is the disk angle of attack of the tail rotor. When the side-slip angle is zero  $\alpha_{TR} = 0$  and

$$\mu_{TR} = \frac{V}{\omega_{TR} R_{TR}} \quad (25)$$

Solution of the propulsive trim problem

implies solving (7), (8), (9), (10), (11), (12) together with

$$\lambda_{MR} = \mu \tan \alpha + \frac{c_T}{2 \sqrt{\mu^2 + \lambda_{MR}^2}} \quad (26)$$

and

$$\lambda_{TR} = \frac{c_{T TR}}{2 \sqrt{\mu_{TR}^2 + \lambda_{TR}^2}} \quad (27)$$

These vehicle equilibrium equations together with the inflow equations, can be written in the form of

$$\{F(\tilde{x})\} = 0 \quad (28)$$

where  $\tilde{x} = \{x\}$  is a vector of trim variables

$$\left\{ \begin{array}{l} \theta_0 \\ \theta_{1C} \\ \theta_{1S} \\ \lambda_{MR} \\ \lambda_{TR} \\ \theta_F \\ \phi_F \end{array} \right\}$$

Equation (28) can be solved numerically using a nonlinear algebraic equation solver, or a suitable iterative procedure.

$$F(T) = P_{\text{ext}} + P_{\text{drag}} + P_{\text{lift}}$$

etc page 81

CPD off simple example (blade as rigid surface)

Trim ... do not converge why? ... harder objective function & target

?? ... need better geometry

(T-12)

## Some Additional Comments on the Solution of the Trim Equations

When solving the trim equations the quantities  $H_{MR}$ ,  $Y_{MR}$  are provided on pages 69-70 of the notes. Thus

$$H_{MR} = \frac{b}{2\pi} \int_0^{2\pi} \int_0^R \frac{dH}{dr} dr d\psi \quad (29)$$

and from Eq(12) page 69

$$dH = dR \cos \psi + (\varphi dL + dD) \sin \psi - \beta dL \cos \psi$$

$$dR = \frac{1}{2} \rho V_R^2 C_D \cos \psi \quad ; \quad V_R = V \cos \alpha F \cos \psi$$

$dR$  - friction drag along the blade.

Similarly, from Eq(14) page 70

$$dY_{MR} = -(\varphi dL + dD) \cos \psi - \beta dL \sin \psi + dR \sin \psi$$

$$Y_{MR} = \frac{b}{2\pi} \int_0^{2\pi} \int_0^R \frac{dY_{MR}}{dr} dr d\psi \quad (30)$$

Equation (28) has a vector of seven quantities that need to be determined, from Eqs (7), (8), (9), (10), (11) and (12) i.e.

- 3 force equilibrium equations
- 3 moment equilibrium equations
- The inflow equation is an additional equation available.

Note that no statement was made about  $\beta_0$ ,  $\beta_{IS}$ ,  $\beta_{IC}$  - to evaluate these quantities, one needs to use Eq(32)-(4) pg 87 of the notes:

$$\beta_0(1+G) = \theta_0 \left( B + \frac{1}{2} \mu^2 E \right) - \lambda C + C \mu \theta_{IS} - \frac{1}{2} \mu D \beta_{IC}$$

$$G \beta_{IC} = \theta_{IC} \left( B + \frac{1}{4} \mu^2 E \right) - \beta_{IS} \left( A + \frac{1}{4} \mu^2 E \right) - \mu \beta_0 C$$

$$G \beta_{IS} = \theta_{IS} \left( B + \frac{3}{2} \mu^2 E \right) + \beta_{IC} \left( A - \frac{1}{4} \mu^2 E \right) + 2 \mu \theta_0 C - \lambda \mu E$$

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Another assumption made is that the TPP - tip path plane is parallel to the hub plane and

$$\theta_{FP} = 0 \quad \text{and} \quad \alpha_F = \theta_F$$

With these assumptions vertical force equilibrium yields

$$W - T_{MR} \cos \alpha_F \cos \phi_F + Y_{MR} \sin \phi_F - H_{MR} \sin \alpha_F + Y_{TR} \sin \phi_F = 0 \quad (1)$$

Longitudinal force equilibrium yields:

$$D + H_{MR} \cos \alpha_F - T_{MR} \sin \alpha_F \cos \phi_F = 0 \quad (2)$$

and lateral force equilibrium implies:

$$Y_{MR} \cos \phi_F + T_{TR} \cos \phi_F + T_{MR} \cos \alpha_F \sin \phi_F = 0 \quad (3)$$

where  $T_{TR}$  is the thrust of the tail rotor.

Pitching moment equilibrium about the hub yields:

$$M_{YMR} + M_{YF} - W(x_{cg} \cos \alpha_F - h \sin \alpha_F) - D(h \cos \alpha_F + x_{cg} \sin \alpha_F) = 0 \quad (4)$$

The last equation contains the assumption that the drag due to the fuselage acts at the c.g.

Rolling moment equilibrium about the hub yields:

$$M_{XMR} + M_{XF} + T_{TR} h_{TR} + W(h \sin \phi_F - y_{cg} \cos \phi_F) = 0 \quad (5)$$

finally the torque moment equilibrium about the shaft, yields:

$$Q_{MR} - Y_{TR} l_{TR} = 0 \quad (6)$$

Using small angle assumptions allows one to simplify Eqs (1)-(6) by assuming  $\sin \alpha_F \approx \alpha_F$ ;  $\cos \alpha_F \approx 1$

From Eq(1)

$$W - \underline{T_{MR}} - \underline{Y_{MR} \phi_F} - \underline{H_{MR} \alpha_F} + \underline{Y_{TR} \phi_F} = 0$$

the underlined terms in the last equation are negligible compared to the first two, thus:

$$W - T_{MR} = 0 \quad (7)$$

Carrying out similar operations on the other equations

yields

$$D + H_{MR} - \bar{T}_{MR} \alpha_F = 0 \quad (8)$$

$$Y_{MR} + \bar{T}_{TR} + T_{MR} \phi_F = 0 \quad (9)$$

$$M_{YMR} + M_{YF} - W(x_{cg} - h\alpha_F) - D(h + x_{cg}\alpha_F) = 0 \quad (10)$$

assuming  $x_{cg}\alpha_F \ll h$  allows one to simplify the last equation

$$M_{YMR} + M_{YF} + W(h\alpha_F - x_{cg}) - hD = 0 \quad (10)$$

$$M_{XMR} + M_{XF} + T_{TR} h_{TR} + W(h\phi_F - Y_{cg}) = 0 \quad (11)$$

$$Q_{MR} - T_{TR} l_{TR} = 0 \quad (12)$$

where  $Q_{MR}$  is the torque of the main rotor.

Recall that the thrust is simply the average of the blade lift during one revolution, multiplied by the number of blades  $N_b$ , thus

$$T_{MR} = \frac{N_b}{2\pi} \int_0^{2\pi} \int_0^R dF_z \, d\psi \quad (13)$$

where  $dF_z = dL \, dr$ .

The thrust coefficient can be written as

$$C_{TMR} = \frac{\bar{T}_{MR}}{\rho(\pi R^2)(\sigma R)^2} = \frac{aG}{2} \frac{1}{2\pi} \int_0^{2\pi} \int_0^R \left[ \left( \frac{U_T}{\sigma R} \right)^2 - \left( \frac{U_p}{\sigma R} \right) \left( \frac{U_T}{\sigma R} \right) \right] dr d\psi \quad (14)$$

Because of the complexity of  $U_p$ ,  $U_T$  and  $\theta$  this equation must be usually treated numerically.

However, by assuming uniform inflow, that is

$\sigma(r, \psi) = \lambda = \text{const}$  over the disk, and

$C$ -chord const;  $C_d = C_{d0}$ , and linear twist