

Major Course Contents

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Part 2: Longitudinal Vehicle Dynamics

Part 3: Vehicle Control Systems

Part 4: Suspensions

Part 5: Three-dimensional rigid body model of a vehicle

Major Course Contents

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- 2.2 Engine model
- 2.3 Transmission
- 2.4 Tire models
- 2.5 Brake

Part 3: Vehicle Control Systems

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Part.2

Longitudinal Vehicle Dynamics

1. Longitudinal Dynamic Model
2. Engine model
3. Transmission
4. Tire models
5. Brake

4. Tire Model

4.1 Pacejka Tire Model

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4.1.6 Self Aligning Moment

4.2 Dugoff's Tire Model

4. Tire Model

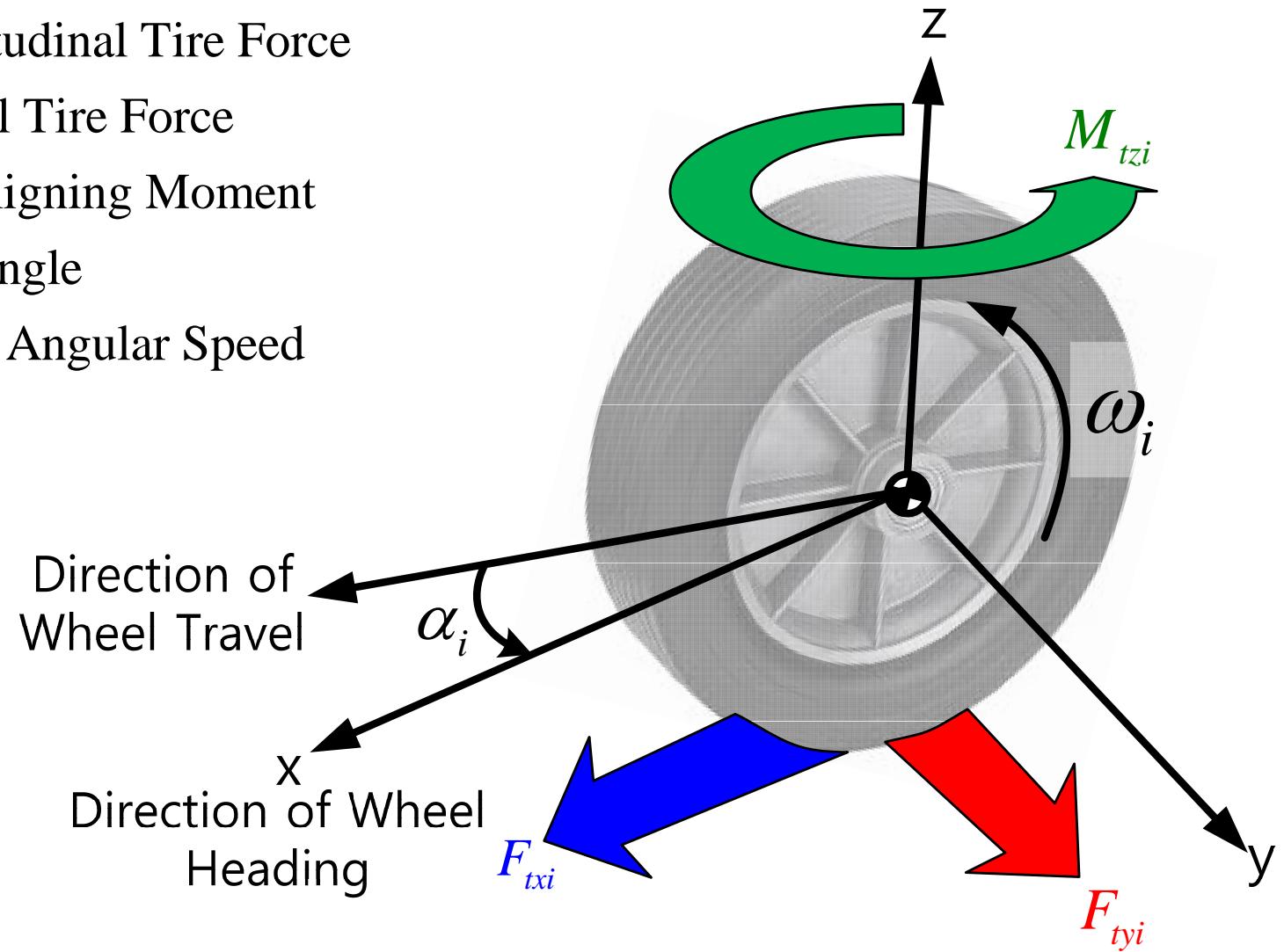
F_{txi} = Longitudinal Tire Force

F_{tyi} = Lateral Tire Force

M_{tzi} = Self Aligning Moment

α_i = Slip Angle

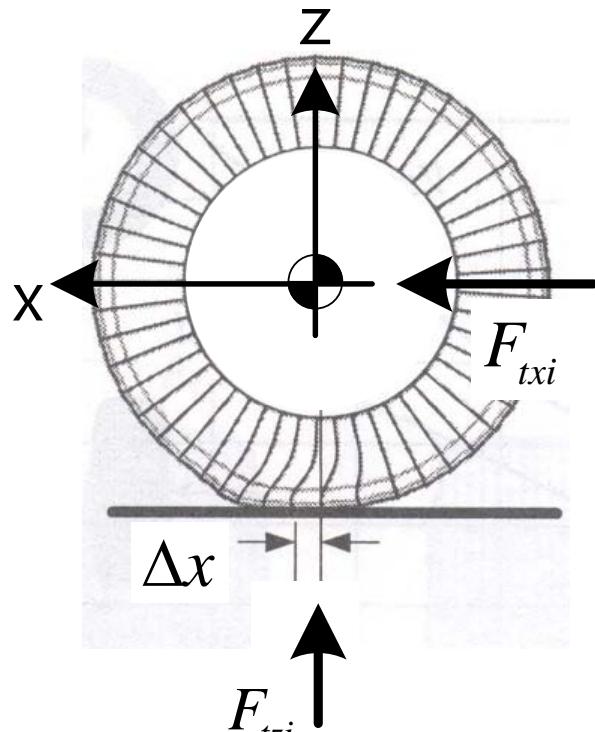
ω_i = Wheel Angular Speed



4. Tire Model

- Tire Deformation
- Longitudinal Tire Force

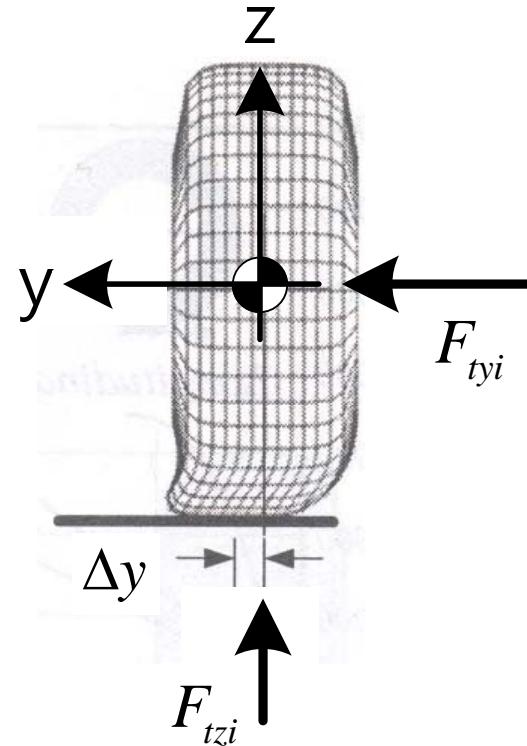
$$F_{txi} \propto \begin{pmatrix} \Delta x \\ F_{tzi} \end{pmatrix}$$



< Longitudinally tire deformation >

- Lateral Tire Force

$$F_{tyi} \propto \begin{pmatrix} \Delta y \\ F_{tzi} \end{pmatrix}$$



< Laterally tire deformation >

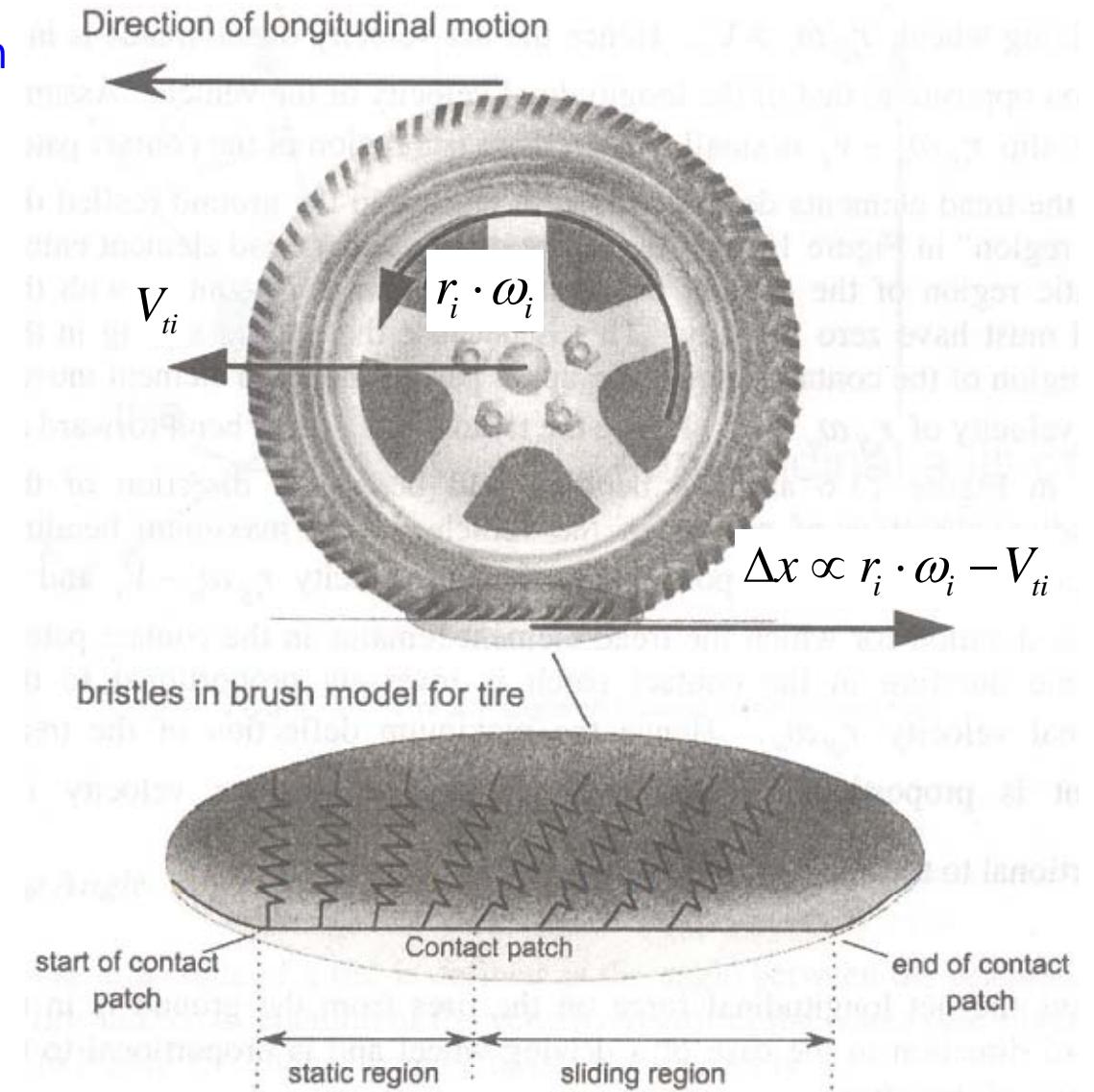
4. Tire Model

- Longitudinally Tire Deformation
- Longitudinal Tire Force

$$F_{txi} \propto \left(\frac{\Delta x}{F_{tzi}} \right)$$

- Longitudinally Tire Deformation

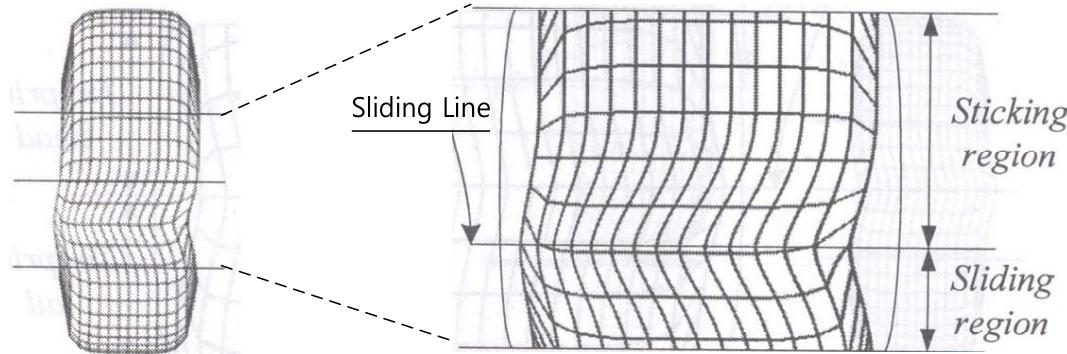
$$\Delta x \propto r_i \cdot \omega_i - V_{ti}$$



REF: Rajesh Rajamani, "Vehicle Dynamics and Control", pp391 ~ 394, Springer, 2006

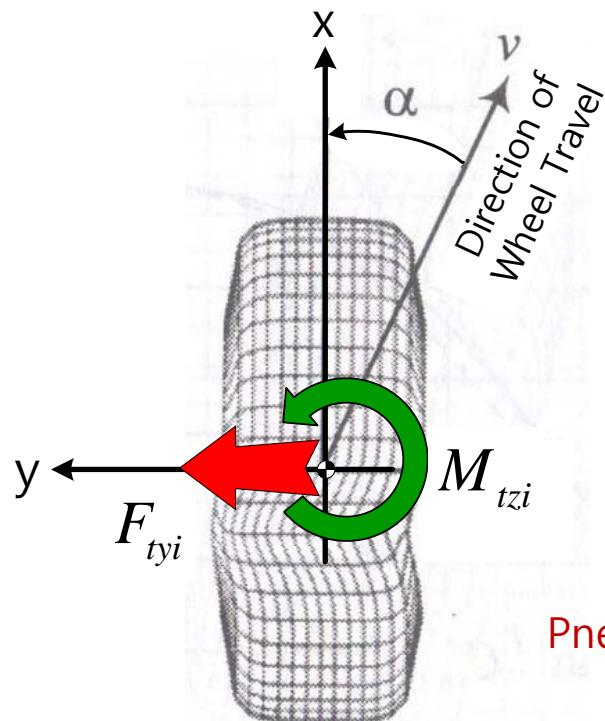
4. Tire Model

- Laterally Tire Deformation



< Bottom view of a laterally deflected and turning tire >

- Lateral Tire Force and Self Aligning Moment



- Lateral Tire Force

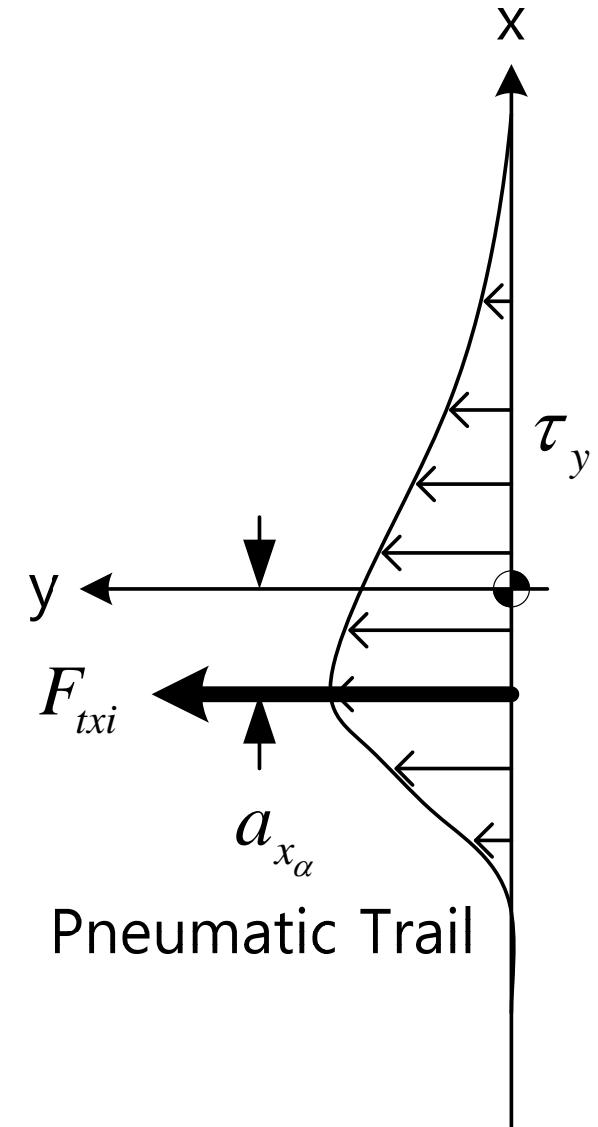
$$F_{tyi} = \int \tau_y dA$$

- Self Aligning Moment

$$M_{tzi} = F_{tyi} \cdot a_{x_\alpha}$$

Pneumatic Trail

- Shear Stress Distribution



4. Tire Model

4.1 Pacejka Tire Model

4.1.1 Slip Angle

4.1.2 Lateral Tire Model

4.1.3 Slip Ratio

4.1.4 Longitudinal Tire Model

4.1.5 Combined Tire Model

4.1.6 Self Aligning Moment

4.2 Dugoff's Tire Model

4.1.1 Slip Angle

- The angle between the orientation of the tire and the orientation of the Wheel

$$\alpha_i = \delta_i - \zeta_i$$

Where, α_i = Tire Slip Angle at i -th Wheel

δ_i = Steering Angle at i -th Wheel

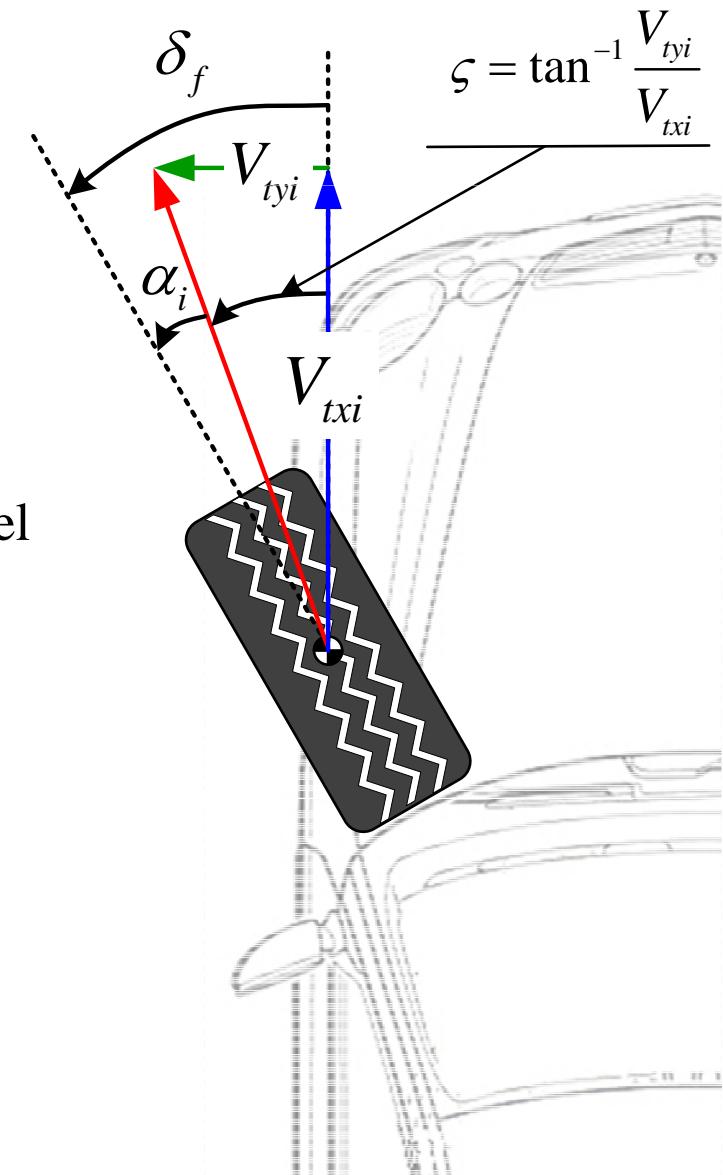
ζ_i = Angle between V_{txi} and V_{tyi} at i -th Wheel

$$\tan(\zeta_1) = \frac{v_y + l_f \cdot \dot{\psi}}{v_x - t_w \cdot \dot{\psi}}$$

$$\tan(\zeta_2) = \frac{v_y + l_f \cdot \dot{\psi}}{v_x + t_w \cdot \dot{\psi}}$$

$$\tan(\zeta_3) = \frac{v_y - l_r \cdot \dot{\psi}}{v_x - t_w \cdot \dot{\psi}}$$

$$\tan(\zeta_4) = \frac{v_y - l_r \cdot \dot{\psi}}{v_x + t_w \cdot \dot{\psi}}$$



4.1.2 Lateral Tire Model

- Lateral Tire Force at the i-th Wheel

$$F_{tyi} = D_y \sin(C_y \tan^{-1}(B_y \Phi_y)) + S_{vy}$$

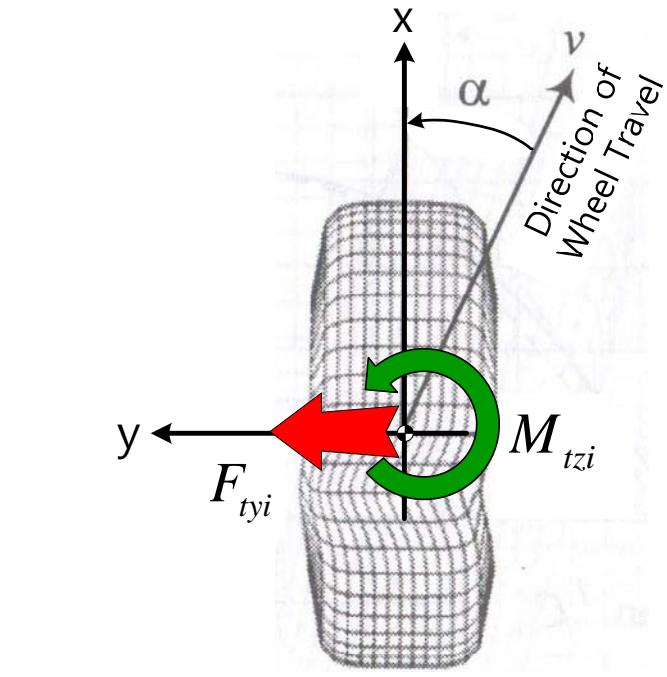
Where, $\Phi_y = (1 - E_y)(\alpha_i + S_{hy}) + \frac{E_y}{B_y} \tan^{-1}(B_y(\alpha_i + S_{hy}))$

$$B_y = 0.22 + \frac{5200 - F_{tzi}}{40000} \quad C_y = 1.26 - \frac{F_{tzi} - 5200}{32750}$$

$$D_y = -0.00003 \cdot F_{tzi}^2 + 1.0096 \cdot F_{tzi} - 22.73$$

$$E_y = -1.6$$

$$S_{hy} = 0$$



$S_{vy} = 0$ < Lateral Tire Force >

- Normal Tire Force at the i-th Wheel

$$F_{tz1} = K_{t1} \cdot (r_{1o} - r_1) + \frac{m \cdot l_r}{2(l_f + l_r)} \cdot g$$

$$F_{tz2} = K_{t2} \cdot (r_{2o} - r_2) + \frac{m \cdot l_r}{2(l_f + l_r)} \cdot g$$

$$F_{tz3} = K_{t3} \cdot (r_{3o} - r_3) + \frac{m \cdot l_f}{2(l_f + l_r)} \cdot g$$

$$F_{tz4} = K_{t4} \cdot (r_{4o} - r_4) + \frac{m \cdot l_f}{2(l_f + l_r)} \cdot g$$

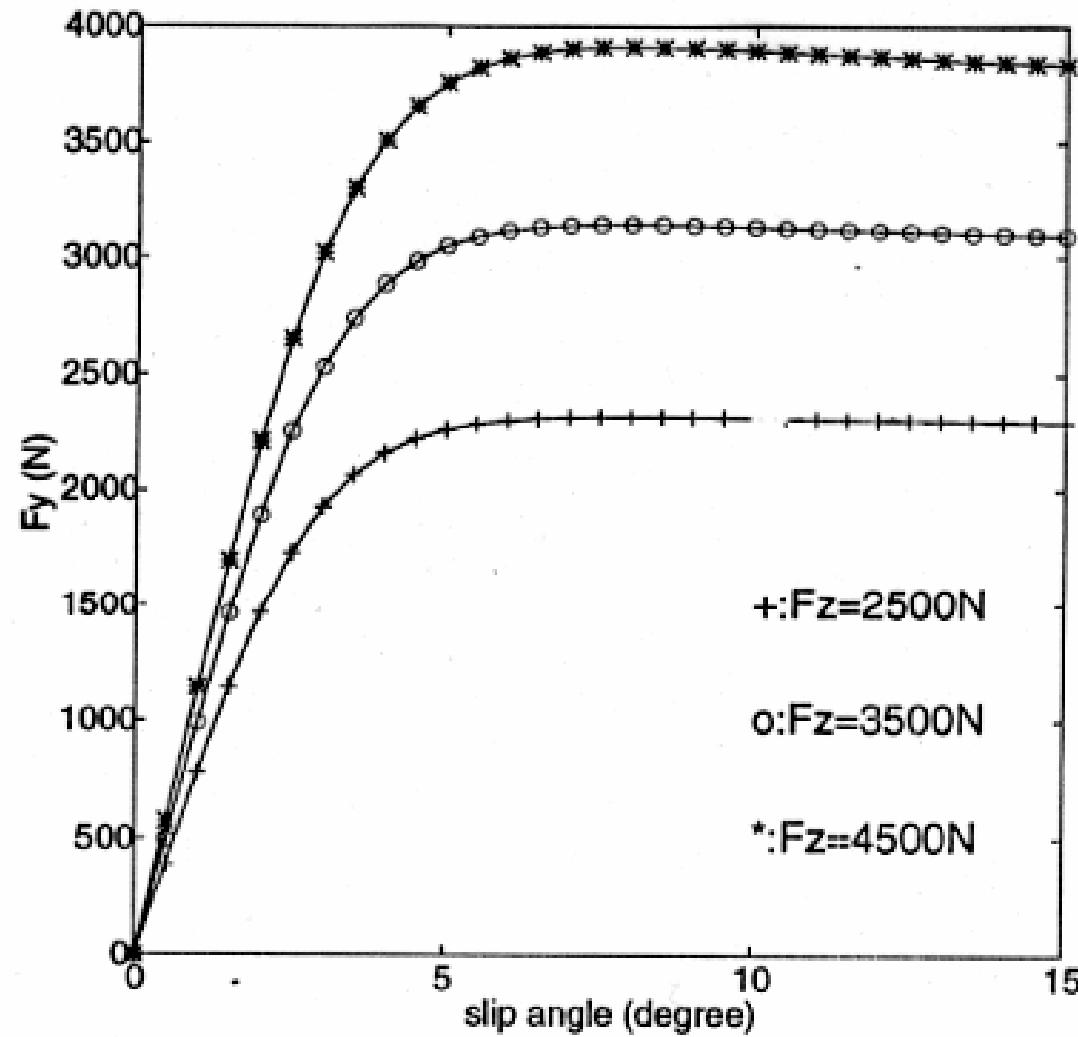
Where, r_{io} = Original Radius of the Tire

r_i = Effective Rolling Radius of the i -th Wheel

K_{ti} = Tire Stiffness at i -th Wheel

4.1.2 Lateral Tire Model

- Slip Angle versus Lateral Tire Force Curve



4.1.3 Slip Ratio

- During Braking

$$\lambda_i = \frac{r_i \cdot \omega_i - V_{ti} \cdot \cos(\alpha_i)}{V_{ti} \cdot \cos(\alpha_i)}$$

- During Traction

$$\lambda_i = \frac{r_i \cdot \omega_i - V_{ti} \cdot \cos(\alpha_i)}{r_i \cdot \omega_i}$$

Where, ω_i = Angular Velocity of the i -th Wheel

r_i = Tire Radius of the i -th Wheel

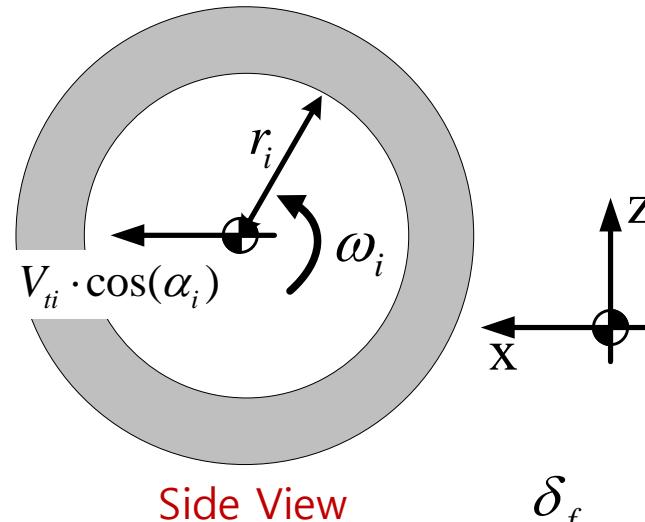
α_i = Tire Slip Angle at i -th Wheel

$$V_{t1} = \sqrt{(v_y + l_f \cdot \dot{\psi})^2 + (v_x - t_w \cdot \dot{\psi})^2}$$

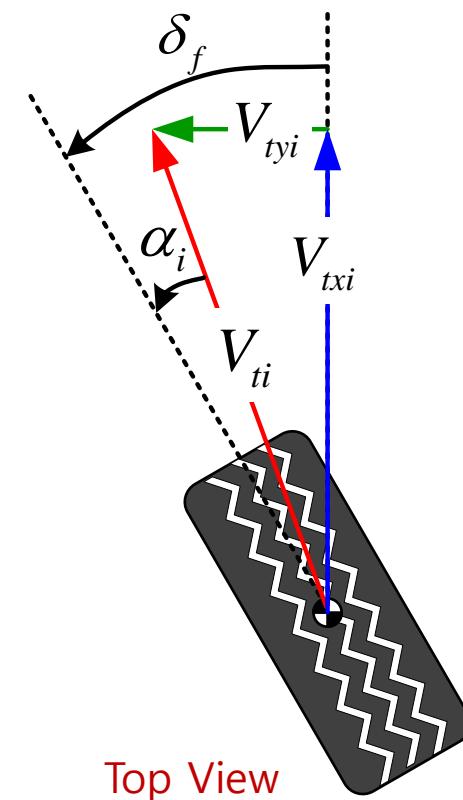
$$V_{t2} = \sqrt{(v_y + l_f \cdot \dot{\psi})^2 + (v_x + t_w \cdot \dot{\psi})^2}$$

$$V_{t3} = \sqrt{(v_y - l_r \cdot \dot{\psi})^2 + (v_x - t_w \cdot \dot{\psi})^2}$$

$$V_{t4} = \sqrt{(v_y - l_r \cdot \dot{\psi})^2 + (v_x + t_w \cdot \dot{\psi})^2}$$



Side View



Top View

4.1.4 Longitudinal Tire Model

- Longitudinal Tire Force at the i-th Wheel

$$F_{txi} = D_x \sin(C_x \tan^{-1}(B_x \Phi_x)) + S_{vx}$$

Where, $\Phi_x = (1 - E_x)(\lambda_i + S_{hx}) + \frac{E_x}{B_x} \tan^{-1}(B_x(\lambda_i + S_{hx}))$

- During Traction ($\lambda_i > 0$)

$$B_x = 22 + \frac{F_{tzi} - 1940}{645}$$

$$E_x = -3.6$$

$$C_x = 1.35 - \frac{F_{tzi} - 1940}{16125}$$

$$S_{hx} = 0$$

$$D_x = 2000 + \frac{F_{tzi} - 1940}{0.956}$$

$$S_{vx} = 0$$

- During Braking ($\lambda_i \leq 0$)

$$B_x = 22 + \frac{F_{tzi} - 1940}{430}$$

$$E_x = 0.1$$

$$C_x = 1.35 - \frac{F_{tzi} - 1940}{16125}$$

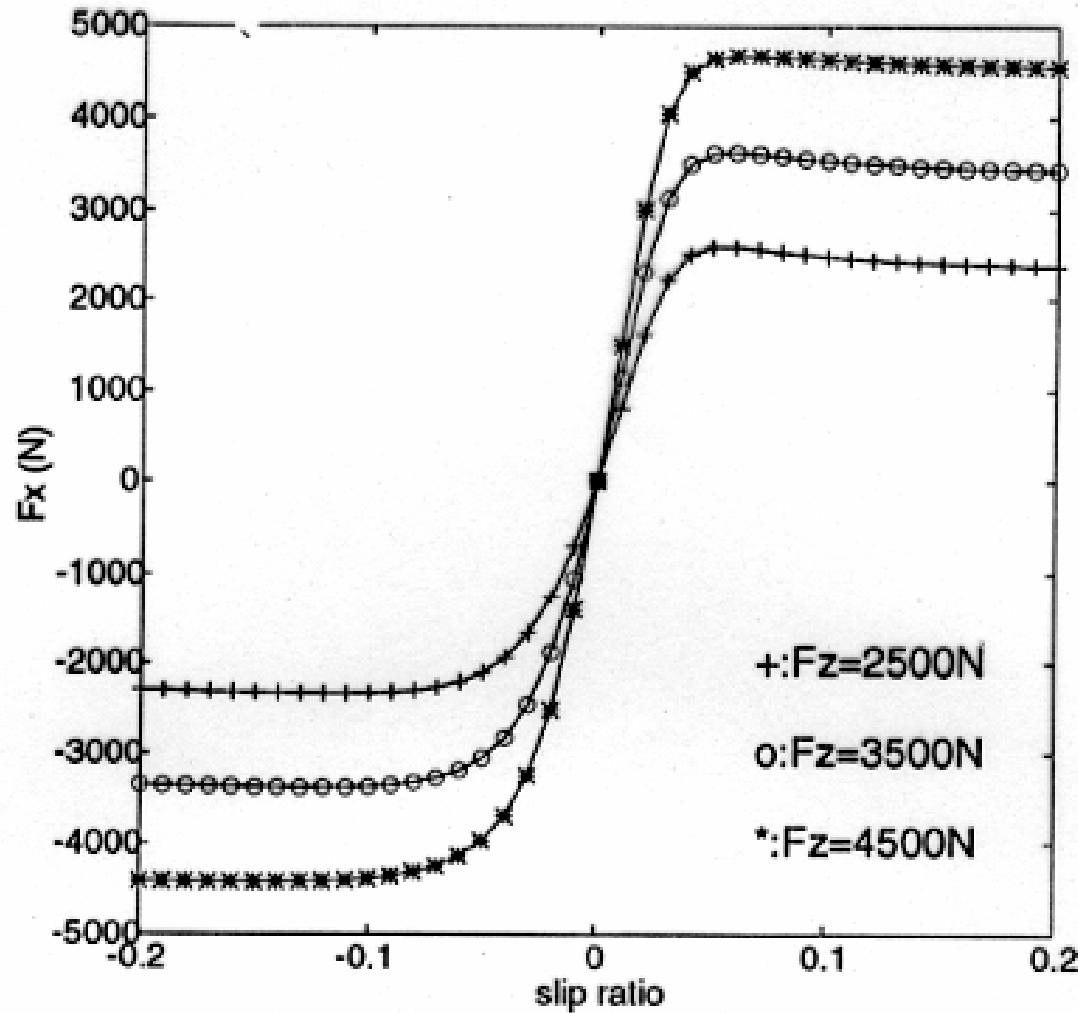
$$S_{hx} = 0$$

$$D_x = 1750 + \frac{F_{tzi} - 1940}{0.956}$$

$$S_{vx} = 0$$

4.1.4 Longitudinal Tire Model

- Slip Ratio versus Longitudinal Tire Force Curve



4.1.5 Combined Tire Model

- Pacejka Tire Model

1) Longitudinal Tire Model: $F_{txi} = F_{tx0}(\lambda_i, F_{tzi})$

2) Lateral Tire Model: $F_{tyi} = F_{ty0}(\alpha_i, F_{tzi})$

▲ There is no correction between Longitudinal and Lateral Tire Model

- Normalized Slip

1) Normalized Slip Ratio: $\lambda_i^* = \frac{\lambda_i}{\lambda_m}$

Where, $\lambda_m = \begin{cases} 0.058 & \text{if } (\lambda > 0) \\ -0.1 & \text{elsewhere} \end{cases}$

2) Normalized Slip Angle: $\alpha_i^* = \frac{\alpha_i}{\alpha_m}$

$$\alpha_m = 6.3 + \frac{F_{tzi} - 650}{3500}$$

- Correction Factor: $\sigma_i^* \triangleq \sqrt{(\lambda_i^*)^2 + (\alpha_i^*)^2}$

- Combined Tire Model based on Pacejka Tire Model

$$F_{txi} = \frac{\lambda_i^*}{\sigma_i^*} F_{tx0}(\sigma_i^* \times \lambda_m, F_{tzi})$$

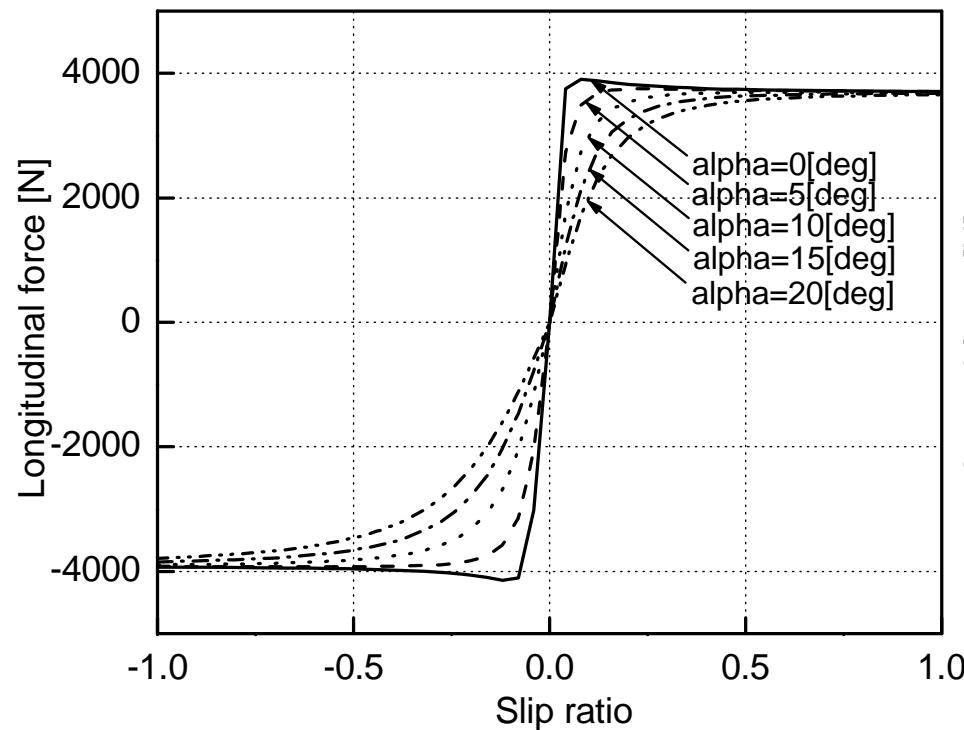
$$F_{tyi} = \frac{\alpha_i^*}{\sigma_i^*} F_{ty0}(\sigma_i^* \times \alpha_m, F_{tzi})$$

4.1.5 Combined Tire Model

- Combined Tire Force

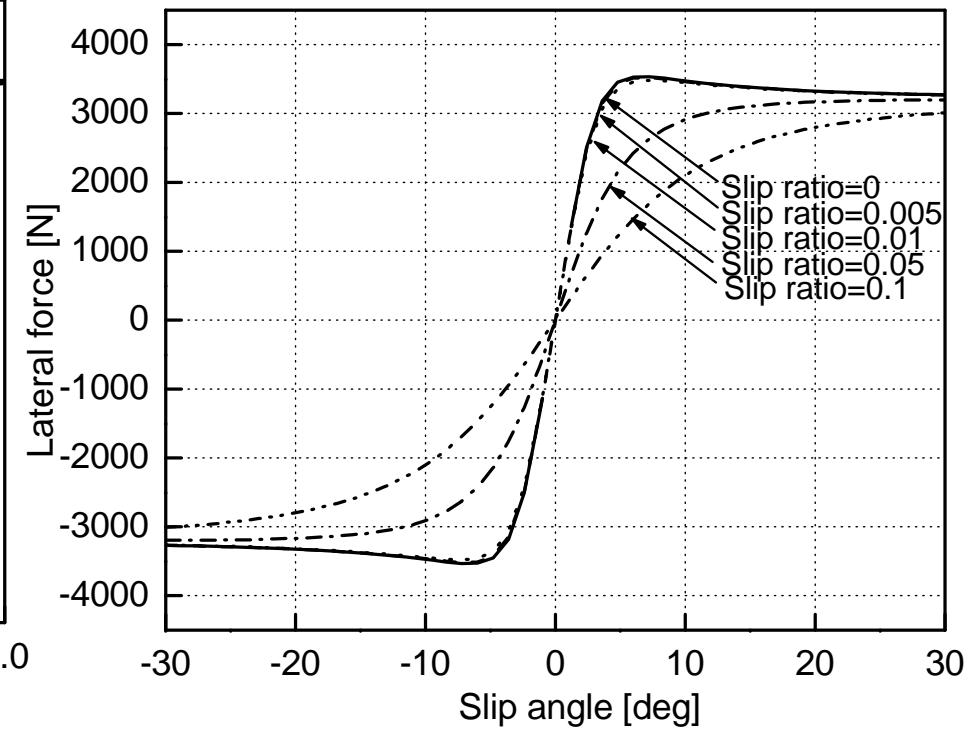
Longitudinal Tire Force

$$F_{txi} = \frac{\lambda_i^*}{\sigma_i^*} F_{tx0}(\sigma_i^* \times \lambda_m, F_{tzi})$$



Lateral Tire Force

$$F_{tyi} = \frac{\alpha_i^*}{\sigma_i^*} F_{ty0}(\sigma_i^* \times \alpha_m, F_{tzi})$$



4.1.6 Self Aligning Moment

- Self Aligning Moment at the i-th Wheel

$$M_{tzi} = D_z \sin(C_z \tan^{-1}(B_z \Phi_z)) + S_{vz}$$

Where, $\Phi_z = (1 - E_z)(\alpha_i + S_{hz}) + \frac{E_z}{B_z} \tan^{-1}(B_z(\alpha_i + S_{hz}))$

$$B_z \cdot C_z \cdot D_z = \frac{(-1.86 \cdot 10^{-6}) \cdot F_{tzi}^2 + (-2.73 \cdot 10^{-3}) \cdot F_{tzi}}{\exp[(0.11 \cdot 10^{-3}) \cdot F_{tzi}]}$$

$$C_z = 2.40$$

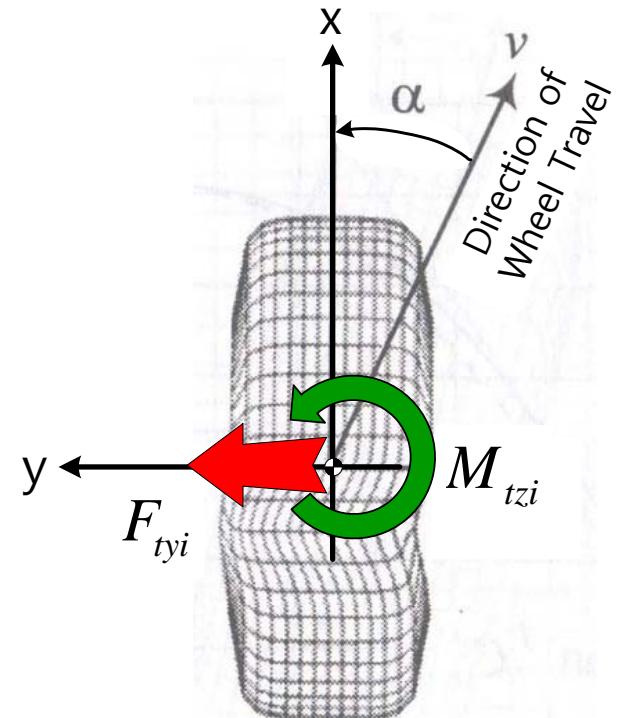
$$D_z = (-2.72 \cdot 10^{-6}) \cdot F_{tzi}^2 + (-2.28 \cdot 10^{-3}) \cdot F_{tzi}$$

$$B_z = \frac{B_z \cdot C_z \cdot D_z}{C_z \cdot D_z}$$

$$E_z = (-0.0 \cdot 10^{-6}) \cdot F_{tzi}^2 + (-0.643 \cdot 10^{-3}) \cdot F_{tzi} - 4.04$$

$$S_{hz} = 0$$

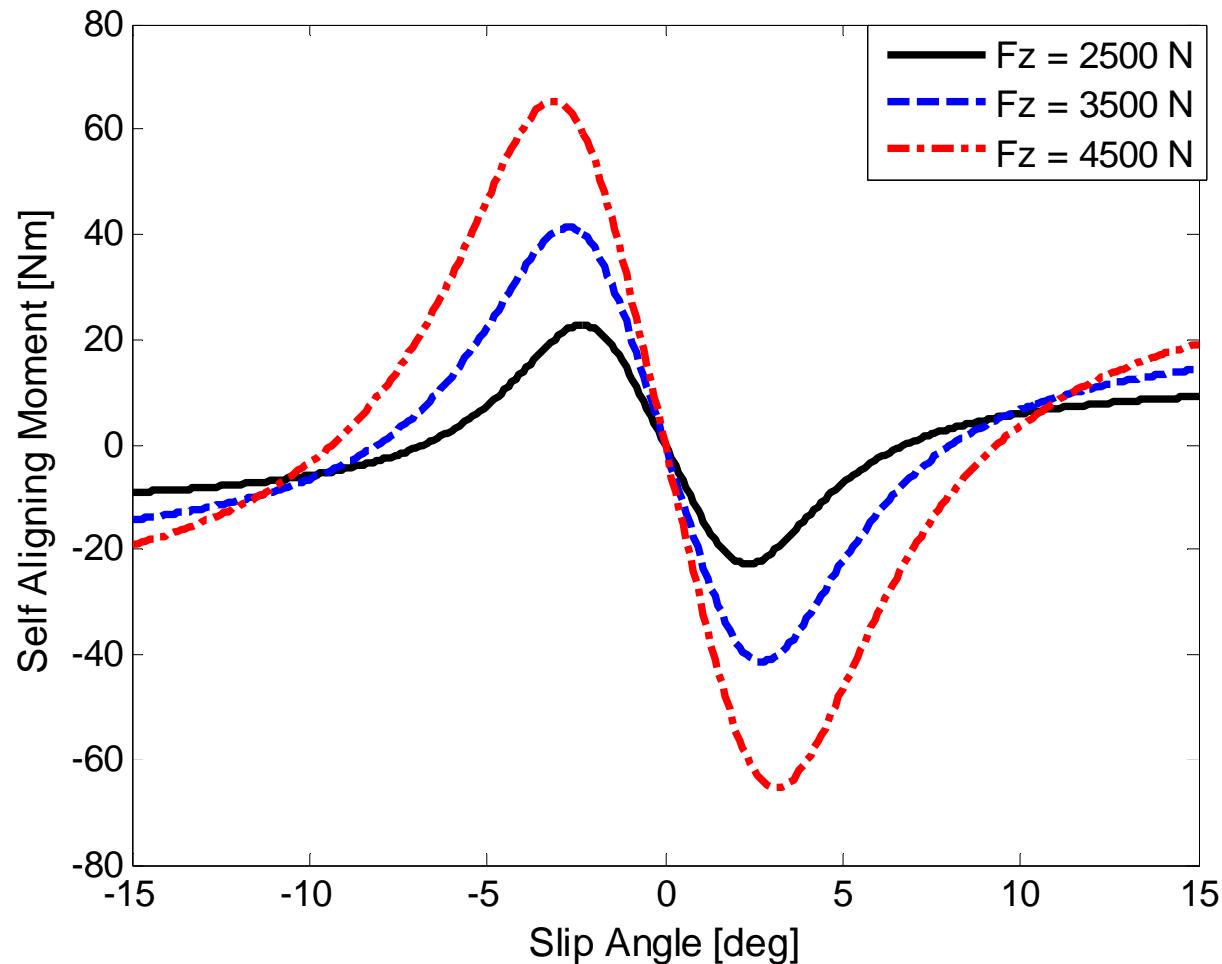
$$S_{vz} = 0$$



< Self Aligning Moment >

4.1.6 Self Aligning Moment

- Slip Angle versus Self Aligning Moment Curve



4. Tire Model

4.1 Pacejka Tire Model

4.1.1 Slip Angle

4.1.2 Lateral Tire Model

4.1.3 Slip Ratio

4.1.4 Longitudinal Tire Model

4.1.5 Combined Tire Model

4.1.6 Self Aligning Moment

4.2 Dugoff's Tire Model

4.2 Dugoff's Tire Model

Longitudinal Tire Force

$$F_{txi} = C_x \cdot \frac{\lambda_i}{1 + \lambda_i} \cdot f(\delta_i)$$

Lateral Tire Force

$$F_{tyi} = C_y \cdot \frac{\tan(\alpha_i)}{1 + \lambda_i} \cdot f(\delta_i)$$

Where,

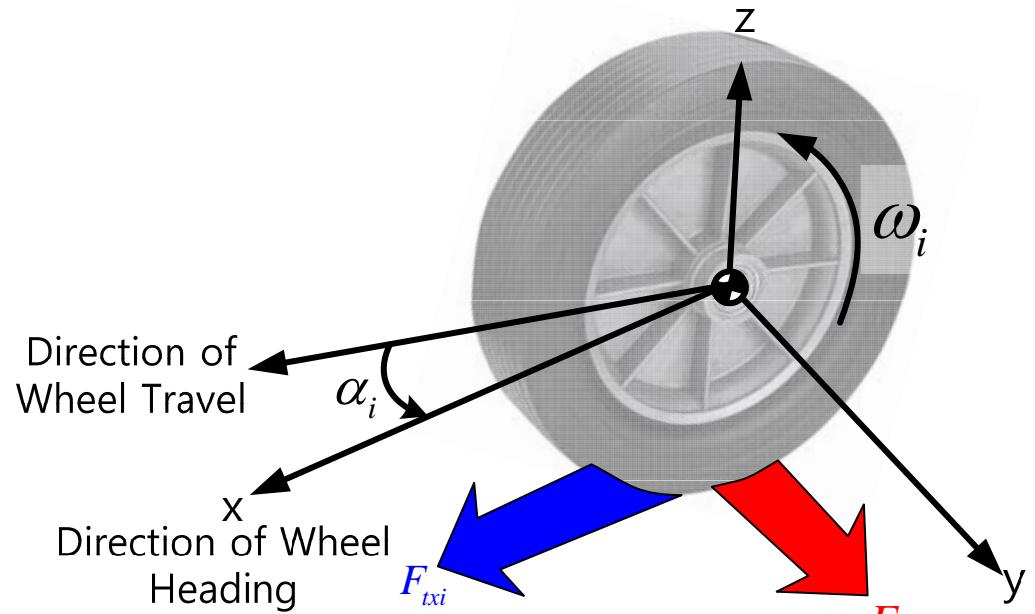
$$\delta_i = \frac{\mu \cdot F_{txi} \cdot (1 + \lambda_i)}{2\sqrt{(C_x \cdot \lambda_i)^2 + (C_y \cdot \tan(\alpha_i))^2}}$$

$$f(\delta_i) = \begin{cases} (2 - \delta_i) \cdot \delta_i & \text{if } (\delta_i < 1) \\ 1 & \text{if } (\delta_i \geq 1) \end{cases}$$

C_x = Longitudinal Tire Stiffness

C_y = Lateral Tire Stiffness

μ = Tire/Road Friction Coefficient

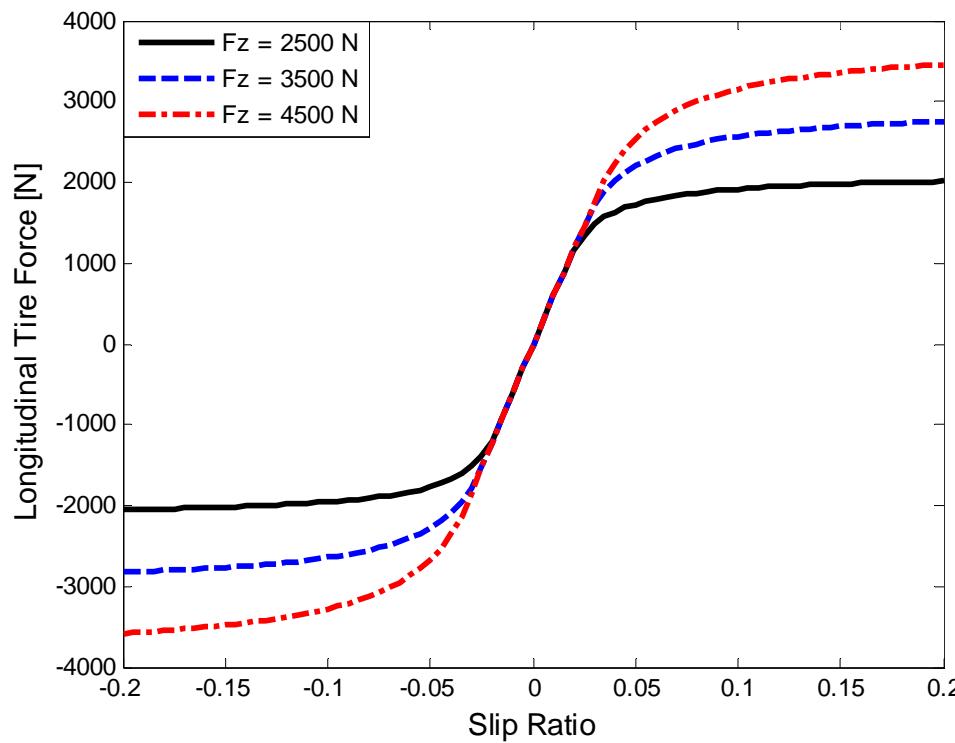


4.2 Dugoff's Tire Model

Longitudinal Tire Force

$$F_{txi} = C_x \cdot \frac{\lambda_i}{1 + \lambda_i} \cdot f(\delta_i)$$

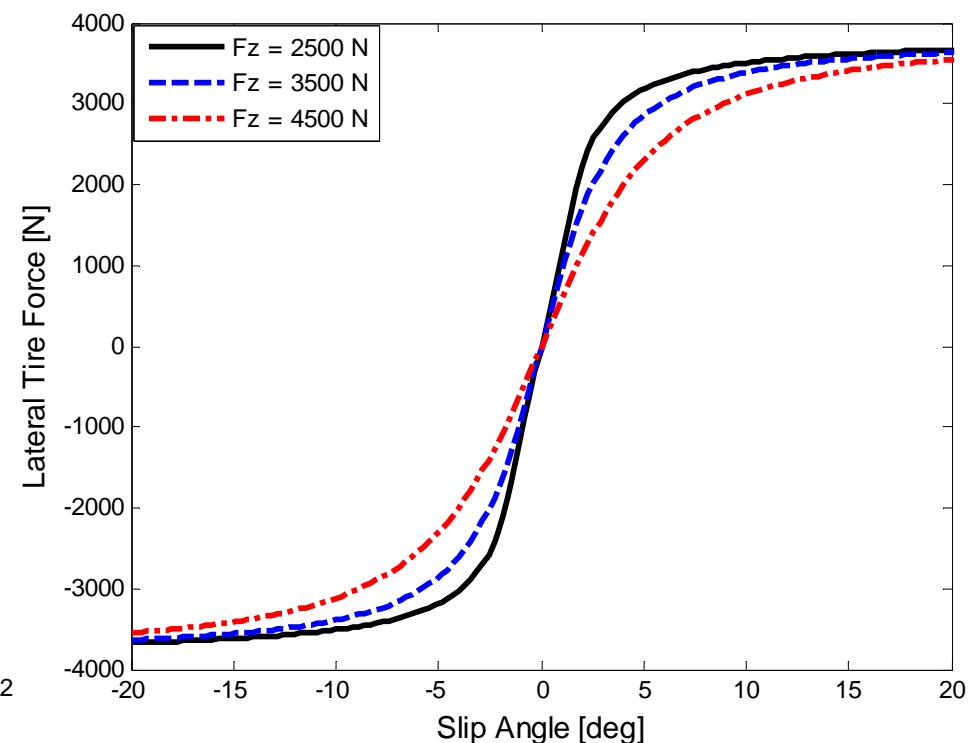
Where, $\alpha_i = 0$



Lateral Tire Force

$$F_{tyi} = C_y \cdot \frac{\tan(\alpha_i)}{1 + \lambda_i} \cdot f(\delta_i)$$

Where, $\lambda_i = 0$

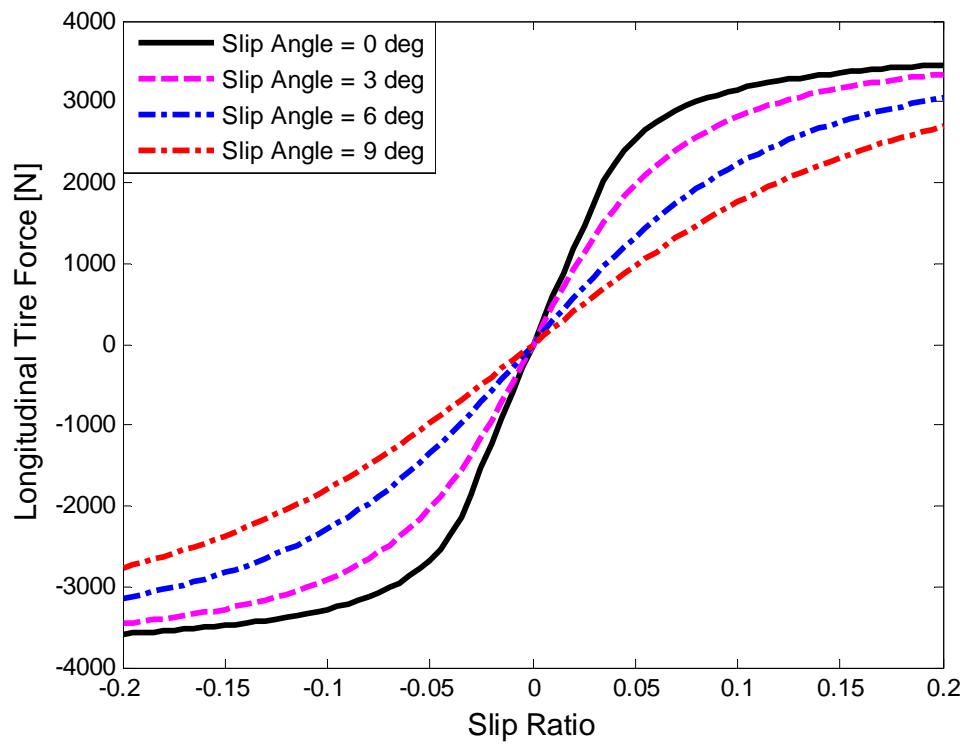


4.2 Dugoff's Tire Model

Longitudinal Tire Force

$$F_{txi} = C_x \cdot \frac{\lambda_i}{1 + \lambda_i} \cdot f(\delta_i)$$

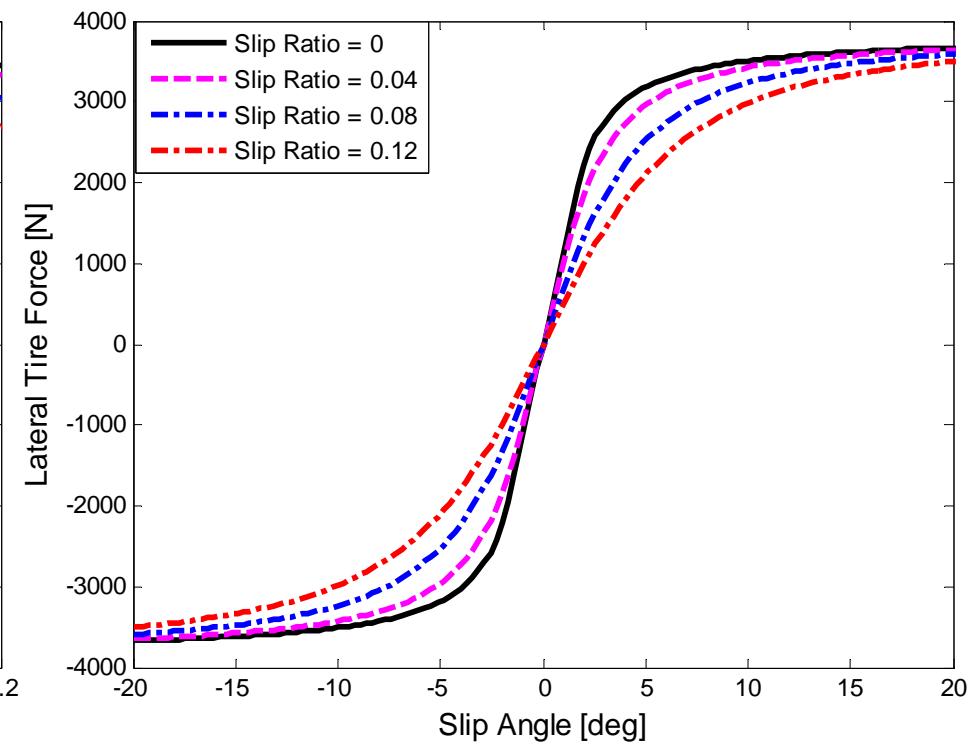
Where, $F_{tz_i} = 4500N$



Lateral Tire Force

$$F_{tyi} = C_y \cdot \frac{\tan(\alpha_i)}{1 + \lambda_i} \cdot f(\delta_i)$$

Where, $F_{tz_i} = 4500N$



Part.2

Longitudinal Vehicle Dynamics

1. Longitudinal Dynamic Model
2. Engine model
3. Transmission
4. Tire models
5. Brake

5. Brake

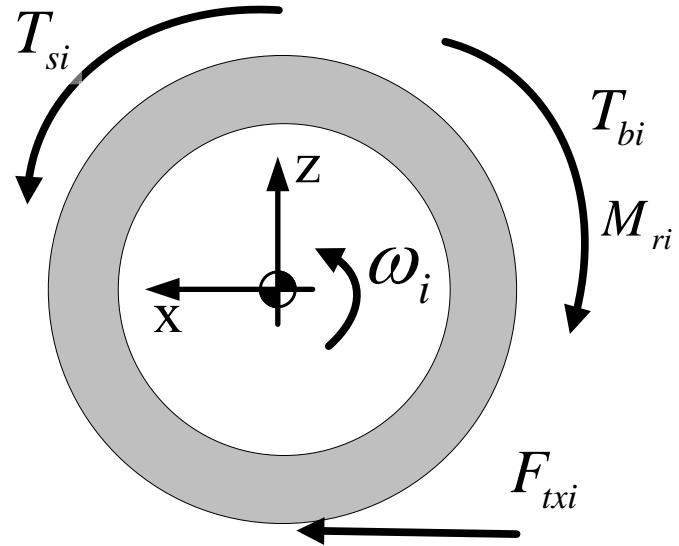
Wheel Dynamics

- Driving Wheel

$$\dot{\omega}_i = \frac{1}{J_w} (T_{si} - r_i \cdot F_{txi} - T_{bi} - M_{ri})$$

- Driven Wheel

$$\dot{\omega}_i = \frac{1}{J_w} (-r_i \cdot F_{txi} - T_{bi} - M_{ri})$$



Where, ω_i = Angular Velocity of the i -th Wheel

r_i = Effective Rolling Radius of the i -th Wheel

J_w = Momentum of Inertia of the i -th Wheel

T_{si} = Driving Torque at i -th Wheel

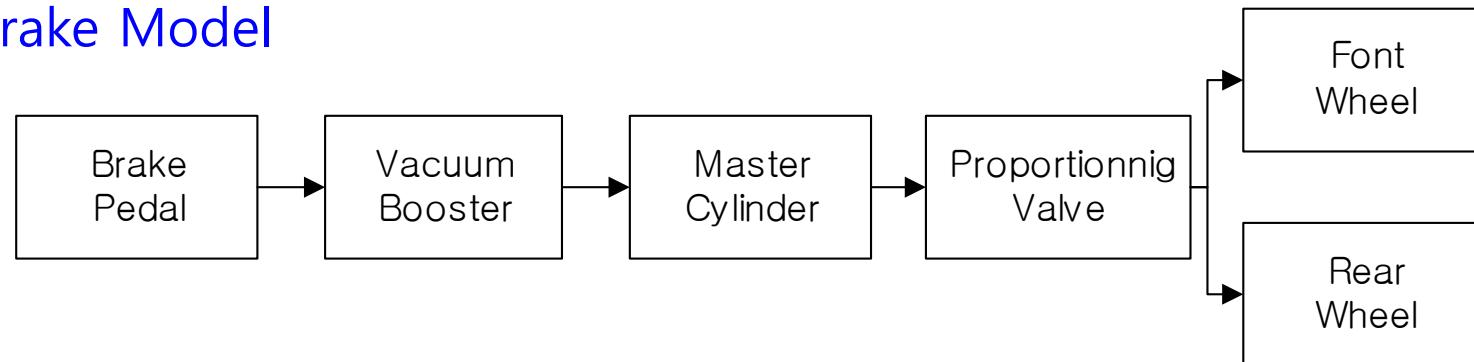
F_{txi} = Longitudinal Tire Force at i -th Wheel

T_{bi} = Brake Torque at i -th Wheel

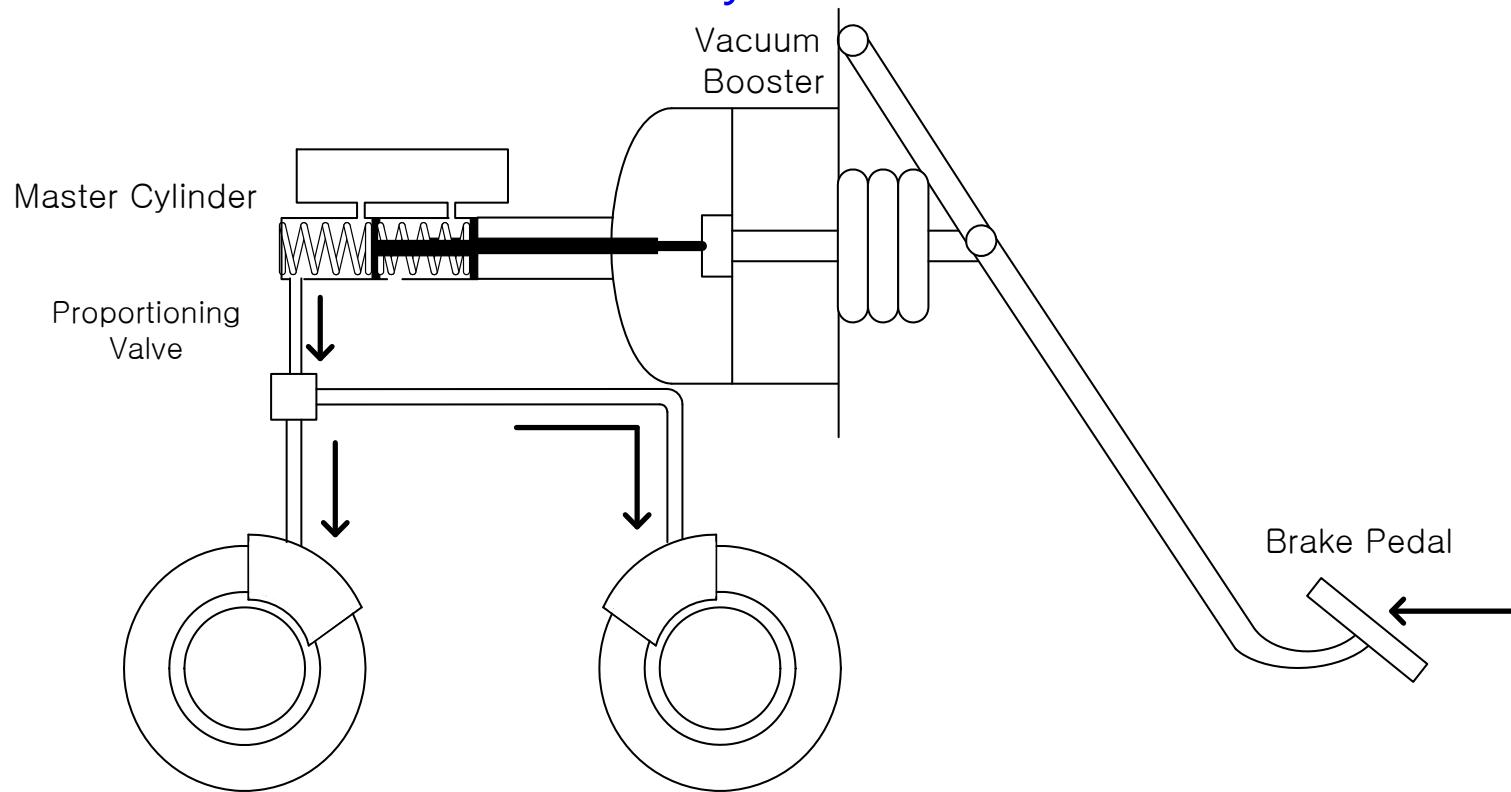
M_{ri} = Rolling Resistance at i -th Wheel

5. Brake

Brake Model

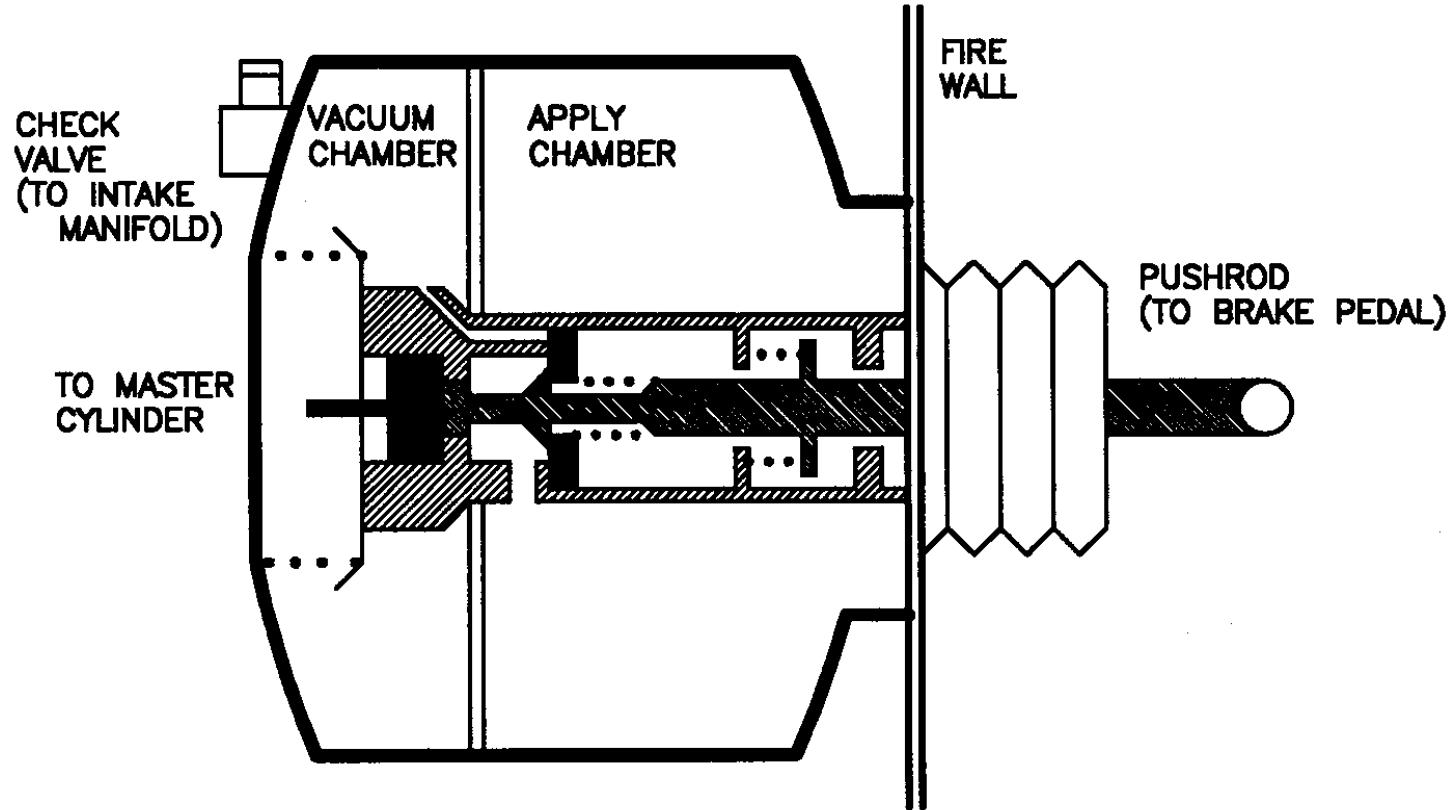


Fundamental structure of a hydraulic brake



5. Brake

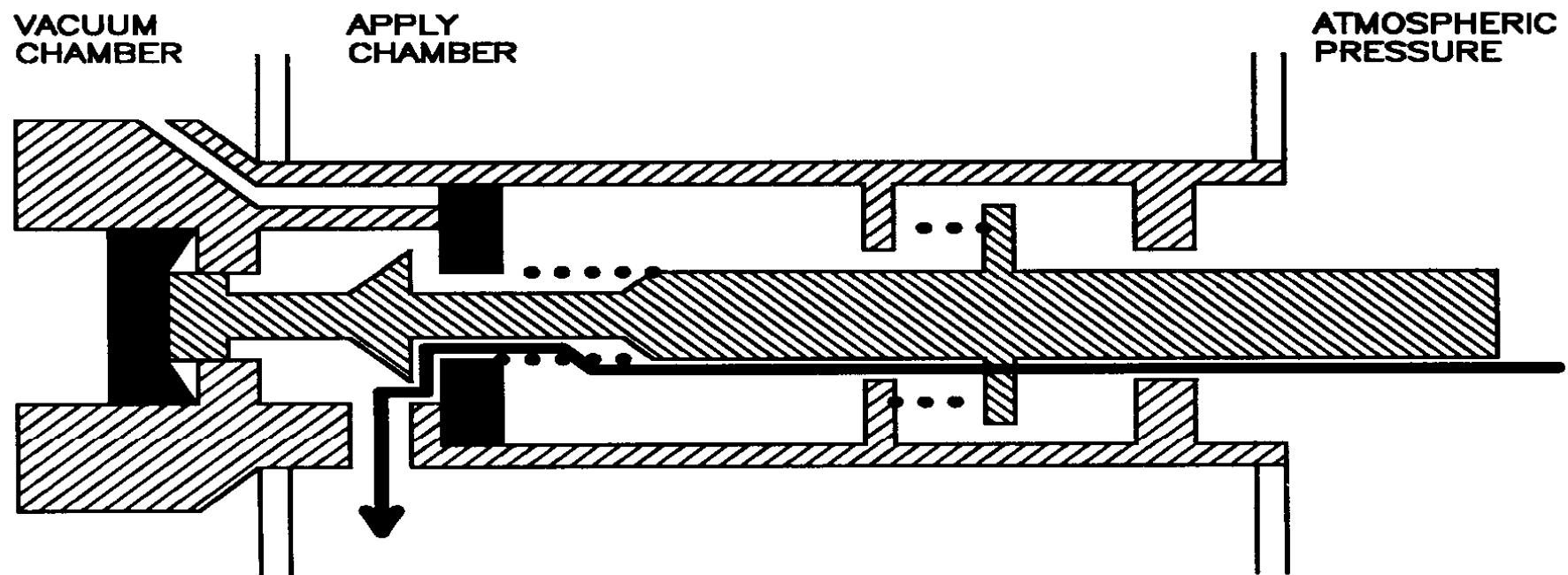
Vacuum Booster



Vacuum Booster Diagram

5. Brake

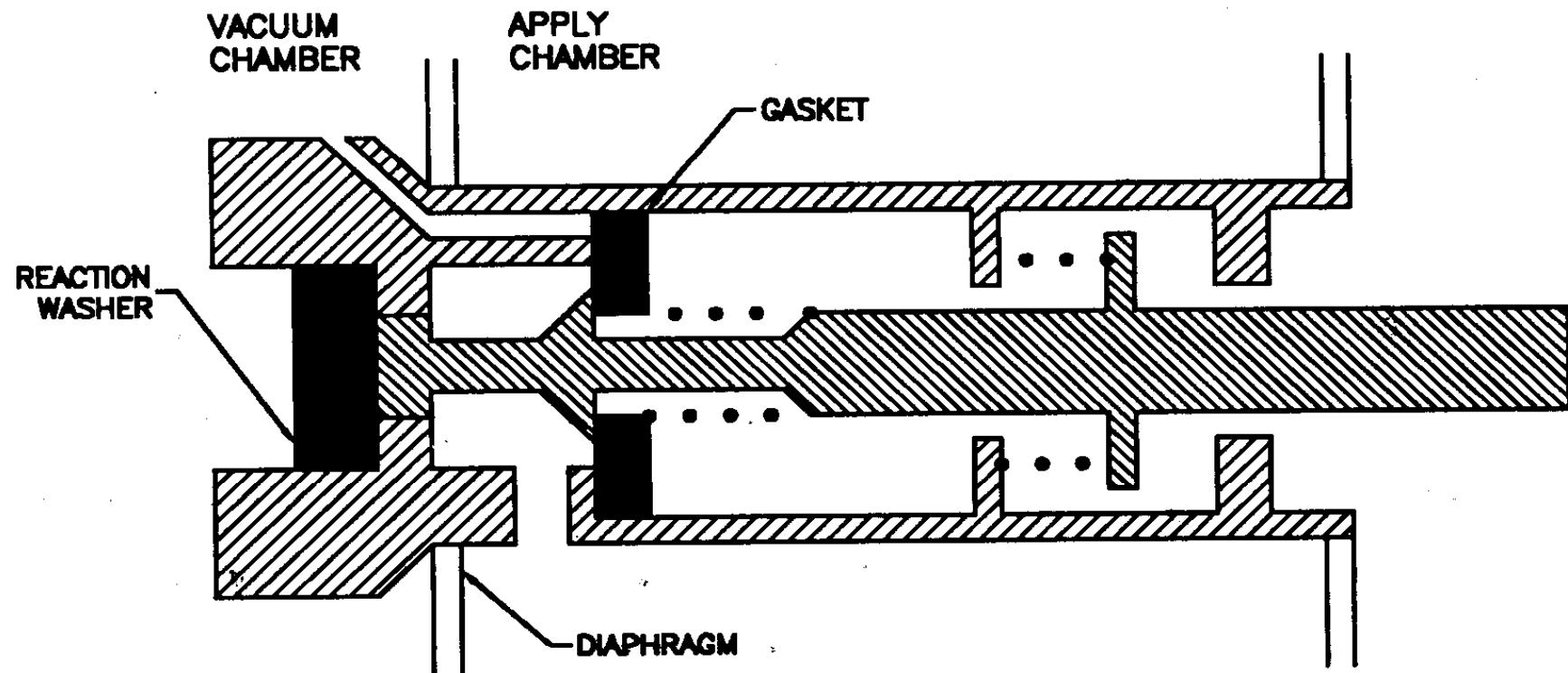
Vacuum Booster Control Valve Model



Control Valve – Apply stage

5. Brake

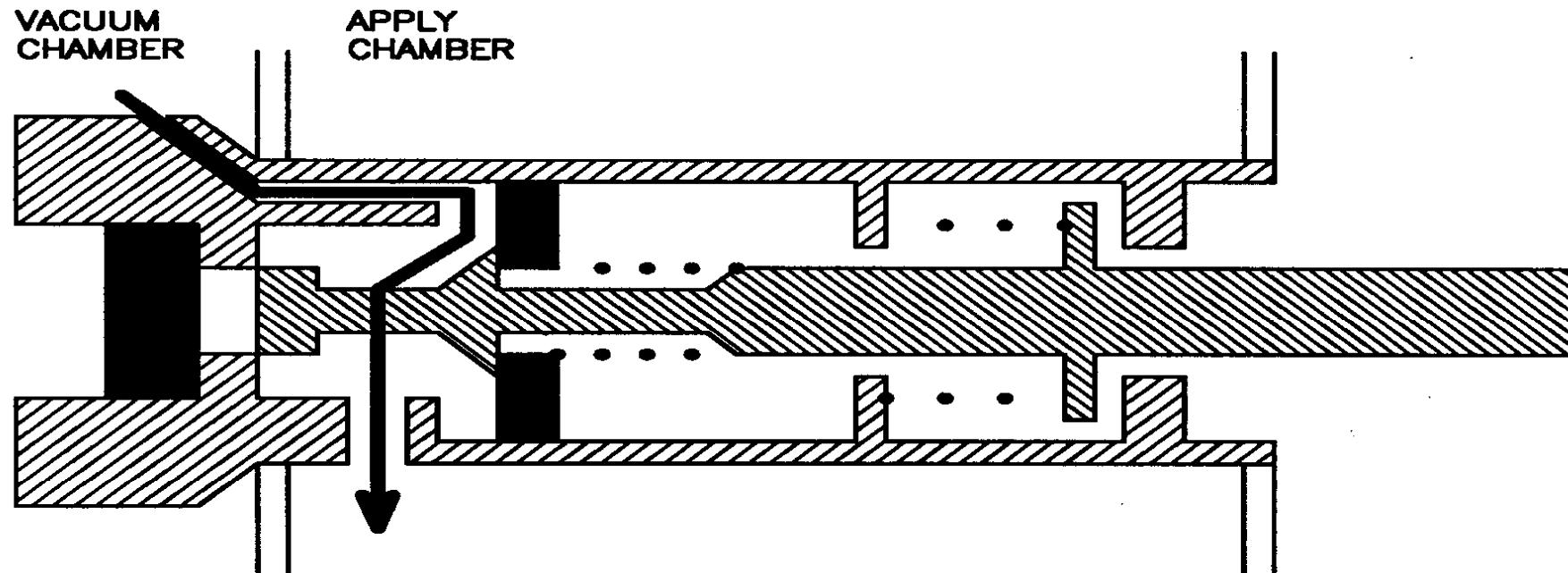
Vacuum Booster Control Valve Model



Control Valve – Hold stage

5. Brake

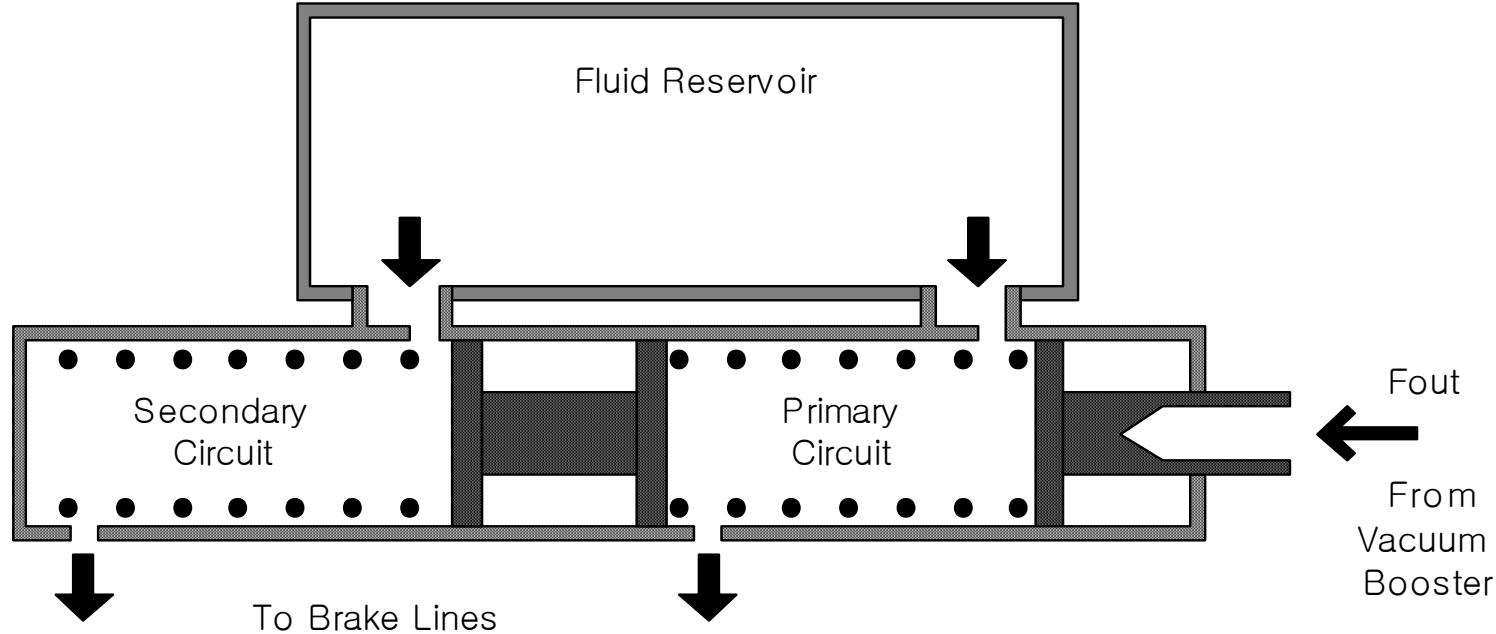
Vacuum Booster Control Valve Model



Control Valve- Release stage

5. Brake

Brake Model



- Equation of motion of master cylinder piston :

$$m_{mc} \ddot{x}_{mc} = -b_{mc} \dot{x}_{mc} - F_{cs} - A_{mc} P_{mc} + F_{out} - \text{sign}(\dot{x}_{mc}) F_{loss}$$

$$x_{mc} = x_{pp}$$

5. Brake

Brake Model

$$F_{out} = \begin{cases} F_d - F_{rs} & F_d > F_{rs} \\ 0 & otherwise \end{cases} : \text{output force of vacuum booster}$$

$$F_{rs} = F_{rs_0} + k_{rs}x_{pp} : \text{return spring force}$$

$$F_{cs} = F_{cs_0} + k_{cs}x_{mc} : \text{master cylinder spring force}$$

$$F_d = A_d(P_A - P_V) : \text{diaphragm force}$$

where , F_{rs_0} : initial return spring force

k_{rs} : return spring constant

x_{mc} : displacement of master cylinder piston

A_{mc} : area of master cylinder piston

F_{cs_0} : initial cylinder spring force

k_{cs} : master cylinder spring constant

P_{mc} : pressure of master cylinder

b_{mc} : master cylinder damping coefficient

5. Brake

Brake Model

Master cylinder pressure

$$\dot{P}_{mc} = \beta \frac{A_{mc} \dot{x}_{mc}}{V_{mc_0} - A_{mc} x_{mc}} - \beta \frac{\sigma_{wf} C_{wf} \sqrt{|P_{mc} - P_{wf}|}}{V_{mc_0} - A_{mc} x_{mc}}$$

Where $\sigma_{wf} = sign(P_{mc} - P_{wf})$

β : bulk modulus

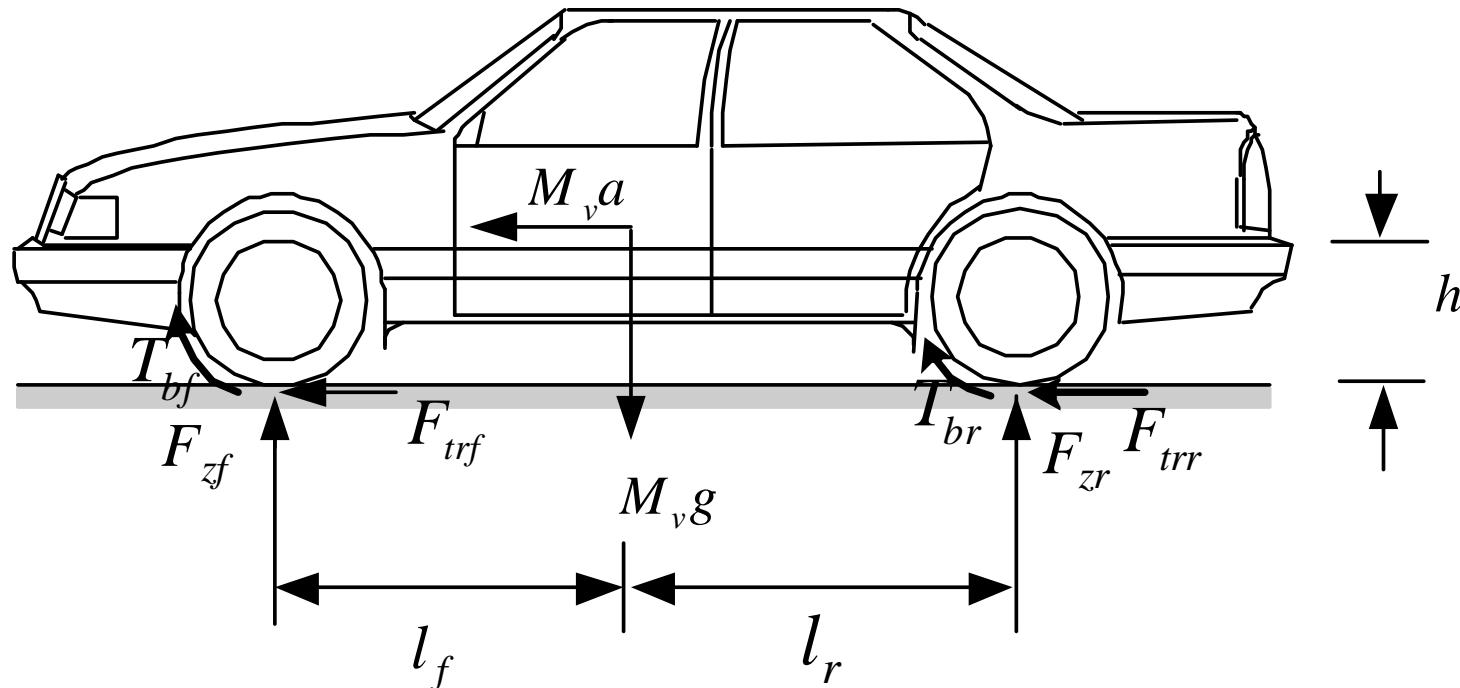
C_{wf} : Fluid flow coefficient

5. Brake

Brake Model

Proportioning valve

- Proportioning valve distribute front and rear traction forces appropriately to the vehicle's ideal traction distribution curve



Free Body Diagram of Vehicle

5. Brake

Brake Model

Force Equilibrium

$$m \cdot g = F_{zf} + F_{zr}$$

Moment Equilibrium

$$\sum M_x = m \cdot a_x \cdot h + F_{zr} \cdot L - m \cdot g \cdot l_f = 0$$

Where , $L = l_f + l_r$

$$F_{zr} \cdot L = m \cdot g \cdot l_f - m \cdot a_x \cdot h$$

$$L \cdot g \cdot F_{zr} = (F_{zr} + F_{zf}) \cdot l_f \cdot g - (F_{zr} + F_{zf}) \cdot a_x \cdot h$$

$$(L \cdot g - l_f \cdot g + a_x \cdot h) \cdot F_{zr} = (l_f \cdot g - a_x \cdot h) \cdot F_{zf}$$

Front/ rear pressure ratio

$$\frac{P_r}{P_f} \propto \frac{F_{zr}}{F_{zf}} = \frac{l_f \cdot g - a_x \cdot h}{l_r \cdot g + a_x \cdot h}$$

5. Brake

Brake Model

Front/Rear Pressure Distribution Equilibrium

$$P_r = \begin{cases} P_{mc} & P_{mc} \leq P_{ch} \\ P_{ch} + r_{rf}(P_{mc} - P_{ch}) & otherwise \end{cases}$$

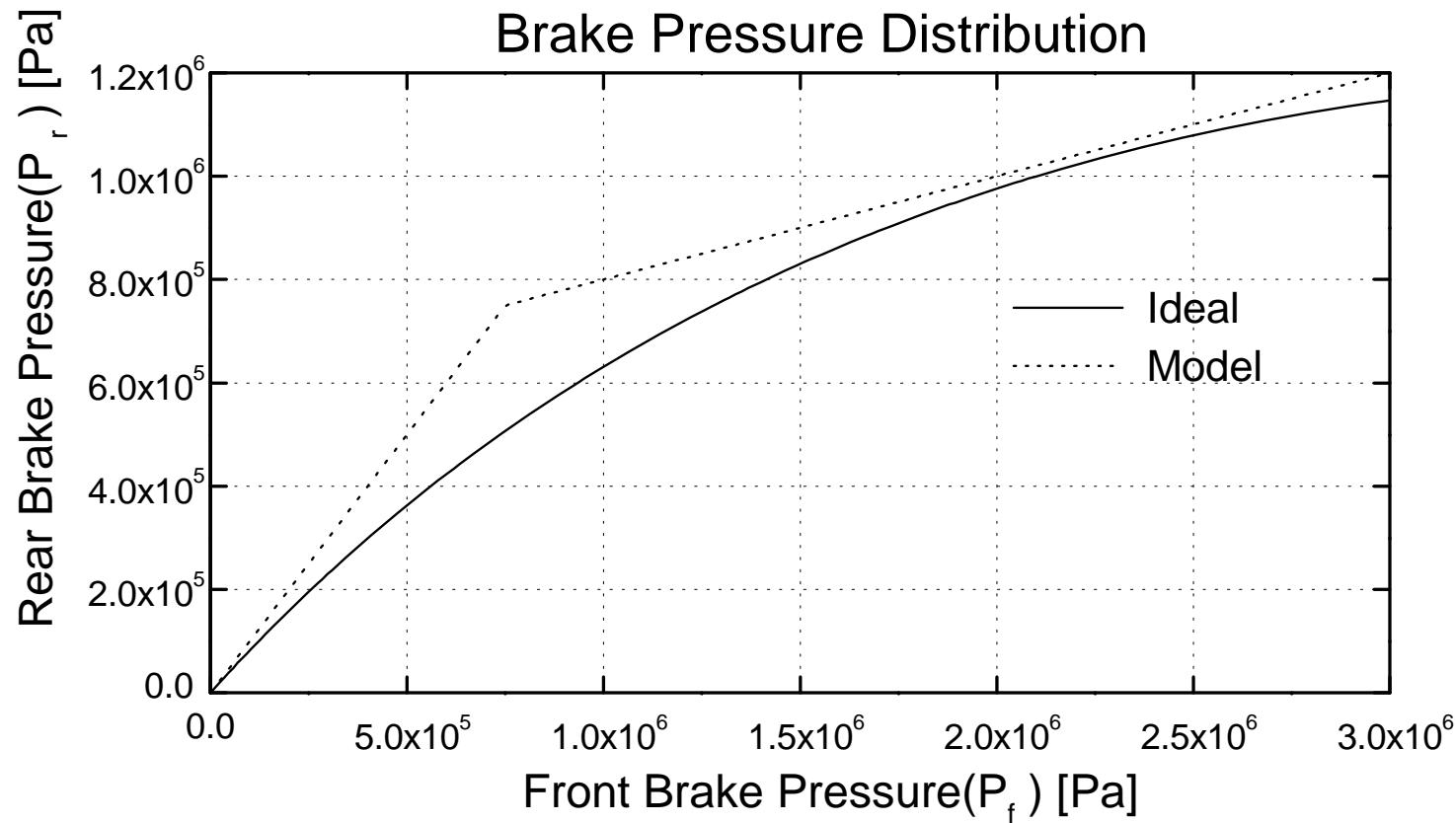
P_{ch} : Knee pressure of proportioning valve

r_{rf} : Area ratio at the proportioning valve

P_r : Rear Brake Pressure

5. Brake

Brake Model



5. Brake

Brake Model

Wheel caliper pressure

- Volumetric flow rate :

$$\dot{V}_{br} = \sigma_{wr} C_{wr} \sqrt{|P_r - P_{wr}|} \quad \sigma_{wr} = sign(P_r - P_{wr})$$

$$\dot{V}_{bf} = \sigma_{wf} C_{wf} \sqrt{|P_{mc} - P_{wf}|} \quad \sigma_{fr} = sign(P_{mc} - P_{wf})$$

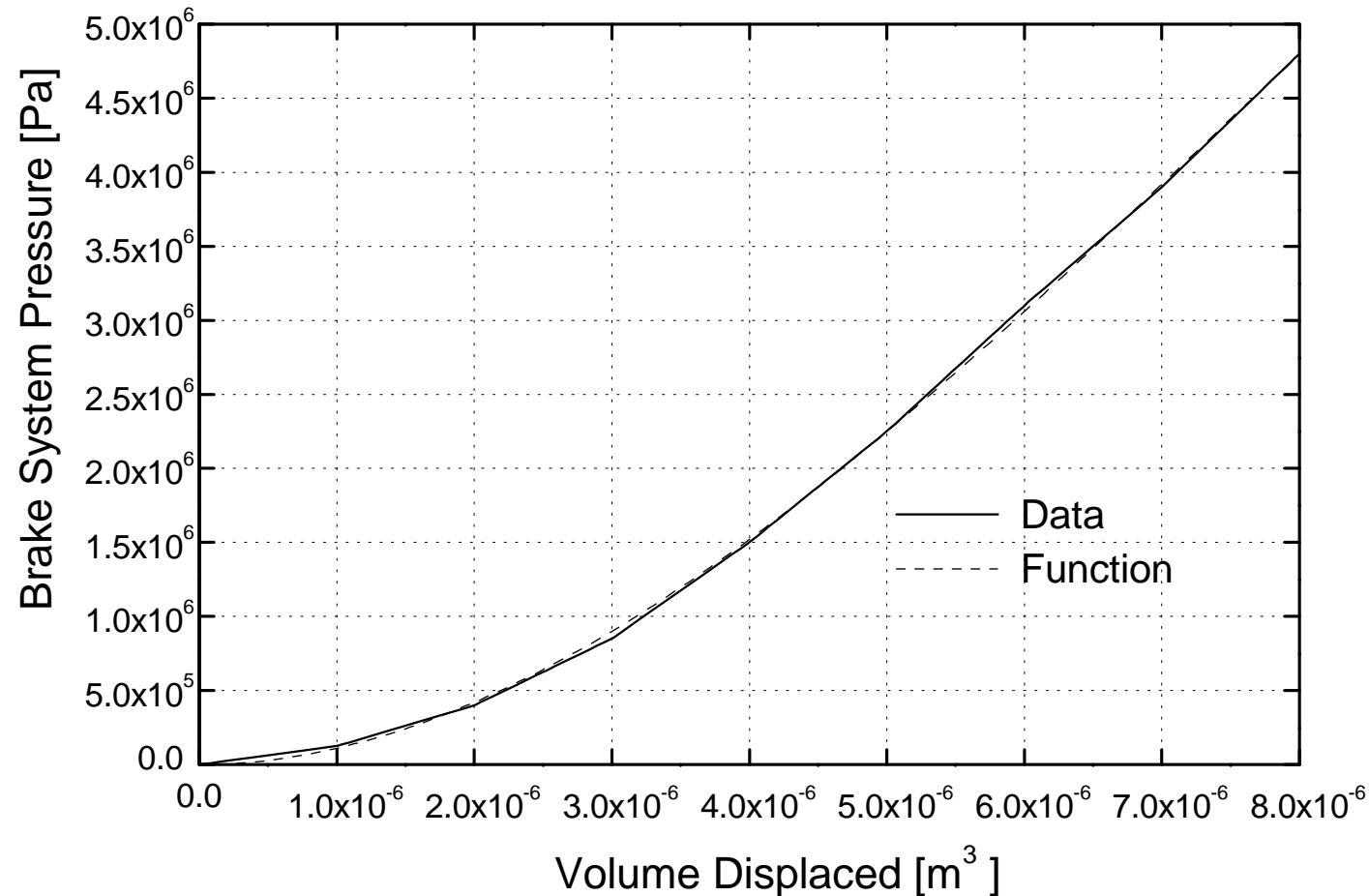
- Wheel caliper pressures :

$$P_{wf} = C_{p2} V_{bf}^2 + C_{p3} V_{bf}^3$$

$$P_{wr} = C_{p2} V_{br}^2 + C_{p3} V_{br}^3$$

5. Brake

Brake Model



5. Brake

Brake Model

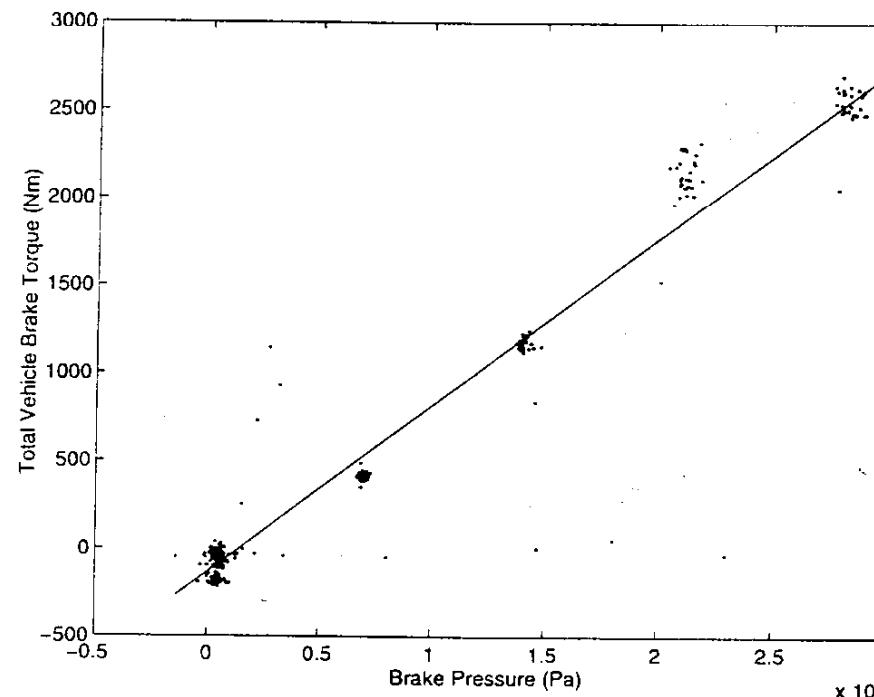
- Relationships between wheel caliper pressure and the brake torque :

$$T_b = K_b \cdot (P_w - P_{th})$$

where, $P_{th} = 0.1 \text{ MPa}$

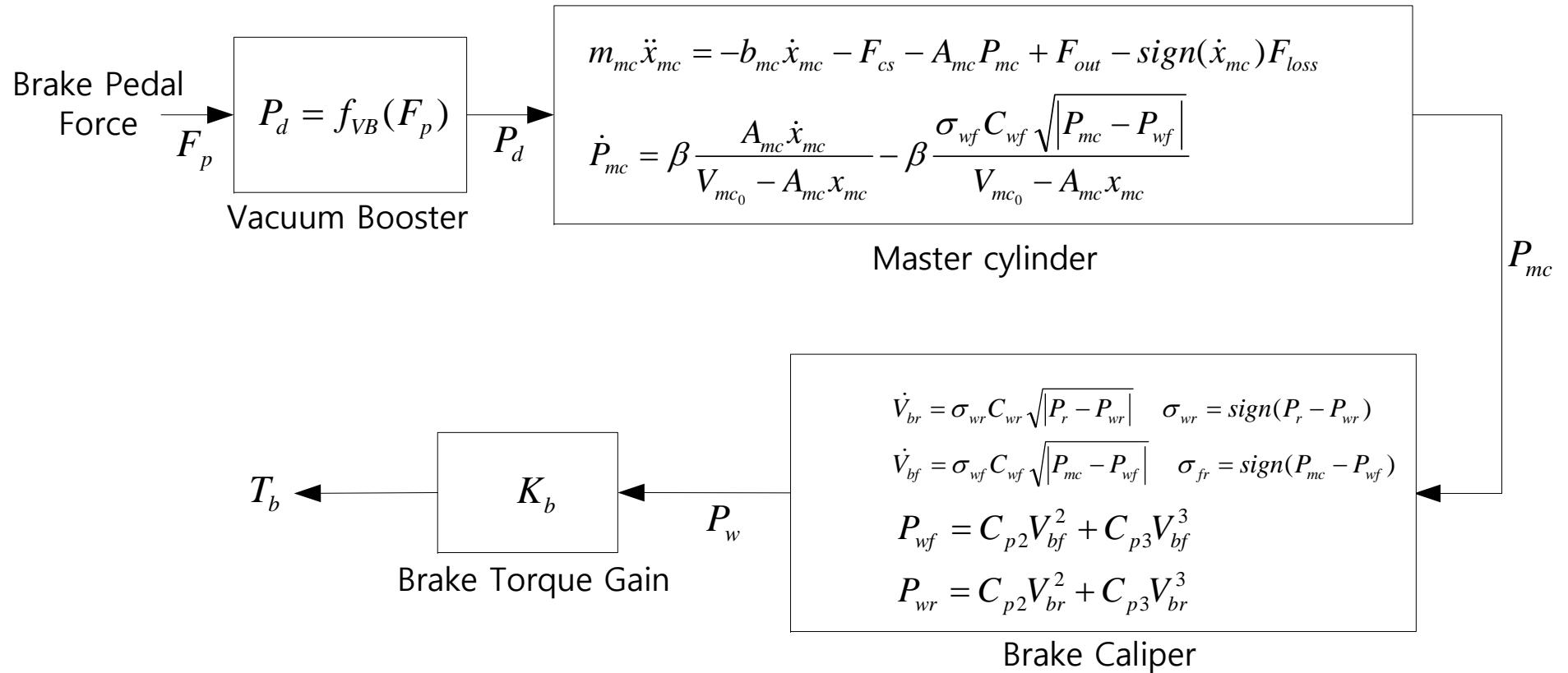
$$K_b = \frac{2500 \text{ N} \cdot \text{m}}{2.8 \times 10^6 \text{ Pa}} = 900 \text{ Nm / MPa}$$

- Brake Torque versus Caliper Pressure



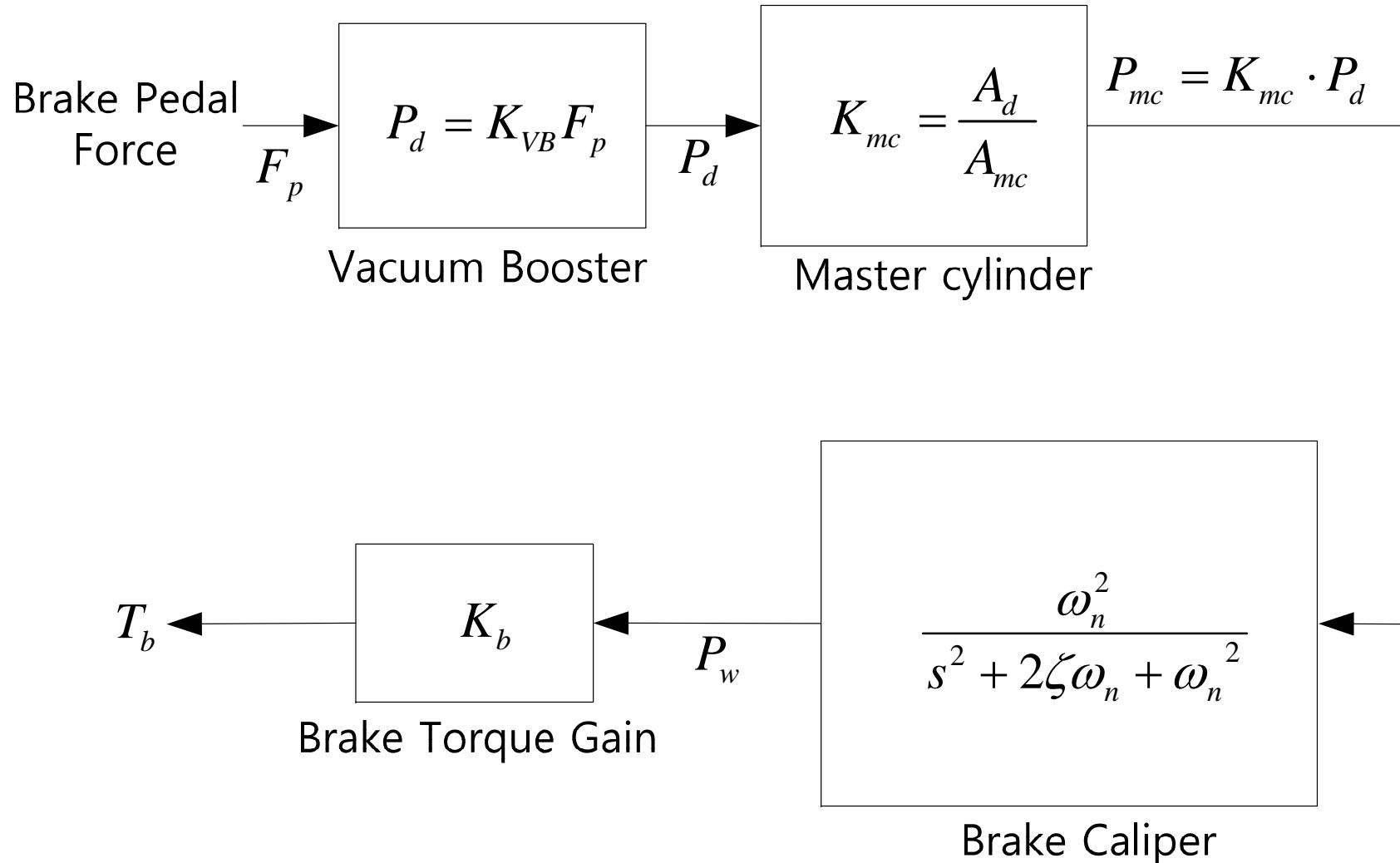
5. Brake

Nonlinear Brake Model



5. Brake

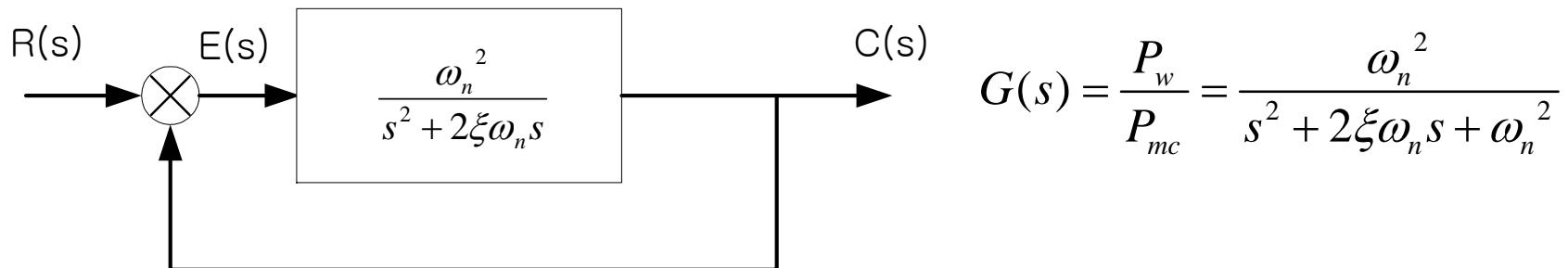
Simplified Brake Model



5. Brake

Simplified Brake Model

Transfer function of second order dynamic



- Definition of State

$$z_1 = P_w$$

$$z_2 = \dot{P}_w$$

- Second order brake model

$$\dot{z}_1 = z_2$$

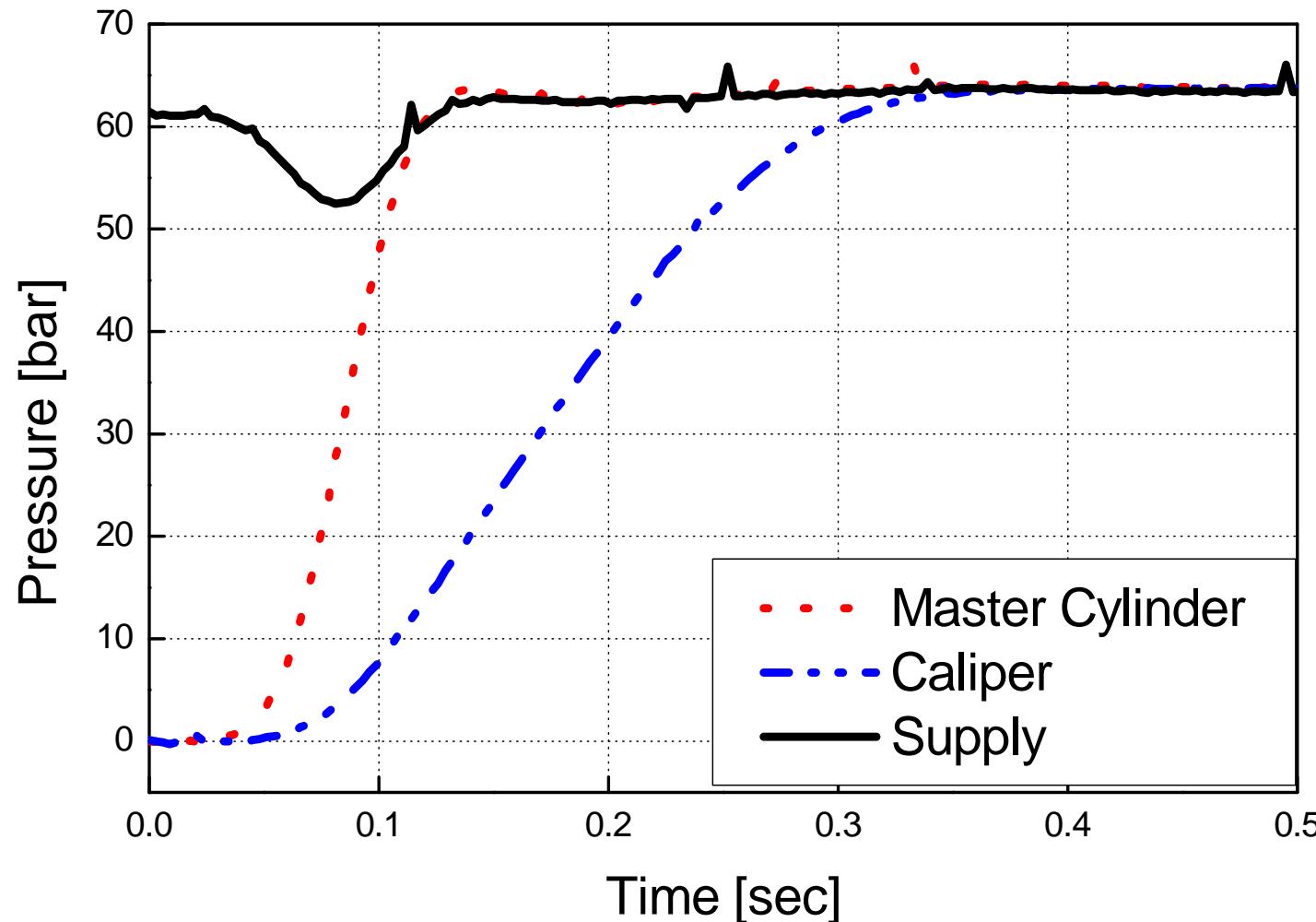
$$\dot{z}_2 = -a_2 z_1 - a_1 z_2 + b_1 u$$

Where u : master cylinder pressure

P_w : brake caliper pressure

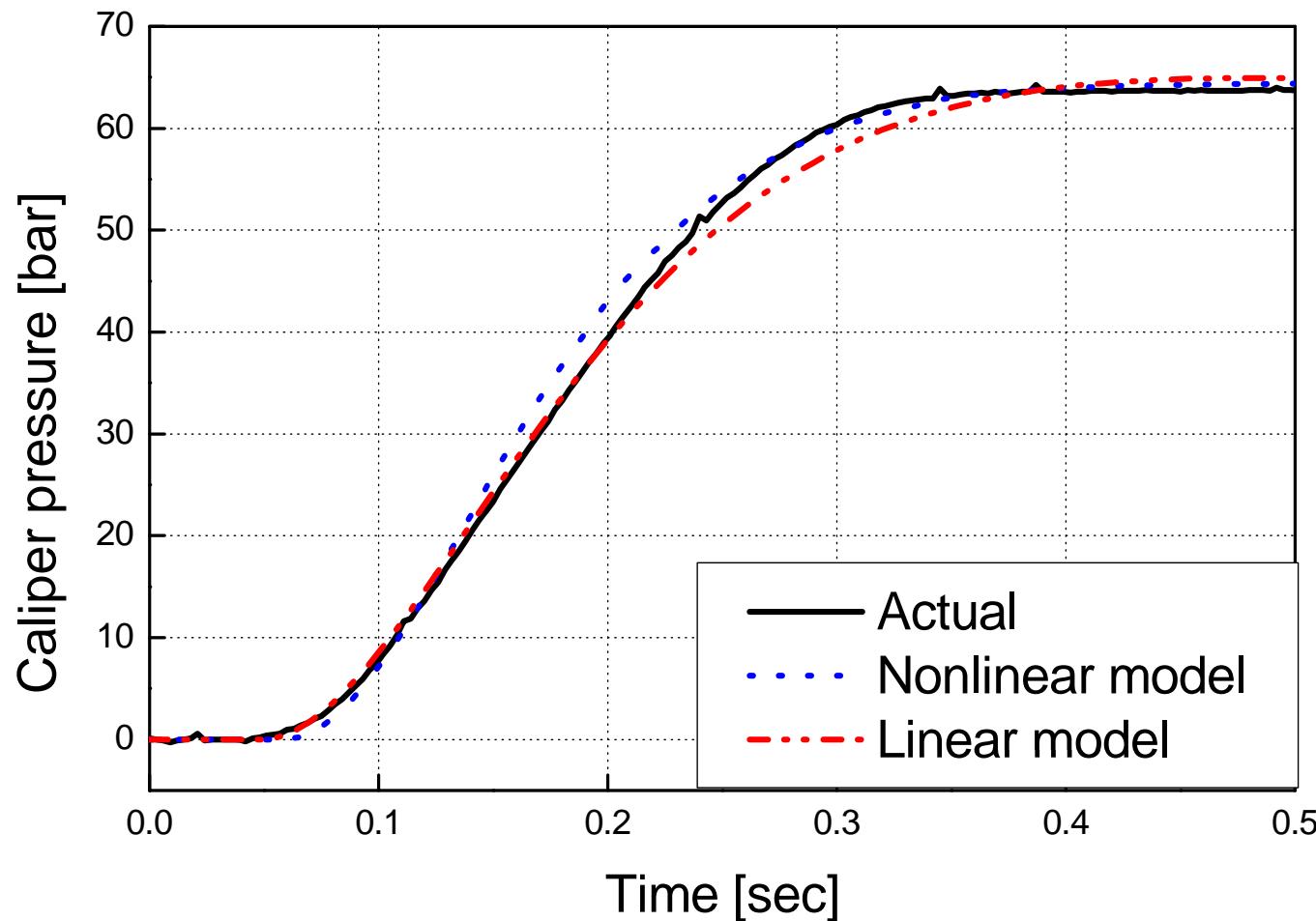
5. Brake

► Brake Test Results



5. Brake

► Brake Model Validation

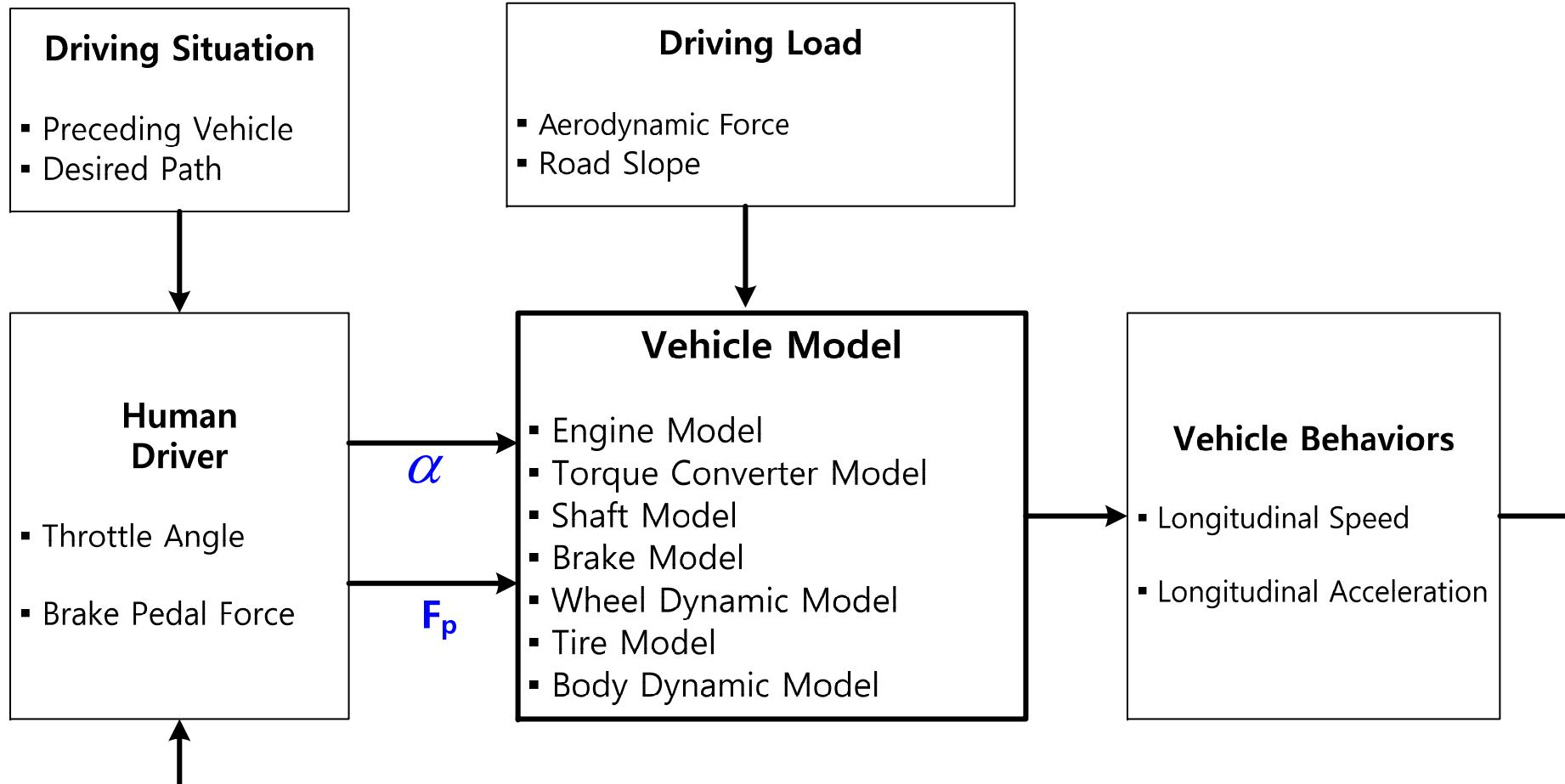


Part.2

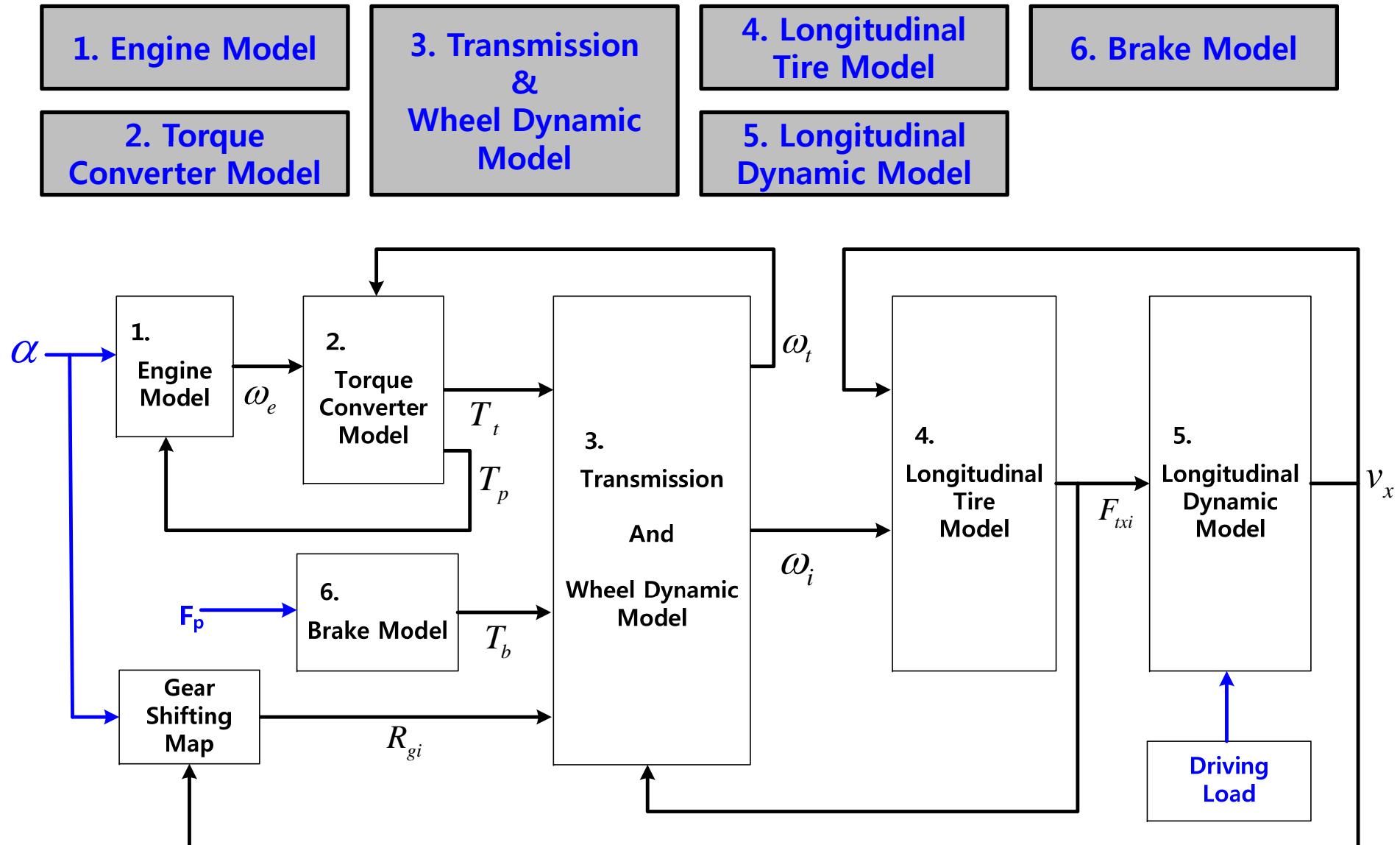
Summary

Longitudinal Vehicle Dynamics

Summary: Longitudinal Vehicle Dynamics



Summary: Longitudinal Vehicle Dynamics



1. Engine model

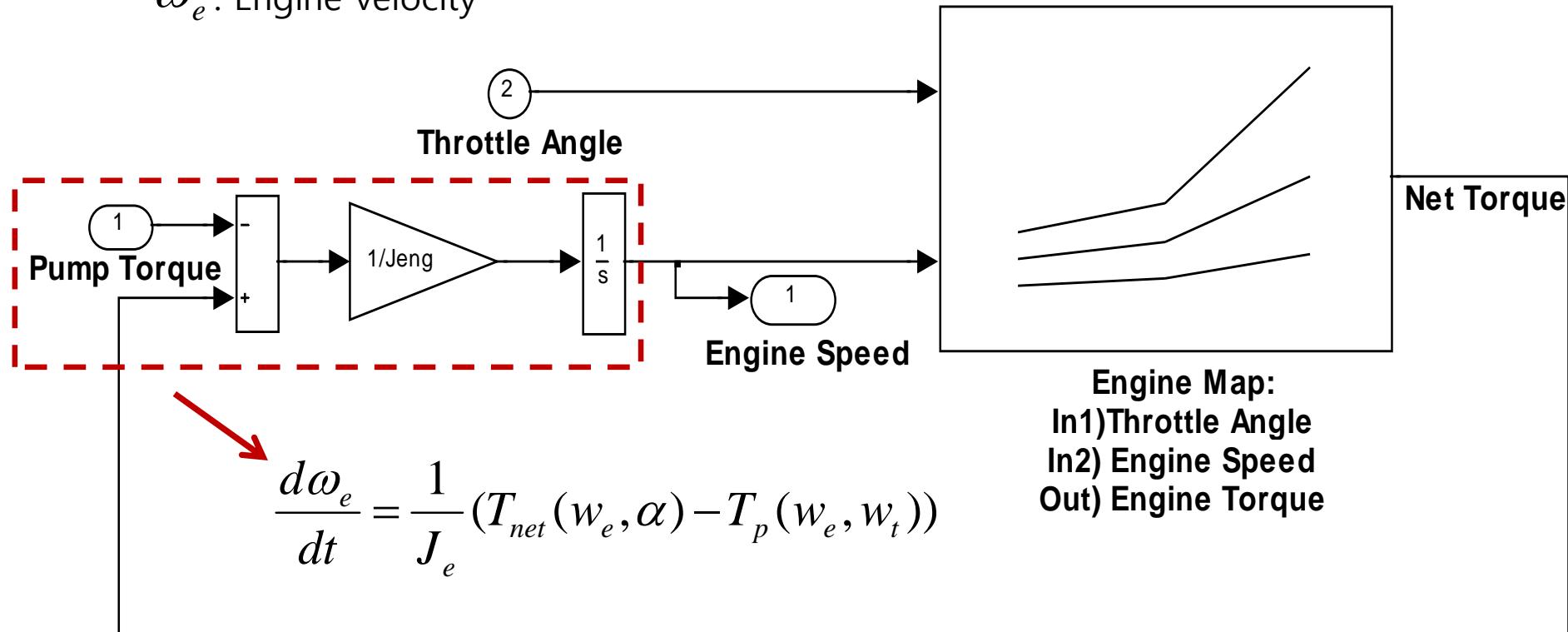
(1) System Input

α : Throttle angle

T_p : Pump torque

(2) System Output

ω_e : Engine velocity



2. Torque Converter Model

(1) System Input

ω_e : Engine velocity

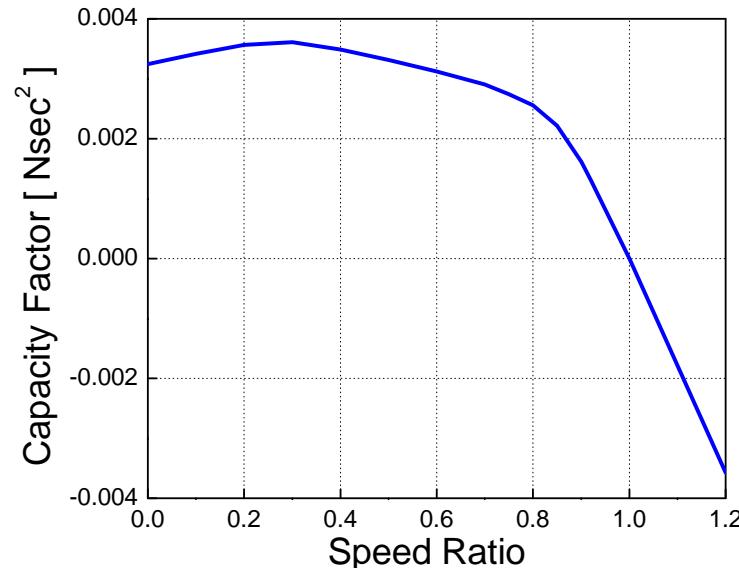
w_t : Turbine velocity

(2) System Output

$$T_p = C_f w_e^2 \text{ : Pump torque}$$

$$T_t = R_t \cdot T_p \text{ : Turbine torque}$$

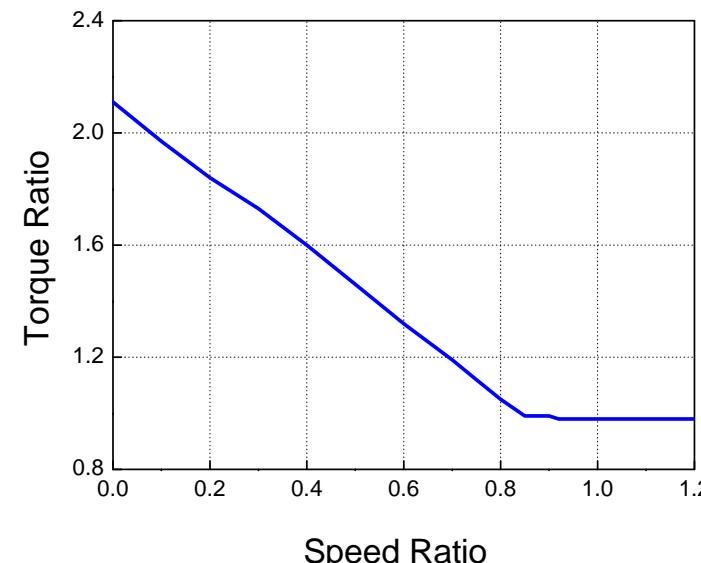
Torque Converter Capacity Factor Map



Where C_f : Capacity Factor

R_t : Torque Ratio

Torque Converter Torque Ratio Map



3. Transmission and Wheel Dynamic Model

(1) System Input

T_t : Turbine Torque

T_b : Brake Torque

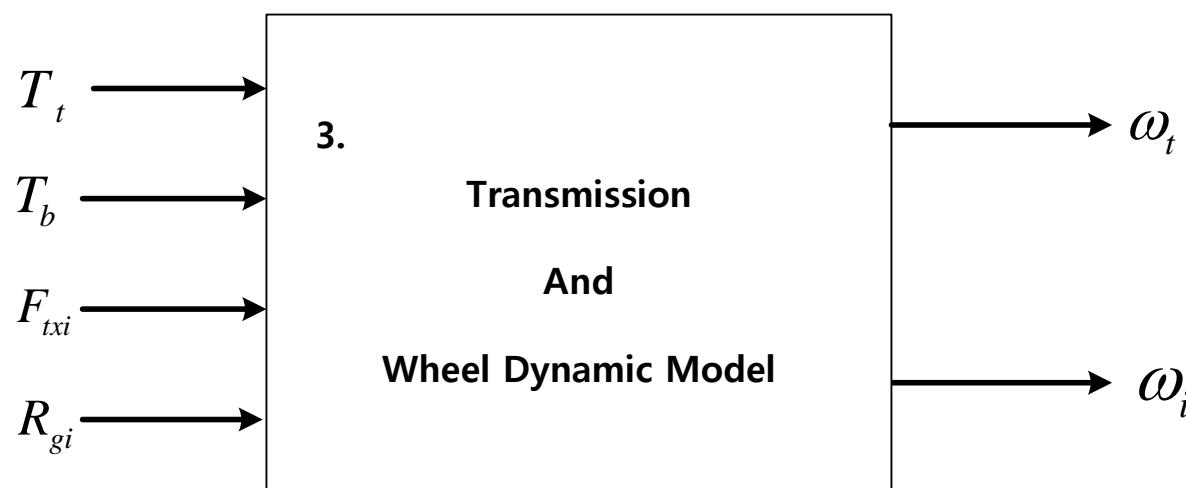
F_{txi} : Longitudinal Tire Force

R_{gi} : i-th gear ratio

(2) System Output

ω_t : Turbine Velocity

ω_i : Wheel Speed at each wheel



3. Transmission and Wheel Dynamic Model

(3) Summary

- Simplified Transmission and Wheel Dynamic Model (Front Wheel)

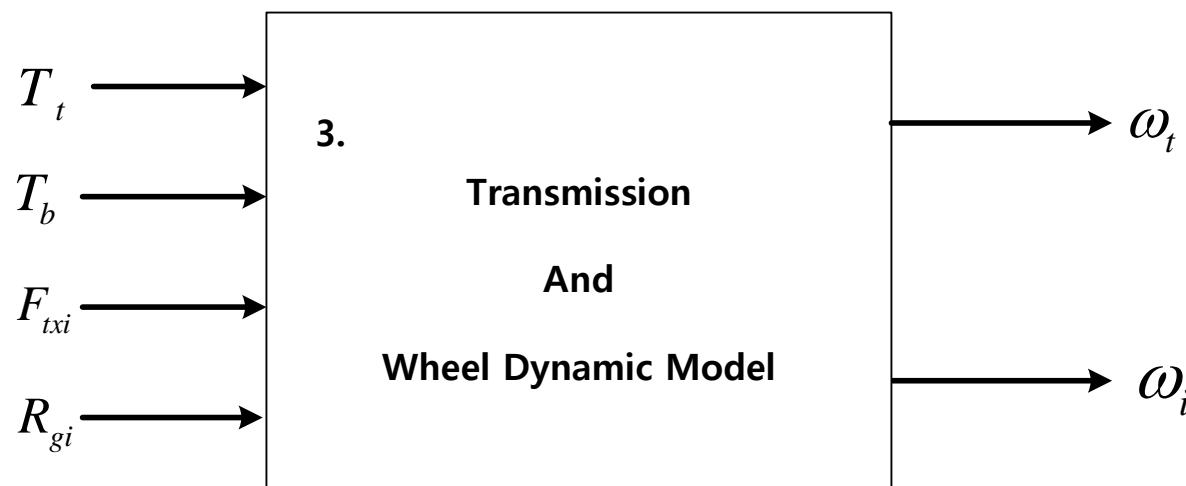
$$\left(\frac{I_{cri}}{2R_d} + R_d \cdot (J_{wL} + J_{wR}) \right) \cdot \frac{dw_f}{dt} = \frac{1}{R_{gi}} T_t - R_d \cdot r \cdot (F_{xfL} + F_{xfR}) - R_d \cdot (T_{bfL} + T_{bfR})$$

- Driven Wheel (Rear Wheel)

$$(J_{wL} + J_{wR}) \cdot \frac{d\omega_r}{dt} = -r \cdot (F_{xrL} + F_{xrR}) - (T_{brL} + T_{brR})$$

- Speed Relation

$$\omega_f = 2\omega_s = 2R_d \cdot w_{cr} = 2R_{gi} \cdot R_d \cdot w_t$$



4. Longitudinal Tire Model

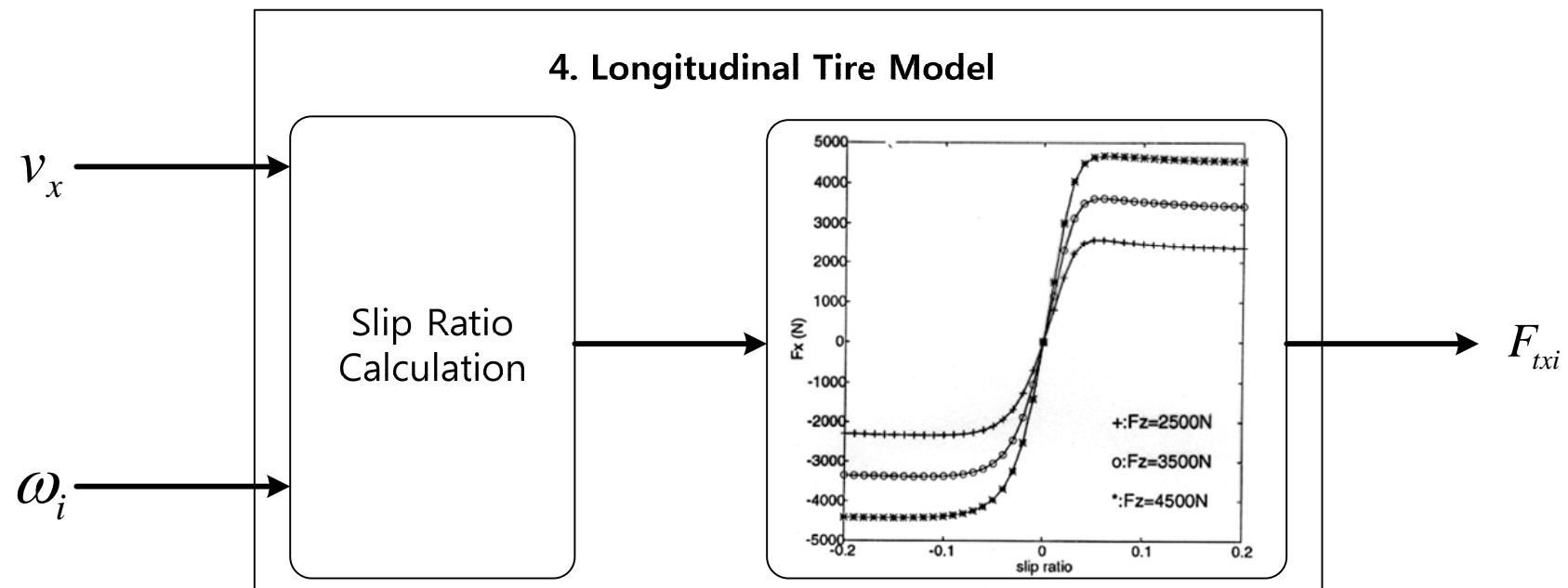
(1) System Input

v_x : Longitudinal Speed

ω_i : Wheel Angular Speed at each wheel

(2) System Output

F_{txi} : Longitudinal Tire Force at each wheel



4. Longitudinal Tire Model

(1) Slip Ratio

- During Braking

$$\lambda_i = \frac{r_i \cdot \omega_i - V_{ti} \cdot \cos(\alpha_i)}{V_{ti} \cdot \cos(\alpha_i)}$$

- During Traction

$$\lambda_i = \frac{r_i \cdot \omega_i - V_{ti} \cdot \cos(\alpha_i)}{r_i \cdot \omega_i}$$

Where, ω_i = Angular Velocity of the i -th Wheel

r_i = Tire Radius of the i -th Wheel

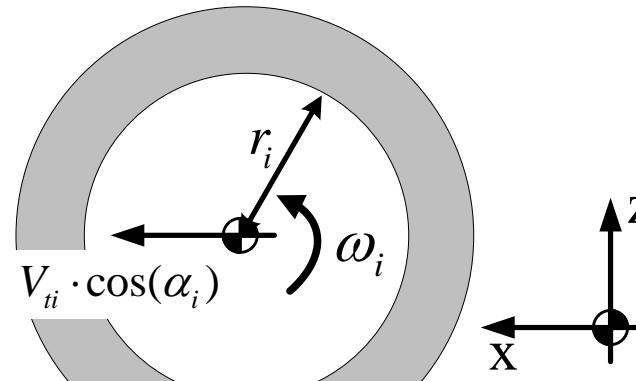
α_i = Tire Slip Angle at i -th Wheel

$$V_{t1} = \sqrt{(v_y + l_f \cdot \dot{\psi})^2 + (v_x - t_w \cdot \dot{\psi})^2}$$

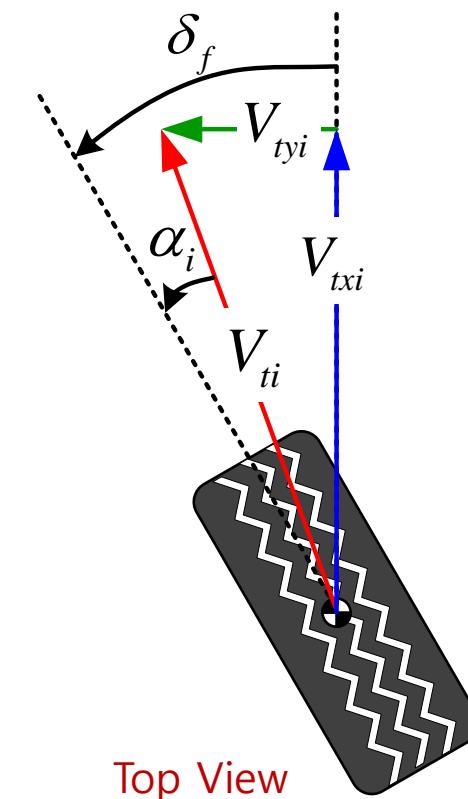
$$V_{t2} = \sqrt{(v_y + l_f \cdot \dot{\psi})^2 + (v_x + t_w \cdot \dot{\psi})^2}$$

$$V_{t3} = \sqrt{(v_y - l_r \cdot \dot{\psi})^2 + (v_x - t_w \cdot \dot{\psi})^2}$$

$$V_{t4} = \sqrt{(v_y - l_r \cdot \dot{\psi})^2 + (v_x + t_w \cdot \dot{\psi})^2}$$



Side View



Top View

4. Longitudinal Tire Model

(2) Tire Model

- Longitudinal Tire Force at the i-th Wheel

$$F_{txi} = D_x \sin(C_x \tan^{-1}(B_x \Phi_x)) + S_{vx}$$

Where, $\Phi_x = (1 - E_x)(\lambda_i + S_{hx}) + \frac{E_x}{B_x} \tan^{-1}(B_x(\lambda_i + S_{hx}))$

- During Traction ($\lambda_i > 0$)

$$B_x = 22 + \frac{F_{tzi} - 1940}{645}$$

$$E_x = -3.6$$

$$C_x = 1.35 - \frac{F_{tzi} - 1940}{16125}$$

$$S_{hx} = 0$$

$$D_x = 2000 + \frac{F_{tzi} - 1940}{0.956}$$

$$S_{vx} = 0$$

- During Braking ($\lambda_i \leq 0$)

$$B_x = 22 + \frac{F_{tzi} - 1940}{430}$$

$$E_x = 0.1$$

$$C_x = 1.35 - \frac{F_{tzi} - 1940}{16125}$$

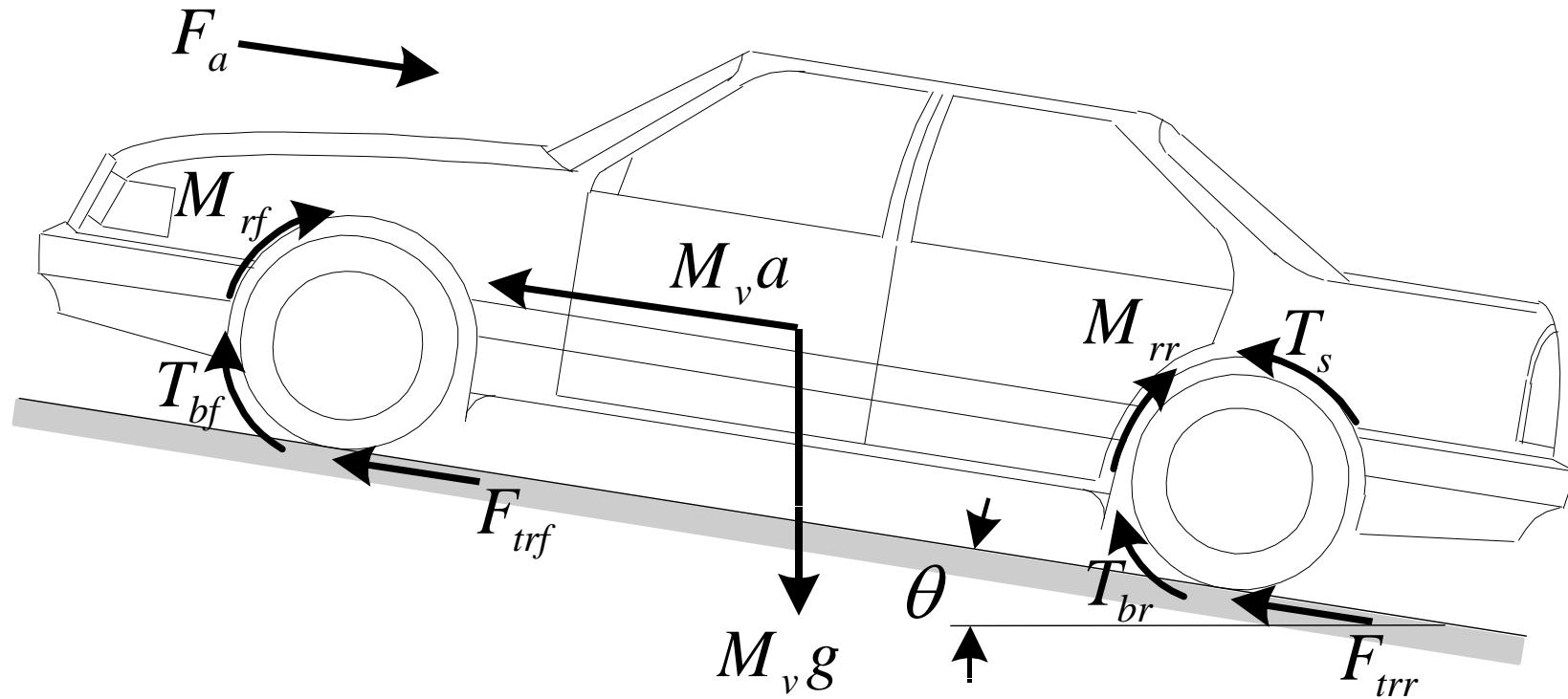
$$S_{hx} = 0$$

$$D_x = 1750 + \frac{F_{tzi} - 1940}{0.956}$$

$$S_{vx} = 0$$

5. Longitudinal Dynamic Model

- Free Body Diagram of Vehicle Body



5. Longitudinal Dynamic Model

Equations of Motion for Vehicle Body

Vehicle Body

$$M_v \frac{dv}{dt} = F_{trf} + F_{trr} - F_L$$

where,

M_v : vehicle mass

F_{trf}, F_{trr} : tractive forces of front/rear wheels

Driving Load

$$F_L = F_a - M_v g \sin \theta$$

F_a : aerodynamic force

g : gravitational constant

J_{wf}, J_{wr} : moment of inertia of front/rear wheel

T_s : driving shaft torque

r_f, r_r : radius of front/rear wheel

M_{rf}, M_{rr} : rolling resistance of front/rear wheel

Rear wheel (driving)

$$J_{wr} \frac{dw_r}{dt} = T_s - r_r F_{trr} - M_{rr} - T_{br}$$

Front wheel (driven)

$$J_{wf} \frac{dw_f}{dt} = -r_f F_{trf} - M_{rf} - T_{bf}$$

T_{bf}, T_{br} : brake torque of front/rear wheel

6 Brake Model

(1) System Input

F_p : Brake Pedal Force

(2) System Output

T_b : Brake Torque

(3) Brake Model

$$T_b = K_b \cdot P_w$$

$$= \frac{K_b}{1 + \tau_b \cdot s} \cdot P_{mc}$$

$$= \frac{K_b}{1 + \tau_b \cdot s} \cdot K_{mc} \cdot P_d = \frac{K_b}{1 + \tau_b \cdot s} \cdot K_{mc} \cdot K_{VB} \cdot F_p$$

