4. Rock strength and deformability

4.1 Introduction

- Behavior of excavation in rock mass depends on relative spacing, orientation and strength of discontinuities and stress level.

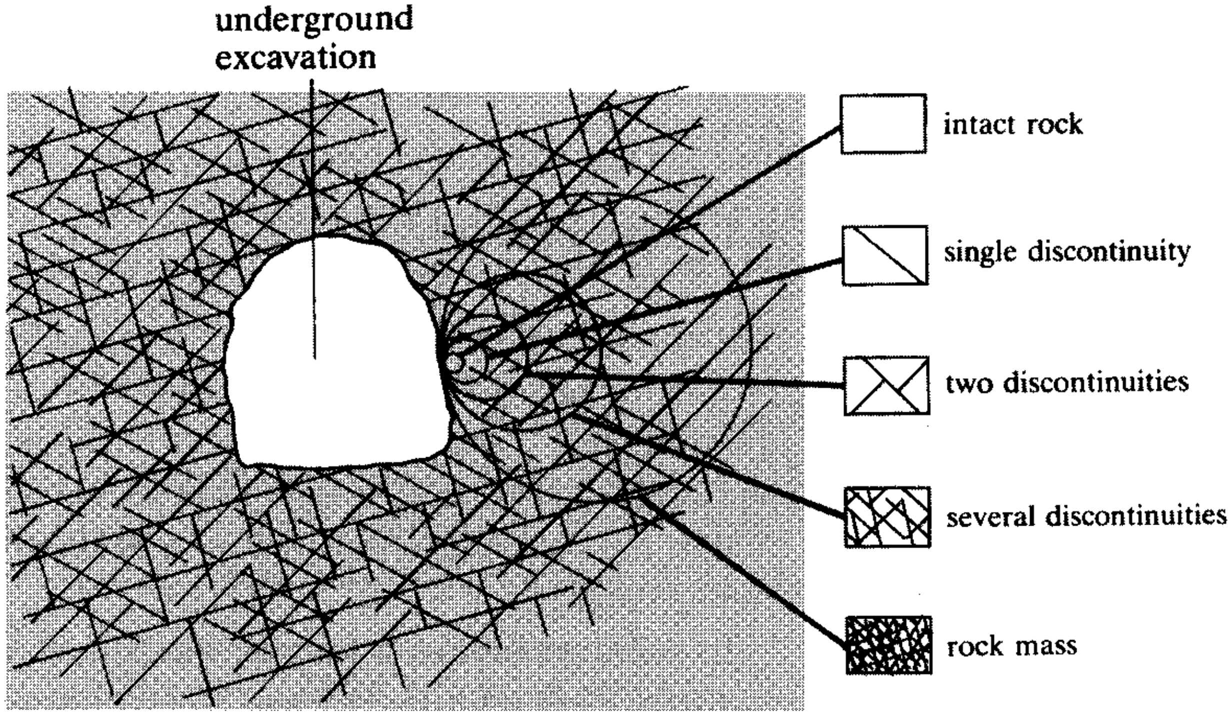
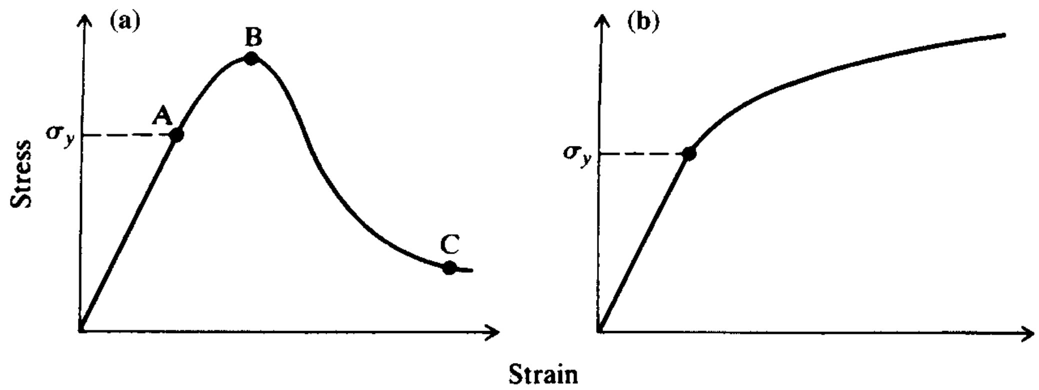


Fig. 4.1

4.2 Concepts and definitions

– Fracture: formation of planes of separation in rock material
– (Peak) Strength: the maximum stress that rock can sustain (point B)
– Residual strength: stress that a damaged rock can still carry (point C)
– Brittle fracture: process by which sudden loss of strength occurs following little or no permanent (plastic) deformation. It is associated with strain-softening or strain-weakening ((a)).
– Ductile deformation: deformation that occurs when the rock can sustain further permanent deformation without losing load-carrying capacity ((b)).



4.2 Concepts and definitions

– Yield: the point where a permanent deformation begins (point A).

– Failure: a state/point where a rock no longer adequately supports the load on it or where an excessive deformation takes place.

– Effective stress: a stress which governs the gross mechanical response of a porous material. It is a function of applied stress and pore (water) pressure.

$$\sigma_{ij}^{'} = \sigma_{ij} - \alpha u \delta_{ij} \quad (\alpha \leq 1,\ u : \text{pore pressure})$$

4.3 Behavior of isotropic rock material in uniaxial compression

- Influence of rock type and condition

– σ_c is one of the most fundamental property of a rock.

– Rock type gives some qualitative indication of its mechanical behavior: a slate shows an anisotropic behavior by cleavage in it; a quartzite is generally strong and brittle.

– σ_c depends on nature and composition of rock as well as test condition:

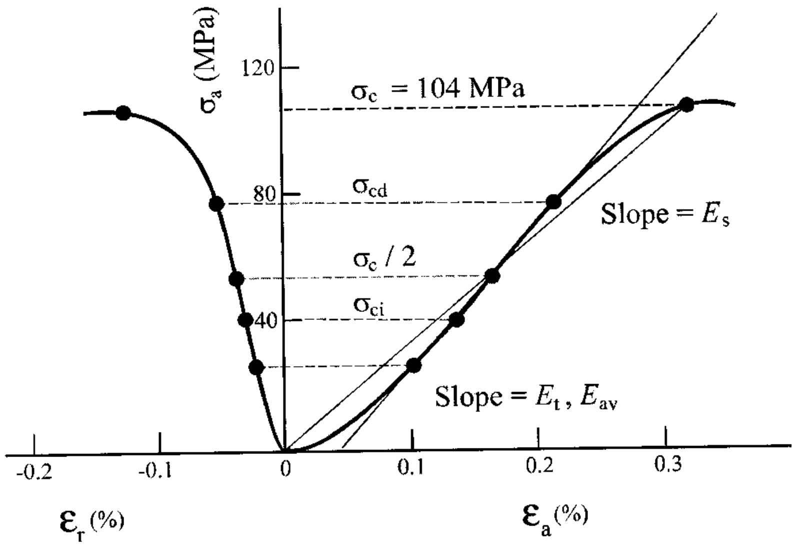
σ_c decreases with increasing porosity, degree of weathering, degree of microfissuring.

- Standard test procedure and interpretation

– Refer to the suggestion by ISRM Commission on Standardization of Laboratory and Field Tests (1979). (p88 of textbook)

4.3 Behavior of isotropic rock material in uniaxial compression

– Four stages of stress-strain response in uniaxial compression: crack closure-elastic deformation-stable crack propagation-unstable/irrecoverable deformation (~peak strength)



– Young's modulus: tangent -, average -, secant -.

– Volumetric strain: $\varepsilon_v = \varepsilon_a + 2\varepsilon_r$

4.3 Behavior of isotropic rock material in uniaxial compression

- End effects and influence of height to diameter ratio

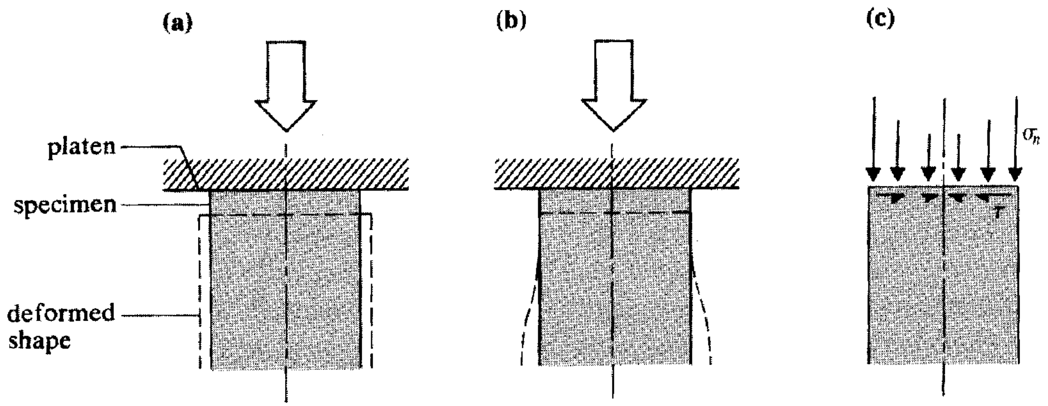
– Different lateral deformation of loading platen and specimen causes lateral restriction at both ends of the specimen (Fig.4.4).

– This effect is subject to H/D ratio of the specimen (Fig.4.5).

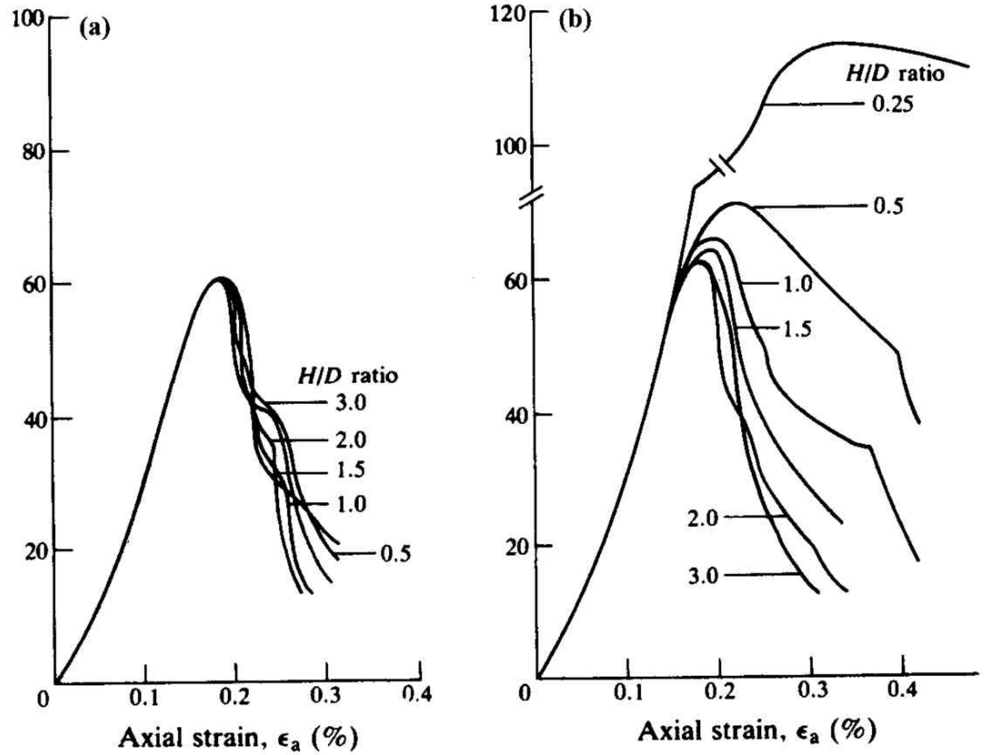
– Brush platen consisting of 3.2 mm^2 steel pins is effective to prevent the end effect, but it is too difficult to be prepared and applied in routine testing.

– Inserting a sheet of soft material or applying a lubricant cause tensile stress at the ends.

– ISRM Commission (1979) recommends treatments of the sample ends except by machining to be avoided.



(a)
(b)
(c)
platen
specimen
deformed
shape
σ_n
τ



(a)
100
80
60
40
20
0
0.1
0.2
0.3
0.4
H/D ratio
3.0
2.0
1.5
1.0
0.5
Axial strain, ϵ_a (%)
(b)
120
100
60
40
20
0
0.1
0.2
0.3
0.4
H/D ratio
0.25
0.5
1.0
1.5
2.0
3.0
Axial strain, ϵ_a (%)

4.3 Behavior of isotropic rock material in uniaxial compression

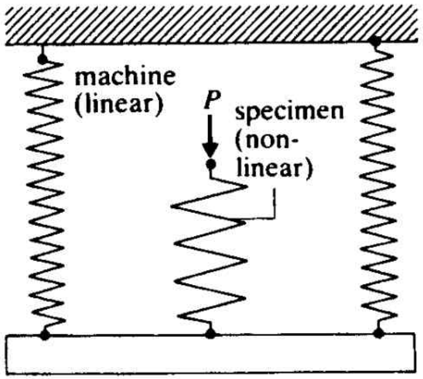
- Influence of testing machine stiffness

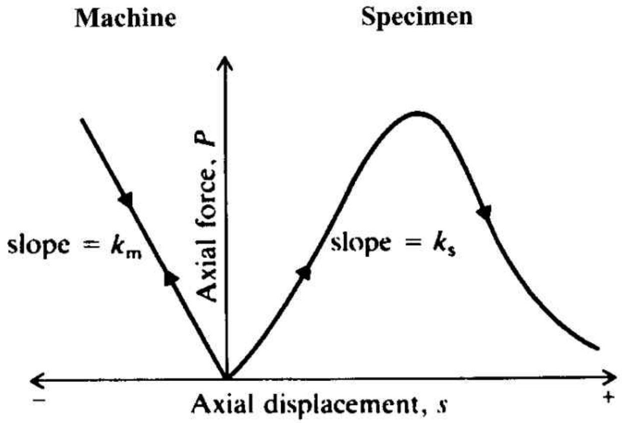
– Machine pillars are extended while a specimen is compressed (Fig.4.6).

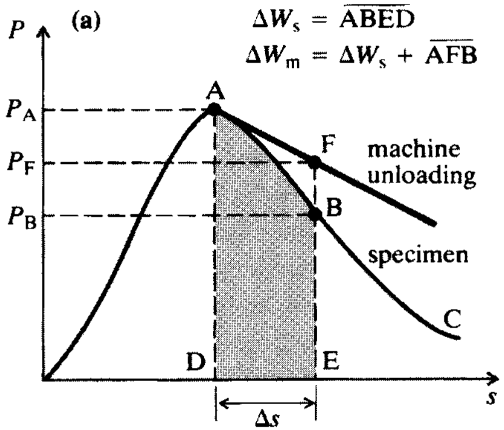
– Test machine should be stiffer than the specimen to observe the post-peak stress-strain behavior (Fig.4.7).

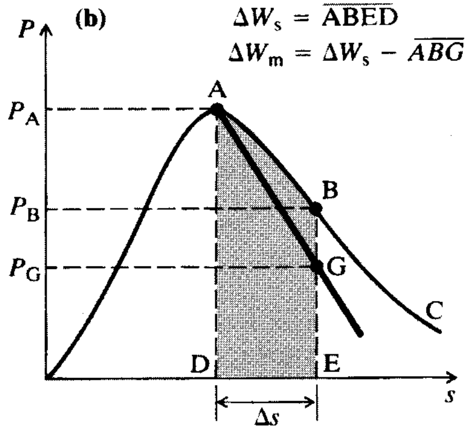
– For brittle rocks of which stiffness is higher than that of a test machine, servo-controlled equipment is required to record the post-peak behavior of the specimen.

– Strain (displacement) rate is serve-controlled for the post-peak behavior (Fig.4.9).









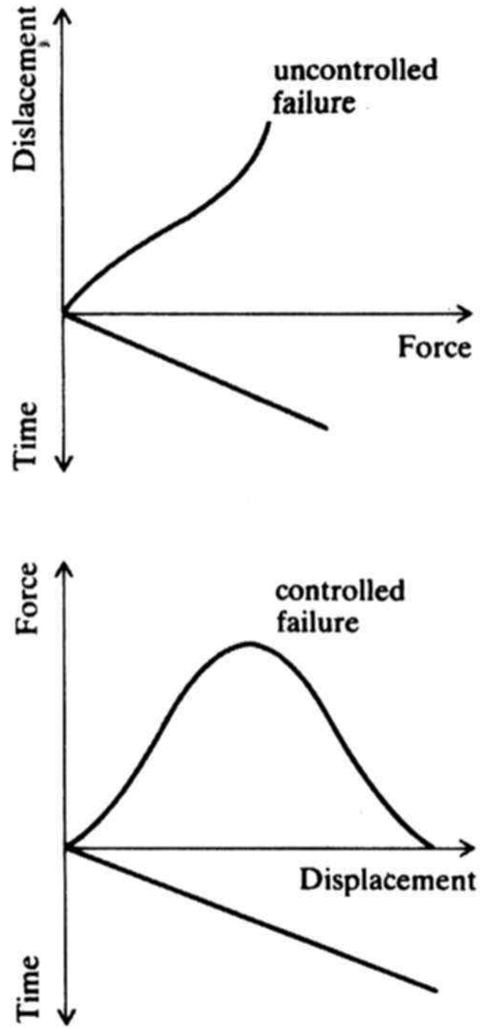


Figure 4.9 Choice between force and displacement as the programmed control variable (after Hudson *et al.*, 1972a).

4.3 Behavior of isotropic rock material in uniaxial compression

- Influence of loading and unloading cycles (Fig.4.13)

– Irrecoverable displacement increases as loading-unloading proceeds in the post-peak region.

– Apparent modulus of rock decreases as loading-unloading proceeds in the post-peak region.

– Few cycles of loading-unloading are recommended in pre-peak range because they show permanent deformation.

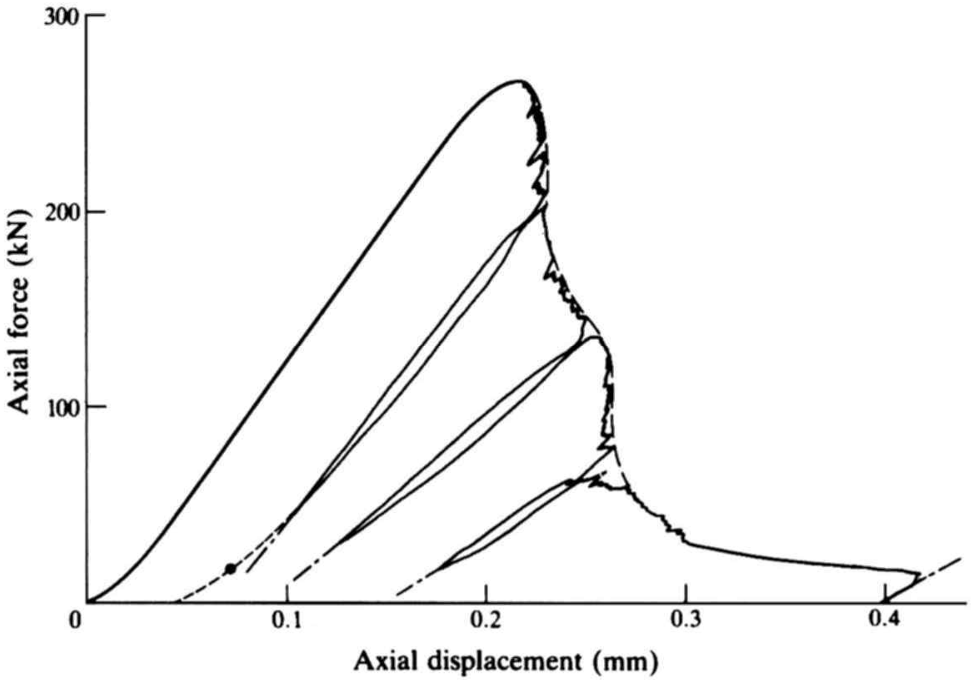
- Point load test (Fig.4.14)

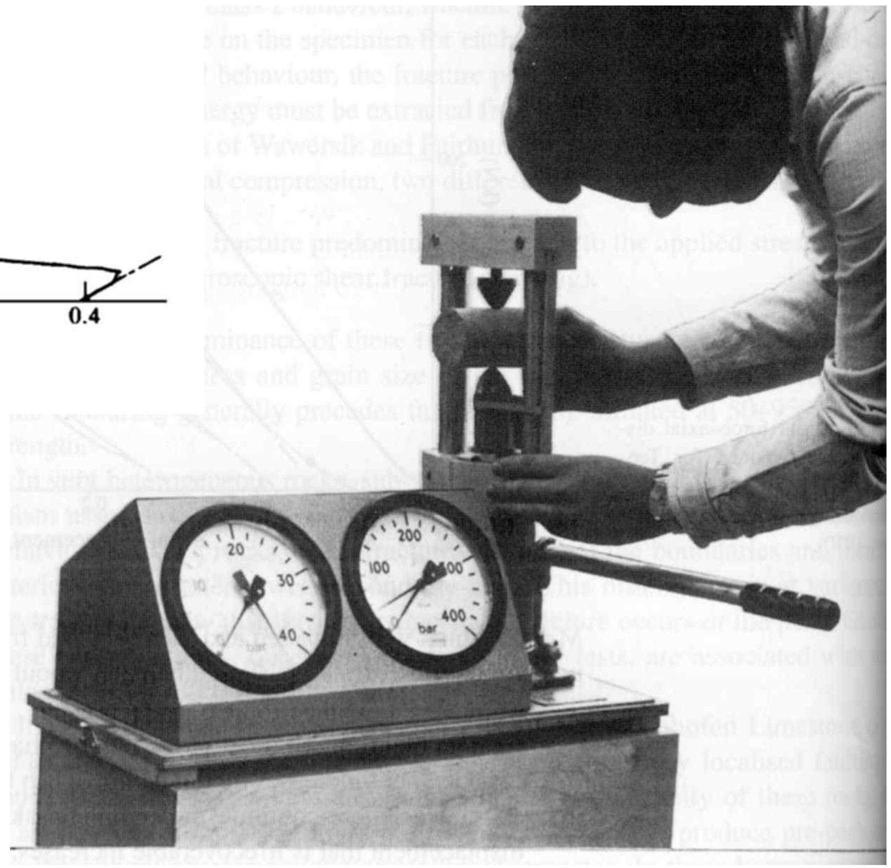
– It is carried out when only an approximate measure of peak strength is required.

– $I_s = P/D_e^2$ ($=\pi P/4A$) is an Uncorrected Point Load Index which depends on D_e.

– $I_{s(50)} = I_s \times (D_e/50)^{0.45}$ is the size-corrected Point Load Strength Index.

– $\sigma_c = (22 \sim 24)\ I_{s(50)}$


300
200
100
Axial force (kN)
0
0.1
0.2
0.3
0.4
Axial displacement (mm)


10
20
30
40
bar
100
200
400
bar

4.4 Behavior of isotropic rock material in multiaxial compression

- Types of multiaxial compression test
  - Biaxial: $\sigma_1 \geq \sigma_2$, $\sigma_3 = 0$.
  - Triaxial: $\sigma_1 > \sigma_2 = \sigma_3$.
  - Polyaxial: $\sigma_1 > \sigma_2 > \sigma_3$.

- Biaxial compression ($\sigma_1 \geq \sigma_2$, $\sigma_3 = 0$)
  - The effect of intermediate principal stress can be neglected so that the uniaxial compressive strength should be used as the rock strength whenever $\sigma_3 = 0$ (Fig.4.15).

- Triaxial compression ($\sigma_1 > \sigma_2 = \sigma_3$)
  - Deformation behavior of rock becomes close to ductile as σ_3 increases (Fig.4.18, 19).
  - Volumetric strain decreases until peak-stress and increases after the peak-stress under relatively lower confining pressure (Fig.4.18).
  - Brittle-ductile transition pressure of granite and quartzite is over 1 GPa.

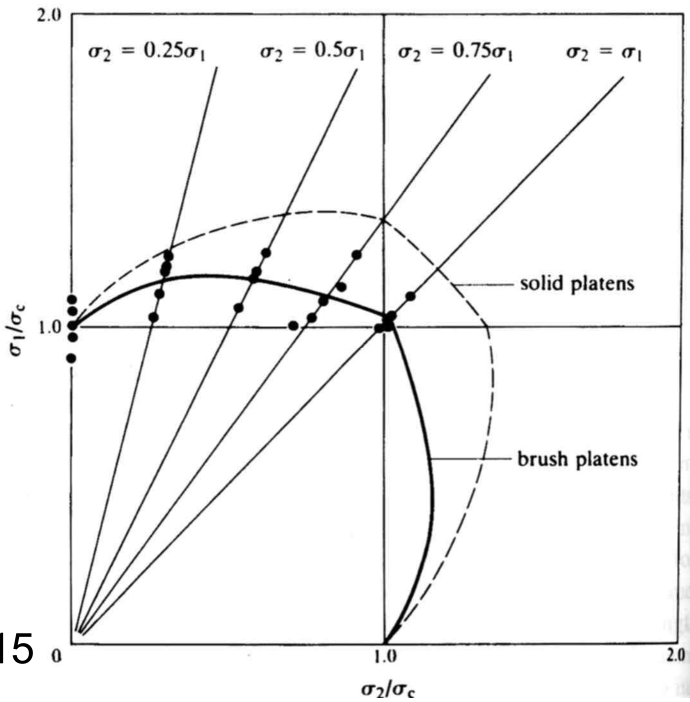


Fig. 4.15

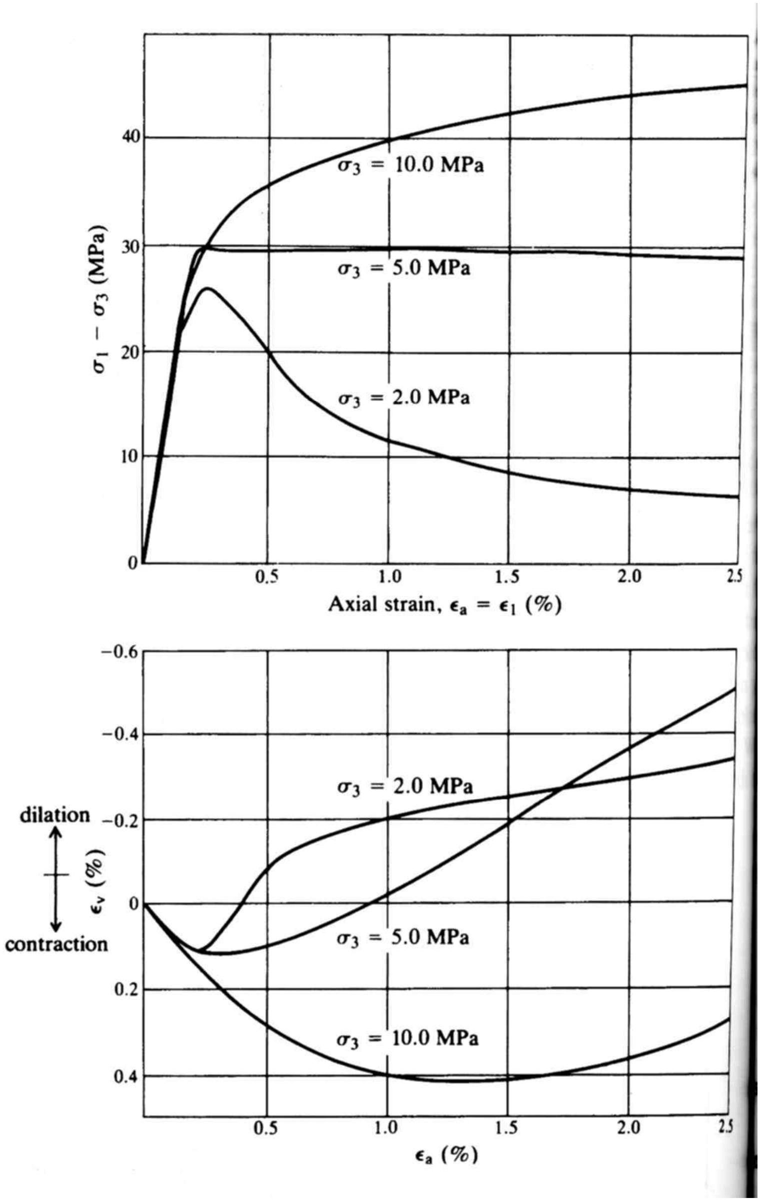


Fig. 4.18

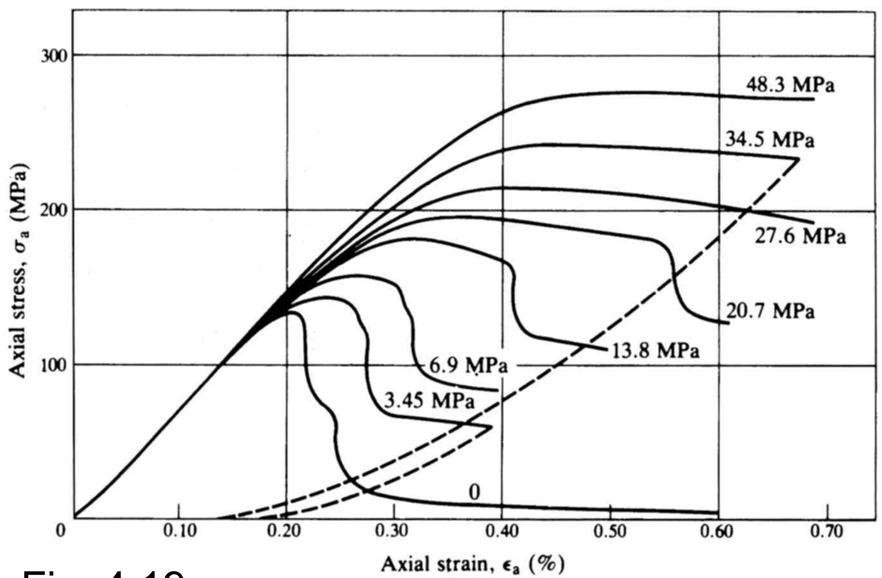


Fig. 4.19

4.4 Behavior of isotropic rock material in multiaxial compression

– Pore pressure makes the rock under confining pressure close to brittle as it increases (Fig.4.20).

– The classical effective stress law by Terzaghi is not well applied to low permeable rocks nor to loading condition with high strain rate.

- Polyaxial compression ($\sigma_1 > \sigma_2 > \sigma_3$)

– End effect is a main obstacle of the test as in the biaxial test.

– σ_2 influences the test result, but it is not as great as σ_3.

- Influence of stress path

– Strength or strain of rock is stress path-independent (Fig.4.21).

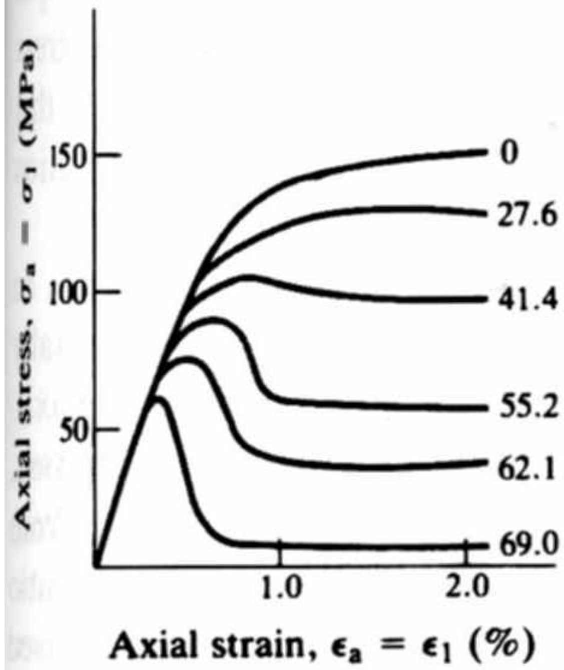


Figure 4.20 Effect of pore pressure (given in MPa by the numbers on the curves) on the stress–strain behaviour of a limestone tested at a constant confining pressure of 69 MPa (after Robinson, 1959).

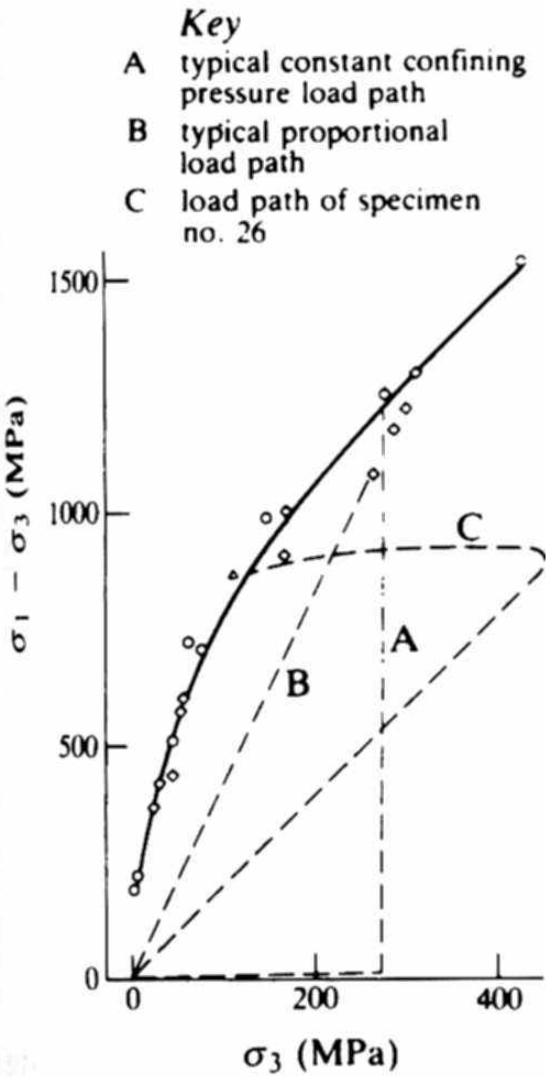


Figure 4.21 Influence of stress path on the peak strength envelope for Westerly Granite (after Swanson and Brown, 1971).

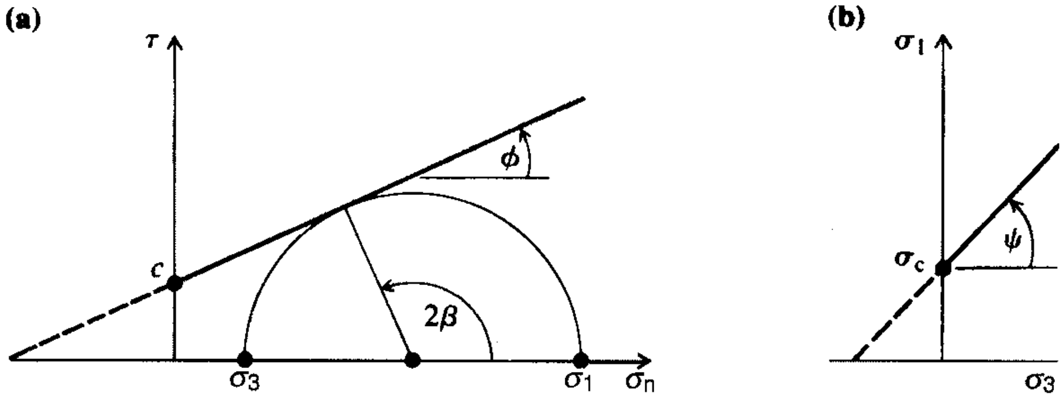
4.5 Strength criteria for isotropic rock material

- Types of strength criterion

– Pore pressure and σ_2 generally affect little on rock failure: $\sigma_1 = f(\sigma_3)$
– Criterion on a particular plane: $\tau = f(\sigma_n)$.

- Coulomb's shear strength criterion

– s = c + σ_n tanΦ (Φ is an internal friction angle)



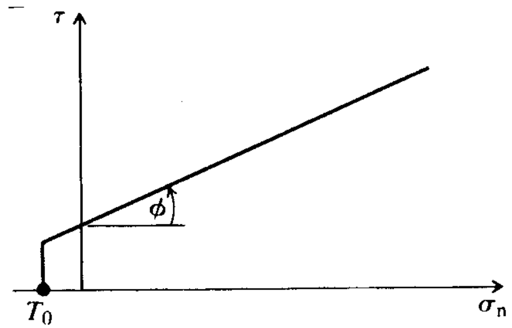
$$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\beta, \quad \tau = \frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\beta$$

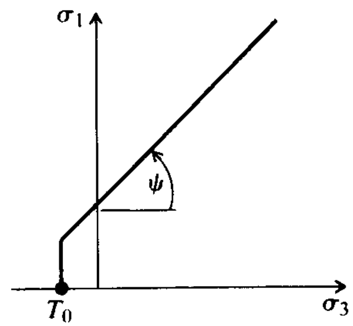
4.5 Strength criteria for isotropic rock material

$$- \ \sigma_1 = \frac{2c + \sigma_3 \left[\sin 2\beta + \tan\phi \left(1 - \cos 2\beta\right)\right]}{\sin 2\beta - \tan\phi \left(1 + \cos 2\beta\right)} \ , \ \beta = \frac{\pi}{4} + \frac{\phi}{2}$$

$$\rightarrow \ \sigma_1 = \frac{2c\cos\phi + \sigma_3\left(1 + \sin\phi\right)}{1 - \sin\phi} = \sigma_c + \sigma_3 \tan\psi$$

$$\sigma_c = \frac{2c\cos\phi}{1 - \sin\phi}, \ \sigma_T = \frac{2c\cos\phi}{1 + \sin\phi}$$





4.5 Strength criteria for isotropic rock material

– Discrepancy between reality and Coulomb’s criterion:

1) Major fractures in failure are not always based on shear failure.

2) Predicted direction of shear failure does not always agree with experimental observations.

3) Experimental failure envelopes are generally non-linear.

- Griffith crack theory

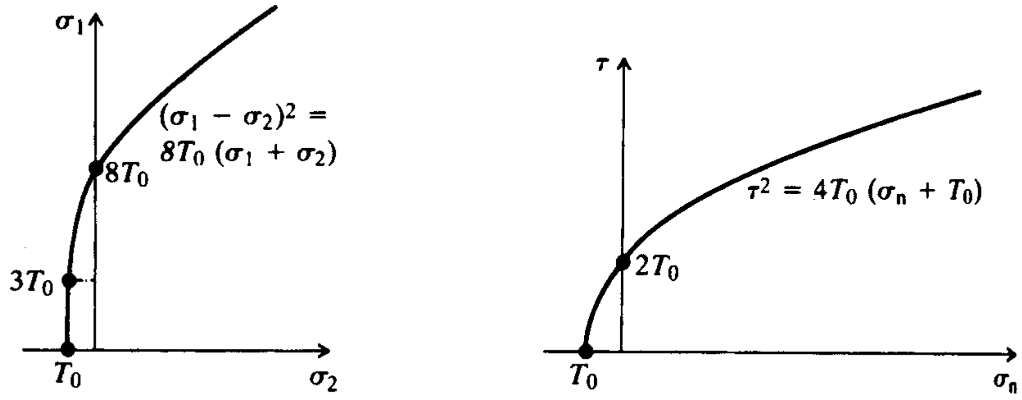
– Energy instability concept on crack extension: A crack will extend only when the total potential energy of the system of applied forces and material decreases or remains constant with an increase in crack length.

$$\sigma \geq \sqrt{\frac{2E\alpha}{\pi c}}$$

σ : tensile stress normal to the crack

α : surface energy per unit area of the crack having an initial length of 2c

4.5 Strength criteria for isotropic rock material

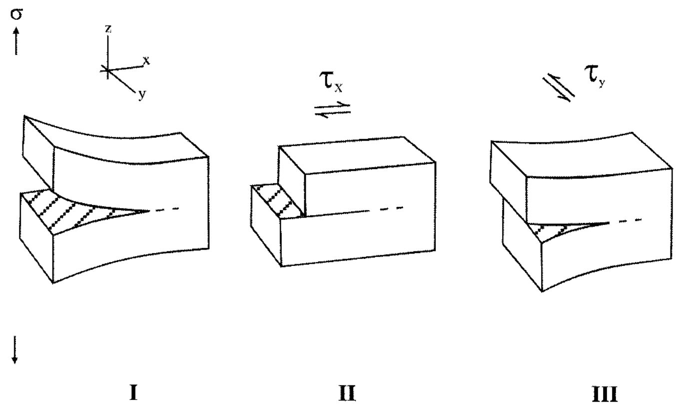


- The classical Griffith criterion does not provide a very good model for the peak strength of rock → a number of modification to Griffith's solution were introduced.

4.5 Strength criteria for isotropic rock material

- Fracture mechanics

– Basic modes of distortion: I (extension, opening), II (in-plane shear) and III (out-of-plane shear)



– Stress intensity factor: a factor to predict the stress state near the tip of a crack caused by a far field stress.

– Fracture toughness: a property which describes the ability of a material containing a crack to resist fracture = critical stress intensity factors (stress intensity factor at crack extension (K_{IC}, K_{IIC}, K_{IIIC})

$$K_I = \sigma\sqrt{\pi c},\ \sigma_z = K_I \big/ \sqrt{2\pi x}$$

4.5 Strength criteria for isotropic rock material

- Empirical criteria

– Bieniawski (1974)'s criterion

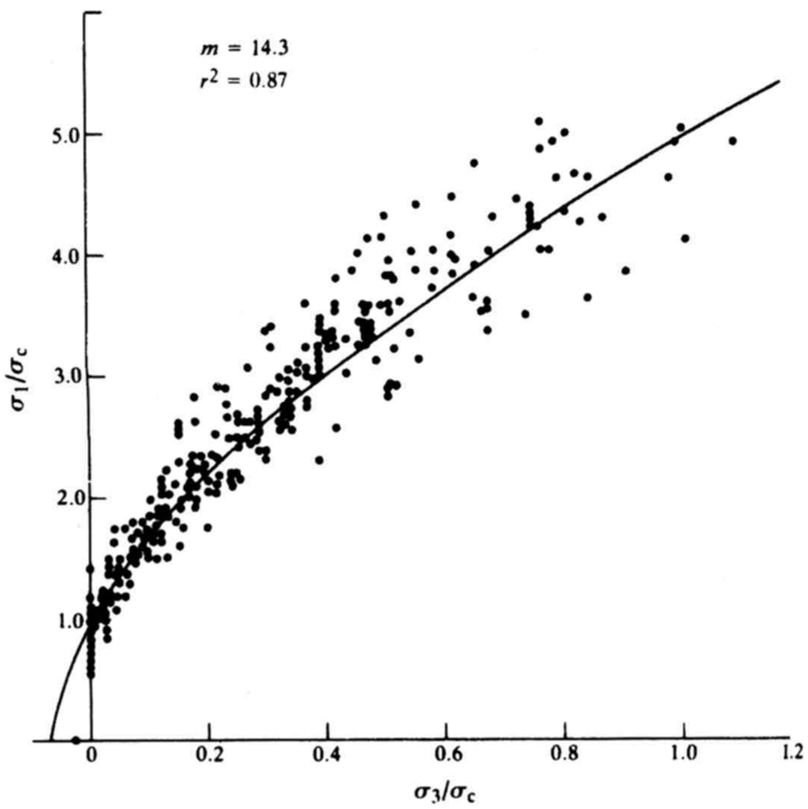
$$\frac{\sigma_1}{\sigma_c} = 1 + A\left(\frac{\sigma_3}{\sigma_c}\right)^k, \quad \frac{\tau_m}{\sigma_c} = 0.1 + B\left(\frac{\sigma_m}{\sigma_c}\right)^c$$

Table 4.1 Constants in Bieniawski's empirical strength criterion (after Bieniawski, 1974).

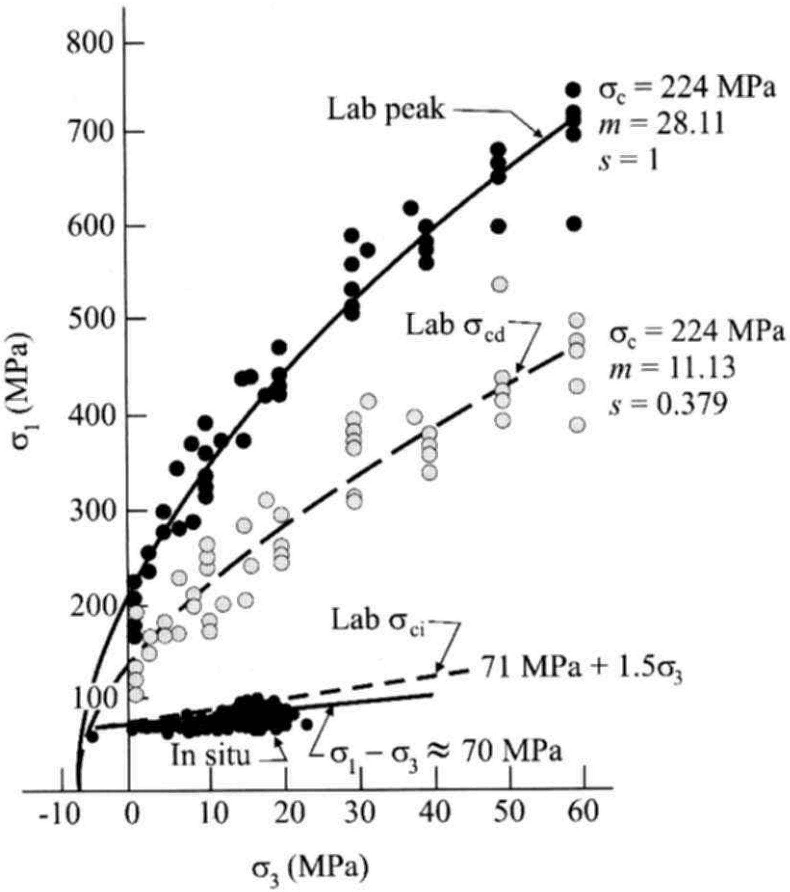
Rock type	A	B
norite	5.0	0.8
quartzite	4.5	0.78
sandstone	4.0	0.75
siltstone	3.0	0.70
mudstone	3.0	0.70

– Hoek & Brown (1980)'s criterion (Fig.4.30, 4.31 & Table 4.2)

$$\frac{\sigma_1}{\sigma_c} = \frac{\sigma_3}{\sigma_c} + \sqrt{m\frac{\sigma_3}{\sigma_c} + s}$$, m varies with rock type and $s = 1$ for intact rock.



m = 14.3
r² = 0.87
σ1/σc
σ3/σc
0
0.2
0.4
0.6
0.8
1.0
1.2
1.0
2.0
3.0
4.0
5.0



Lab peak
σc = 224 MPa
m = 28.11
s = 1
Lab σcd
σc = 224 MPa
m = 11.13
s = 0.379
Lab σci
71 MPa + 1.5σ3
In situ
σ1 − σ3 ≈ 70 MPa
σ1 (MPa)
σ3 (MPa)
800
700
600
500
400
300
200
100
-10
0
10
20
30
40
50
60

Table 4.2 Variation of the constant m_i for intact rock by rock group (after Hoek, 2003).

Rock Type	Class	Group	Texture: Coarse	Texture: Medium	Texture: Fine	Texture: Very fine
SEDIMENTARY	Clastic		Conglomerates* Breccias*	Sandstones 17± 4	Siltstones 7 ± 2 Greywackes (18 ± 3)	Claystones 4 ± 2 Shales (6 ± 2) Maris (7 ± 2)
SEDIMENTARY	Non-Clastic	Carbonates	Crystalline Limestone (12 ± 3)	Sparitic Limestones (10 ± 2)	Micritic Limestones (9 ± 2)	Dolomites (9 ± 3)
SEDIMENTARY	Non-Clastic	Evaporites		Gypsum 8 ± 2	Anhydrite 12 ± 2	
SEDIMENTARY	Non-Clastic	Organic				Chalk 7 ± 2
METAMORPHIC	Non Foliated		Marble 9 ± 3	Hornfels (19 ± 4) Metasandstone (19 ± 3)	Quartzites 20 ± 3	
METAMORPHIC	Slightly foliated		Migmatite (29 ± 3)	Amphibolites 26 ± 6	Gneiss 28 ± 5	
METAMORPHIC	Foliated **			Schists 12 ± 3	Phyllites (7 ± 3)	Slates 7 ± 4
IGNEOUS	Plutonic	Light	Granite 32 ± 3	Diorite 25 ± 5		
IGNEOUS	Plutonic	Light	Granodiorite (29 ± 3)			
IGNEOUS	Plutonic	Dark	Gabbro 27 ± 3	Dolerite (16 ± 5)		
IGNEOUS	Plutonic	Dark	Norite 20 ± 5			
IGNEOUS	Hypabyssal		Porphyries (20 ± 5)		Diabase (15 ± 5)	Periodotite (25 ± 5)
IGNEOUS	Volcanic	Lava		Rhyolite (25 ± 5) Andesite 25 ± 5)	Dacite (25 ± 3) Basalt (25 ± 5)	
IGNEOUS	Volcanic	Pyroclastic	Agglomerate (19 ± 3)	Breccia (19 ± 5)	Tuff (13 ± 5)	

4.6 Strength of anisotropic rock material in triaxial compression

- Peak strength of transversely isotropic rock

– Max. differential stress vs. inclination angle (α): Fig. 4.33

– Simplified theoretical approach: Fig. 4.34(b)

$$\left(\sigma_1-\sigma_3\right)_s=\frac{2\left(c_w+\sigma_3\tan\phi_w\right)}{\left(1-\tan\phi_w\cot\beta\right)\sin 2\beta}\quad \text{(when a weak plane fails)}$$

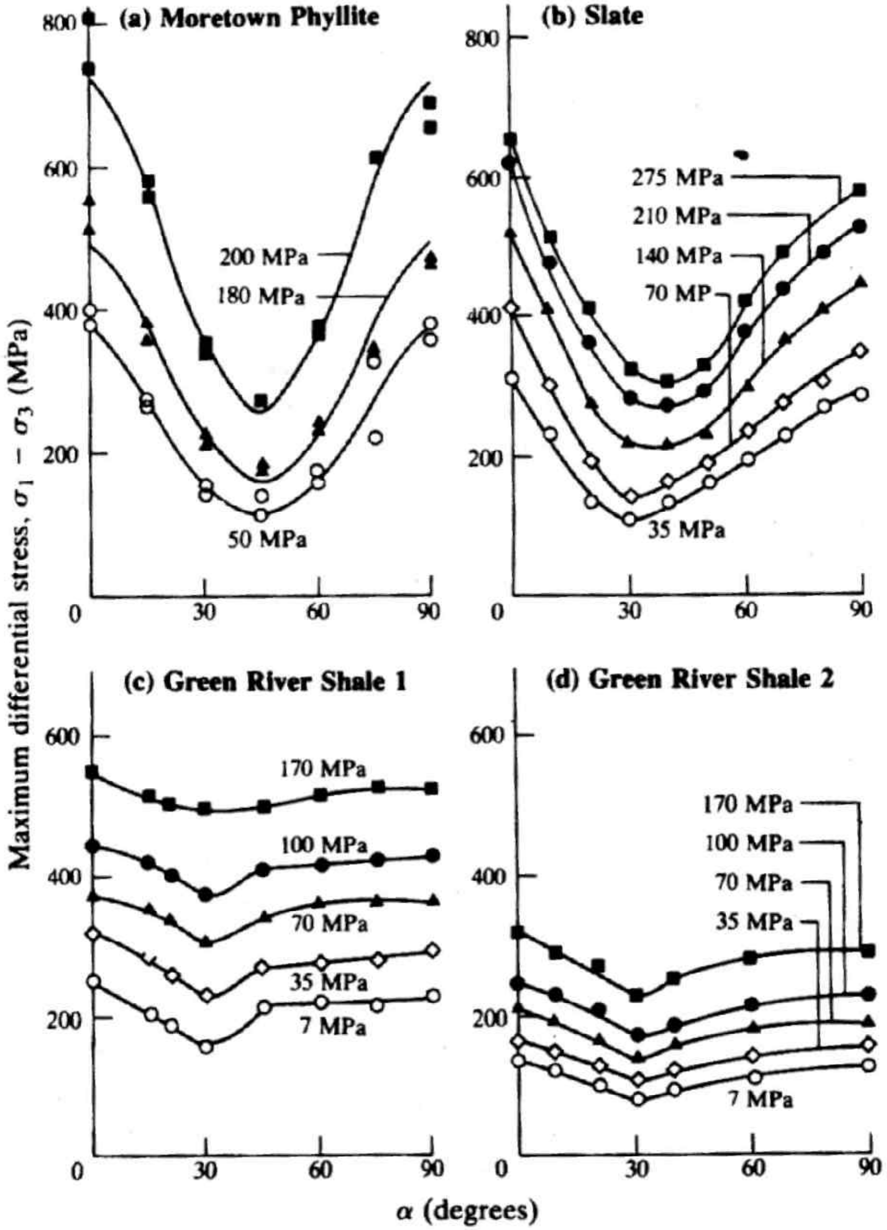
$$\left(\sigma_1-\sigma_3\right)_s>0\ \rightarrow\ \beta\neq 90^o,\ \beta>\phi_w\ \left(\because 1-\tan\phi_w\cot\beta>0\right)$$

– Plateau in Fig. 4.34(b) exists because the role of weak planes in failure is not considered when the failure occurs out of the weak planes.

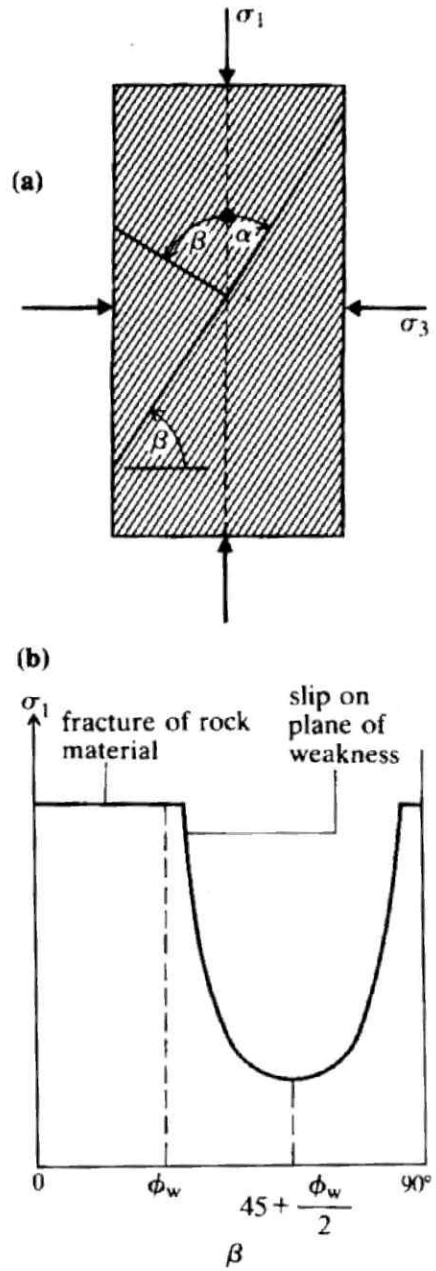
– Functions of c_w and $\tan\phi_w$ have been proposed to correct the plateau:

$$c_w=A-B\left[\cos 2\left(\alpha-\alpha_{c0}\right)\right]^n$$

$$\tan\phi_w=C-D\left[\cos 2\left(\alpha-\alpha_{\phi 0}\right)\right]^m$$



(a) Moretown Phyllite
200 MPa
180 MPa
50 MPa
(b) Slate
275 MPa
210 MPa
140 MPa
70 MP
35 MPa
(c) Green River Shale 1
170 MPa
100 MPa
70 MPa
35 MPa
7 MPa
(d) Green River Shale 2
170 MPa
100 MPa
70 MPa
35 MPa
7 MPa
Maximum differential stress, $\sigma_1 - \sigma_3$ (MPa)
α (degrees)

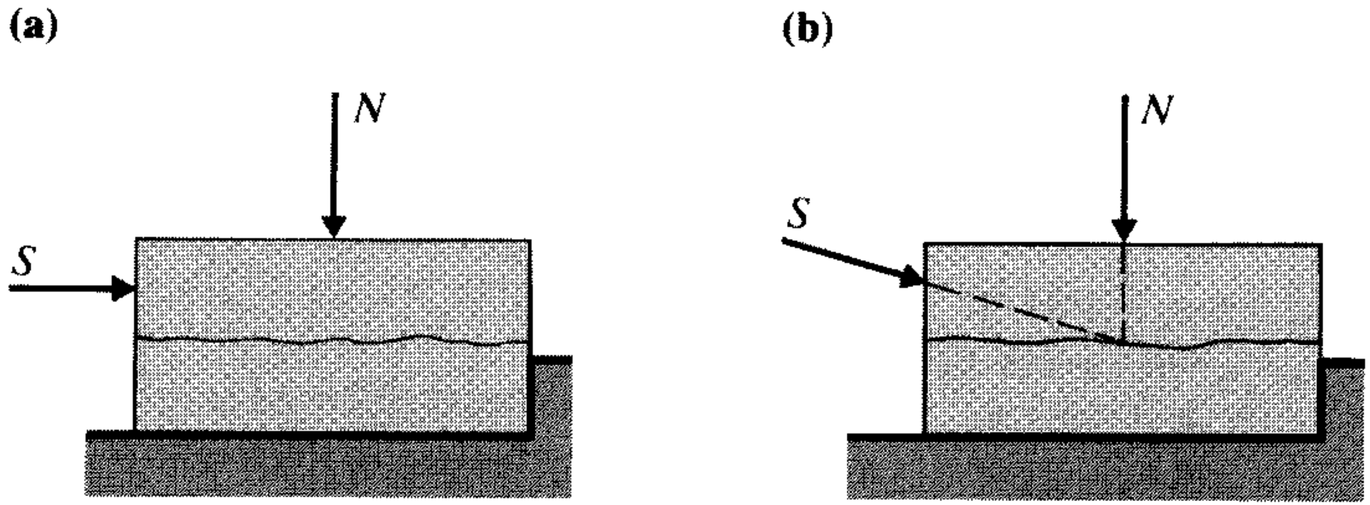


(a)
σ_1
σ_3
α
β
(b)
σ_1
fracture of rock material
slip on plane of weakness
0
ϕ_w
$45 + \frac{\phi_w}{2}$
90°
β

4.7 Shear behavior of discontinuities

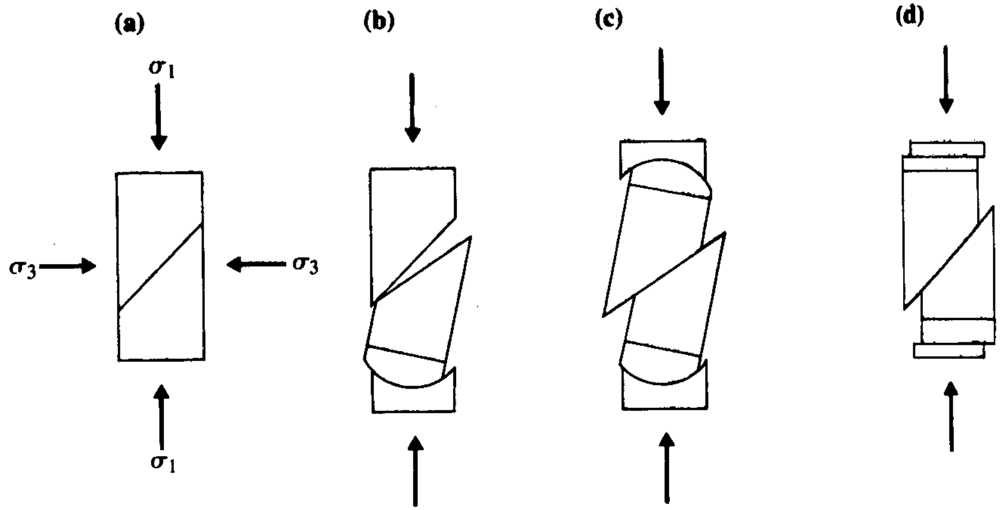
- Shear testing

– Direct shear tests with shear force (a) parallel or (b) inclined (10° ~15°) to the discontinuity: the latter is not proper to the test with very low normal stress.



4.7 Shear behavior of discontinuities

– A core specimen with a discontinuity in a triaxial cell can be used for the test
– Stage testing has been devised to overcome the difficulty in preparing several specimens containing similar discontinuities: disc seats lubricated with a molybdenum disulphide grease are recommended.

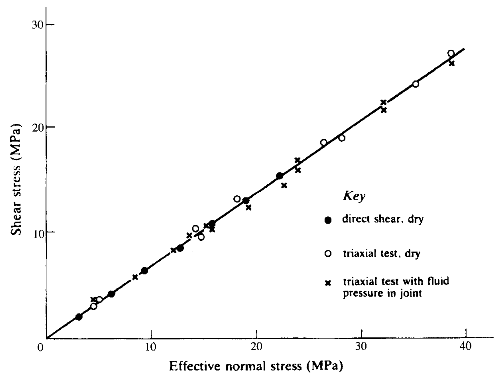


4.7 Shear behavior of discontinuities

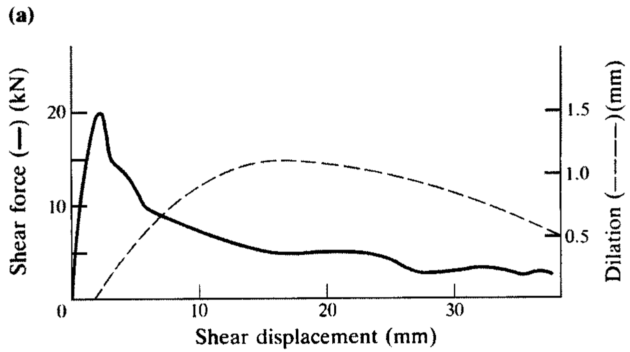
- Influence of surface roughness on shear strength

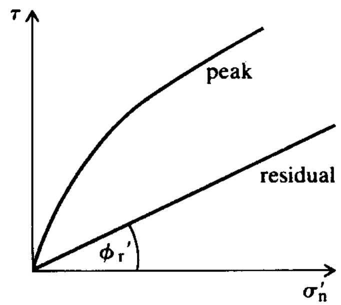
– Shear strength envelope of a smooth discontinuity surface

$$s = \sigma_n \tan\phi$$



– Shear strength envelope of a rough joint





4.7 Shear behavior of discontinuities

- Influence of surface roughness on shear strength

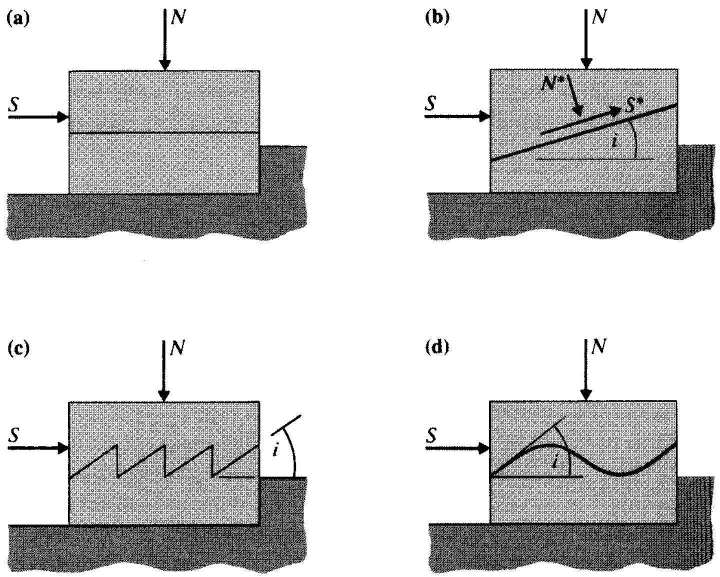
– Shear strength envelope of a sawtooth-shaped joint

$$\frac{S}{N} = \tan\phi \text{ : a flat joint}$$

$$\frac{S'}{N'} = \frac{S\cos i - N\sin i}{N\cos i + S\sin i} = \tan\phi \text{ : an inclined joint}$$

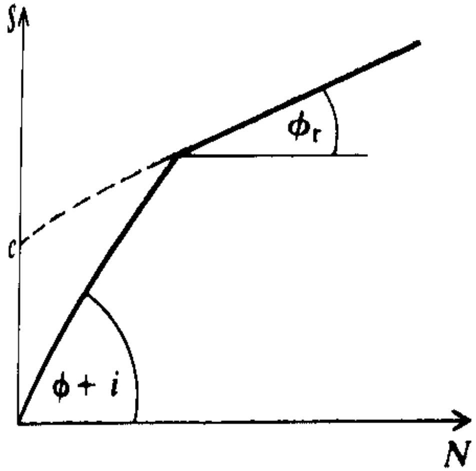
$$\frac{S/N\cos i - \sin i}{\cos i + S/N\sin i} = \tan\phi$$

$$\frac{S}{N} = \tan(\phi + i)$$

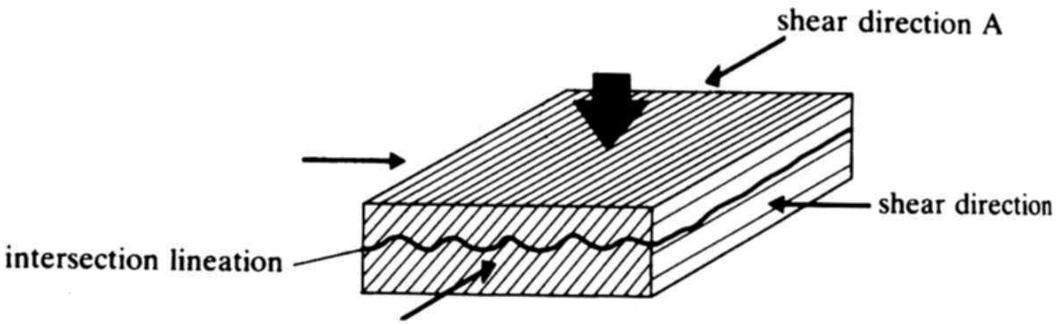


4.7 Shear behavior of discontinuities

– Considering the shearing off of asperities



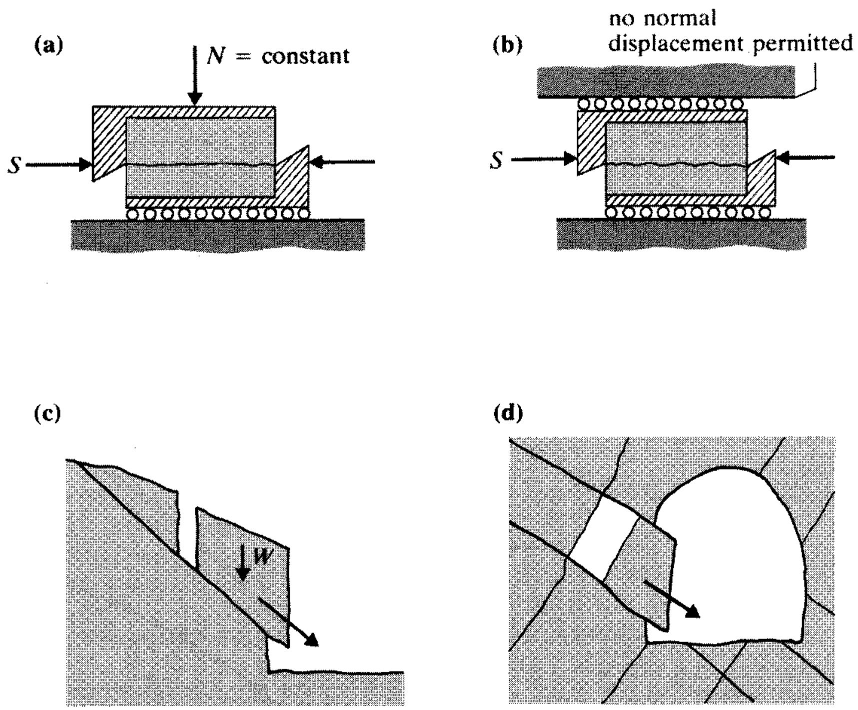
– Shear strength envelope of anisotropic rocks



4.7 Shear behavior of discontinuities

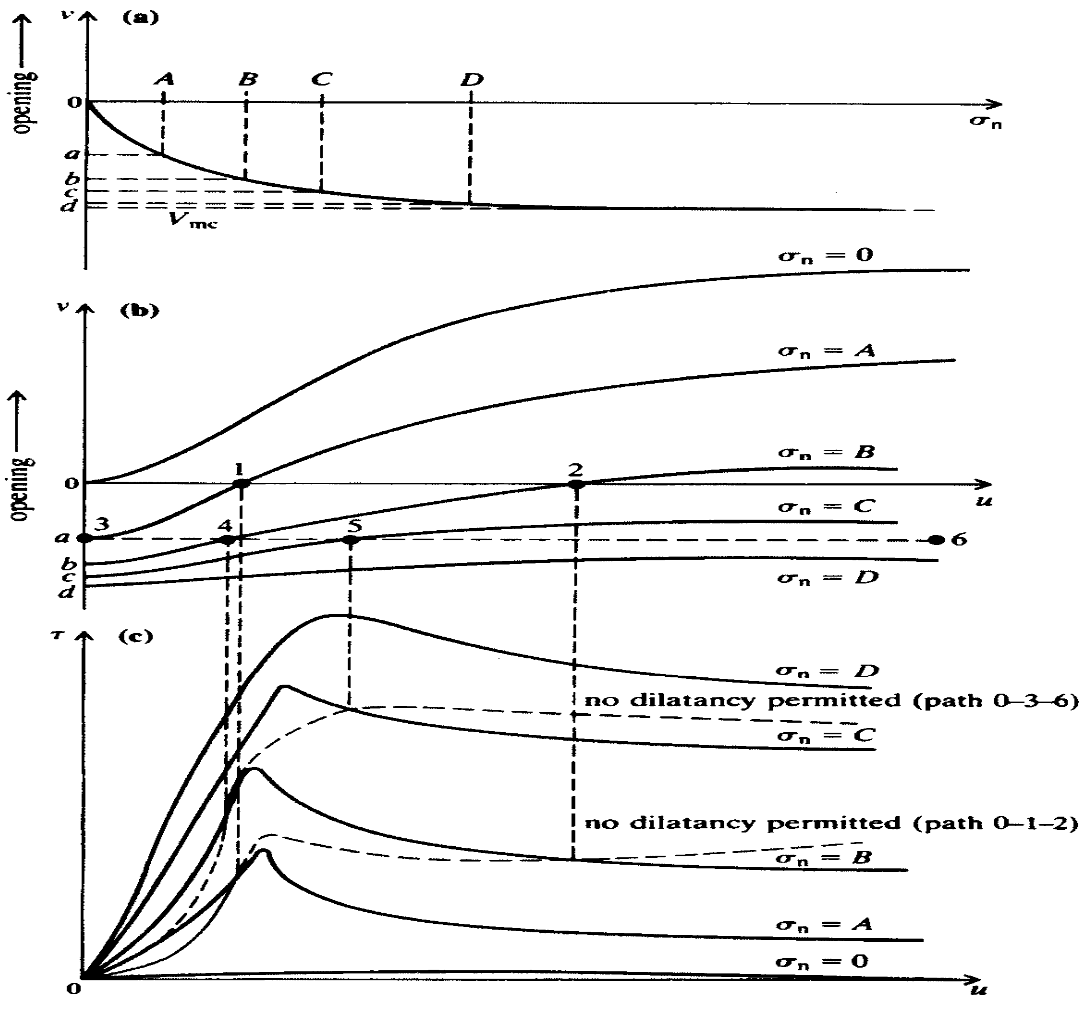
- Interrelation between dilatancy and shear strength

– Constant normal stress (controlled normal force) vs. constant normal strain (controlled normal displacement)



4.7 Shear behavior of discontinuities

– Normal stress-displacement & shear displacement-dilatancy-shear stress



4.7 Shear behavior of discontinuities

- Influence of scale

$$\tau = \sigma_n \tan\left[\phi_r + JRC\log_{10}\left(\frac{JCS}{\sigma_n}\right)\right] \rightarrow \sigma_n \tan\left[\phi_r + i\right]$$

$JRC\log_{10}\left(\frac{JCS}{\sigma_n}\right)$: net roughness component

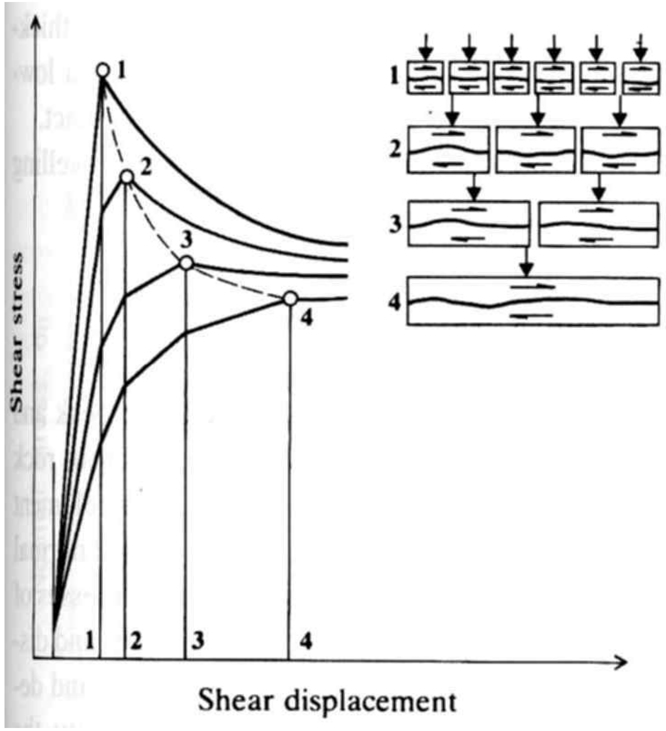
JRC : geometrical component ($\downarrow$ as scale $\uparrow$)

$\frac{JCS}{\sigma_n}$: asperity failure component ($\downarrow$ as scale $\uparrow$)

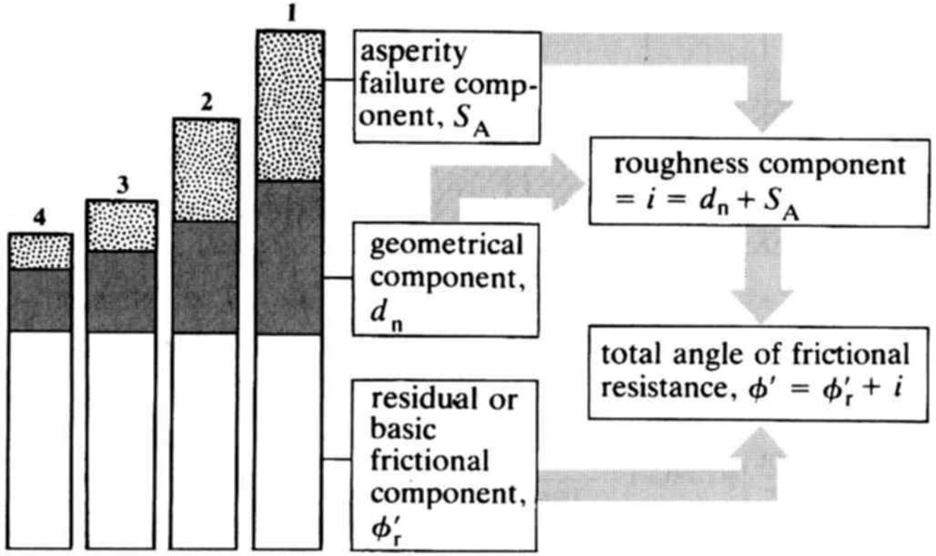
– Refer to Fig.4.46

- Infilled discontinuities

– Filling materials which are soft and weak decrease both stiffness and shear strength



Shear stress
Shear displacement
1
2
3
4



4
3
2
1
asperity failure component, S_A
geometrical component, d_n
residual or basic frictional component, ϕ'_r
roughness component $= i = d_n + S_A$
total angle of frictional resistance, $\phi' = \phi'_r + i$

4.8 Models of discontinuity strength and deformation

- Coulomb friction, linear deformation model

– Appropriate for smooth discontinuities such as faults at residual strength (Fig.4.47)

$$\tau = c + \sigma_n \tan\phi$$

- Barton-Bandis model

– Normal stiffness of a joint is highly dependent on normal stress (Fig.4.48)

– Contribution of roughness to shear strength decreases during post-peak shearing due to mismatch and wear.

$$\tau = \sigma_n \tan\left[\phi_r + JRC \log_{10}\left(\frac{JCS}{\sigma_n}\right)\right]$$

- Continuous-yielding joint model

– Plastic shear displacement causes reduction of mobilized friction angle.

– Normal and shear stiffnesses of a joint are function of normal stress.

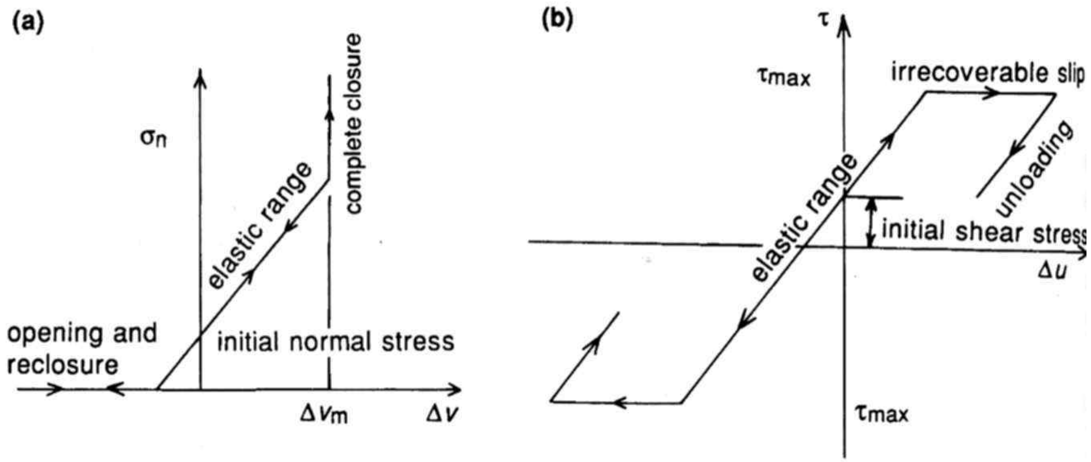


Fig.4.47

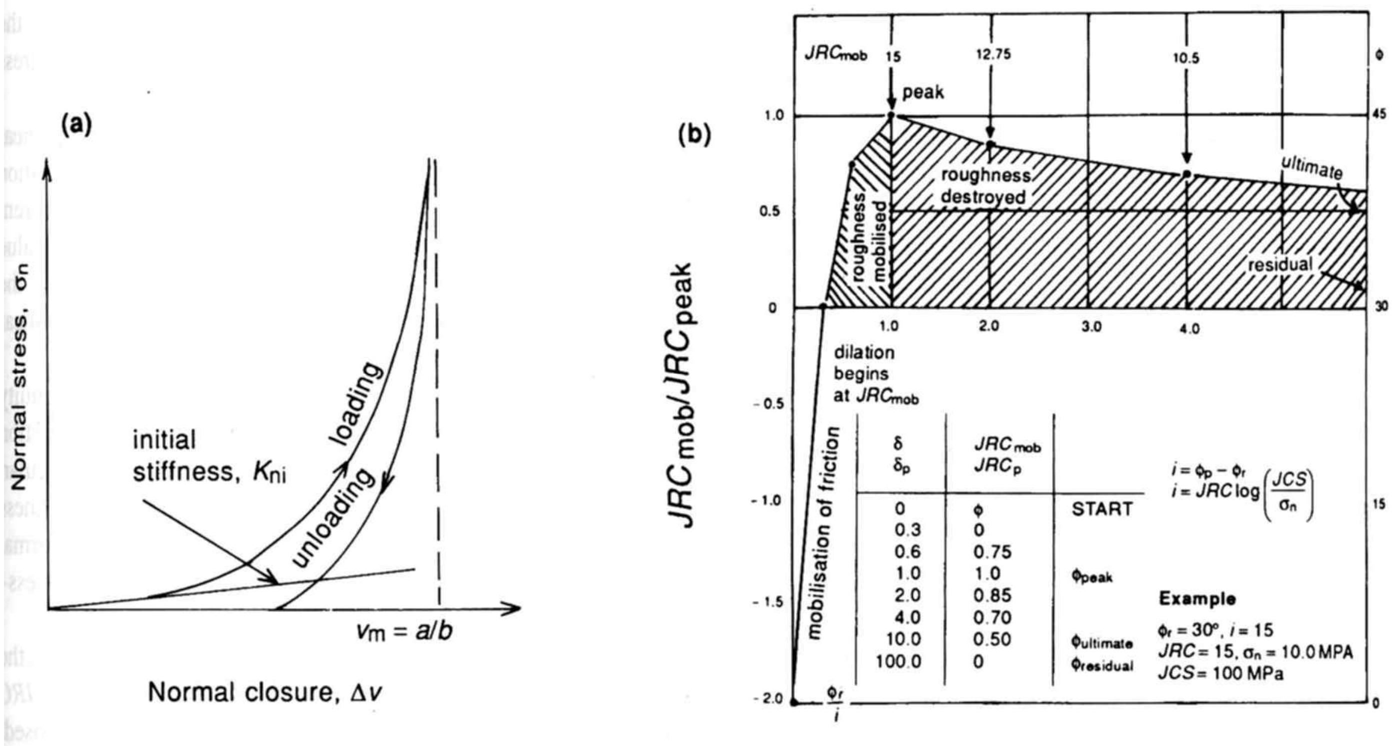


Fig.4.48

4.9 Behavior of discontinuous rock masses

- Strength

– Overall strength of a multiply jointed rock mass: almost isotropic (Fig.4.49)

- Generalized Hoek-Brown peak strength criterion for rock mass

$$\sigma_n' = \sigma_3' + \left(m_b \sigma_c \sigma_3' + s\sigma_c^2\right)^a$$

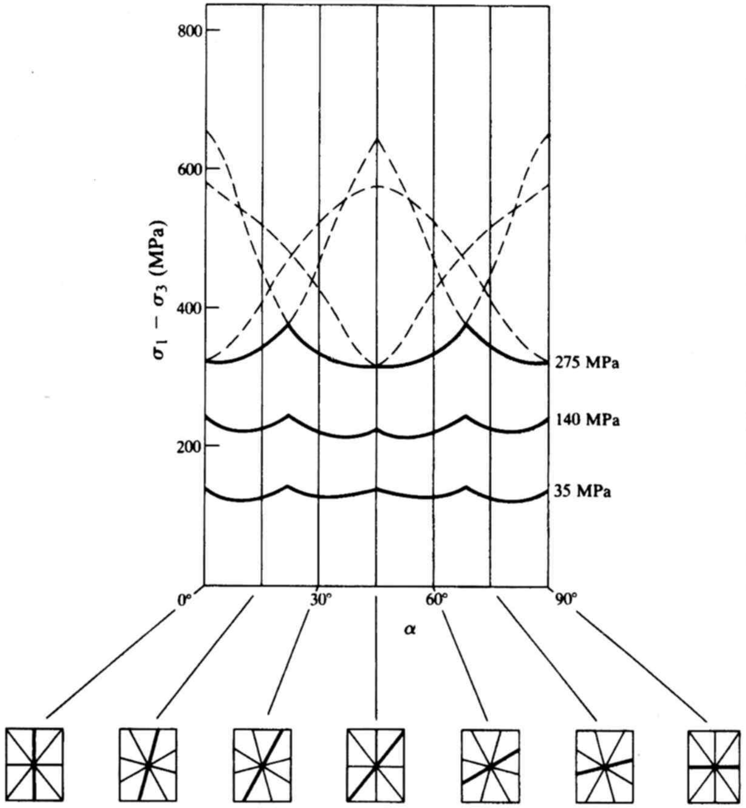
$$where \;\; m_b = m_i \exp\left\{\frac{GSI - 100}{28 - 14D}\right\}$$

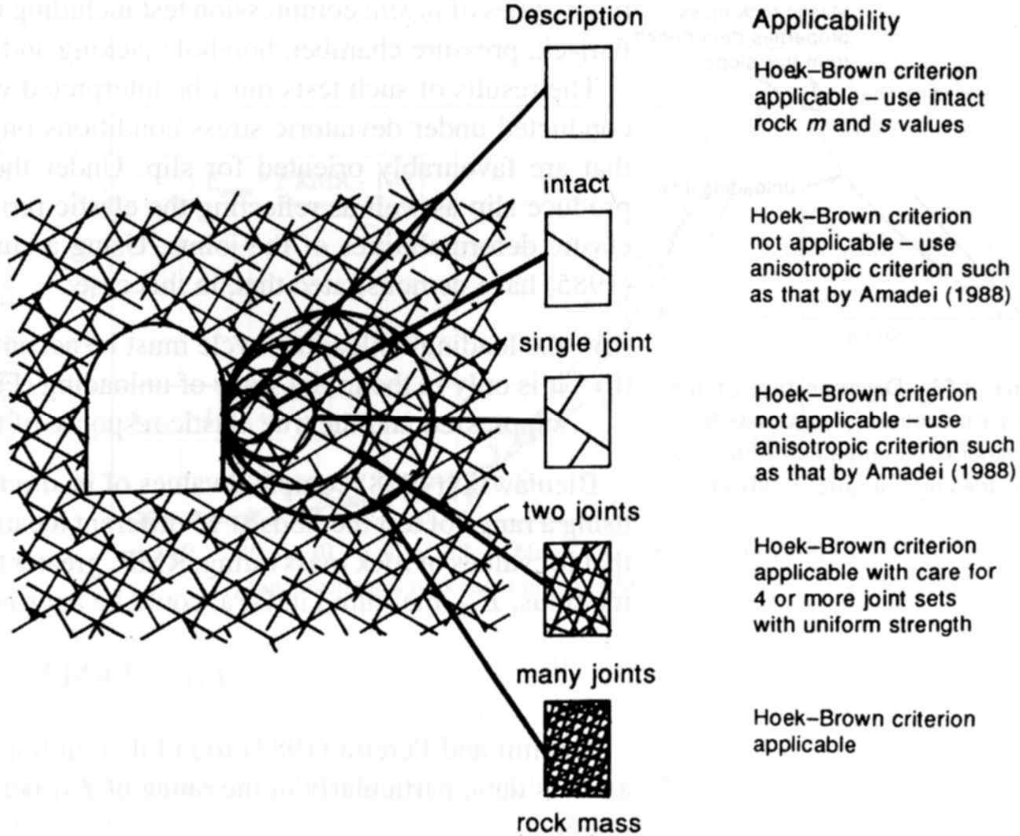
$$s = \exp\left\{\frac{GSI - 100}{9 - 3D}\right\} \quad \left(\text{D} = 0 \text{ for undisturbed, D} = 1 \text{ for very disturbed } \textit{in situ} \text{ rock}\right)$$

$$a = 0.5 + \left(\exp^{-GSI/15} - \exp^{-20/3}\right)\big/6 \quad (a \approx 0.5 \text{ for } GSI > 50,\ a \to 0.65 \text{ for very low } GSI)$$

$$\sigma_{cm} = \sigma_c s^a, \quad \sigma_{tm} = -s\sigma_c \big/ m_b \quad (\text{ from } \sigma_1' = \sigma_3' = \sigma_{tm})$$

– Applicable to short-term peak strength criterion of sensibly isotropic rock masses (Fig.4.51)


800
600
400
200
$\sigma_1 - \sigma_3$ (MPa)
0°
30°
60°
90°
α
275 MPa
140 MPa
35 MPa


Description
Applicability
intact
Hoek–Brown criterion applicable – use intact rock *m* and *s* values
single joint
Hoek–Brown criterion not applicable – use anisotropic criterion such as that by Amadei (1988)
two joints
Hoek–Brown criterion not applicable – use anisotropic criterion such as that by Amadei (1988)
many joints
Hoek–Brown criterion applicable with care for 4 or more joint sets with uniform strength
rock mass
Hoek–Brown criterion applicable

4.9 Behavior of discontinuous rock masses

- Deformability

– Elastic constants of transversely isotropic rock mass:

$$E_1 = E, \quad \frac{1}{E_2} = \frac{1}{E} + \frac{1}{K_n S} \quad (S\text{: spacing})$$

$$\nu_1 = \nu, \quad \nu_2 = \frac{E_2}{E}\nu$$

$$\frac{1}{G_2} = \frac{1}{G} + \frac{1}{K_s S}$$

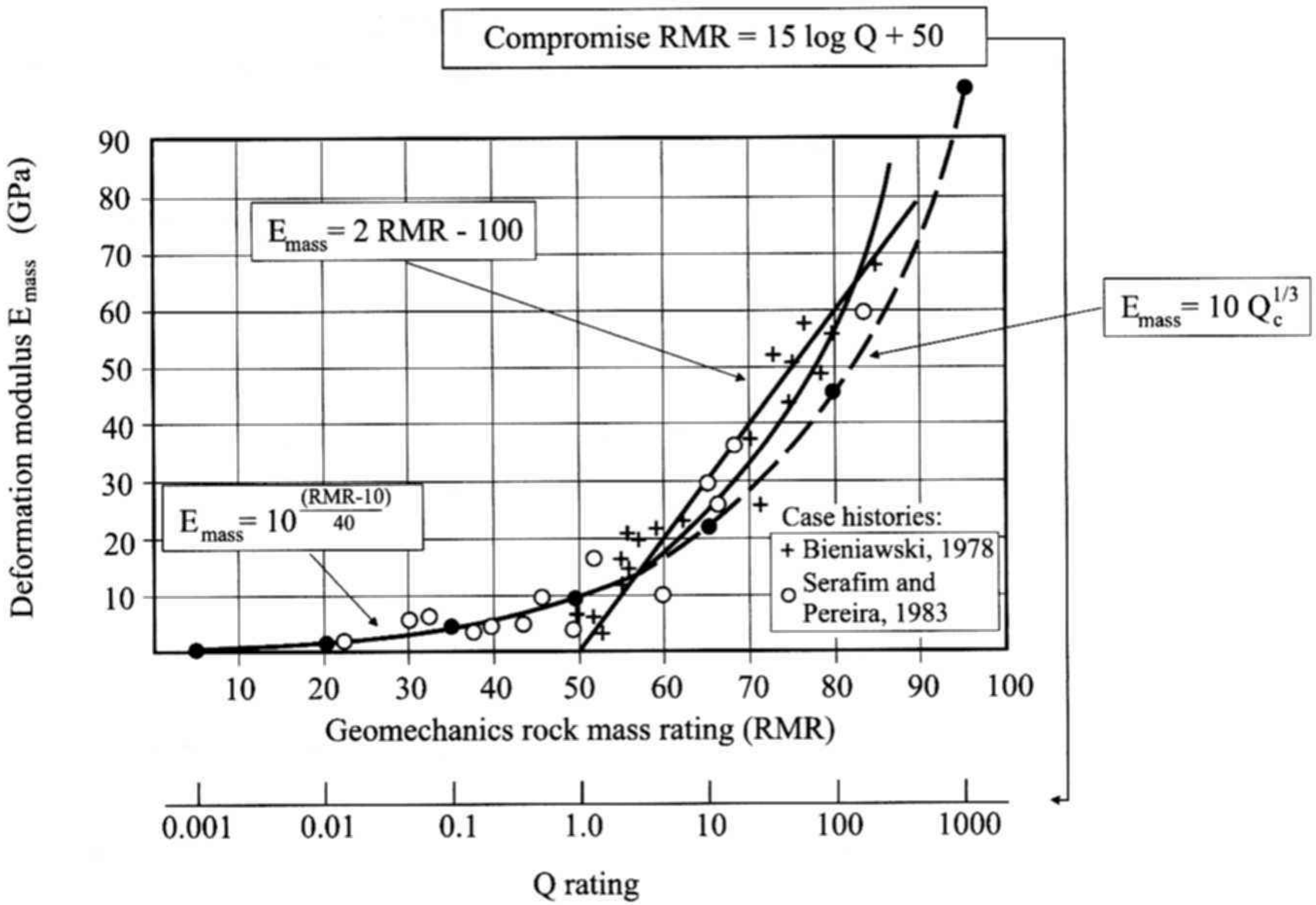
– Deformation modulus estimated from rock classification indices (Fig.4.53)

$$E_M = 2(RMR) - 100, \quad E_M = 10^{\frac{RMR-10}{40}}$$

$$E_M = 10 Q_c^{1/3} \quad \left(Q_c = Q \frac{\sigma_c}{100} \right)$$

$$E_M = (1 - D/2)\sqrt{\frac{\sigma_c}{100}}\, 10^{\frac{(GSI-10)}{40}}$$

$$V_p \approx 3.5 + \log_{10} Q_c$$



Compromise RMR = 15 log Q + 50
E_{mass} = 2 RMR - 100
$E_{mass} = 10\,Q_c^{1/3}$
$E_{mass} = 10^{\frac{(RMR-10)}{40}}$
Case histories:
+ Bieniawski, 1978
o Serafim and Pereira, 1983
Deformation modulus E_{mass} (GPa)
Geomechanics rock mass rating (RMR)
Q rating