Introduction to Nuclear Fusion (409.308A, 3 Credits)

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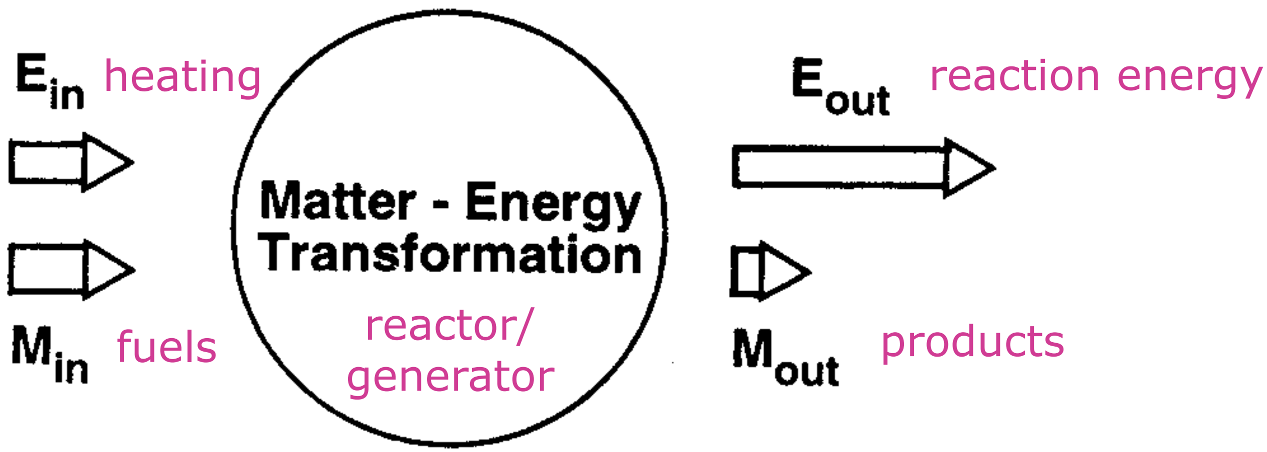
Matter and Energy



- Hydro-electric process
- Chemical reactions (combustion)
- Fission process
- Fusion process

- Total energy conservation including rest mass energy

$$E_{in} + M_{in} \rightarrow E_{out} + M_{out}$$

- If $\Delta m = M_{out} - M_{in} < 0$, then we can get $E_{out} > E_{in}$.

Mass Defect Energy of Nuclear Reaction

reactants products

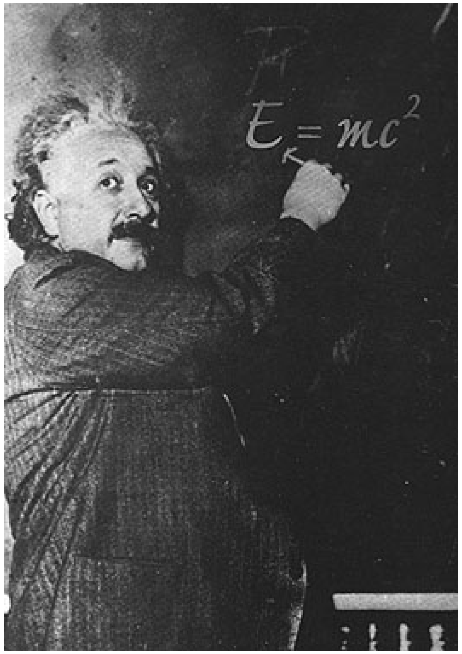
$$a + b \rightarrow d + e$$

$$\Delta m_{ab} = (m_d + m_e) - (m_a + m_b)$$

$\Delta m_{ab} < 0$: exothermic or exoergic
$\Delta m_{ab} > 0$: endothermic or endoergic

$$Q_{ab} = (-\Delta m)_{ab} c^2$$

Einstein's mass-energy relation



$$E^*_{before} = E^*_{after}$$

$$(E_{k,a} + m_a c^2) + (E_{k,b} + m_b c^2) = (E_{k,d} + m_d c^2) + (E_{k,e} + m_e c^2)$$

Mass Defect Energy of Nuclear Reaction

For $E_{k,a} + E_{k,b} << Q_{ab}$

$$Q_{ab} \approx E_{k,d} + E_{k,e} = \frac{1}{2} m_d v_d^2 + \frac{1}{2} m_e v_e^2$$

Momentum conservation for reactions with CM at rest

$$m_d v_d = m_e v_e$$

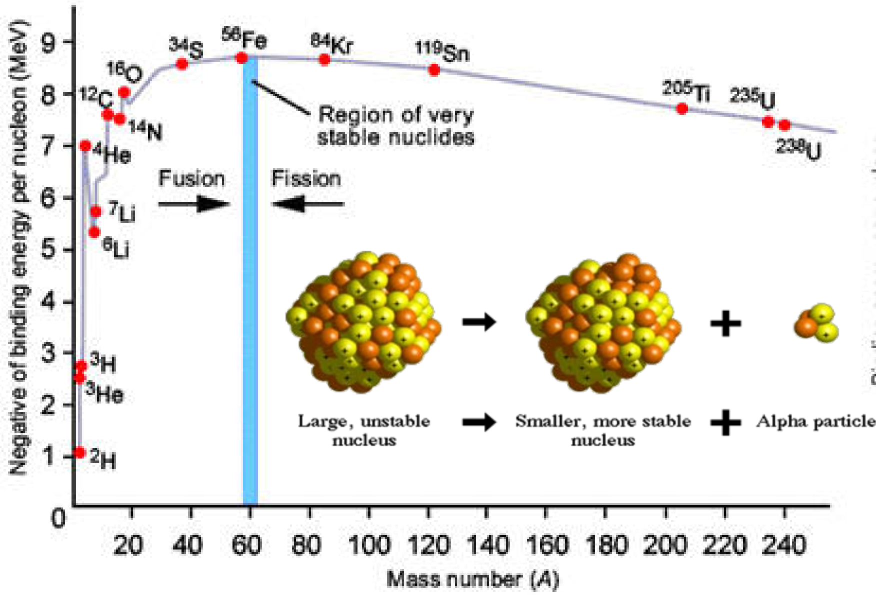
$$E_{k,d} \approx \left(\frac{m_e}{m_d + m_e} \right) Q_{ab}, \quad E_{k,e} \approx \left(\frac{m_d}{m_d + m_e} \right) Q_{ab}$$

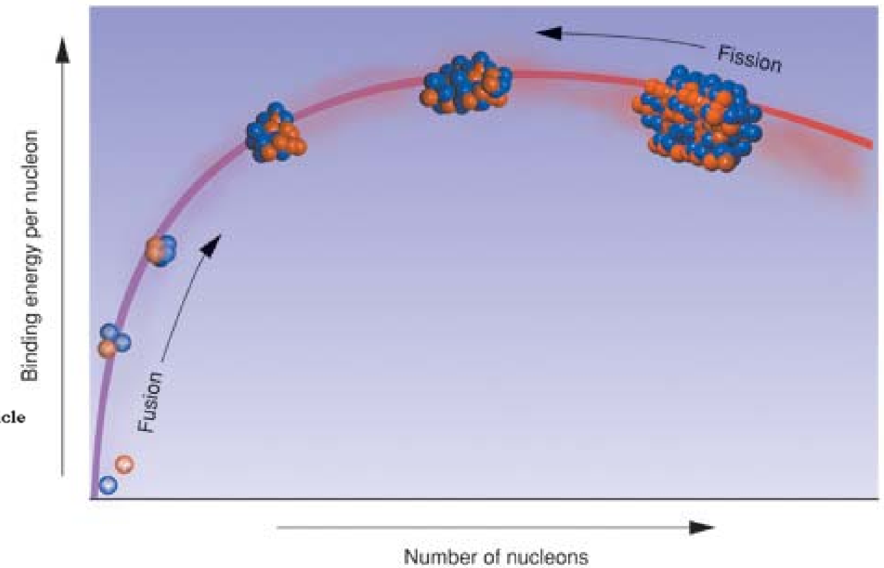
Ex) d-t fusion reaction $d + t \rightarrow n + \alpha + 17.6\,\mathrm{MeV}$

$$Q_{dt} = 17.6\,\mathrm{MeV}$$

$$E_{k,n} \approx \frac{4}{5} Q_{dt} \approx 14.1\,\mathrm{MeV}, \; E_{k,\alpha} \approx \frac{1}{5} Q_{dt} \approx 3.5\,\mathrm{MeV}$$

Binding Energy for an Assembled Nucleus





- The amount of energy released when a particular isotope is formed.
- The strength of the bonding is measured by the binding energy per nucleon where "nucleon" is a collective name for neutrons and protons, sometimes called the mass defect per nucleon.
- The difference in mass is equivalent to the energy released in forming the nucleus.

$$Zp + (A-Z)n \rightarrow {}^{A}_{Z}X + B.E.$$

$$B.E. \equiv -[(m_X - Zm_p + (A-Z)m_n)]c^2 = -\Delta mc^2$$

$\Delta m < 0$: released energy (exothermic or exoergic

Fusion in Nature

- **Fusion reactions by which stars convert hydrogen to helium**

- The PP (proton-proton) chain: in stars the mass of the Sun and less

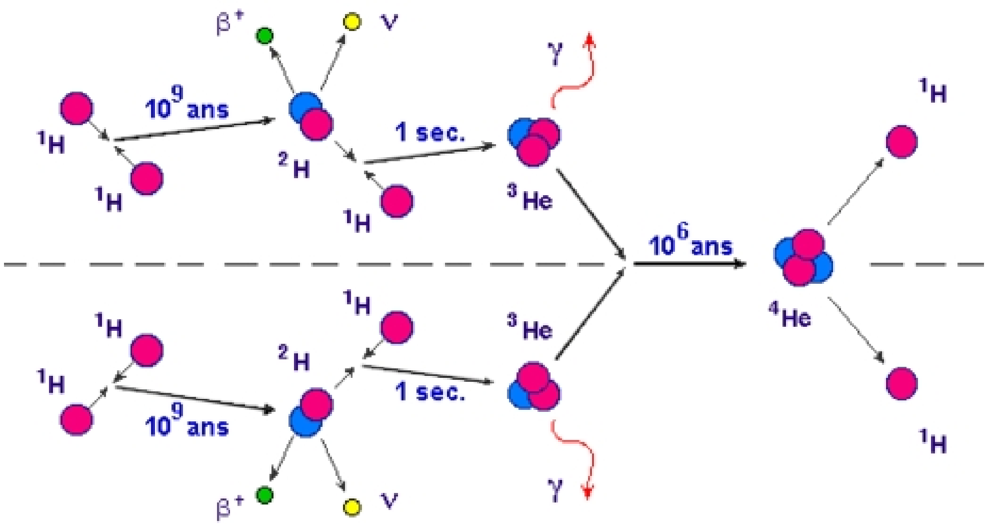
- The CNO cycle (Bethe-Weizsäcker-cycle): in more massive stars

Nobel prize in physics 1967
“for his contribution to the theory of nuclear reactions, especially his discoveries concerning the energy production in stars”

Hans Albrecht Bethe
(1906. 7. 2 – 2005. 3. 6)

Fusion in Nature

- **The PP (Proton-Proton) Chain**



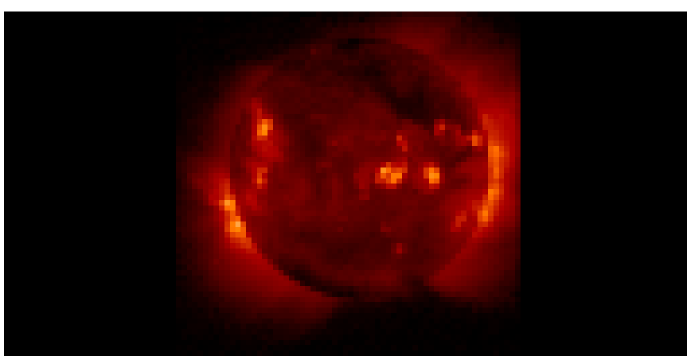
$$p + p \rightarrow d + \beta^{+} + \nu + 1.2\ MeV$$

$$p + d \rightarrow {}^{3}He + 5.5\ MeV$$

$${}^{3}He + {}^{3}He \rightarrow {}^{4}He + 2p + 12.9\ MeV$$

$${}^{3}He + {}^{4}He \rightarrow {}^{7}Be + 1.6\ MeV$$

$${}^{7}Be + \beta^{-} \rightarrow {}^{7}Li + 0.06\ MeV$$



Sun $\leq 15 \times 10^{6}\ K$

- Only 1.7% of ^{4}He nuclei being produced in the Sun are born in the CNO cycle

Fusion in Nature

- **CNO (Carbon-Nitrogen-Oxygen) Cycle**

$$^{12}C + p \rightarrow {}^{13}N + 1.9\,MeV$$

$$^{13}N \rightarrow {}^{13}C + \beta^{+} + \nu + 1.5\,MeV$$

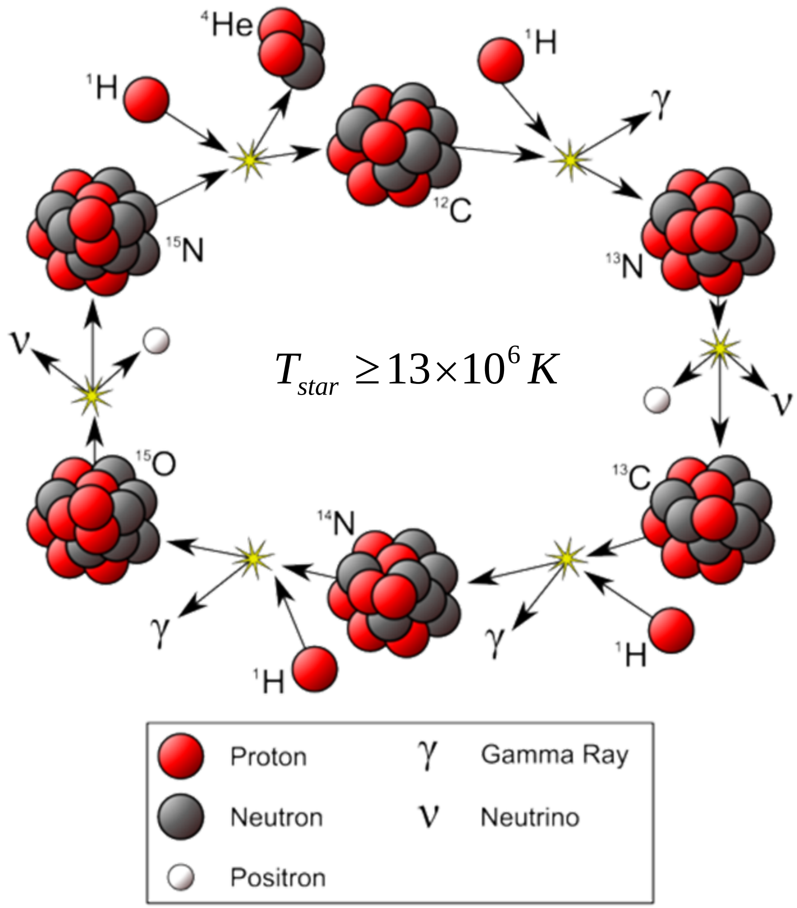
$$^{13}C + p \rightarrow {}^{14}N + 7.6\,MeV$$

$$^{14}N + p \rightarrow {}^{15}O + 7.3\,MeV$$

$$^{15}O \rightarrow {}^{15}N + \beta^{+} + \nu + 1.8\,MeV$$

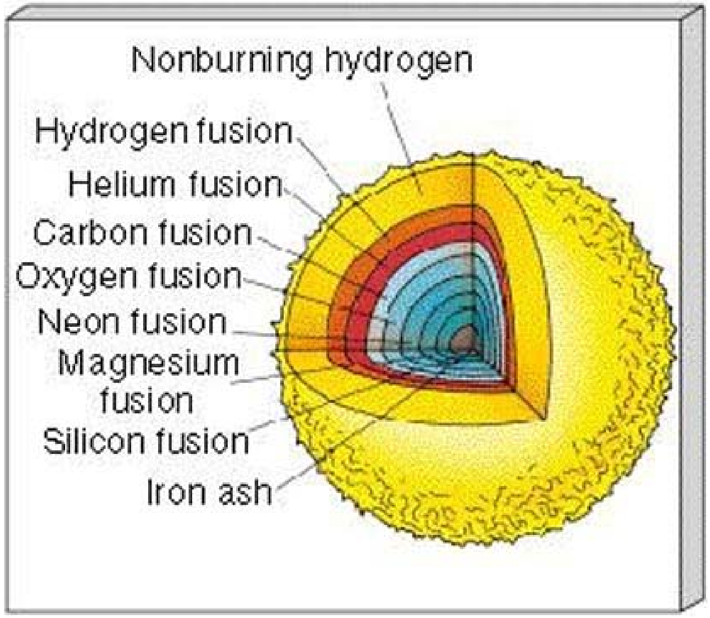
$$^{15}N + p \rightarrow {}^{12}C + \alpha + 5.0\,MeV$$

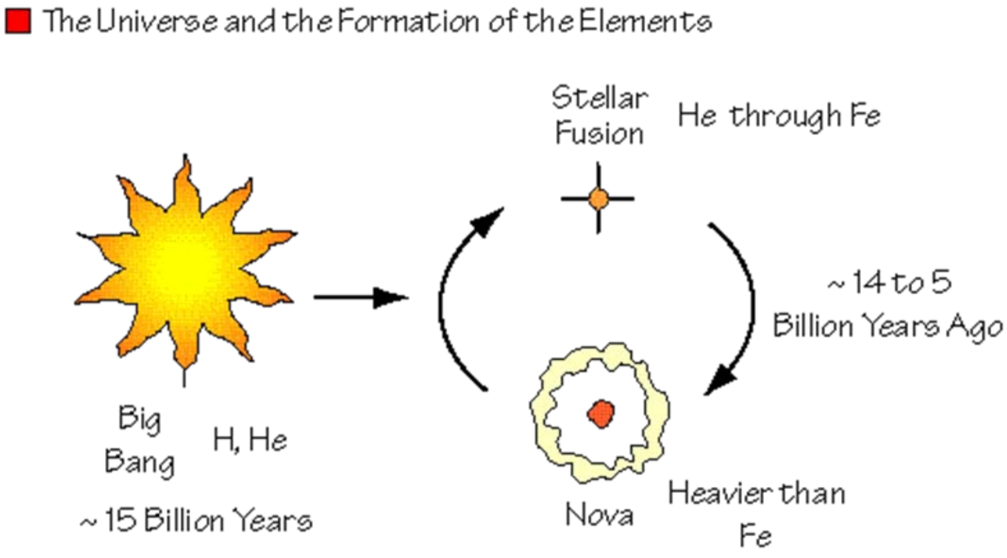
$$4p \rightarrow \alpha + 2\beta^{+} + 2\nu + 25.1\,MeV$$



Fusion in Nature

Layers of Fusion in a Star





http://jcconwell.wordpress.com/2009/07/20/formation-of-the-elements/
http://eqseis.geosc.psu.edu/~cammon/HTML/Classes/IntroQuakes/Notes/earth_origin_lecture.html

Fusion in Nature

Dynamic universe birth of the milky way galaxy

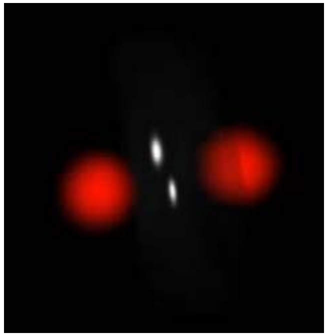
Physical Characterisation of Fusion Reaction

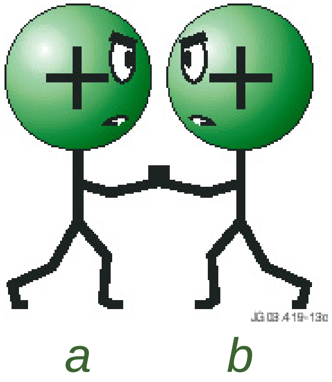
- $a+b \rightarrow (ab) \rightarrow d+e+Q_{ab}$

- (*ab*) : a complex short-lived dynamic state which disintegrates into products *d* and *e*.

→ The energetics are determined according to nucleon kinetics analysis, with nuclear excitation and subsequent gamma ray emission known to play a comparatively small role in fusion processes at the energies of interest envisaged for fusion reactors.

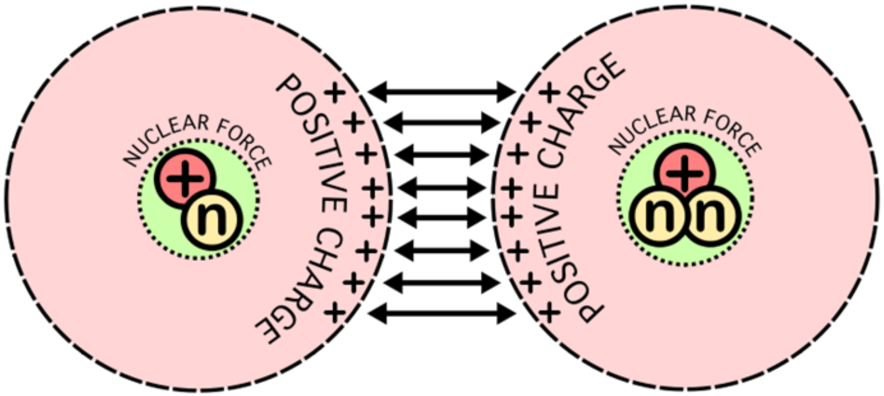
Physical Characterisation of Fusion Reaction





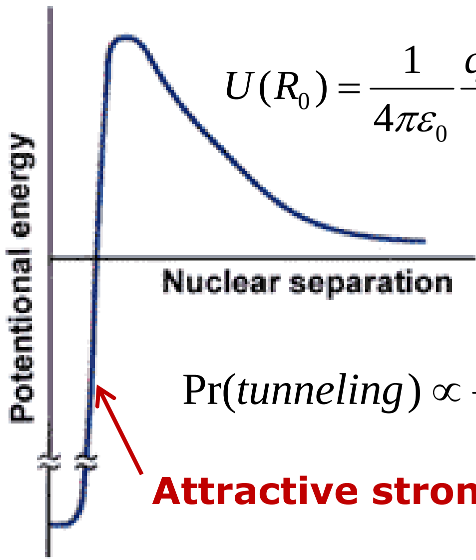
$$F_{g,a} = -G\frac{m_a m_b}{r^3}\vec{r}$$

$$F_{c,a} = \frac{1}{4\pi\varepsilon_0}\frac{q_a q_b}{r^3}\vec{r}$$



- The electrostatic force caused by positively charged nuclei is very strong over long distances, but at short distances the nuclear force is stronger.
- As such, the main technical difficulty for fusion is getting the nuclei close enough to fuse. Distances not to scale.

Physical Characterisation of Fusion Reaction

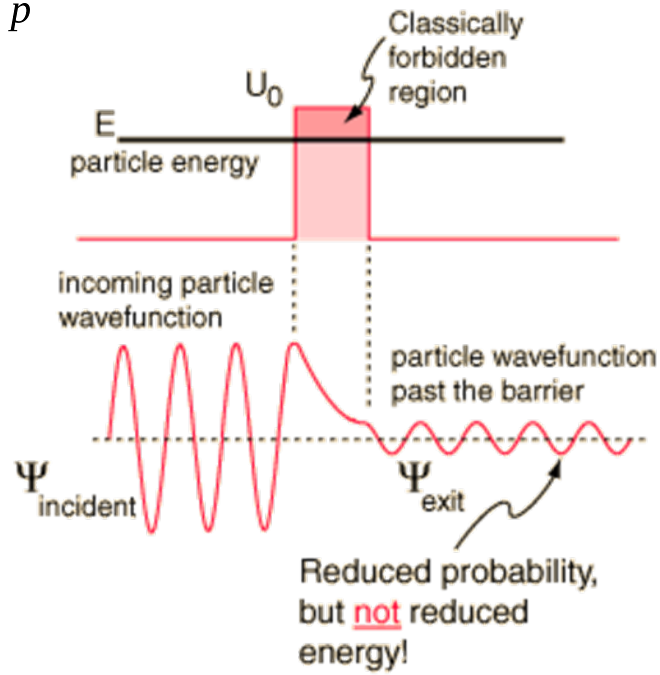


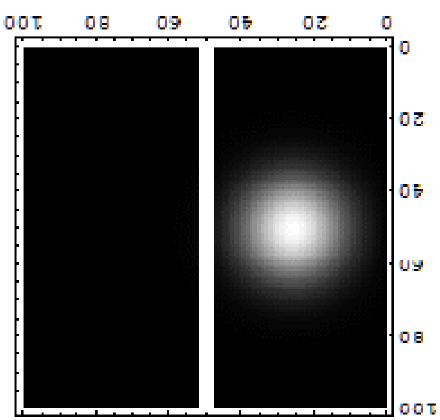
$$U(R_0) = \frac{1}{4\pi\varepsilon_0}\frac{q_a q_b}{R_0} \sim 0.4\,\text{MeV for } d, t, p$$

$$\Pr(tunneling) \propto \frac{1}{v_r}\exp[-\gamma\frac{q_a q_b}{v_r}]$$

Attractive strong nuclear force

Finite potential barrier





$$E << V_0$$

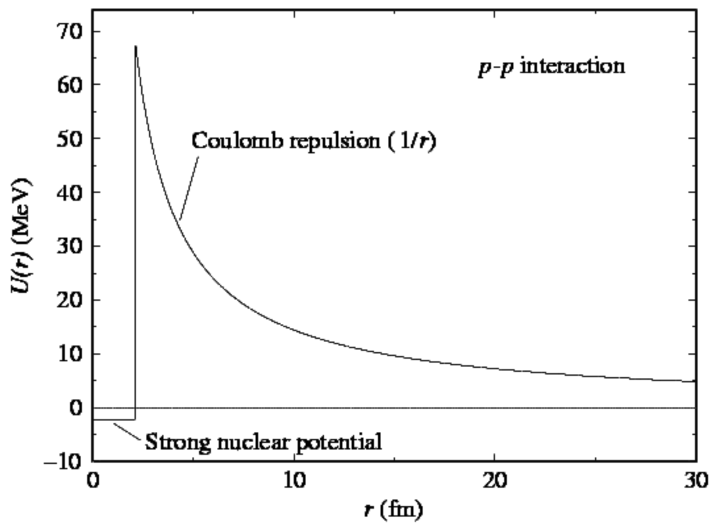
$$|T|^2 = \frac{(2k_0/\kappa)^2}{(2k_0/\kappa)^2 + \left(\kappa^2 + k_0{}^2\right)^2 \sinh^2 \kappa a}$$

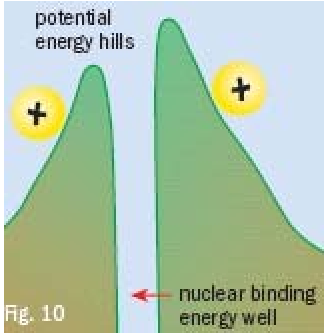
$$\cong \left(\frac{4k_0\kappa}{\kappa^2 + k_0{}^2}\right)^2 e^{-2\kappa a} \qquad \neq 0$$

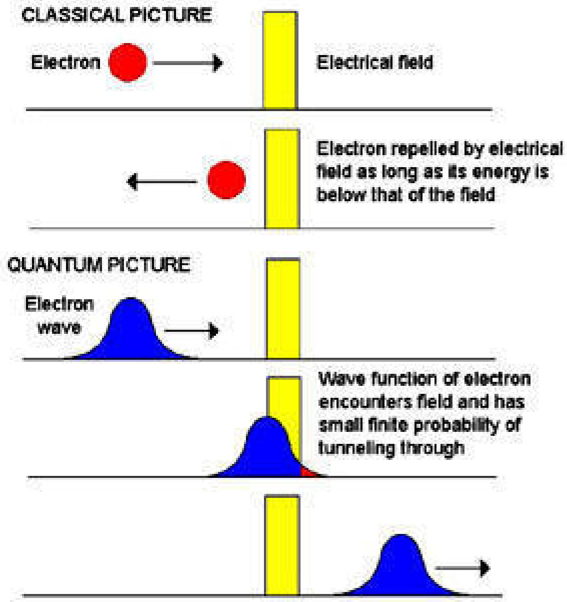
Reflection and tunneling of an electron wavepacket directed at a potential barrier

A. B. Balantekin and N. Takigawa, 'Quantum tunneling in nuclear fusion', Rev. Mod. Phys. 70, 77 (1998).

Physical Characterisation of Fusion Reaction

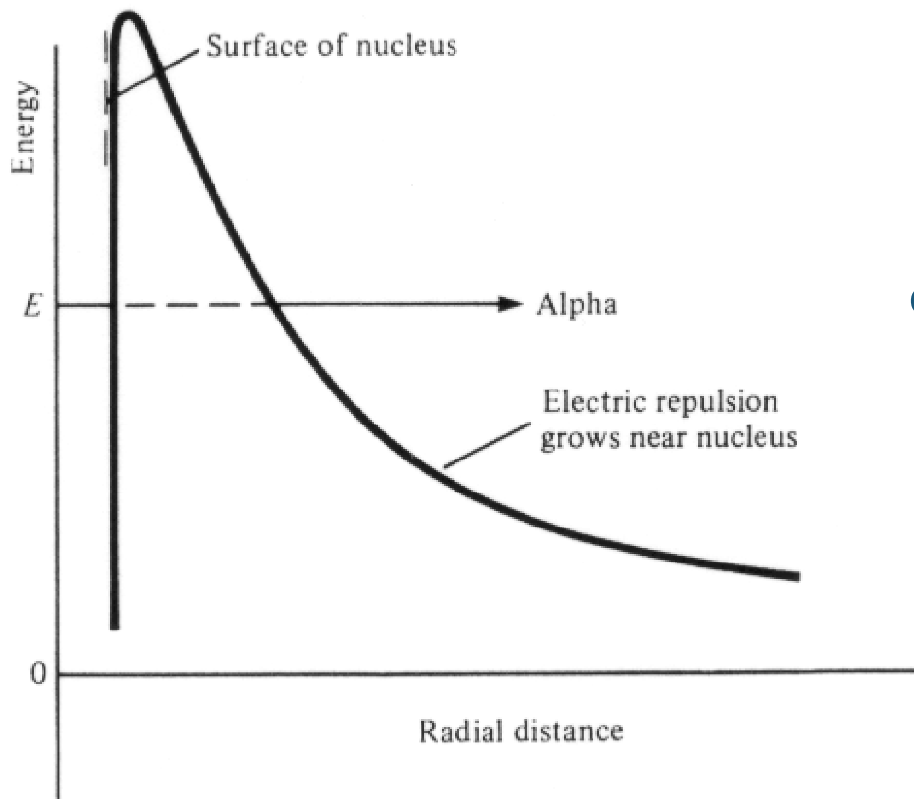




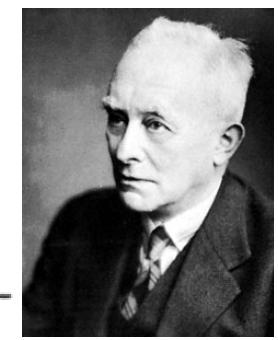


Physical Characterisation of Fusion Reaction

Potential barrier around a uranium nucleus presented to an alpha particle. The central well is due to the average nuclear attraction of all the nucleons and the hill is due to the electric repulsion of the protons. Alpha particles with energy *E* trapped inside the nuclear well may still escape to become alpha rays, by quantum mechanically tunnelling through the barrier.



George Gamow
(1904-1968)



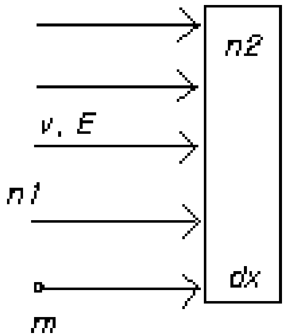
Max Born
(1882-1970)

- By 1928, George Gamow had solved the theory of the alpha decay of a nucleus via tunneling. After attending a seminar by Gamow, Max Born recognized the generality of quantum-mechanical tunneling.

(Max Born, Nobel Prize in Physics 1954)

Fusion Reaction Cross Sections

- Beam-target collisions (Binary interactions)



- For fixed target

$$m = m_1, \quad v = v_1, \quad E = m_1 v_1^2 / 2$$

- For moving target

$$m = m_r, \quad v = |v_1 - v_2|, \quad E = E_{CM}$$

$$dn_1 = -\sigma_{12}(E) n_1 n_2 dx$$

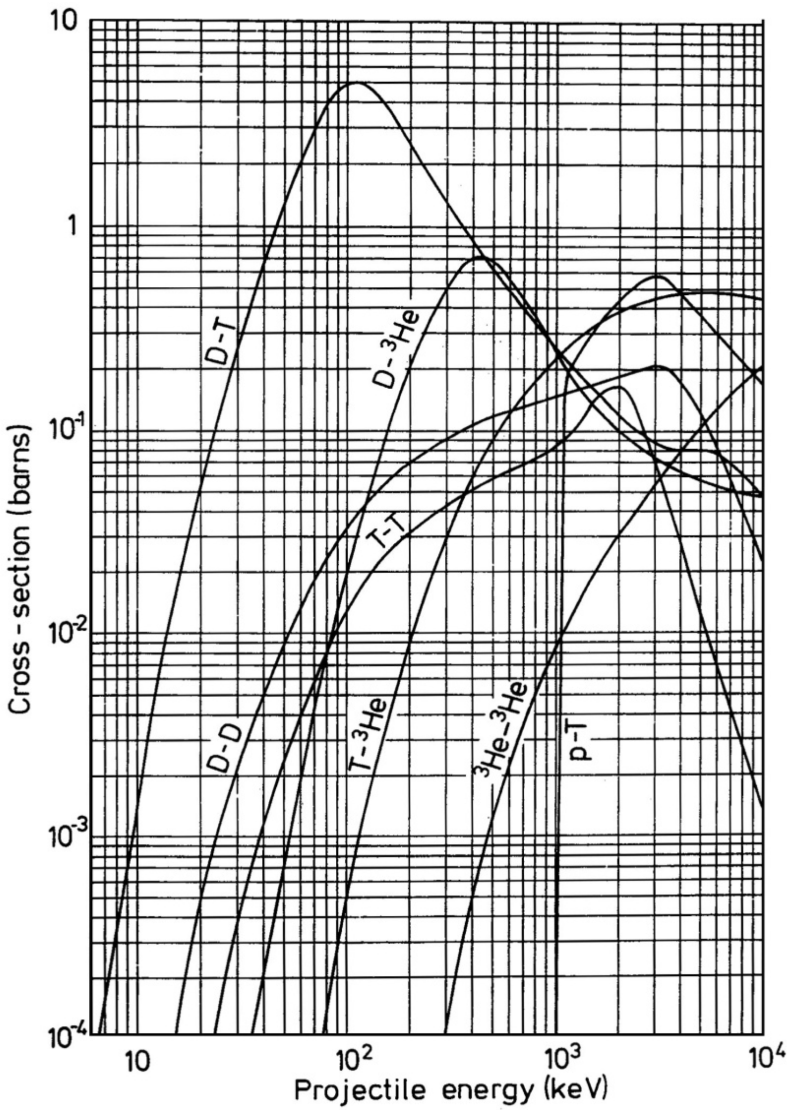
- Fusion cross section for low energy $E_{CM} < U(R_0)$ by quantum mechanical tunneling process:

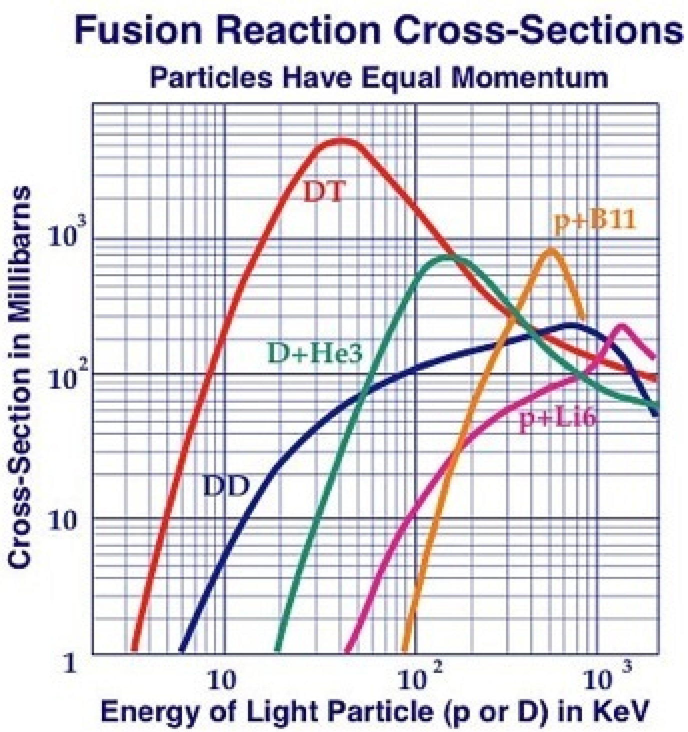
$$\sigma_{12}(E) = \frac{A}{E} e^{-B/\sqrt{E}}$$

Gamow theory (1938)

$$A = const., \quad B = 2^{-1/2} \pi m_r^{1/2} Z_1 Z_2 e^2 / h\varepsilon_0$$

Fusion Reaction Cross Sections



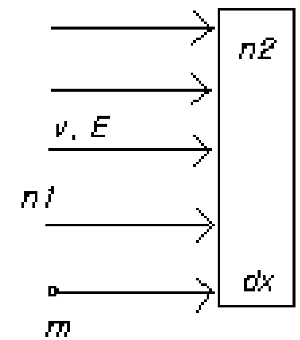


Fusion Reaction Rate Parameter (Reactivity or σ-*v* Parameter)

- σ-*v* parameter

$$<\sigma v>_{ab}=\int_{v_a}\int_{v_b}\sigma_{ab}\left(\left|v_a-v_b\right|\right)\left|v_a-v_b\right|F_a(v_a)F_b(v_b)d^3v_a d^3v_b$$

$$dn_1=-\sigma_{12}(E)n_1n_2dx$$



- Fusion reaction rate density

$$dR_{fu}\equiv\frac{d}{dt}(-dn)=dn_1dn_2\sigma_{12}(v)v$$

$$dn_1=n_1f_1(v_1)d^3v_1$$

$$dn_2=n_2f_2(v_2)d^3v_2$$

$f_{1,2}$: normalised distribution function

$$R_{fu}=\int_{v_a}\int_{v_b}\sigma_{fu}\left(\left|v_a-v_b\right|\right)\left|v_a-v_b\right|N_aF_a(v_a)N_bF_b(v_b)d^3v_a d^3v_b$$

$$=N_aN_b\int_{v_a}\int_{v_b}\sigma_{fu}\left(\left|v_a-v_b\right|\right)\left|v_a-v_b\right|F_a(v_a)F_b(v_b)d^3v_a d^3v_b$$

$$F_x(v_x)\rightarrow M_x(v_x)$$

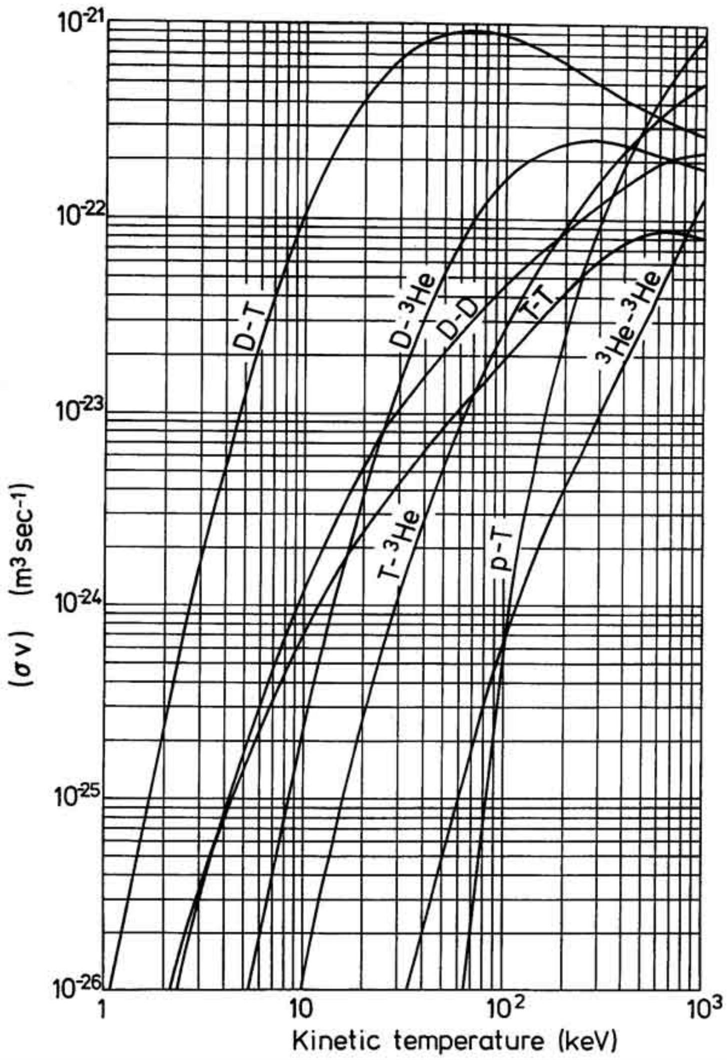
Thermodynamic equilibrium

$$R_{fu}=N_aN_b<\sigma v>_{ab}$$

- Fusion power density

$$P_{fu}=R_{fu}Q_{fu}=N_aN_b<\sigma v>_{ab}Q_{fu}$$

Fusion Reaction Rate Parameter (Reactivity or σ-*v* Parameter)



Both species at the same temperatures

Fusion Fuels

- **Possible fusion reactions**

$$d + t \rightarrow n + \alpha + 17.6\ \mathrm{MeV}$$

$$d + d \rightarrow p + t + 4.1\ \mathrm{MeV}$$

$$\rightarrow n + {}^{3}He + 3.2\ \mathrm{MeV}$$

$$d + {}^{3}He \rightarrow p + \alpha + 18.3\ \mathrm{MeV}$$

$$p + {}^{6}Li \rightarrow {}^{3}He + \alpha + 4.0\ \mathrm{MeV}$$

$$p + {}^{9}Be \rightarrow \alpha + {}^{6}Li + 2.1\ \mathrm{MeV}$$

$$\rightarrow d + 2\alpha + 0.6\ \mathrm{MeV}$$

$$p + {}^{11}B \rightarrow 3\alpha + 8.7\ \mathrm{MeV}$$

$$d + {}^{6}Li \rightarrow {}^{7}Be + n + 3.4\ \mathrm{MeV}$$

$$\rightarrow {}^{7}Li + p + 5.0\ \mathrm{MeV}$$

$$\rightarrow p + \alpha + t + 2.6\ \mathrm{MeV}$$

$$\rightarrow 2\alpha + 22.3\ \mathrm{MeV}$$

$$\rightarrow {}^{3}He + \alpha + n + 1.8\ \mathrm{MeV}$$

$$t + t \rightarrow 2n + \alpha + 11.3\ \mathrm{MeV}$$

$${}^{3}He + {}^{3}He \rightarrow 2p + \alpha + 12.9\ \mathrm{MeV}$$

$$t + {}^{3}He \rightarrow n + p + \alpha + 12.1\ \mathrm{MeV}$$

Fusion Fuels

- **Choice of a fusion reaction as a fuel in a fusion reactor**

- Availability of fusion fuels

- Requirements for attaining a sufficient reaction rate density

- **D-T reaction: 1st generation** $d + t \rightarrow n + \alpha + 17.6\,\mathrm{MeV}$

- Considered for the first generation of fusion reactors

- Ample supply of deuterium: d/(p+d)~1/6700 in the world's oceans, fresh water lakes, rivers

- Scarce of tritium: radioactive β^- decay with a half life of 12.3 years. total steady state atmospheric and oceanic quantity produced by cosmic radiation ~ 50 kg

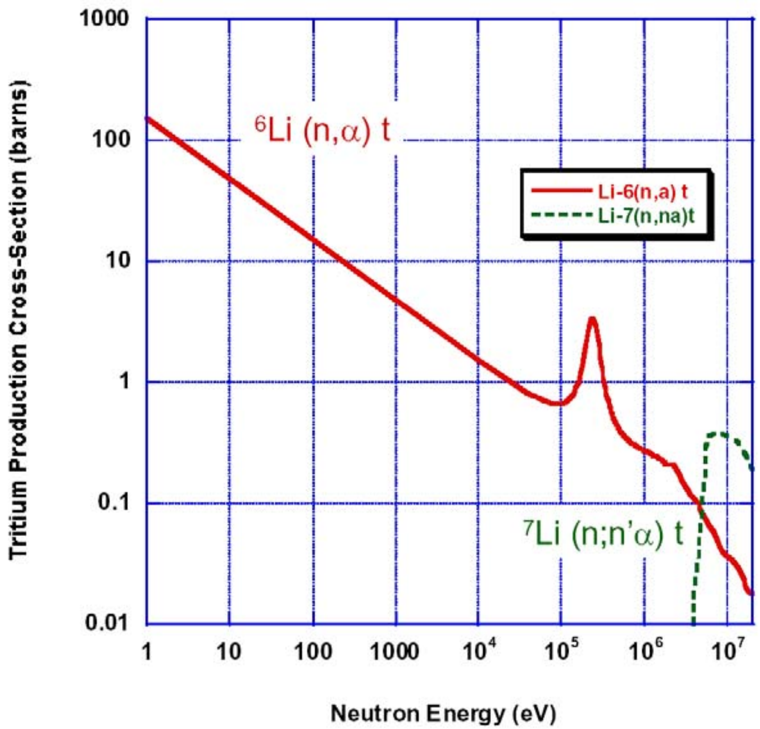
Fusion Fuels

- **D-T reaction: 1st generation**

- Tritium breeding

$$n + {}^{6}Li \rightarrow t + {}^{4}He + 4.78\ \text{MeV}$$ 7.42% of natural Li

$$n + {}^{7}Li \rightarrow t + {}^{4}He + n' - 2.47\ \text{MeV}$$ 92.58% of natural Li



The ^{7}Li(n,n′α)t reaction is a threshold reaction and requires an incident neutron energy in excess of 2.8 MeV.

D-T Burn

- $d+t \to n+\alpha$, Q_{dt}=17.6 MeV

$$R_{dt}(\vec{r},t) = \int\int N_d(\vec{r},\vec{v}_d,t) N_t(\vec{r},\vec{v}_t,t) \sigma_{dt}\left(\left|\vec{v}_d-\vec{v}_t\right|\right)\left|\vec{v}_d-\vec{v}_t\right| d^3v_d\, d^3v_t$$

$$= N_d^* N_t^* \int\int f_d(\vec{r},\vec{v}_d,t) f_t(\vec{r},\vec{v}_t,t) \sigma_{dt}\left(\left|\vec{v}_d-\vec{v}_t\right|\right)\left|\vec{v}_d-\vec{v}_t\right| d^3v_d\, d^3v_t$$

$$= N_d(\vec{r},t) N_t(\vec{r},t) \frac{\int\int f_d(\vec{r},\vec{v}_d,t) f_t(\vec{r},\vec{v}_t,t) \sigma_{dt}\left(\left|\vec{v}_d-\vec{v}_t\right|\right)\left|\vec{v}_d-\vec{v}_t\right| d^3v_d\, d^3v_t}{\int\int f_d(\vec{r},\vec{v}_d,t) f_t(\vec{r},\vec{v}_t,t) d^3v_d\, d^3v_t}$$

$$= N_d(\vec{r},t) N_t(\vec{r},t) < \sigma v >_{dt} (\vec{r},t)$$

$$P_{dt}(\vec{r},t) = R_{dt}(\vec{r},t) Q_{dt} = N_d(\vec{r},t) N_t(\vec{r},t) < \sigma v >_{dt} (\vec{r},t) Q_{dt}$$

D-D Burn Modes

- D-D reactions and side reactions

$$d + d \rightarrow p + t + 4.1\,\text{MeV}$$

$$\rightarrow n + {}^{3}He + 3.2\,\text{MeV}$$

$$d + t \rightarrow n + \alpha + 17.6\,\text{MeV}$$

$$d + {}^{3}He \rightarrow p + \alpha + 18.3\,\text{MeV}$$

$$t + t \rightarrow 2n + \alpha + 11.3\,\text{MeV}$$

$${}^{3}He + {}^{3}He \rightarrow 2p + \alpha + 12.9\,\text{MeV}$$

$$t + {}^{3}He \rightarrow n + p + \alpha + 12.1\,\text{MeV}$$

$$\rightarrow d + \alpha + 14.3\,\text{MeV}$$

D-D Burn Modes

- PURE-D Mode

$$d + d \rightarrow p + t + 4.1\,\mathrm{MeV}$$ Channel - t

$$\rightarrow n + {}^3He + 3.2\,\mathrm{MeV}$$ Channel - ^{3}He

$$R_{dd,t} = \frac{N_d^2}{2} < \sigma v >_{dd,t}$$

$$R_{dd,{}^3He} = \frac{N_d^2}{2} < \sigma v >_{dd,{}^3He}$$

$$< \sigma v >_{dd} = < \sigma v >_{dd,t} + < \sigma v >_{dd,{}^3He}$$

$$< \sigma v >_{dd,t} \approx < \sigma v >_{dd,{}^3He} \approx \frac{1}{2} < \sigma v >_{dd}$$

At temperatures of common interest

D-D Burn Modes

	a_1	a_2	…	a_x	…	a_{Na}
b_1						
b_2						
…						
b_y				(a_x,b_y)		
…						
b_{Nb}						

Interaction between N_a a-type and N_b b-type particles

$$R_{ab} = N_a N_b < \sigma v >_{ab}$$

	a_1	a_2	…	a_x	…	a_{Na}
a_1						
a_2						
…						
a_x				(a_x,a_x)		
…						
a_{Na}						

Interaction between N_a a-type particles

$$R_{aa} = \frac{N_a(N_a - 1)}{2} < \sigma v >_{aa}$$

$$\approx \frac{N_a^2}{2} < \sigma v >_{aa}$$

D-D Burn Modes

- Semi-Catalyzed-D cycle (SCAT-D Mode)

$$d + d \rightarrow t + p + 4.1\,\text{MeV}$$

Channel - t

reaction link

$$d + t \rightarrow n + \alpha + 17.6\,\text{MeV}$$

$$d + d \rightarrow {}^{3}He + n + 3.2\,\text{MeV}$$

Channel - ^{3}He

$$5d \rightarrow 2n + {}^{3}He + \alpha + p + 24.9\,\text{MeV}$$

Providing $R_{dd,t} = R_{dt}$

$$\frac{N_d^2}{2} < \sigma v >_{dd,t} = N_d N_t < \sigma v >_{dt}$$

$$\frac{N_t}{N_d} = \frac{1}{2}\frac{< \sigma v >_{dd,t}}{< \sigma v >_{dt}} \approx \frac{1}{4}\frac{< \sigma v >_{dd}}{< \sigma v >_{dt}}$$

D-D Burn Modes

- Catalyzed-D cycle (CAT-D Mode)

$$d + d \rightarrow t + p + 4.1\,\mathrm{MeV}$$

Channel - t

reaction link

$$d + t \rightarrow n + \alpha + 17.6\,\mathrm{MeV}$$

$$d + d \rightarrow {}^{3}He + n + 3.2\,\mathrm{MeV}$$

Channel - ^{3}He

reaction link

$$d + {}^{3}He \rightarrow \alpha + p + 18.3\,\mathrm{MeV}$$

$$6d \rightarrow 2n + 2\alpha + 2p + 43.2\,\mathrm{MeV}$$

D-D Burn Modes

- General D-D initiated fusion linkage processes

$$d + d \rightarrow t(1.0) + p(3.1)$$

$$d + t \rightarrow n(14.1) + \alpha(3.5)$$

$$t + t \rightarrow n(5.0) + n(5.0) + \alpha(1.3)$$

$$t + {}^{3}He \rightarrow p(5.7) + \alpha(1.3) + n(5.1)$$

$$d + d \rightarrow {}^{3}He(0.8) + n(2.4)$$

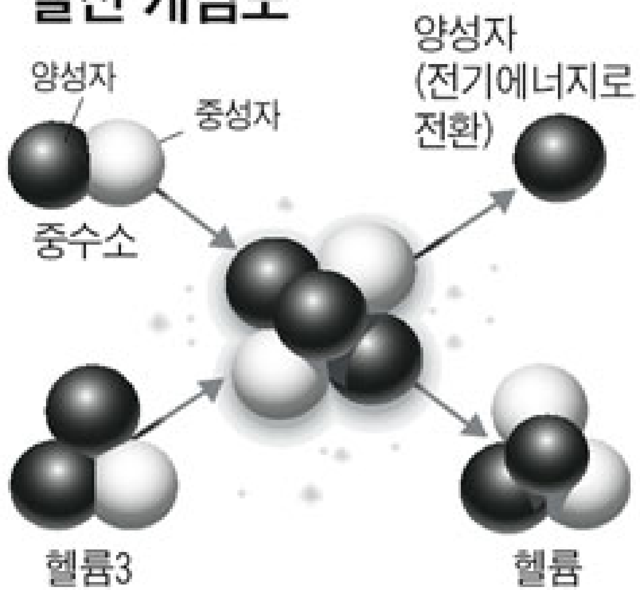
$$d + {}^{3}He \rightarrow \alpha(3.7) + p(14.6)$$

$${}^{3}He + {}^{3}He \rightarrow p(5.7) + p(5.7) + \alpha(1.4)$$

D-³He Fusion

$$d + {}^{3}He \rightarrow p + \alpha + 18.3\,\mathrm{MeV}$$

헬륨3를 이용한 핵융합 발전 개념도



헬륨3 차세대 핵융합 발전의 연료.
헬륨3의 원자는 양성자 2개와 중성자 1개로 이뤄져 있으며, 중수소(양성자 1개 중성자 1개)와 핵융합을 하면 정상적인 헬륨 원자(양성자 2개, 중성자 2개)가 되면서 강한 에너지를 가진 양성자를 방출한다.

1000MW급 발전소 가동을 위한 연료별 소모량 단위:kg/day

석탄	8,640,000
우라늄(235)	3
헬륨3	0.2

D-^{3}He Fusion

- An attainable "clean" fusion reaction, direct energy conversion

- Tritium, neutron: problems of radiological safety,
  first wall endurance, shielding and induced radioactivity

- Higher reaction temperature required
- More severe Bremsstrahlung radiation
- Scarce ^{3}He: ^{3}He/(^{3}He+^{4}He)~10^{-6}

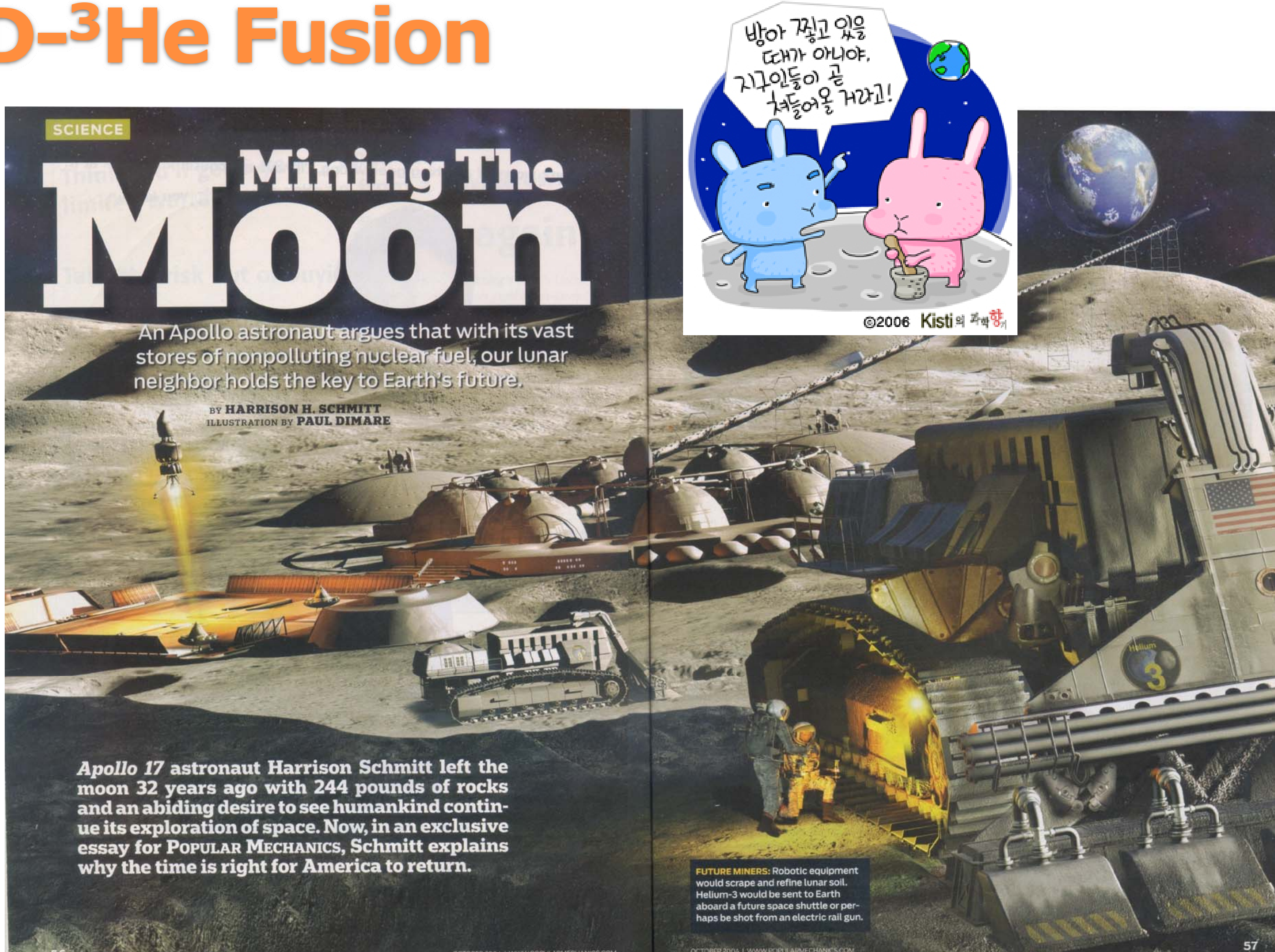
  cf) Lunar Rock

$$t \rightarrow {}^3He + \beta^-, \quad \tau_{1/2} = 12.3\ years$$

$$d + d \rightarrow n + {}^3He$$

$$d + {}^3He \rightarrow p + \alpha + 18.3\,\mathrm{MeV}, \quad R_{d^3He} = <\sigma v>_{d^3He} N_d N_{^3He}$$

D-^{3}He Fusion



SCIENCE

Mining The Moon

An Apollo astronaut argues that with its vast stores of nonpolluting nuclear fuel, our lunar neighbor holds the key to Earth's future.

BY HARRISON H. SCHMITT
ILLUSTRATION BY PAUL DIMARE

***Apollo 17* astronaut Harrison Schmitt left the moon 32 years ago with 244 pounds of rocks and an abiding desire to see humankind continue its exploration of space. Now, in an exclusive essay for POPULAR MECHANICS, Schmitt explains why the time is right for America to return.**

FUTURE MINERS: Robotic equipment would scrape and refine lunar soil. Helium-3 would be sent to Earth aboard a future space shuttle or perhaps be shot from an electric rail gun.

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D-³He Fusion

- An attainable "clean" fusion reaction, direct energy conversion
- Tritium, neutron: problems of radiological safety, first wall endurance, shielding and induced radioactivity
- Higher reaction temperature required
- More severe Bremsstrahlung radiation
- Scarce ^3He: $^3He/(^3He+^4He) \sim 10^{-6}$

cf) Lunar Rock

$$t \rightarrow {}^3He + \beta^-, \quad \tau_{1/2} = 12.3\ years$$

$$d + d \rightarrow n + {}^3He$$

Control on high temperature and ^3He and d fuel ions

$$d + {}^3He \rightarrow p + \alpha + 18.3\,MeV, \quad R_{d^3He} = <\sigma v>_{d^3He} N_d N_{^3He}$$

unclean side reactions

$$d + d \rightarrow t + p + 4.1\,\mathrm{MeV}, \quad R_{dd,t} = <\sigma v>_{dd,t} \frac{N_d^2}{2}$$

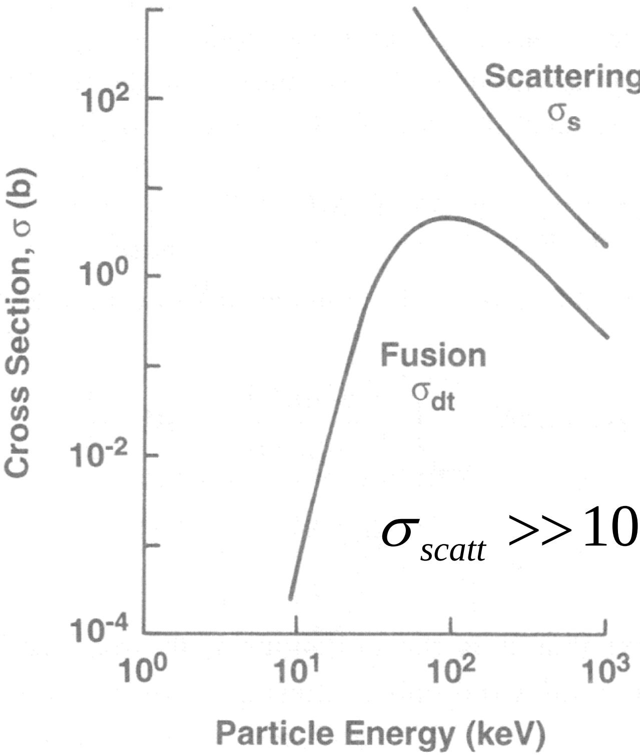
$$d + d \rightarrow {}^3He + n + 3.2\,\mathrm{MeV}, \quad R_{dd,^3He} = <\sigma v>_{dd,^3He} \frac{N_d^2}{2}$$

$$\frac{R_{d^3He}}{R_{dd,t}} = 2\frac{<\sigma v>_{d^3He}}{<\sigma v>_{dd,t}} \frac{N_{^3He}}{N_d}$$

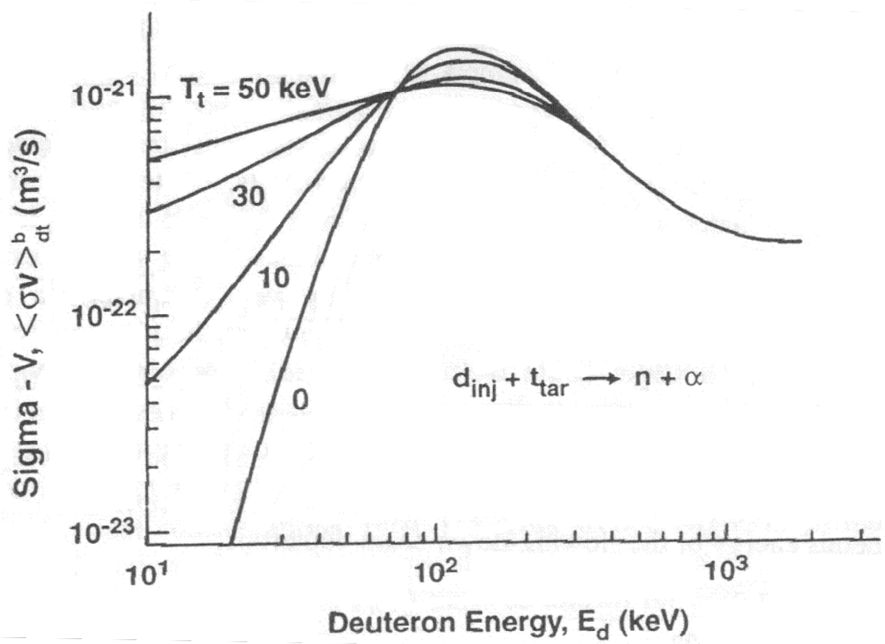
$$\frac{R_{d^3He}}{R_{dd,^3He}} = 2\frac{<\sigma v>_{d^3He}}{<\sigma v>_{dd,^3He}} \frac{N_{^3He}}{N_d}$$

Beam-target Fusion

- Beam-target collisions (Binary interactions)



$$\sigma_{scatt} >> 100\sigma_{dt}$$



- loss energy >> fusion energy
- Fusion by beam-target collisions are not proper for practical energy-producing fusion reactors

Confinement needed!