재료의 전자기적 성질

PART II.
ELECTRICAL PROPERTIES OF MATERIALS

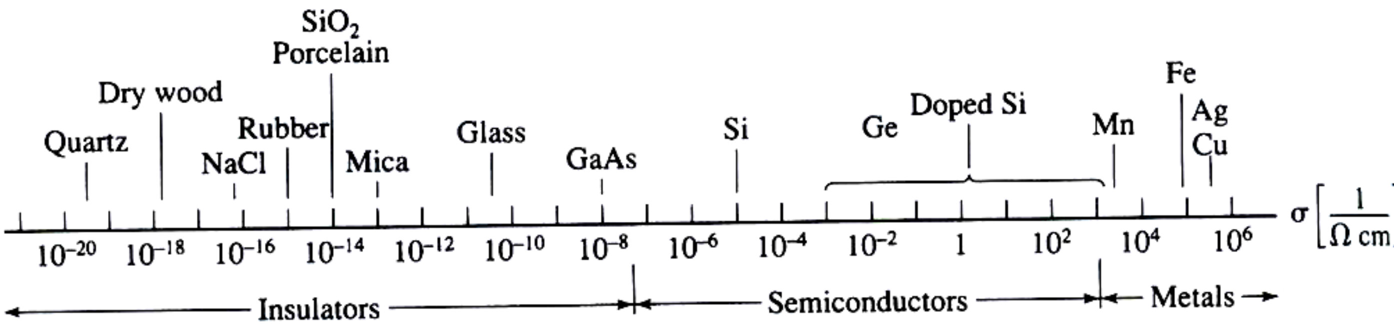
4

Electrical Conduction in Metals And Alloys

Chapter 7 in the textbook

7.1 Introduction and Survey

$$V = RI, \quad j = \sigma \mathrm{E}, \Rightarrow \quad j = \frac{I}{A},$$



$$\mathrm{E} = \frac{V}{L} \quad , j = N\upsilon e, \quad R = \frac{L\rho}{A}, \Rightarrow \quad \rho = \frac{1}{\sigma}.$$

V / cm

$\#$ of electrons/cm$^2 \cdot$sec

$amphere(A) = coulomb(C)/\sec.$

$\# electrons / cm^2 \cdot \sec = A / cm^2$

$\sigma(1/\Omega \cdot cm)$

7.2 Conductivity-Classical Electron Theory

Understanding of electrical conduction

As Drude did,

A free electron gas or plasma ; valence electrons of individual atoms in a crystal

What is plasma?

$$N_a = \frac{N_0 \delta}{M},$$

For a monovalent metal,

N_a : # of atoms / cm^3

N_0 : Avogadro constant (#/mole)

δ : density (gram/cm^3)

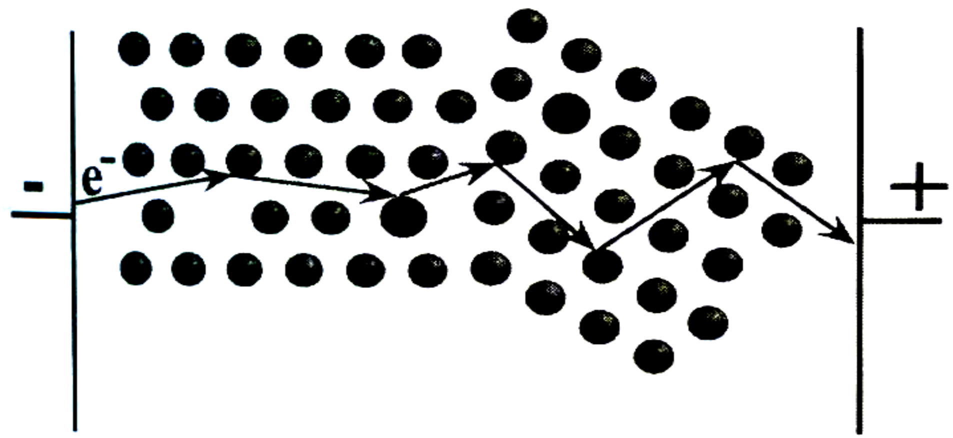
M : atomic mass of element (gram/mole)

Without electric field ?

If E is applied

For a free electron

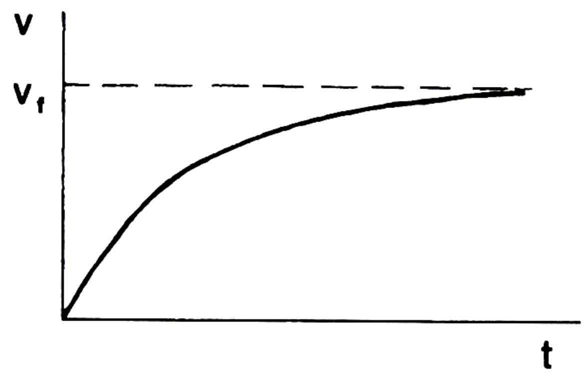
$$F = ma = m\frac{d\upsilon}{dt} = e\mathrm{E},$$



(a)

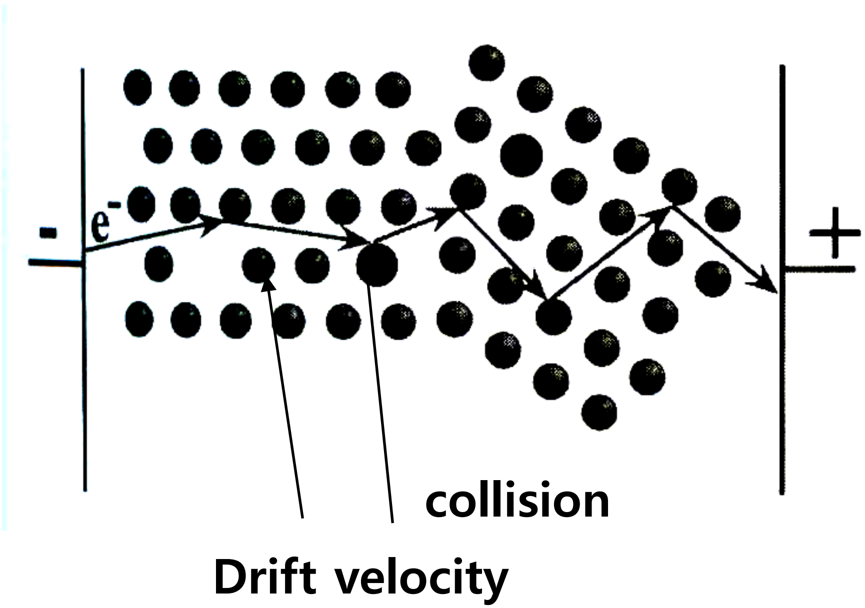
1) Acceleration,
2) Constant velocity

Free electron model should be modified.



(b)

In a crystal,



Friction force (or damping force)

$$\gamma\upsilon$$

Equation of Motion

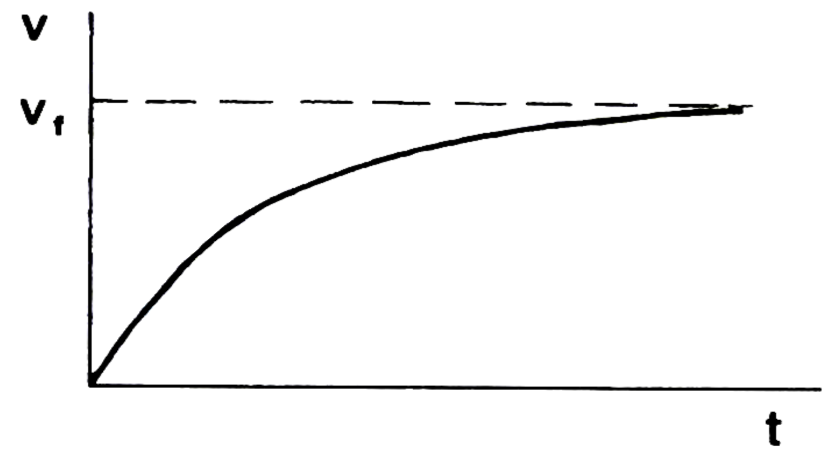
$$m\frac{d\upsilon}{dt}+\gamma\upsilon=eE,$$

For the steady state

$$\upsilon = \upsilon_{\mathrm{f}}\,, i.e., d\upsilon / dt = 0$$

$$m\cancel{\frac{d\upsilon}{dt}} + \gamma\upsilon = e\mathrm{E},$$

$$\gamma\upsilon_{\mathrm{f}} = e\mathrm{E}, \qquad \gamma = \frac{e\mathrm{E}}{\upsilon_{\mathrm{f}}}.$$



(b)

Final drift velocity

$$m\frac{d\upsilon}{dt} + \frac{eE}{\upsilon_{\mathrm{f}}}\upsilon = eE.$$

$$\upsilon = \upsilon_{\mathrm{f}}\left[1 - \exp\left(-\left(\frac{eE}{m\upsilon_f}t\right)\right)\right]. \qquad \tau = \frac{m\upsilon_{\mathrm{f}}}{eE},$$

Where, t is a relaxation time: average time between two consecutive collisions

$$\upsilon = \upsilon_{\mathrm{f}}\left[1 - \exp\left(-\left(\frac{t}{\tau}\right)\right)\right].$$

Final drift velocity

$$\tau = \frac{m\upsilon_{\mathrm{f}}}{e\mathrm{E}}, \quad \Rightarrow \quad \upsilon_{\mathrm{f}} = \frac{\tau e \mathrm{E}}{m}.$$

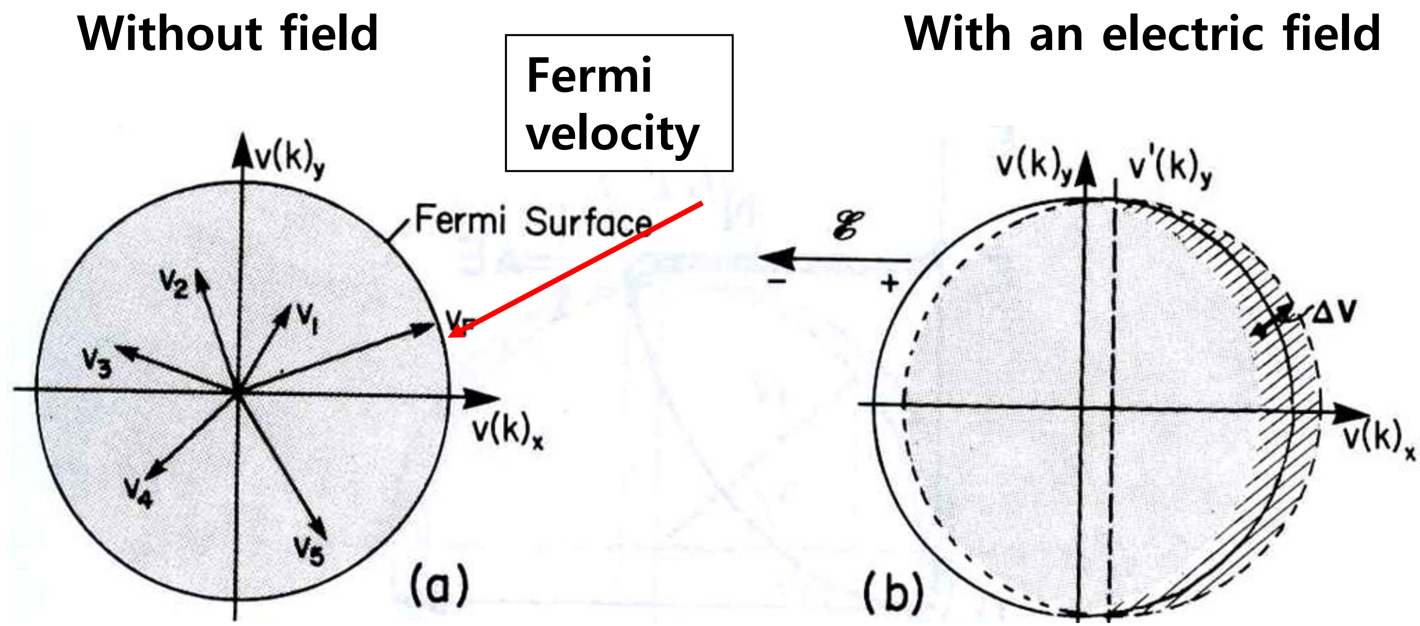
$$j = N_{\mathrm{f}}\upsilon_{\mathrm{f}}e = \sigma\mathrm{E}.$$

$$\boxed{\therefore\ \sigma = \frac{N_{\mathrm{f}}e^2\tau}{m}.}$$

Mean free path between two consecutive collisions

$$l = \upsilon\tau. \qquad (\upsilon \neq \upsilon_{\mathrm{f}})$$

7.3 Conductivity-Quantum Mechanical Considerations



At equilibrium, no net velocity

What difference between Class. and QM

The maximum velocity that the electrons can have is the Fermi velocity (i.e., the velocity of electrons at the Fermi energy)

Only specific electrons participate in conduction and that these electrons drift with a high velocity (v_F)

$$j = \upsilon e N.$$

$$j = \upsilon_{\mathrm{F}} e N'. \qquad N' \neq N$$

$$N' = N\left(E_F\right)\Delta E$$

$$j = \upsilon_F e N\left(E_F\right)\Delta E = \upsilon_F e N\left(E_F\right)\frac{dE}{dk}\Delta k.$$

$$E = \frac{\hbar^2}{2m}k^2.$$

$$\frac{dE}{dk} = \frac{\hbar^2}{m}k = \frac{\hbar^2 p}{m\hbar} = \frac{\hbar m \upsilon_F}{m} = \hbar \upsilon_F$$

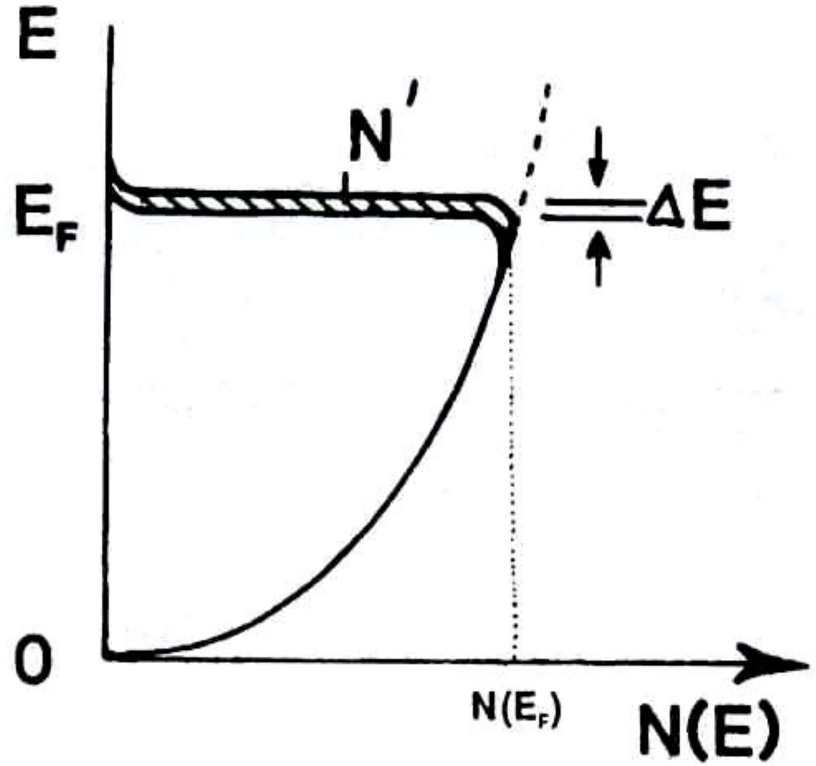


Fig.7.4 Population density

$$j = \upsilon_F^2 e N(E_F)\hbar\Delta k.$$

$$F = m\frac{d\upsilon}{dt} = \frac{d(m\upsilon)}{dt} = \frac{dp}{dt} = \hbar\frac{dk}{dt} = e\mathrm{E},$$

$$dk = \frac{e\mathrm{E}}{\hbar}dt, \quad \Delta k = \frac{e\mathrm{E}}{\hbar}\Delta t = \frac{e\mathrm{E}}{\hbar}\tau,$$

$$\boxed{j = \upsilon_F^2 e^2 N(E_F)\mathrm{E}\tau.}$$

Population density at E_F

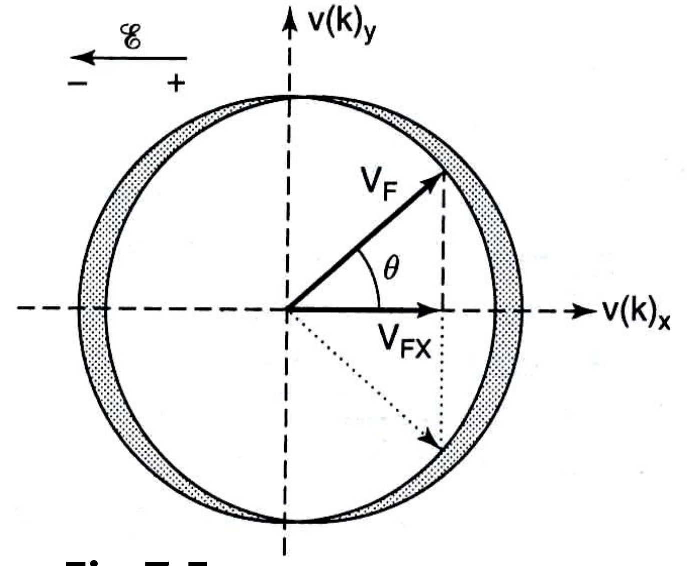


Fig.7.5

$$j = e^2 N(E_F)\mathrm{E}\tau\int_{-\pi/2}^{+\pi/2}(\upsilon_F\cos\theta)^2\frac{d\theta}{\pi} = e^2 N(E_F)\mathrm{E}\tau\frac{\upsilon_F^2}{\pi}\int_{-\pi/2}^{+\pi/2}\cos^2\theta d\theta$$

$$= e^2 N(E_F)\mathrm{E}\tau\frac{\upsilon_F^2}{\pi}\left[\frac{1}{4}\sin 2\theta + \frac{\theta}{2}\right]_{-\pi/2}^{+\pi/2},$$

$$j = \frac{1}{2}e^2 N(E_F)\mathrm{E}\tau\upsilon_F^2.$$

For a spherical Fermi surface,

$$j = \frac{1}{3} e^2 \upsilon_F^2 \tau N\left(E_F\right) \mathrm{E}.$$

Fermi velocity, relaxation time, population density at Fermi energy

Since, σ = j/E

$$\sigma = \frac{1}{3} e^2 \upsilon_F^2 \tau N\left(E_F\right). \quad \textbf{vs} \quad \sigma = \frac{N_{\mathrm{f}} e^2 \tau}{m}.$$

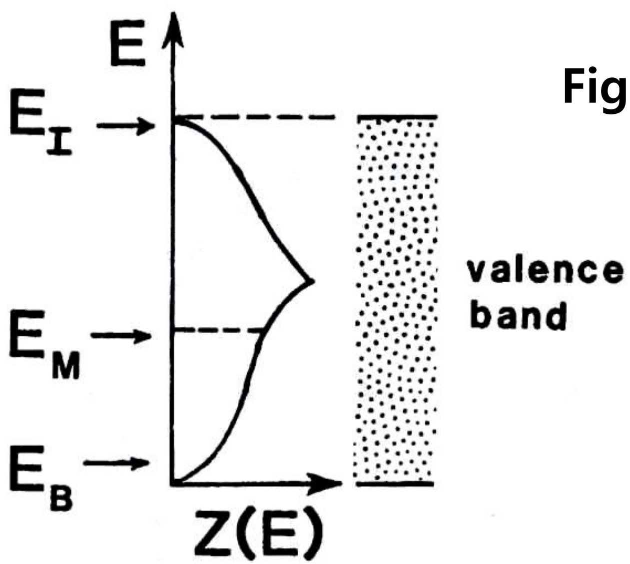


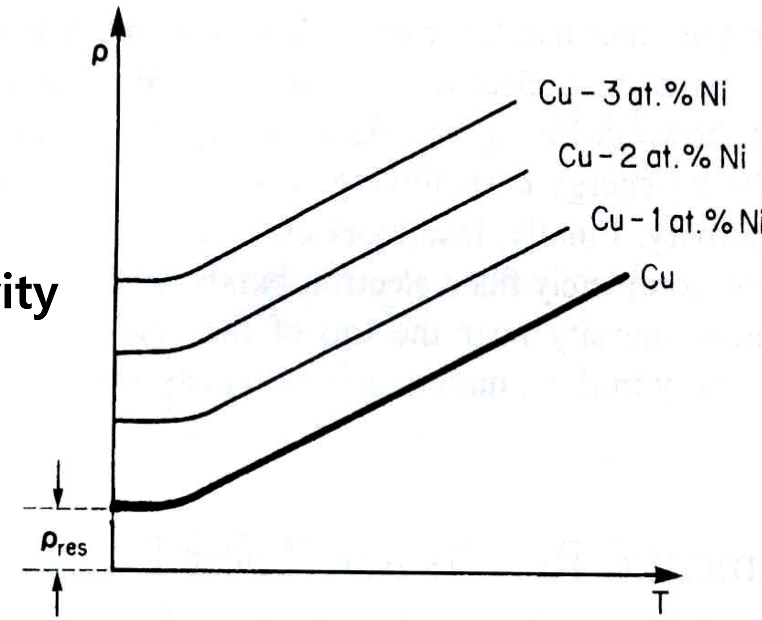
Fig.7.6

7.4 Experimental Results and Their Interpretation

7.4.1 For Pure Metals

$$\rho_2 = \rho_1\left[1+\alpha\left(T_2 - T_1\right)\right],$$

Linear temperature coefficient of resistivity



Matthiessen's rule

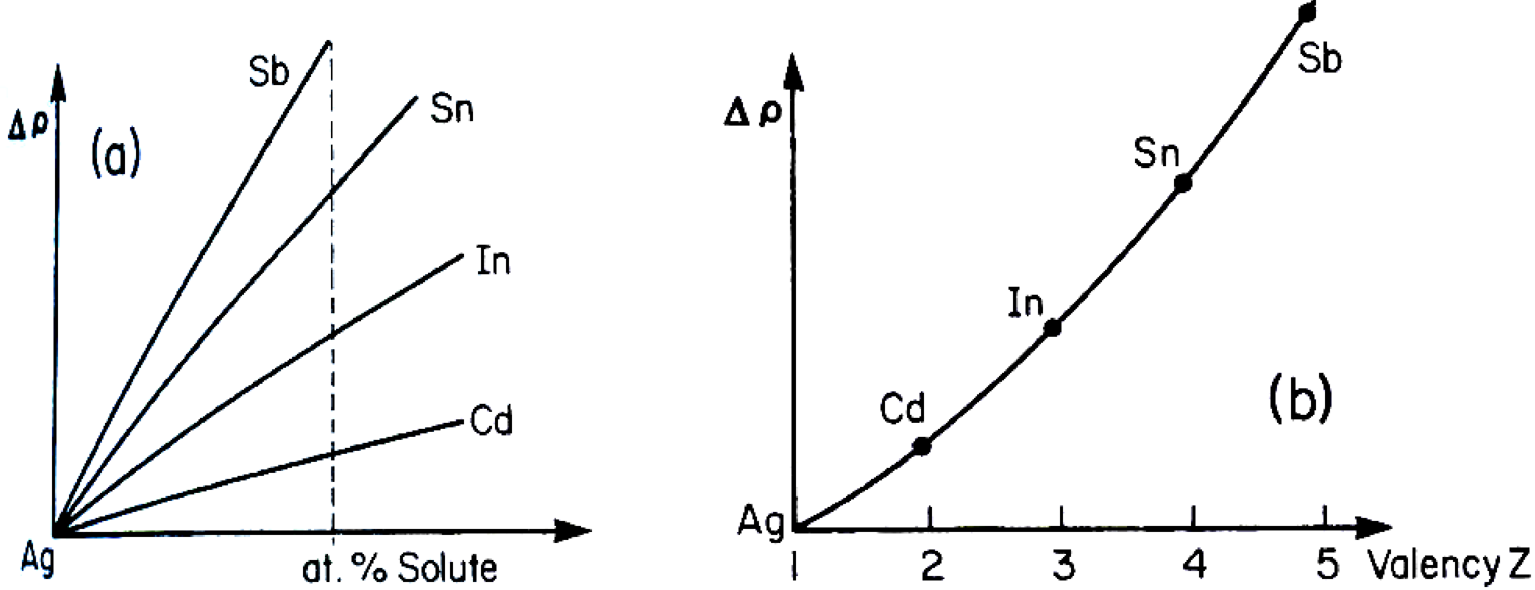
$$\rho = \rho_{\text{th}} + \rho_{\text{imp}} + \rho_{\text{def}} = \rho_{\text{th}} + \rho_{\text{res}}.$$

ρ_{th} **Ideal resistivity**

$\rho_{\text{res}} \neq f(T)$

7.4.2 For alloys

Linde's rule ∝(valence electrons)$^{1/2}$



For dilute single-phase alloys

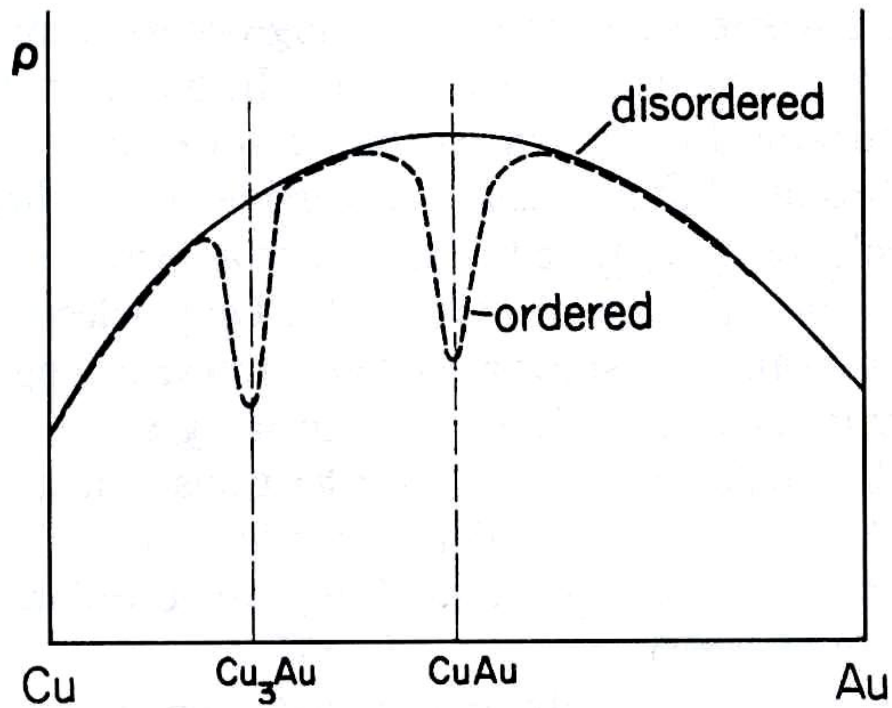
Atoms of different size cause a variation in the lattice parameters local charge valence alter the position of the Fermi energy.

$$\rho_{res} = X_A \rho_A + X_B \rho_B + C X_A X_B$$

(Nordheim's rule)
for transition metal alloys

Two phase mixture: Matthiessen's rule

7.4.3 Ordering vs Disordering



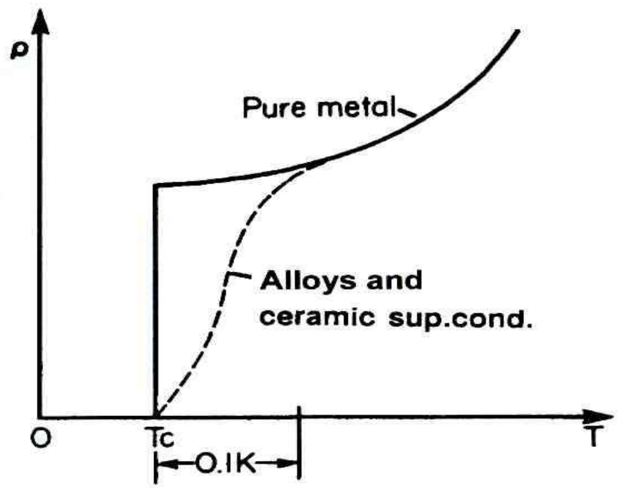
7.5 Superconductivity

Table 7.1. Critical Temperatures of Some Superconducting Materials.

Materials	T_c [K]	Remarks
Tungsten	0.01	—
Mercury	4.15	H.K. Onnes (1911)
Sulfur-based organic superconductor	8	S.S.P. Parkin et al. (1983)
Nb_3Sn and Nb–Ti	9	Bell Labs (1961), Type II
V_3Si	17.1	J.K. Hulm (1953)
Nb_3Ge	23.2	(1973)
La–Ba–Cu–O	40	Bednorz and Müller (1986)
$YBa_2Cu_3O_{7-x}$ [a]	92	Wu, Chu, and others (1987)
$RBa_2Cu_3O_{7-x}$ [a]	~92	R = Gd, Dy, Ho, Er, Tm, Yb, Lu
$Bi_2Sr_2Ca_2Cu_3O_{10+\delta}$	113	Maeda et al. (1988)
$Tl_2CaBa_2Cu_2O_{10+\delta}$	125	Hermann et al. (1988)
$HgBa_2Ca_2Cu_3O_{8+\delta}$	134	R. Ott et al. (1995)

[a] The designation "1-2-3 compound" refers to the molar ratios of rare earth to alkaline earth to copper. (See chemical formula.)

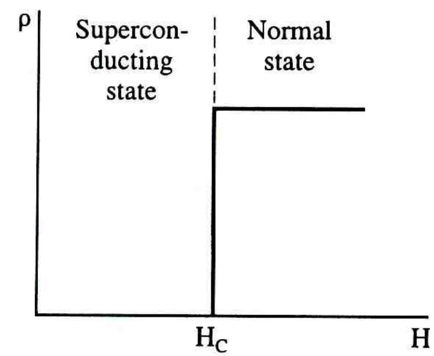
7.5.1 Experimental Results

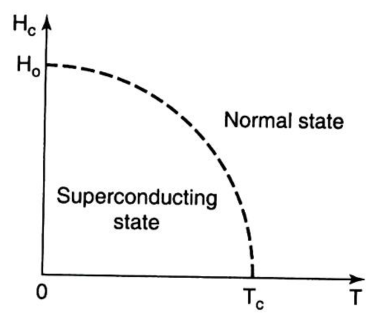


$$m_a^{\alpha} \cdot T_c = \text{const.},$$

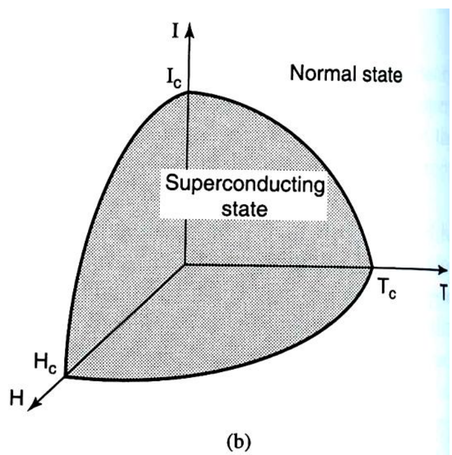
m_a: atomic mass
α: material constant

$$H_c = H_0\left(1 - \frac{T^2}{T_c^2}\right),$$



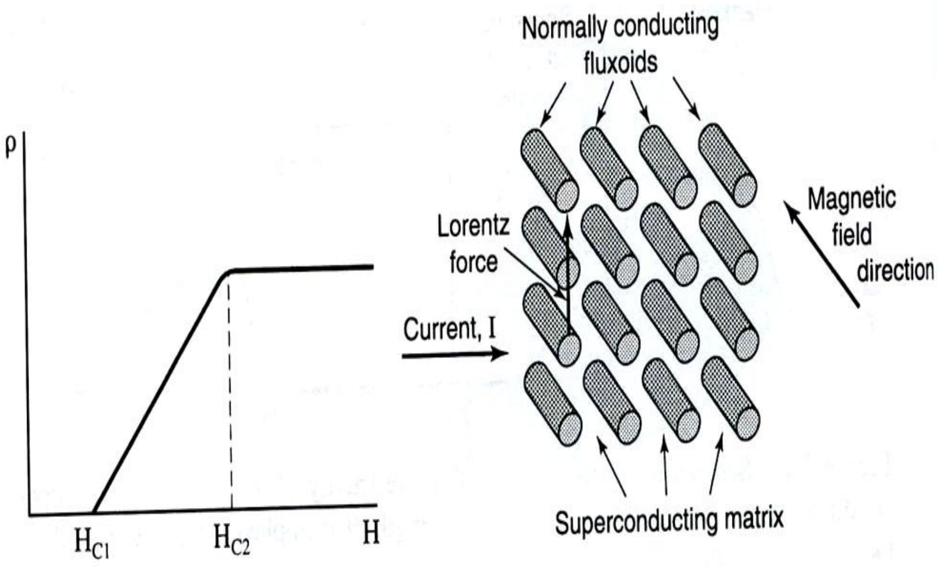


(a)

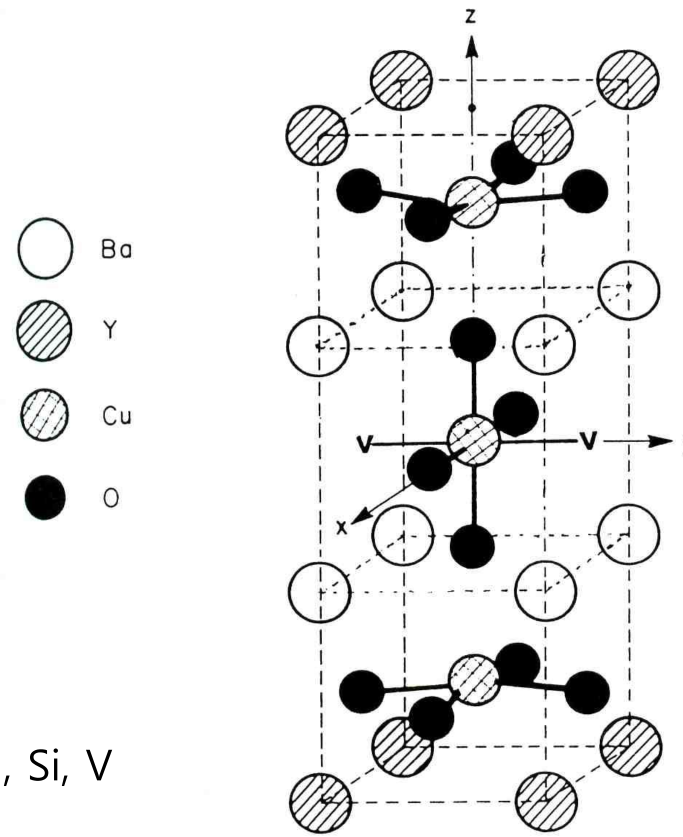


(b)

Type I superconductors



(a) (b)



Type II superconductors:

Transition metals and alloys consisting of Nb, Al, Si, V
Pb, Sn, Ti, Nb_3Sn, Nb-Ti

Contains small circular regions called vortices or fluxoids.

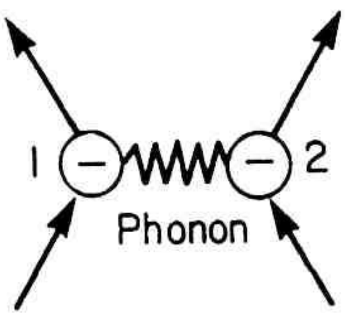
Flux quantum:

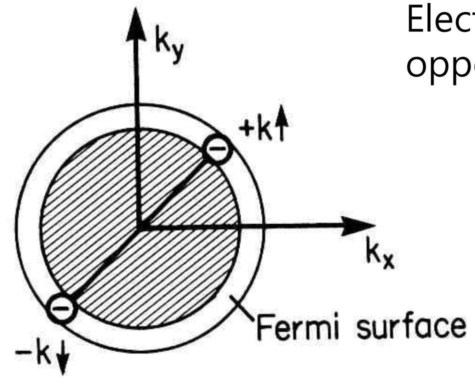
$$\phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15} (\mathrm{T} \cdot \mathrm{m}^2).$$

7.5.2 Theory

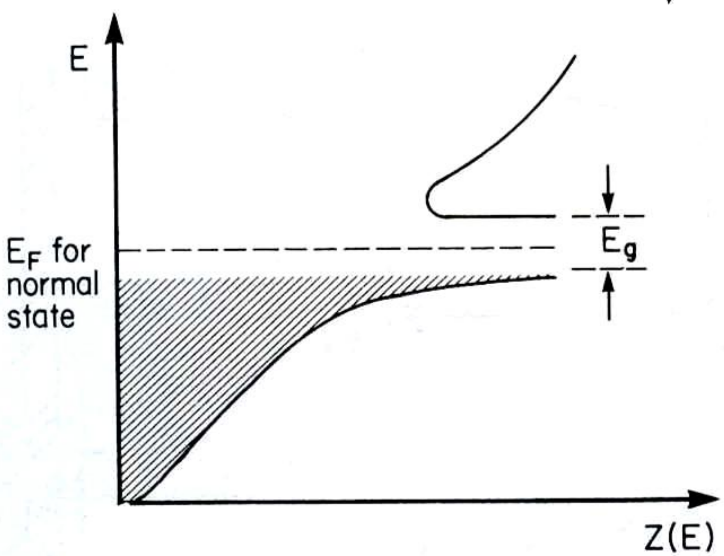
Postulate superelectrons that experience no scattering, having zero entropy (perfect order),
And have long coherence lengths.

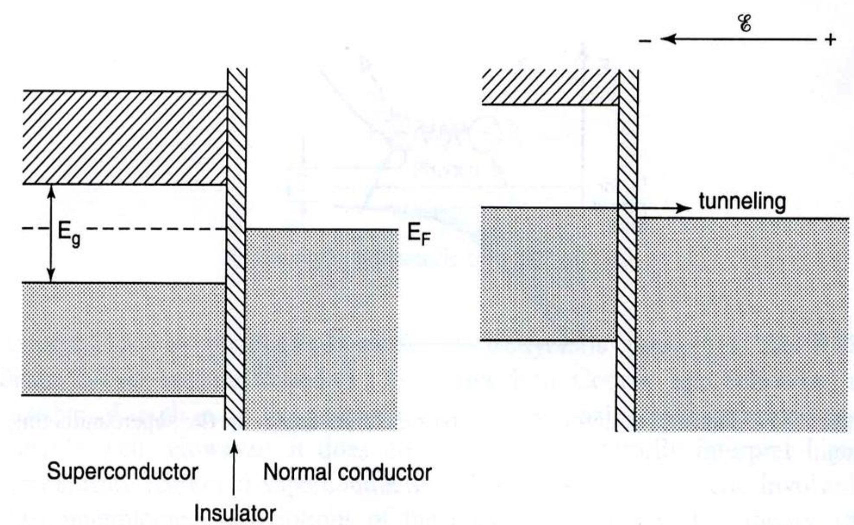
BCS theory: Cooper pair (pair of electrons that has a lower energy than two individual electrons





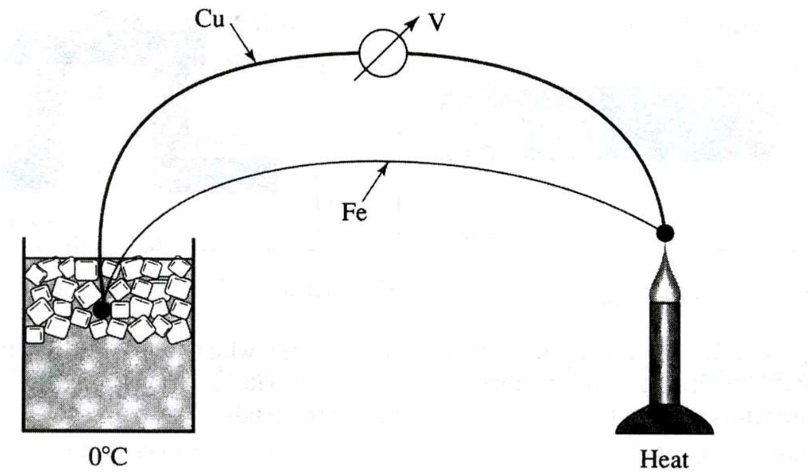
Electrons having opposite momentum and opposite spin forms cooper-pair





(a) (b)

7.6 Thermoelectric Phenomena



$$\frac{\Delta V}{\Delta T} = S$$

Thermoelectric power, or Seebeck coefficient