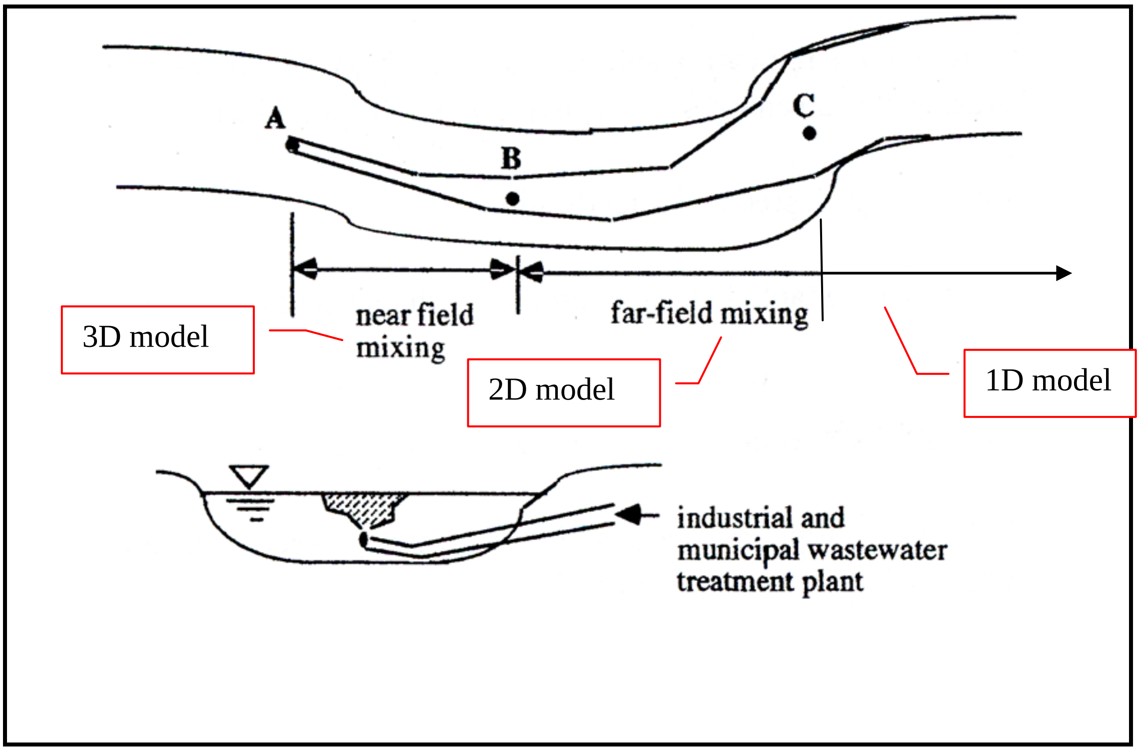
Chapter 5 Mixing in Rivers

Contents

Objectives:

- Discuss turbulent diffusion
- Study transverse mixing in the 2^{nd} stage mixing
- Discuss process of longitudinal dispersion for the analysis of final stage



Consider a stream of effluent discharged into a river.

What happens can be divided into three stages:

Stage I: near field mixing

~ initial momentum and buoyancy determine mixing near the outlet

~ vertical mixing is usually completed

→ Ch.9 Turbulent jets and plumes

Ch.10 Design of ocean wastewater discharge system

Stage II: two-dimensional mixing (longitudinal + lateral mixing)

~ waste is mixed across the receiving channel primarily by turbulence in the receiving stream

→ dye mixing across the Columbia River (Fig. 5.6)

Stage III: Longitudinal dispersion

~ process of longitudinal shear flow dispersion erases any longitudinal concentration variations

~ we could apply Taylor's analysis of longitudinal dispersion

[Re] Two phases of hydrodynamics mixing processes

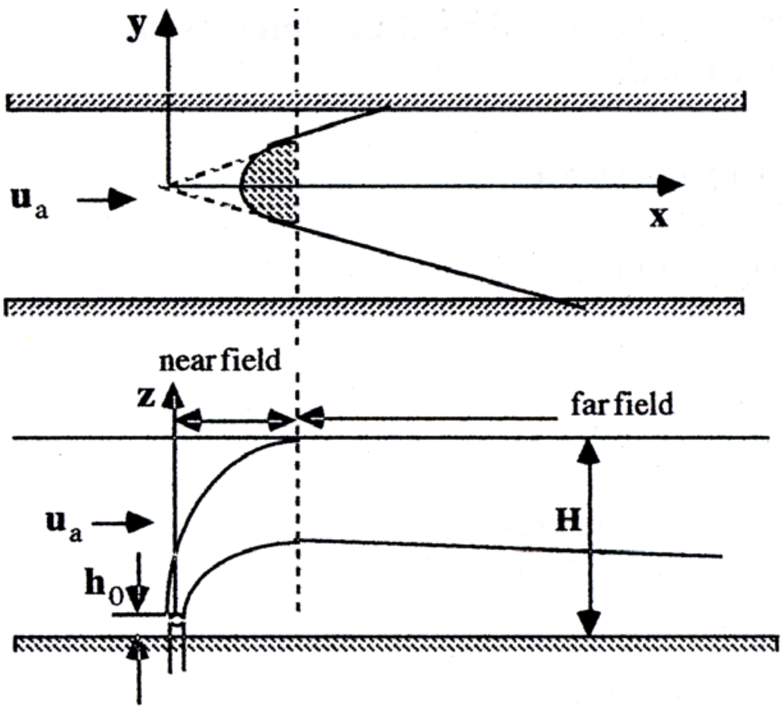
1) Near field: mixing is controlled by the initial jet characteristics of momentum flux, buoyancy flux, and outfall geometry

2) Far field: source characteristics are less important, mixing is controlled by buoyant spreading motions and passive diffusion due to ambient turbulence

Far field = Stage II+ Stage III

→ deal with a source of tracer without its own momentum or buoyancy

[Re] Analysis of near field mixing



▪ Multiport diffuser

- linear structure consisting of many closely spaced ports, or nozzles, through which wastewater effluent is discharged at high velocity into the receiving water body

jet

- attractive engineering solution to the problem of managing wastewater discharge in an environmentally sound way
- offer high degree of initial dilution
- optimally adapted to the assimilative characteristic of the water body

· Thermal diffuser: heated water discharge from the once-through cooling systems of nuclear power plant and fossil fuel power plant

$$S \sim 10$$

· Wastewater diffuser: wastewater discharge from the sewage treatment plants

$$S \sim 100$$

▪ Three groups of parameters for jet analysis

1) receiving water flow patterns – ambient water depth, velocity, density stratification
2) pollutant discharge flow characteristics – discharge velocity (momentum), flow rate, density of pollutant (buoyancy)
3) diffuser characteristics - single/multi ports, submerged/surface discharge, alignment of port

▪ Jet analysis model: CORMIX (Cornell Mixing Zone Expert System)
VISJET

▪ Water-quality policy in USA

Office of Water (1991) "Technical support document for water quality-based toxics control," Washington, DC.

~ regulations on toxic control with higher initial mixing requirements

regulatory mixing zone (RMZ)
= limited area or volume of water where initial dilution of an aqueous pollutant discharge occurs
- regulator = U.S. Environmental Protection Agency
- should predict the initial dilution of a discharge and extent of its mixing zone
- toxic dilution zone (TDZ) for toxic substances
- regularly mixing zone (RMZ) for conventional pollutants

streams, rivers	lakes, estuaries
Florida: RMZ ≤ 800m and ≤ 10% total length	≤ 125,600 m^2 and ≤ 10% surface area
Michigan: RMZ ≤ 1/4 cross-sectional area	≤ 1000 ft radius
West Virginia: RMZ ≤ 20～33% cross-sectional area and ≤ 5～10 times width	≤ 300 ft any direction

5.1 Turbulent Mixing in Rivers

5.1.1 The Idealized Case of a Uniform, Straight, Infinitely Wide Channel of Constant Depth

Consider mixing of source of tracer without its own momentum or buoyancy in a straight channel of constant depth and great width

The turbulence is homogeneous, stationary because the channel is uniform.

→ The important length scale is depth.

From Eq. (3.40), turbulent mixing coefficient is given as

$$\varepsilon = \ell_L \left[\overline{u'^2}\right]^{\frac{1}{2}} \qquad (1)$$

where ε = turbulent mixing coefficient

ℓ_L = Lagrangian length scale $\approx d$ (a)

$\left[\overline{u'^2}\right]^{\frac{1}{2}}$ = intensity of turbulence

Experiments (Lauffer, 1950) show that

turbulent intensity $\propto$ shear stress on the wall (b)

For dimensional reasons use shear velocity

$$u^* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{gdS} \qquad (5.1)$$

where τ_0 = shear stress on the channel bottom

[Re] shear stress (Henderson, 1966)

~ bottom shear stress is evaluated by a force balance

$$\tau_0 = \rho g d S$$

where S = slope of the channel

Substitute (a) & (b) into (1)

$$\varepsilon \propto d\, u^*$$

$$\boxed{\varepsilon = \alpha\, d\, u^*}$$

→ turbulence will not be isotropic

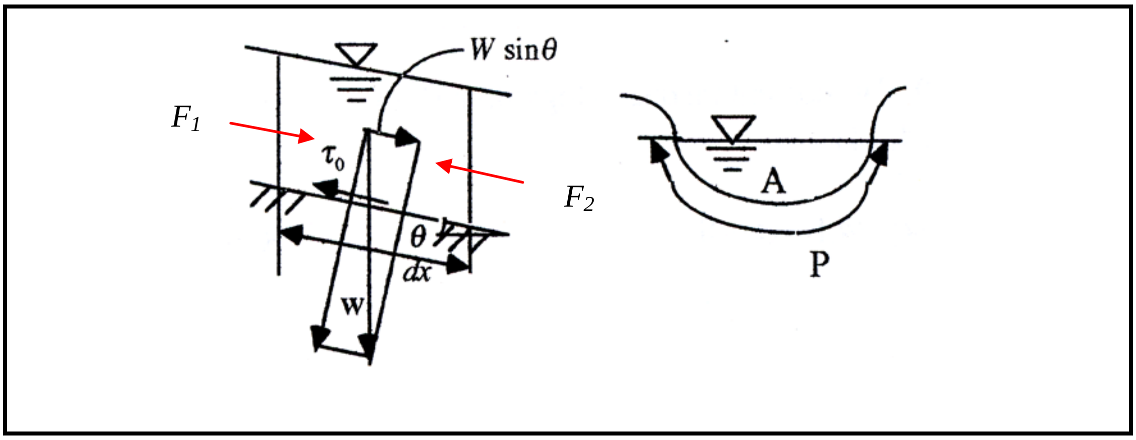
i) vertical mixing, ε_v

~ influence of surface and bottom boundaries

ii) transverse and longitudinal mixing, $\varepsilon_t, \varepsilon_l$

~ no boundaries to influence flow

[Re] Shear stress and shear velocity



Apply Newton's 2nd law of motion to uniform flow

$$\boxed{\vec{a}=0}$$

$$\boxed{F_1=F_2}$$

$$\Sigma\vec{F}=m\vec{a}$$

$$F_1-bottom\,shear+W\sin\theta-F_2=0$$

$$-\tau_0 Pdx+\rho gAdx\sin\theta=0$$

$$\tau_0=\rho g\frac{A}{P}\sin\theta$$

where P = wetted perimeter

Set $S=\tan\theta\approx\sin\theta$

R = hydraulic radius $=\dfrac{A}{P}$

Then $\tau_0=\gamma RS$

For very wide channel (b>>d)

$$R=\frac{bd}{b+2d}=\frac{d}{1+2\dfrac{d}{b}}\approx d$$

$$\boxed{\tau_0=\gamma dS}$$

5.1.1.1 Vertical Mixing

Vertical mixing coefficient in 3D model

→ no dispersion effect by shear flow

i) vertically varying coefficient:

The vertical mixing coefficient for momentum can be derived from logarithmic law velocity profile (Eq. 4.43).

$$\varepsilon_v = \kappa d u^* (z/d)\left[1-(z/d)\right] \tag{5.2}$$

The Reynolds analogy states that the same coefficient can be used for transport of mass.

→ verified by Jobson and Sayre (1970)

[Re] Velocity profiles:

- vertical profile of u-velocity ~ logarithmic
- vertical profile of v-velocity ~ linear/cubic → might be neglected because v-velocity is relatively small compared to u-velocity

ii) depth-averaged coefficient

Average Eq. (5.2) over the depth, taking $\kappa = 0.4$

$$\boxed{\overline{\varepsilon_v} = \frac{1}{d}\int_0^d \kappa d u^* \left(\frac{z}{d}\right)\left[1-\left(\frac{z}{d}\right)\right] dz = \frac{\kappa}{6} d u^* = 0.067 d u^*} \tag{5.3}$$

[Cf] For atmospheric boundary layer: $\overline{\varepsilon_v} = 0.05 d u^*$

where d = depth of boundary layer; u^* = shear velocity at the earth surface

5.1.1.2 Transverse Mixing

(1) Transverse mixing coefficient in 3D model

～ no dispersion effect by shear flow, turbulence effect only

→ vertically varying coefficient

For infinitely wide uniform channel, there is no transverse velocity profile.

～ not possible to establish a transverse analogy of Eq. (5.2)

→ need to know velocity profiles:

- transverse profile of *u*-velocity ～ parabolic/beta function
- transverse profile of *w*-velocity ～ might be neglected because *w*-velocity is usually very small

[Re] Turbulent diffusion coefficient for 3-D flow

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + w\frac{\partial c}{\partial z} = \frac{\partial}{\partial x}(\varepsilon_x \frac{\partial c}{\partial x}) + \frac{\partial}{\partial y}(\varepsilon_y \frac{\partial c}{\partial y}) + \frac{\partial}{\partial z}(\varepsilon_z \frac{\partial c}{\partial z})$$

Consider shear stress tensor

$$\begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}$$

Now, consider velocity gradients for each turbulent diffusion coefficient

$$\tau_{xz} = \rho\varepsilon_v \frac{du}{dz} \quad \tau_{yz} = \rho\varepsilon_v \frac{dv}{dz} \quad \cancel{\sigma_{zz}} = \rho\varepsilon_v \frac{dw}{dz}$$

$$\tau_{xy} = \rho\varepsilon_t \frac{du}{dy} \quad \cancel{\sigma_{yy}} = \rho\varepsilon_t \frac{dv}{dy} \quad \tau_{zy} = \rho\varepsilon_t \frac{dw}{dy}$$

$$\sigma_{xx} = \rho\varepsilon_l \frac{du}{dx} \quad \tau_{yx} = \rho\varepsilon_l \frac{dv}{dx} \quad \tau_{zx} = \rho\varepsilon_l \frac{dw}{dx}$$

1) vertical mixing

- vertical profile of u-velocity ~ logarithmic
- vertical profile of v-velocity ~ linear/cubic → might be neglected because v-velocity is relatively small compared to u-velocity

2) transverse mixing

- transverse profile of u-velocity ~ parabolic/beta function
- transverse profile of w-velocity → might be neglected because w-velocity is usually very small

3) longitudinal mixing

- longitudinal profile of v-velocity ~ linear/cubic
- longitudinal profile of w-velocity → might be neglected because w-velocity is usually very small

(2) Transverse mixing coefficient in 2D model

Depth-averaged 2D model is

$$\frac{\partial \bar{c}}{\partial t} + \bar{u}\frac{\partial \bar{c}}{\partial x} + \bar{v}\frac{\partial \bar{c}}{\partial y} = K_L \frac{\partial^2 \bar{c}}{\partial x^2} + K_T \frac{\partial^2 \bar{c}}{\partial y^2}$$

Include dispersion effect by shear flow due to vertical variation of *v*-velocity

$$v = v(z) = \bar{v} + v'$$

- depth-averaged coefficient

→ rely on experiments (Table 5.1 for results of 75 separate experiments)

$$K_T = \varepsilon_t \cong 0.15 d u^* \tag{5.4}$$

Researchers (Okoye, 1970; Lau and Krishnappan, 1977) proposed that

$$K_T = \alpha d u^*$$

$$\alpha = f(W/d)$$

5.1.1.3 Longitudinal Mixing

(1) Longitudinal mixing coefficient in 3D model

~ no dispersion effect by shear flow, turbulence effect only

→ ε_l ~ longitudinal analogy of Eq. (5.2)

(2) Longitudinal mixing coefficient in 2D model

~longitudinal turbulent mixing is the same rate as transverse mixing because there is an equal lack of boundaries to inhibit motion

~ However, longitudinal mixing by turbulent motion is unimportant because shear flow dispersion coefficient caused by the velocity gradient (vertical variation of *u*-velocity) is much bigger than mixing coefficient caused by turbulence alone

Use Elder's result for depth-averaged longitudinal dispersion coefficient

$$K_l = 5.93 d u^* \approx 40 \varepsilon_t$$

[Re] Aris (1956) showed that coefficients due to turbulent mixing and shear flow are additive.

$$K_l + \varepsilon_\ell \rightarrow K_L$$

→ can neglect the longitudinal turbulent diffusion coefficient

(3) Longitudinal dispersion coefficient in 1D model

See Section 5.2

→ $K1_l + K2_l + \varepsilon_\ell \rightarrow K$

where $K1_l$ ~ due to lateral variation of *u*-velocity; $K2_l$ ~ due to vertical variation of *u*-velocity

5.1.2 Mixing in Irregular Channels and Natural Streams

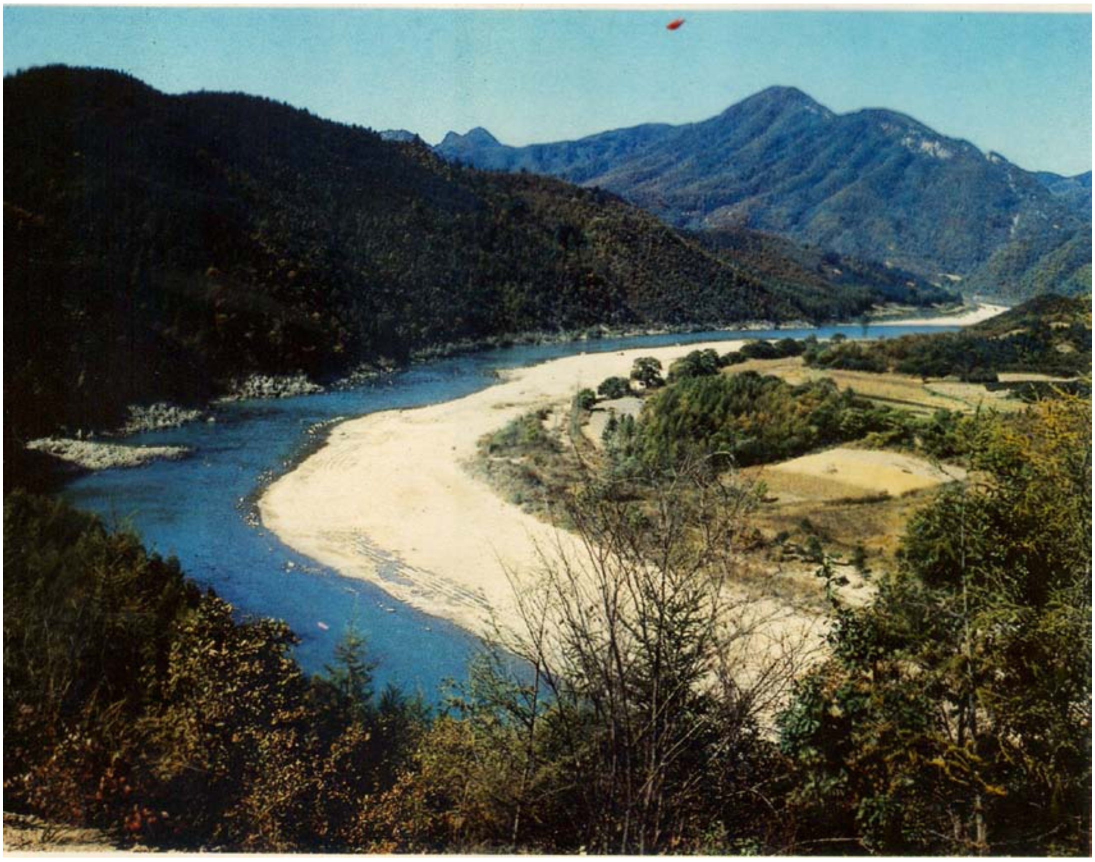
5.1.2.1 Mixing in natural channels

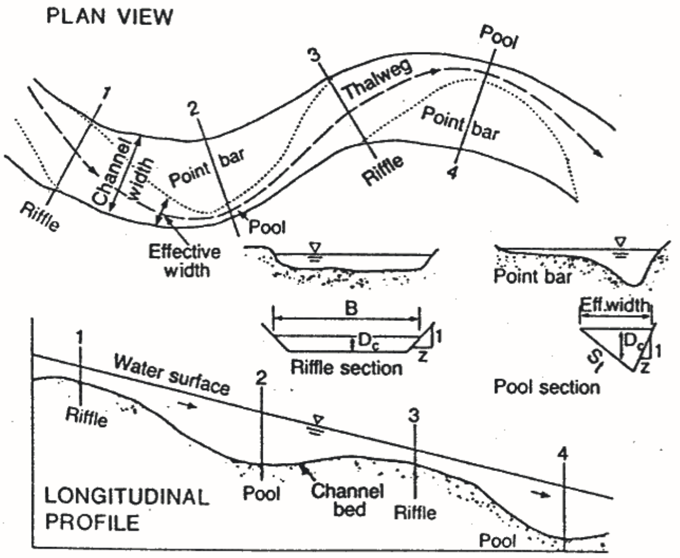
Natural streams differ from uniform rectangular channels:

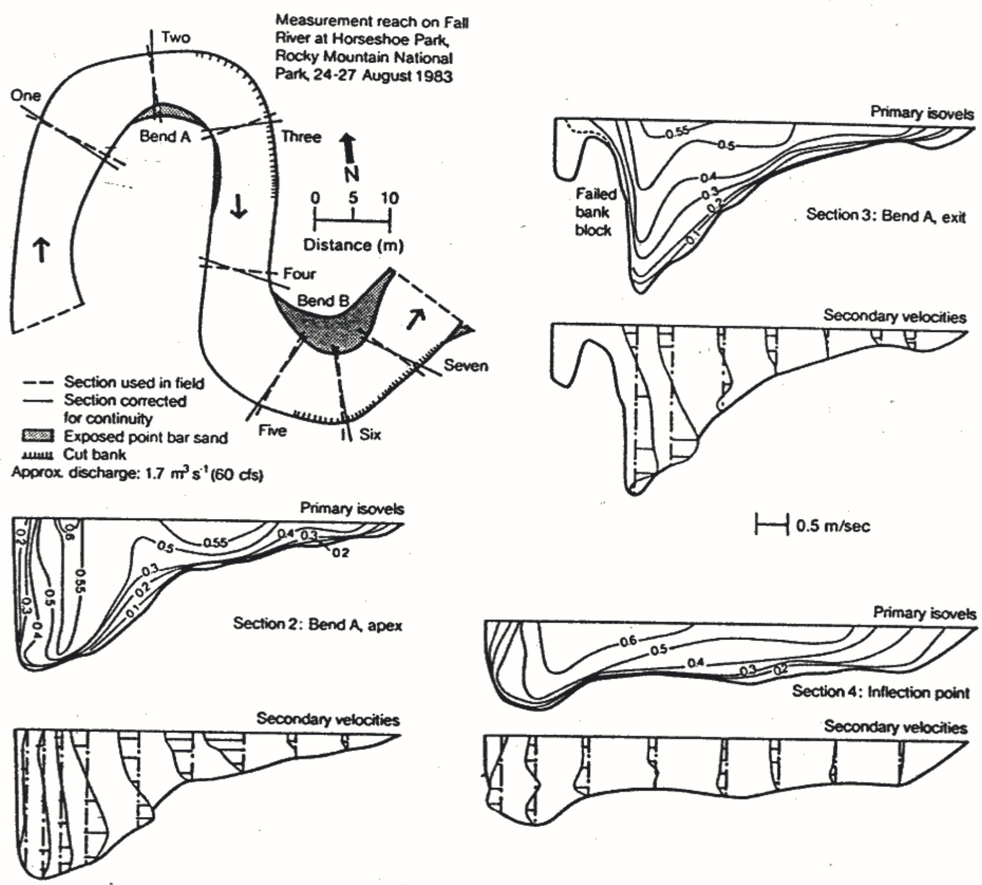
- depth may vary irregularly → pool and riffle sequences

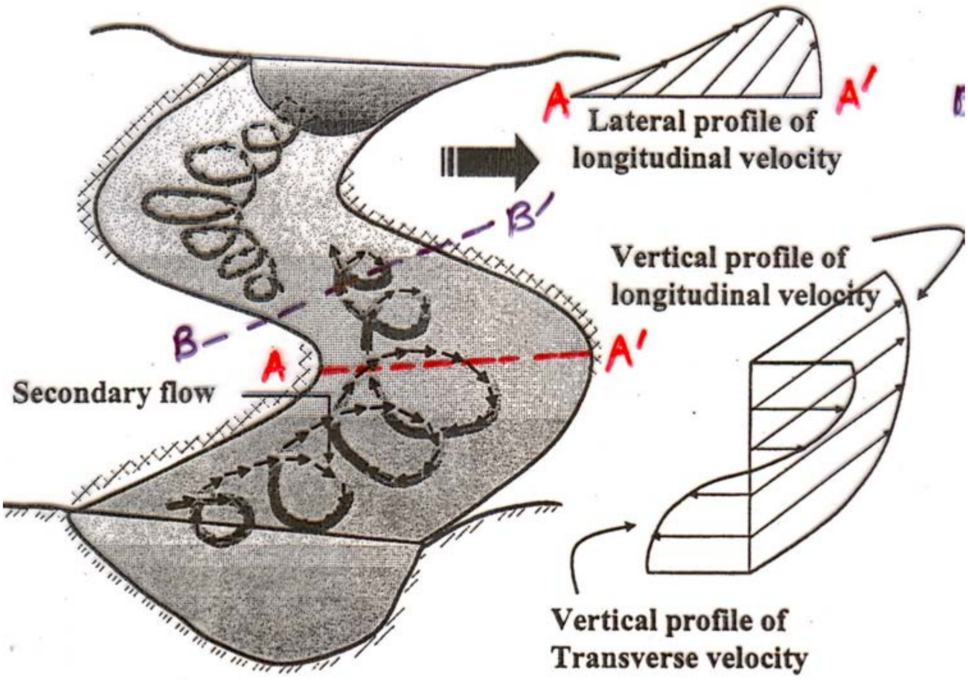
- the channel is likely to curve → meandering rivers

- there may be large sidewall irregularities → groins, dikes




PLAN VIEW
Pool
Thalweg
Point bar
Riffle
Channel width
Point bar
Riffle
Pool
Effective width
Point bar
Eff.width
B
Riffle section
Pool section
Water surface
Riffle
Pool
Channel bed
Riffle
Pool
LONGITUDINAL PROFILE


Measurement reach on Fall River at Horseshoe Park, Rocky Mountain National Park, 24-27 August 1983
One
Two
Three
Bend A
N
Distance (m)
Four
Bend B
Five
Six
Seven
Section used in field
Section corrected for continuity
Exposed point bar sand
Cut bank
Approx. discharge: 1.7 m³ s⁻¹ (60 cfs)
Primary isovels
Failed bank block
Section 3: Bend A, exit
Secondary velocities
0.5 m/sec
Primary isovels
Section 2: Bend A, apex
Secondary velocities
Primary isovels
Section 4: Inflection point
Secondary velocities



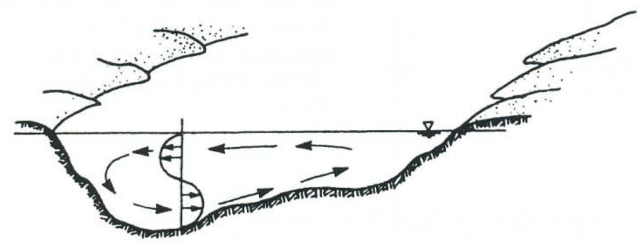


Figure 5.2 An illustration of the cross-sectional component of velocity in a curve, showing the velocity profile used to obtain Eq. (5.5).

i) Vertical mixing coefficient

These have not much influence on vertical mixing since scale of vertical motion is limited by the local depth, *d*

$$\varepsilon_v = 0.067 du^*$$

ii) Transverse mixing coefficient

Transverse mixing is strongly affected by the channel irregularities because they are capable of generating a wide variety of transverse motions

→ transverse dispersion enhanced by vertical variation of *v*-velocity

Transverse mixing in open channels with curves and irregular sides

→ see Table 5.2

$$0.3 < \frac{\varepsilon_t}{du^*} < 0.7$$

1) effect of channel irregularity

~ the bigger the irregularity, the faster the transverse mixing

2) effect of channel curvature

~ secondary flow causes transverse dispersion due to shear flow

~ when a flow rounds a bend, the centrifugal forces induce a flow towards the outside bank at the surface, and a compensating reverse flow near the bottom.
→ secondary flow generates

Fischer (1969) predict a transverse dispersion coefficient based on the transverse shear flow

$$\frac{K_T}{du^*} = 25\left(\frac{\bar{u}}{u^*}\right)^2 \left(\frac{d}{R}\right)^2 \qquad (5.5)$$

where R = radius of curvature

Yotsukura and Sayre (1976) revised Eq. (5.5) (Fig.5.3)

$$\frac{K_T}{du^*} \propto \left(\frac{\bar{u}}{u^*}\right)^2 \left(\frac{W}{R}\right)^2$$

where W = channel width

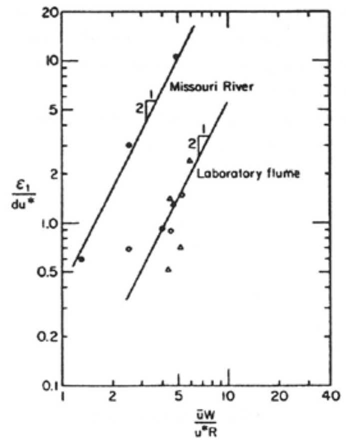
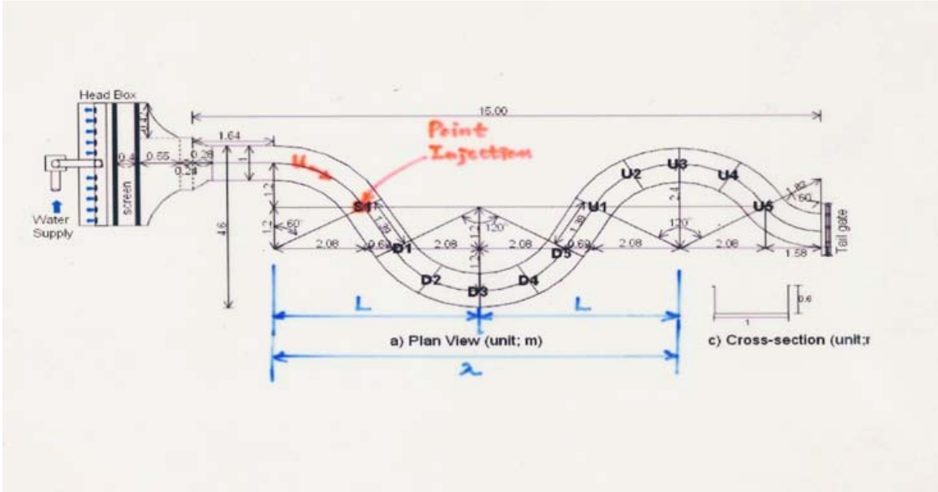


Figure 5.3 The effect of channel curvature on the transverse mixing coefficient. Data sources: ● Sayre and Yeh (1973) and Yotsukura *et al.* (1970), ○ Chang (1971), △ Fischer (1969). [After Yotsukura and Sayre (1976).]

For straight, uniform channels, $K_T = 0.15du^*$

For natural channels with side irregularities, $K_T = 0.4du^*$

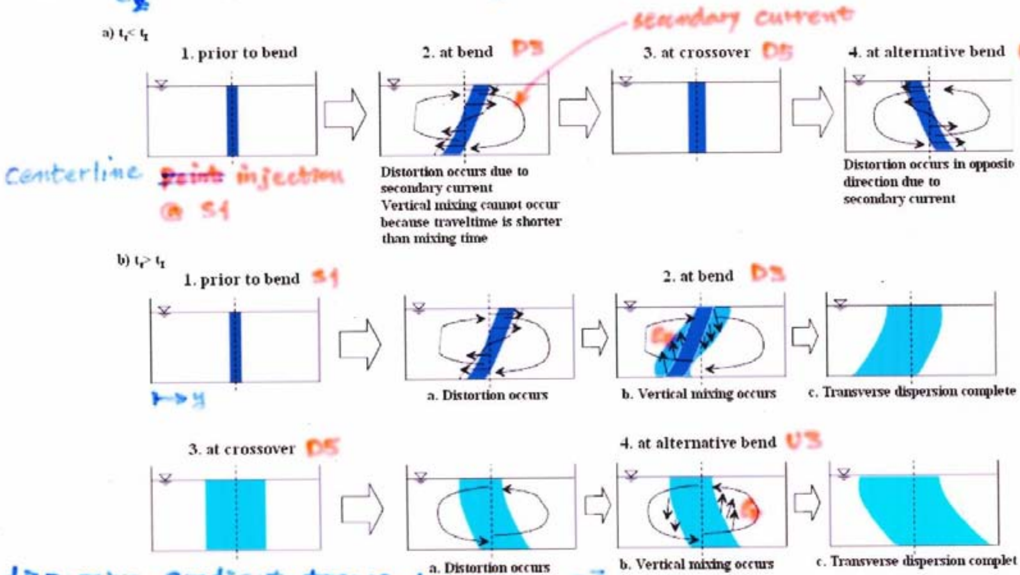
For meandering channels with side irregularities, $K_T = (0.6 \pm 50\%)du^*$



◎ Transverse Dispersion due to secondary current

$t_t = \frac{L}{U}$ (travel time)

$t_I = \frac{H^2}{\epsilon_z}$ (vertical mixing time)



For dispersive gradient transport; $\overline{u'c'} \propto \frac{\partial \bar{c}}{\partial y}$

그림 4.3 이차류가 횡혼합에 미치는 영향

$$\frac{t_t}{t_I} = \frac{\frac{L}{U}}{\frac{H^2}{\epsilon_z}} = \frac{\frac{L}{U}}{\frac{H^2}{0.07 H u_*}} = 0.07 \frac{u_*}{U}\frac{L}{H} > 1 \qquad \therefore \ \frac{u_*}{U}\frac{L}{H} > 14$$

Additional transverse mixing coeff. $\Delta\alpha$

$$\Delta\alpha = 25\left(\frac{UH}{U_* R_c}\right)^2 \qquad R_c = \text{radius of curvature}$$

after a 'initial period'

Criterion whether secondary current is strong enough

$$\Delta\alpha = 25\left(\frac{UH}{U_* R_c}\right)^2 > 10\% \text{ of } \alpha \text{ for straight irregular channel}$$

$$= (0.1)(0.4) \qquad \left(\frac{\varepsilon_t}{d u_*} = 0.4\right)$$

$$\therefore \frac{UH}{U_* R_c} > 0.04$$

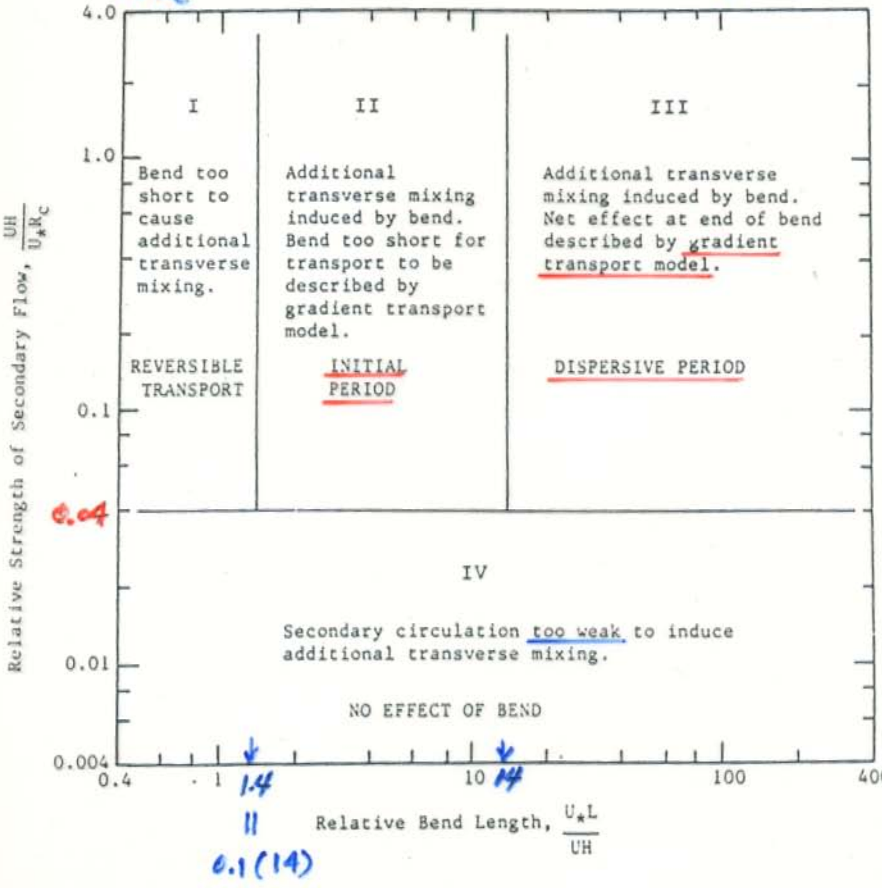


Fig. 3.2 Classification of Lateral Transport Processes in Curved Channels

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5.1.2.2 2D depth-averaged model

Depth-averaged 2D model is

$$\frac{\partial \overline{c}}{\partial t} + \overline{u}\frac{\partial \overline{c}}{\partial x} + \overline{v}\frac{\partial \overline{c}}{\partial y} = K_L \frac{\partial^2 \overline{c}}{\partial x^2} + K_T \frac{\partial^2 \overline{c}}{\partial y^2}$$

$$K_T = K_t + \varepsilon_t$$

K_t = transverse dispersion coefficient due to vertical profile of v-velocity

ε_t = transverse dispersion coefficient due to transverse profile of u-velocity

• Transverse dispersion coefficient in meandering channels

- Baek et al. (2006) - observation
- Baek and Seo (2008) - prediction

• Transverse dispersion coefficient in natural streams

- Seo et al. (2006) - observation
- Jeon et al. (2007) - prediction
- Baek and Seo (2010) – observation

- Jeon et al. (2007)

$$\frac{K_T}{du^*} = a\left(\frac{\overline{u}}{u^*}\right)^b \left(\frac{W}{d}\right)^c \left(\frac{d}{R_C}\right)^d S_n^{\ e}$$

a=0.029; b=0.463; c=0.299; d=0; e=0.733

- Baek and Seo (2008)

$$\frac{K_T}{du^*} = 0.04\left(\frac{\overline{u}}{u^*}\right)^2 \left(\frac{W}{R}\right)^2 \left(\left|\frac{x}{2L_C}\sin(2\pi\frac{x}{L_C})\right| + \frac{1}{2}\right)^2 I$$

[Re] Determination of dispersion coefficients for 2-D modeling

1) Observation – calculation of observed concentration curves from field data
2) Prediction – estimation of dispersion coefficient using theoretical or empirical equations

Observation Method	
Moment method	Simple moment method
	Stream-tube moment method
Routing procedure	2-D routing method
	2-D stream-tube routing method

Prediction Method	
Theoretical equation for K_T	Use vertical profile of *v*-velocity
	Baek & Seo (2008)
Empirical equation for K_T	Use mean hydraulic data
	Fischer (1969) Yotsukura & Sayre (1976) Jeon et al. (2007)

5.1.3 Computation of concentration distributions

Compute the distribution of concentration downstream from a continuous effluent discharge in a flowing stream

In most of the natural streams the flow is much wider than it is deep; a typical channel dimension might be 30 m wide by 1 m deep, for example.

Recall that the mixing time is proportional to the square of the length divided by the mixing coefficient,

$$T \propto \frac{(length)^2}{\varepsilon}$$

$$\frac{W}{d} \cong \frac{30}{1} = 30$$

$$\frac{\varepsilon_t}{\varepsilon_v} = \frac{0.6du^*}{0.067du^*} \approx 10$$

$$\therefore \frac{T_t}{T_v} = \frac{(W)^2}{\varepsilon_t} / \frac{(d)^2}{\varepsilon_v} = \left(\frac{W}{d}\right)^2 \frac{\varepsilon_v}{\varepsilon_t} = \left(\frac{30}{1}\right)^2 \left(\frac{1}{10}\right) = 90 \approx 10^2$$

$$\therefore T_t \approx 10^2 T_v$$

→ vertical mixing is instantaneous compared to transverse mixing

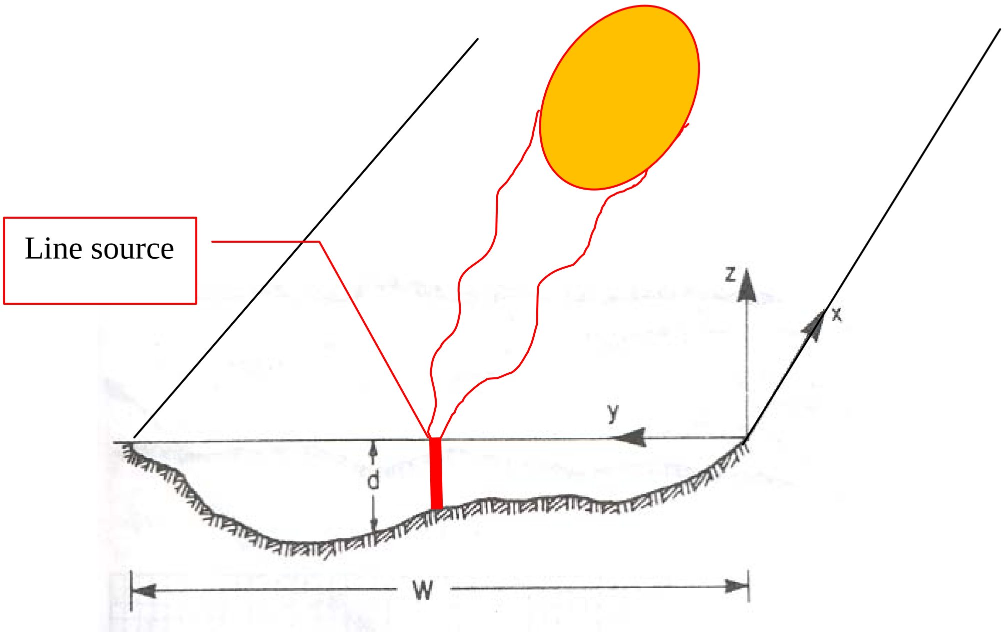
Thus, in most practical problems, we can start assuming that the effluent is uniformly distributed over the vertical.

→ analyze the two-dimensional spread from a uniform line source

Now consider the case of a rectangular channel of depth d into which is discharged $\dot{M}$ units of mass (per time) in the form of line source.

~ is equivalent to a point source of strength $\dot{M}/d$ in a two-dimensional flow

→ maintained source in 2D



Recall Eq. (2.68)

$$C = \frac{\dot{M}/d}{\bar{u}\sqrt{4\pi\varepsilon_t \frac{x}{\bar{u}}}} \exp\left(-\frac{y^2\bar{u}}{4\varepsilon_t x}\right) \tag{5.7}$$

i) For very wide channel, when $t >> 2\varepsilon_t / \bar{u}^2$

→ use Eq. (5.7)

ii) For narrow channel, consider effect of boundaries

$$\frac{\partial C}{\partial y} = 0 \; at \; y = 0 \; and \; y = W$$

→ method of superposition

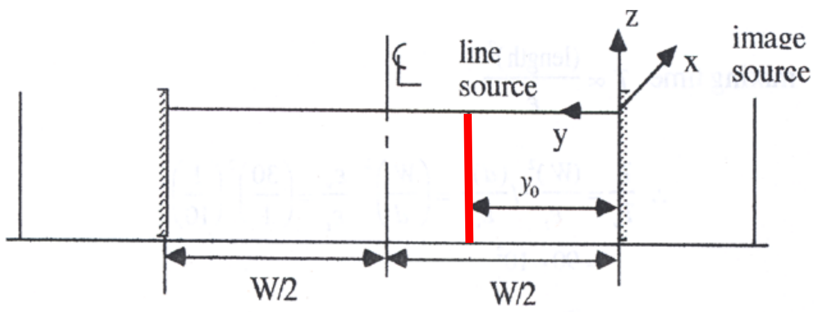
Define dimensionless quantities by setting

$$C_0 = \frac{\dot{M}}{\bar{u}dW} = \text{mass rate / volume of ambient water}$$

~ concentration after cross-sectional mixing is completed

$$x' = \frac{x\varepsilon_t}{\bar{u}W^2}$$

$$y' = y/W$$



Then Eq. (5.7) becomes

$$C = \frac{\frac{\dot{M}}{\bar{u}dW}}{\sqrt{\frac{4\pi\varepsilon_t x}{\bar{u}W^2}}} \exp\left(-\frac{(\frac{y}{W})^2}{\frac{4\varepsilon_t x}{\bar{u}W^2}} \right)$$

$$= \frac{C_0}{\sqrt{4\pi x'}} \exp\left(-\frac{y'^2}{4x'} \right)$$

$$\frac{C}{C_0} = \frac{1}{(4\pi x')^{1/2}} \exp\left(-\frac{y'^2}{4x'} \right)$$

If the source is located at $y = y_0 (y' = y'_0)$

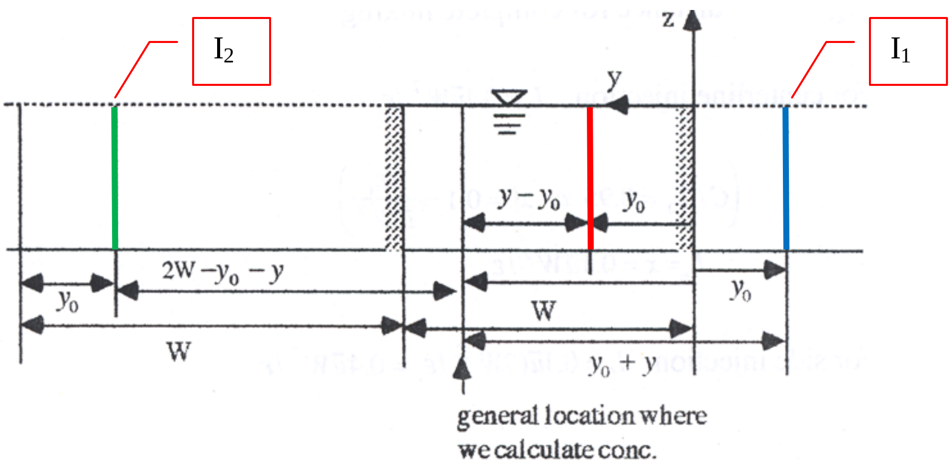
Consider real and image sources, then superposition gives the downstream concentration distribution as

real $\quad$ I_1 $\quad$ I_2

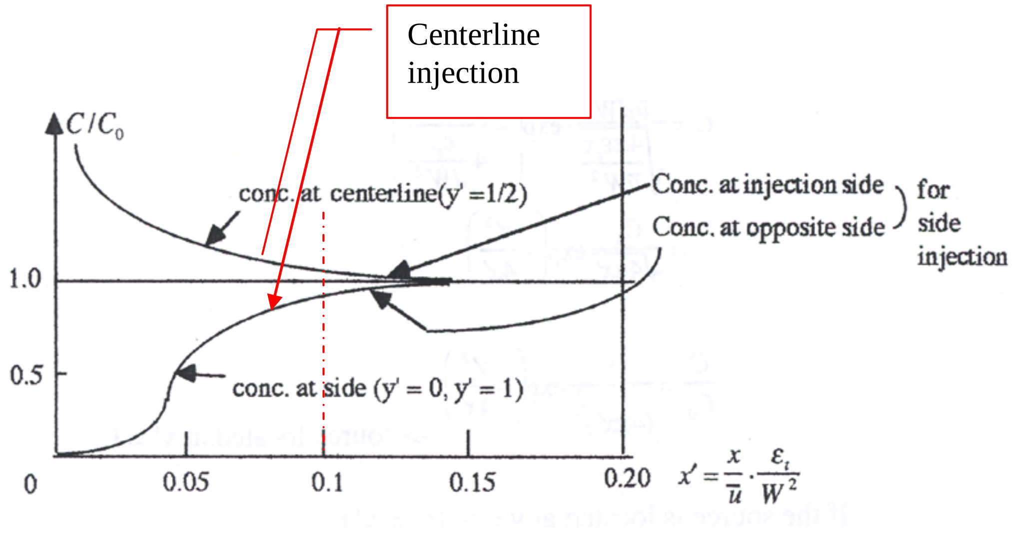
$$\frac{C}{C_0} = \frac{1}{(4\pi x')^{\frac{1}{2}}}\left[\exp\left(-\left\{\frac{(y' - y'_0)^2}{4x'}\right\}\right) + \exp\left(-\left\{\frac{(y' + y'_0)^2}{4x'}\right\}\right) + \exp\left(-\left\{\frac{(y' - 2 + y'_0)^2}{4x'}\right\}\right) + \bullet\bullet\bullet\right]$$

$$= \frac{1}{(4\pi x')^{\frac{1}{2}}} \sum_{n=-\infty}^{\infty} \left\{\exp\left[-(y' - 2n + y'_0)^2 / 4x'\right] + \exp\left[-\left(y' - 2n + y'_0\right)^2 / 4x'\right]\right\} \quad (5.9)$$

Sum for $n = 0, \pm 1, \pm 2$



Continuous centerline discharge: $y'_0 = 1/2$



From this figure, for *x'* greater than about 0.1 the concentration is within 5 % of its mean value everywhere on the cross section.

Thus, the longitudinal distance for complete transverse mixing for centerline injection is

$$\boxed{L_c = 0.1\bar{u}W^2/\varepsilon_t} \tag{5.10}$$

[Re] $\dfrac{C}{C_0} = 0.95\, at\, x' = 0.1 = \dfrac{x\varepsilon_t}{\bar{u}W^2}$

$$L_c = x = 0.1\bar{u}W^2/\varepsilon_t$$

For side injection, the width over which mixing must take place is twice that for a centerline injection

$$\boxed{L = 0.1\bar{u}(2W)^2/\varepsilon_t = 0.4\bar{u}W^2/\varepsilon_t} \tag{5.10a}$$

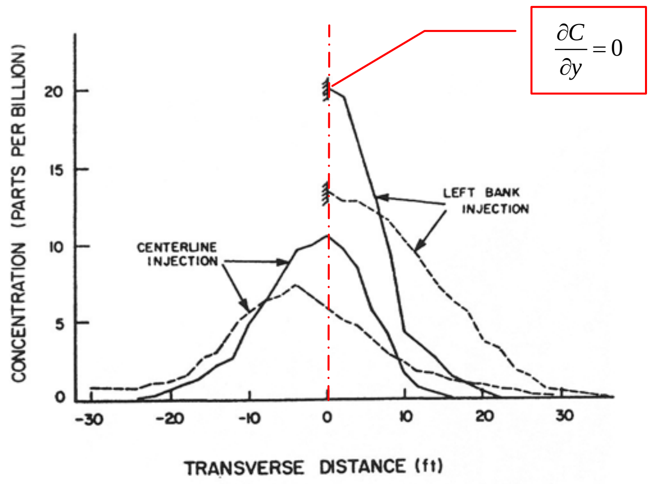


Figure 5.7 Observed concentrations downstream from centerline and side injections of dye in Fischer's (1967b) experiment. ——— 400 ft downstream from injection, --- 1000 ft downstream from injection. Transverse distance is measured from the centerline for the centerline injection and from a point 1 ft from the left bank for the left bank injection.

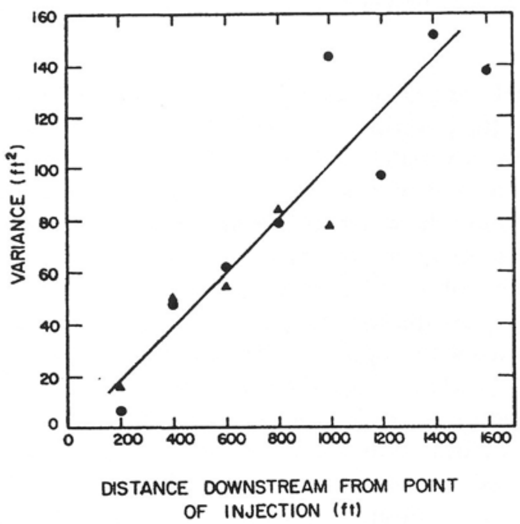


Figure 5.8 Variances of the concentration distributions shown in Fig. 5.7 (and others not shown) versus distance downstream from point of injection. ● Side injection. ▲ Centerline injection.

[Ex 5.1] Spread of a plume from a point source

An industry discharges effluent;

C = 200ppm

$$Q = 3MGPD = \frac{3\times 10^6 GPD}{7.48G/ft^3 \times 24 \times 3600} = 4.64\,CFS$$

Thus, rate of mass input is

$$\dot{M} = QC = 4.64(200ppm) = 928\ CFSppm$$

Centerline injection in very wide, slowly meandering stream

$$d = 30\,ft;\quad \bar{u} = 2\,fps;\quad u^* = 0.2\,fps$$

Determine the width of the plume, and maximum concentration 1000 ft downstream from discharge assuming that the effluent is completely mixed over the vertical.

[Sol]

For meandering stream,

$$\varepsilon_t = 0.6du^* = 0.6(30)(0.2) = 3.6\,ft^2/s$$

Use Eq.(5.7) for line source

Peak concentration

Exponential decay

$$C = \frac{\dot{M}}{\bar{u}d\left(\frac{4\pi\varepsilon_t x}{\bar{u}}\right)^{\frac{1}{2}}}\exp\left(-\frac{y^2\bar{u}}{4\varepsilon_t x}\right) \tag{5.7}$$

Compare with normal distribution; $C = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{y^2}{2\sigma^2}\right)$

$$\exp\left(-\frac{y^2}{\frac{4\varepsilon_t x}{\bar{u}}}\right) = \exp\left(-\frac{y^2}{2\sigma^2}\right)$$

$$\sigma^2 = \frac{2\varepsilon_t x}{\bar{u}}$$

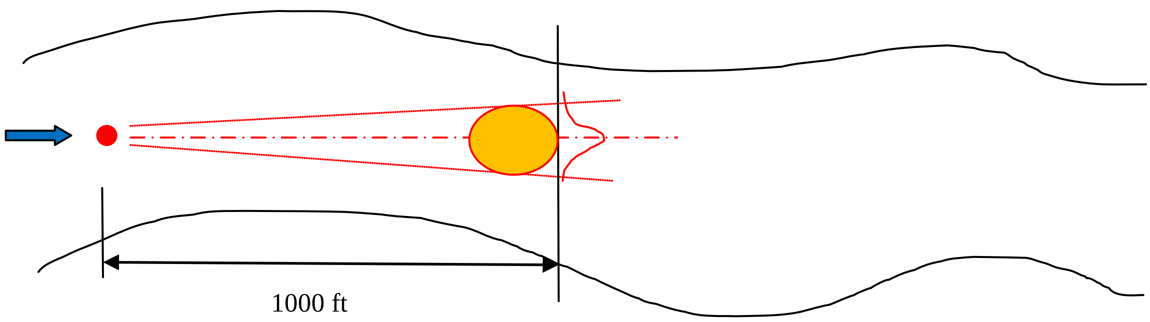
$$\sigma = \sqrt{\frac{2\varepsilon_t x}{\bar{u}}}$$

a) width of plume can be approximate by 4σ (includes 95% of total mass)

$$b = 4\sigma = 4\sqrt{\frac{2\varepsilon_t x}{\bar{u}}} = 4\sqrt{\frac{2(3.6)(1000)}{2}} = 240\ ft$$

b) maximum concentration

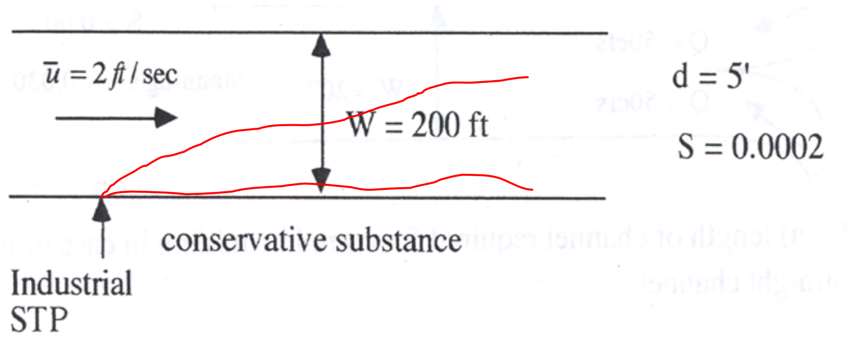
$$C_{\max} = \frac{\dot{M}}{\bar{u}d\left(\frac{4\pi\varepsilon_t x}{\bar{u}}\right)^{\frac{1}{2}}} = \frac{928 CFSppm}{(2\ fps)(30\ ft)\left(\frac{4\pi \times 3.6\ ft^2/s \times 1000\ ft}{2\ ft/s}\right)^{\frac{1}{2}}}$$



[Ex 5.2] Mixing across a stream

→ consider boundary effect

Given:



Find: length of channel required for "complete mixing" as defined to mean that the concentration of the substance varies by no more than 5% over the cross section

[Sol]

Shear velocity

$$u^* = \sqrt{gdS} = \sqrt{32.2(5)(0.0002)} = 0.18\, ft/s$$

For uniform, straight channel

$$\varepsilon_t = 0.15du^* = 0.15(5)(0.18) = 0.135\, ft^3/s$$

For complete mixing from a <u>side discharge</u>

Very long distance for a real channel

$$L = 0.4\bar{u}W^2 / \varepsilon_t$$

$$= 0.4(2)(200)^2 / 0.135 = 237{,}000\, ft \approx 45 mile \approx 72 km$$

[Ex 5.3] Blending of two streams

Compute the mixing of two streams which flow together at a smooth junction so that the streams flow side by side until turbulence accomplishes the mixing.

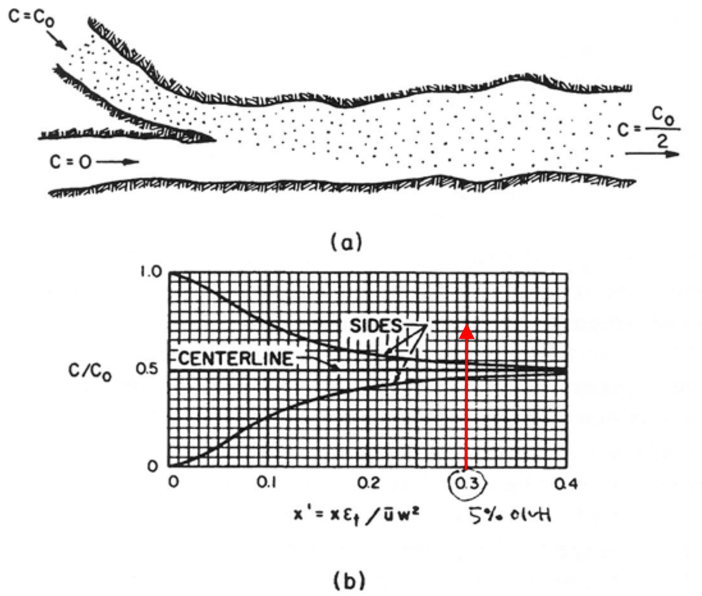


Figure 5.9 (a) Blending of two streams of equal discharge and (b) concentrations on the centerline and at each side downstream of the junction point, as given by Eq. (5.12).

Given:

$$Q = 50\, ft^3 / s; W = 20\, ft; S = 0.001; n = 0.030$$

Find:

a) length of channel required for complete mixing for uniform <u>straight channel</u>

b) length of channel required for complete mixing for <u>curved channel</u> with a radius of 100 ft

[Sol]

The velocity and depth of flow can be found by solving Manning's formula

$$\bar{u} = \frac{1.49}{n} R^{\frac{2}{3}} S^{\frac{1}{2}}$$

R = hydraulic radius = A/P

$$Q = A\bar{u} = \frac{1.49}{n} A R^{2/3} S^{1/2} = \frac{1.49}{n} \frac{A^{5/3}}{P^{2/3}} S^{1/2}$$

$$100 = \frac{1.49}{0.030} \frac{\left(20d\right)^{5/3}}{\left(20+2d\right)^{2/3}} \left(0.001\right)^{1/2} = 145.41 \frac{d^{5/3}}{\left(10+d\right)^{2/3}}$$

$$d^{5/3} = 0.688\left(10+d\right)^{2/3}$$

$$d = 0.799\left(10+d\right)^{2/5}$$

By trial-error method, $d = 2.2\ ft$

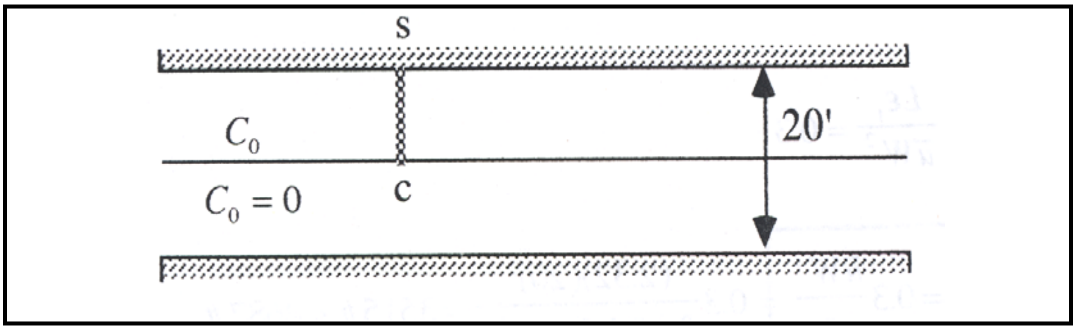
$$R = \frac{2.2\left(20\right)}{\left(20+4.4\right)} = 1.8\ \text{ ft}$$

$$\bar{u} = \frac{1.49}{0.030}\left(\frac{2.2 \times 20}{20+4.4}\right)^{2/3} \left(0.001\right)^{1/2} = 2.32\ ft/s$$

$$\therefore u^* = \sqrt{gRS} = \sqrt{32.2\left(1.8\right)\left(0.001\right)} = 0.24\ fps$$

$$\varepsilon_t = 0.15 d u^* = 0.15(2.2)(0.24) = 0.079\ \text{ft}^2/\text{s}$$

For the case of blending of two streams, there is a tracer whose concentration is C_0 in one stream and zero in the other.

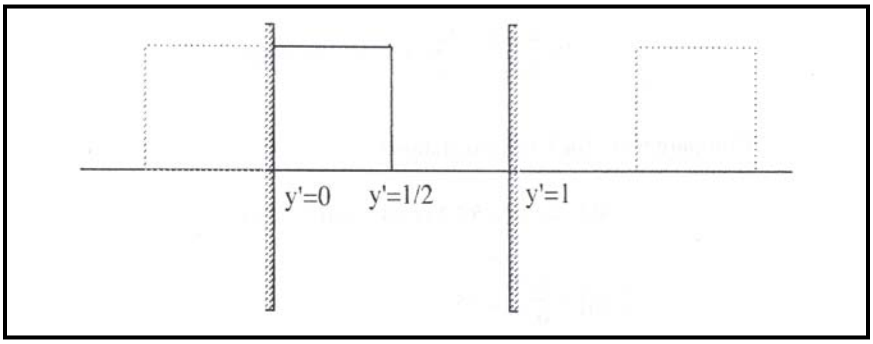


If the steams were mixed completely the concentration would be 1/2 C_0 everywhere on the cross section.

The initial condition may be considered to consist of a uniform distribution of unit inputs in one-half of the channel.

→ The exact solution can be obtained by superposition of solutions for the step function in an unbounded system [Eq. (2.33)].

Consider sources ranging $y'_0 = 0 \sim 1/2$



Method of images gives

$$\frac{C}{C_0} = \frac{1}{2}\sum_{n=-\infty}^{\infty}\left(erf\,\frac{y' + 1/2 + 2n}{\sqrt{4x'}} - erf\,\frac{y' - 1/2 + 2n}{\sqrt{4x'}} \right)$$

where $y' = y/W; x' = \dfrac{x\varepsilon_t}{\overline{u}W^2}$

From Fig. 5.9, maximum deviation in concentration is 5% of the mean when $x' \approx 0.3$.

$$x' = \frac{L\varepsilon_t}{\overline{u}W^2} = 0.3$$

$$L = 0.3\frac{\overline{u}W^2}{\varepsilon_t} = 0.3\frac{(2.32)(20)^2}{0.15(2.2)(0.24)} = 3515\, ft < 4687\, ft$$

[Re] For side injection only

$$L = 0.4\frac{\overline{u}W^2}{\varepsilon_t} = 0.4\frac{(2.32)(20)^2}{0.15(2.2)(0.24)} = 4687\, ft$$

For curved channel

$$\frac{\varepsilon_t}{du^*} = 25\left(\frac{\overline{u}}{u^*}\right)^2\left(\frac{d}{R}\right)^2$$

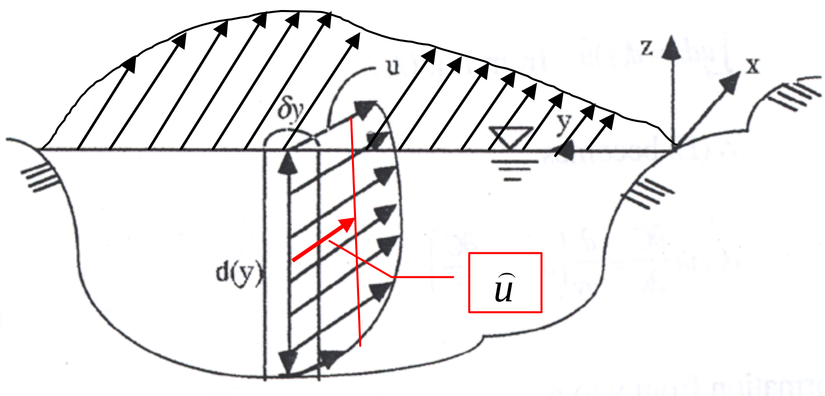
$$\therefore \varepsilon_t = 25\left(\frac{2.23}{0.24}\right)^2\left(\frac{2.2}{100}\right)^2(2.2)(0.24) = 0.60\, ft^2/s$$

$$L = 0.3\frac{\overline{u}W^2}{\varepsilon_t} = \frac{0.3(2.32)(20)^2}{0.60} = 464\, ft$$

5.1.4 Cumulative Discharge Method

Analysis was presented assuming a uniform flow of constant velocity everywhere in the channel.

However, in real rivers, the downstream velocity varies across the cross section, and there are irregularities along the channel.



Use cumulative discharge method by Yotsukura and Sayre (1976)

Define velocity averaged over depth at some value of *y* as

$$\hat{u} = \frac{1}{d(y)} \int_{-d(y)}^{0} u dz \qquad \text{(a)}$$

Then, <u>cumulative discharge</u> is given as

$$\boxed{q(y) = \int_0^y dq = \int_0^y d(y)\hat{u} dy} \qquad \text{(b)}$$

$$q(y) = 0 \quad at \quad y = 0 \qquad \text{(c)}$$

$$q(y) = Q \quad at \quad y = W$$

[Cf] $\bar{u}$ = cross-sectional average velocity

Now, derive depth-averaged 2D equation for transverse diffusion assuming steady-state concentration distribution and neglecting longitudinal mixing and *v*-velocity

$$\cancel{\frac{\partial C}{\partial t}}+u\frac{\partial C}{\partial x}+\cancel{v\frac{\partial C}{\partial y}}=\cancel{\frac{\partial}{\partial x}\left(\varepsilon_l\frac{\partial C}{\partial x}\right)}+\frac{\partial}{\partial y}\left(\varepsilon_t\frac{\partial C}{\partial y}\right) \qquad \text{(d)}$$

Integrate (d) over depth

$$\int_{-d}^{0}u\frac{\partial C}{\partial x}dz=\int_{-d}^{0}\frac{\partial}{\partial y}(\varepsilon_t\frac{\partial C}{\partial y})dz \qquad \text{(e)}$$

From Eq.(a)

$$\int_{-d}^{0}udz=d\left(y\right)\hat{u}$$

Eq. (e) becomes

$$d\left(y\right)\hat{u}\frac{\partial C}{\partial x}=\frac{\partial}{\partial y}\left(d\left(y\right)\varepsilon_t\frac{\partial C}{\partial y}\right)$$

$$\frac{\partial C}{\partial x}=\frac{1}{d\left(y\right)\hat{u}}\frac{\partial}{\partial y}\left(d\left(y\right)\varepsilon_t\frac{\partial C}{\partial y}\right) \qquad \text{(f)}$$

$$\boxed{\frac{\partial q}{\partial y}=\frac{\partial}{\partial y}\left[\int_0^y d(y)\hat{u}d\,y\right]=d(y)\hat{u}}$$

Transformation from *y* to *q* gives

$$\frac{\partial}{\partial y}=\frac{\partial q}{\partial y}\frac{\partial}{\partial q}=d\left(y\right)\hat{u}\frac{\partial}{\partial q} \qquad \text{(g)}$$

Substituting Eq. (g) into Eq.(f) yields

$$\frac{\partial C}{\partial x}=\frac{1}{\cancel{d\left(y\right)\hat{u}}}\cancel{d\left(y\right)\hat{u}}\frac{\partial}{\partial q}\left(d\left(y\right)\varepsilon_t\left(d\left(y\right)\hat{u}\frac{\partial C}{\partial q}\right)\right)=\frac{\partial}{\partial q}\left(d^2\left(y\right)\varepsilon_t\hat{u}\frac{\partial C}{\partial q}\right)$$

If we set $\varepsilon_q = d^2 \varepsilon_t \hat{u} \cong$ constant diffusivity, then equation becomes

$$\boxed{\frac{\partial C}{\partial x} = \varepsilon_q \frac{\partial^2 C}{\partial q^2}}$$

→ Fickian Diffusion equation; Gaussian solution in the *x*-*q* coordinate system

- Advantage of *x*-*q* coordinate system

\- A fixed value of q is attached to a <u>fixed streamline</u>, so that the coordinate system shifts back and forth within the cross section along with the flow.

→ simplifies interpretation of tracer measurements in meandering stream

→ Transformation from transverse distance to cumulative discharge as the independent variable essentially <u>transforms meandering river into an equivalent straight river</u>.

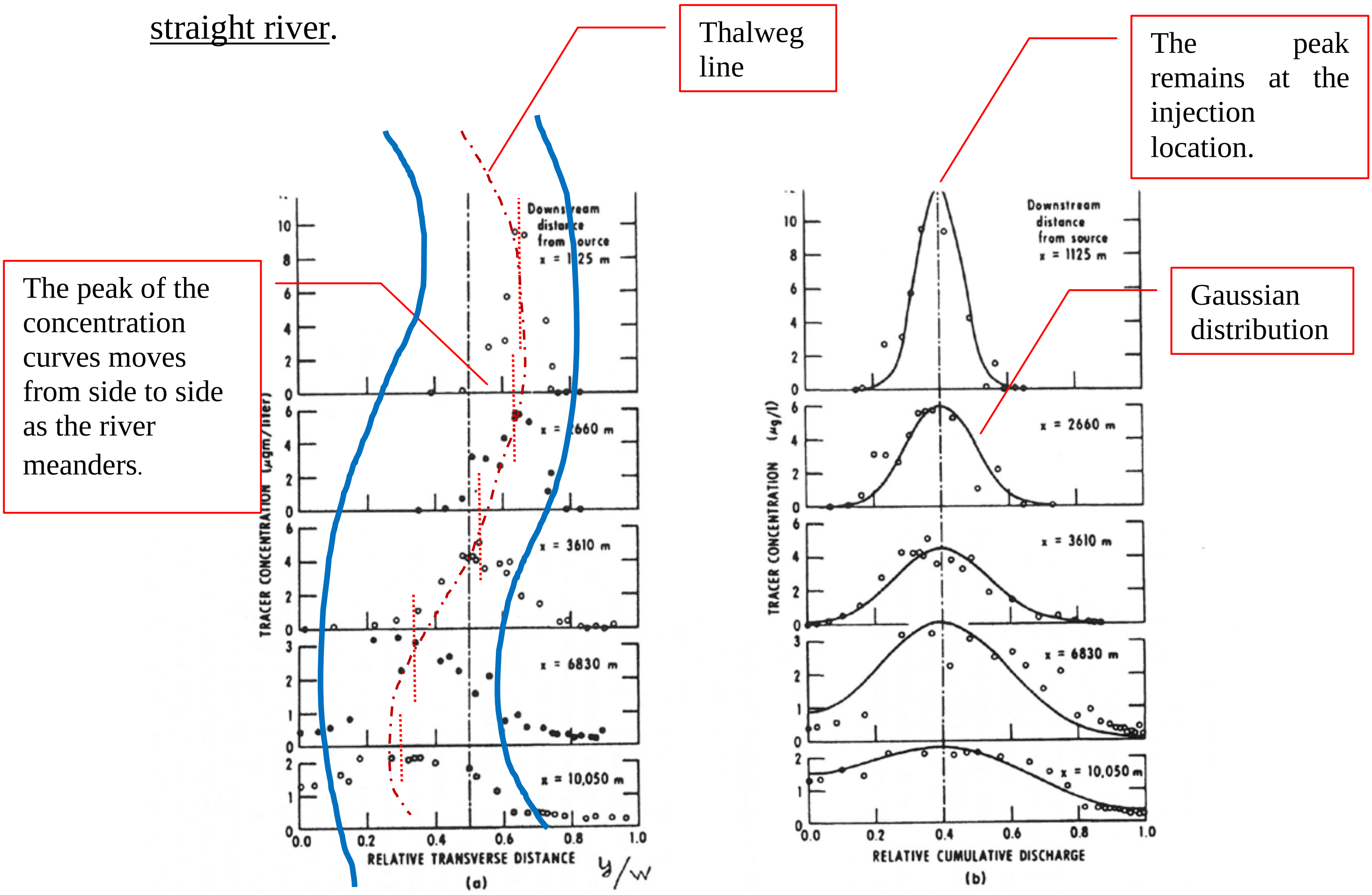


Figure 5.10 Transverse distributions of dye observed in the Missouri River near Blair, Nebraska, by Yotsukura *et al.* (1970), plotted (a) versus actual distance across the stream and (b) versus relative cumulative discharge. [After Yotsukura and Sayre (1976)].

5.2 Longitudinal Dispersion in Rivers

After a tracer has mixed across the cross section, the final stage in the mixing process is the reduction of longitudinal gradients by longitudinal dispersion.

The longitudinal dispersion may be neglected when effluent is discharged at a constant rate $\rightarrow$ Streeter-Phelps equation for BOD-DO analysis

There are, however, practical cases where longitudinal dispersion is important.

$\rightarrow$ accidental spill of a quantity of pollutant

output from a STP which has a daily cyclic variation

· 1D dispersion equation

$$\frac{\partial \overline{C}}{\partial t} + \overline{u}\frac{\partial \overline{C}}{\partial x} = K\frac{\partial^2 \overline{C}}{\partial x^2}$$

$\rightarrow$ apply shear flow dispersion theory to evaluate the longitudinal dispersion coefficient K

5.2.1 Theoretical Derivation of Longitudinal Dispersion Coefficient

Elder's analysis

- dispersion due to vertical variation of u-velocity (logarithmic profile)

$$u(z) = \overline{u} + \frac{u^*}{\kappa}\left\{1 + \ln\left[z + d / d\right]\right\}$$

$$\boxed{K = 5.93 d u^*}$$

Elder's equation does not describe longitudinal dispersion in real streams (1D model).

Experimental results shows $K >> 5.93du^{*} \rightarrow$ Table 5.3

1) Fischer (1967) - Laboratory channel

$$\frac{K}{du^{*}} = 150 \sim 392$$

2) Fischer (1968) - Green-Duwamish River

$$\frac{K}{du^{*}} = 120 \sim 160$$

3) Godfrey and Frederick (1970)

– natural streams in which radioactive tracer Gold-198 was used

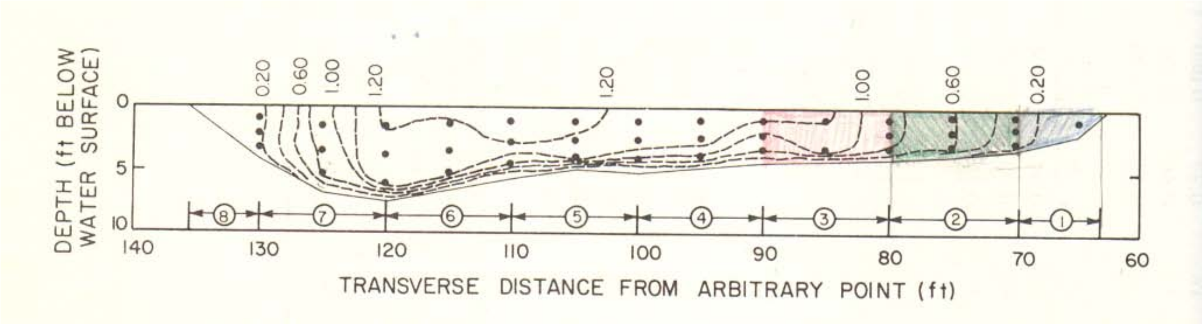
$$\frac{K}{du^{*}} = 140 \sim 500$$

4) Yotsukura et al. (1970) - Missouri river

$$\frac{K}{du^{*}} = 7500$$

Fischer's model (1966, 1967)

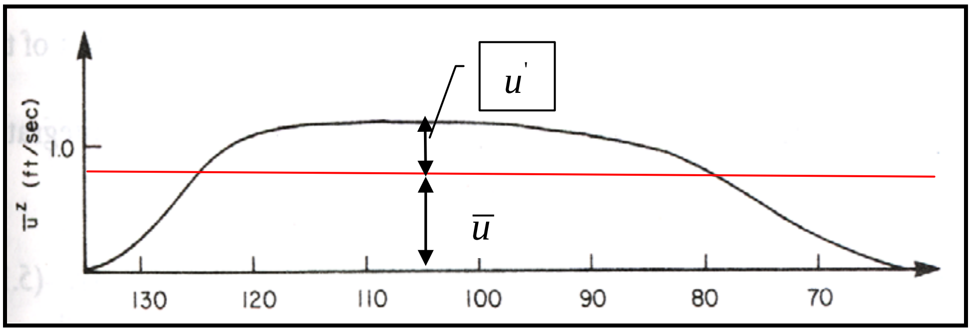
He show that the reason that Elder's result does not apply to 1D model is because of <u>transverse variation</u> of across the stream.



Vertical velocity profile, $u(z)$ is approximately logarithmic.

Now, consider transverse variation of depth-averaged velocity

$$\hat{u}(y) = \frac{1}{d(y)} \int_{-d(y)}^{0} u(y,z)\,dz$$



Transverse velocity profile would be approximated by parabolic, polynomial, or beta function.

3. Shear Flow Dispersion

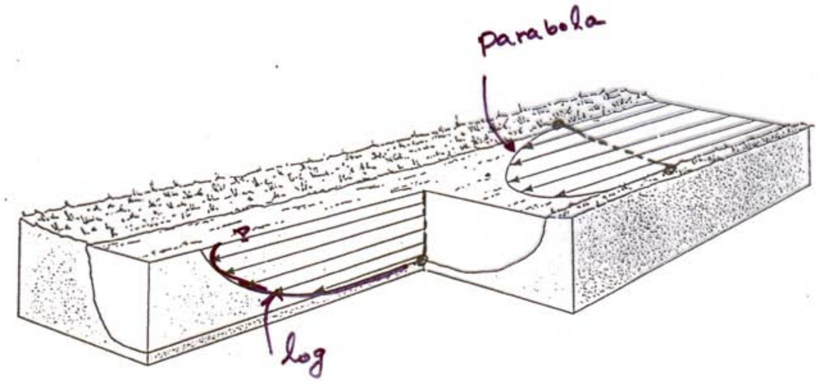
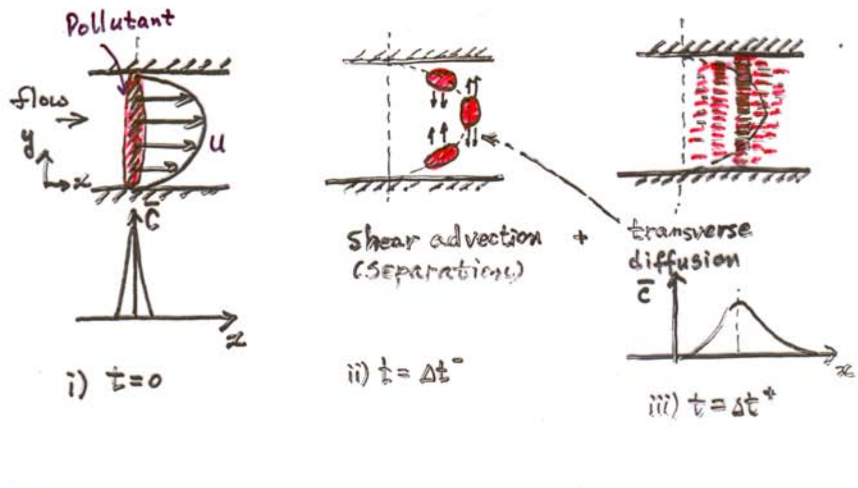


Figure 10.5
Variations in the velocity of flow in natural stream channels occur both horizontally and vertically. Friction reduces the velocity along the floor and sides of the channels. The maximum velocity in a straight channel is near the top and center of the channel.



$\hat{u}(y)$ is a shear flow velocity profile extending over the stream width *W*, whereas $u(z)$, the profile used in Elder's analysis, extends only over the depth of flow *d*.

Remember that longitudinal dispersion coefficient is proportional to the square of the distance over which the shear flow profile extends.

Eq. (4.26): $$K = \frac{h^2 \overline{u'^2}}{E} I$$

$$K \propto h^2$$

where *h* = characteristic length, *W* or *d*

Say that $W/d \approx 10$

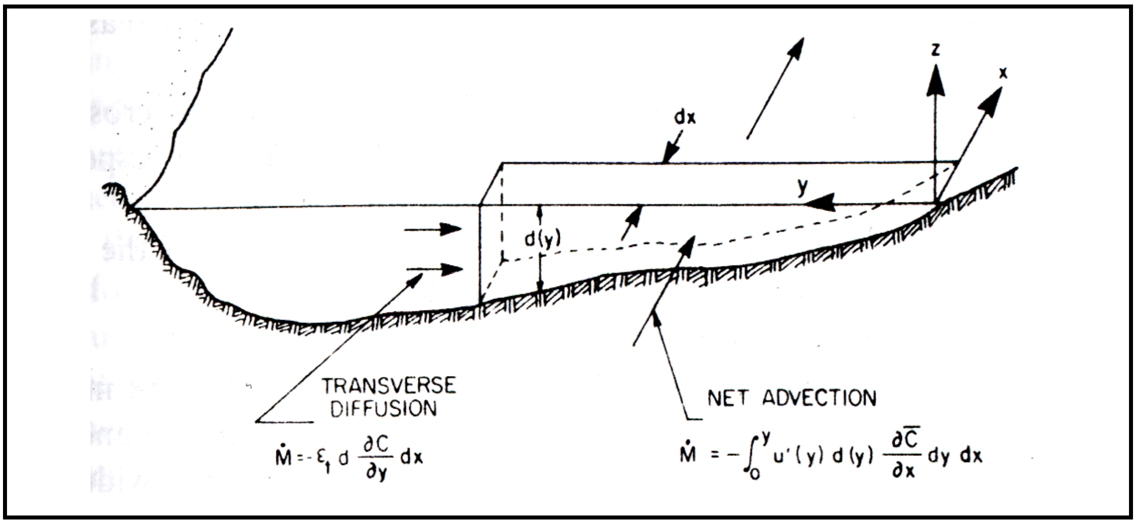
Therefore,

$$\boxed{K_W \approx 100 K_d}$$

→ Transverse profile *u(y)* is 100 or more times as important in producing longitudinal dispersion as the vertical profile.

→ The dispersion coefficient in a real stream (1D model) should be obtained by neglecting the vertical profile entirely and applying Taylor's analysis to the transverse velocity profile.

Balance of diffusion and advection



Let $u'(y) = \hat{u}(y) - \bar{u}$

$C'(y) = \hat{C}(y) - \bar{C}$

$\bar{u}$ = cross-sectional average velocity = U

Equivalent of Eq. (4.35) is

$$u'(y)\frac{\partial \bar{C}}{\partial x} = \frac{\partial}{\partial y}\varepsilon_t \frac{\partial C'}{\partial y} \qquad \text{(a)}$$

Shear advection

Transverse diffusion

Integrate Eq. (a) over the depth

$$\int_{-d}^{0} u'(y)\frac{\partial \bar{C}}{\partial x} dz = \int_{-d}^{0} \frac{\partial}{\partial y}\varepsilon_t \frac{\partial C'}{\partial y} dz \qquad \text{(b)}$$

$$u'(y)\, d(y)\frac{\partial \bar{C}}{\partial x} = \frac{\partial}{\partial y} d(y)\,\varepsilon_t \frac{\partial C'}{\partial y} \qquad \text{(c)}$$

Integrate Eq. (c) w.r.t. y

$$\int_0^y u^{'}(y)\,d(y)\frac{\partial \bar{C}}{\partial x}dy = d\varepsilon_t\frac{\partial C^{'}}{\partial y} \qquad (5.15)$$

$$\frac{\partial C^{'}}{\partial y} = \frac{1}{d\varepsilon_t}\int_0^y u^{'}(y)\,d(y)\frac{\partial \bar{C}}{\partial x}dy \qquad \text{(d)}$$

Integrate Eq. (d) w.r.t. y

$$C^{'} = \int_0^y \frac{1}{d\varepsilon_t}\int_0^y u^{'}(y)\,d(y)\frac{\partial \bar{C}}{\partial x}dydy \qquad \text{(e)}$$

Eq. (4.27)

$$K = -\frac{1}{A\frac{\partial \bar{C}}{\partial x}}\int_A u^{'}C^{'}dA \qquad \text{(f)}$$

Substitute Eq. (e) into Eq. (f)

$$K = -\frac{1}{A}\frac{1}{\cancel{\frac{\partial \bar{C}}{\partial x}}}\cancel{\frac{\partial \bar{C}}{\partial x}}\int_A u^{'}\int\frac{1}{d\varepsilon_t}\int du^{'}dydydA$$

Substitute $dA = dy\,d$

$$\boxed{K = -\frac{1}{A}\int_0^W u^{'}d\int_0^y \frac{1}{\varepsilon_t d}\int_0^y u^{'}d\;dydydy} \qquad (5.16)$$

This result is only an estimate because it is based on the concept of a <u>uniform flow in a constant cross section</u>.

[Re] $K1_l + K2_l + \varepsilon_\ell \rightarrow K$

where $K1_l$ ~ due to lateral variation of *u*-velocity;

$K2_l$ ~ due to vertical variation of *u*-velocity

- Simplified equation

Let $d' = d/\overline{d}$; $u'' = \dfrac{u'}{\sqrt{\overline{\overline{u'^2}}}}$; $\varepsilon'_t = \dfrac{\varepsilon_t}{\overline{\overline{\varepsilon_t}}}$; $y' = \dfrac{y}{W}$

Overbars mean cross-sectional average; $\overline{d}$ = cross-sectional average depth

Then

$$\boxed{K = \frac{W^2 \overline{u'^2}}{\overline{\varepsilon_t}} I} \tag{5.17}$$

where I is dimensionless integral given as

$$I = -\int_0^1 u'' d' \int_0^{y'} \frac{1}{\varepsilon'_t d'} \int_0^{y'} u'' dy' dy' dy'$$

Compare with Eq. (4.26)

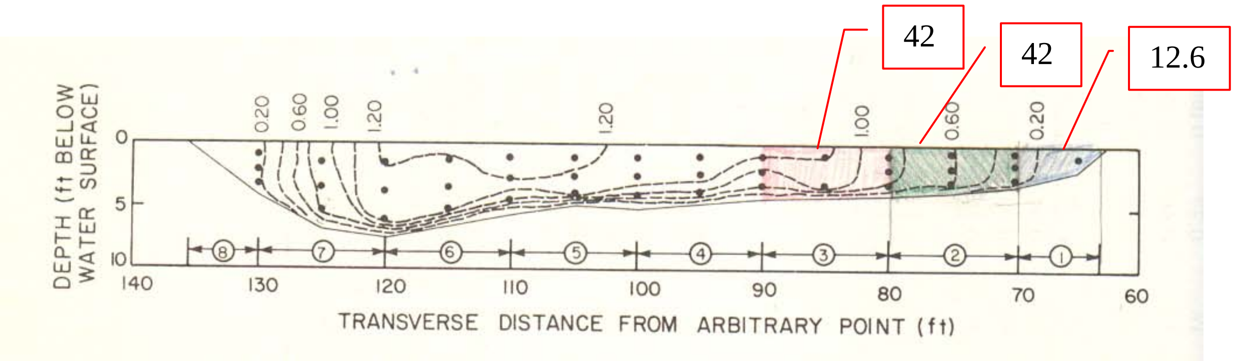
$$K = \frac{h^2 \overline{u'^2}}{E} I$$

[Example 5.4] cross-sectional distribution of velocity (Fig.5.11) of Green-Duwamish at Renton Junction

$$\varepsilon_t = 0.133\, ft^2 / \sec$$

Estimate longitudinal dispersion coefficient

Solution: divide whole cross section into 8 subarea



$$K = -\frac{1}{A}\int_0^W u' d \int_0^y \frac{1}{\varepsilon_t d}\int_0^y du' dy dy dy$$

→ perform <u>inner integral first</u>

Column 2: transverse distance to the end of subarea

Column 4: $\Delta A = d\,\Delta y$

Column 46: $\Delta Q = \hat{u}\,\Delta A$

Column 8: $Relative\,\Delta Q = u'\,\Delta A$

Column 9: Cumulative of $Relative\,\Delta Q = u'\,\Delta A$

Column 11: $\int_0^y \frac{1}{\varepsilon_t d}\int_0^y du' dy dy = \sum Col(10)\frac{\Delta y}{\varepsilon_t d}$

Column 13: $\int_0^W u' d \int_0^y \frac{1}{\varepsilon_t d}\int_0^y du' dy dy dy = Col(8) \times Col(12)$

$$K = -\frac{1}{A} Cumulative\, of\;\, Col(13)$$

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	(1)	(2)	(3)	(4)	(5) $\hat{u}$ (측정치)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
2								Rel. ΔQ =						
3			측정치	ΔA =	Subarea mean	ΔQ=	u' =	u' * ΔA,	$\int_0^y u' dA$	Average	$\int_0^y \frac{1}{E_t d}\int_0^y du' dy dy$	Average		=(13)의 누가치
4		y,	d,	d*Δy,	Velocity, $\hat{u}$	$\hat{u}$*ΔA,	$\hat{u}$ - ū,	CFS	(8)을 누가한 값	of	(누가치)	of		
5	Subareas	ft	ft	ft2	ft/s	CFS	fps	(4)*(7)		(9)		(11)	(8)*(12)	Σ(13)
6		63							0		0			0
7	1		1.8	12.6	0.105	1.323	-0.796	-10.026		-5.013		-73	735	
8		70		=1.8(7)		=0.105(12.6)			-10.026		-147			735
9	2		4.2	42	0.526	22.092	-0.375	-15.738		-17.895		-307	4828	
10		80							-25.764		-467			5563
11	3		4.2	42	0.986	41.412	0.085	3.582		-23.973		-682	-2441	
12		30							-22.182		-896			3121
13	4		4.8	48	1.091	52.368	0.190	9.134		-17.616		-1034	-9445	
14		100							-13.049		-1172			-6323
15	5		5.2	52	1.196	62.192	0.295	15.355		-5.371		-1211	-18593	
16		110							2.306		-1250			-24916
17	6		6.6	66	1.148	75.768	0.247	16.321		10.466		-1190	-19423	
18		120							18.627		-1130			-44339
19	7		6.4	64	0.766	49.024	-0.135	-8.622		14.316		-1046	9022	
20		130							10.005		-962			-35317
21	8		2	12	0.067	0.804	-0.834	-10.005		5.002		-906	9063	
22		136							0.000		-849			-26254
23														
24	Sum=		A =	338.6	Q =	304.983	0	0.000						
25														
26	(6-1) ū =		Q/A=	0.90	FPS			K = -	(-26254)/A =	77.54	ft²/s			
27														
28		E_t =	0.133	ft²/s										

(5) given in P.128

$$K = -\frac{1}{A}\int_0^W u'd \underbrace{\int_0^y \frac{1}{E_t d} \underbrace{\int_0^y du'\, dy}_{(9)}\, dy}_{(11)}\, dy \quad (14)$$

(5.16) → Inner integral first

(11): $(-5.013)(7)/(0.133)(1.8) = -146.6$

$-146.6 + (-17.895\times 10)/(0.133\times 4.2) = -467.0$

(9) $\int_0^y du'dy = \Sigma du'\Delta y = \Sigma u'\Delta A$ ($\because$ $d\Delta y = \Delta A$)

(11) $\int_0^y \frac{1}{E_t d}\int_0^y du'\,dy\,dy = \Sigma \int_0^y du'dy\,\frac{\Delta y}{E_t d} = \Sigma(10) \times \Delta y / E_t d$

(14) $\Sigma\, \underbrace{u'\underbrace{d\Delta y}_{\Delta A}}_{\text{Rel. }\Delta Q = (8)} \underbrace{\left[\int \frac{1}{E_t d}\int du' dy\, dy\right]}_{(12)} = \Sigma(8) \times (12)$

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
subarea	y (ft)	d (측정치) (ft)	$\Delta A = d \times \Delta y$ (ft^2)	$\hat{u}$ Stream mean velocity (측정치) (ft/s)	$\Delta Q = \hat{u} \times \Delta A$ (CFS)	$u' = \hat{u} - \bar{u}$ (fps)	Rel. $\Delta Q = \hat{u} \times \Delta A$ (CFS) (4)*(7)	$\int_0^y u'\,dA$ (8)을 누가한 값	Average of (9)
	63							0	
1		1.8	12.6	0.105	1.323	-0.796	-10.026		-5.013
	70		=1.8(7)		=0.105(12.6)			-10.026	
2		4.2	42	0.526	22.092	-0.375	-15.738		-17.895
	80							-25.764	
3		4.2	42	0.986	41.412	0.085	3.582		-23.973
	90							-22.182	
4		4.8	48	1.091	52.368	0.190	9.134		-17.616
	100							-13.049	
5		5.2	52	1.196	62.192	0.295	15.355		-5.371
	110							2.306	
6		6.6	66	1.148	75.768	0.247	16.321		10.466
	120							18.627	
7		6.4	64	0.766	49.024	-0.135	-8.622		14.316
	130							10.005	
8		2	12	0.067	0.804	-0.834	-10.005		5.002
	136							0.000	
Sum		$A =$	338.6	$Q =$	304.98		0.000		
		$\varepsilon_t =$	0.133 ft^2/s	$\bar{u} = Q / A =$	0.90 fps		$K =$	-(-26254)/A = 77.54 ft^2/s	

(5) given in p.128

(9) $\int_0^y du'\,dy = \sum du'\Delta y = \sum u'\Delta A \quad (\because d\Delta y = \Delta A)$

(5.16) : Inner integral first

$$\int_0^y \frac{1}{\varepsilon_t d}\int_0^y du'\,dy\,dy = \sum \int_0^y du'\,dy \frac{\Delta y}{\varepsilon_t d} = \sum (10) \times \Delta y / \varepsilon_t d$$

$$(-5.013)(7)/(0.133)(1.8) = -146.6$$

$$K = -\frac{1}{A}\int_0^W \underbrace{u'd \underbrace{\int_0^y \frac{1}{\varepsilon_t d} \underbrace{\int_0^y du'\,dy}_{(9)}\,dy}_{(11)}\,dy}_{(14)}$$

(11)

(11):

(14) $\sum \underbrace{u' \underbrace{d\Delta y}_{\Delta A}}_{\text{Rel. } \Delta Q = (8)} \underbrace{\left[\int_0^y \frac{1}{\varepsilon_t d}\int_0^y du'\,dy\,dy\right]}_{(12)} = \sum (8) \times (12)$

$$-146.6 + (-17.895)(7)/(0.133 \times 4.2) = -467.0$$

Homework Assignment #5-1

Due: Two weeks from today

1. Estimate the longitudinal dispersion coefficient using the cross-sectional distribution of velocity measured in the field using Eq. (5.16). Take S (channel slope) = 0.00025 for natural streams.
2. Compare this result with Elder's analysis and Fischer's approximate formula, Eq. (5.19).

Table 1 Cross-sectional Velocity Distribution at Ottawa in the Fox River, Illinois

Station	Y from left bank (ft)	Depth, d (ft)	Mean Velocity (ft/sec)
1	0.00	0.0	0.00
2	4.17	1.4	0.45
3	7.83	3.0	0.68
4	11.50	3.7	1.05
5	15.70	4.7	0.98
6	22.50	5.3	1.50
7	29.83	6.2	1.65
8	40.83	6.7	2.10
9	55.50	7.0	1.80
10	70.17	6.5	2.40
11	84.83	6.3	2.55
12	99.50	6.8	2.45
13	114.17	7.4	2.20
14	132.50	7.3	2.65

15	150.83	7.1	2.70
16	169.16	7.4	2.35
17	187.49	7.8	2.65
18	205.82	7.8	2.80
19	224.15	7.8	2.60
20	242.48	6.6	2.50
21	260.81	6.3	2.30
22	279.14	6.2	2.35
23	297.47	6.6	2.30
24	315.80	6.0	2.65
25	334.13	5.5	2.50
26	352.46	5.4	2.10
27	370.79	5.2	2.25
28	389.12	5.5	2.30
29	407.45	5.7	1.50
30	416.62	3.2	1.30
31	422.00	0.0	0.00

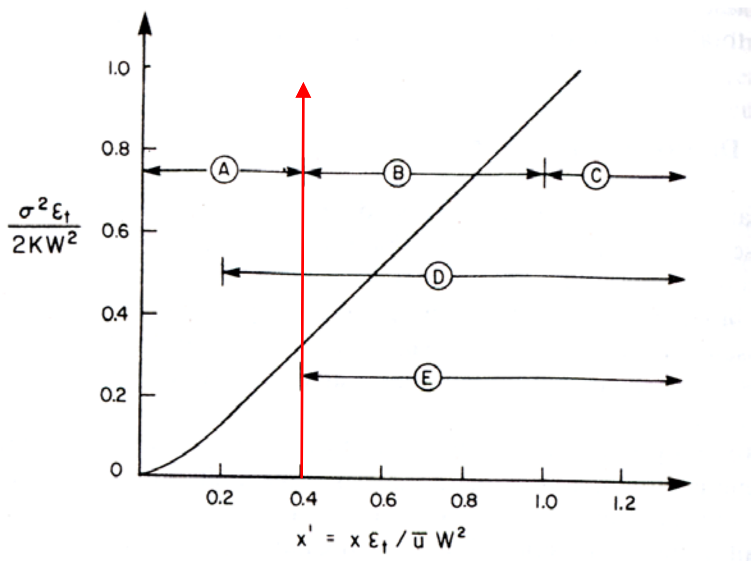
5.2.2 Dispersion in Real Streams

So far the analyses have been limited to uniform channels because Taylor's analysis assumes that everywhere along the stream the cross section is the same.

Real streams have bends, sandbars, side pockets, pools and riffles, bridge piers, man-made revetments.

→ Every irregularities contribute to dispersion.
→ It is not suitable to apply Taylor's analysis to real streams with these irregularities.

5.2.2. 1 Limitation of Taylor's analysis



Taylor's analysis cannot be applied until after the initial period.

Numerical experiments showed that in a uniform channel the variance of dispersing cloud behaves as a line as shown in Fig. 5.14.

A) generation of skewed distribution: $x^{'} (= \dfrac{x}{\bar{u}W^2 / \varepsilon_t}) < 0.4$ (initial period)

B) decay of the skewed distribution: $0.4 < x^{'} < 1.0$

C) approach to Gaussian distribution: $1.0 < x^{'}$

D) zone of linear growth of the variance: $0.2 < x^{'}$; $\dfrac{\partial \sigma^2}{\partial t} = 2D$

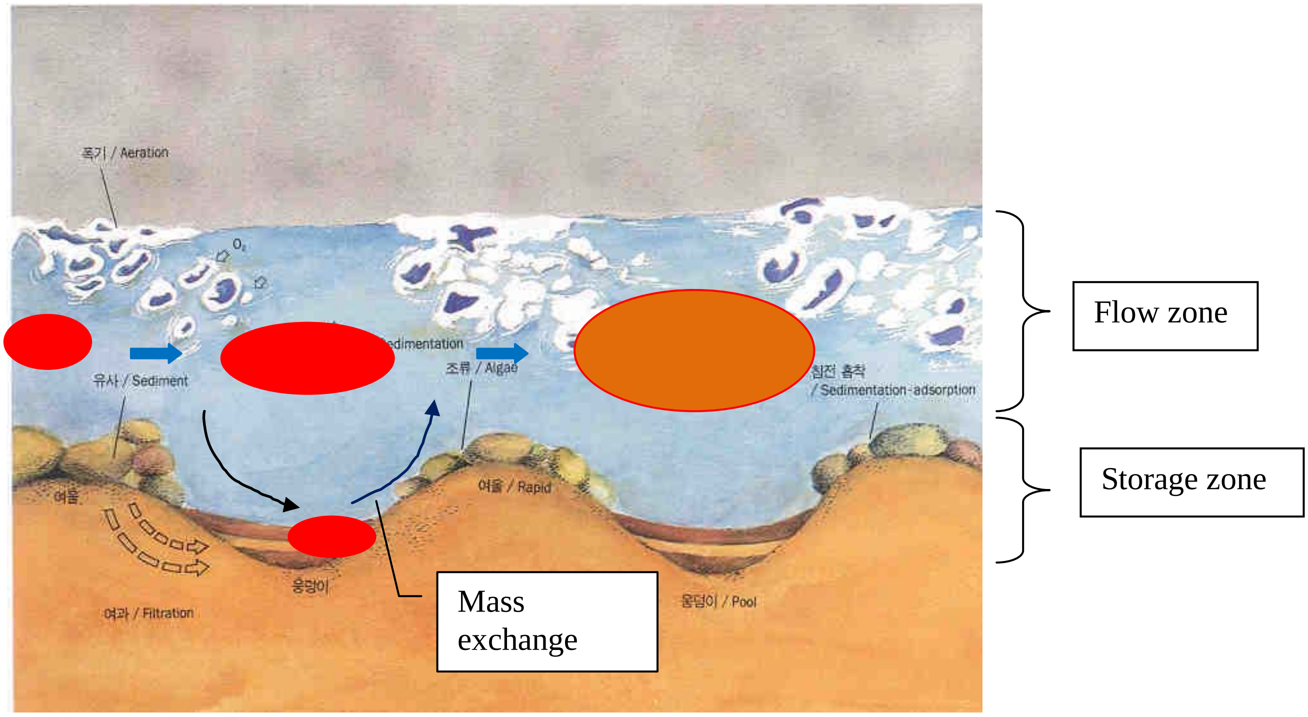
E) zone where use of the routing procedure is acceptable: $0.4 < x^{'}$

Analytical solution of 1D advection-dispersion model

5.2.2.2 Two-zone Models

Irregularities in real streams increase the length of the initial period, and produce long tail on the observed concentration distribution due to detention of small amounts of effluent cloud and release slowly after the main cloud has passed.

Pockets of dye are retained in small irregularities along the side of the channel. The dye is released slowly from these pockets, and causes measurable concentrations of dye to be observed after the main portion of the cloud has passed.


폭기 / Aeration
유사 / Sediment
조류 / Algae
침전 흡착
/ Sedimentation-adsorption
여울
여울 / Rapid
웅덩이
여과 / Filtration
웅덩이 / Pool
Mass exchange
Flow zone
Storage zone

· Field studies

Godfray and Frederick (1974)
Nordin and Savol (1974)
Day (1975)
Legrand-Marcq and Laudelot (1985)

showed <u>nonlinear behavior of variance</u> for times beyond the initial period. (increased faster than linearly with time)

$$\sigma^2 = f\left(t^{1.4}\right)$$

→ skewed concentration distribution
→ cannot apply Taylor's analysis

· Effect of storage zones (dead zones)

1) increases the length of the initial period

2) increases the magnitude of the longitudinal dispersion coefficient

• Two zone models
~ divide stream area into two zones

Flow zone: advection, dispersion, reaction, mass exchange

$$A_F \frac{\partial C_F}{\partial t} + U_F A_F \frac{\partial C_F}{\partial x} = \frac{\partial}{\partial x}\left(K A_F \frac{\partial C_F}{\partial y} \right) + F$$

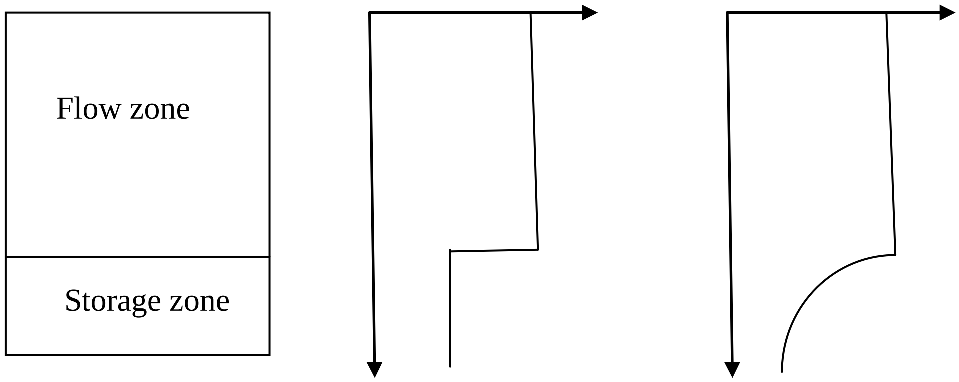
Storage zone: vortex, dispersion, reaction, mass exchange

$$A_S \frac{\partial C_S}{\partial t} = -F$$

Introduce auxiliary equation for mass exchange term *F*

Exchange model: $F = k(C_F - C_S)P$

Diffusion model: $F = -\varepsilon_y \frac{\partial C_S}{\partial y}\bigg|_{y=0}$



- Dead zone model

Hays et al (1967)

Valentine and Wood (1977, 1979), Valentine (1978)

Tsai and Holley (1979)

Bencala and Waters (1983), Jackman et al (1984)

▪ Storage zone model

Seo (1990), Seo and Maxwell (1991, 1992)
Seo and Yu (1993)
Seo & Cheong (2001), Cheong & Seo (2003)

· Effect of bends

1) Bends increase the rate of transverse mixing.

2) Transverse velocity profile induced by meandering flow increase longitudinal dispersion coefficient significantly because the velocity differences across the stream are accentuated.

(3) Effect of alternating series of bends depends on the ratio of the cross-sectional diffusion time to the time required for flow round the bend.

$$\gamma = \frac{W^2 / \varepsilon_t}{L / \bar{u}} \tag{5.18}$$

where L= length of the curve

$\gamma \le 25 = \gamma_0 \rightarrow K = K_0$ → no effect due to alternating direction

$$\gamma > 25 \rightarrow K = K_0 \frac{\gamma_0}{\gamma}$$

K_0 = dispersion coefficient for the steady-state concentration profile, Eq. (5.16)

5.2.3 Estimating and Using the Dispersion Coefficient

Observation – calculation of observed values from field data

Prediction – estimation of dispersion coefficient by theoretical or empirical equations

5.2.3.1 Observation of dispersion coefficient

1) Change of moment method

$$K = \frac{{\sigma_{x2}}^2 - {\sigma_{x1}}^2}{2(t_2 - t_1)} = U^2 \frac{{\sigma_{t2}}^2 - {\sigma_{t1}}^2}{2(\bar{t}_2 - \bar{t}_1)} \qquad (2.30a)$$

where σ_x^2 = variance of *C*-*x* curve;

σ_t^2 = variance of *C*-*t* curve;

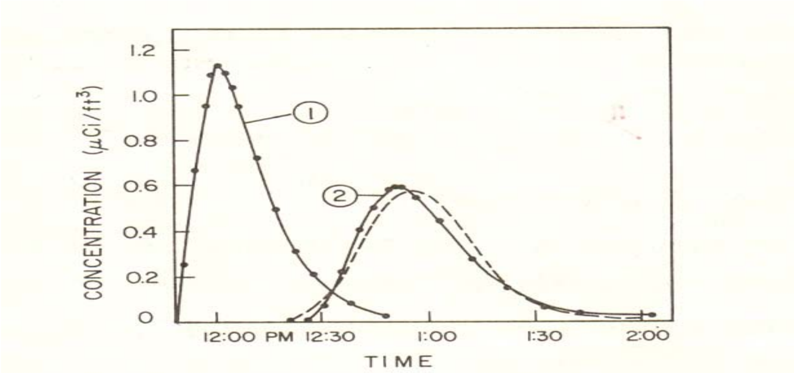
$\bar{t}_1$ = centroid of *C*-*t* curve at $x = x_1$

It is difficult to compute a meaningful value of variance when concentration distributions are skewed because of long tails on the observed distributions.

2) Routing procedure (Fischer, 1968)

→ match a downstream observation of passage of a tracer cloud to the prediction based on the upstream observation using an analytical solution.

We can use this procedure only when $x' > 0.4$



Predicted concentration distribution at downstream station is obtained according to the solution of one-dimensional Fickian dispersion model.

$$C^{p}(x_2,t)=\int_{-\infty}^{\infty} C(x_1,\tau)\frac{exp\left[-\dfrac{\bar{u}^2(\bar{t}_2-\bar{t}_1-t+\tau)^2}{4K(\bar{t}_2-\bar{t}_1)}\right]}{\sqrt{4\pi K(\bar{t}_2-\bar{t}_1)}}\,\bar{u}\,d\tau \qquad (5.20)$$

where

$\bar{t}_1$ = mean time of passage at the upstream station (x_1)

$\bar{t}_2$ = mean time of passage at downstream station (x_2)

τ = timelike variable of integration

$C(x_1,\tau)$ = upstream observed concentration-time curve

Compare $C^{p}(x_2,t)$ with $C(x_2,t)$ [= downstream observed concentration curve] until it fits together with varying dispersion coefficient *K*.

Then, the <u>best fit value</u> is regarded as the observed dispersion coefficient

5.2.3.2 Prediction of dispersion coefficient

1) Theoretical equation

$$K = -\frac{1}{A}\int_0^W u^{'} d \int_0^y \frac{1}{\varepsilon_t d}\int_0^y du^{'} dy dy dy \tag{5.16}$$

Seo and Baek (2004)

~ use beta function for transverse profile of *u*-velocity

$$\frac{u}{U} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\left(\frac{y}{W}\right)^{\alpha-1}(1-\frac{y}{W})^{\beta-1}$$

$$K = \gamma \frac{U^2 W^2}{du^*}$$

2) Empirical equation

· Fischer (1975)

$$K^{'} = \frac{\overline{Iu^{'2}h^2}}{E} \tag{4.26}$$

Select $I = 0.07(0.054 \sim 0.10)$

$h = 0.7W\left(0.5 \sim 1.0W\right)$

$\overline{u^{'2}} = 0.2\overline{u}^2(0.17 \sim 0.25)$

$$E = \varepsilon_t = 0.6du^*$$

Then (4.26) becomes

$$K = 0.01\frac{U^2W^2}{du^*} \tag{5.19}$$

· Seo and Cheong (1998)

Use dimensional analysis to find significant factors

Include dispersion by shear flow and dispersion by storage effects

$$\frac{K}{du^*} = a\left(\frac{U}{u^*}\right)^b \left(\frac{W}{d}\right)^c$$

Fischer (1975):	a=0.011; b=2.0; c=2.0
Liu (1979):	a=0.18; b=0.5; c=2.0
Iwasa and Aya (1991):	a=2.0; b=0; c=1.5
Koussis and Rodrguez-Mirasol (1998):	a=0.6; b=0; c=2.0
Seo and Cheong (1998):	a=5.92; b=1.43; c=0.62

[Ex 5.5]   Dispersion of slug (instantaneous input)

Given:

$M = 10lb$ (Rhodamine WT dye); $\overline{u} = 0.90\, ft/s$; $W = 73\, ft$; $A = 338.6$

$\overline{d} = 4.46\, ft$, (weighted average)

$\varepsilon_t = 0.133\, ft^2/s$

$$u^* = \frac{\varepsilon_t}{0.4d} = \frac{0.133}{0.4(4.64)} = 0.072\, ft/s$$

Find:

(a) K by Eq. (5.19)

(b) length of initial zone in which Taylor's analysis does not apply

(c) length of dye cloud at the time that peak passes =20,000 ft

(d) C_{peak} at x =20,000ft

[Solution]

(a) Eq. (5.19)

$$K = 0.011\overline{u}^2 W^2 / du^*$$

$$= 0.011(0.90)^2 (73)^2 / (4.46)(0.072)$$

$$= 142.1\, ft^2/s$$

$$K(5.19)/K(5.16) = 142.1/77.5 = 1.83$$

[Cf] K by Seo & Cheong (1998)

$$\frac{K}{du^*} = 5.92\left(\frac{U}{u^*}\right)^{1.43}\left(\frac{W}{d}\right)^{0.62} = 294\, ft^2/s$$

→ include dispersion by shear flow dispersion and <u>storage effects</u>

(b) initial period

$$x = 0.4\bar{u}W^2/\varepsilon_t = 0.4(0.90)(73)^2/(0.133) = 14,424\,ft$$

(c) length of cloud

$$x' = x\varepsilon_t/\bar{u}W^2 = \frac{(20,000)(0.133)}{(0.90)(73)^2} = 0.55$$

- decay of skewed concentration distribution

→ assume Gaussian distribution

$$\frac{d\sigma^2}{dt} = 2K$$

From Fig.5.14

$$\frac{\sigma^2\varepsilon_t}{2KW^2} = (x' - 0.07)$$

$$\sigma^2 = 2K(W^2/\varepsilon_t)(x' - 0.07)$$

$$= 2(142)(73)^2/0.133(0.55 - 0.07) = 5.46\times10^{-6}\ ft^2$$

$$\therefore \sigma = 2.337$$

length of cloud $= 4\sigma = 4(2,337) = 9,348\,ft$

(d) peak concentration

$$C_{\max} = \frac{M}{A\sqrt{4\pi Kx/\bar{u}}} = \frac{10}{(338.6)\sqrt{4\pi(142)(20,000)/(0.90)}} = 4.69\times10^{-6}\,lb/\,ft^3$$

$$= 4.69\times10^{-6}\times\frac{453.6g}{0.0283m^3} = 75.1\times10^{-3}\,g/m^3 (= mg/l = ppm)$$

$$= 75.1ppb$$

Homework Assignment #5-2

Due: Two weeks from today

Concentration-time data listed in Table 2 are obtained from dispersion study by Godfrey and Fredrick (1970).

1) Plot concentration vs. time

2) Calculate time to centroid, variance, skew coefficient.

3) Calculate dispersion coefficient using the change of moment method and routing procedure.

4) Compare and discuss the results.

Test reach of the stream is straight and necessary data for the calculation of dispersion coefficient are

$$\bar{u} = 1.70\,ft/s; \qquad W = 60\,ft;$$

$$d = 2.77\,ft; \qquad u^{*} = 0.33\,ft/s$$

Table-2 Time-concentration data for Copper Creek, Virginia

Section 1 x=630ft		Section 2 x=3310ft		Section 3 x=5670ft		Section 4 x=7870ft		Section 5 x=11000ft		Section 6 x=13550ft	
T (hr)	C/C_0	T (hr)	C/C_0	T (hr)	C/C_0	T (hr)	C/C_0	T (hr)	C/C_0	T (hr)	C/C_0
1111.5	0.00	1125.0	0.00	1138.0	0.00	1149.0	0.00	1210.0	0.00	1226.0	0.00
1112.5	2.00	1126.0	0.15	1139.0	0.12	1152.0	0.26	1215.0	0.05	1231.0	0.07
1112.5	16.50	1127.0	1.13	1140.0	0.30	1155.0	0.67	1220.0	0.25	1236.0	0.22
1113.0	13.45	1128.0	2.30	1143.0	1.21	1158.0	0.95	1225.0	0.52	1241.0	0.40

1113.5	7.26	1128.5	2.74	1145.0	1.61	1200.0	1.09	1228.0	0.64	1245.0	0.50
1114.0	5.29	1129.0	2.91	1147.0	1.64	1202.0	1.13	1231.0	0.70	1249.0	0.58
1115.0	3.37	1129.5	2.91	1149.0	1.56	1204.0	1.10	1234.0	0.72	1251.0	0.59
1116.0	2.29	1130.0	2.80	1153.0	1.26	1206.0	1.04	1237.0	0.71	1253.0	0.59
1117.0	1.54	1131.0	2.59	1158.0	0.86	1208.0	0.95	1240.0	0.65	1257.0	0.54
1118.0	1.03	1133.0	2.18	1203.0	0.53	1213.0	0.72	1244.0	0.55	1304.0	0.44
1120.0	0.40	1137.0	1.34	1208.0	0.30	1218.0	0.50	1248.0	0.45	1313.0	0.27
1124.0	0.10	1143.0	0.60	1213.0	0.17	1223.0	0.31	1258.0	0.24	1323.0	0.14
1128.0	0.04	1149.0	0.23	1218.0	0.10	1228.0	0.21	1308.0	0.12	1333.0	0.06
1133.0	0.02	1158.0	0.08	1228.0	0.04	1238.0	0.08	1318.0	0.06	1343.0	0.03
1138.0	0.00	1208.0	0.03	1238.0	0.01	1248.0	0.02	1333.0	0.03	1403.0	0.02
-	-	1218.0	0.00	1248.0	0.00	1300.0	0.00	1353.0	0.00	1423.0	0.00