Chapter 1 Fundamentals

1.1 Scope of Fluid Mechanics

1.2 Historical Perspective

1.3 Physical Characteristics of the Fluid State

1.4 Units, Density, Weight Density, Specific Volume, and Specific Gravity

1.5 Compressibility, Elasticity

1.6 Viscosity

1.7 Surface Tension, Capillarity

1.8 Vapor Pressure

Objectives

1.1 Scope of Fluid Mechanics

$$\text{• Problems of}\quad\left.\begin{array}{l}\text{water supply}\\ \text{flood prevention}\\ \text{navigation}\\ \text{water power}\\ \text{irrigation}\end{array}\right\}\longrightarrow\ \text{need to know fluid phenomena}$$

1.2 Historical Perspective

- d'Alembert (1744)

"The theory of fluids must necessarily be based upon experiment"

$$\text{• d'Alembert paradox}\ \left\{\begin{array}{l}\text{theory - ideal, inviscid fluid}\\ \text{practice - real fluid}\end{array}\right.$$

$$\text{• Two schools}\ \left\{\begin{array}{l}\text{theoretical group}\ \rightarrow\ \text{hydrodynamics}\\ \text{practical group}\ \rightarrow\ \text{hydraulics}\end{array}\right.$$

- Navier and Stokes

  → general equations for viscous fluid → equation of motion

[Re] Navier-Stokes equation

Claude-Louis Navier (1785-1836, French engineer) and George Gabriel Stokes (1819-1903, UK mathematician & physicist)

- model the weather, ocean currents, water flow in a pipe, the air's flow around a wing, and motion of stars inside a galaxy
- design of aircraft and cars, the study of blood flow, the design of power stations, the analysis of pollution,
- exact solution - one of the seven most important open problems in mathematics

• New problems in modern times

- Dispersion of man's wastes in lakes, rivers, and oceans

→ *Environmental Hydraulics* → www.ehlab.re.kr

1.3 Physical Characteristics of the Fluid State

• state:		solid		
	fluid	liquid	increasing spacing and latitude of particle motion ↓	↑ increasing inter molecular cohesive force
		gaseous		
		plasma		

- fluid – continuum → no voids or holes

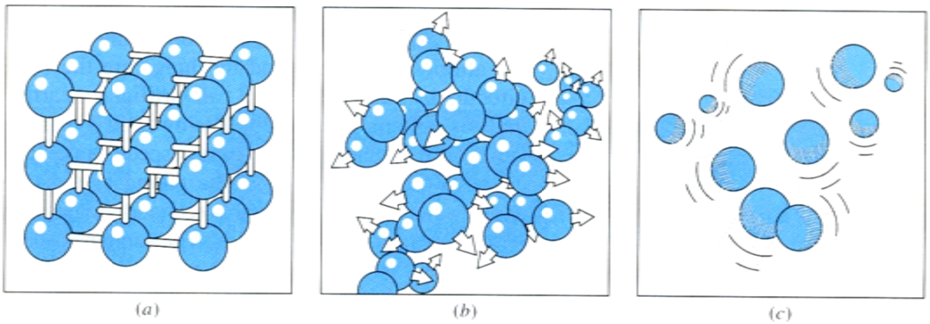


FIGURE 1–5
The arrangement of atoms in different phases: (*a*) molecules are at relatively fixed positions in a solid, (*b*) groups of molecules move about each other in the liquid phase, and (*c*) molecules move about at random in the gas phase.

- Classification of states by stress-strain relationships

stress	**strain**	
	solid	fluid
tension	elastic deformation → permanent distortion	unable to support tension (surface tension)
compression		elastic deformation (compressible fluid)
shear (tangential forces)		permanent distortion or flow (change shape) to infinitesimal shear stress

* Fluid does not resist any small shearing stress → "Flow occurs"

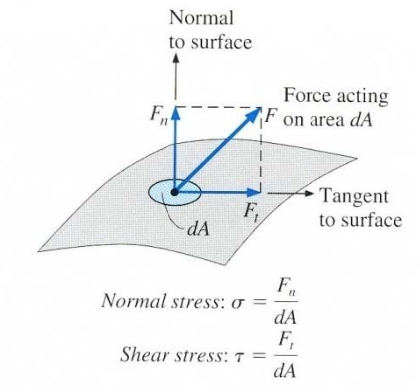


FIGURE 1–3

The normal stress and shear stress at the surface of a fluid element. For fluids at rest, the shear stress is zero and pressure is the only normal stress.

- Comparison between real fluid and ideal fluid

stress	**real fluid (viscous fluid)**		**ideal fluid (non-viscous fluid)**
	in motion	at rest	at rest / in motion
compression (pressure)	○	○	○
shear	○	×	×

- Comparison between compressible fluid and incompressible fluid

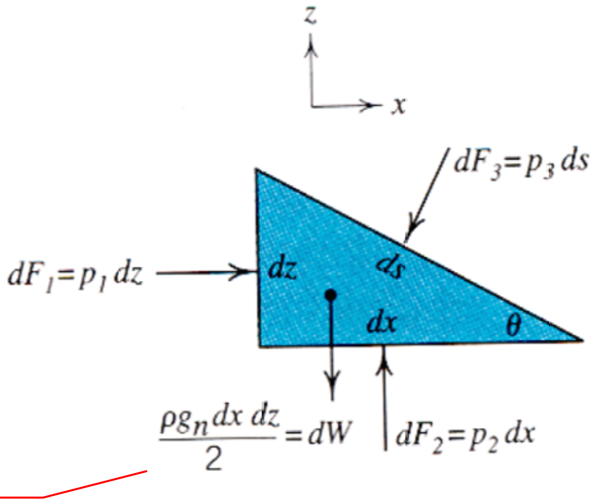
incompressible fluid	**compressible fluid**
① Compressibility is of small important.	① Compressibility is predominant.
② Liquids and gases may be treated similarly.	② Behavior of liquids and gases is quite dissimilar.
③ Fluid problems may be solved with the principles of mechanics.	③ Thermodynamics and heat transfer concepts must be used as well as principles of mechanics.

- Properties of pressure (compression)

① Pressure must be transmitted to solid boundaries normal to those boundaries. → Fig. 1.1

② At a point, pressure has the same magnitude in all directions. → Fig. 1.2

③ Pressure is a scalar quantity.



Weight
$W = \gamma Vol.$

Fig. 1.2

[Pf]

$\sum F = 0$

Apply Newton's law for static equilibrium

$$\sum F_x = p_1 dz - p_3 ds \sin\theta = 0 \qquad \text{(a)}$$

$$\sum F_z = p_2 dx - \rho g dx dz / 2 - p_3 ds \cos\theta = 0 \qquad \text{(b)}$$

Substitute following relations into Eq. (a) & (b)

$$dz = ds \sin\theta$$

$$dx = ds \cos\theta$$

$$\therefore \quad (a):\ p_1 ds \sin\theta - p_3 ds \sin\theta = 0 \rightarrow p_1 = p_3$$

$$(b):\ p_2 ds\cos\theta - \rho g\frac{dz}{2}ds\cos\theta - p_3 ds\cos\theta = 0$$

$\therefore \quad p_2 = p_3 + \frac{1}{2}\rho g dz$

As $dz \to 0$ then $p_2 \approx p_3$

$\therefore \quad p_1 = p_2 = p_3$ at a point $(dx = dz = 0)$

1.4 Units, Density, Weight Density, Specific Volume, and Specific Gravity

- SI units - SI system – metric system

- Basic dimensions and units

U.S. customary system

Dimension	SI unit	English system (FSS)
Length (L)	metre (m)	feet (ft)
Mass (M)	kilogram (kg)	slug (-)
Time (t)	second (s)	second (s)
Temp. (T)	kelvin (K)	degree Rankine (°R)

- Frequency (f): hertz (HZ = s^{-1})

- Force, F

→ introduce Newton's 2nd law of motion

$$F = ma$$

Force = mass × acceleration

$$a = v/t = L/t^2 \qquad \left[Lt^{-2}\right] \qquad (\mathrm{m/s^2})$$

$$v = L/t \qquad \left[Lt^{-1}\right] \qquad (\mathrm{m/s})$$

$$\therefore F \rightarrow 1\mathrm{kg \cdot m/s^2} = 1\mathrm{N}(Newton)$$

- Energy, E (work)

$$E = FL \rightarrow \mathrm{kg \cdot m^2/s^2} = J(Joule)$$

- Power, P

$$P = E/t \rightarrow J/s = \mathrm{kg \cdot m^2/s^3}$$

- Pressure, p;   Stress, σ, τ

$$p = F/A \rightarrow \mathrm{N/m^2} = \mathrm{Pa\ (pascal)} = \mathrm{kg/m \cdot s^2}$$

- Temperature, T: degree Celsius (°C)

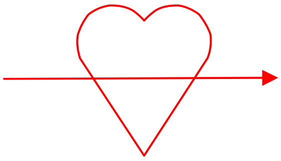
- Density, ρ

= mass per unit volume

~ depends on the number of molecules per unit of volume

~ decreases with increasing temperature

$$\rho = \frac{M}{V} \rightarrow \mathrm{kg/m^3}$$



- Specific weight (weight density), γ

= weight (force) per unit volume

$$\gamma = \frac{W}{V} \rightarrow \mathrm{N/m^3} = \mathrm{kg/m^2 \cdot s^2}$$

[Re]

$W = Mg$ (Newton's 2nd law of motion)

g = acceleration due to gravity

$$\therefore \gamma = \rho g \tag{1.1}$$

- Specific volume=volume per unit mass=$1/\rho$
- Specific gravity, s.g. , ~ r. d. (relative density)

  = ratio of density of a substance to the density of water at a specified temperature and pressure

$$s.g. = \frac{\rho_f}{\rho_w} = \frac{\gamma_f}{\gamma_w}$$

[Re] s.g. of sea water = 1.03

s.g. of soil = 2.65

s.g. of mercury = 13.6

- For water at 5 °C (p. 694, App. 2)

	SI	English system
ρ	1,000 kg/m^3	1.94 slugs/ft^3
γ	9,806 N/m^3	62.4 lb/ft^3
g	9.81 m/s^2	32.2 ft/s^2

- Advantage of SI system and English FSS system

① It distinguishes between force(F) and mass(M).

② It has no ambiguous definitions.

- Greek Alphabet

α	Alpha	angle
β	Beta [beitə]	angle
γ, Γ	Gamma	specific weight, circulation
δ, Δ	Delta	thickness of boundary layer
ε	Epsilon	eddy viscosity, height of surface roughness
ζ	Zeta	
η	Eta	
θ, Θ	Theta	
ι	Iota [aioutə]	
κ	Kappa [kæpə]	
λ, Λ	Lambda	
μ	Mu [mju:]	dynamic viscosity
ν	Nu	kinematic viscosity
ξ	Xi [gzai, ksai]	vorticity
o	Omicron	
π	Pi [pai]	
ρ	Rho	mass density
σ, Σ	Sigma	Sigma Xi, Scientific Research Society, 1886 honor society for scientists & engineers
τ	Tau	shear
υ, Υ	Upsilon	
φ, Φ	Phi [fai]	Phi Beta Kappa

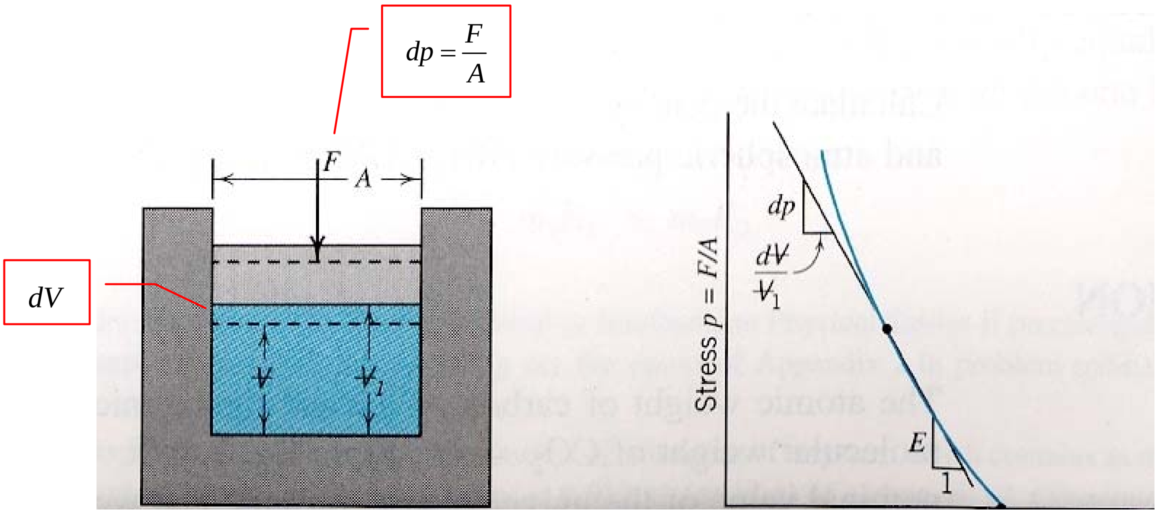
χ	Chi [kai]	
ψ, Ψ	Psi [psai, sai]	stream function
ω, Ω	Omega	angular velocity

- Prefixes

E	exa	10^{18}
P	peta	10^{15}
T	tera	10^{12}
G	giga	10^{9}
M	mega	10^{6}
k	kilo	10^{3}
h	hecto	10^{2}
da	deca	10^{1}
d	deci	10^{-1}
c	centi	10^{-2}
m	milli	10^{-3}
μ	micro	10^{-6}
n	nano	10^{-9}
p	pico	10^{-12}
f	femto	10^{-15}
a	atto	10^{-18}

1.5 Compressibility, Elasticity

- Elastic behavior to compression



$dp = \frac{F}{A}$

dV

Fig. 1.3

Strain $= \frac{V}{V_1}$

- Compressibility ≡ change in volume due to change in pressure

solid - modulus of elasticity, E (N/m^2)

fluid - bulk modulus

Minus means that increase in pressure causes decrease in volume

- Stress strain curve (E↑, difficult to compress) → Fig. 1.3 (see Table 1-1)

$$dp \propto \frac{dV}{V_1} \rightarrow dp = -E\frac{dV}{V_1}$$

$$E = -\frac{dp}{\frac{dV}{V_1}} = -V_1\frac{dp}{dV} \neq const = fn(p,T) \rightarrow p\uparrow \;\rightarrow\; E\uparrow$$

$$C = \frac{1}{E} = -\frac{dV}{V_1}\frac{1}{dp}$$

= modulus of compressibility (m^2/N)

[Re] large E/small C → less compressible

- incompressible fluid (inelastic): $E = \infty, C \ll 1$

→ constant density ρ=const.

~ water

- compressible fluid

→ changes in density → variable density

~ gas

[Table 1.1] Bulk modulus of water, E $(10^6\ \mathrm{N/m^2})$

Pressure $10^6\ \mathrm{N/m^2}$	**Temperature, °C**				
	0°	20°	50°	100°	150°
0.1	1950	2130	2210	2050	
10.0	2000	2200	2280	2130	1650
30.0	2110	2320	2410	2250	1800
100.0	2530	2730	$\underline{\underline{2840}}$	2700	2330

- E increases as pressure increases.
- E is maximum at about 50 °C.

→ The water has minimum compressibility at about 50 °C.

[Table A2-1]

Compressibility		**Modulus of Elasticity, E (kPa)**	
steel	1/80 of water	water	2,170,500
mercury	1/12.5 of water	sea water	2,300,000
nitric acid	6 of water	mercury	26,201,000

- For the case of a fixed mass of liquid at constant temperature

$$E = -V_1 \frac{dp}{dV}$$

$$\frac{\Delta V}{V_1} \approx -\frac{\Delta p}{E}$$

$$\frac{V_2 - V_1}{V_1} \approx -\frac{p_2 - p_1}{E}$$

[Ex] For water; $E = 2,200 \times 10^6 \,\mathrm{Pa}$ @ 20˚C

$$p_2 = p_1 + 7 \times 10^6 \,\mathrm{Pa}$$

$$\therefore \quad \frac{V_2 - V_1}{V_1} \approx -\frac{p_2 - p_1}{E} = -0.0032$$

$$\therefore \quad V_2 = (1 - 0.0032)\, V_1$$

$\Delta V \approx 0.3\%$ decrease

→ water is incompressible

1.6 Viscosity

[Re] From Wikipedia

Viscosity is a measure of the resistance of a fluid which is being deformed by either shear stress or tensile stress.

Viscosity ~ "thickness" or "internal friction"

- water ~ "thin", having a lower viscosity
- honey ~ "thick", having a higher viscosity

The less viscous the fluid is, the greater its ease of movement (**fluidity**).

Viscosity describes a fluid's internal resistance to flow and may be thought of as a measure of fluid friction.

For example, high-viscosity felsic magma will create a tall, steep stratovolcano, because it cannot flow far before it cools, while low-viscosity mafic lava will create a wide, shallow-sloped shield volcano.

All real fluids (except superfluids) have some resistance to stress and therefore are **viscous**, but a fluid which has no resistance to shear stress is known as an **ideal fluid** or **inviscid fluid**.

- Two types of fluid motion (real fluid)

1) laminar flow:

- viscosity plays a dominant role
- fluid elements or particles slide over each other in layers (laminar)
- molecular diffusion

[Ex] flow in a very small tube, a very thin flow over the pavement, flow in the laminar flow table

2) turbulent flow:

- random or chaotic motion, eddies of various sizes are seen
- common in nature (streams, rivers, pipes)
- large scale mixing between the layers

[Ex] flows in the water supply pipe, flows in the storm sewer pipe, flows in the and canals and streams

- Reynolds number

$$\mathrm{Re} = \frac{Vd}{\nu}$$

Diameter of pipe,
Depth of stream

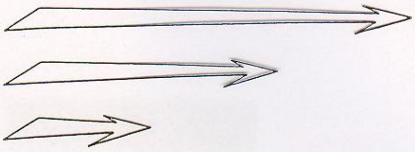
where V = flow velocity; d = characteristic length; ν = kinematic viscosity

- Reynolds experiments

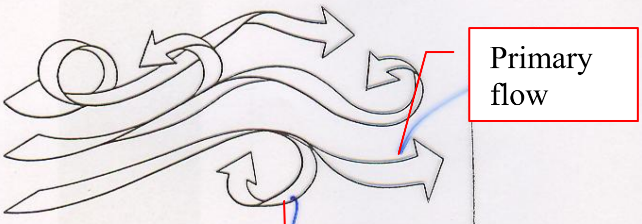
laminar flow: $\mathrm{Re} < 2{,}100$

transition: $2{,}100 < \mathrm{Re} < 4{,}000$

turbulent flow: $\mathrm{Re} > 4{,}000$



(A) In laminar flow, water particles move in parallel lines.



(B) In turbulent flow, many secondary eddies are superposed on the main stream flow.

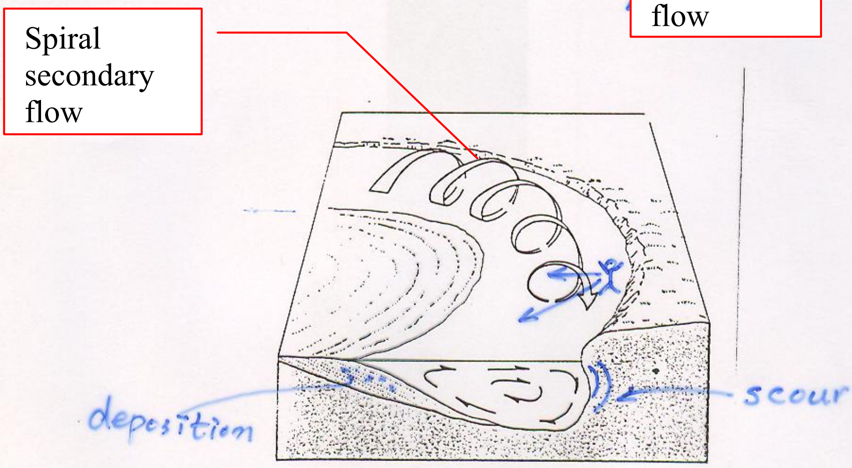
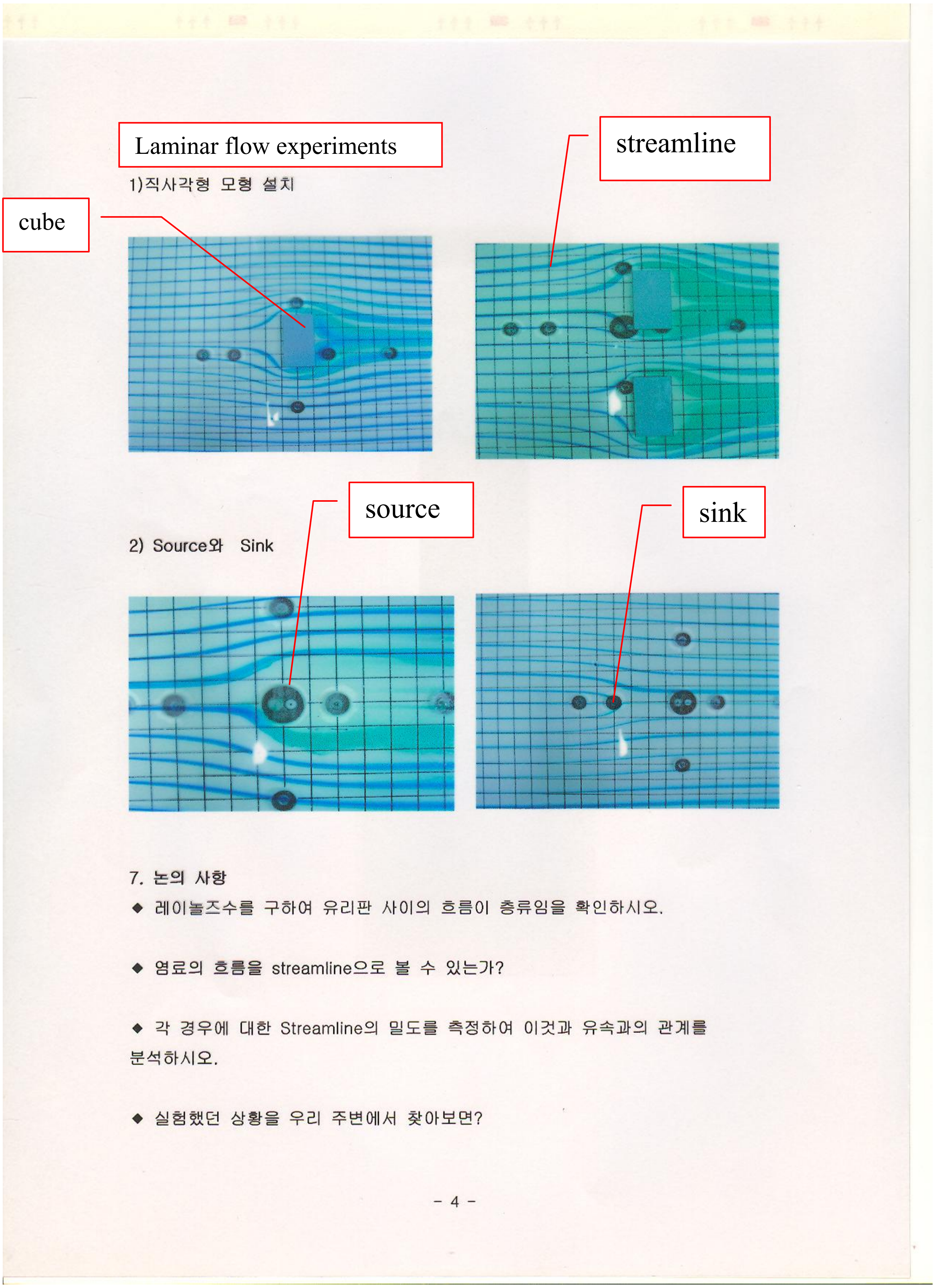


Figure 10.6(A)
Flow in a curved channel follows a corkscrew pattern. Water on the outside of the bend is forced to flow faster than that on the inside of the curve. This difference in velocity, together with normal frictional drag on the channel walls, produces a corkscrew flow pattern. As a result, erosion occurs on the outer bank, and deposition occurs on the inside of the bend. These processes produce an asymmetrical channel, which slowly migrates laterally.



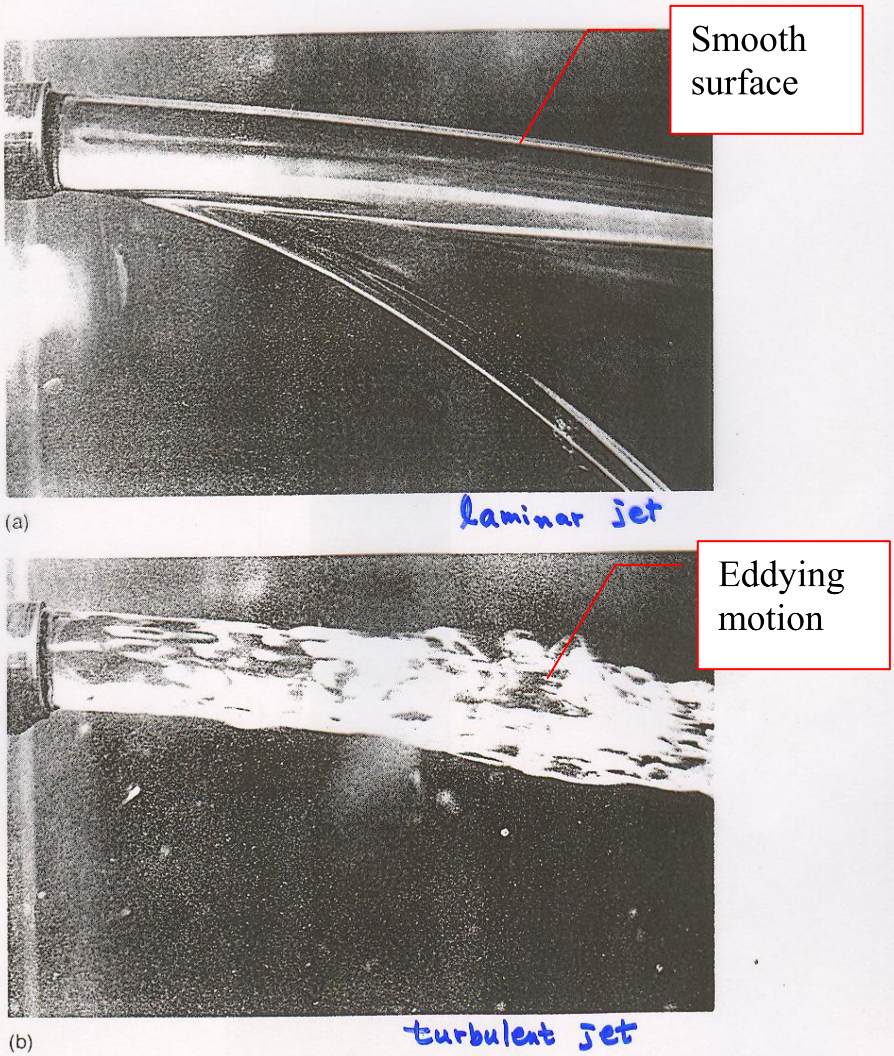
1)직사각형 모형 설치

2) Source와 Sink

7. 논의 사항

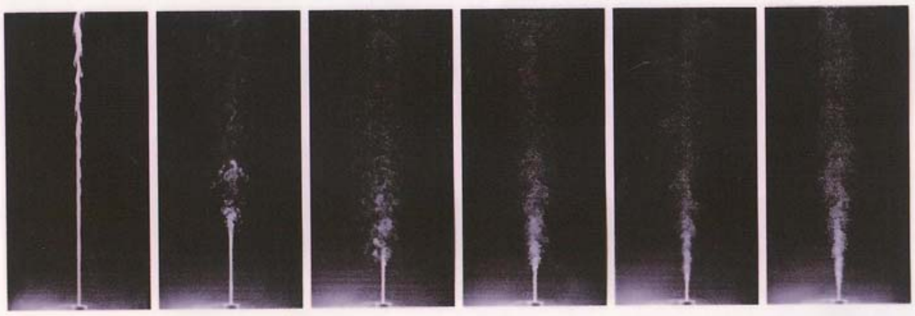
- ◆ 레이놀즈수를 구하여 유리판 사이의 흐름이 층류임을 확인하시오.
- ◆ 염료의 흐름을 streamline으로 볼 수 있는가?
- ◆ 각 경우에 대한 Streamline의 밀도를 측정하여 이것과 유속과의 관계를 분석하시오.
- ◆ 실험했던 상황을 우리 주변에서 찾아보면?

- 4 -



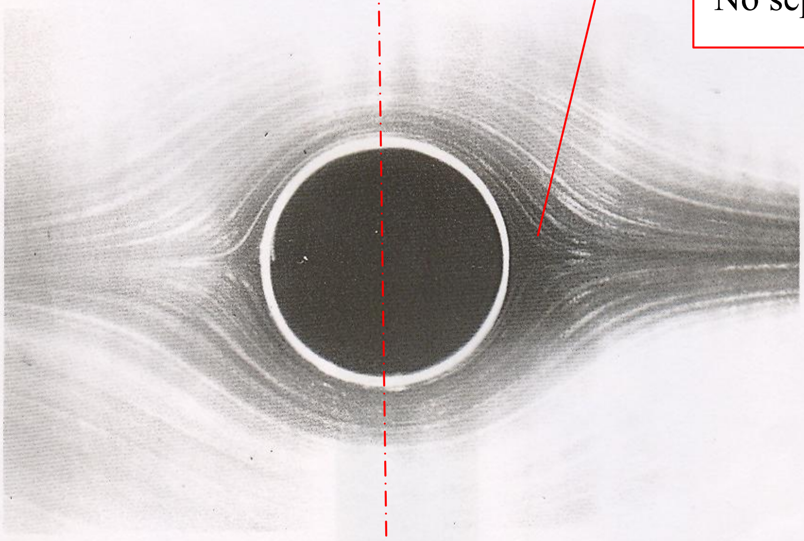
Two flow conditions for a liquid flowing from a long smooth tube. The laminar jet (a) has a smooth surface and the turbulent jet (b) shows the effect of eddying motion. Both are at the same flow rate. Exposure time about 1/200 sec.

Round jet tests using PIV (Particle Image Velocimetry)



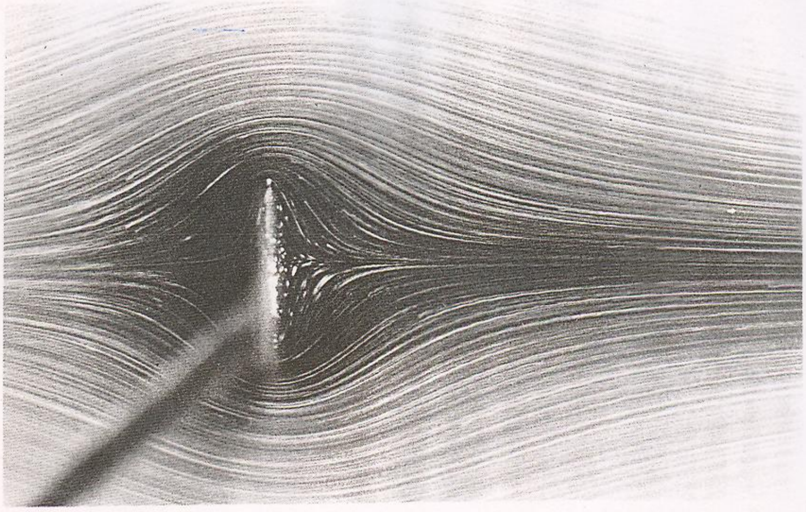
(R100J, Re =177) (R200J, Re = 437) (R400J, Re =1,305) (R500J, Re = 2,163) (R600J, Re =3,208) (R900J, Re =5,142)

Figs. Evolution of Round Jet with Increase of Reynolds Number (Instantaneous Images)

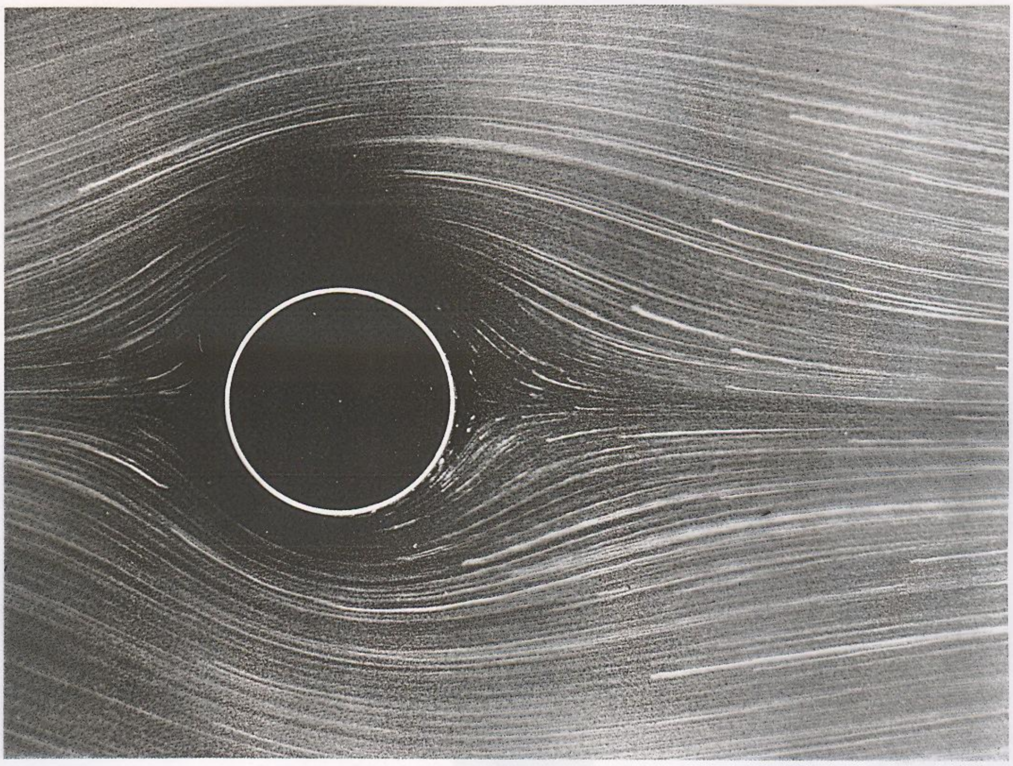


Symmetric shape,
No separation

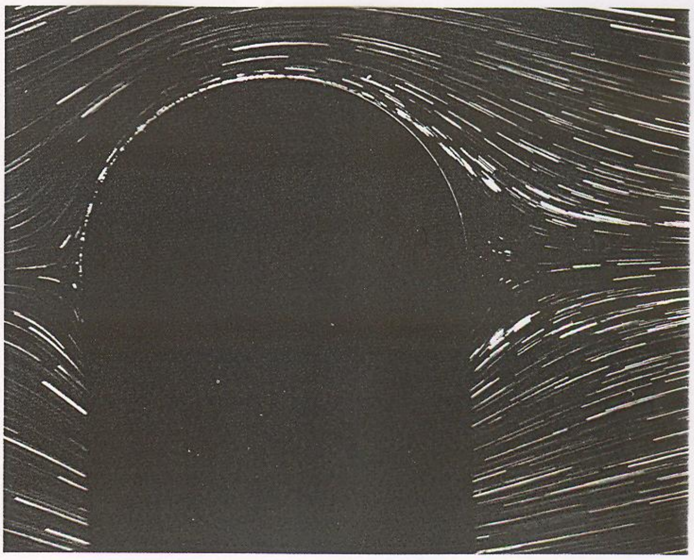
6. Uniform flow past a circular cylinder at $R=0.16$. That the flow is from left to right can scarcely be deduced from the streamline pattern, because in the limit of zero Reynolds number the flow past a solid body is reversible, and hence symmetric about a symmetric shape. It resembles superficially the pattern of potential flow in figure 1, but the disturbances to the uniform stream die off much more slowly. The flow of water is shown by aluminum dust. *Photograph by Sadatoshi Taneda*



7. Uniform flow normal to a plate at $R=0.334$. The streamline pattern is still almost symmetric fore-and-aft at this higher Reynolds number. It is possible, however, that the flow has separated over the rear. Aluminum dust shows the flow of glycerine. *Taneda 1968*

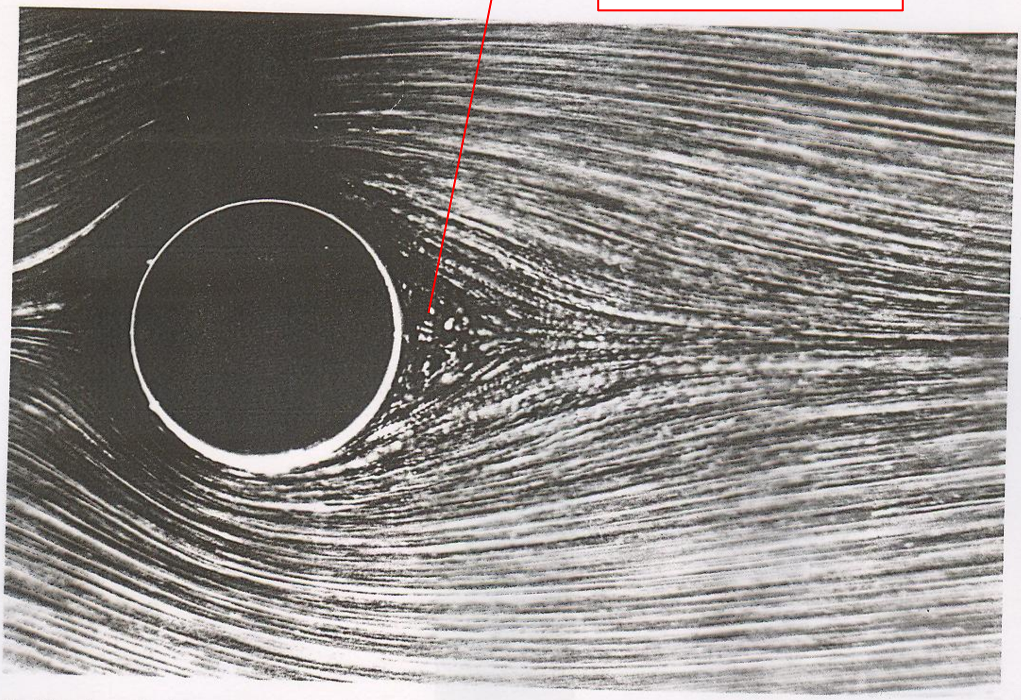


24. Circular cylinder at R**=1.54.** At this Reynolds number the streamline pattern has clearly lost the fore-and-aft symmetry of figure 6. However, the flow has not yet separated at the rear. That begins at about $R=5$, though the value is not known accurately. Streamlines are made visible by aluminum powder in water. *Photograph by Sadatoshi Taneda*

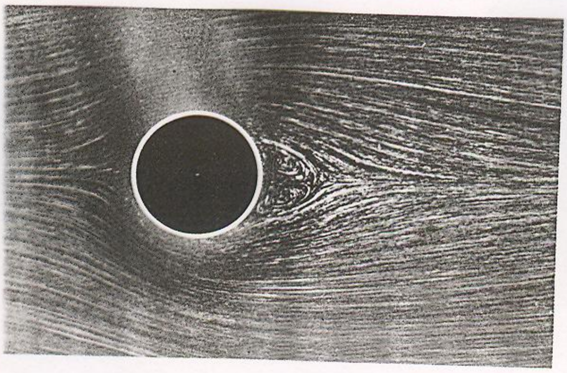


25. Sphere at R**=9.8.** Here too, with wall effects negligible, the streamline pattern is distinctly asymmetric, in contrast to the creeping flow of figure 8. The fluid is evidently moving very slowly at the rear, making it difficult to estimate the onset of separation. The flow is presumably attached here, because separation is believed to begin above $R=20$. Streamlines are shown by magnesium cuttings illuminated in water. *Photograph by Madeleine Coutanceau and Michele Payard*

Separation, eddy formation

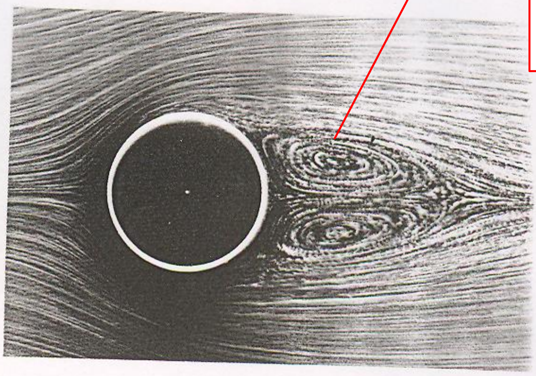


40. Circular cylinder at R=9.6. Here, in contrast to figure 24, the flow has clearly separated to form a pair of recirculating eddies. The cylinder is moving through a tank of water containing aluminum powder, and is illuminated by a sheet of light below the free surface. Extrapolation of such experiments to unbounded flow suggests separation at R=4 or 5, whereas most numerical computations give R=5 to 7. *Photograph by Sadatoshi Taneda*



41. Circular cylinder at R=13.1. The standing eddies become elongated in the flow direction as the speed increases. Their length is found to increase linearly with Reynolds number until the flow becomes unstable above R=40. *Taneda 1956a*

Growth of eddy



42. Circular cylinder at R=26. The downstream distance to the cores of the eddies also increases linearly with Reynolds number. However, the lateral distance between the cores appears to grow more nearly as the square root. *Photograph by Sadatoshi Taneda*

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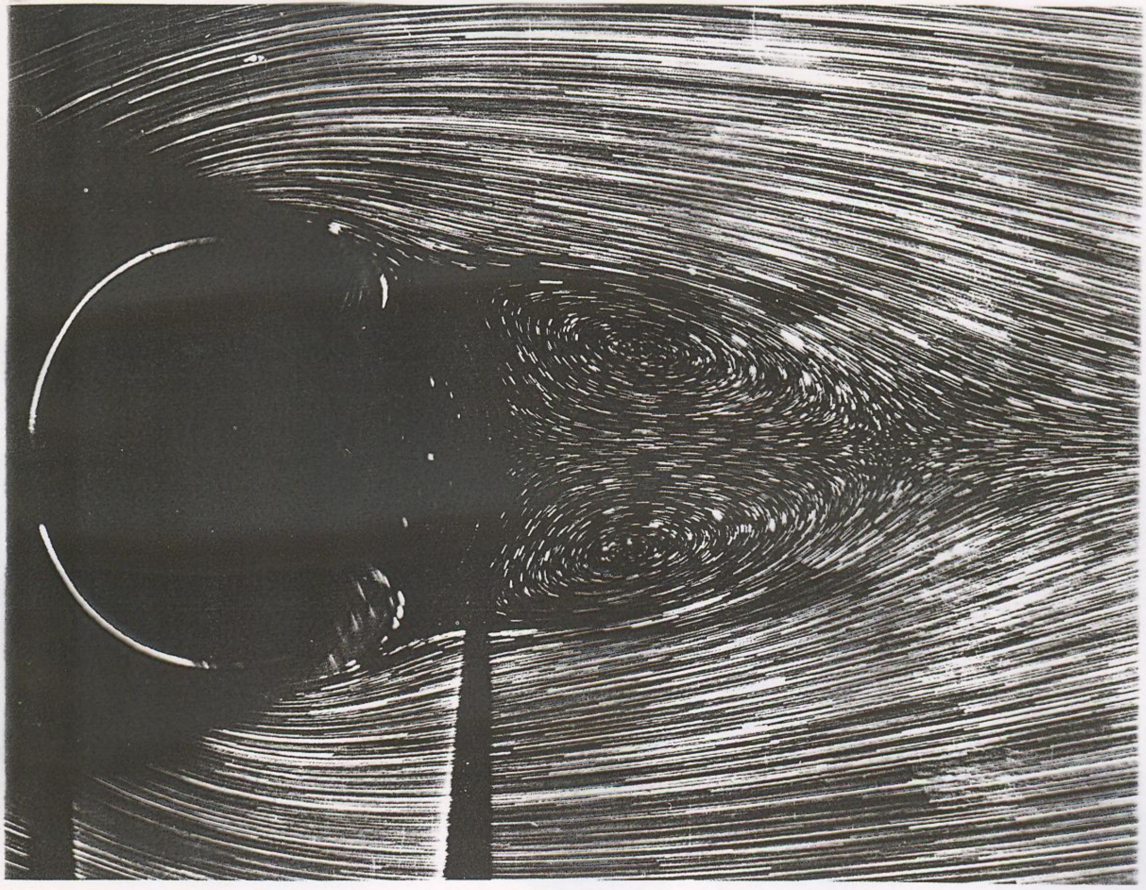
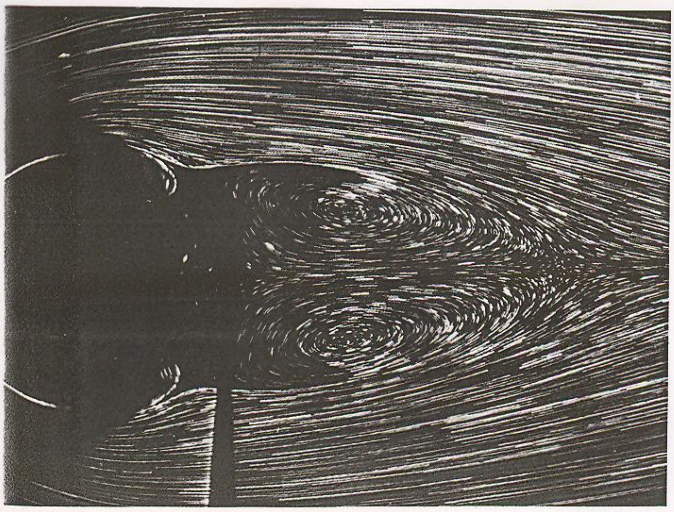
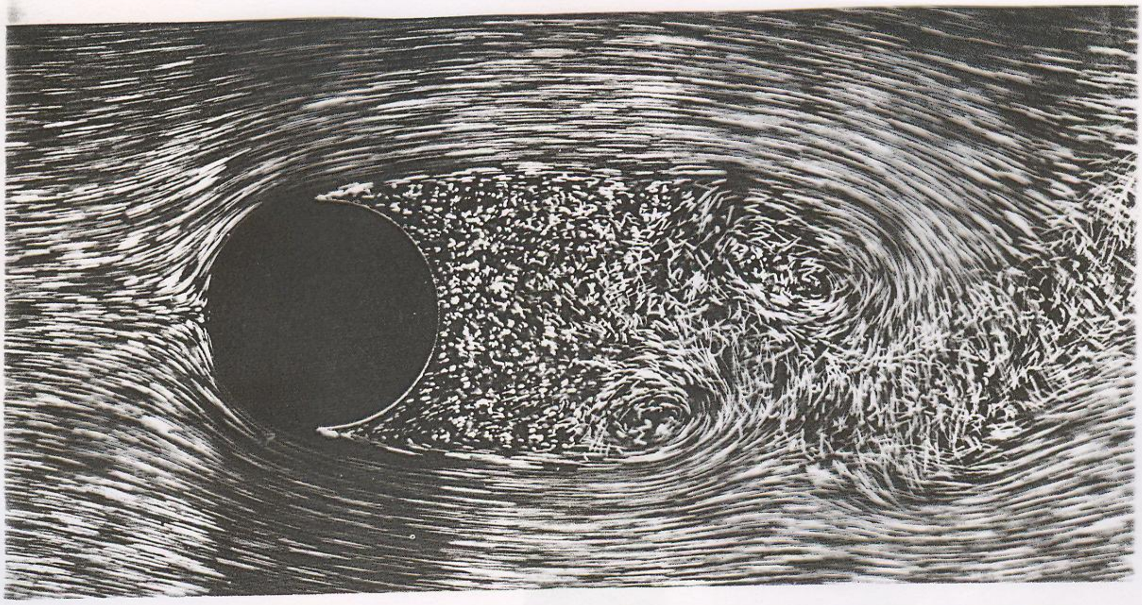


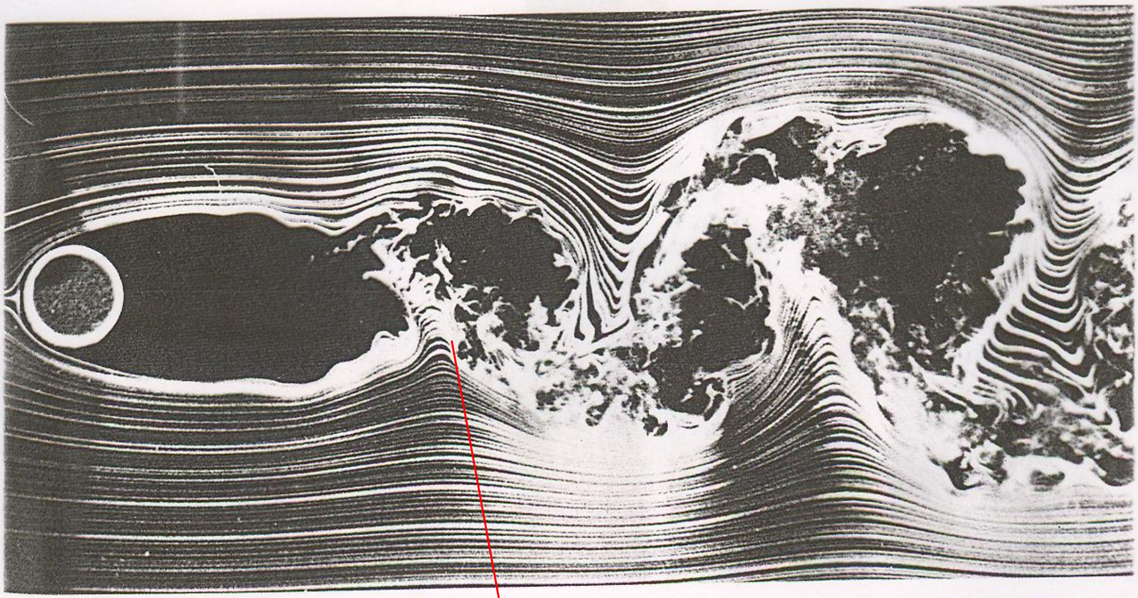
43. Circular cylinder at $R=24.3$. A different view of the flow is obtained by moving a cylinder through oil. Tiny magnesium cuttings are illuminated by a sheet of light from an arc projector. The two dark wedges below the circle are an optical effect. The lengths of the particle trajectories have been measured to find the velocity field to within two per cent. *Coutanceau & Bouard 1977*



44. Circular cylinder at $R=30.2$. The flow is here still completely steady with the recirculating wake more than one diameter long. The walls of the tank, 8 diameters away, have little effect at these speeds. *Photograph by Madeleine Coutanceau and Roger Bouard*



47. Circular cylinder at $R=2000$**.** At this Reynolds number one may properly speak of a boundary layer. It is laminar over the front, separates, and breaks up into a turbulent wake. The separation points, moving forward as the Reynolds number is increased, have now attained their upstream limit, ahead of maximum thickness. Visualization is by air bubbles in water. *ONERA photograph, Werlé & Gallon 1972*

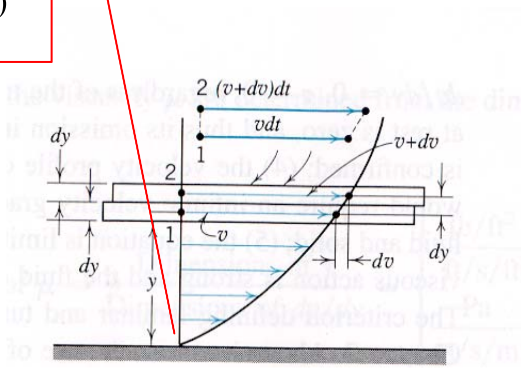


48. Circular cylinder at $R=10{,}000$**.** At five times the speed of the photograph at the top of the page, the flow pattern is scarcely changed. The drag coefficient consequently remains almost constant in the range of Reynolds number spanned by these two photographs. It drops later when, as in figure 57, the boundary layer becomes turbulent at separation. *Photograph by Thomas Corke and Hassan Nagib*

Karman vortex street – periodic shedding of vortices in sinuous form

- laminar flow

no velocity at the boundary (no slip)



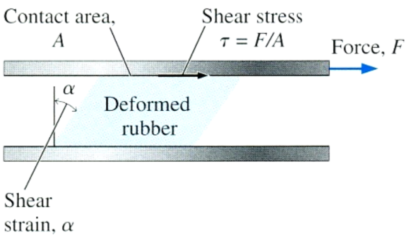


FIGURE 1–2
Deformation of a rubber eraser placed between two parallel plates under the influence of a shear force.

Fig 1.4

- strain = relative displacement

$$= \frac{d_2 - d_1}{dy} = \frac{dvdt}{dy} = \frac{dv}{dy}dt$$

[Cf] solid mechanics

$$\tau_{yx} = G\frac{d\zeta}{dy}$$

total angular displacement

[Re] $$d_2 = v_2 dt; d_1 = v_1 dt$$
$$d_2 - d_1 = (v_2 - v_1)dt$$

- Experiment has shown that, in many fluids, shearing (frictional) stress per unit of contact area, τ is proportional to the time rate of relative strain.

$$\therefore \tau \propto \frac{dv}{dy}dt / dt = \frac{dv}{dy} \text{(velocity gradient)}$$

$$\boxed{\tau = \mu\frac{dv}{dy}} \rightarrow \text{Newton's equation of viscosity} \qquad (1.12)$$

Large μ → sticky, difficult to flow

where μ = coefficient of viscosity

= dynamic (absolute) viscosity

- viscosity = measure of fluid's resistance to shear or angular deformation

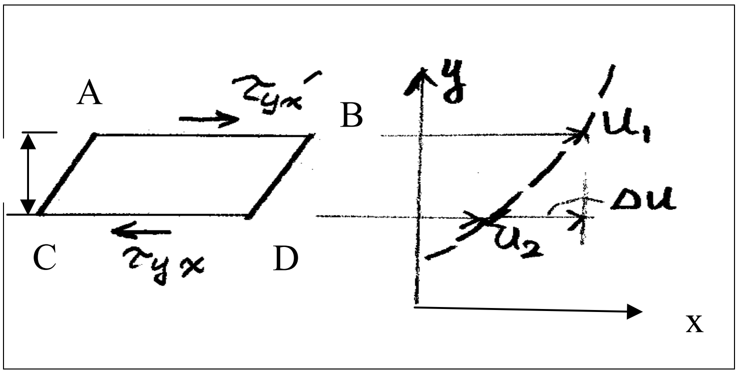
= internal resistance of a fluid to motion (fluidity)

[Re] Friction forces result from

- cohesion for liquid

- momentum interchange between molecules for gas

[Re] angular deformation due to tangential stress



•rate of angular deformation

(i) displacement of AB relative to CD

$$=\left(u+\frac{du}{dy}\Delta y\right)\Delta t-u\Delta t=\frac{du}{dy}\Delta y\Delta t$$

(ii) angular displacement of AC

$$=\frac{du}{dy}\Delta y\Delta t/\Delta y=\frac{du}{dy}\Delta t$$

(iii) time rate of angular deformation

$$= \frac{du}{dy}\Delta t / \Delta t = \frac{du}{dy}$$

$$\boxed{\tau = \mu \frac{dv}{dy}}$$

- dynamic viscosity, μ

$$\tau = F / A$$

$$[\tau] = \left[MLT^{-2} / L^2\right]\left[ML^{-1}T^{-2}\right] = \mathrm{kg/(m \cdot s^2)} = \mathrm{Pa}$$

$$\left[\frac{dv}{dy}\right] = \left[\frac{LT^{-1}}{L}\right] = \left[T^{-1}\right]$$

$$\therefore [\mu] = \left[\tau / \frac{dv}{dy}\right] = \left[\frac{ML^{-1}T^{-2}}{T^{-1}}\right] = \left[ML^{-1}T^{-1}\right] = \mathrm{kg/m \cdot s} = \mathrm{N \cdot s / m^2} = \mathrm{Pa \cdot s}$$

$$\Rightarrow 1\, poises(Poiseuille) = 10^1\, \mathrm{Pa \cdot s}$$

- kinematic viscosity, ν

$$\nu = \frac{\mu}{\rho} \tag{1.13}$$

$$[\nu] = \left[\frac{ML^{-1}T^{-1}}{ML^{-3}}\right] = \left[L^2T^{-1}\right] = \mathrm{m^2/s}$$

1 $\mathrm{m^2/s}$ = 10^4 stokes =10^6 centistokes

- Remarks on Eq. (1.12)

① τ, μ are independent of pressure. [Cf] friction between two moving solids

② Shear stress τ (even smallest τ) will cause flow (velocity gradient).

③ Shearing stress in viscous fluids at rest will be zero.

$$\frac{dv}{dy} = 0 \rightarrow \tau = 0 \text{ regardless of } \mu$$

④ At solid boundary, $\frac{dv}{dy} \neq \infty$ $(\rightarrow \tau \neq \infty$ (no infinite shear))

→ Infinite shearing stress between fluid and solid is not possible.

⑤ Eq. 1.12 is limited to laminar (non-turbulent) fluid motion in which viscous action is predominant.

[Cf] turbulent flow

$$\tau = \varepsilon \frac{dv}{dy}$$

$\boxed{\varepsilon \; \square \; \mu}$

where ε = eddy viscosity

⑥ Velocity at a solid boundary is zero.

→ No slip condition (continuum assumption)

- Newtonian and non-Newtonian fluids

i) Newtonian fluid ~ water

ii) Non-Newtonian fluid ~ plastic, blood, suspensions, paints, polymer solutions → rheology

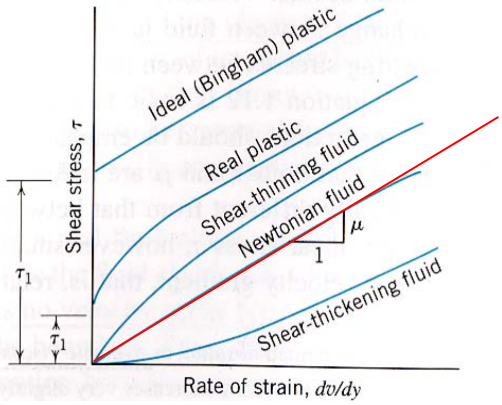


Fig. 1.5

- Non-Newtonian fluid

1) $\tau - \tau_1 = \mu \dfrac{dv}{dy}$ plastic, τ_1 = threshold

2) $\tau = K\left(\dfrac{dv}{dy}\right)^n$ $n > 1$ Shear-thickening fluid

$n < 1$ Shear-thinning fluid

- Couette flow: laminar flow in which the shear stress is constant

thin fluid film between two large flat plates

thin fluid film between the surfaces of coaxial cylinders

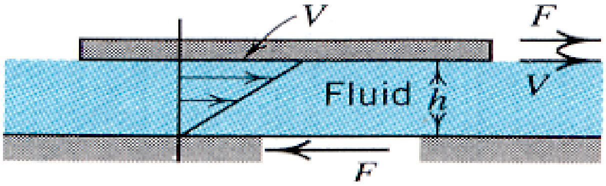


Fig. 1.7

$\dfrac{dv}{dy} = \dfrac{V}{h}$ ~ linear velocity gradient

$\therefore \ \tau = \mu \dfrac{V}{h}$ ~ constant

- Turbulent flow

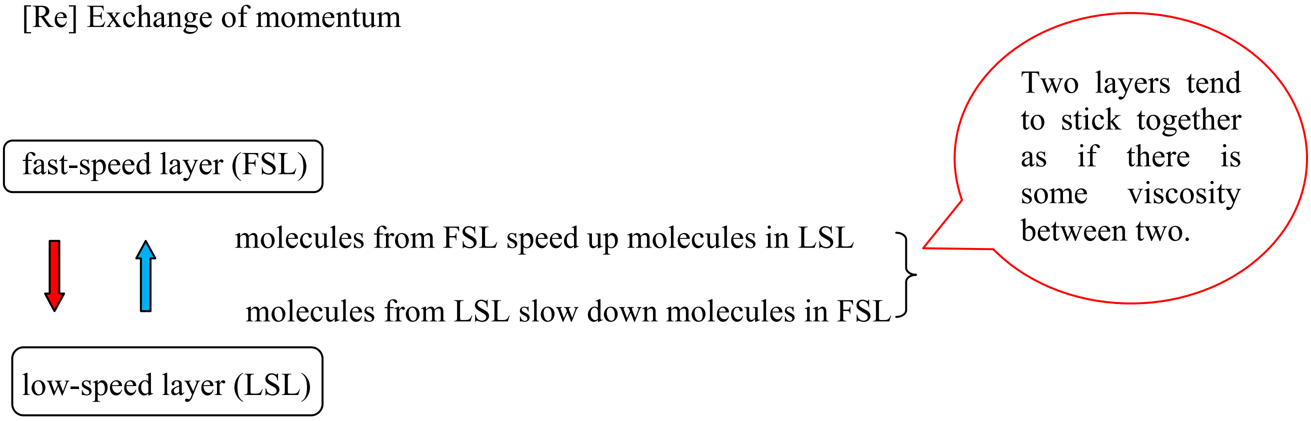
$$\tau = (\mu + \varepsilon)\frac{dv}{dy}$$

ε = eddy viscosity = viscosity due to turbulent factor

- Mechanism of viscosity for liquid and gas

	gas	**liquid**
main cause of viscosity	exchange of molecule's momentum → interchange of molecules between the fluid layers of different velocities	intermolecular cohesion
effect of temperature variation	temp↑ → molecular activity↑ → viscosity↑ → shearing stress↑	temp↑ → cohesion↓ → viscosity↓ → shear stress↓

[Re] Exchange of momentum



1) exchange of momentum : exchange momentum in either direction from high to low or from low to high momentum due to random motion of molecules

2) transport of momentum : transport of momentum from layers of high momentum (high velocity, mv) to layers of low momentum

1.7 Surface Tension, Capillarity

- surface tension
  - occur when the liquid surfaces are in contact with another fluid (air) or solid
  - f_n (relative sizes of intermolecular cohesive and adhesive forces to another body)
  - as temp↑ → cohesion↓ → $\sigma\downarrow$ ☞ Table A2.4b, p. 694

- some important engineering problems related to surface tension
  - capillary rise of liquids in narrow spaces
  - mechanics of bubble formation
  - formation of liquid drops
  - small models of larger prototype → dam, river model

- surface tension, σ (F/L, N/m)
  - force per unit length
  - force attracting molecules away from liquid

Consider static equilibrium

$$\sum F = 0 \quad \text{(forces normal to the element } a,b,c,d\text{)}$$

$$(p_i - p_0)dxdy = 2\sigma dy\sin\alpha + 2\sigma dx\sin\beta$$

where p_i = pressure inside the curvature; p_o = pressure inside the curvature

$$\sin\alpha = \frac{dx}{2R_1}, \quad \sin\beta = \frac{dy}{2R_2} \quad \left[dx = 2(R_1\sin\alpha)\right]$$

$$\therefore\ p_i - p_0 = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{1.15}$$

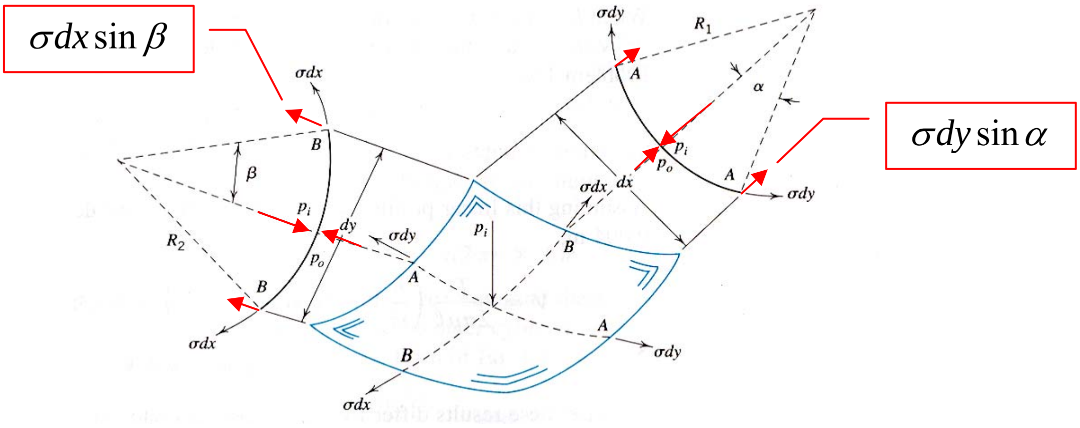


Fig. 1.9

- Cylindrical capillary tube

  - due to both cohesion and adhesion

  cohesion < adhesion → rise (water)

  cohesion > adhesion → depression (mercury)

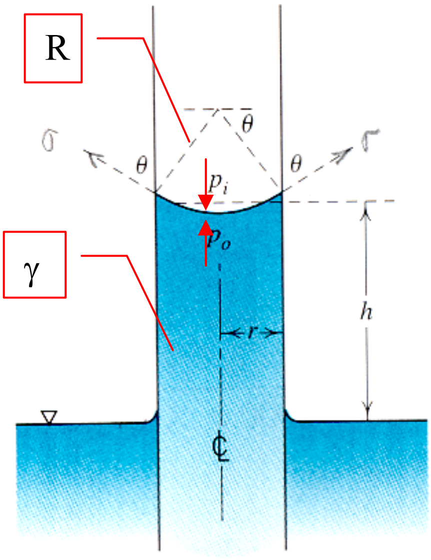


Fig. 1.10

For a small tube, given conditions are as follows

$R_1 = R_2 = R$ (liquid surface ≈ section of sphere) ← Ch. 2

$p_0 = -\gamma h$ (hydrostatic pressure)

$p_i = 0$ (atmospheric)

Substitute above conditions into Eq. 1.15: $$p_i - p_0 = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{1.15}$$

$$\therefore \gamma h = \sigma \frac{2}{R}$$

By the way, $r = R\cos\theta$

$$\therefore \gamma h = \sigma \frac{2}{r/\cos\theta} = \frac{2\sigma\cos\theta}{r}$$

$$\boxed{h = \frac{2\sigma\cos\theta}{\gamma r}} \tag{1.16}$$

in which h = capillary rise → $r\uparrow$ → $h\downarrow$

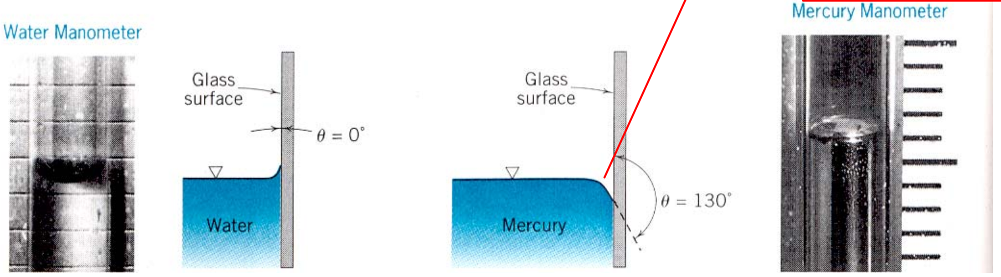
θ = angle of contact

r = radius of tube ≤ 2.5 mm for spherical form

[Ex] water and mercury → Fig. 1.11

If $r > 12$ mm, h is negligible for water.

cohesion > adhesion
→ depression



- Pressure measurement using tubes in hydraulic experiments → Ch.2 manometer

~ capillarity problems can be avoided entirely by providing tubes large enough to render the capillarity correction negligible.

- Fomation of curved surface, droplet

  - At free liquid surface contacting the air, cohesive forces at the outer layer are not balanced by a layer above.

  →The surface molecules are pulled tightly to the lower layer.

  →Free surface is curved.

[Ex] Surface tension force supports small loads (water strider).

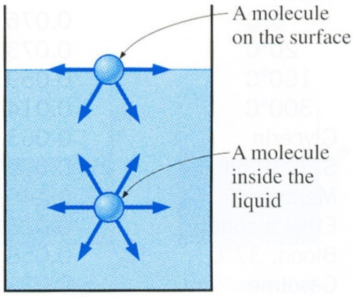
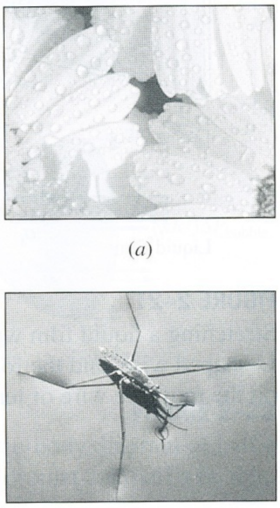


FIGURE 2–20
Attractive forces acting on a liquid molecule at the surface and deep inside the liquid.


(*a*)

(*b*)

FIGURE 2–19
Some consequences of surface tension.

[IP 1.10] For a droplet of water (20 °C), find diameter of droplet

Given: $p_i - p_0 = 1.0$ kPa

At 20°C, $\sigma = 0.0728$ N/m ← App. 2

[Sol]

$$p_i - p_0 = \sigma\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \frac{2\sigma}{R} \tag{1.15}$$

$$\therefore \quad 1\times10^3\ \text{N/m}^2 = 2(0.0728)\cdot\frac{1}{R}$$

$$\therefore \quad R = 0.000146\text{m} = 0.146\text{mm} \rightarrow d = 0.292\ \text{mm}$$

[IP 1.11] Find height of capillary rise in a clean glass tube of 1 mm diameter if the water temperature is 10°C or 90°C.

[Sol]

From App. 2 Table A 2.4b;

@ 10°C σ=0.0742 N/m, $\gamma = 9.804$ kN/m^3

@ 90°C σ=0.0608 N/m, $\gamma = 9.466$ kN/m^3

Use Eq. 1.16

For water, $\theta = 0^\circ$

$$h = \frac{2\sigma\cos\theta}{\gamma r} \tag{1.16}$$

$$\therefore \quad h_{10} = \frac{2(0.0742)(1)}{9804(0.0005)} = 0.030\text{m=30mm}$$

$$h_{90} = \frac{2(0.0608)(1)}{9466(0.0005)} = 0.026\text{m=26mm}$$

1.8 Vapor Pressure

- vapor pressure = partial pressure exerted by ejected molecules of liquid

  → Table A2.1 and A2.4b

- liquids ~ tend to vaporize or evaporate due to molecular thermal vibrations (molecular activity)

  → change from liquid to gaseous phase

temperature↑ → molecular activity↑ → vaporization↑ → vapor pressure↑

- volatile liquids:

  ~ easy to vaporize → high vapor pressure

  gasoline: $p_v = 55.2$ kPa at 20 °C

  water: $p_v = 2.34$ kPa at 20 °C

  mercury: $p_v = 0.00017$ kPa at 15.6 °C

- mercury : low vapor pressure and high density = difficult to vaporize

  → suitable for pressure-measuring devices

- Cavitation: App. 7 (p. 672)

  In a flow fluid wherever the local pressure falls to the vapor pressure of the liquid,

  local vaporization occurs.

  In the interior and/or boundaries of a liquid system

  → Cavities are formed in the low pressure regions.

  High velocity region

  → The cavity contains a swirling mass of droplets and vapor.

  → Cavities are swept downstream into a region of high pressure.

→ Then, cavities are collapses suddenly.

→ surrounding liquid rush into the void together

→ it causes erosion (pitting) of solid boundary surfaces in machines, and vibration

→ boundary wall receives a blow as from a tiny hammer

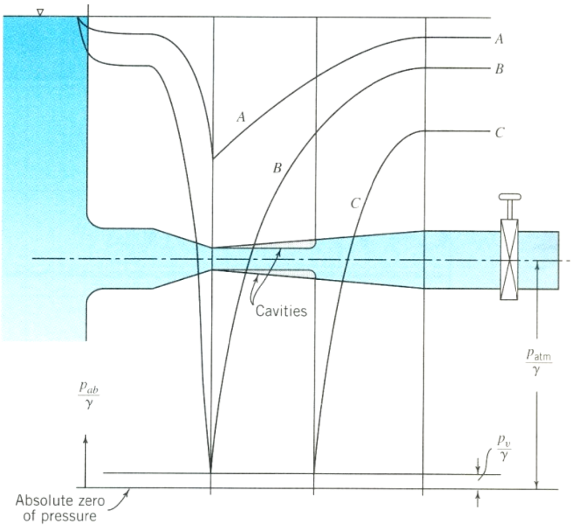


Fig. A.1

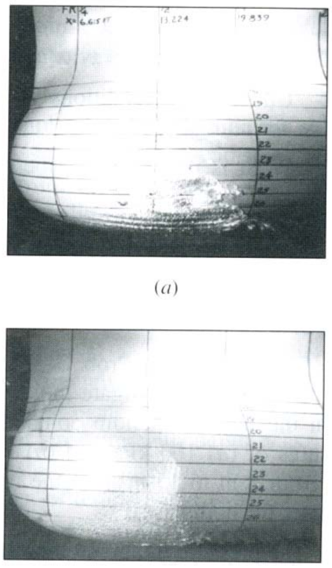
- Prevention of cavitation

~ cavitation is of great importance in the design of high-speed hydraulic machinery such as turbines, pumps, in the overflow and underflow structures of high dams, and in high-speed motion of underwater bodies (submarines, hydrofoils).

Body cavitation

→ design improved forms of boundary surfaces

→ predict and control the exact nature of cavitation → set limits



(a)

(b)

FIGURE 2–28

- Boiling:

= rapid rate of vaporization caused by an increase in temperature

= formation of vapor bubbles throughout the fluid mass

~ occur (whatever the temperature) when the external absolute pressure imposed on the liquid is equal to or less than the vapor pressure of the liquid

~ boiling point = f (imposed pressure, temp.)

$$p_{atm} \leq p_v \rightarrow \text{boiling occurs}$$

Table A2.4b

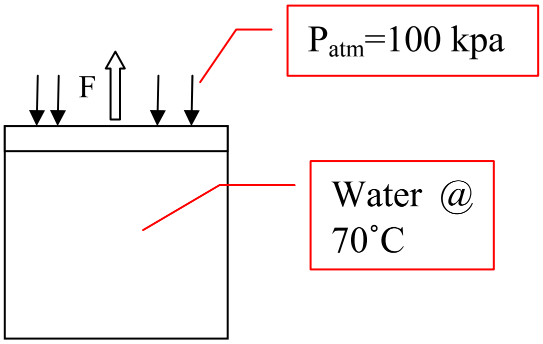
Table A2.5b

[Ex] boiling point of water

altitude (El. m)	Temp. (°C)	p_v (kPa), absolute	p_{atm} (kPa), absolute	boiling point (°C)	remark
m.s.l.	100	101.3	101.3	100	
12,000	60	19.9	19.4	60	undercooked

- Evaporation: When the space surrounding the liquid is too large, the liquid continues to vaporize until the liquid is gone and only vapor remains at a pressure less than or equal p_v.

[IP 1.12] For a vertical cylinder of diameter 300 mm, find min. force that will cause the water boil.



[Sol] From Table A2.4b; p_v=31.16 kPa at 70 °C

For water to boil; $p^{'} \le p_v$=31.16,

$$\therefore \quad p^{'} = 100 - \frac{F}{A} = 31.16$$

$$\therefore \quad F = (100 - 31.16)\frac{\pi(0.3)^2}{4} = 4.87 \text{ kN}$$

Homework Assignment # 1

Due: 1 week from today

Prob. 1.2

Prob. 1.10

Prob. 1.27

Prob. 1.46

Prob. 1.49

Prob. 1.58

Prob. 1.69

Prob. 1.72

Prob. 1.82