Deformation of Concrete Structures

Fall 2010

Dept of Architecture

1st week

Overview of design codes

Course Preview

- Code provisions
- Fib
- Euro
- ACI
- Basics in Deformation
- Axial deformation
- Flexural deformation
- Creep and Shrinkage
- Shear deformation of Plane stress elements

Text book and References

1. Concrete Structures : Stresses and Deformation by A. Ghali and R Favre
2. Chap. 11 in Reinforced Concrete Structures by R Park and T. Paulay
3. Ductility of Reinforced Concrete Structures CEB Bulletin 242
4. Strength and Deformation of Structural Concrete by Kaufmann

Objectives of Course

1. Understanding of Concepts in Design Codes
2. Basic Concepts of Deformation of Concrete Structures: Axial and Bending
3. Ductility
4. Plane stress elements in shear
5. Mechanisms based Deformation Capacity Estimation

Code Provisions-Fib

- 7.6 Verification of Serviceability of RC and PC structures

7.6.1 Requirement

7.6.2 Design Criteria

7.6.3 Stress limitation

7.6.4 Limit state of cracking

7.6.5 Limit state of deformation

7.6.1 Requirement

- Stresses
- Crack width
- Deformation
- vibrations

7.6.2 Design criteria

- Characteristic value
- Combination value
- Frequent value
- Quasi-permanent value

7.2. DESIGN CRITERIA

For the verification of serviceability limit states all direct and indirect actions (loads or imposed or restrained deformations) should be taken into account.

The design criteria depend on the type of SLS and are given in clause 1.6.6.2.

The partial safety coefficients are taken equal to 1.0.

The combination of loads to be considered depends on the type of SLS and on the specific problem. It is suitable to utilize one of the combinations given in clause 1.6.6.5, i.e.

- rare combination
- frequent combination
- quasi-permanent combination.

Prestressing forces should be considered as permanent actions.

The relevant values of the prestressing force depend on the type of SLS and the problem considered. Prestressing force values to be considered are suggested in section 4.6.

For structural analysis any appropriate method may be used, which takes account of the material behaviour under service loads.

7.6.3 stress limitation

- Tensile stresses in concrete
- Compressive stresses in concrete
- Tensile stresses in the steel

7.3. STRESS LIMITATION

Under service load conditions the limitation of stresses may be required for

- tensile stresses in concrete
- compressive stresses in concrete
- tensile stresses in steel.

The limitation of tensile stresses in concrete is an adequate measure to reduce the probability of cracking.

The limitation of compressive stresses in concrete should avoid excessive compression, producing irreversible strains and longitudinal cracks.

Tensile stresses in reinforcement should be limited with an appropriate safety margin below the yielding strength, preventing uncontrolled cracking.

In calculating the stress, account shall be taken of whether the section is expected to crack under service loads and also of the effects of creep, shrinkage and relaxation of prestressing steel. Other indirect actions which could influence the stress, such as temperature, should also be considered.

7.3.1. Tensile stresses in concrete

Depending on the stress limit chosen different limit states may be required, but the LS of decompression is considered to be the most significant. Stresses may be calculated on the basis of a homogeneous uncracked concrete section (state I). The contribution of reinforcement to the area and section modulus of the cross-section may be taken into account.

7.3.1.1. Limit state of decompression

The limit state of decompression is defined as the state where all axial concrete stresses are below or equal to zero.

7.3.2. Compressive stresses in concrete

Excessive compressive stress in the concrete under service load may lead to longitudinal cracks and high and hardly predictable creep, with serious consequences to prestress losses. When such effects are likely to occur, measures should be taken to limit the stresses to an appropriate level.

If the stress does not exceed $0.6f_{ck}(t)$

- under the rare combination, longitudinal cracking is unlikely to occur
- under the quasi-permanent combination, creep and the corresponding prestress losses can be correctly predicted.

If under the quasi-permanent combination the stress exceeds $0.4f_{ck}(t)$, the non-linear model shall be used for the assessment of creep (see clause 2.1.6.4.3(d)).

7.3.3. Steel stresses

Tensile stresses in the steel under serviceability conditions which could lead to inelastic deformation of the steel shall be avoided as this will lead to large, permanently open, cracks.

7.6.4 Limit state of cracking

- Requirement
- Design criteria vs. cracking
- Limitation of crack width
- Calculation of crack width in RC members

7.4. LIMIT STATE OF CRACKING

7.4.1. Requirements

It should be ensured that, with an adequate probability, cracks will not impair the serviceability and durability of the structure.

Cracks do not, per se, indicate a lack of serviceability or durability; in reinforced concrete structures, cracking may be inevitable due to tension, bending, shear, torsion (resulting from either direct loading or restraint of imposed deformations), without necessarily impairing serviceability or durability.

However, the following specific requirements should generally be respected.

7.4.1.1. Function requirements

The function of the structure should not be harmed by the cracks formed.

In relevant cases, nominal crack width limits may be agreed with the client, unless reference is made to more simplified design means.

7.4.1.2. Durability

The durability of the structure during its intended lifetime should not be harmed by the cracks formed.

7.6.5 Limit states of deformation

- General
- Deformations due to bending with or w/o axial force
- Instantaneous deflection
- Long-term deflections

7.5. LIMIT STATES OF DEFORMATION

7.5.1. General

7.5.1.1. Requirements

In-service deformations (deflections and rotations) may be harmful to

- the appearance of the structure
- the integrity of non-structural parts
- the proper function of the structure or its equipment.

To avoid harmful effects of deformations appropriate limiting values should be respected.

7.5.1.2. Combination of actions

The combinations of actions to be considered depend on the criteria in question and are defined in section 7.2.

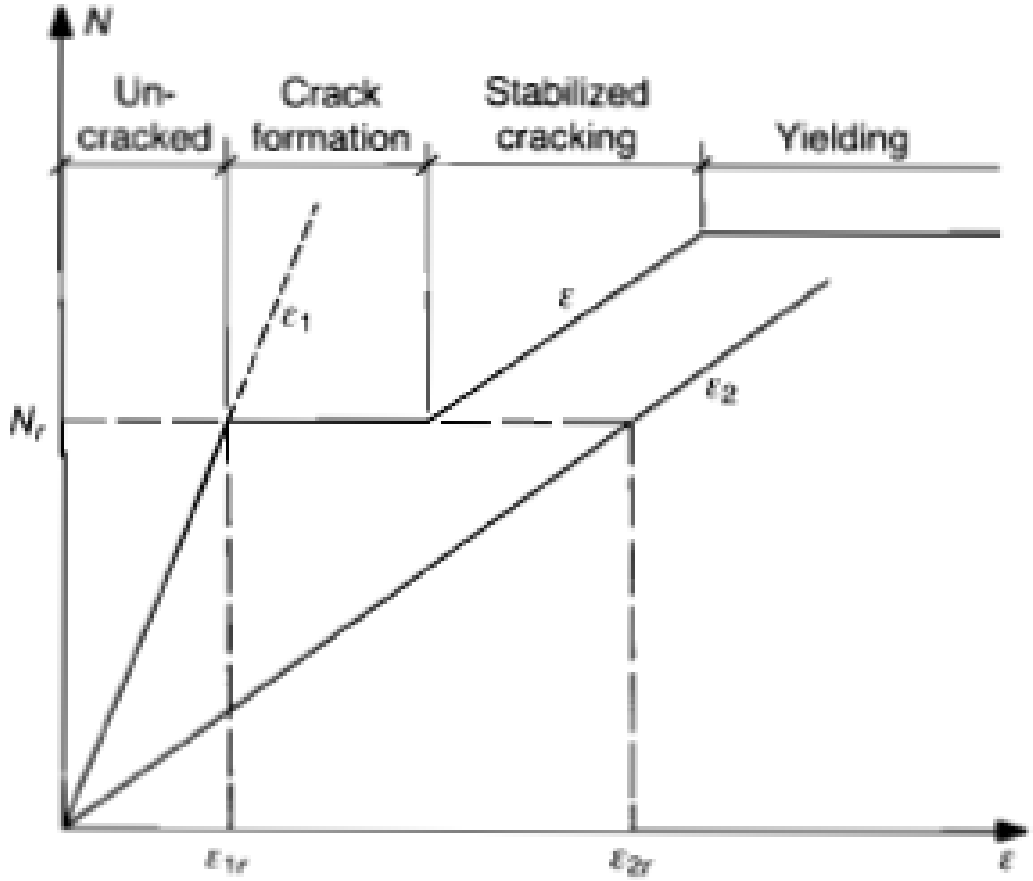


Fig. 7.5.2. Instantaneous mean strain

Euro code

Section 7 Serviceability limit state

7.1 General

7.2 Stress limitation

7.3 Crack control

7.4 Deflection control

- The **SLS** for the selected design situations during execution needs to be verified, as appropriate, in accordance with EN 1990.
- The criteria associated with the **SLS** during execution should take into account the ***requirements for the completed structure***.
- Operations which can cause *excessive cracking* and/or *early deflection during execution* and which may adversely affect the durability, fitness for use and/or aesthetic appearance in the final stage has to be avoided.

The combinations of actions should be established in accordance with EN 1990. In general, the relevant combinations of actions for transient design situations during execution are:

- the **characteristic** combination
- the **quasi-permanent** combination

SLS: combinations of actions.

Characteristic combination (irreversible SLS)

$$\sum_{j\geq 1} G_{k,j} \text{"+"} P \text{"+"} Q_{k,1} \text{"+"} \sum_{i>1} \psi_{0,i} Q_{k,i}$$

Quasi-permanent combination (reversible SLS)

$$\sum_{j\geq 1} G_{k,j} \text{"+"} P \text{"+"} \sum_{i\geq 1} \psi_{2,i} Q_{k,i}$$

Deformation of cracked concrete in tension

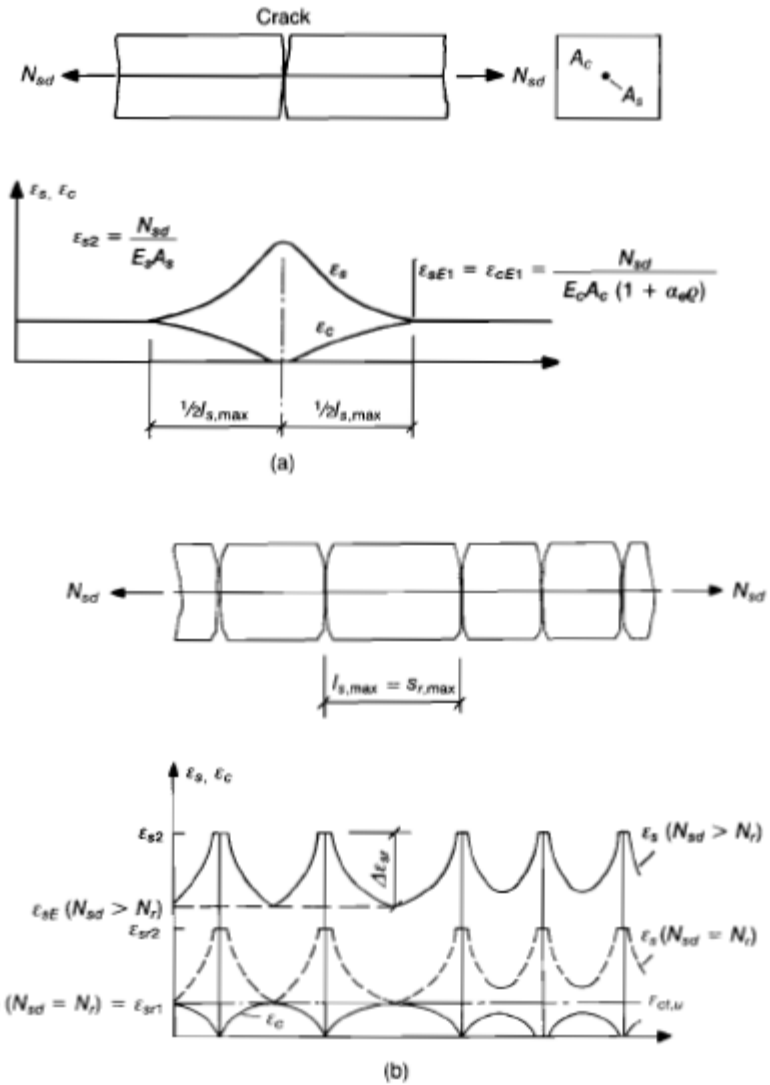


Fig. 7.4.1. Strains for calculating the crack spacing and the average strains: (a) for single cracks; (b) for stabilized cracking

1) N before cracking lower than the following value

$$N_r = f_{ct}\left(A_c + nA_s\right) = f_{ct}A_1$$

$$n = \frac{E_s}{E_c}$$

$$\sigma_{sr} = \frac{N_r}{A_s}$$

Stress in steel

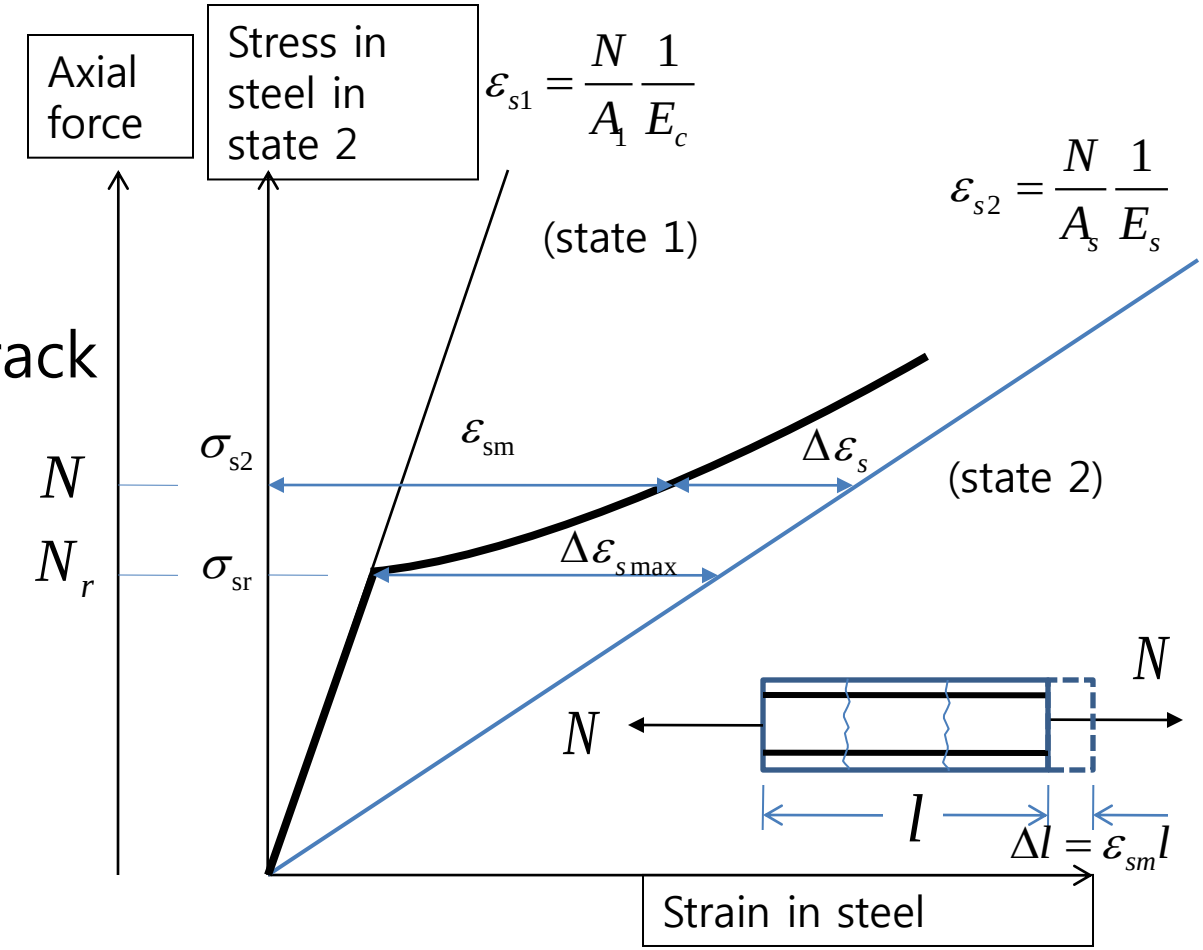
2) After cracking

$$N > N_r$$

Stress in steel at a crack

$$\sigma_{s2} = \frac{N}{A_s}$$

$$\varepsilon_{s2} = \frac{N}{A_s}\frac{1}{E_s}$$



$$\varepsilon_{s1} = \varepsilon_{c1} = \frac{N}{E_c\left(A_c + nA_s\right)} = \frac{N}{E_c A_1}$$

$$\Delta\varepsilon_s = \underbrace{\varepsilon_{s2}}_{\text{Fully cracked}} - \underbrace{\varepsilon_{sm}}_{\text{Mean steel strain}}$$

Assume strain difference has hyperbolic variation with stress in steel

$$\Delta\varepsilon_s = \Delta\varepsilon_{s\max}\frac{\sigma_{sr}}{\sigma_{s2}}$$

$$\boxed{\Delta\varepsilon_{s\max} = \left(\varepsilon_{s2} - \varepsilon_{s1}\right)\frac{\sigma_{sr}}{\sigma_{s2}}}$$

$$\begin{aligned}
\varepsilon_{sm} &= \varepsilon_{s2} - \Delta\varepsilon_s \\
&= \varepsilon_{s2} - \Delta\varepsilon_{s\max}\frac{\sigma_{sr}}{\sigma_{s2}} \\
&= \varepsilon_{s2} - \left(\varepsilon_{s2} - \varepsilon_{s1}\right)\left(\frac{\sigma_{sr}}{\sigma_{s2}}\right)^2 \\
&= \left(\frac{\sigma_{sr}}{\sigma_{s2}}\right)^2 \varepsilon_{s1} + \left[1 - \left(\frac{\sigma_{sr}}{\sigma_{s2}}\right)^2\right]\varepsilon_{s2}
\end{aligned}$$

Let $\zeta = 1 - \left(\frac{\sigma_{sr}}{\sigma_{s2}} \right)^2$

$$\varepsilon_{sm} = \left(1 - \zeta\right) \varepsilon_{s1} + \zeta \varepsilon_{s2}$$

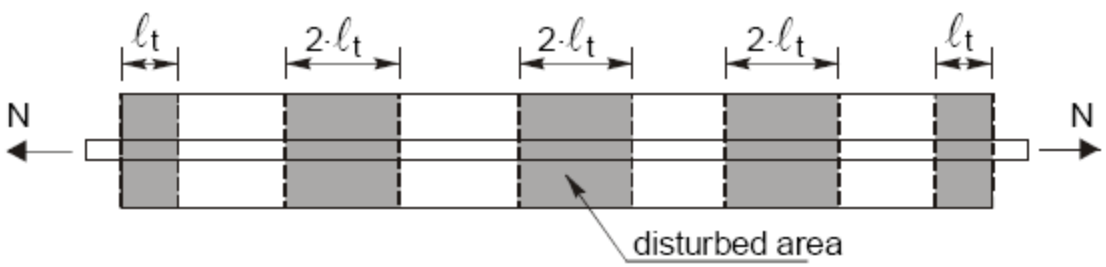
$$\zeta = 1 - \beta_1 \beta_2 \left(\frac{\sigma_{sr}}{\sigma_{s2}} \right)^2$$

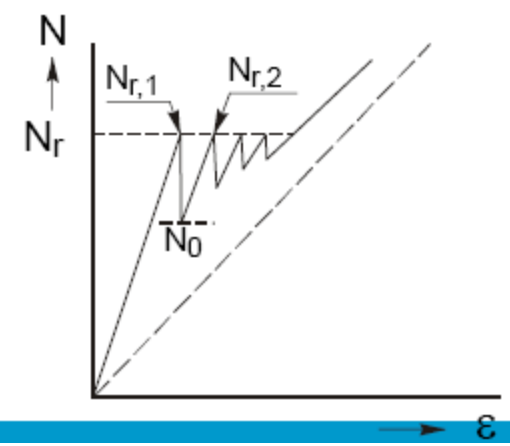
β_1 : bond effect

β_2 : loading characteristics

Theory of crack width control (4)

When more cracks occur, more disturbed regions are found in the concrete tensile bar. In the N-ε relation this stage (the "crack formation stage" is characterized by a "zig-zag"-line ($N_{r,1}$-$N_{r,2}$). At a certain strain of the bar, the disturbed areas start to overlap.
If no intermediate areas are left, the concrete cannot reach the tensile strength anymore, so that no new cracks can occur. The "crack formation stage" is ended and the stabilized cracking stage starts. No new cracks occur, but existing cracks widen.

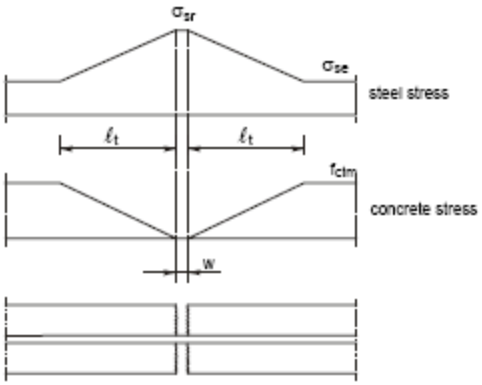




EC-formulae for crack width control (1)

For the calculation of the maximum (or characteristic) crack width, the difference between steel and concrete deformation has to be calculated for the largest crack distance, which is $s_{r,max} = 2l_t$. So

$$w_k = s_{r,max}\left(\varepsilon_{sm} - \varepsilon_{cm}\right) \qquad \text{Eq. (7.8)}$$



where

$s_{r,max}$ is the maximum crack distance

and

$(\varepsilon_{sm} - \varepsilon_{cm})$ is the difference in deformation between steel and concrete over the maximum crack distance.

Accurate formulations for $s_{r,max}$ and $(\varepsilon_{sm} - \varepsilon_{cm})$ will be given

EC-2 formulae for crack width control (2)

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \dfrac{f_{ct,eff}}{\rho_{p,eff}}(1+\alpha_e \rho_{p,eff})}{E_s} \geq 0{,}6\frac{\sigma_s}{E_s} \qquad \text{Eq. 7.0}$$

where: σ_s is the stress in the steel assuming a cracked section

α_e is the ratio E_s/E_{cm}

$\rho_{p,eff} = (A_s + \xi A_p)/A_{c,eff}$ (effective reinforcement ratio including eventual prestressing steel A_p

ξ is bond factor for prestressing strands or wires

k_t is a factor depending on the duration of loading (0,6 for short and 0,4 for long term loading)

EC-3 formulae for crack width control (4)

Maximum final crack spacing $s_{r,max}$

$$s_{r,max} = 3.4c + 0.425\, k_1 k_2 \frac{\phi}{\rho_{p,eff}} \qquad \text{(Eq. 7.11)}$$

where c is the concrete cover

Φ is the bar diameter

k_1 bond factor (0,8 for high bond bars, 1,6 for bars with an effectively plain surface (e.g. prestressing tendons)

k_2 strain distribution coefficient (1,0 for tension and 0,5 for bending: intermediate values van be used)

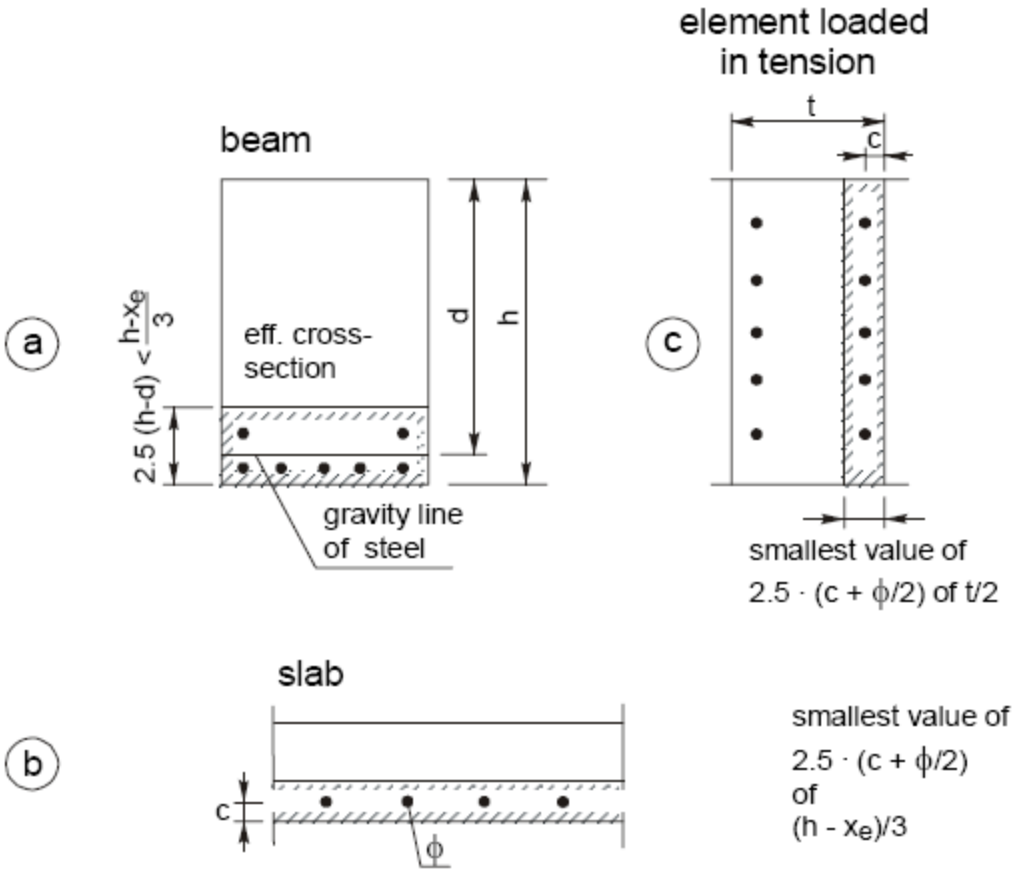
EC-2 requirements for crack width control (recommended values)

Exposure class	RC or unbonded PSC members	Prestressed members with bonded tendons
	Quasi-permanent load	Frequent load
X0,XC1	0.3	0.2
XC2,XC3,XC4	0.3	
XD1,XD2,XS1,XS2, XS3		Decompression

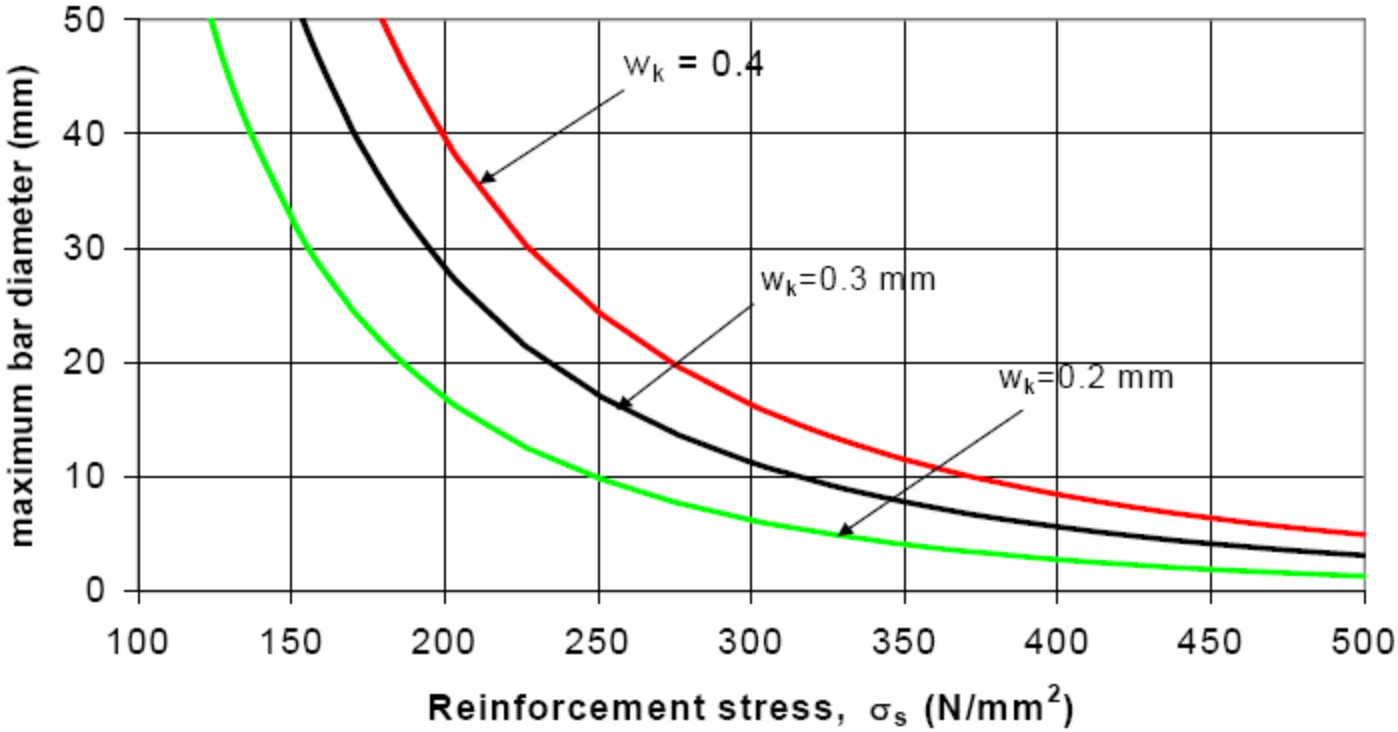
EC-2 formulae for crack width control (5)

In order to be able to apply the crack width formulae, basically valid for a concrete tensile bar, to a structure loaded in bending, a definition of the "effective tensile bar height" is necessary. The effective height $h_{c,ef}$ is the minimum of:

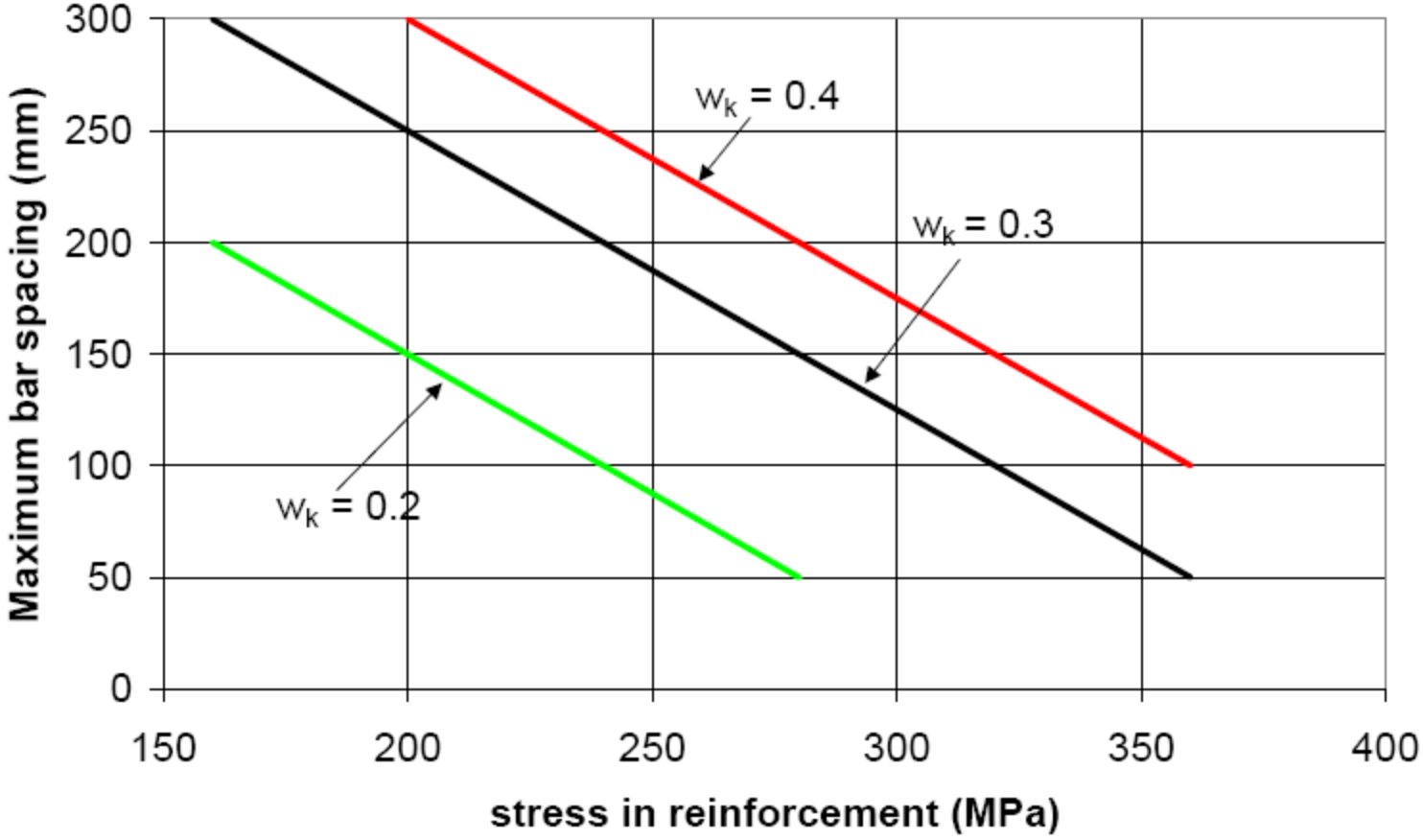
2,5 (h-d)
(h-x)/3
h/2



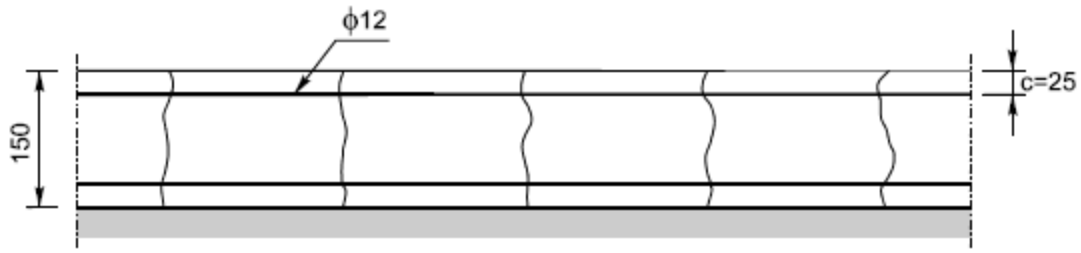
Maximum bar diameters for crack control (simplified approach 7.3.3)



Maximum bar spacing for crack control (simplified approach 7.3.3)



Example (1)



Continuous concrete road

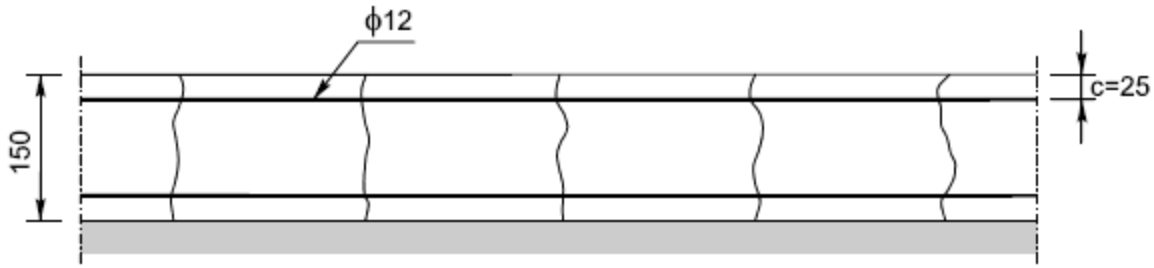
Data: Concrete C20/25, f_{ctm} = 2,2 MPa, shrinkage $\varepsilon_{sh}=0{,}25\cdot10^{-3}$, temperature difference in relation to construction situation $\Delta T=25^0$. Max. crack width allowed = 0,2mm.

Calculation

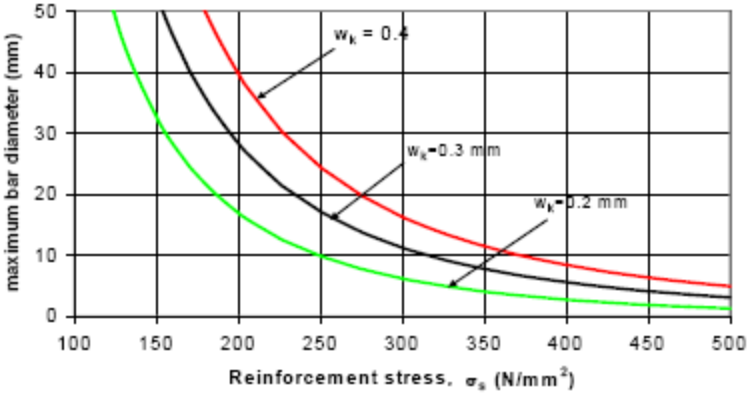
The maximum imposed deformation (shrinkage + temperature) is ε_{tot} = $0{,}50\cdot10^{-3}$. Loading is slow, so $E_{c,\infty}=E_c/(1+\varphi) \cong 30.000/(1+2)$ = 10.000 MPa. At **ε_{tot} = 0,50 ·10^{-3} a concrete tensile strength of 5 MPa applies, so the road is cracked.**

Cont. →

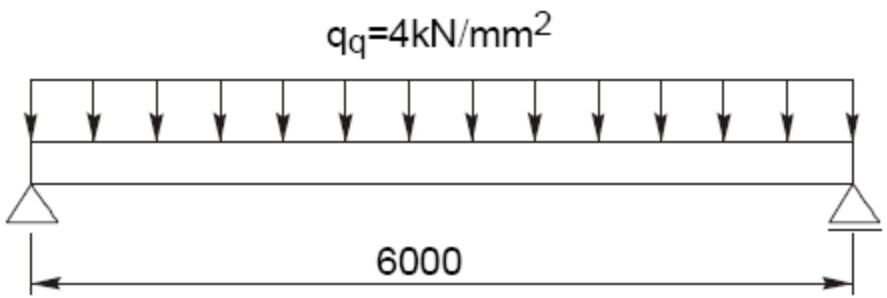
Example (1, cont.)

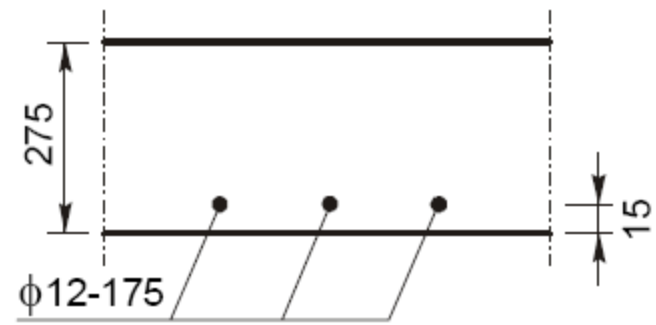


For imposed deformation the "crack formation stage" applies. So, the load will not exceed the cracking load, which is $N_{cr} = A_c(1+n\rho)f_{ctm} \cong 1{,}1A_cf_{ctm} = 330$ kN for b = 1m. From the diagram at the right it is found that a diameter of 12mm would require a steel stress not larger than 225 MPa. To meet this requirement d =12mm bars at distances 150mm, both at top and bottom, are required.



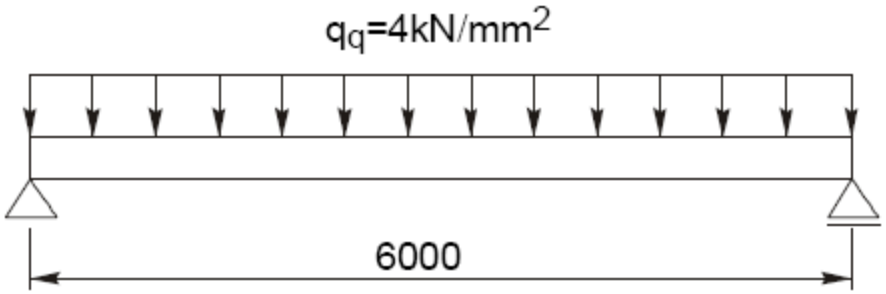
Example (2)

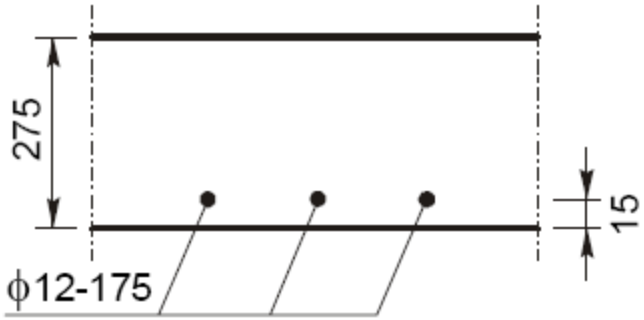




A slab bearing into one direction is subjected to a maximum variable load of 4KN/m^2. It should be demonstrated that the maximum crack width under the quasi permanent load combination is not larger than 0,4mm. (The floor is a part of a shopping centre: the environmental class is X0) (cont.→)

Example (2)

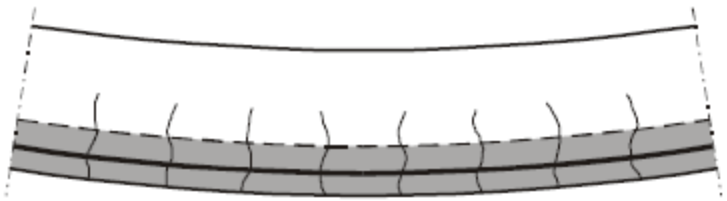


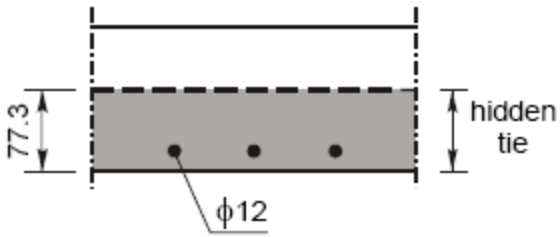


The governing load for the quasi-permanent load combination is: $q = q_g + \psi_2 \cdot q_{var.} = (0.275 \cdot 2500) + 0{,}6 \cdot 400 = 928$ kg/m^2. The maximum bending moment is then $M = 9{,}28 \cdot 6^2/8 = 41{,}8$ kNm/m′. For this bending moment the stress in the steel is calculated as $\sigma_s = 289$ MPa.

Cont.→

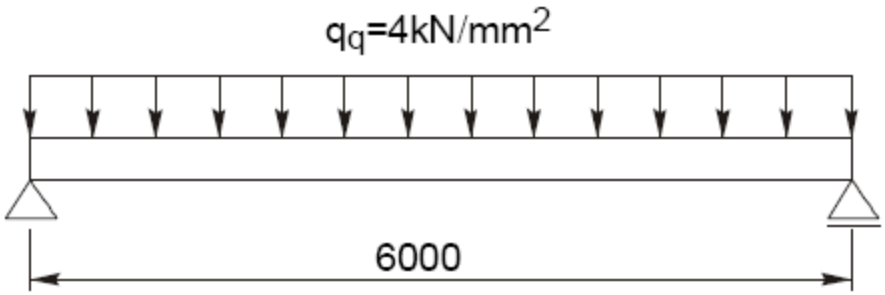
Example (2)

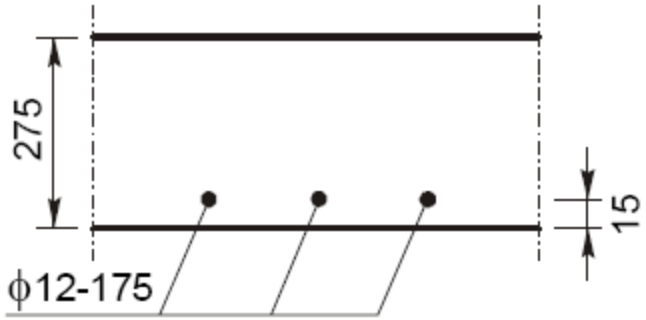




The effective height of the tensile tie is the minimum of 2,5(c+ ϕ/2) of (h-x)/3, where x = height of compression zone, calculated as 44mm. So, the governing value is (h-x)/3 = 77 mm. The effective reinforcement ratio is then ρ_{eff} = (113/0,175)/(77·1000)=0,83·10^{-2}. The crack distance $s_{r,amx}$ (Eq. 7.11) is found to be 245mm. For the term (ε_{sm}-ε_{cm}) a value 1,0·10^{-3} is found. This leads to a cracks width equal to w_k= 0,25 mm, which his smaller than the required 0,4mm.

Example (2)





A slab bearing into one direction is subjected to a maximum variable load of 4KN/m^2. It should be demonstrated that the maximum crack width under the quasi permanent load combination is not larger than 0,4mm. (The floor is a part of a shopping centre: the environmental class is X0) (cont.→)

Reasons for controling deflections (1)

Appearance

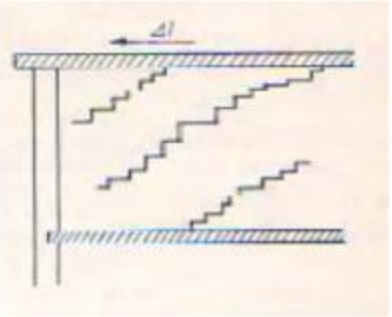
Deflections of such a magnitude that members appear visibly to sag will upset the owners or occupiers of structures. It is generally accepted that a deflection larger than span/250 should be avoided from the appearance point of view. A survey of structures in Germany that had given rise to complaints

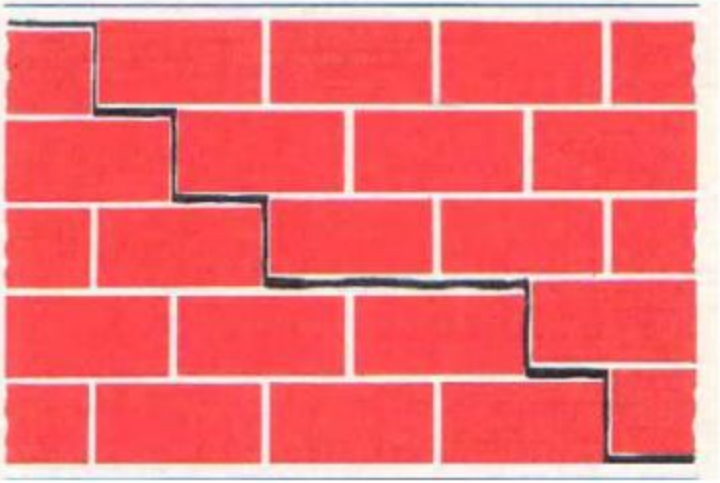
produced 50 examples. The measured sag was less than span/250 in only two of these.

Reasons for controling deflections (2)

<u>Damage to non-structural Members</u>

An important consequence of excessive deformation is damage to non structural members, like partition walls. Since partition walls are unreinforced and brittle, cracks can be large (several millimeters). The most commonly specified limit deflection is span/500, for deflection occurring after construction of the partitions. It should be assumed that all quasi permanent loading starts at the same time.

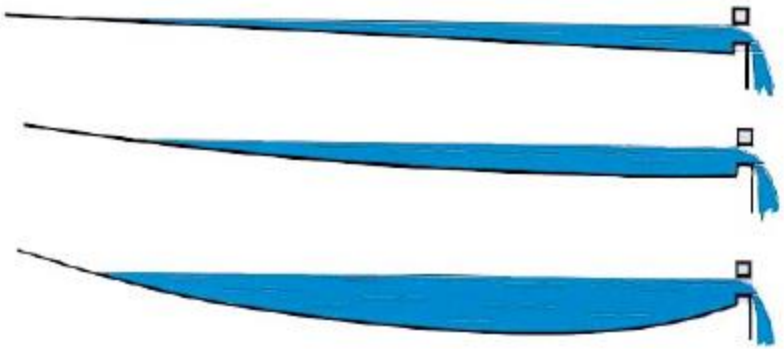




Reasons for controling deflections (3)

Collapse

In recent years many cases of collapse of flat roofs have been noted. If the rainwater pipes have a too low capacity, often caused by pollution and finally stoppage, the roof deflects more and more under the weight of the water and finally collapses. This occurs predominantly with light roofs. Concrete roofs are less susceptible for this type of damage

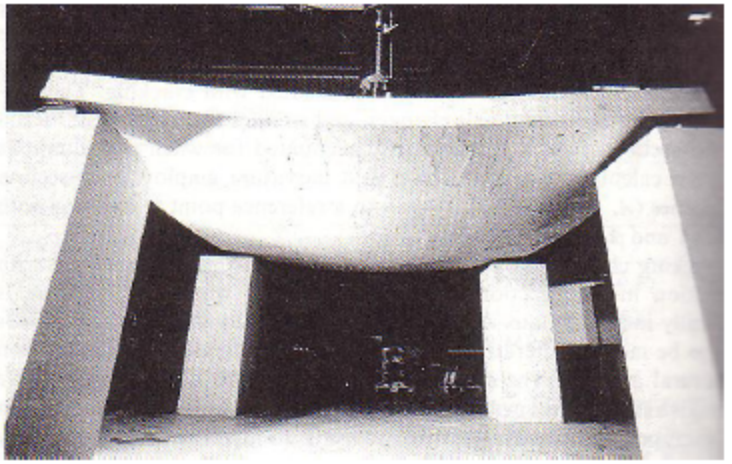


EC-2 Control of deflections

<u>Deflection limits according to chapter 7.4.1</u>

- Under the quasi permanent load the deflection should not exceed span/250, in order to avoid impairment of appearance and general utility

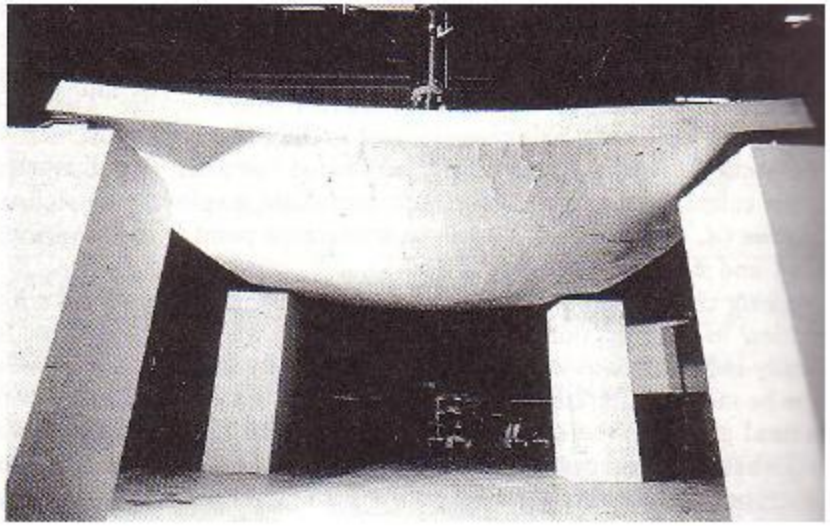
- Under the quasi permanent loads the deflection should be limited to span/500 after construction to avoid damage to adjacent parts of the structure



EC-2: SLS - Control of deflections

<u>Control of deflection can be done in two ways</u>

- By calculation
- By tabulated values



Calculating the deflection of a concrete member

The deflection follows from:

$$\delta = \zeta\, \delta_{II} + (1 - \zeta)\delta_{I}$$

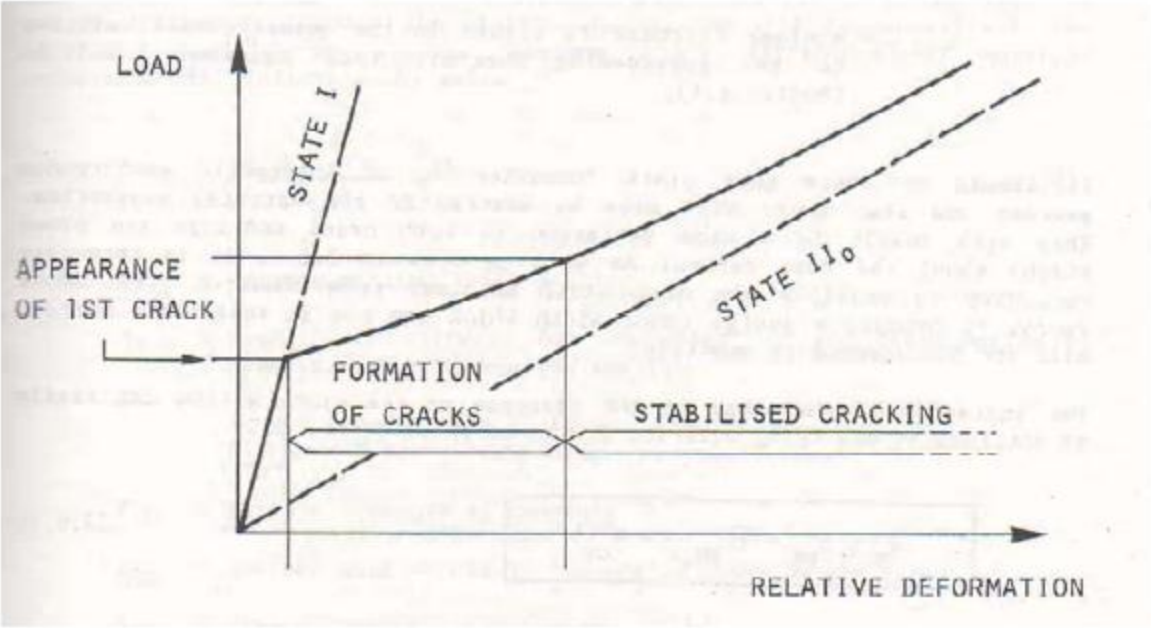
δ deflection
δ_I deflection fully cracked
δ_{II} deflection uncracked
ζ coefficient for tension stiffening (transition coefficient)

$$\zeta = 1 - \beta\, (\sigma_{sr}/\sigma_s)^2$$

σ_{sr} steel stress at first cracking
σ_s steel stress at quasi permanent service load
β 1,0 for single short-term loading
0,5 for sustained loads or repeated loading

Calculating the deflection of a concrete member

The transition from the uncracked state (I) to the cracked state (II) does not occur abruptly, but gradually. From the appearance of the first crack, realistically, a parabolic curve can be followed which approaches the line for the cracked state (II).



Calculating the deflection of a concrete member

For pure bending the transition factor

$$\xi = 1 - \beta(\sigma_s / \sigma_r)^2$$

can as well be written as

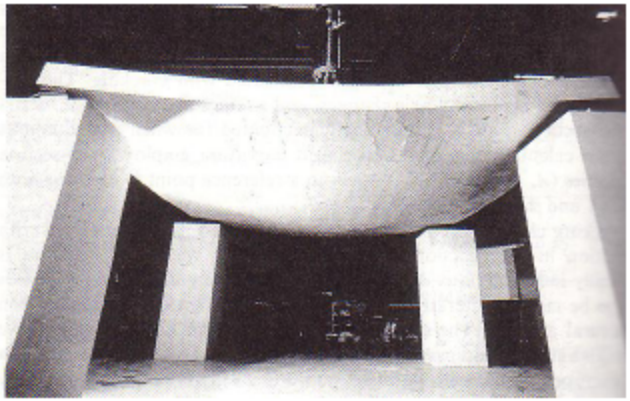
$$\xi = 1 - \beta(M_{cr} / M)^2$$

where M_{cr} is the cracking moment and M is the applied moment

Calculating the deflection of a concrete member

7.4.3 (7)

"The most rigorous method of assessing deflections using the method given before is to compute the curvatures at frequent locations along the member and then calculate the deflection by numerical integration.



In most cases it will be acceptable to compute the deflection twice, assuming the whole member to be in the uncracked and fully cracked condition in turn, and then interpolate using the expression:

$$\xi = 1 - \beta (M_{cr} / M)^2$$

Cases where detailed calculation may be omitted

In order to simplify the design, expressions have been derived, giving limits of l/d for which no detailed calculation of the deflection has to be carried out.

These expressions are the results of an extended parameter analysis with the method of deflection calculation as given before. The slenderness limits have been determined with the criteria $\delta < L/250$ for quasi permanent loads and $\delta < L/500$ for the additional load after removing the formwork

The expressions, which will be given at the next sheet, have been calculated for an assumed steel stress of 310 MPa at midspan of the member. Where other stress levels are used, the values obtained by the expressions should be multiplied with $310/\sigma_s$

Calculating the deflection of a concrete member

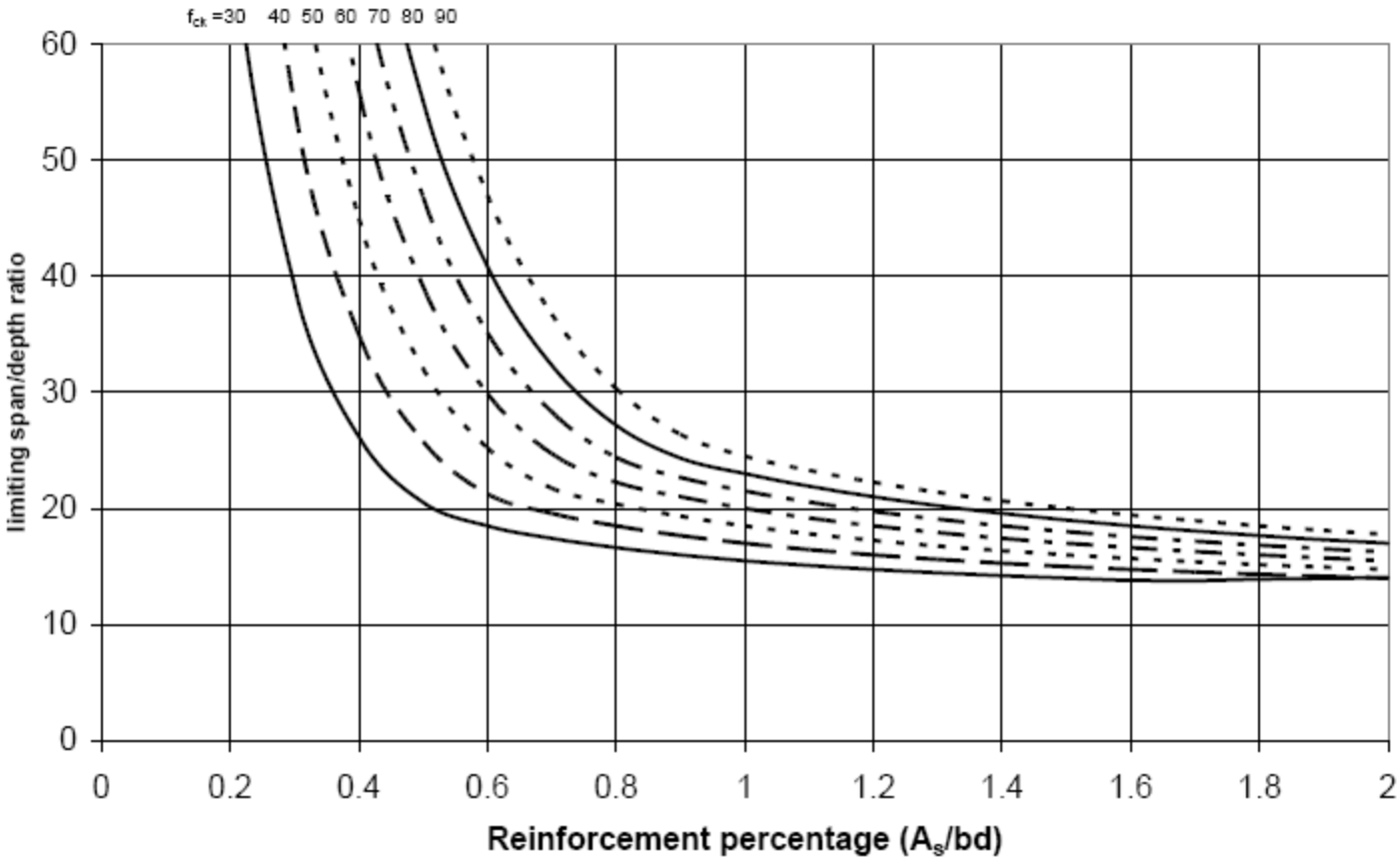
For span-depth ratios below the following limits no further checks is needed

$$\frac{l}{d} = K\left[11 + 1{,}5\sqrt{f_{ck}}\,\frac{\rho_0}{\rho} + 3{,}2\sqrt{f_{ck}}\left(\frac{\rho_0}{\rho} - 1\right)^{3/2}\right] \qquad \text{if } \rho \le \rho_0 \qquad (7.16.a)$$

$$\frac{l}{d} = K\left[11 + 1{,}5\sqrt{f_{ck}}\,\frac{\rho_0}{\rho - \rho'} + \frac{1}{12}\sqrt{f_{ck}}\sqrt{\frac{\rho'}{\rho_0}}\right] \qquad \text{if } \rho > \rho_0 \qquad (7.16.b)$$

l/d	is the limit span/depth
K	is the factor to take into account the different structural systems
ρ_0	is the reference reinforcement ratio = $\sqrt{f_{ck}}\ 10^{-3}$
ρ	is the required tension reinforcement ratio at mid-span to resist the moment due to the design loads (at support for cantilevers)
ρ'	is the required compression reinforcement ratio at mid-span to resist the moment due to design loads (at support for cantilevers)

Previous expressions in a graphical form (Eq. 7.16):



Limit values for l/d below which no calculated verification of the deflection is necessary

The table below gives the values of K (Eq.7.16), corresponding to the structural system. The table furthermore gives limit l/d values for a relatively high (ρ=1,5%) and low (ρ=0,5%) longitudinal reinforcement ratio. These values are calculated for concrete C30 and σ_s = 310 MPa and satisfy the deflection limits given in 7.4.1 (4) and (5).

Structural system	K	ρ = 0,5%	ρ = 1,5%
Simply supported slab/beam	1,0	l/d=14	l/d=20
End span	1,3	l/d=18	l/d=26
Interior span	1,5	l/d=20	l/d=30
Flat slab	1,2	l/d=17	l/d=24
Cantilever	0,4	l/d= 6	l/d=8

Annexes (all informative)

Annex K – Effect of temperature on the properties of concrete

Annex L – Calculation of strians and stresses in concrete sections subjected to restrained imposed deformations

Annex M – Calculation of crack widths due to restraint of imposed deformations

Annex N – Provision of movement joints

Annex K – Effect of temperature in the properties of concrete.

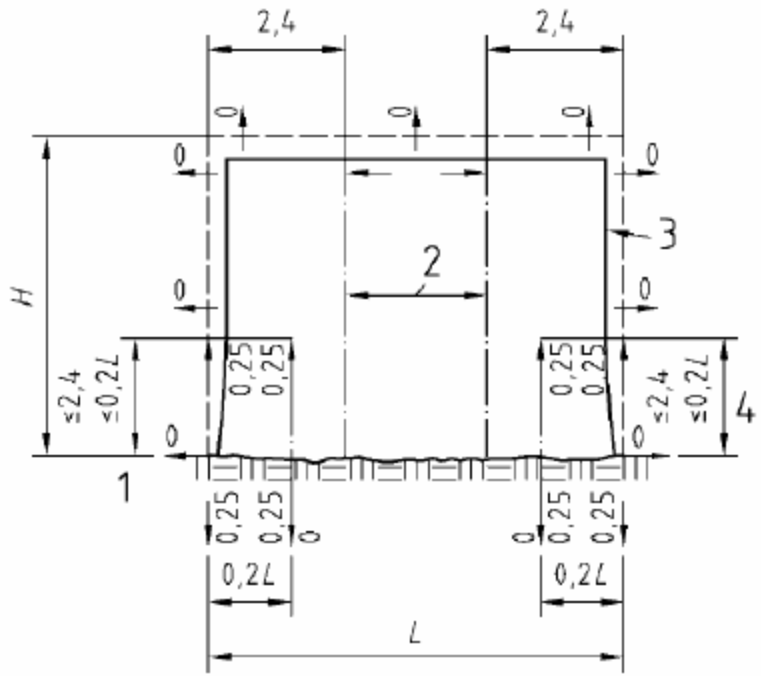
- Material enhancements given for sub zero temperatures – not always conservative to ignore.
- For elevated temperatures reference to fire part, to avoid duplication.
- Methods presented to calculate increased creep (and transitional thermal strain) and reduced elastic modulus.

Annex L – Calculation of the strains and stresses in concrete sections subjected to restrained imposed deformations

Actual strain $\varepsilon_{az} = (1-R_{ax})\varepsilon_{i_{av}}$

Stress in concrete $\sigma_z = E_{c,eff}\,(\varepsilon_{i_{av}} - \varepsilon_{az})$

Restraint Factor

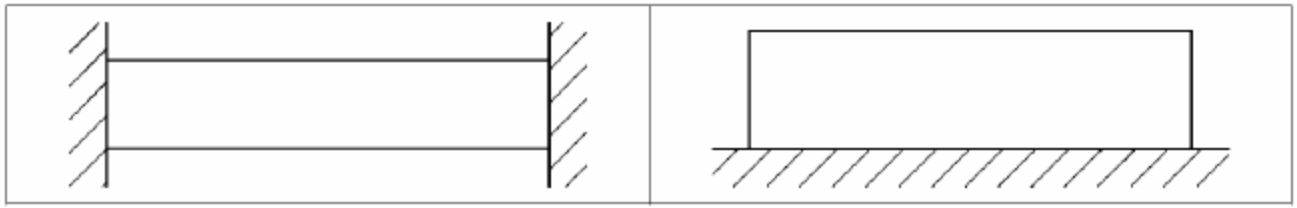


(a) Wall on base

Restraint Factor

Annex M – Calculation of crack widths due to restraint of imposed deformations

Two case considered:

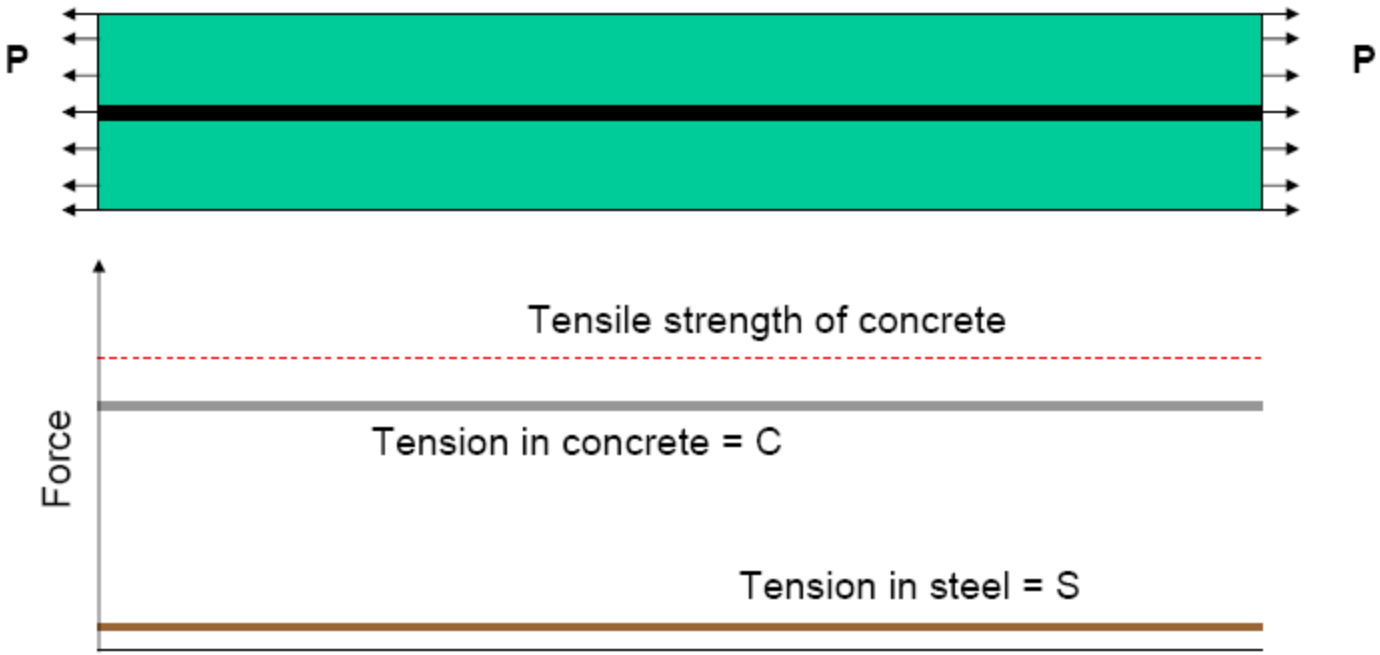


(a) restraint of a member at its ends

(b) restraint along one edge

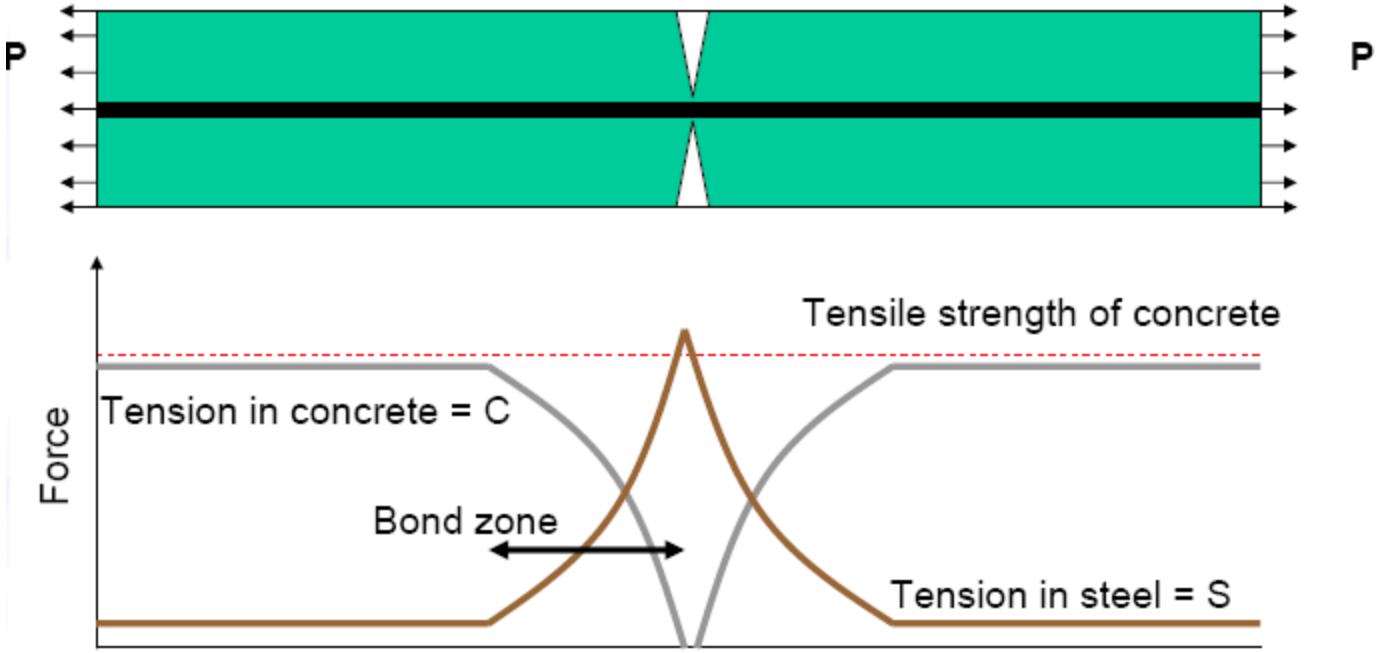
Figure M.1 — Types of restraint to walls

End Restraint



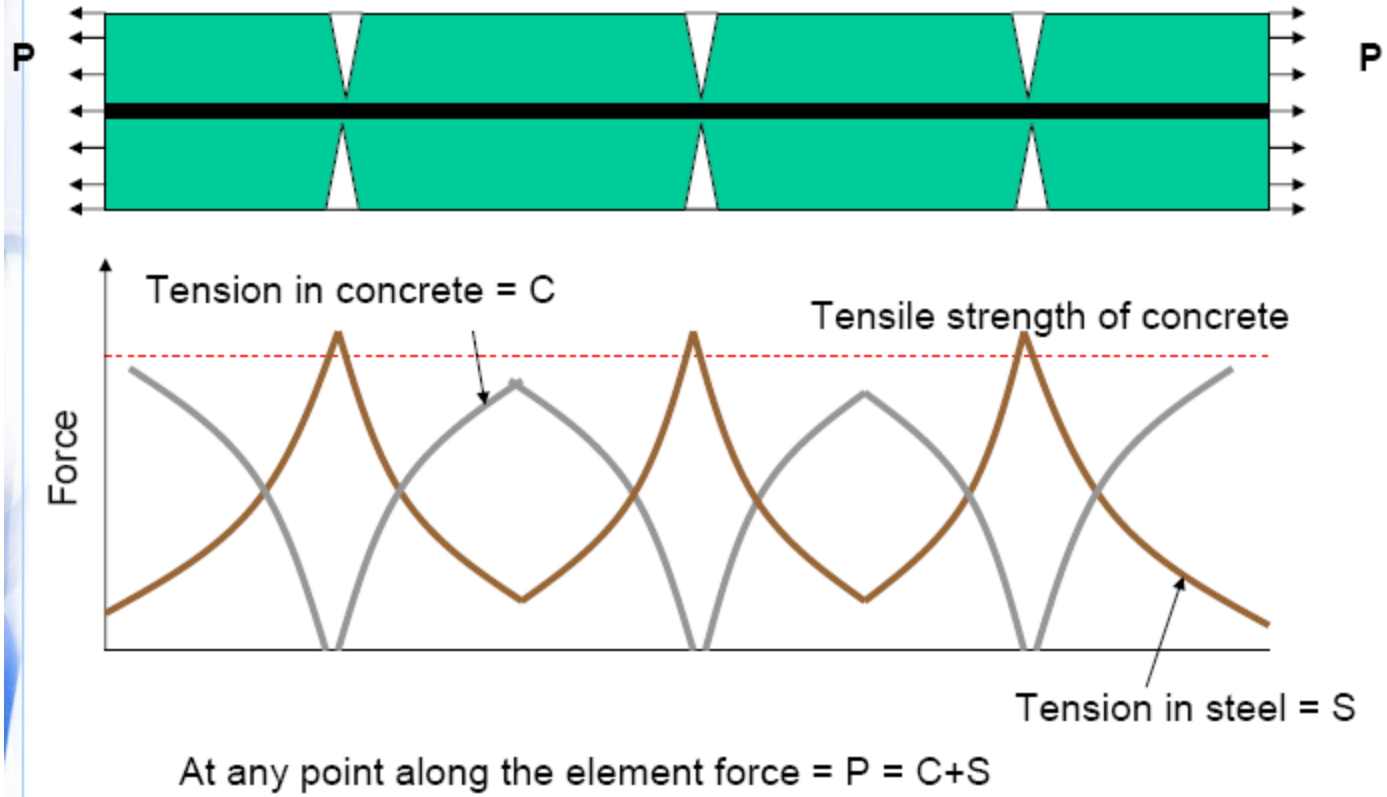
At any point along the element force = P = C+S

End Restraint



At any point along the element force = P = C+S

End Restraint



At any point along the element force = P = C+S

$$\varepsilon_{sm}-\varepsilon_{cm}=0{,}5\alpha_e k_c k f_{ct,eff}(1+1/(\alpha_e\rho))/E_s$$

Edge Restraint

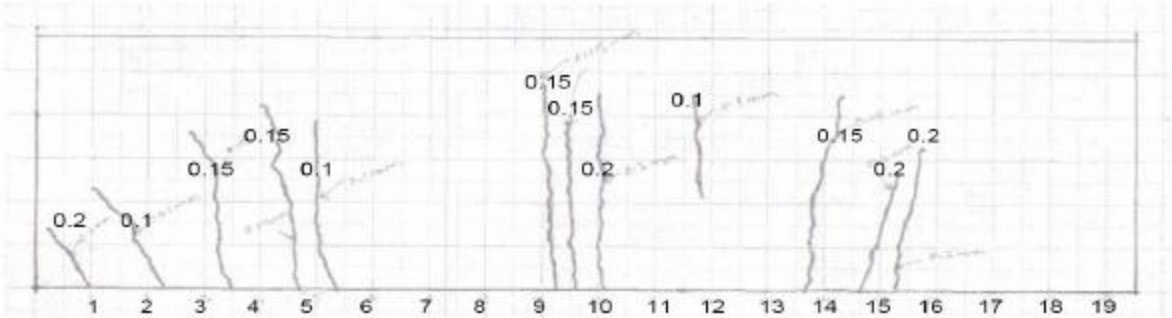


Figure 2 *Crack pattern observed in a 19.5m long wall*

From Bamforth

$$\varepsilon_{sm} - \varepsilon_{cm} = R_{ax}\ \varepsilon_{free}$$

National Choices

- Definition of w_{k1} (crack width limit for tightness class 1 structures)
- X_{min} depth of section to remain in compression for tightness class 2 structures
- κ maximum duct size related to wall thikness
- t_1 and t_2 minimum wall thicknesses for class 0 and class 1 or 2 structures respectively.

ACI code provisions (318-08)

Chap. 9.5 control of deflection

Chap. 10.6 Distribution of flexural reinforcement in beams and one-way slabs

Basics in deformation

- Axial deformation
- Cracks
- Tension stiffening
- Bond
- Virtual work method
- Uncracked and cracked sections

1) N before cracking lower than the following value

$$N_r = f_{ct}\left(A_c + nA_s\right) = f_{ct}A_1$$

$$n = \frac{E_s}{E_c}$$

$$\sigma_{sr} = \frac{N_r}{A_s}$$

Stress in steel

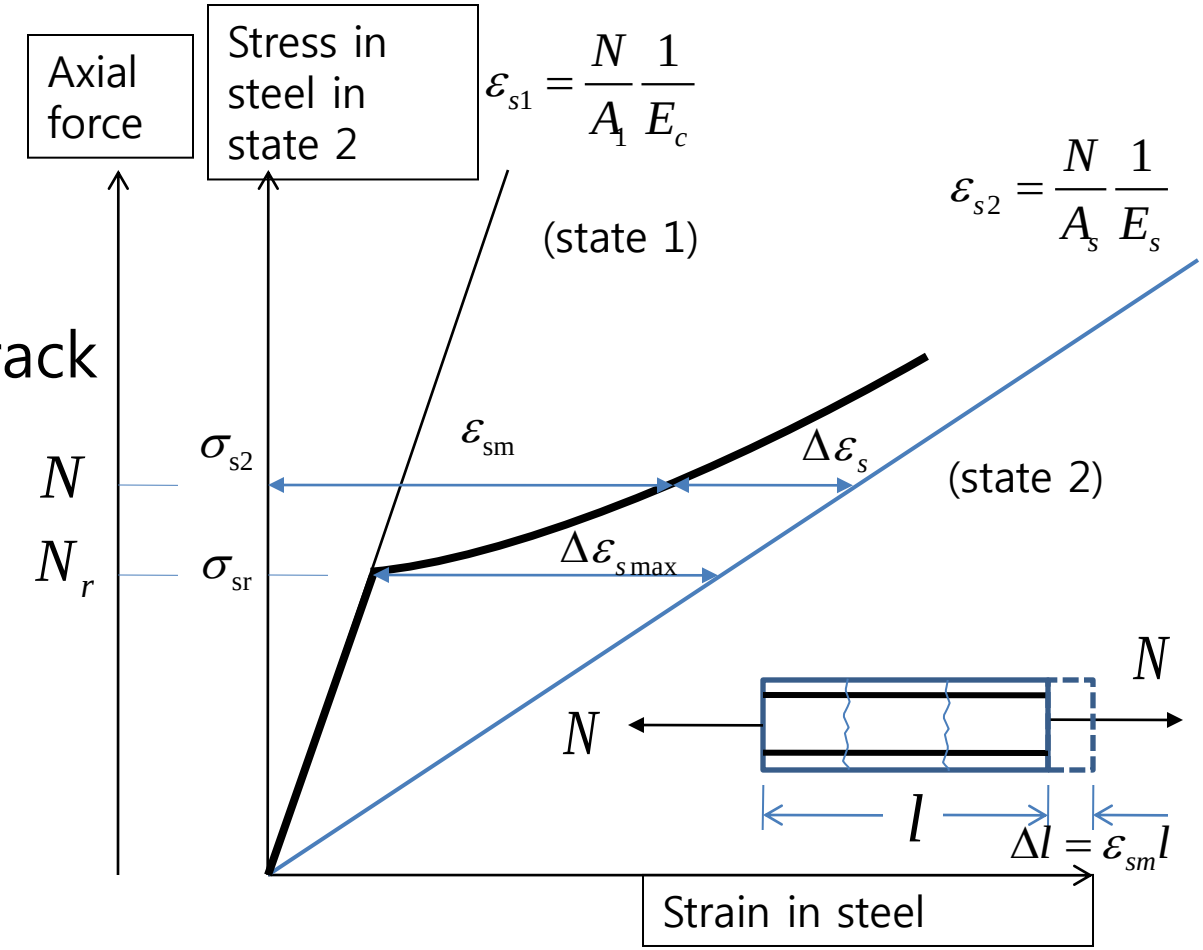
2) After cracking

$$N > N_r$$

Stress in steel at a crack

$$\sigma_{s2} = \frac{N}{A_s}$$

$$\varepsilon_{s2} = \frac{N}{A_s}\frac{1}{E_s}$$



$$\varepsilon_{s1} = \varepsilon_{c1} = \frac{N}{E_c\left(A_c + nA_s\right)} = \frac{N}{E_c A_1}$$

$$\Delta\varepsilon_s = \underbrace{\varepsilon_{s2}}_{\text{Fully cracked}} - \underbrace{\varepsilon_{sm}}_{\text{Mean steel strain}}$$

Assume strain difference has hyperbolic variation with stress in steel

$$\Delta\varepsilon_s = \Delta\varepsilon_{s\max}\frac{\sigma_{sr}}{\sigma_{s2}}$$

$$\boxed{\Delta\varepsilon_{s\max} = \left(\varepsilon_{s2} - \varepsilon_{s1}\right)\frac{\sigma_{sr}}{\sigma_{s2}}}$$

$$\begin{aligned}
\varepsilon_{sm} &= \varepsilon_{s2} - \Delta\varepsilon_s \\
&= \varepsilon_{s2} - \Delta\varepsilon_{s\max}\frac{\sigma_{sr}}{\sigma_{s2}} \\
&= \varepsilon_{s2} - \left(\varepsilon_{s2} - \varepsilon_{s1}\right)\left(\frac{\sigma_{sr}}{\sigma_{s2}}\right)^2 \\
&= \left(\frac{\sigma_{sr}}{\sigma_{s2}}\right)^2 \varepsilon_{s1} + \left[1 - \left(\frac{\sigma_{sr}}{\sigma_{s2}}\right)^2\right]\varepsilon_{s2}
\end{aligned}$$

Let $\zeta = 1 - \left(\frac{\sigma_{sr}}{\sigma_{s2}} \right)^2$

$$\varepsilon_{sm} = \left(1 - \zeta\right) \varepsilon_{s1} + \zeta \varepsilon_{s2}$$

$$\zeta = 1 - \beta_1 \beta_2 \left(\frac{\sigma_{sr}}{\sigma_{s2}} \right)^2$$

β_1 : bond effect

β_2 : loading characteristics

Long-term deflection

1. Creep of concrete
2. Shrinkage
3. Relaxation of prestressing steel
4. Creep superposition
5. Aging Coefficient
6. Relaxation of concrete
7. Adjusted elasticity modulus