Deformation of Concrete

Fall 2010

Dept of Architecture
Seoul National University

2nd week

Creep and Shrinkage

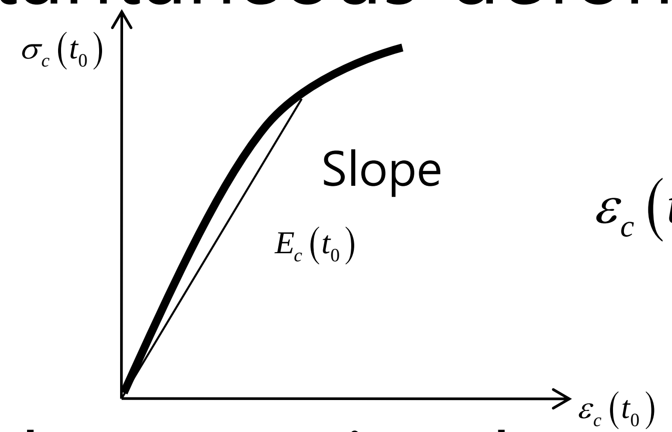
Chap. 1 Creep and Shrinkage of concrete and relaxation of steel

1.1 introduction

1. Subject to change for long time period of time
2. Creep, shrinkage, and relaxation
3. Time function for strain and stress
4. Age of concrete, length of the period after loading, environment, shape of concrete members

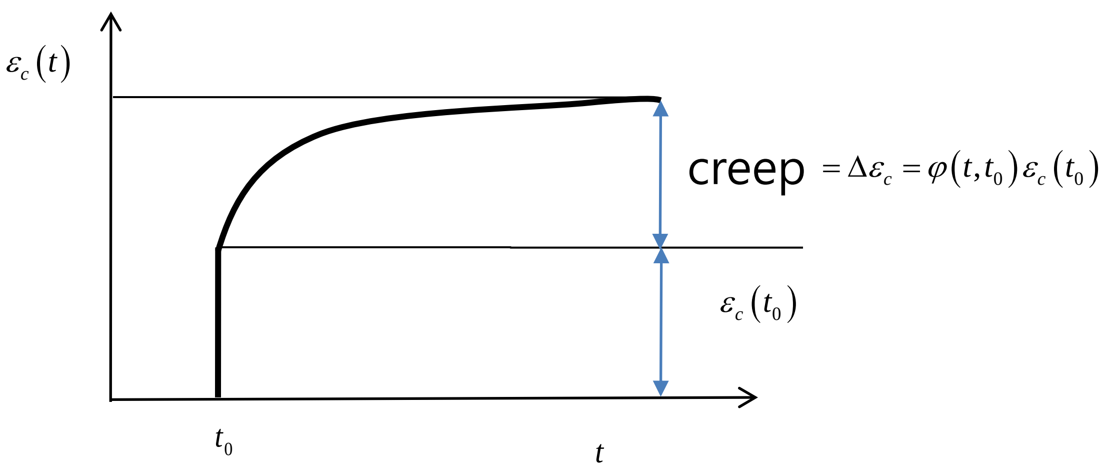
1.2 Creep of concrete

1. Instantaneous deformation



$$\varepsilon_c(t) = \varepsilon_c(t_0) + \Delta\varepsilon_c = \frac{\sigma_c(t_0)}{E_c(t_0)}\left[1 + \varphi(t, t_0)\right]$$

2. Under sustained stress



1.3 Shrinkage of concrete

1. Drying shrinkage
2. Free shrinkage

$$\varepsilon_{cs}\left(t,t_s\right)=\varepsilon_{cs0}\beta_s\left(t-t_s\right)$$

1.4 Relaxation of prestressed steel

- Relaxation under constant strain as in a constant-length test

$$\frac{\Delta\sigma_{pr}}{\sigma_{p0}} = -\frac{\log(\tau - t_0)}{10}\left(\frac{\sigma_{p0}}{f_{py}} - 0.55\right)$$

- The ultimate intrinsic relaxation

$$\frac{\Delta\sigma_{pr\infty}}{\sigma_{p0}} = -\eta(\lambda - 0.4)^2 \qquad \lambda = \frac{\sigma_{p0}}{f_{ptk}}$$

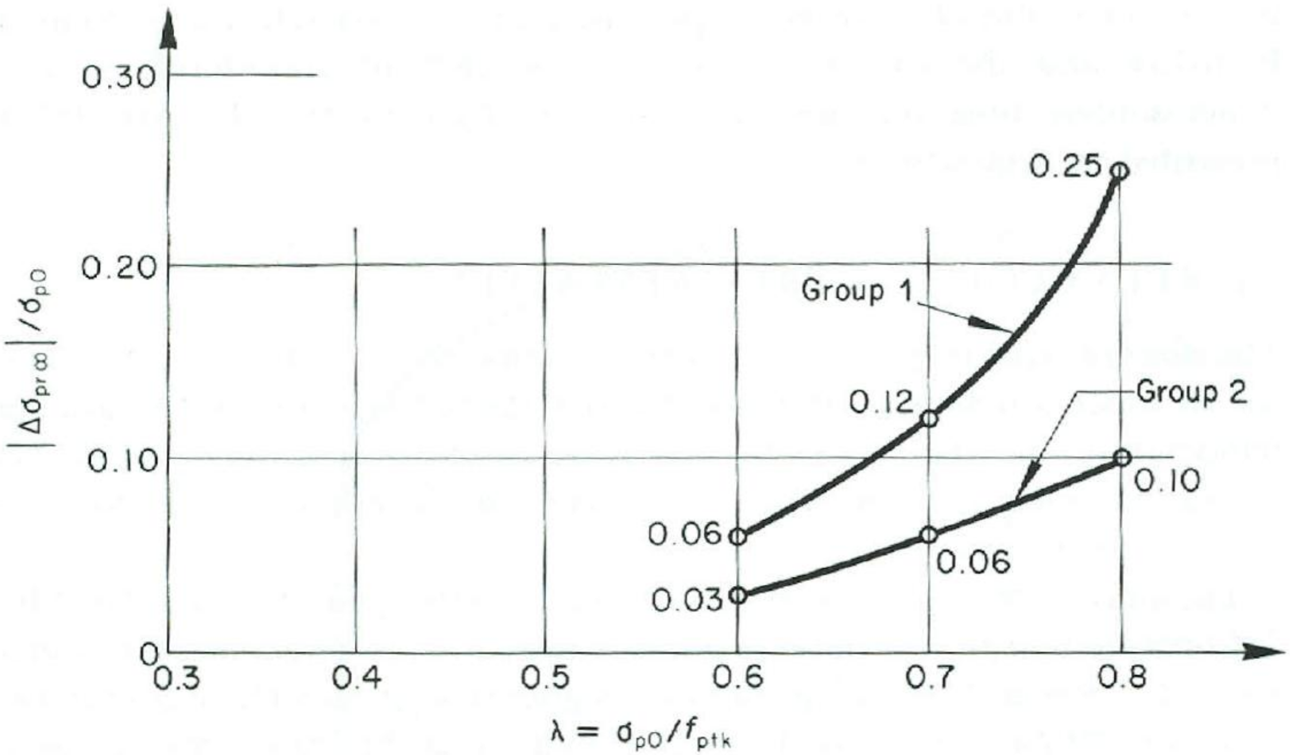


Fig. 1.3 Very-long-term intrinsic relaxation of prestressing steel according to MC-78. $|\Delta\sigma_{pr\infty}|$ = absolute value of the final intrinsic relaxation value; σ_{p0} = initial stress; f_{ptk} = characteristic tensile strength

1.5 Reduced relaxation

- To consider creep and shrinkage of prestressed concrete

$$\Delta\bar{\sigma}_{pr} = \chi_r \Delta\sigma_{pr}$$

$$\Omega = -\left(\frac{\Delta\sigma_{ps} - \Delta\sigma_{pr}}{\sigma_{p0}} \right)$$

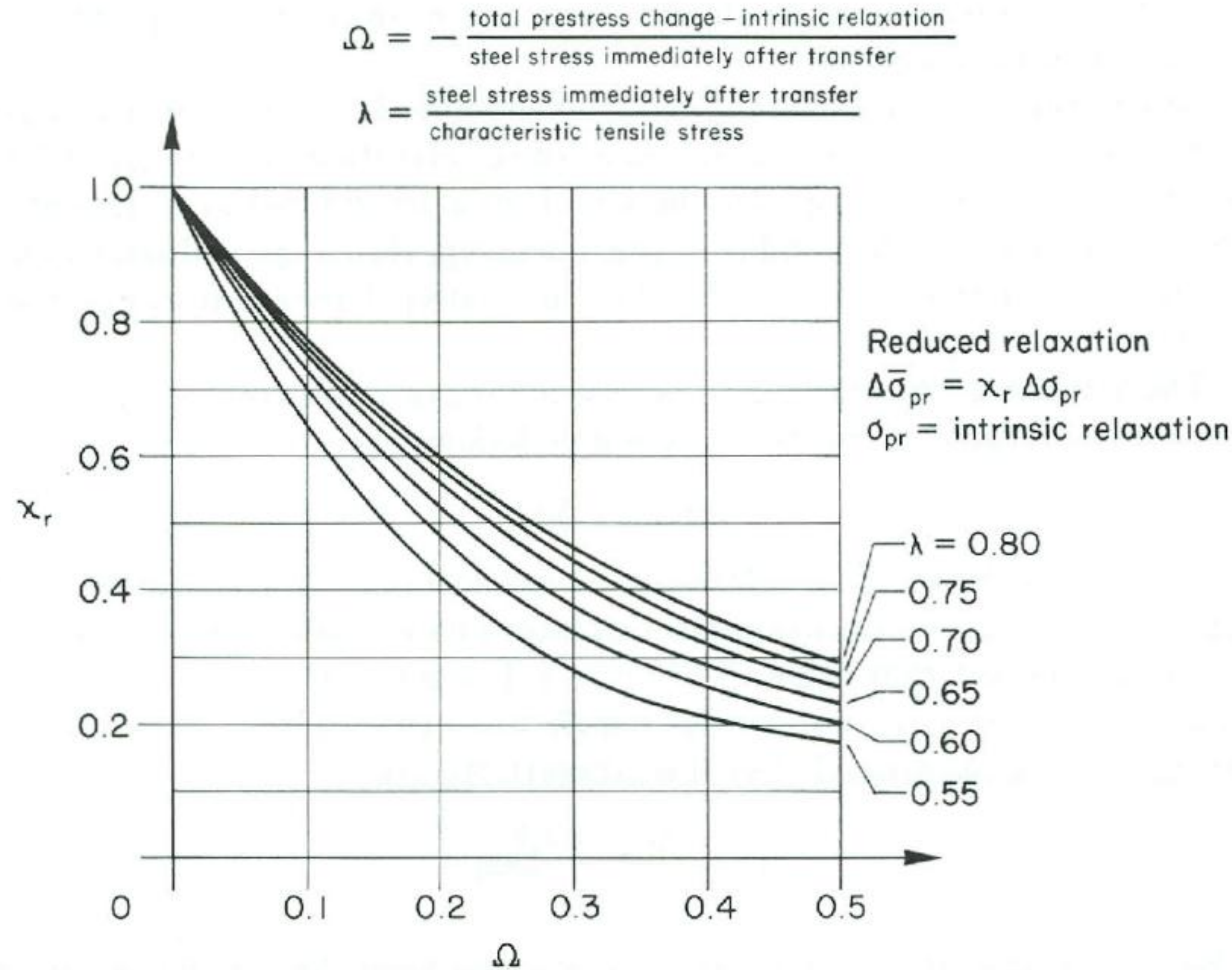


Fig. 1.4 Relaxation reduction coefficient χ_r

Appendix B gives the derivation of the relaxation reduction coefficient values in Table 1.1 and the graphs in Fig. 1.4. The values given in the table and the graphs may be approximated by Equation (B. 11).

1.6 Creep superposition

$$\varepsilon_c(t) = \sigma_c(t_0)\frac{1+\varphi(t,t_0)}{E_c(t_0)} + \int_0^{\Delta\sigma_c(t)} \frac{1+\varphi(t,\tau)}{E_c(\tau)}\, d\sigma_c(\tau) + \varepsilon_{cs}(t,t_0)$$

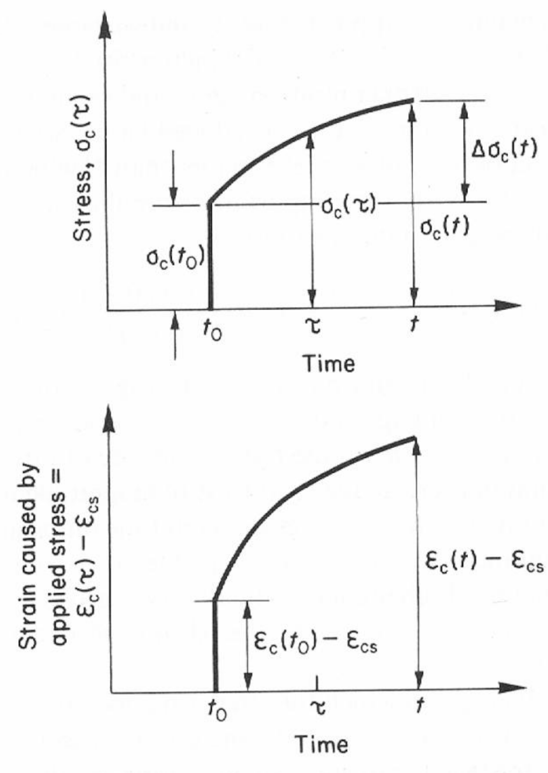


Fig. 1.5 Stress versus time and strain versus time for a concrete member subjected to uniaxial stress of magnitude varying with time

1.7 Aging Coefficient

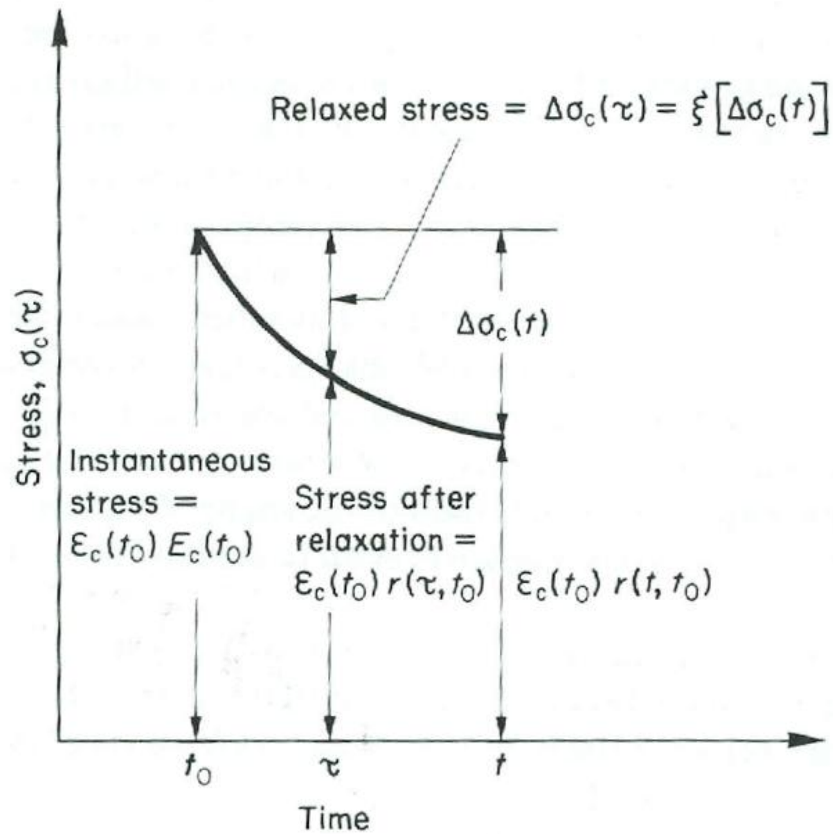


Fig. 1.6 Variation of stress with time due to a strain ε_c, imposed at age t_0 and maintained constant thereafter (phenomenon of relaxation)

$$\varepsilon_c(t) = \sigma_c(t_0)\frac{1+\varphi(t,t_0)}{E_c(t_0)} + \int_0^{\Delta\sigma_c(t)} \frac{1+\varphi(t,\tau)}{E_c(\tau)}\, d\sigma_c(\tau) + \varepsilon_{cs}(t,t_0)$$

$$\varepsilon_c(t) = \sigma_c(t_0)\frac{1+\varphi(t,t_0)}{E_c(t_0)} + \Delta\sigma_c(t)\frac{1+\chi\varphi(t,t_0)}{E_c(t_0)} + \varepsilon_{cs}(t,t_0)$$

1.8 Equation for Aging coefficient

Stress variation

$$\xi_1 = \frac{\sigma_c(\tau) - \sigma_c(t_0)}{\Delta\sigma_c(t)}$$

Differentiation

$$\frac{d\xi_1}{d\tau} = \frac{d\sigma_c(\tau)/d\tau}{\Delta\sigma_c(t)} \Rightarrow \frac{d\sigma_c(\tau)}{d\tau} = \Delta\sigma_c(t)\frac{d\xi_1}{d\tau}$$

Substitution

$$\varepsilon_c(t) = \sigma_c(t_0)\frac{1+\varphi(t,t_0)}{E_c(t_0)} + \Delta\sigma_c(t)\int_{t_0}^{t}\frac{1+\varphi(t,\tau)}{E_c(\tau)}\frac{d\xi_1}{d\tau}d\tau + \varepsilon_{cs}(t,t_0)$$

$$\varepsilon_c(t) = \sigma_c(t_0)\frac{1+\varphi(t,t_0)}{E_c(t_0)} + \Delta\sigma_c(t)\frac{1+\chi\varphi(t,t_0)}{E_c(t_0)} + \varepsilon_{cs}(t,t_0)$$

$$\chi(t,t_0) = \frac{E_c(t_0)}{\varphi(t,t_0)}\int_{t_0}^{t}\frac{1+\chi\varphi(t,t_0)}{E_c(t_0)}\frac{d\xi_1}{d\tau}d\tau - 1$$

1.9 Relaxation of concrete

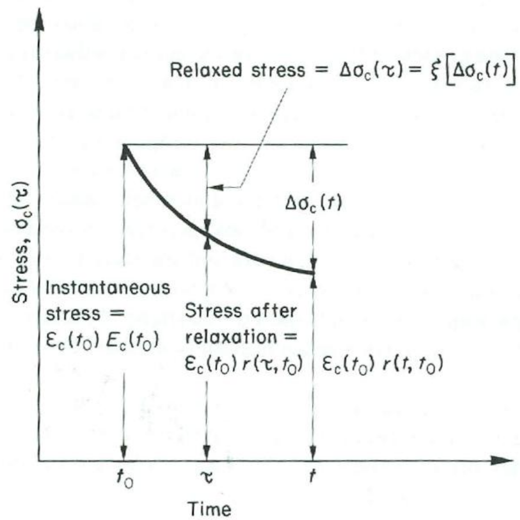


Fig. 1.6 Variation of stress with time due to a strain ε_c, imposed at age t_0 and maintained constant thereafter (phenomenon of relaxation)

Instantaneous stress

$$\sigma_c(t_0) = \varepsilon_c E_c(t_0)$$

Stress at any time

$$\sigma_c(t) = \varepsilon_c r(t, t_0)$$

Magnitude of relaxed stress

$$\Delta\sigma_c(\tau) = \xi\left[\Delta\sigma_c(t)\right]$$

A dimensionless shape function

$$\xi = \frac{\Delta\sigma_c(\tau)}{\Delta\sigma_c(t)}$$

$$\Delta\sigma_c(\tau) = \sigma_c(\tau) - \sigma_c(t_0)$$

$$\Delta\sigma_c(t) = \sigma_c(t) - \sigma_c(t_0)$$

$$\varepsilon_c(t) = \sigma_c(t_0)\frac{1+\varphi(t,t_0)}{E_c(t_0)} + \Delta\sigma_c(t)\frac{1+\chi\varphi(t,t_0)}{E_c(t_0)} + \varepsilon_{cs}(t,t_0) \tag{1.10}$$

$$\sigma_c(t_0) = \varepsilon_c E_c(t_0)$$

$$\varepsilon_c(t) = \varepsilon_c\frac{1+\varphi(t,t_0)}{1} + \varepsilon_c\left[r(t,t_0) - E_c(t_0)\right]\frac{1+\chi\varphi(t,t_0)}{E_c(t_0)}$$

$$\chi(t,t_0) = \frac{1}{1-r(t,t_0)/E_c(t_0)} - \frac{1}{\varphi(t,t_0)}$$

1.10 Step-by-step calculation of relaxation function

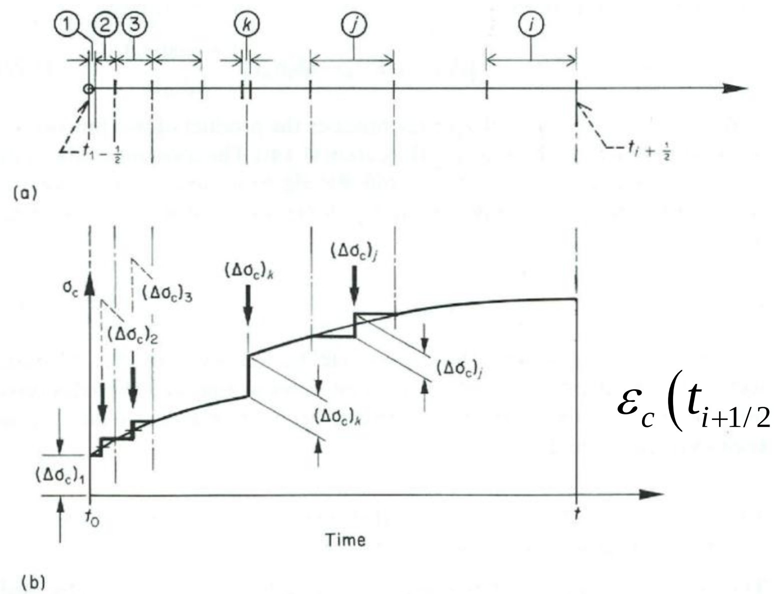


Fig. 1.7 Division of: (a) time into intervals and (b) stress into increments for step-by-step analysis

$$\varepsilon_c(t) = \sigma_c(t_0)\frac{1+\varphi(t,t_0)}{E_c(t_0)} + \int_0^{\Delta\sigma_c(t)} \frac{1+\varphi(t,\tau)}{E_c(\tau)} d\sigma_c(\tau) + \varepsilon_{cs}(t,t_0)$$

$$\varepsilon_c(t_{i+1/2}) = \sum_{j=1}^{i}\left((\Delta\sigma_c)_j \frac{1+\varphi(t_{i+1/2},t_j)}{E_c(t_j)} \right) + \varepsilon_{cs}(t_{i+1/2},t_0)$$

Separation of the last term

$$\varepsilon_c(t_{i+1/2}) = (\Delta\sigma_c)_i \frac{1+\varphi(t_{i+1/2},t_i)}{E_c(t_i)} + \sum_{j=1}^{i-1}\left((\Delta\sigma_c)_j \frac{1+\varphi(t_{i+1/2},t_j)}{E_c(t_j)} \right) + \varepsilon_{cs}(t_{i+1/2},t_0)$$

1.11 Age-adjusted elasticity

$$\varepsilon_c(t) = \sigma_c(t_0)\frac{1+\varphi(t,t_0)}{E_c(t_0)} + \Delta\sigma_c(t)\frac{1+\chi\varphi(t,t_0)}{E_c(t_0)} + \varepsilon_{cs}(t,t_0)$$

$$\varepsilon_c(t) = \sigma_c(t_0)\frac{1+\varphi(t,t_0)}{E_c(t_0)} + \frac{\Delta\sigma_c(t)}{\overline{E}_c(t,t_0)} + \varepsilon_{cs}(t,t_0)$$

$$\overline{E}_c(t,t_0) = \frac{E_c(t_0)}{1+\chi\varphi(t,t_0)}$$

$$\Delta\varepsilon_c(t) = \frac{\Delta\sigma_c(t)}{\overline{E}_c(t,t_0)}$$