Risk Management and Decision Analysis

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Recall

Bayes' Theorem

- Based on the symmetry of the definition of conditional probability, we can predict the **posterior probability** based on the **prior information**

$$\boxed{P(A|B) = \frac{P(A) \times P(B|A)}{P(B)}}$$

Modeling Uncertainty: Statistics

- Concept of Moments
- Central Limit Theorem
- Types of Probability Distribution
- Discrete Approximation: Bracket Median Method, Pearson-Tukey 3 points Method

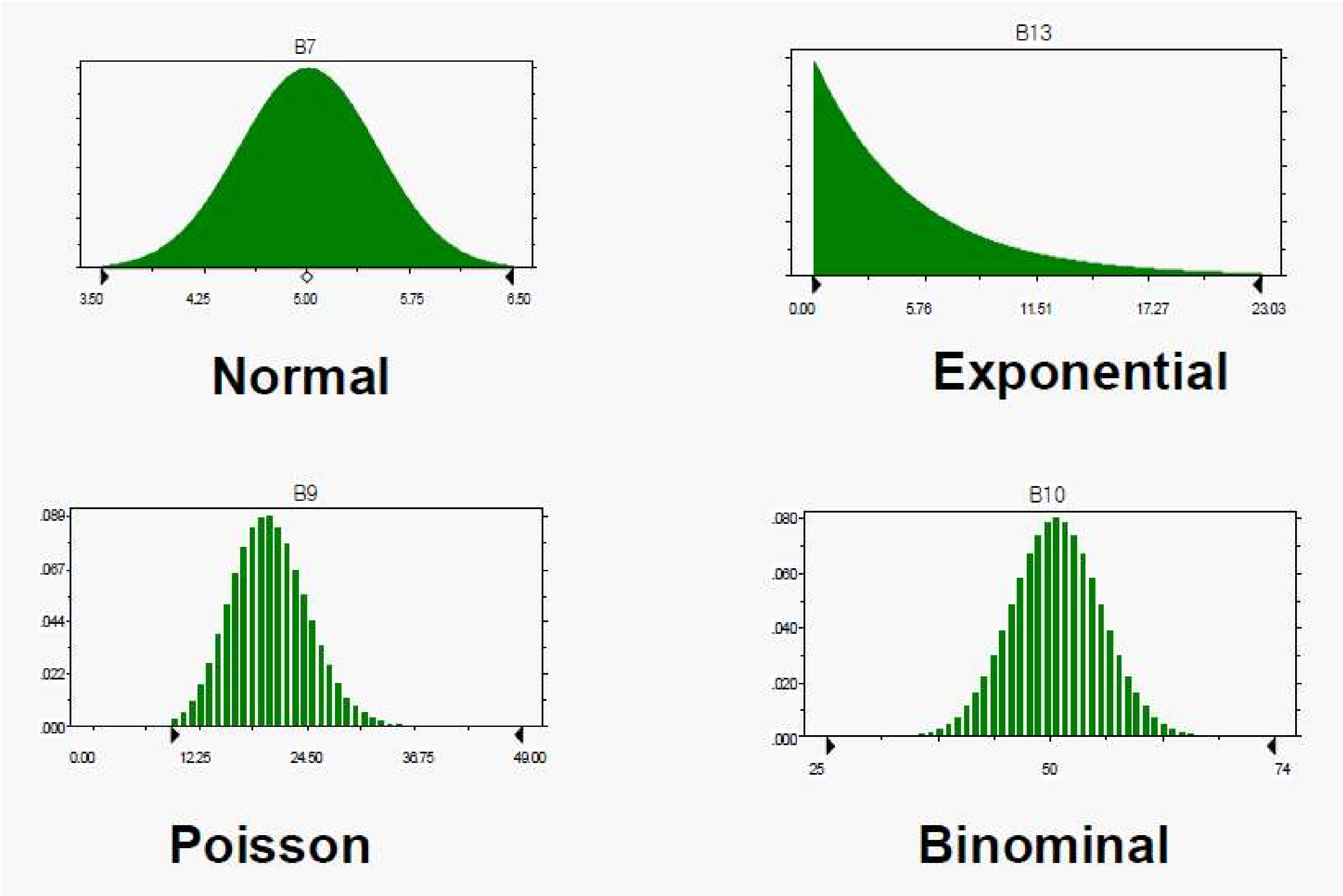
PART I

MODELING UNCERTAINTY

- Data Fitting

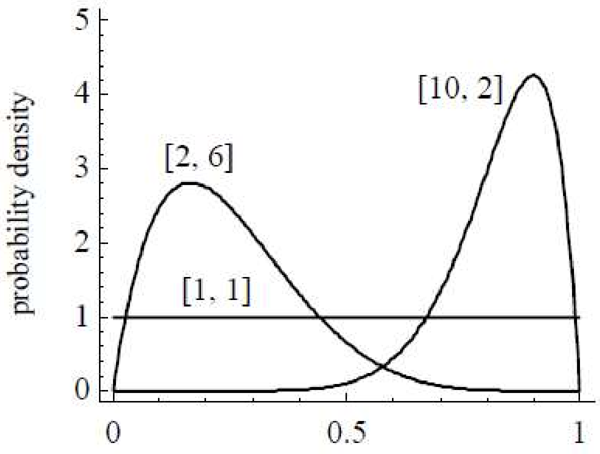
Variables & Distributions

Types of Distribution I

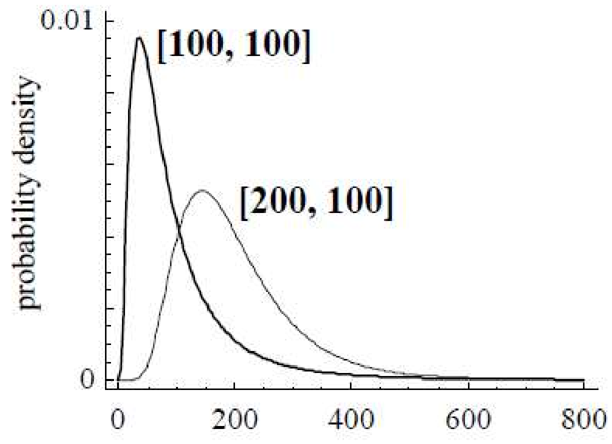


Variables & Distributions

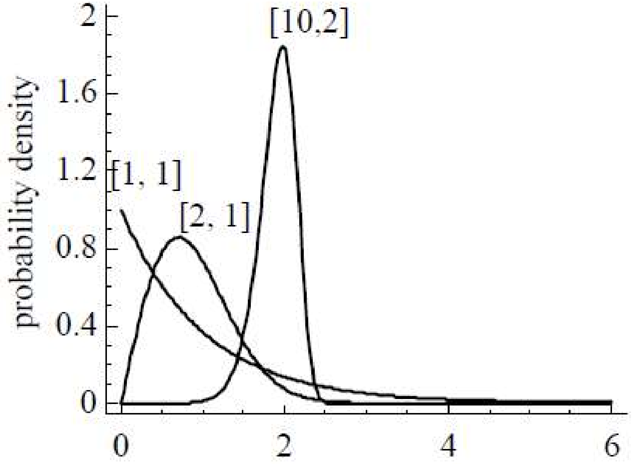
Types of Distribution II



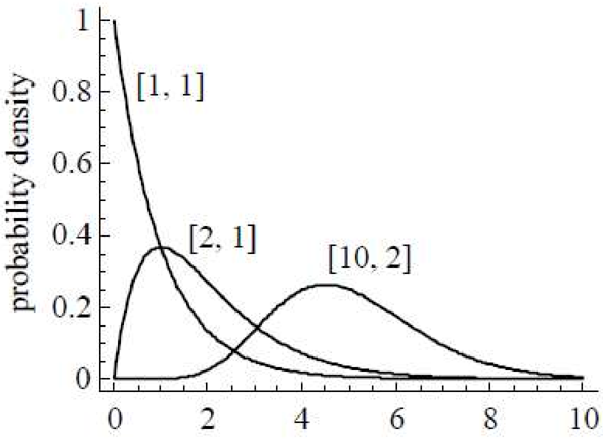
Beta



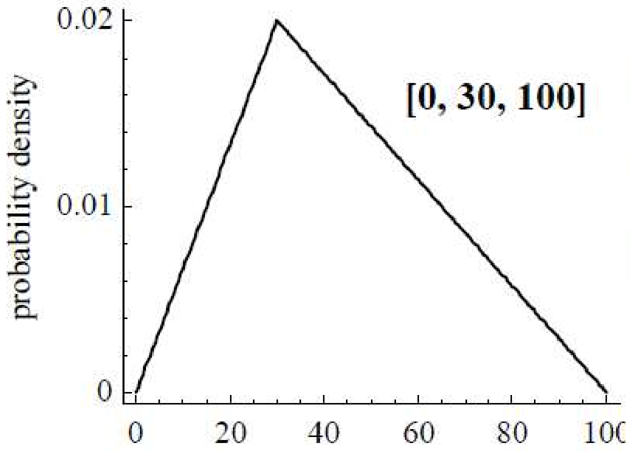
Lognormal



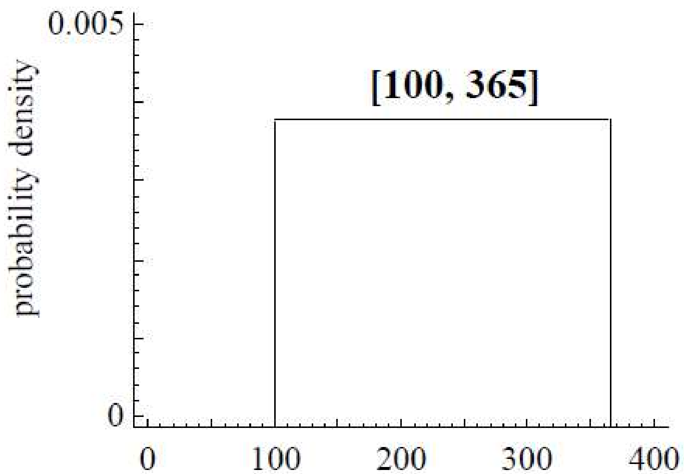
Weibull



Gamma



Triangular



Uniform

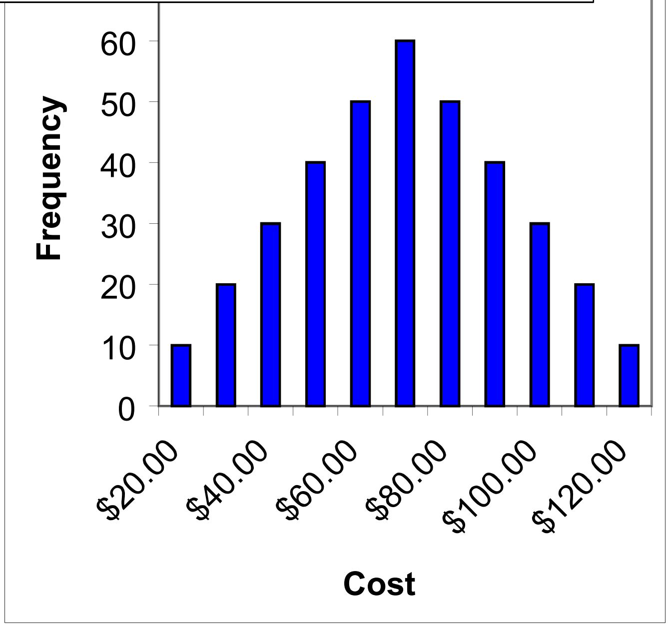
Data fitting is the procedure of selecting a statistical distribution that best fits to a data set generated by some random process.

Data Fitting

Original Data

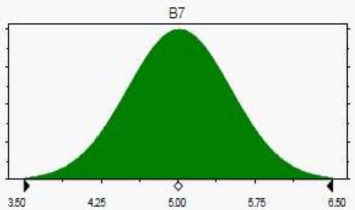
(Frequency)	Cost $
10	$ 20.00
20	$ 30.00
30	$ 40.00
40	$ 50.00
50	$ 60.00
60	$ 70.00
50	$ 80.00
40	$ 90.00
30	$ 100.00
20	$ 110.00
10	$ 120.00
Total 360	
Average	$ 70.00

Descriptive Statistics

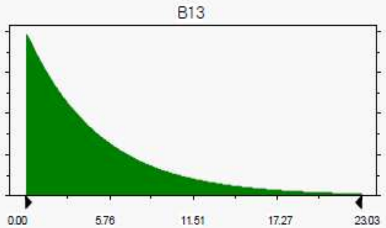


?

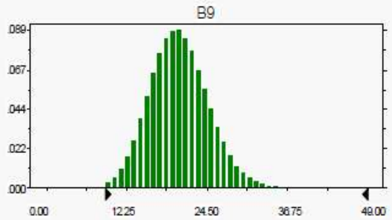
Distribution



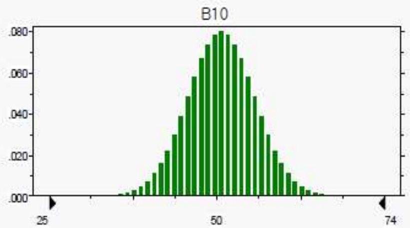
Normal



Exponential

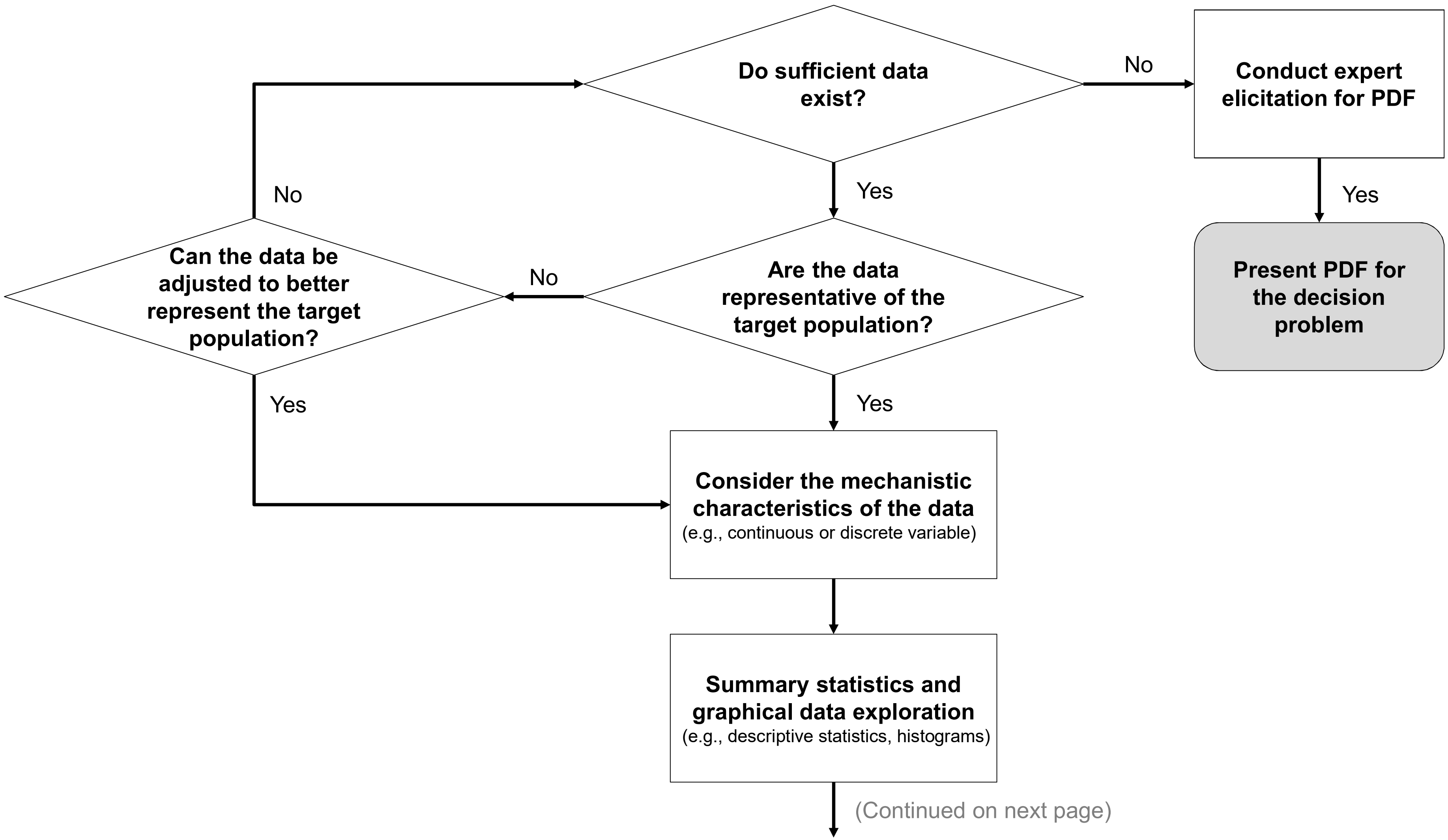


Poisson



Binominal

Data Fitting Process



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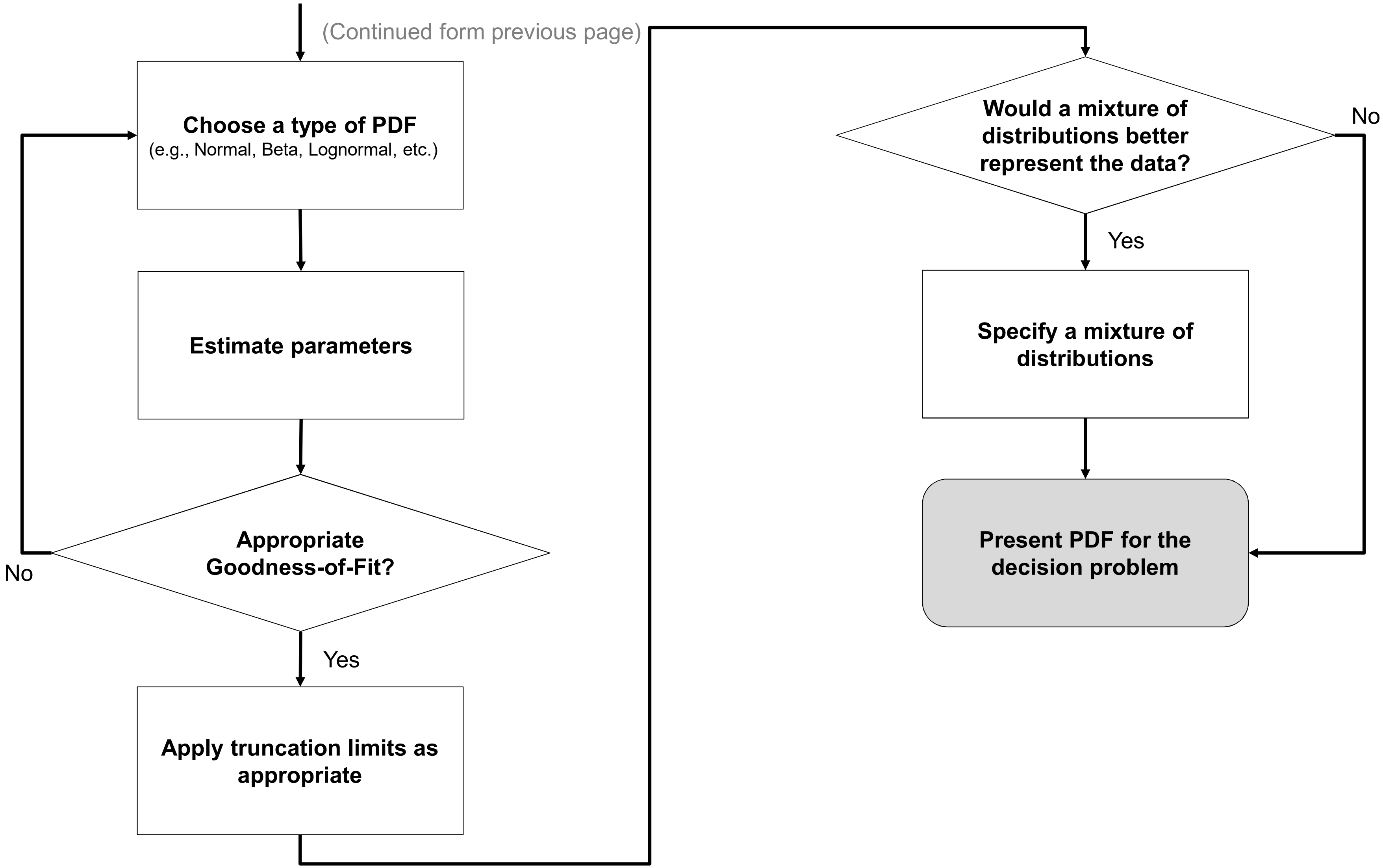
Data Fitting Process



What is data fitting? Why is it important?

- Data fitting is the procedure of selecting a statistical distribution that best fits to a data set generated by some random process.

- Probability distributions can be viewed as a tool for dealing with uncertainty: you use distributions to perform specific calculations, and apply the results to make well-grounded business decisions.

- However, if you use a wrong tool, you will get wrong results. If you select and apply an inappropriate distribution (the one that doesn't fit to your data well), your subsequent calculations will be incorrect, and that will certainly result in wrong decisions.

- Data fitting allows you to develop valid models of random processes you deal with, protecting you from potential time and money loss which can arise due to invalid model selection, and enabling you to make better business decisions.

Choose a Type of Distribution

- You cannot "just guess" and use any other particular distribution without testing several alternative models as this can result in analysis errors.

- In most cases, you need to fit two or more distributions, compare the results, and select the most valid model. The “candidate” distributions you fit should be chosen depending on the nature of your probability data.
  - For example, if you need to analyze the time between failures of technical devices, you should fit non-negative distributions such as Exponential or Weibull, since the failure time cannot be negative.

- You can also apply some other identification methods based on properties of your data.
  - For example, you can build a histogram and determine whether the data are symmetric, left-skewed, or right-skewed, and use the distributions which have the same shape.

Choose a Type of Distribution

- To actually fit the "candidate" distributions you selected, you need to employ statistical methods allowing to estimate distribution parameters based on your sample data.
- The solution of this problem involves the use of certain algorithms implemented in specialized software.
- After the distributions are fitted, it is necessary to determine how well the distributions you selected fit to your data.
- This can be done using the specific goodness of fit tests or visually by comparing the empirical (based on sample data) and theoretical (fitted) distribution graphs.
- As a result, you will select the most valid model describing your data.

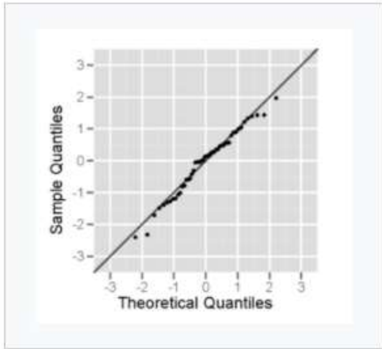
Normal Distribution – Assumption of Normality

- The Normal distribution is one of the oldest, the most well-known, and frequently used distributions.
- Normal distribution assumptions are important to note because so many researches rely on assuming a distribution to be normal. In most cases, the assumption of normality is a reasonable one to make.
- The reason for the normal distribution assumptions is that this is usually the simplest mathematical model that can be used. In addition, it is surprisingly ubiquitous and it occurs in most natural and social phenomena. This is why the assumption of normality is usually a good first approximation.
- The assumption of normality is valid in most cases but when it is not, it could lead to serious trouble. Also, since this assumption is made so inherently, it is hard to spot and sometimes difficult to question. Therefore care must be taken to ensure that the researcher is aware of not just the assumption of normality but in fact all the assumptions that go into a statistical analysis. This will help define the scope of the research and if something is not as expected, one can find the reason for the discrepancy.

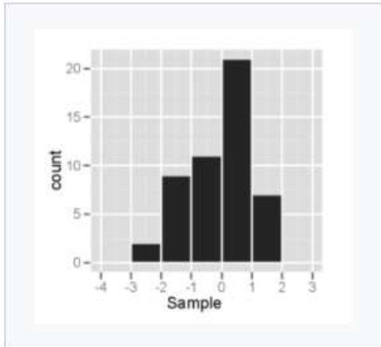
Normal Distribution – Evaluating Normality

- There are statistical tests that a researcher can undertake which help determine whether the normal distribution assumptions are valid or not.
  - One quick way is to compare the sample means to the real mean. For a normally distributed population, the sampling distribution is also normal when there are sufficient test items in the samples.

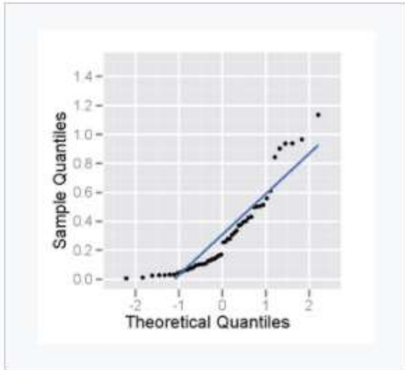
- There are both graphical and statistical methods for evaluating normality.
  - Graphical methods include the histogram and normality plot (The data are plotted against a theoretical normal distribution in such a way that the points should form an approximate straight line).



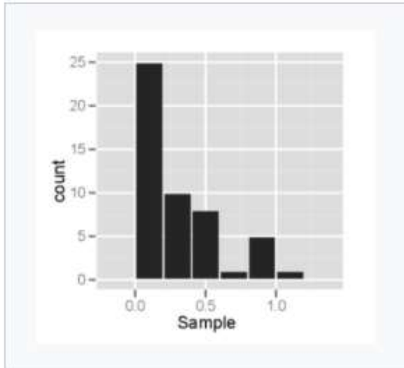
Normal probability plot of a sample from a normal distribution – it looks fairly straight, at least when the few large and small values are ignored.



Histogram of a sample from a normal distribution – it looks fairly symmetric and unimodal



Normal probability plot of a sample from a right-skewed distribution – it has an inverted C shape.



Histogram of a sample from a right-skewed distribution – it looks unimodal and skewed right.

Normal Distribution – Evaluating Normality

Q-Q Plot

두 데이터의 분위수(Quantile)와 분위수를 그려서 두 데이터가 같은 분포를 따르는지 판단 → 데이터와 정규분포를 비교하여 정규성 검정에 활용

*그리는 방법: 두 데이터를 오름차순으로 Sorting 해서 같은 분위수를 가진 숫자 둘을 x, y로 해서 Plot

```
> SP=sample(1:1000,50)
> SP
 [1] 544 110  12 576 631 816 496 150 184 913  97
[12] 172 822 604 876 497  90 372 710 562 727  44
[23] 997 870 485 935 255 321 558 883 149 703 171
[34] 654 891 910 414 765  56 711 316 384 324 753
[45] 487 635 458 600 437 299
```

```
> SP_norm
 [1]  0.679951138 -0.608127463  1.533355558
 [4] -0.531255859  1.832681815 -0.572747918
 [7]  0.585023110 -0.198374745  0.126205900
[10] -0.017061023  0.015871186 -0.076208099
[13] -0.489279480 -0.836422802 -0.575932423
[16]  1.210258851 -0.055865188  1.713292910
[19] -1.053883859  0.581943602  0.143578258
[22]  1.020568170  0.644320752  0.388051422
[25]  0.726307792 -0.490604274  0.484533305
[28]  0.228335362  0.973336953 -0.949986306
[31]  0.020909686  1.958810854  0.002770274
[34]  0.887778067  1.064505737  0.638031588
[37] -0.713016597 -0.821891220 -0.946939897
[40]  2.290853302  0.192030345 -1.091702432
[43]  2.389921209 -2.053085830  0.293937389
[46] -0.727549121  0.975691424  0.491614750
[49]  0.219295085 -2.470632443
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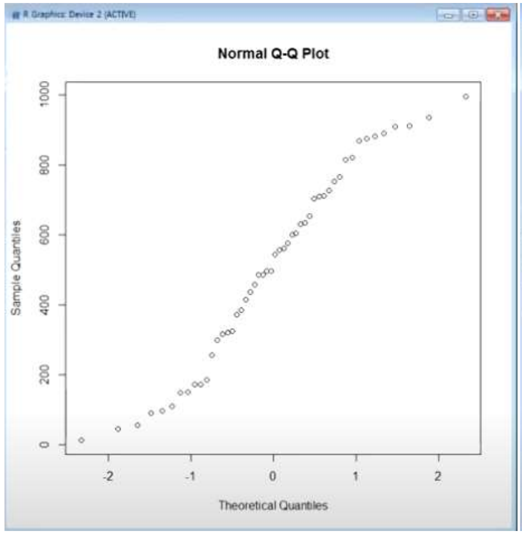
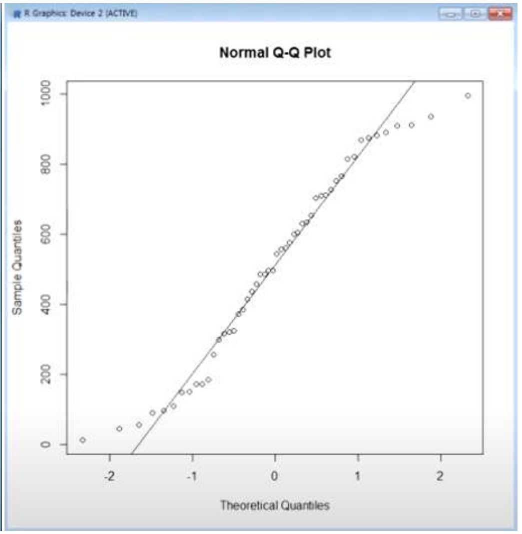
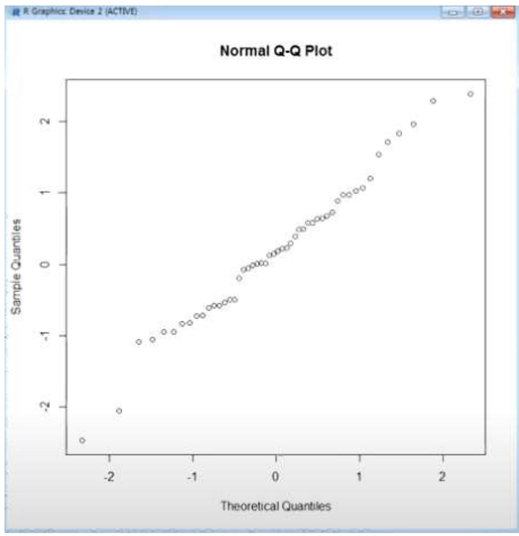
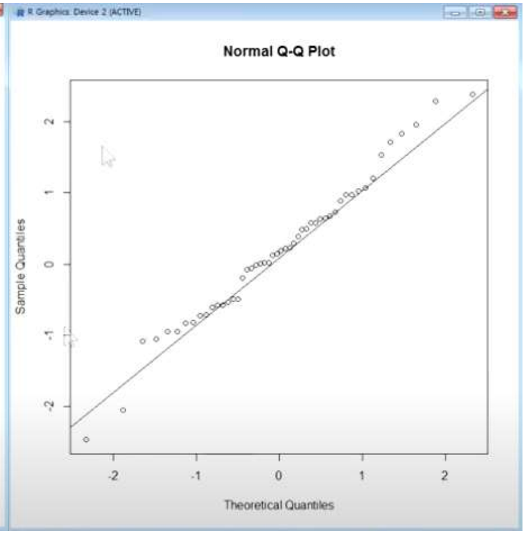




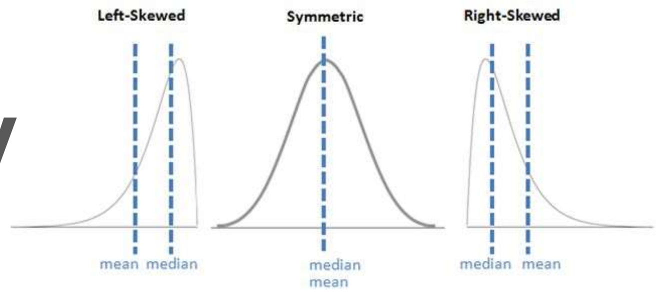




Normal Distribution – Evaluating Normality

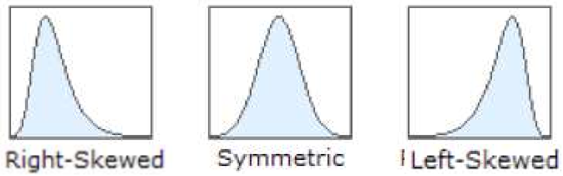
- There are both graphical and statistical methods for evaluating normality.
  - Statistical methods include diagnostic hypothesis tests for normality, and a rule of thumb(경험식) that says a variable is reasonably close to normal if its skewness and kurtosis have values between –1.0 and +1.0.
  - None of the methods is absolutely definitive.

Normal Distribution – Normality



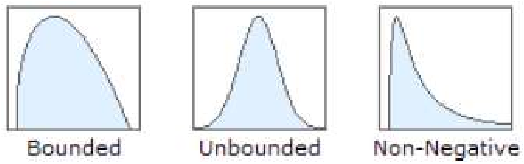
1) Is it symmetric?

- The probability density function of the Normal distribution is symmetric about its mean value, and this distribution cannot be used to model right-skewed or left-skewed data.



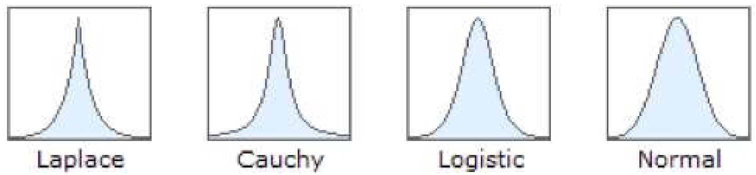
2) Is it unbounded?

- The Normal distribution is defined on the entire real axis (-Infinity, +Infinity), and if the nature of your data is such that it is bounded or non-negative (can only take on positive values), then this distribution is almost certainly not a good fit.



3) Is its shape constant?

- The shape of the Normal distribution does not depend on the distribution parameters. Even if your data is symmetric by nature, it is possible that it is best described by one of the heavy-tailed models.



Normal Distribution – Transformations

- When a variable is not normally distributed, we can create a transformed variable and test it for normality. If the transformed variable is normally distributed, we can substitute it in our analysis.

- Three common transformations are: the logarithmic transformation (x to log(x)), the square root transformation (x to sqrt(x)), and the inverse transformation (quantile function, switch x and y).

- All of these change the measuring scale on the horizontal axis of a histogram to produce a transformed variable that is mathematically equivalent to the original variable.

EDA (Explanatory Data Analysis)

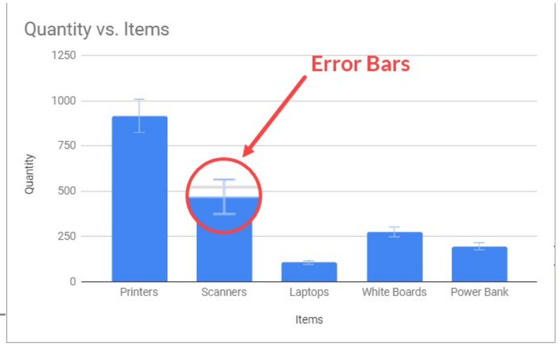
EDA includes:

1) **Descriptive statistics** (numerical summaries): mean, median, range, variance, standard deviation, etc.

2) **Chi-square**

   - This method compares number of observations found in discrete classes to that predicted by the proposed model.
   - Best suited for discrete random variables.
   - Generally, p-value greater than 0.05 indicates a close fit (at the 95% significant level).

EDA (Explanatory Data Analysis)

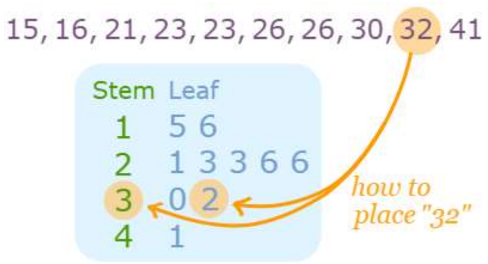


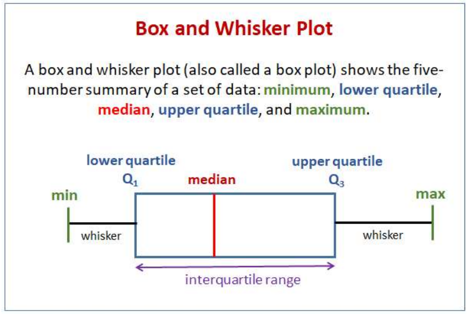
EDA includes:

3) Kolmogorov-Smirnov & Shapiro-Wilk tests:

- These methods test whether one distribution (e.g. your dataset) is significantly different from another (e.g. a normal distribution) and produce a numerical answer, yes or no.
- Use the Shapiro-Wilk test if the sample size is between 3 and 2,000 and the Kolmogorov-Smirnov test if the sample size is greater than 2,000.
- Unfortunately, in some circumstances, both of these tests can produce misleading results, so "real" statisticians prefer graphical plots to tests.

4) **Graphical methods:** frequency distribution histograms, stem & leaf plots, scatter plots, box & whisker plots, normal probability plots (PP for CDF and QQ for Quantiles plots), graphs with error bars, etc.





Goodness-of-Fit Test: Chi-square Test

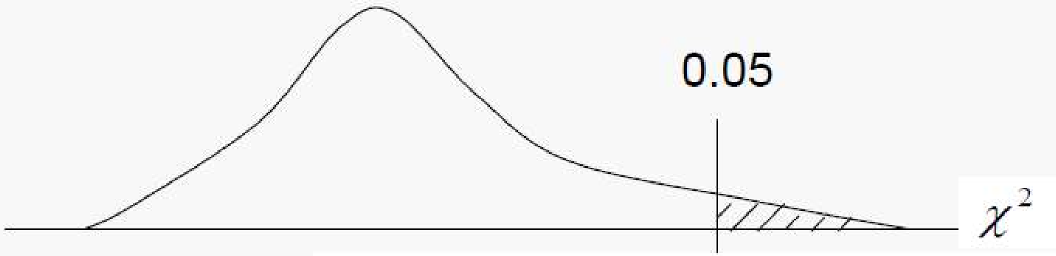
- The **chi-square test** is used to test if a sample of data came from a population with a specific distribution (모집단의 표본이 모집단을 잘 대표하고 있는지 검정).
- Another way of looking at that is to ask if the frequency distribution fits a specific pattern.
- Two values are involved, an observed value (관측값), which is the frequency of a category from a sample, and the expected frequency (기대값), which is calculated based upon the claimed distribution.
- The idea is that if the observed frequency is really close to the claimed (expected) frequency, then the square of the deviations will be small.
  - The square of the deviation is divided by the expected frequency to weight frequencies.

$$\chi^2 = \Sigma\,(\text{관측값} - \text{기댓값})^2 / \text{기댓값}$$

  - A difference of 10 may be very significant if 12 was the expected frequency, but a difference of 10 is not very significant at all if the expected frequency was 1,200.

Goodness-of-Fit Test: Chi-square Test

- If the sum of these weighted squared deviations is small, the observed frequencies are close to the expected frequencies and there would be no reason to reject the claim that it came from that distribution.
- Only when the sum is large is the a reason to question the distribution. Therefore, the chi-square goodness-of-fit test is always a right tail test.

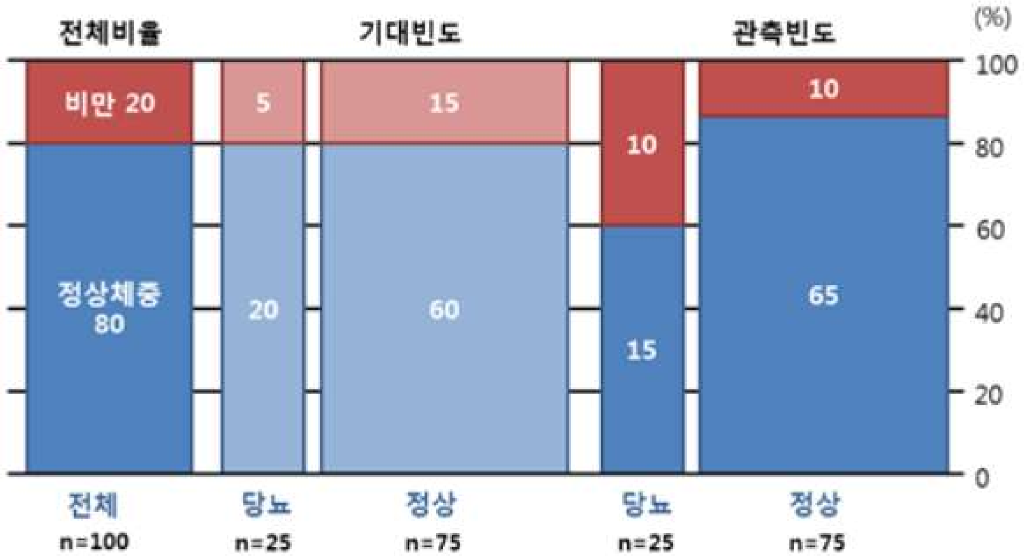


If Estimated $\chi^2 \geq$ Tabulated χ^2 (from statistics book)

It is not an assumed distribution – reject!

	당뇨	정상	전체
비만	10 (40.0%)	10 (13.3%)	20 (20.0%)
정상체중	15 (60.0%)	65 (86.7%)	80 (80.0%)
전체	25 (100%)	75 (100%)	100 (100%)

처음의 예제로 돌아가 보자. 위의 표는 당뇨 환자 25명과 당뇨가 없는 정상인 75명을 대상으로 비만 유무를 조사한 결과이다. 이렇게 실제로 조사에 의해 관찰된 빈도를 **관측빈도**라고 한다. 비만의 비율은 당뇨 환자 중에는 40%, 당뇨가 없는 정상인에서는 13.3%이다. 전체 인구집단을 놓고 보면 20%이다. 두 집단의 비만 비율이 통계적으로 차이가 전혀 없다면 당뇨 환자 25명 중에서 20%(5명)가 비만이고, 정상인 75명 중에서도 20%(15명)가 비만일 것으로 기대할 수 있을 것이다. 이렇듯 두 변수 사이에 연관성이 없다는 가정 하에 예상되는 빈도를 **기대빈도**라고 한다. 이 두 빈도의 실제 크기를 반영하여 다음과 같은 그림을 그려보면 기대빈도와 관측빈도의 그 차이를 확연히 느낄 수 있다.



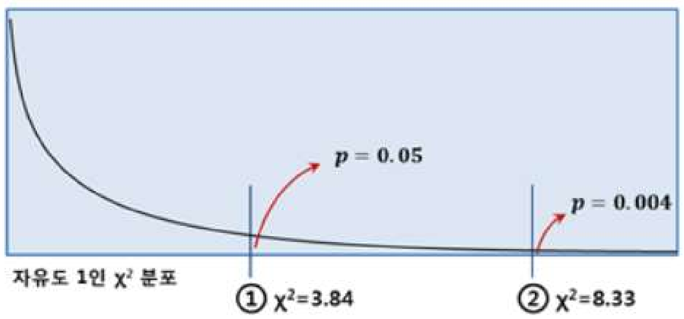
카이제곱 검정의 가설 검정

귀무가설 H_0	두 변수는 연관성이 없다.
대립가설 H_1	두 변수는 연관성이 있다.

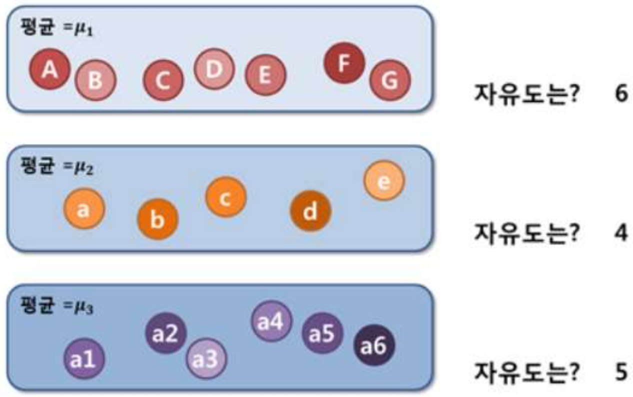
2x2 분할표에서 각 셀의 "(관측빈도-기대빈도)^2/기대빈도"의 합은 자유도 1의 카이제곱 분포를 따르는 것이 알려져 있다. 두 변수 사이의 연관성이 전혀 없다는 귀무가설 하에서 검정통계량의 값이 클수록 이러한 현상이 관찰될 가능성(p value)은 적어지게 된다.

$$\chi^2 = \sum \frac{(\text{관측빈도} - \text{기대빈도})^2}{\text{기대빈도}} = \frac{(+5)^2}{5} + \frac{(-5)^2}{20} + \frac{(-5)^2}{15} + \frac{(+5)^2}{60} = 8.33$$

당뇨와 비만의 경우로 돌아가서 위와 같이 검정통계량 카이제곱값을 직접 계산하면 을 구할 수 있다. 실제 자유도 1인 카이제곱 분포에서 카이제곱값=3.84인 현상이 관찰될 가능성이 5%이며(p value=0.05), 카이제곱값이 8.33인 현상이 관찰될 확률은 0.4%에 불과하므로(p value=0.004), 귀무가설을 기각하고 당뇨와 비만 사이에 연관성이 있다는 대립가설을 채택할 수 있다.



자유도란 실질적으로 독립인 값들의 개수를 의미한다. 예를 들어 평균이 m인 10개의 자료 중에서 9개의 값은 아무 값이나 자유롭게 취할 수 있지만, 평균이 정해져 있다면 마지막 남은 1개의 값은 다른 값들에 의해 정의되므로 자유도는 9이다. 즉 자료의 개수가 n인 경우 자유도는 n-1이다.



그럼 이제 2x2 분할표에서 자유도를 따져보자. 다음은 100명의 환자를 대상으로 당뇨와 고혈압 유무를 각각 조사한 결과이다. a, b, c, d, 4개의 값 중 한 개의 값만 구해지면 각각의 테이블의 계산에 의해 나머지 값은 자동으로 정해지게 된다. 즉 자유도는 1이다.

	당뇨	정상	전체
고혈압	*a*	*b*	20
정상	*c*	*d*	80
전체	25	75	100

자유도는? 1

그럼 이번에는 당뇨, 내당능장애, 정상의 3개 열과 고혈압, 정상의 2개 행으로 구분된 3x2 교차표를 살펴보자. a, b, c, d, e, f, 6개의 값 중 2개의 값만 정해지면 나머지 값을 모두 채울 수 있다. 이렇듯 nxm 교차표는 (n-1)x(m-1)의 자유도를 따르는 것으로 알려져 있다.

	당뇨	내당능장애	정상	전체
고혈압	*a*	*b*	*c*	20
정상	*d*	*e*	*f*	80
전체	25	25	50	100

자유도는? 2

Goodness-of-Fit Test: Chi-square Test

- The chi-square test is defined for the hypothesis:

  H_0: The data follow a specified distribution.

  H_a: The data do not follow the specified distribution.

- Test Statistic: For the chi-square goodness-of-fit computation, the data are divided into k bins and the test statistic is defined as

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

  where, O is the observed frequency and E is the expected frequency.

Goodness-of-Fit Test: Chi-square Test

Assumptions:

- The data are obtained from a random sample.
- The expected frequency of each category must be at least 5.
  - This goes back to the requirement that the data be normally distributed. You're simulating a multinomial experiment (using a discrete distribution) with the goodness-of-fit test (and a continuous distribution), and if each expected frequency is at least five then you can use the normal distribution to approximate (much like the binomial).

카이제곱 검정의 시행에도 전제 조건이 있다. 기대빈도가 5 미만인 셀이 전체의 20%를 넘는 경우 카이제곱 분포를 따른다고 가정을 할 수 없다. 가장 빈번한 자료의 형태인 2x2 분할표는 모두 4개의 셀로 구성되므로 한 셀만 기대빈도가 5 미만이어도 카이제곱 검정을 사용할 수 없다. 이렇게 기대빈도가 5 이하인 셀이 20%를 넘는 경우에는 카이제곱 분포를 가정하는 카이제곱 검정 대신에 다음에 이어서 배울 Fisher의 정확한 검정을 시행하게 된다.

Fisher's Exact Test: 조합을 통해 모든 경우의 수를 직접 따져서 검정

Goodness-of-Fit Test: Chi-square Test

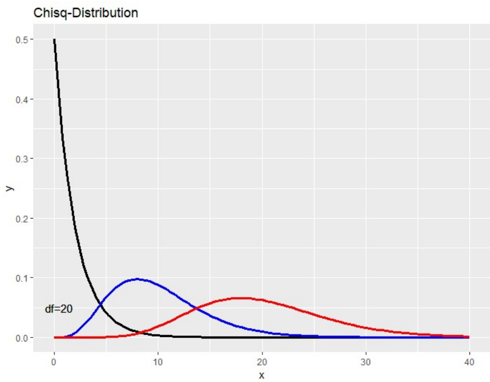
Properties of the Goodness-of-Fit Test

- The data are the observed frequencies. This means that there is only one data value for each category.

- The degrees of freedom is one less than the number of categories, not one less than the sample size.

- It is always a right tail test.

- It has a chi-square distribution.



자유도가 증가할수록 정규분포에 수렴

- The value of the test statistic doesn't change if the order of the categories is switched.

Goodness-of-Fit Test: Chi-square Test

Example #1:

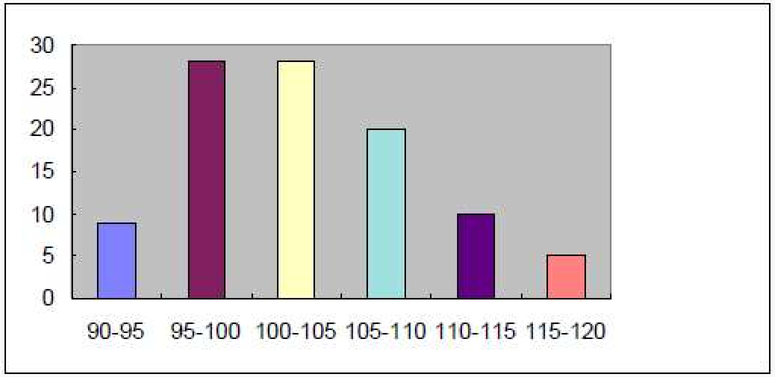
	DATA SET 3		sort
1	102.05		90.38
2	90.38		91.14
3	96.81		92.47
4	101.27		93.61
5	102.22		93.88
6	101.14		94.22
7	98.60		94.22
8	106.70		94.25
9	99.44		94.34
10	105.06		95.32
11	105.06		95.42
12	113.84		96.33
13	99.10		96.72
14	112.13		96.81
15	100.03		97.27
16	105.19		97.38
17	92.47		97.40
18	100.96		97.91
19	112.35		98.23
20	97.27		98.24
21	106.00		98.28
22	111.88		98.32
23	96.33		98.44
24	94.22		98.56
25	103.86		98.60
26	94.22		99.10
27	96.72		99.19
28	95.42		99.21
29	117.40		99.23
30	99.70		99.36
31	107.24		99.42
32	105.08		99.44
33	106.93		99.49
34	113.48		99.60
35	98.56		99.70
36	98.28		99.83
37	94.34		99.95
38	113.23		100.03
39	110.11		100.16
40	99.19		100.65
41	109.20		100.73
42	98.23		100.96
43	98.44		101.05
44	98.24		101.14
45	104.95		101.16
46	116.54		101.23
47	100.65		101.27
48	101.23		101.63
49	99.49		101.83
50	108.03		102.02
51	97.38		102.05
52	100.16		102.22
53	109.69		102.22
54	116.87		102.37
55	99.21		102.85
56	110.19		102.88
57	101.05		103.20
58	102.02		103.77
59	98.32		103.78
60	107.06		103.86

Original data

Grouping data

Rank order	Frequency
90-95	9
95-100	29
100-105	28
105-110	17
110-115	10
115-120	7
sum	100

Plotting



Goodness-of-Fit Test: Chi-square Test

Example #1:

- Estimating Chi-square.

Group	Observed Frequency	Expected Frequency		Chi-Square	
		Normal	Poisson	Normal	Poisson
90-95	9	9.9	13.5	0.082	1.500
95-100	29	21.3	27.1	2.784	0.133
100-105	28	30.9	27.1	0.272	0.030
105-110	17	24.5	18.0	2.296	0.056
110-115	10	10.6	9.0	0.034	0.111
115-120	7	2.5	5.3	8.100	0.545
Sum				**13.567**	**2.375**

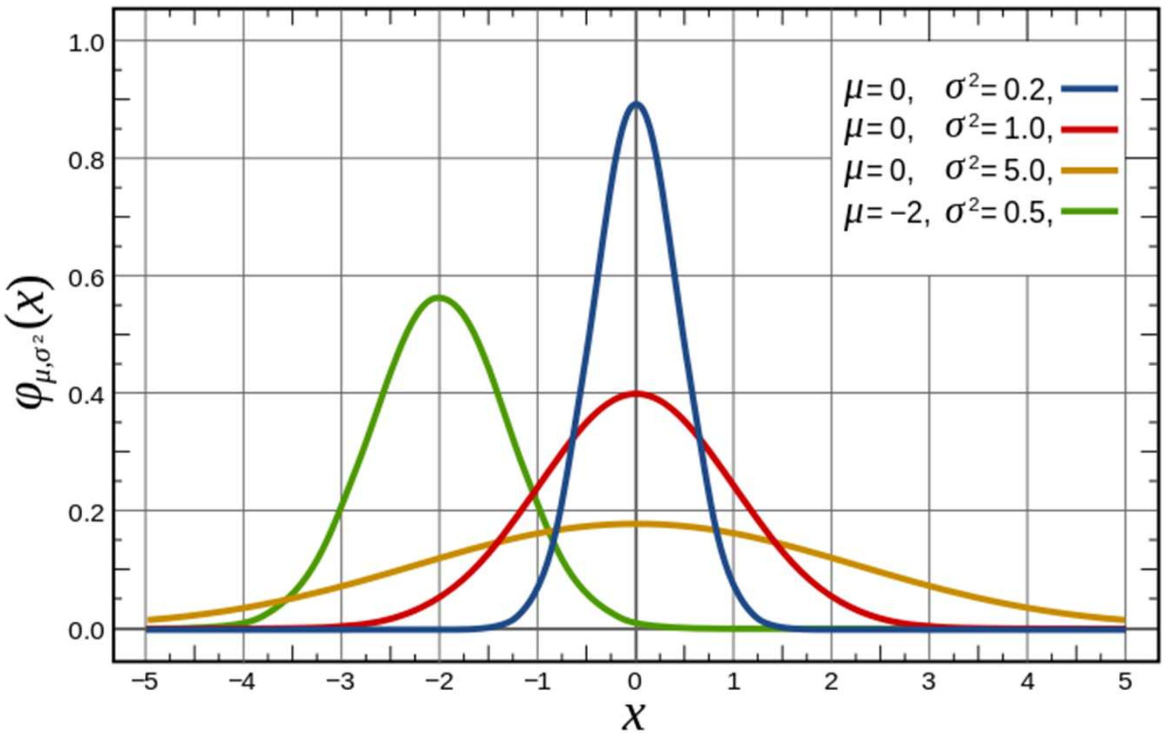
From the definition of the distribution (statistics book)

Goodness-of-Fit Test: Chi-square Test

Example #1:

- Estimating Chi-square
  - Normal distribution:

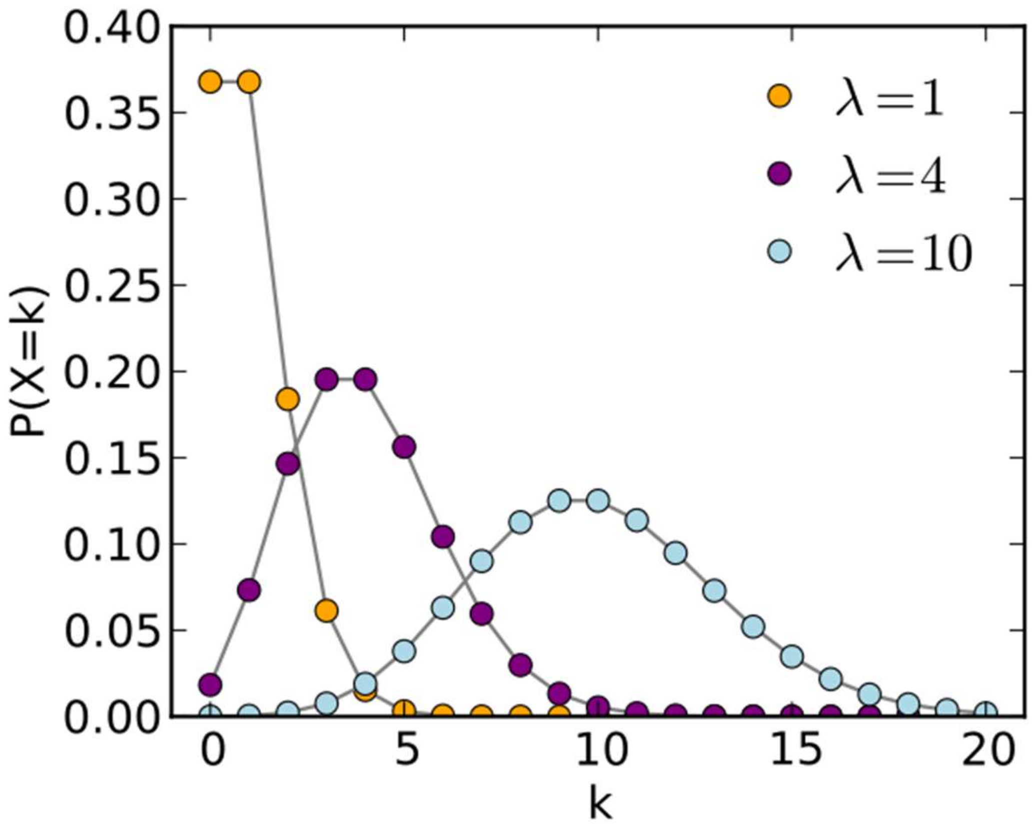
$$N(\mu, \sigma^2), f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Goodness-of-Fit Test: Chi-square Test

Example #1:

- Estimating Chi-square
  - Poisson distribution: $\lambda = E(X) = Var(X),\ \Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$



Goodness-of-Fit Test: Chi-square Test

Example #1:

- Estimating Chi-square.

From statistics book

Group	Observed Frequency	Expected Frequency		Chi-Square	
		Normal	Poisson	Normal	Poisson
90-95	9	9.9	13.5	0.082	1.500
95-100	29	21.3	27.1	2.784	0.133
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115-120	7	2.5	5.3	8.100	0.545
Sum				**13.567**	**2.375**

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

Goodness-of-Fit Test: Chi-square Test

Example #1:

- Estimating Chi-square

From statistics book

Group	Observed Frequency	Expected Frequency		Chi-Square	
		Normal	Poisson	Normal	Poisson
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110-115	10	10.6	9.0	0.034	0.111
115-120	7	2.5	5.3	8.100	0.545
Sum				**13.567**	**2.375**

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

- If Normal distribution is assumed, chi-square is 13.567.
- If Poisson distribution is assumed, chi-square is 2.375.

Goodness-of-Fit Test: Chi-square Test

Example #1:

- Estimating Chi-square
  - Assume 5% level of significance, degree of freedom $v = k - 1 - m$ (where, $k =$ number of data groups, $m =$ number of population parameters from the sample).
  - In the Normal distribution, tabulated Chi-square value is 7.814 under 5% level of significance and degree of freedom $v = 6 - 1 - 2 = 3$.
  - In the Poisson distribution, tabulated Chi-square value is 9.488 under 5% level of significance and degree of freedom $v = 6 - 1 - 1 = 4$.

Goodness-of-Fit Test: Chi-square Test

Example #1:

- **Reference: Chi-square Table**

표 6. χ^2 분포표

d.f. \ α	.995	.990	.975	.950	.050	.025	.010	.005
1	392704×10^{-10}	157088×10^{-9}	982069×10^{-9}	393214×10^{-8}	3.84146	5.02389	6.63490	7.87944
2	.0100251	.0201007	.0506356	.102587	5.99147	7.37776	9.21034	10.5966
3	.0717212	.114832	.215795	.351846	7.81473	9.34840	11.3449	12.8381
4	.206990	.297110	.484419	.710721	9.48773	11.1433	13.2767	14.8602
5	.411740	.554300	.831211	1.145476	11.0705	12.8325	15.0863	16.7496
6	.675727	.872085	1.237347	1.63539	12.5916	14.4494	16.8119	18.5476
7	.989265	1.239043	1.68987	2.16735	14.0671	16.0128	18.4753	20.2777
8	1.344419	1.646482	2.17973	2.73264	15.5073	17.5346	20.0902	21.9550
9	1.734926	2.087912	2.70039	3.32511	16.9190	19.0228	21.6660	23.5893
10	2.15585	2.55821	3.24697	3.94030	18.3070	20.4831	23.2093	25.1882
11	2.60321	3.05347	3.81575	4.57481	19.6751	21.9200	24.7250	26.7569
12	3.07382	3.57056	4.40379	5.22603	21.0261	23.3367	26.2170	28.2995
13	3.56503	4.10691	5.00874	5.89186	22.3621	24.7356	27.6883	29.8194
14	4.07468	4.66043	5.62872	6.57063	23.6848	26.1190	29.1413	31.3193
15	4.60094	5.22935	6.26214	7.26094	24.9958	27.4884	30.5779	32.8013
16	5.14224	5.81221	6.90766	7.96164	26.2962	28.8454	31.9999	34.2672
17	5.69724	6.40776	7.56418	8.67176	27.5871	30.1910	33.4087	35.7185
18	6.26481	7.01491	8.23075	9.39046	28.8693	31.5264	34.8053	37.1564
19	6.84398	7.63273	8.90655	10.1170	30.1435	32.8523	36.1908	38.5822

Goodness-of-Fit Test: Chi-square Test

Example #1:

- **Reference: Parameters of Distributions**

Information / Constraints	Distribution Shape
[a, b]	uniform
[a, m, b]	triangular
[a, b, α_1, α_2, β]	beta
[μ, σ]	normal
γ	exponential
[a, b, μ, σ]	Johnson Sb, Lognormal
[α, β]	gamma

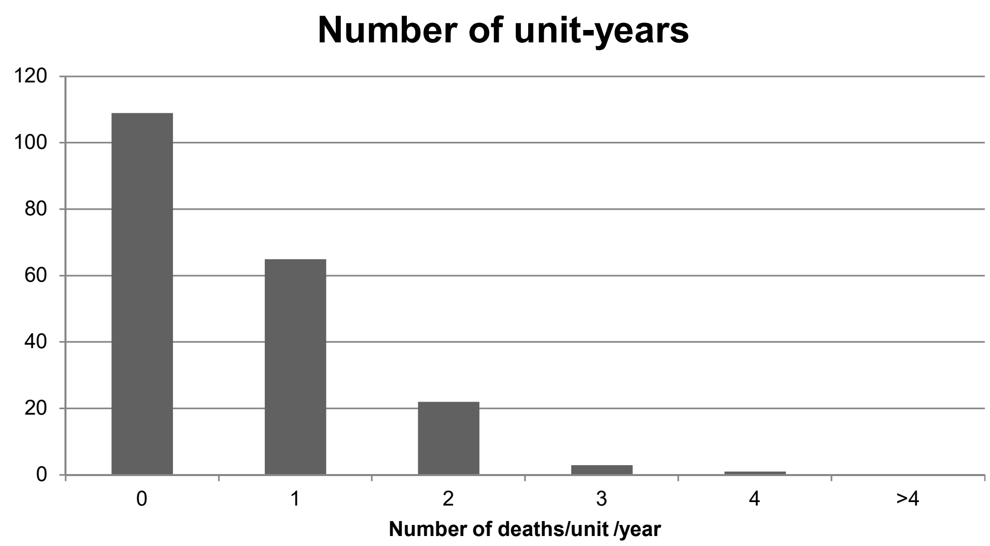
a=minimum, b=maximum, m=mode, α=shape parameter, μ=mean, σ=standard deviation, γ=average rate of occurrence of events, β=scale,

Goodness-of-Fit Test: Chi-square Test

Example #2: Prussian Cavalry (프로이센의 기병대) getting kicked in the head

- X: the number of fatalities per regiment/year in the Prussian cavalry due to horse kicks

Number of deaths/unit/year	Number of unit/years
0	109
1	65
2	22
3	3
4	1
>4	0
Total	**200**



Poisson Distribution?

Goodness-of-Fit Test: Chi-square Test

Example #2: Prussian Cavalry getting kicked in the head

- To test this with a goodness of fit test, we must first know how to generate the null distribution.
- The problem is that we don't have an a priori expectation for the rate of horse-kick fatalities, and we must therefore estimate the rate from the data itself.
- The average number of kicking deaths per year is :

$$\frac{(109 \times 0) + (65 \times 1) + (22 \times 2) + (3 \times 3) + (1 \times 4)}{200} = 0.61 deaths/unit/year$$

- So we can use this as our estimate of the rate of kicking fatalities.

Goodness-of-Fit Test: Chi-square Test

Example #2: Prussian Cavalry getting kicked in the head

- Expected relative frequency
  - Poisson distribution:

$$\lambda = 0.61 = E(X) = Var(X), \qquad \Pr(X = k) = \frac{0.61^k e^{-0.61}}{k!}$$

Number of deaths/unit /year	Expected relative frequency
0	0.54
1	0.33
2	0.10
3	0.02
4	0.003
>4	**0.0004**

Goodness-of-Fit Test: Chi-square Test

Example #2: Prussian Cavalry getting kicked in the head

- From this we can calculate the expected frequencies of the numbers of deaths per year, given the Poisson distribution:

Number of deaths/unit /year	Expected relative frequency	Expected count (relative freq. $\times$ total number)
0	0.54	109
1	0.33	66
2	0.10	20
3	0.02	4
4	0.003	1
>4	0.0004	0
Total		**200**

Goodness-of-Fit Test: Chi-square Test

Example #2: Prussian Cavalry getting kicked in the head

- Then, we must combine across classes to ensure $E \geq 5$:

Number of deaths/unit /year	Observed	Expected
0	109	109
1	65	66
2	22	20
>2	4	5
Total	**200**	**200**

- So now there are 4 classes and we have estimated one parameter (the average rate) from the data, we have 4 - 1 - 1 = 2 d.f..
- We can calculate that $\chi^2 = 0.415$, and the critical value of χ^2 with 2 d.f. and 5% level of significance is $\chi^2 = 5.991$, we are not in the tail of the distribution, and we cannot reject the null hypothesis that the deaths are occurring at random. In fact the match to the Poisson distribution is remarkably good.

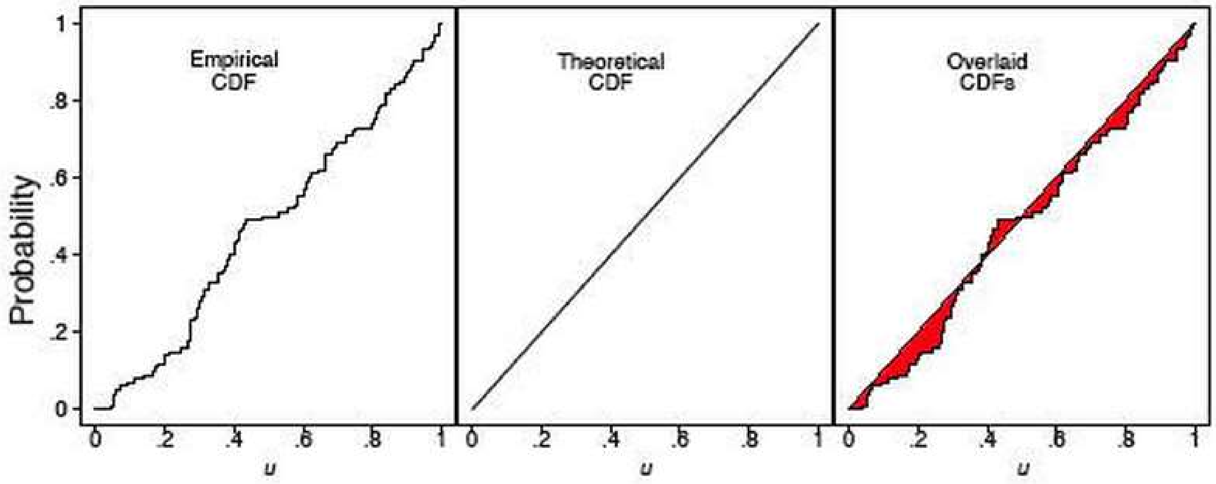
Goodness-of-Fit Test: K-S Test

- The **Kolmogorov-Smirnov test**, also called the **Kolmogorov-Smirnov test**, is a goodness-of-fit test which tests whether a given distribution is not significantly different from one hypothesized (ex., on the basis of the assumption of a normal distribution).
- It is a more powerful alternative to chi-square goodness-of-fit tests when its assumptions are met.
- Whereas the chi-square test of goodness-of-fit tests whether in general the observed distribution is not significantly different from the hypothesized one, the K-S test tests whether this is so even for the most deviant values of the criterion variable. Thus it is a more stringent (엄격한) test.

Chi-square Test	Kolmogorov-Smirnov Test
기대값과 관측값의 차이	누적분포함수(CDF)의 차이
이산형데이터. 연속형일 경우 그룹화	연속형데이터
표본크기가 커야함	필요 표본크기가 상대적으로 작음

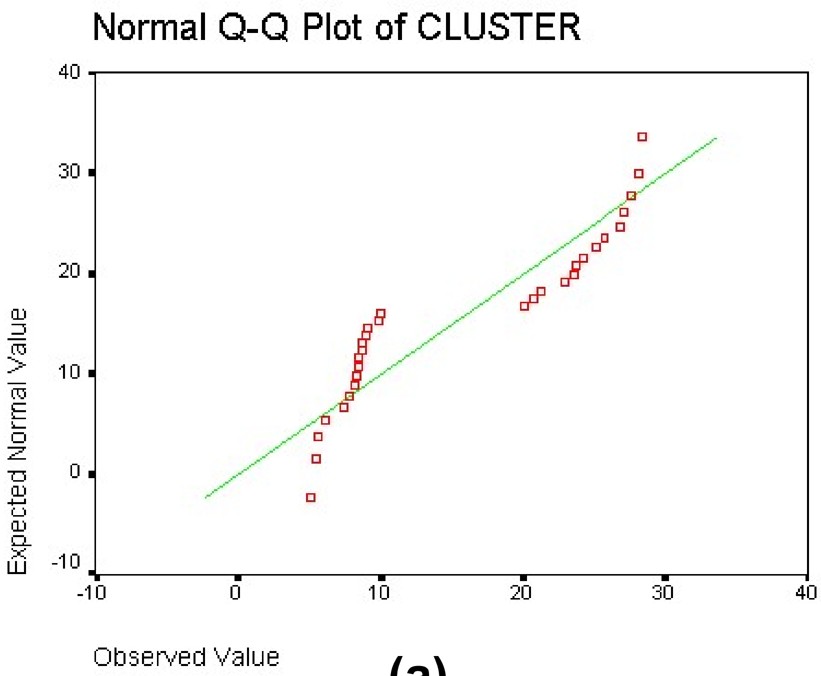
Quantile-Quantile Plots (Q-Q Plot)

- The assumption of a normal model for a population of samples will be required in order to perform certain inference procedures. Histogram can be used to get an idea of the shape of a distribution.
- However, there are more sensitive tools for checking if the shape is close to a normal model – a Q-Q Plot.
- Q-Q Plot is a plot of the percentiles (or quantiles) of a standard normal distribution (or any other specific distribution) against the corresponding percentiles of the observed data.
- If the observations follow approximately a normal distribution, the resulting plot should be roughly a straight line with a positive slope.

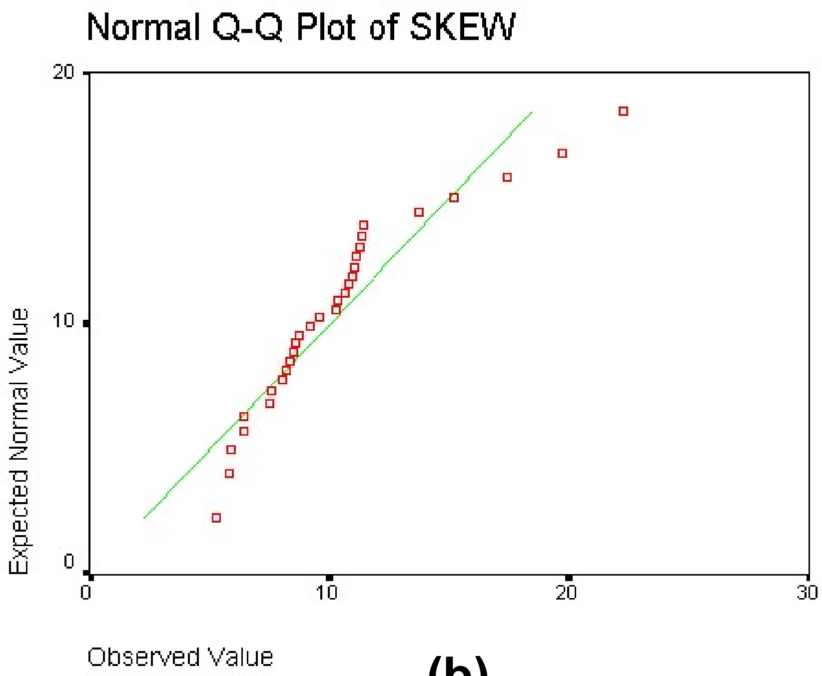


Quantile-Quantile Plots (Q-Q Plot)

- The graphs below are examples for which a normal model for the response is not reasonable.
  - The Q-Q plot (a) indicates the existence of two clusters of observations.
  - The Q-Q plot (b) shows an example where the shape of distribution appears to be skewed right.



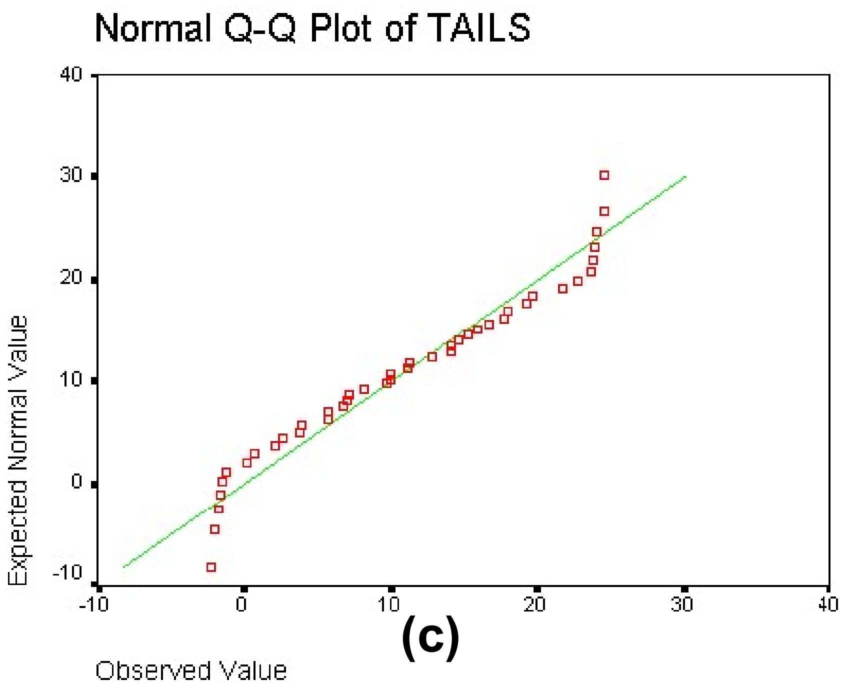
(a)



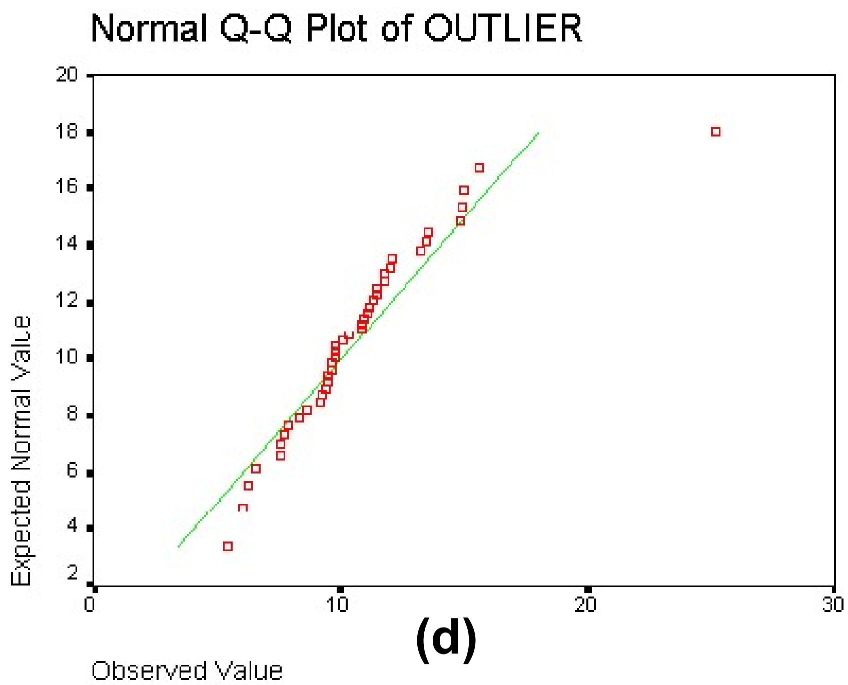
(b)

Quantile-Quantile Plots (Q-Q Plot)

- The graphs below are examples for which a normal model for the response is not reasonable.
  - The Q-Q plot (c) shows evidence of an underlying distribution that has heavier tails compared to those of a normal distribution.
  - The Q-Q plot (d) shows evidence of an underlying distribution which is approximately normal except for one large outlier that should be further investigated.



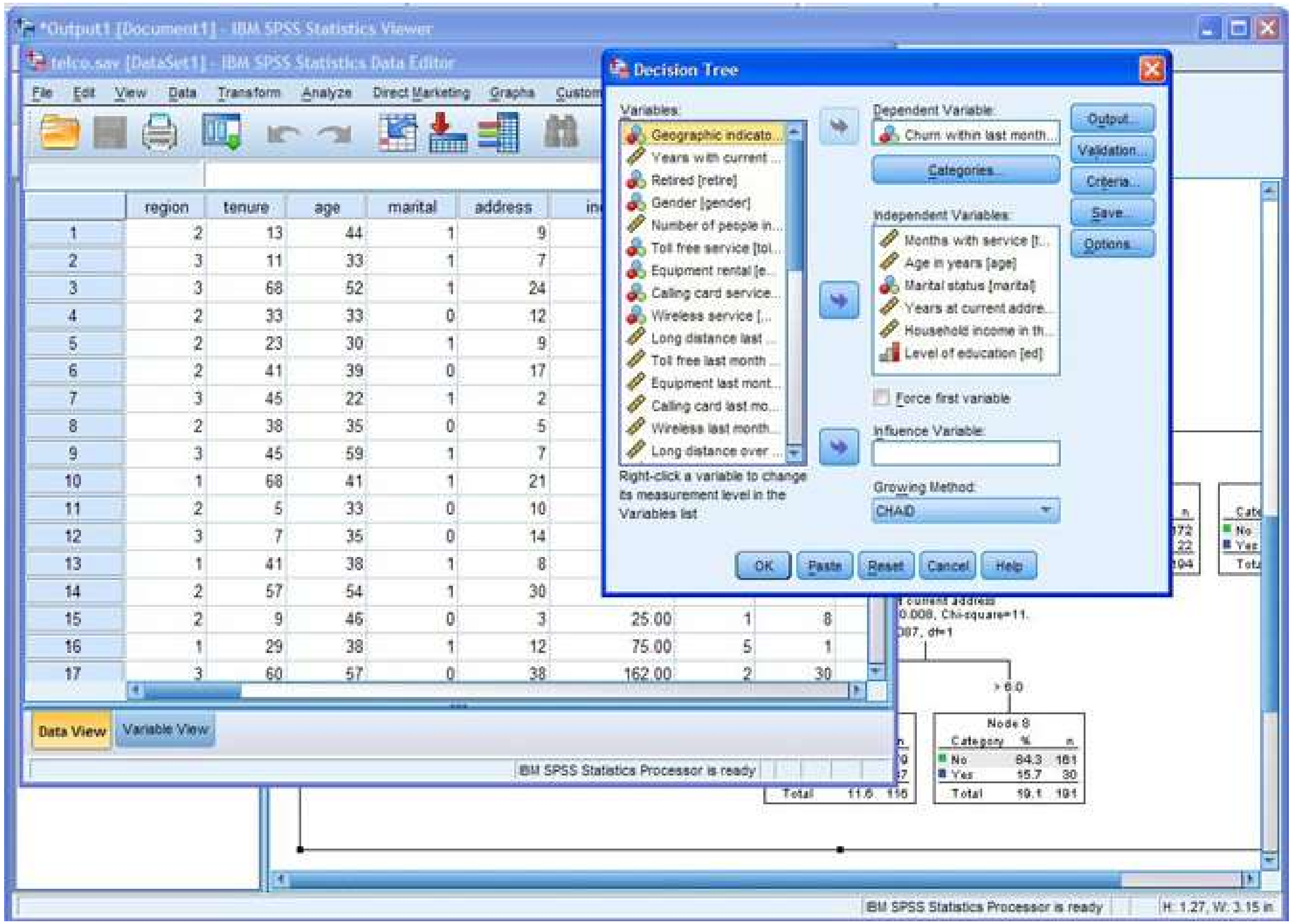
(c)



(d)

Using S/W

- SPSS



Using S/W

- SPSS

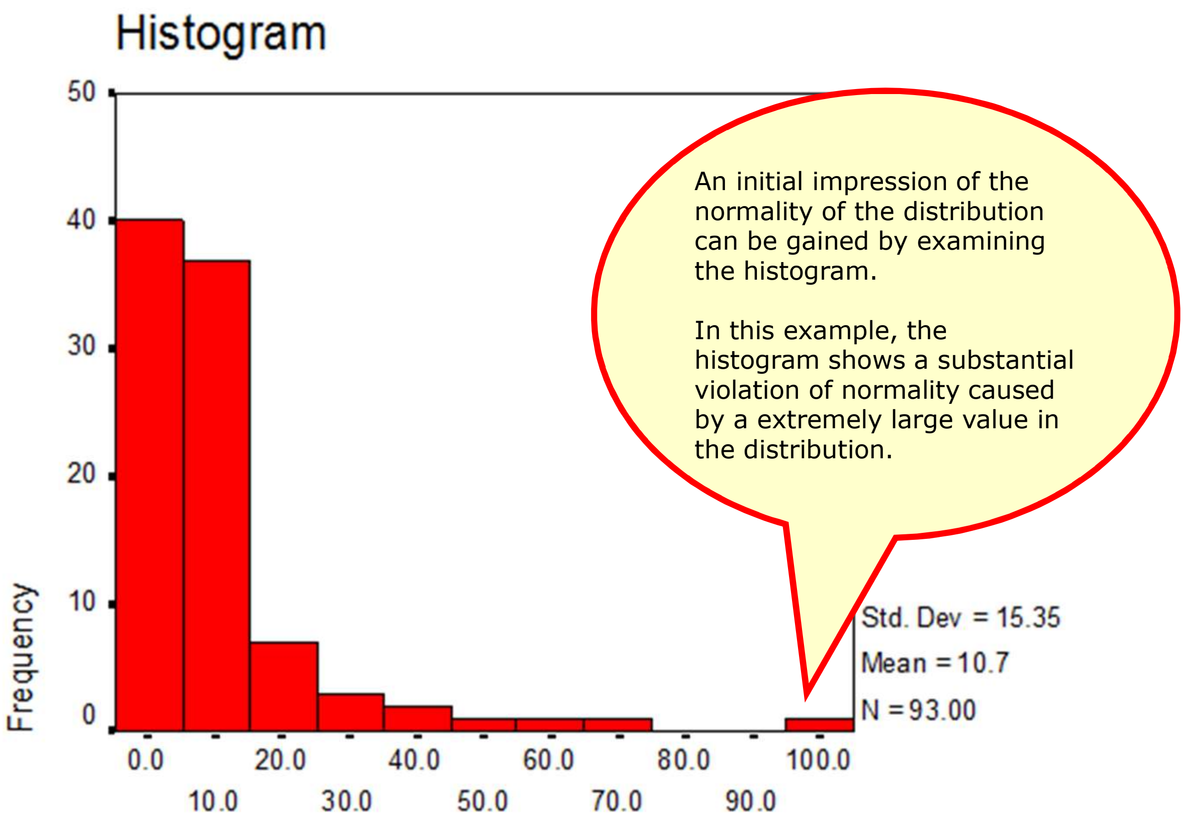
Descriptives

			Statistic	Std. Error
TOTAL TIME SPENT ON THE INTERNET	Mean		10.731	1.5918
	95% Confidence Interval for Mean	Lower Bound	7.570	
		Upper Bound	13.893	
	5% Trimmed Mean		8.295	
	Median		5.500	
	Variance		235.655	
	Std. Deviation		15.3511	
	Minimum		.2	
	Maximum		102.0	
	Range		101.8	
	Interquartile Range		10.200	
	Skewness		3.532	.250
	Kurtosis		15.614	.495

The skewness and kurtosis for the variable both exceed the rule of thumb criteria of 1.0. The variable is not normally distributed.

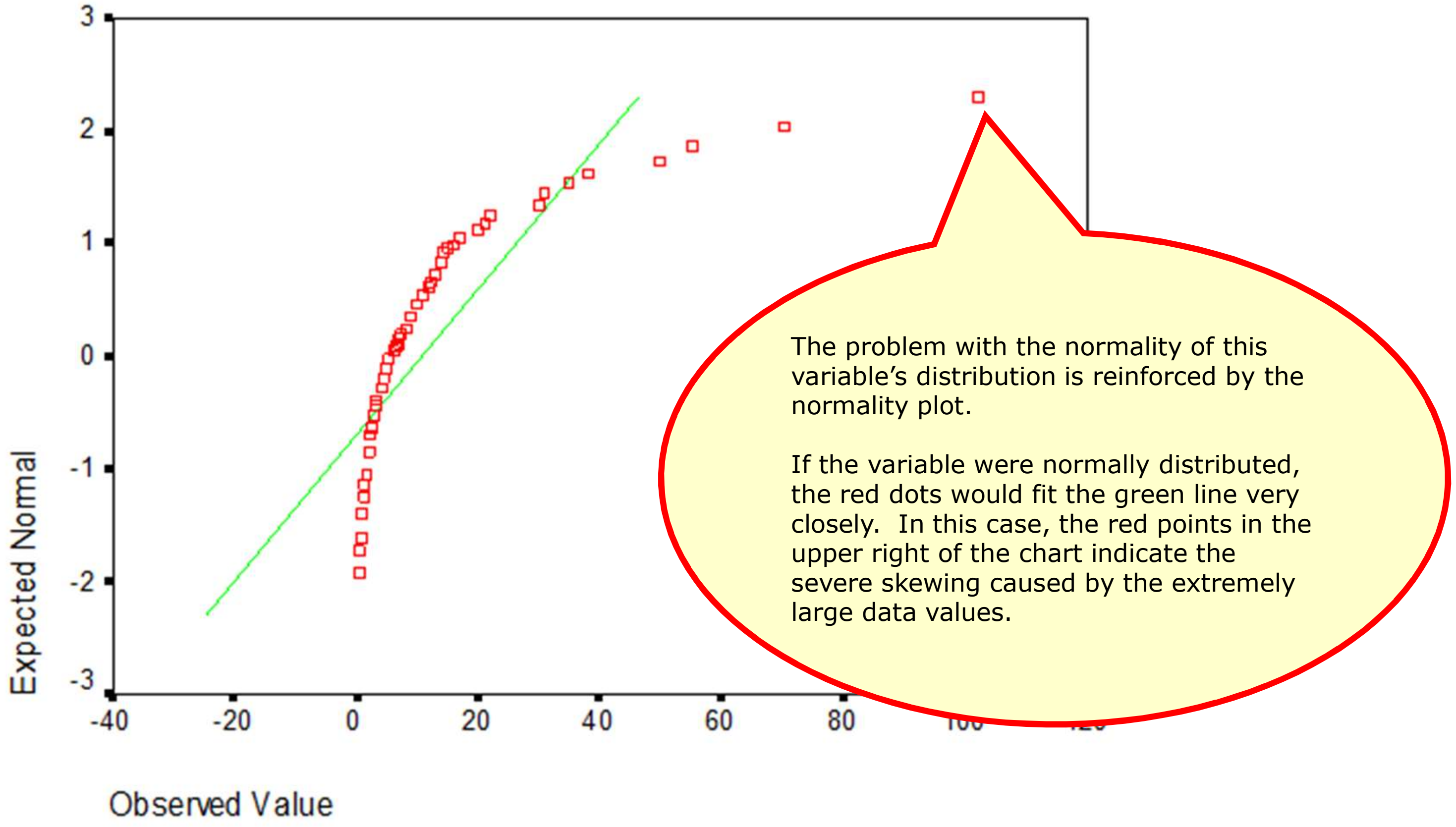
Using S/W

- SPSS



Using S/W

- SPSS



Using S/W

- SPSS

Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
TOTAL TIME SPENT ON THE INTERNET	.246	93	.000	.606	93	.000

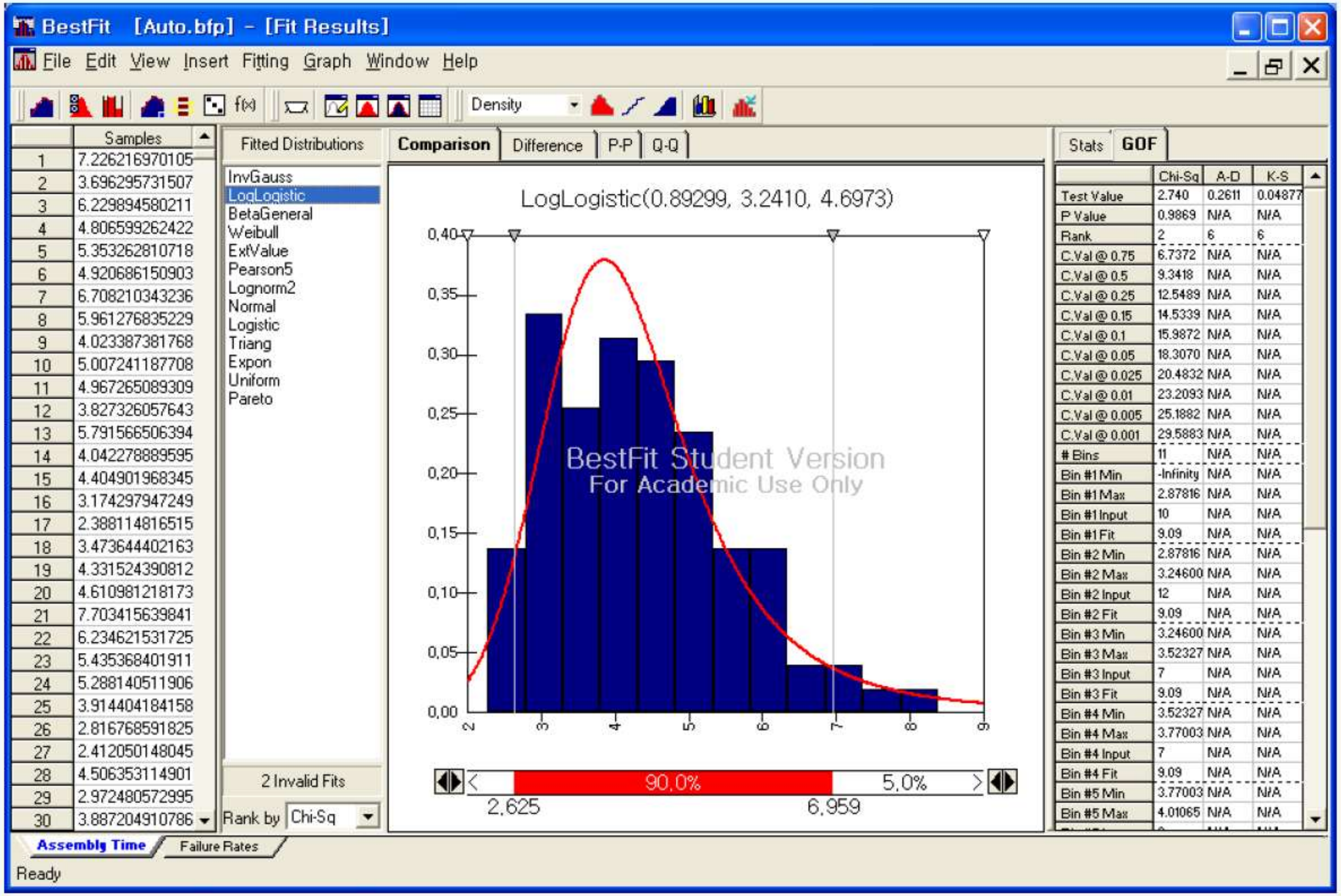
a. Lilliefors Significance Correction

Since the sample size is larger than 50, we use the Kolmogorov-Smirnov test. If the sample size were 50 or less, we would use the Shapiro-Wilk statistic instead.

The null hypothesis for the test of normality states that the actual distribution of the variable is equal to the expected distribution, i.e., the variable is normally distributed. Since the probability associated with the test of normality is < 0.001 is less than or equal to the level of significance (0.01), we reject the null hypothesis and conclude that total hours spent on the Internet is not normally distributed. (Note: we report the probability as <0.001 instead of .000 to be clear that the probability is not really zero.)

Using S/W

- Best fit (embedded in “@Risk” program)

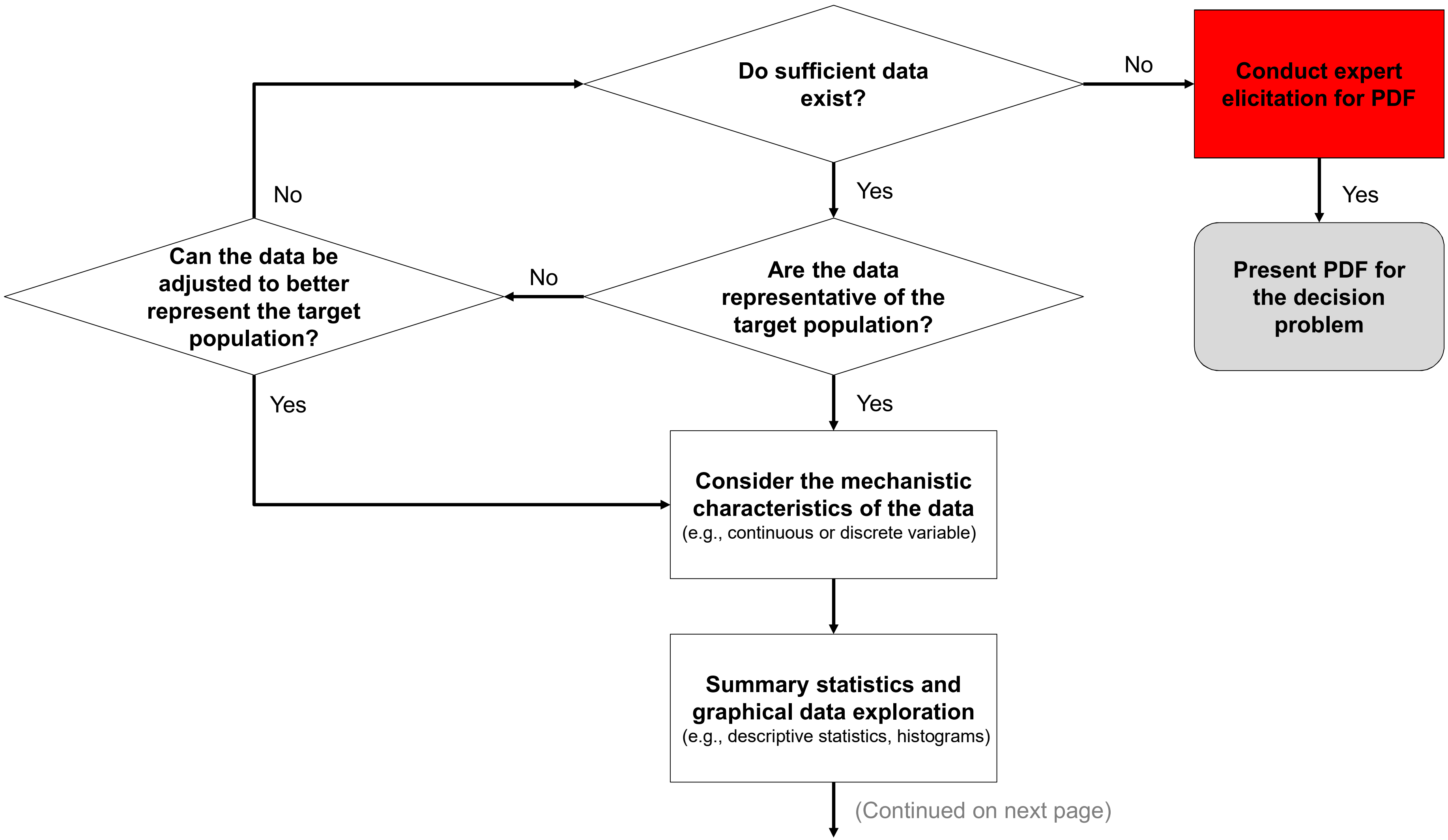


PART II

MODELING UNCERTAINTY

- Using Subjective Assessment

Expert's Judgement



Expert's Judgement

- Unfortunately, in many cases, directly applicable historical data concerning the risk are not available inadequate amount, and a subjective assessment will be required.

  - Contractors are generally reluctant to document or record data as they come from the field during construction or as the project proceeds.

  - Even if they do so, the data are incomplete.

  - Hence, available data are mainly <u>subjective in nature</u> and must be obtained through careful questioning of ***experts*** or people with the relevant knowledge.

1. Risk Perception

- Different experts draw contradictory conclusions

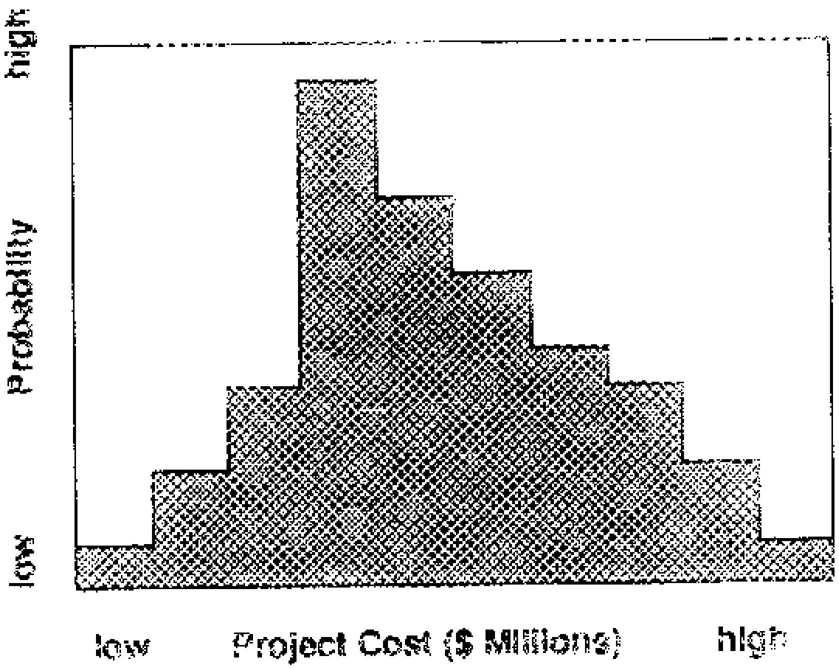
- How people perceive risk?
  - Personal background
  - Experiences (expert vs. novice), domain specific knowledge
  - Very sensitive to certain kindness of specific risks
  - Conservative or Speculative: Manager or estimator's perspective? Cost risk or Revenue risk?

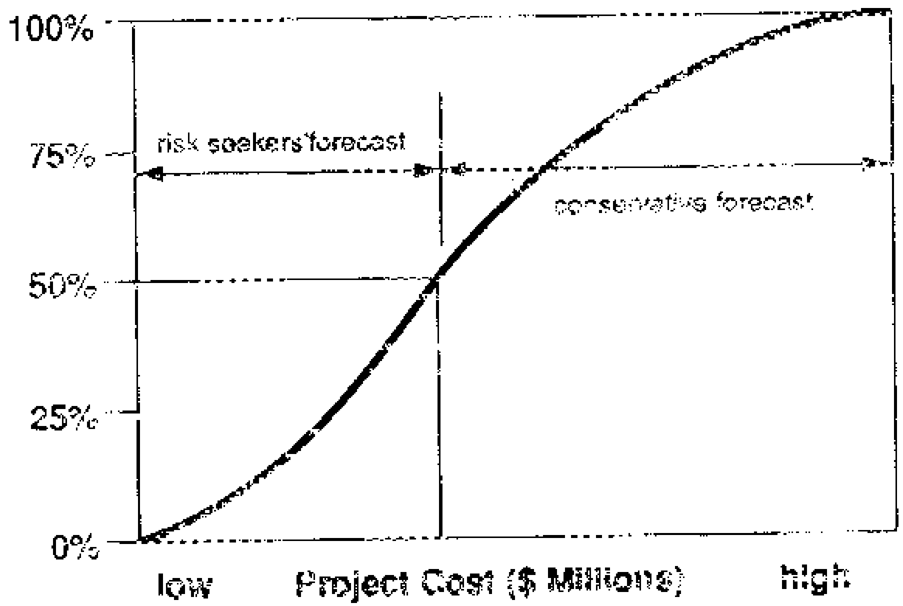
- Risk messages are difficult to formulate in accurate, clear, and not misleading way

- How to communicate risk information to the public or higher levels of decision makers? → **Big issue**

1. Risk Perception

- **Mak and Raftery (1992)**
  - Risk Attitude in Forecast of Costs

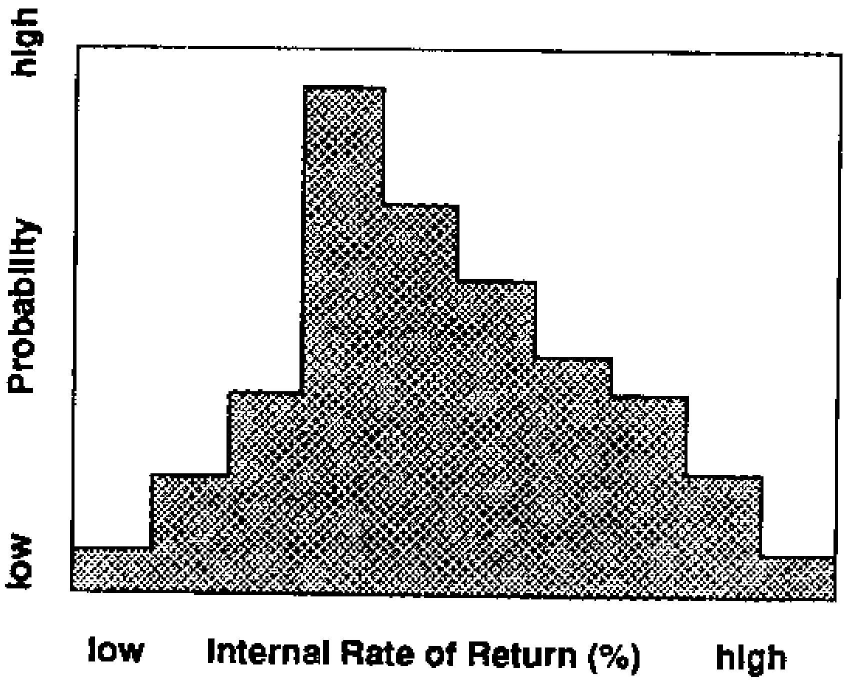


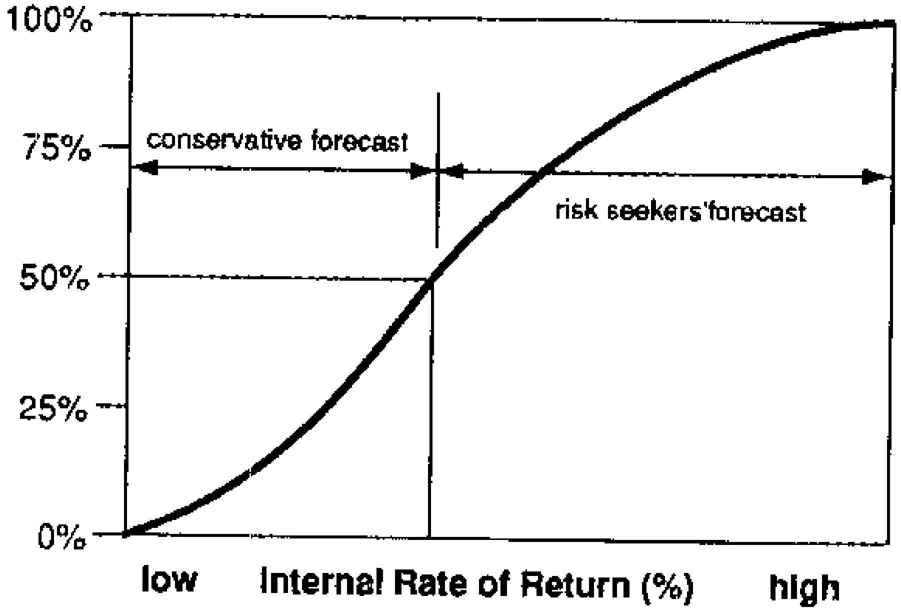


* Mak, S. and J. Raftery (1992). "Risk attitude and systematic bias in estimating and forecasting." *Construction Management & Economics*, 10(4): 303-320.

1. Risk Perception

- **Mak and Raftery (1992)**
  - Risk Attitude in Forecast of Return

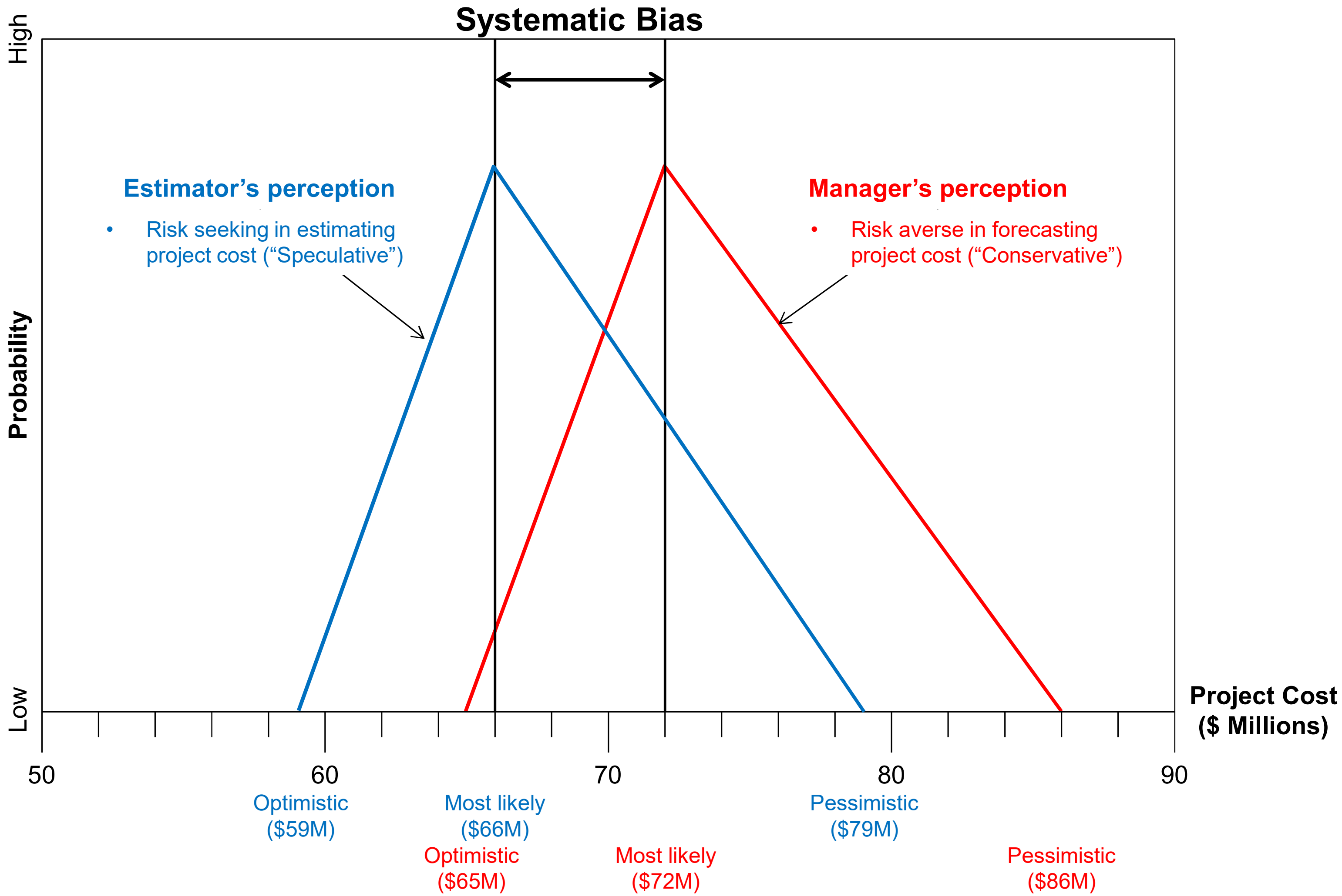




* Mak, S. and J. Raftery (1992). "Risk attitude and systematic bias in estimating and forecasting." *Construction Management & Economics*, 10(4): 303-320.

1. Risk Perception

- **Mak and Raftery (1992)**



* Mak, S. and J. Raftery (1992). "Risk attitude and systematic bias in estimating and forecasting." *Construction Management & Economics*, 10(4): 303-320.

1. Risk Perception

- **Variation in common language**
  [Survey by DSMC (1983)]
  - Need for common language in uncertain (risky) situations
  - 23 military experts interpreting various phrases
  - How to reduce error and inconsistency in elicit uncertain information?

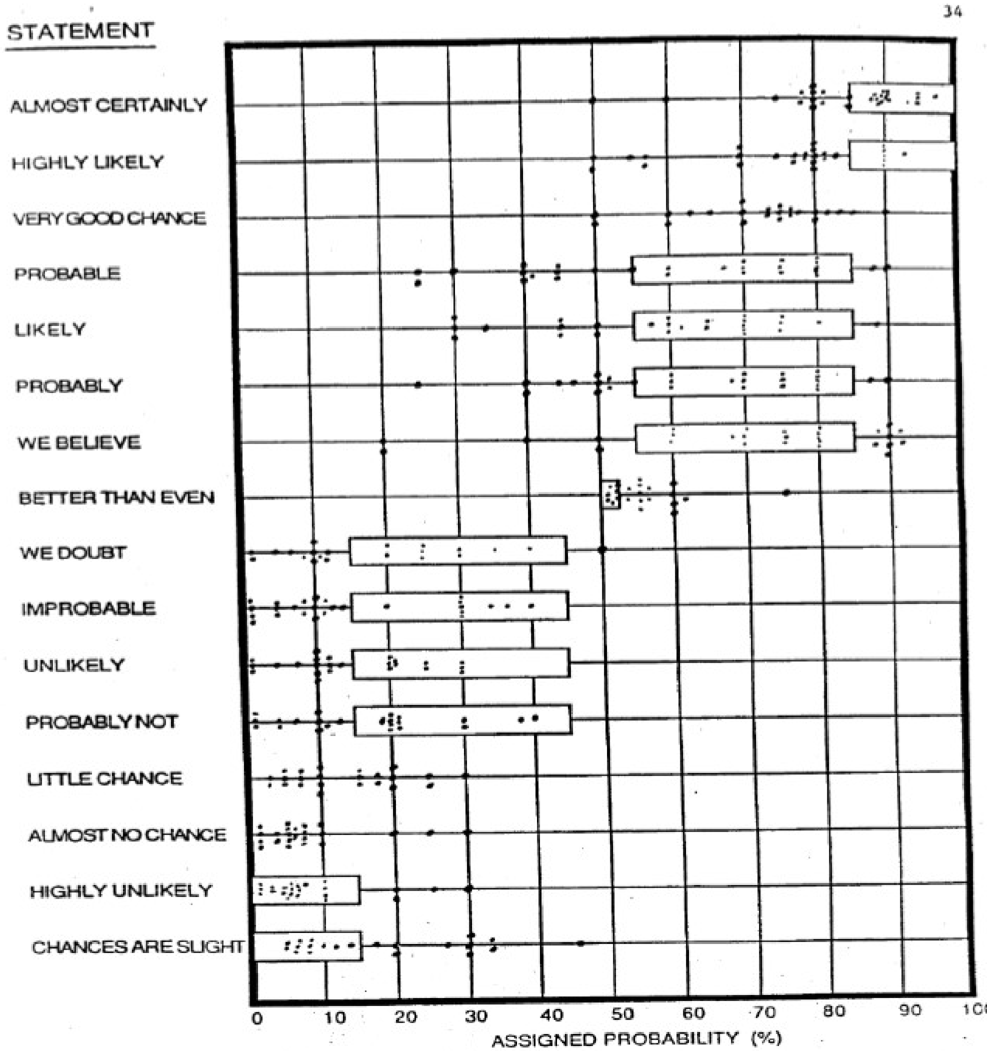


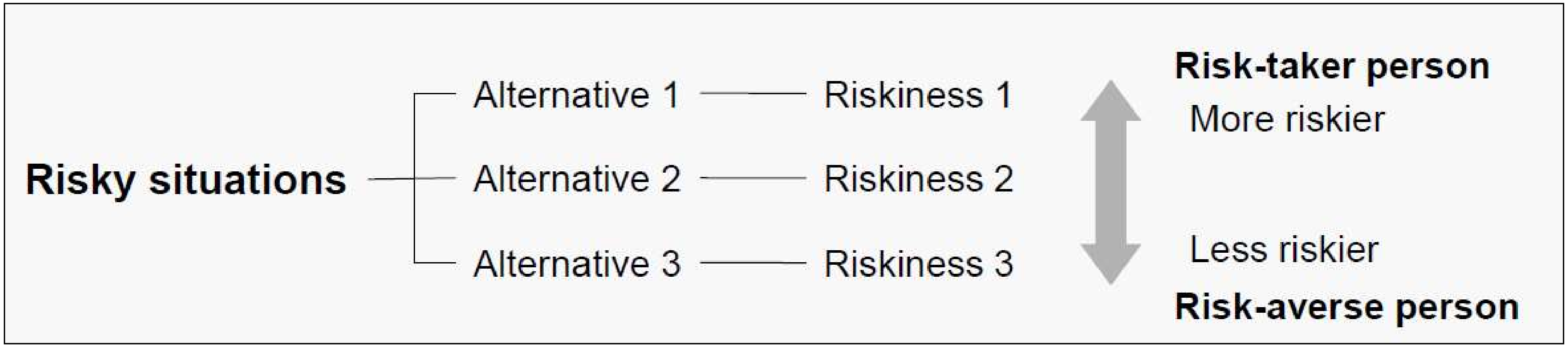
Figure III-IV: What Uncertainty Statements Mean To Different Readers
After Defense Systems Management College [1983]

* Defense Systems Management College (1983). “Risk Assessment Techniques, A Handbook for Program Management Personnel," Defense Systems Management College (Fort Belvoir), July.

2. Risk Preferences/ Risk Attitudes

- Different people have different attitudes toward risk
  - Make different choices within the same risky context

- Three types
  - Risk averse: Individuals who are afraid of risk or are sensitive to risk
  - Risk seeking (or taking): person those who try to seek the risky alternatives
  - Risk neutral: person who ignores risk aspects of alternatives



2. Risk Preferences/ Risk Attitudes

Components of Risk	Risk Averter Requires	Risk Taker Accepts
Magnitude of Potential Loss	Low Maximum loss Low stakes,commitment Low variability in pay offs More information on losses More control over losses	Higher Maximum loss Higher stakes,commitment Higher variability in pay offs Less information on losses Less control over losses
Chances of Potential Loss	Low chance of loss Familiar environment Few uncertain events More information on chances More control over uncertain events Low uncertainty	Higher chance of loss Unfamiliar environment Many uncertain events Less information on chances Less control over uncertain events Higher uncertainty
Exposure to Potential Loss	Low exposure Shared responsibility More information on exposure More control over exposure	Higher exposure Sole responsibility Less information on exposure Less control over exposure
Other Risk Components	Control by self Contingency plans Consensus Exit from risky situations	Control by others No Contingency plans Conflict Participation in risky situations

2. Risk Preferences/ Risk Attitudes

- **Situation:**
  - We are not all expected monetary value (EMV) decision makers.
  - Some of us are risk-preferring and some of us are risk-averse.
  - **How can we factor this into the analysis?**

2. Risk Preferences/ Risk Attitudes

- **Are you a risk averter or a risk taker?**

Game #1	Game #2
✓ Win $30 /w p=0.5	✓ Win $2,000 /w p=0.5
✓ Lose $1 /w p=0.5	✓ Lose $1,900 /w p=0.5

- **Which game do you prefer?**
  - Game #1: EMV=$14.5, SD=14.53
  - Game #2: EMV=$50, SD=1,900.7

2. Risk Preferences/ Risk Attitudes

Framing Effects

- Suppose that we are preparing for an outbreak of an unusual disease that is expected to **kill 6,000 people**.
- Two alternatives are possible, and scientific estimates of the consequences are as follows:
  - If **program A** is adopted, **2,000 people** will be **saved**.
  - If **program B** is adopted,

    1/3 probability that **6,000 people** will be **saved**.

    2/3 probability that **no people** will be **saved**.
- **Which program would you favor?**

2. Risk Preferences/ Risk Attitudes

Framing Effects

- Here are other alternative programs with the same situation (disease that is expected to **kill 6,000 people**).
- Two alternatives are possible, and scientific estimates of the consequences are as follows:
  - If **program C** is adopted, **4,000 people** will **die**.
  - If **program D** is adopted,

    1/3 probability that **nobody** will **die**.

    2/3 probability that **6,000 people** will **die**.
- **Which program would you favor?**

People behave differently against gain (i.e. SAVE) and loss (i.e. DIE) even though the results are the same.

2. Risk Preferences/ Risk Attitudes

Framing Effects (Tversky and Kahneman, 1983)

- The framing effect is an example of cognitive bias, in which people react to a particular choice in different ways depending on how it is presented; e.g. as a loss or as a gain.
- People tend to avoid risk when a positive frame is presented but seek risks when a negative frame is presented.
- Gain and loss are defined in the scenario as descriptions of outcomes (e.g. lives lost or saved, disease patients treated and not treated, lives saved and lost during accidents, etc.).
- Prospect theory shows that a loss is more significant than the equivalent gain, that a sure gain (certainty effect and pseudo-certainty effect) is favored over a probabilistic gain, and that a probabilistic loss is preferred to a definite loss. One of the dangers of framing effects is that people are often provided with options within the context of only one of the two frames.

* Tversky, A. and Kahneman, D. (1983). "Extensional Versus Intuitive Reasoning: The Conjunction Fallacy in Probability Judgment." *Psychological Review*, 90(4), pp. 293–315.

2. Risk Preferences/ Risk Attitudes

Framing Effects (Tversky and Kahneman, 1983)

- **Examples of Framing Effects**
  - 6,000 deaths are more than three times as bad as 2,000 deaths. By contrast, 6,000 savings is less than three times as good as 2,000 savings.
  - The large airplane accident (350 fatalities) is more than twice as serious as small one with the half of the number of fatalities (175 fatalities).
  - War involving one million casualties will be more than twice as bad as one involving half million casualties.

* Tversky, A. and Kahneman, D. (1983). "Extensional Versus Intuitive Reasoning: The Conjunction Fallacy in Probability Judgment." *Psychological Review*, 90(4), pp. 293–315.

2. Risk Preferences/ Risk Attitudes

Framing Effects (Tversky and Kahneman, 1983)

- **Examples of Framing Effects**
  - Concerns about a death accident by crashing at the same airplane make husband and wife to decide flying on separate airplanes.
    - ❖ By doing so, they reduce the probability that both of them will die at the same accident.
    - ❖ However, it simultaneously increase the probability that at least one of them die.
      - ➢ Crash of BOTH planes: $p(a) \times p(b) = 0.01 \times 0.01 = 0.0001$ → **Prob. of both die at the same time**
      - ➢ Crash of ONE plane: $p(a) + p(b) = 0.01 + 0.01 = 0.02$ → **Prob. of one of them die**
  - **WHY?**
    - ❖ Dislike of both dying is more than twice as serious as the dislike of one of them dying.

* Tversky, A. and Kahneman, D. (1983). "Extensional Versus Intuitive Reasoning: The Conjunction Fallacy in Probability Judgment." *Psychological Review*, 90(4), pp. 293–315.

2. Risk Preferences/ Risk Attitudes

Can Risk Attitude Be Measured?

- **Yes it can, but imperfectly.**
  - As is true of any measurement, the measurement of risk attitude is subject to error and possibly bias, especially as we are dealing with an inherently subjective construct.

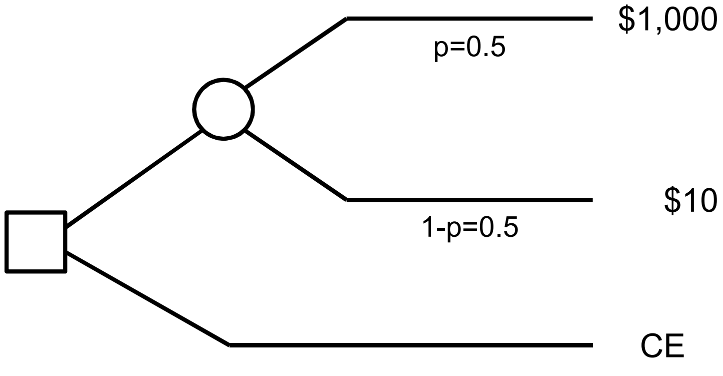
- **A variety of measurement methods have been developed, each with its own merits.**
  - Different methods sometimes produce discordant (조화롭지 못한/서로 다른) results: this may be viewed not necessarily as a problem, but as an opportunity to learn about, and reconcile (조화/조율하다), possible inconsistencies in risk taking.

2. Risk Preferences/ Risk Attitudes

Assessing Risk Attitude

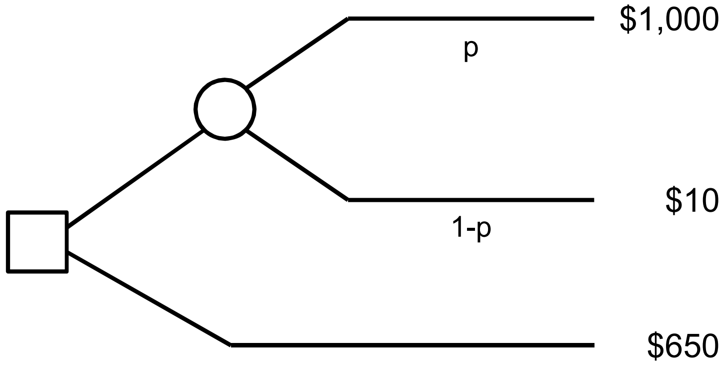
- **Two methods are available.**

Certainty-Equivalent (CE 확실성등가)



A Reference Lottery (CE method)

Probability-Equivalent (PE 확률등가)



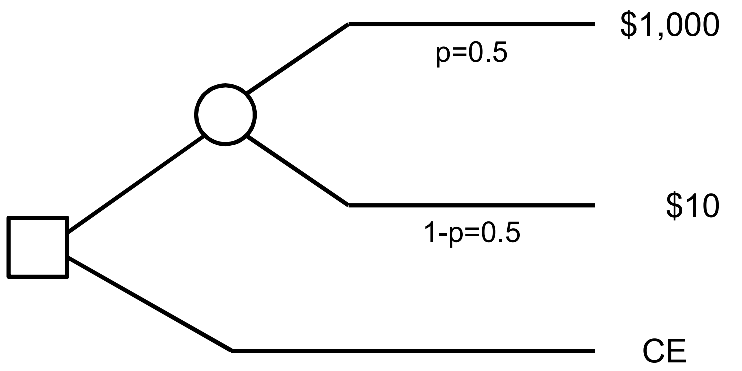
A Reference Lottery (PE method)

- **Subjects are ask for specifying how much sure payoff must be received to make them indifferent between;**
  - Sure payoff (CE) 확실한 페이
  - Expected value (EV) of the given risky investment that is not certain

2. Risk Preferences/ Risk Attitudes

Assessing Risk Attitude – (1) CE method

- **Criteria to decide attitude toward risk**
  - If one answer CE would be less than the EMV ($505), it indicates that he/she prefers the sure payoffs (확실성 담보) rather than risky option (opportunity to earn more payoff).
  - On the other hand, if the CE would be greater than the EMV ($505), this would imply that the decision maker would be a risk-seeker who is not intent to release the risky opportunity (불확실) without being paid a higher CE.



A Reference Lottery (CE method)

2. Risk Preferences/ Risk Attitudes

Assessing Risk Attitude – (1) CE method

- **Exercise**
  - Imagine you have a ticket (opportunity) to play the following bonus game.
  - One of your friends is interested in taking your place.
  - You can trade/sell this game for a sure price to your friend.

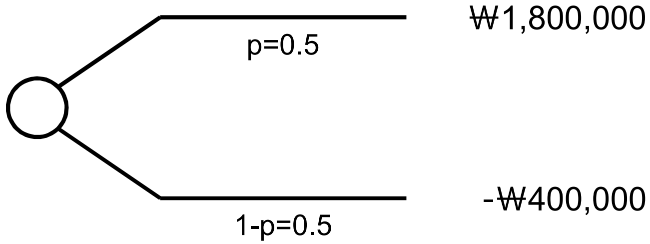
2. Risk Preferences/ Risk Attitudes

Assessing Risk Attitude – (1) CE method

- **Exercise:** How much are you willing to sell this game (opportunity)?

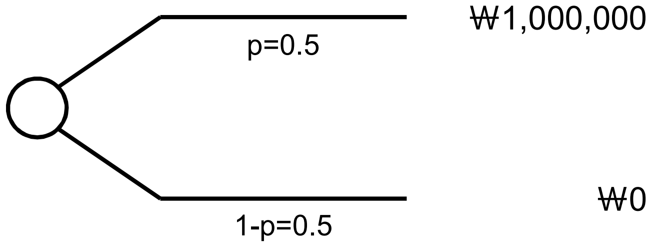
Certainty-Equivalent of Games??

Game #1



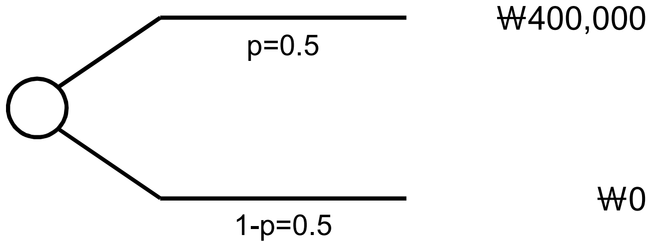
EMV: ₩700,000
CE: ₩500,000 (risk averse)
₩900,000 (risk take)

Game #2



EMV: ₩500,000
CE: ₩300,000
₩700,000

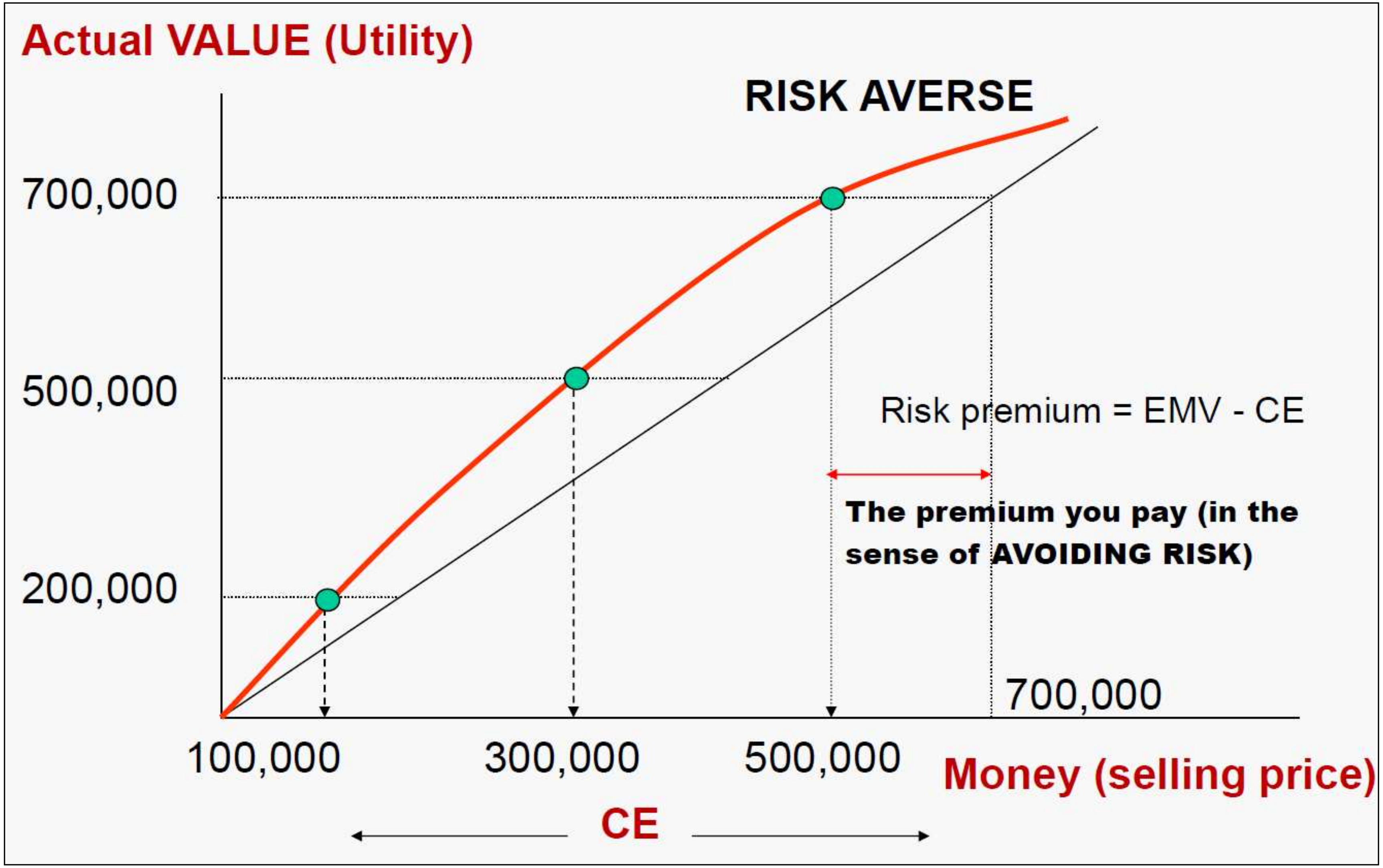
Game #3



EMV: ₩200,000
CE: ₩100,000
₩300,000

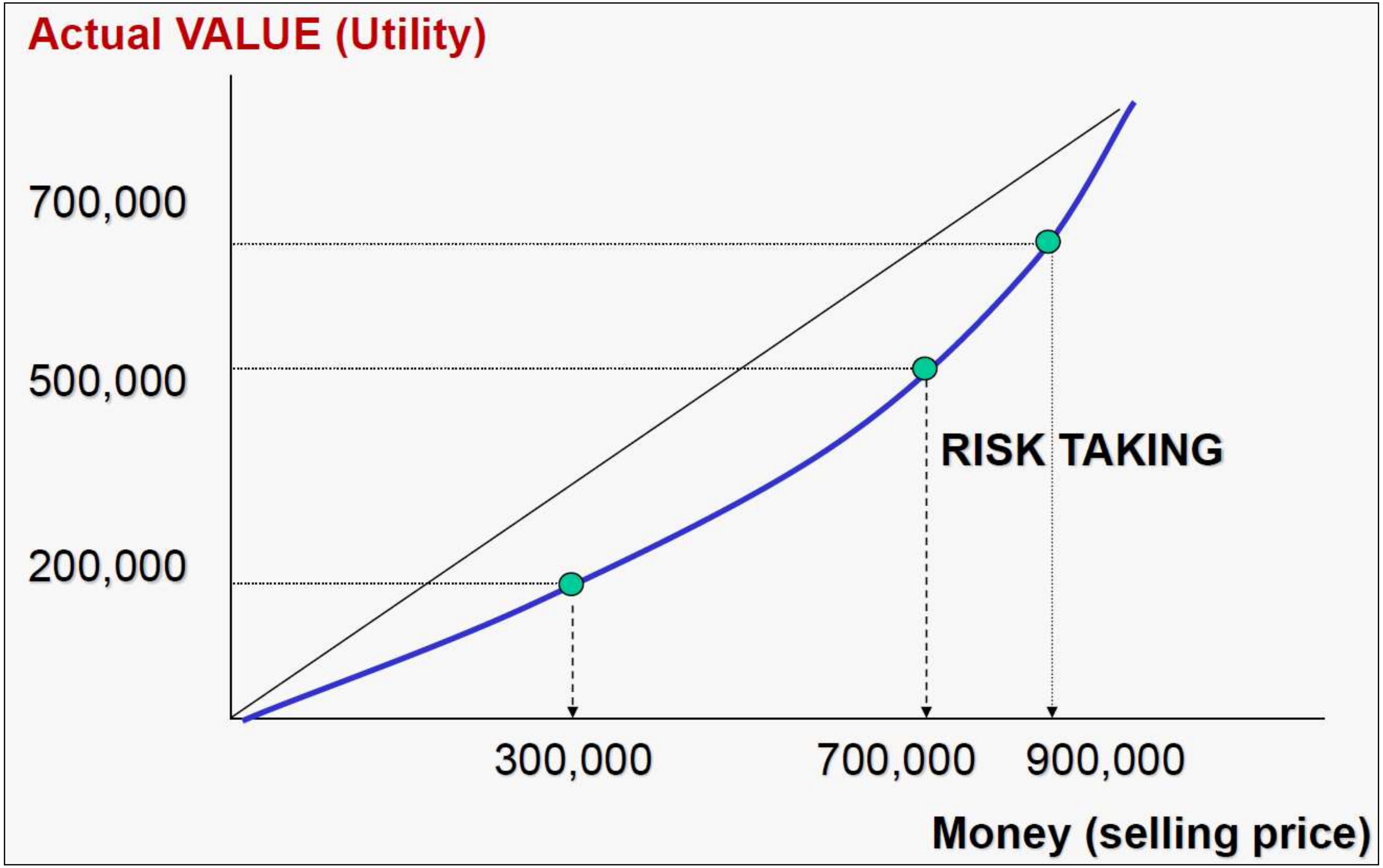
2. Risk Preferences/ Risk Attitudes

Assessing Risk Attitude – (1) CE method



2. Risk Preferences/ Risk Attitudes

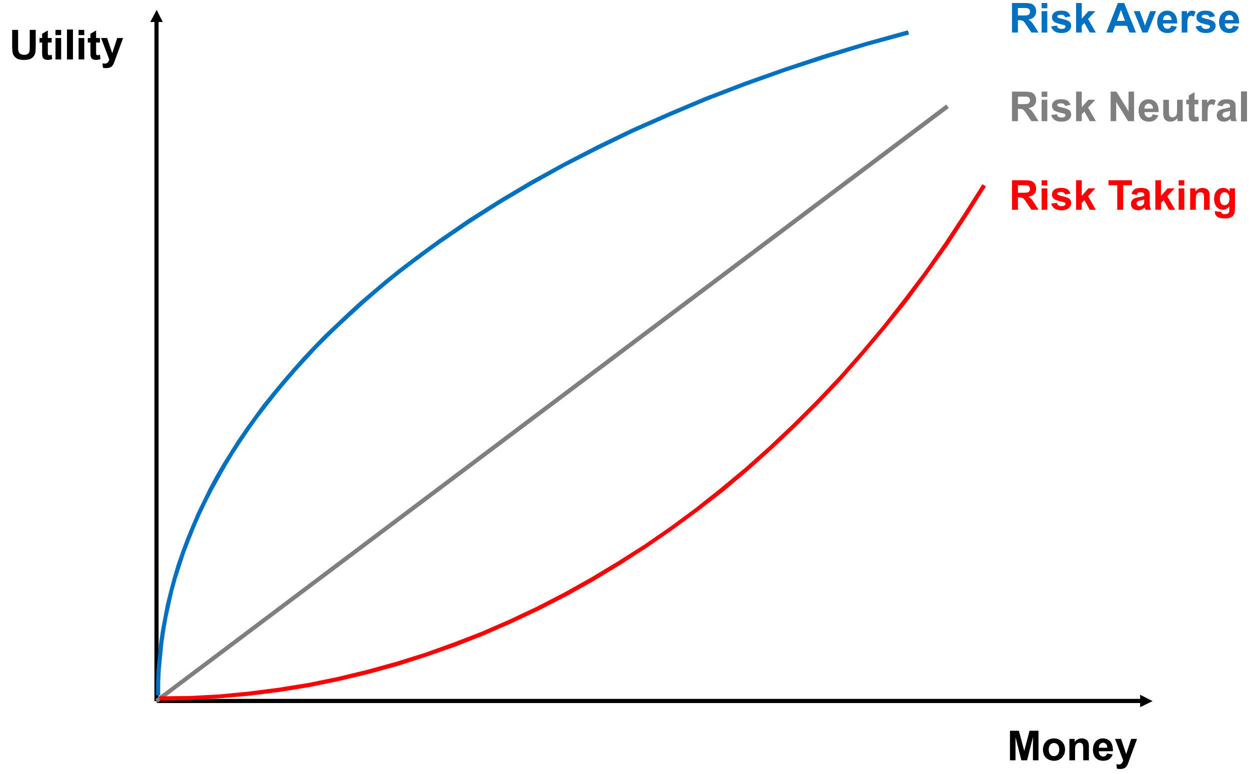
Assessing Risk Attitude – (1) CE method



2. Risk Preferences/ Risk Attitudes

Assessing Risk Attitude – (1) CE method

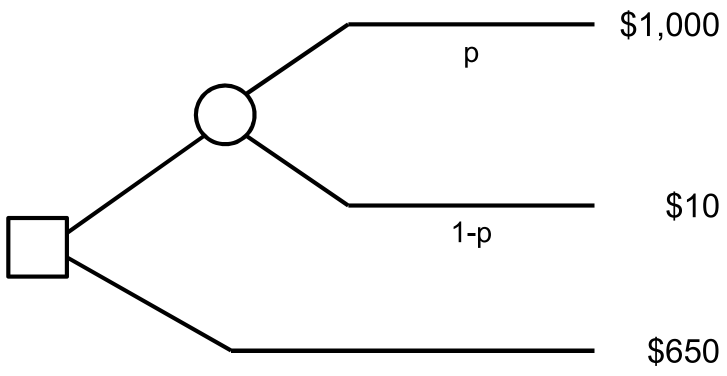
- Typical shapes of risk attitude



2. Risk Preferences/ Risk Attitudes

Assessing Risk Attitude – (2) PE method

- To find the utility for \$650, adjust p until you are indifferent between the sure \$650 and the gamble.
  - $U(\$650) = p \times U(\$1{,}000) + (1 - p) \times U(\$10)$
  - If risk neutral, $p = \frac{1-0}{1{,}000-} \times (650 - 10) = 0.646$
  - If you answered $p = 0.87$, $U(\$650) = 0.87 > 0.646$ → risk averse



A Reference Lottery (PE method)

2. Risk Preferences/ Risk Attitudes

Assessing Risk Attitude

- **DOSPERT(Domain-Specific Risk-Taking) Scale**
  - DOSPERT is a psychometric scale that assesses risk taking in five content domains: financial decisions (separately for investing versus gambling), health/safety, recreational, ethical, and social decisions.
  - Respondents rate the likelihood that they would engage in domain-specific risky activities (Part I).
  - An optional Part II assesses respondents' perceptions of the magnitude of the risks and expected benefits of the activities judged in Part I.

https://www8.gsb.columbia.edu/decisionsciences/research/tools/dospert

https://sites.google.com/a/decisionsciences.columbia.edu/dospert/home

- Weber, E. U., Blais, A.-R., & Betz, N. (2002). A domain-specific risk-attitude scale: Measuring risk perceptions and risk behaviors. Journal of Behavioral Decision Making, 15, 263-290.
- Blais, A.-R., & Weber, E. U. (2006) A Domain-Specific Risk-Taking (DOSPERT) scale for adult populations. Judgment and Decision Making, 1, 33-47.

2. Risk Preferences/ Risk Attitudes

Some phenomena

- The subjective values for gains and losses are non-linear, so-called normally concave(오목) for gains and convex(볼록) for losses.
- Gains increase more slowly than losses decrease (“loss aversion” – a loss of the certain dollars is more aversive than a gain of the same dollars is attractive).

2. Risk Preferences/ Risk Attitudes

Risk Takers and Risk Averters

- It can be varied from person to person.
- Several factors affect the personal risk propensity:
  ① Personal characteristics
  ② Financial status
  ③ Business background (small venture firm or large scale firm)
  ④ Personal experience (seniority, experience, years)
  ⑤ Social & cultural differences, nationalities, etc.

2. Risk Preferences/ Risk Attitudes

Literatures on risk attitude

- **De Neufville, R., Hani, E.N., and Lesage, Y. (1977)** “Tendering Models: Effects of Bidders Risk Aversion.” ASCE *Journal of Construction Division*, 103(CO1), pp. 57-70.
  - Contractors behave differently when dealing with small and large projects, and when operating in good years or bad so that they are most risk averse toward larger projects (esp. in lean years(흉년)) and bid relatively lower.

2. Risk Preferences/ Risk Attitudes

Literatures on risk attitude

- **MacCrimmon, K. R., and Wehrung, D. A. (1986)** *Taking risks: The management of uncertainty*. New York: Free Press. - surveys in various business contexts
  - Decision makers are more risk averse in opportunity situations than in threat situations.
  - Decision makers are extremely risk averse when the chance of loss it too high.
  - Both Canadian and American managers believe that Canadians are more risk averse, but there was no significant evidence.
  - Generally, older decision makers with longer seniority in their firms were more risk averse.
- Basically, these findings are analogous to the works from "**Tversky and Kahneman (1983)**".

2. Risk Preferences/ Risk Attitudes

Literatures on risk attitude

- **Taylor (1991)** concluded that subjects are more motivated to avoid losses than to obtain the equivalent gains.
- **Brown (1998)** also demonstrated that individuals gain knowledge of forecasting with greater accuracy when threatened by a loss than motivated with more gains (referred to as “loss avoidance”).
- **Weber and Hsee (1998)** indicated a cultural impact on the risk aversion demonstrating that Chinese is more risk averse than is the case of American or Polish.

Q & A


Street Calculus
Risk factors
black
male
aggressive body language
baseball cap on backwards
short hair
baggy jacket
wrong neighborhood
RF = 3
Mitigating Factors
female
over 40
loafers
tie and coat
whistling Sondheim
Fed Ex envelope
polo shirt
MF = 4
4 > 3
Risk: Acceptable.
Risk Factors
black
white
male
long hair
wrong neighborhood
police officer
baseball cap
Rf = 3
Mitigating factors
female
white
briefcase
groceries
humming Motown
tie + coat
over 40
MF = 4
4 > 3
Risk: Acceptable.
Good evening.
Good evening.