# Engineering Economic Analysis

2019 SPRING

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Chap. 14

# CONSUMER SURPLUS

# Consumer Surplus

- Measure how consumers are affected by changes in the economic environment?
- Three such measures are:
  - Consumer’s Surplus
  - Equivalent Variation, and
  - Compensating Variation

# Consumer Surplus

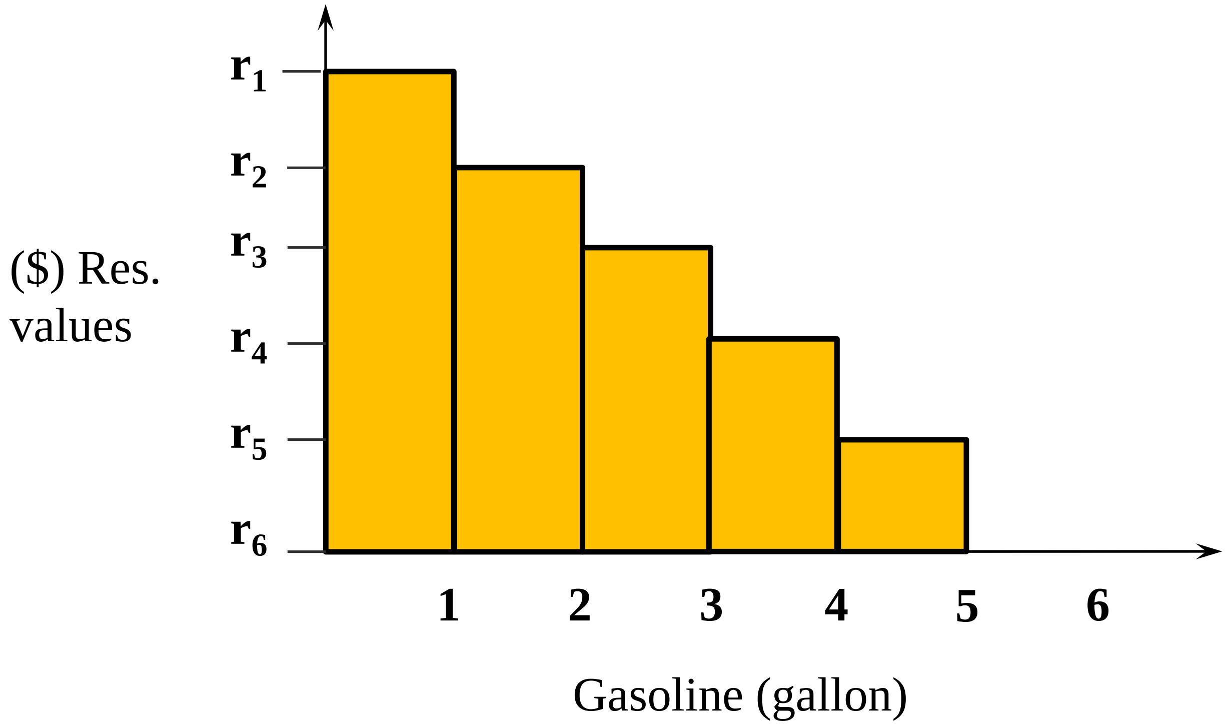
- Suppose gasoline can be bought only in lumps of one gallon.
- Use $r_1$ to denote the most a single consumer would pay for a 1st gallon – call this her **reservation price** for the 1st gallon.
- Now that she has one gallon, use $r_2$ to denote the most she would pay for a 2nd gallon – this is her reservation price for the 2nd gallon.

# Consumer Surplus

- Generally, if she already has $n-1$ gallons of gasoline then $r_n$ denotes the most she will pay for an nth gallon.
- $r_n$ is the dollar equivalent of the marginal utility of the nth gallon.

# Consumer Surplus

Reservation Price Curve for Gasoline

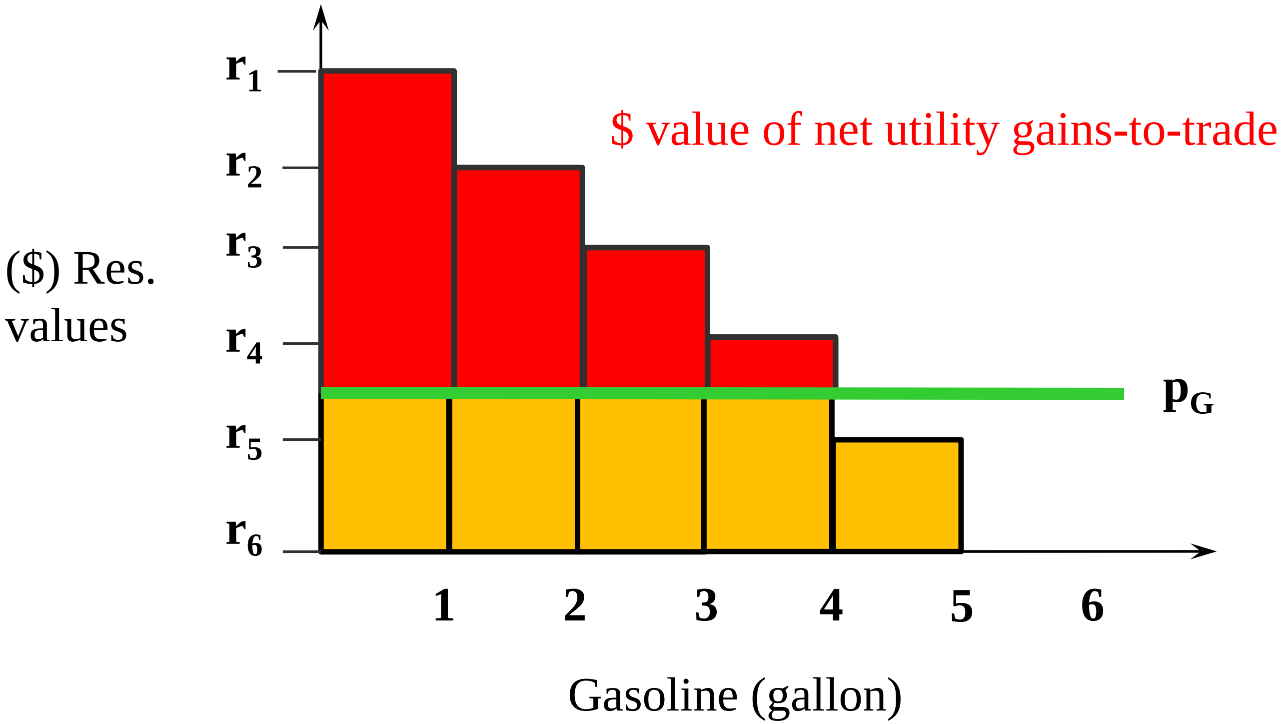


# Consumer Surplus

- What is the monetary value of our consumer's gain-to-trading in the gasoline market at a price of $\$P_G$?

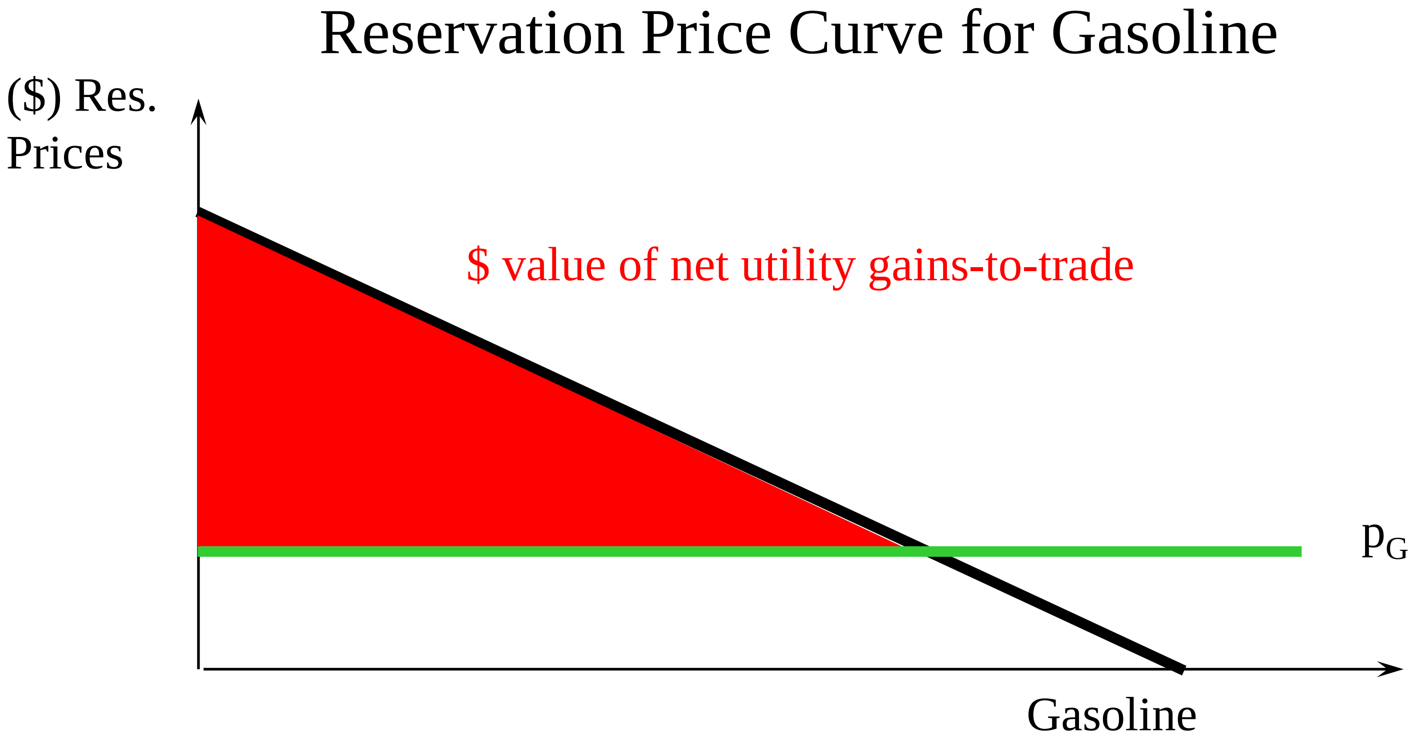
# Consumer Surplus

Reservation Price Curve for Gasoline



# Consumer Surplus

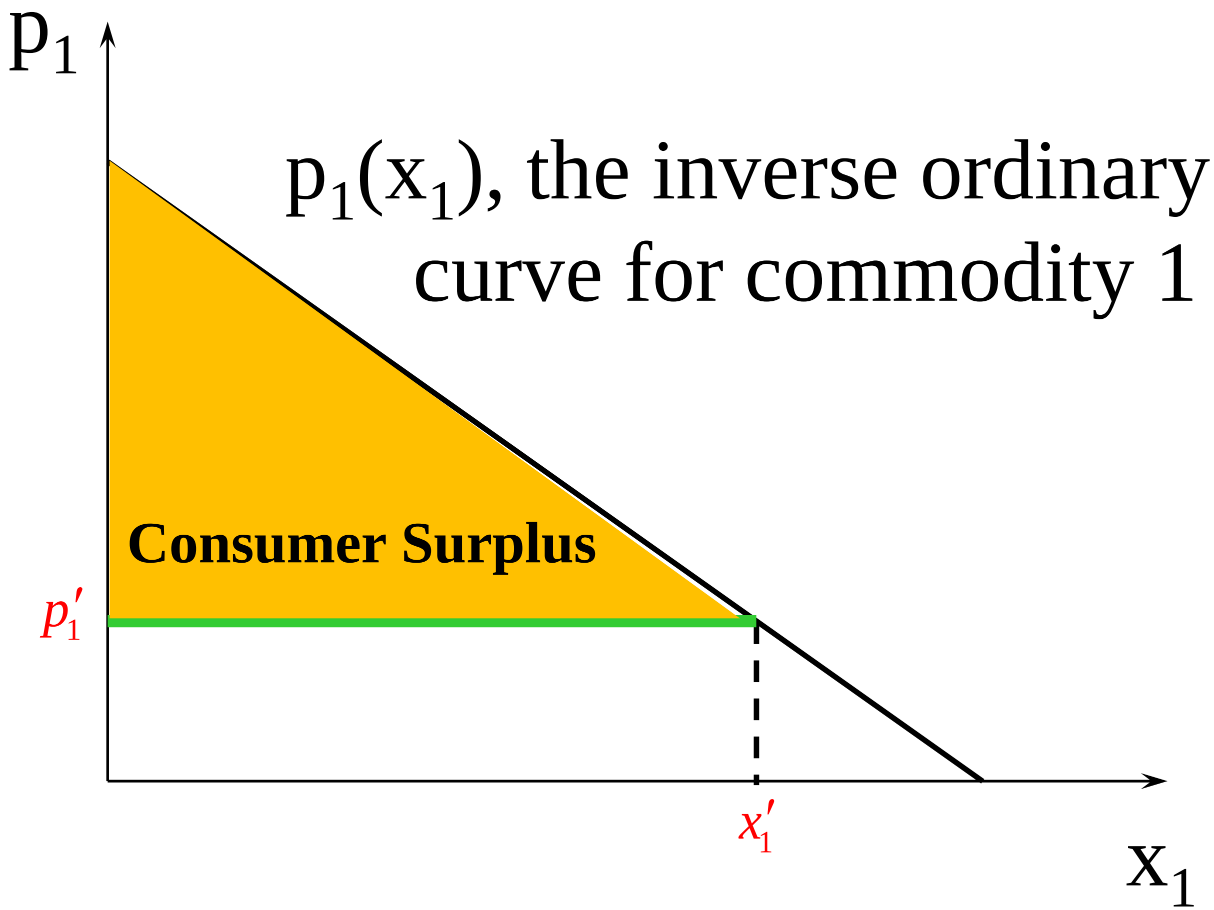
- Gasoline can be purchased in any quantity then …



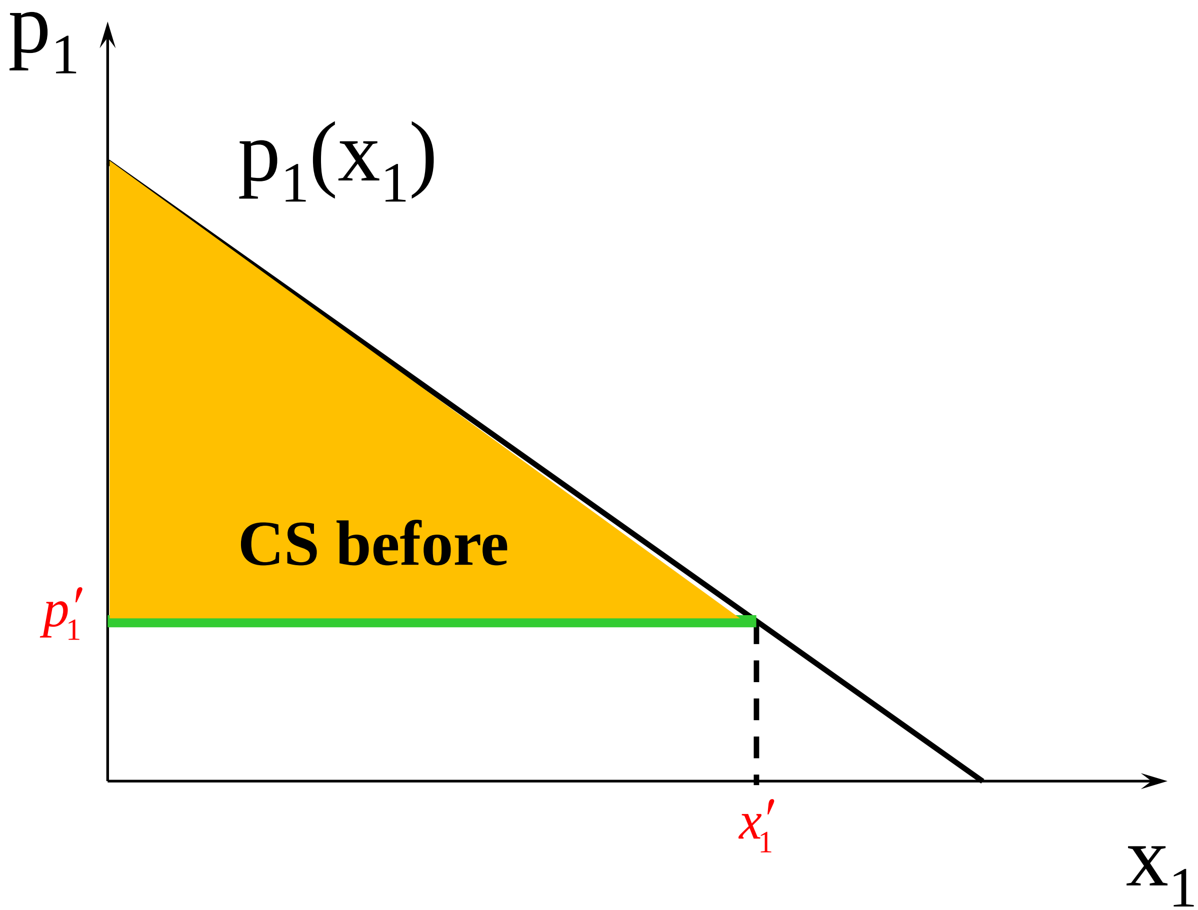
# Consumer Surplus

- Unfortunately, estimating a consumer’s reservation-price curve is difficult,
- So, as an approximation, the reservation-price curve is replaced with consumer’s ordinary demand curve.

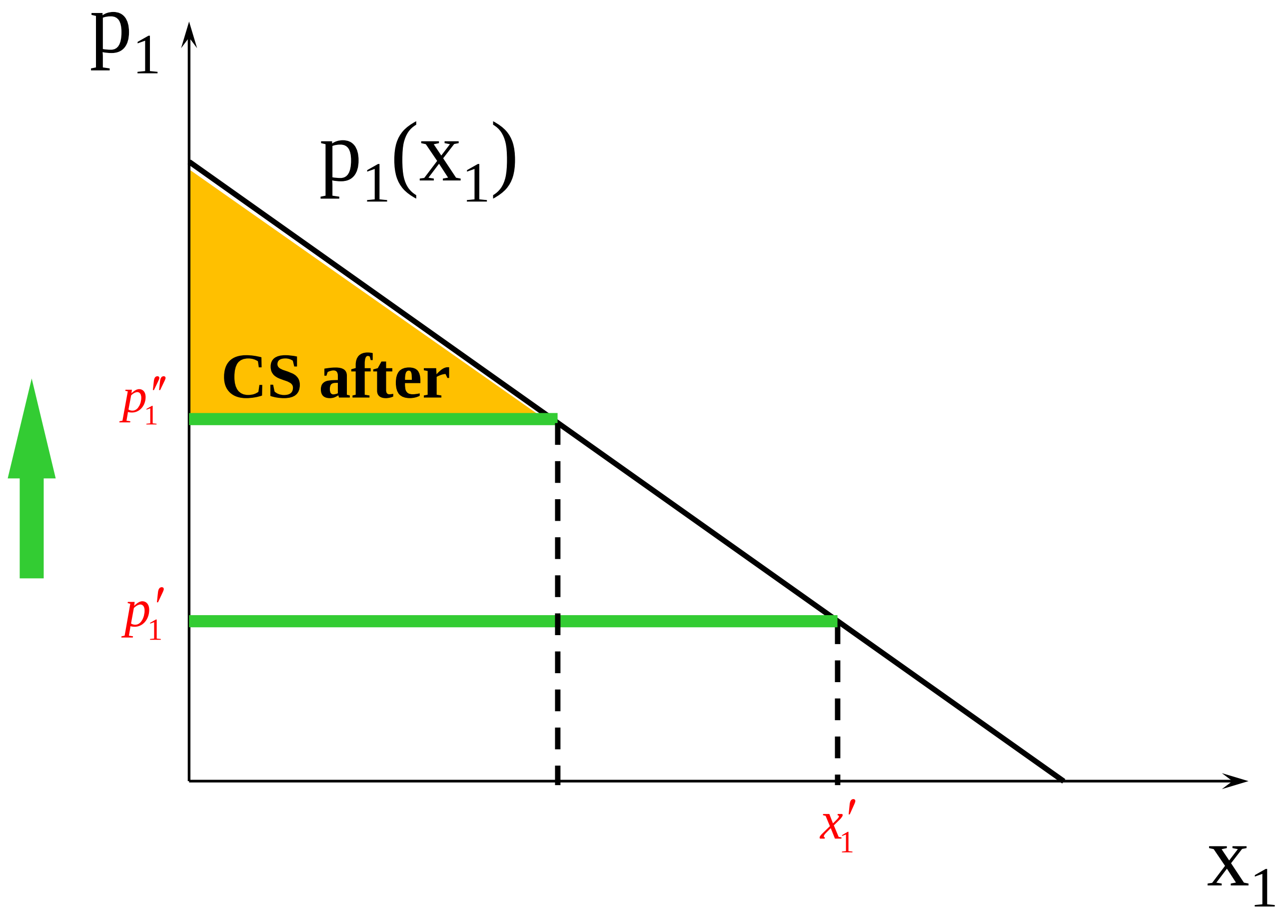
# Consumer's Surplus



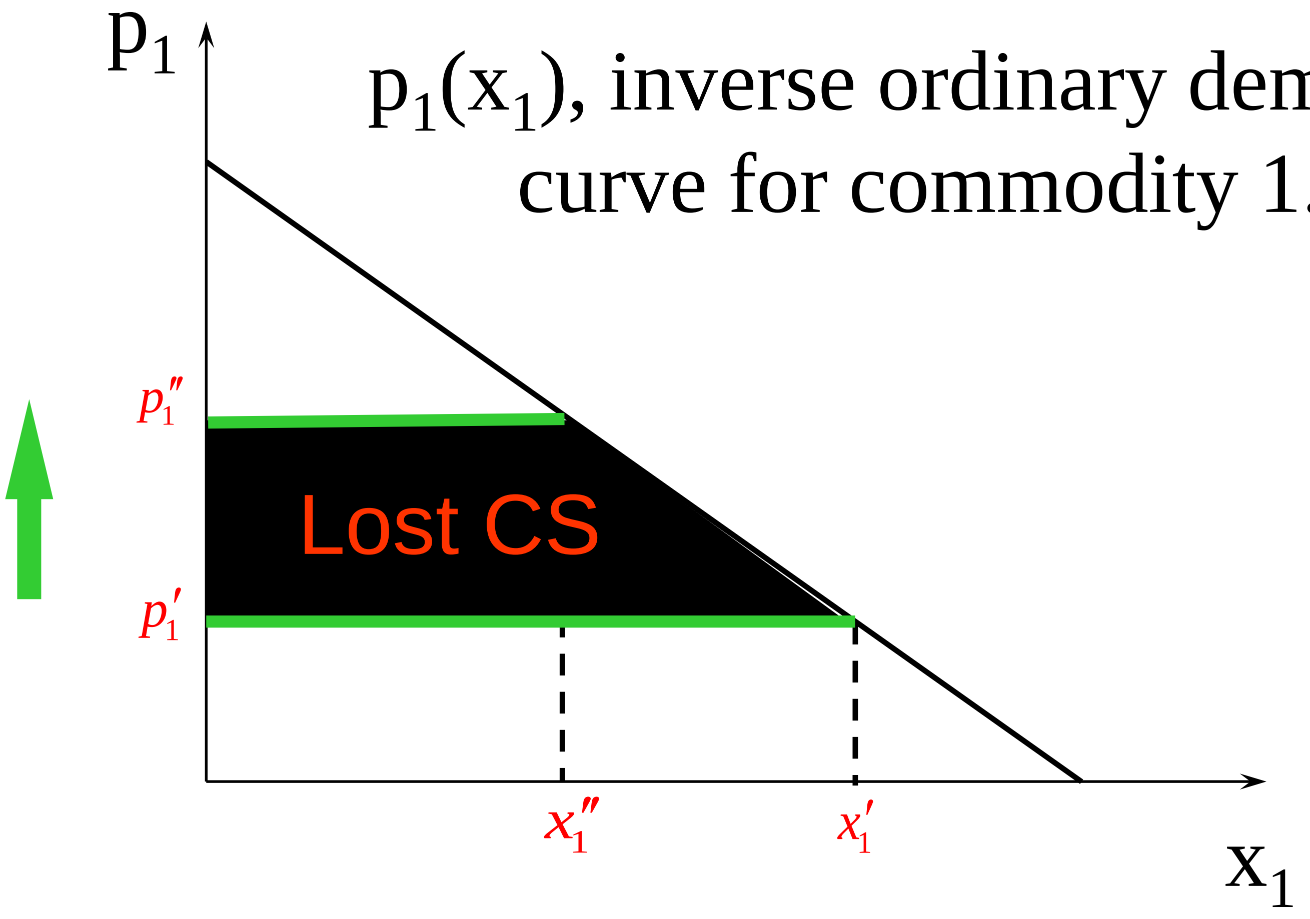
# Change in Consumer's surplus



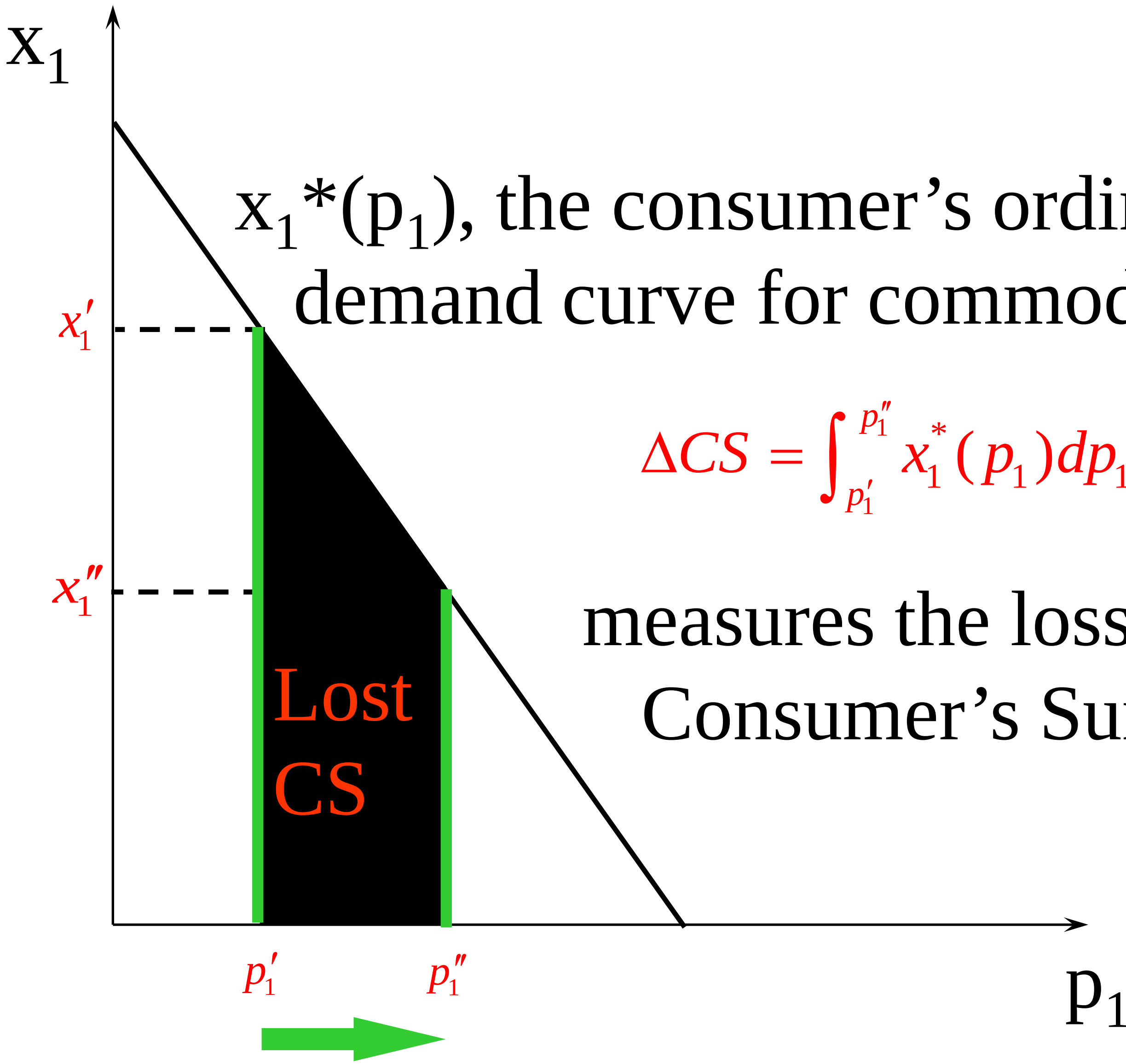
# Change in Consumer's surplus



# Change in Consumer's surplus



# Consumer Surplus




$x_1^*(p_1)$, the consumer's ordinary demand curve for commodity 1.

$$\Delta CS = \int_{p_1'}^{p_1''} x_1^*(p_1)\,dp_1$$

measures the loss in Consumer's Surplus.

# Compensating and Equivalent variations

- Measure how consumers are affected by changes in the economic environment?
  - Consumer surplus (based on demand function)
  - **Compensating variations**
  - **Equivalent variations**
- A measure of the changes in utility resulting from some environment change
  - From status quo $(\tilde{p}^0, m^0)$ to proposed change $(\tilde{p}', m')$

$$\textit{Welfare change} = v(\tilde{p}', m') - v(\tilde{p}^0, m^0)$$

Just ordinal measure!

- Need to have monetary measure?

# Compensating and Equivalent variations

- Money metric utility function $\mu(\tilde{q}:\tilde{p},m)$
  - How much money income the consumer would need at price $\tilde{q}$ to be as well off as he would be facing $(\tilde{p},m)$

$$\mu(\tilde{q}:\tilde{p},m)=e\left(\tilde{q},v\left(\tilde{p},m\right)\right)$$

  - Then

$$\begin{aligned} \textit{Welfare change} &= v(\tilde{p}',m')-v(\tilde{p}^0,m^0) \\ &= \mu(\tilde{q}:\tilde{p}',m')-\mu(\tilde{q}:\tilde{p}^0,m^0) \\ &= e(\tilde{q},v(\tilde{p}',m'))-e(\tilde{q},v(\tilde{p}^0,m^0)) \end{aligned}$$

- According to the choice of base prices

If we let $\tilde{q}=\tilde{p}^0$, then equivalent variations$(EV)$

If we let $\tilde{q}=\tilde{p}'$, then compensating variations$(CV)$

# Compensating and Equivalent variations

- Equivalent variations

$$EV = \mu(\tilde{p}^0 : \tilde{p}', m') - \mu(\tilde{p}^0 : \tilde{p}^0, m^0)$$
$$= \mu(\tilde{p}^0 : \tilde{p}', m') - m^0$$

  - What income change at the current prices would be equivalent to the proposed change in terms of its impact on utility

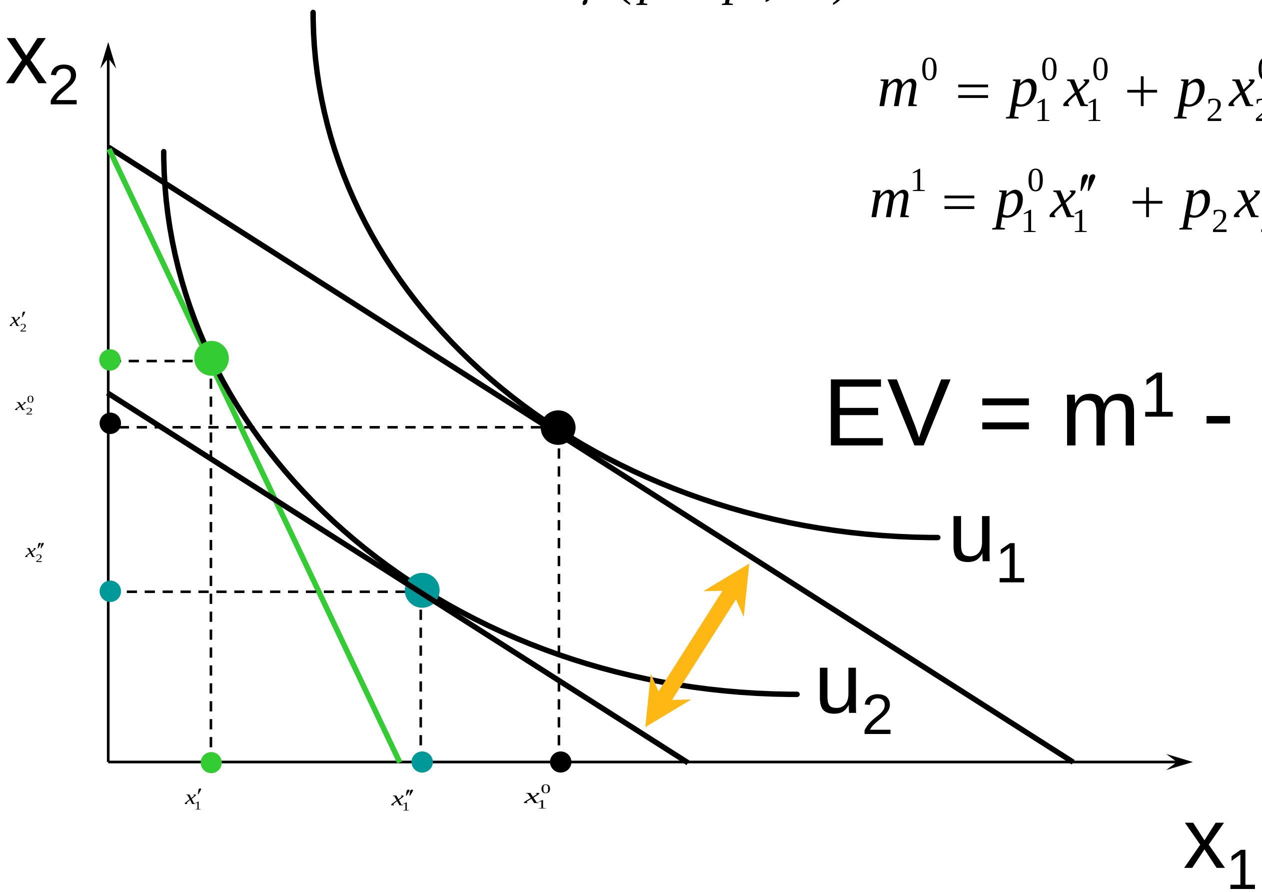
- Two-good case
  - $p_1$ rises.
  - Q: What is the least extra income that, at the original prices, just restores the consumer's original utility level?
  - A: The Equivalent Variation.

# Compensating and Equivalent variations

$$EV = \mu(\tilde{p}^0 : \tilde{p}', m') - \mu(\tilde{p}^0 : \tilde{p}^0, m^0)$$
$$= \mu(\tilde{p}^0 : \tilde{p}', m') - m^0$$

$$m^0 = p_1^0 x_1^0 + p_2 x_2^0$$

$$m^1 = p_1^0 x_1'' + p_2 x_2''$$




EV = m[1] - m[0]

# Compensating and Equivalent variations

- Compensating variations

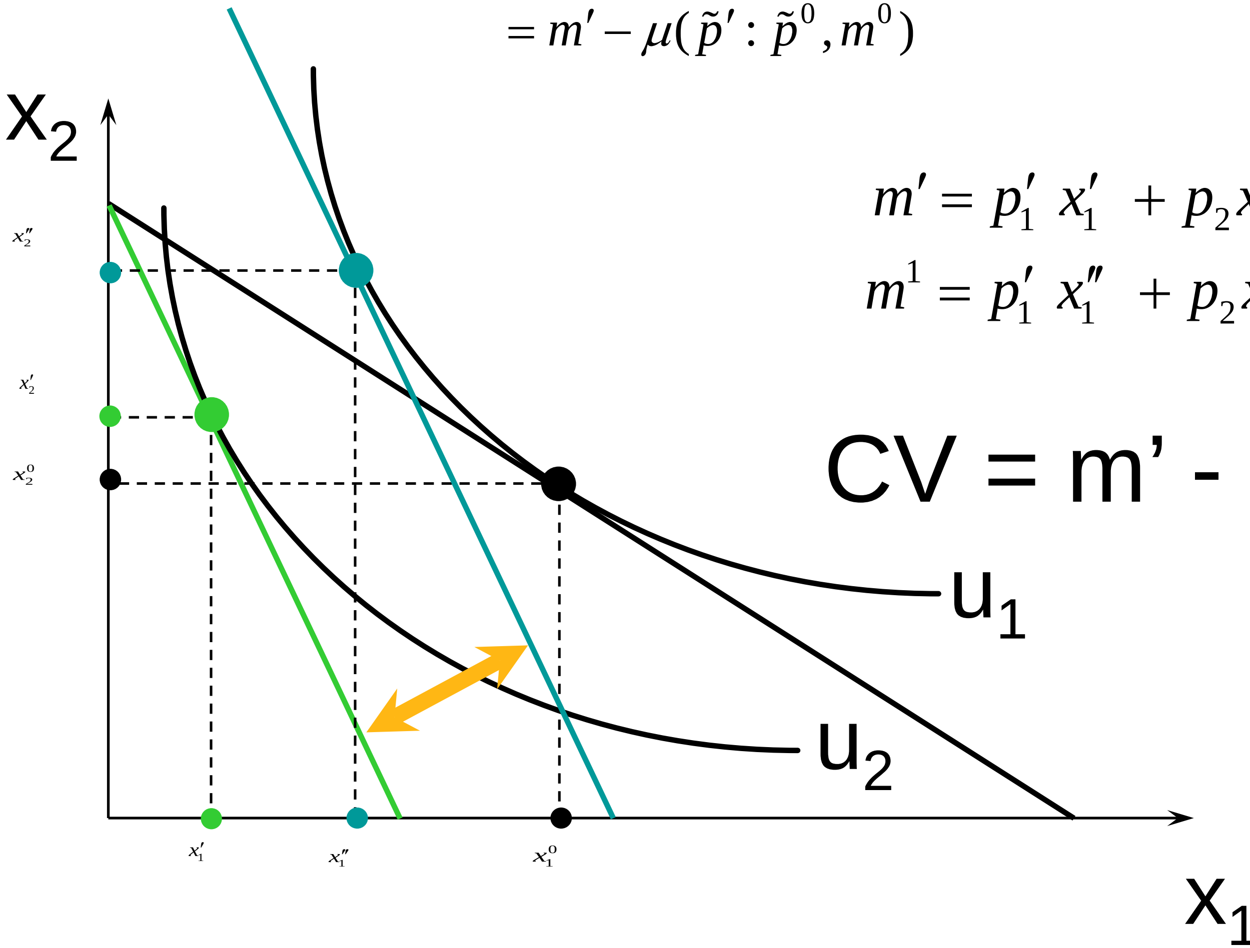
$$CV = \mu(\tilde{p}' : \tilde{p}', m') - \mu(\tilde{p}' : \tilde{p}^0, m^0)$$
$$= m' - \mu(\tilde{p}' : \tilde{p}^0, m^0)$$

  - What income change at the after-change prices would be necessary to compensate the consumer for the price change

- Two-good case
  - $p_1$ rises.
  - Q: What is the least extra income that, at the new prices, just restores the consumer's original utility level?
  - A:  The Compensating Variation.

# Compensating and Equivalent variations

$$CV = \mu(\tilde{p}' : \tilde{p}', m') - \mu(\tilde{p}' : \tilde{p}^0, m^0)$$
$$= m' - \mu(\tilde{p}' : \tilde{p}^0, m^0)$$




$$m' = p_1' \; x_1' \; + p_2 x_2'$$
$$m^1 = p_1' \; x_1'' \; + p_2 x_2''$$

CV = m' - m[1]

# Compensating and Equivalent variations

- In general, the magnitudes of EV and CV are different, but the sign is always the same!

# Chap. 15

# MARKET DEMAND

# From Individual to Market Demand Functions

- Consumer i' s ordinary demand function for good j is

$$x_i^j(\tilde{p}, m_i)$$

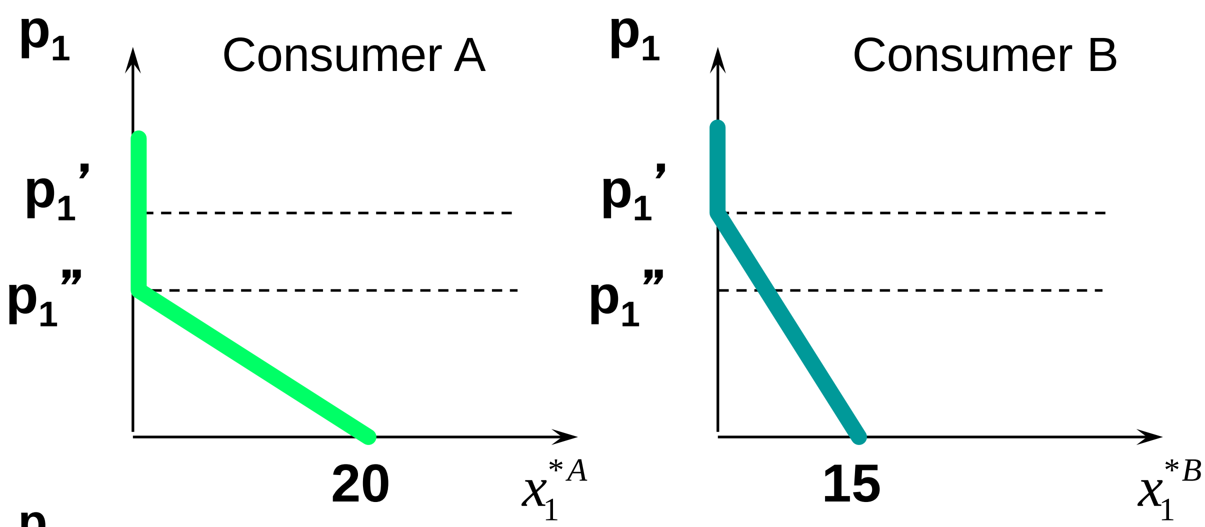
- Market demand function for good j is

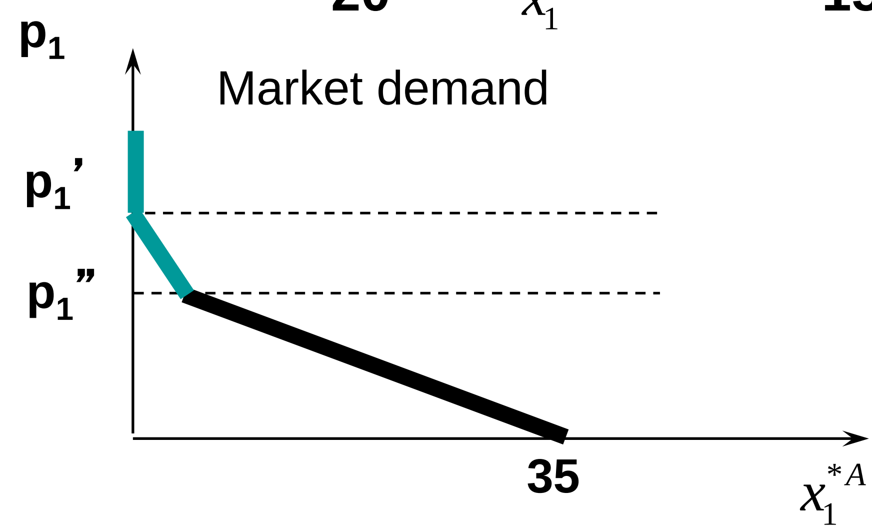
$$X^j(\tilde{p}, \tilde{m}) = \sum_{i=1}^{n} x_i^j(\tilde{p}, m_i)$$

  - Depends on prices and the distribution of incomes

- The market demand curve is the "horizontal sum" of the individual consumers' demand curves.

# Market Demand Curve




**The "horizontal sum" of the demand curves of individuals A and B.**

# Inverse Demand Function

- P(X): measures what the market price of the good would have to be for X units of it to be demanded
- Note that, at the optimal choice of consumption, MRS = $p_1/p_2$
  - If all consumers are facing the same prices, then all consumers have the same MRS at the optimum.
- Thus P(X) measures the MRS, or the marginal willingness to pay, of every consumer who is purchasing the good.

# Elasticity

- Elasticity measures the "sensitivity" of one variable with respect to another.

$$\varepsilon_{x,y} = \frac{\%\Delta x}{\%\Delta y} = \frac{\Delta x / x}{\Delta y / y}$$

- Own price elasticity

$$\varepsilon = \frac{\Delta q / q}{\Delta p / p} = \frac{p}{q}\frac{\Delta q}{\Delta p}$$

$$= \frac{p}{q}\frac{dq}{dp} = \frac{d \ln q(p)}{d \ln p}$$

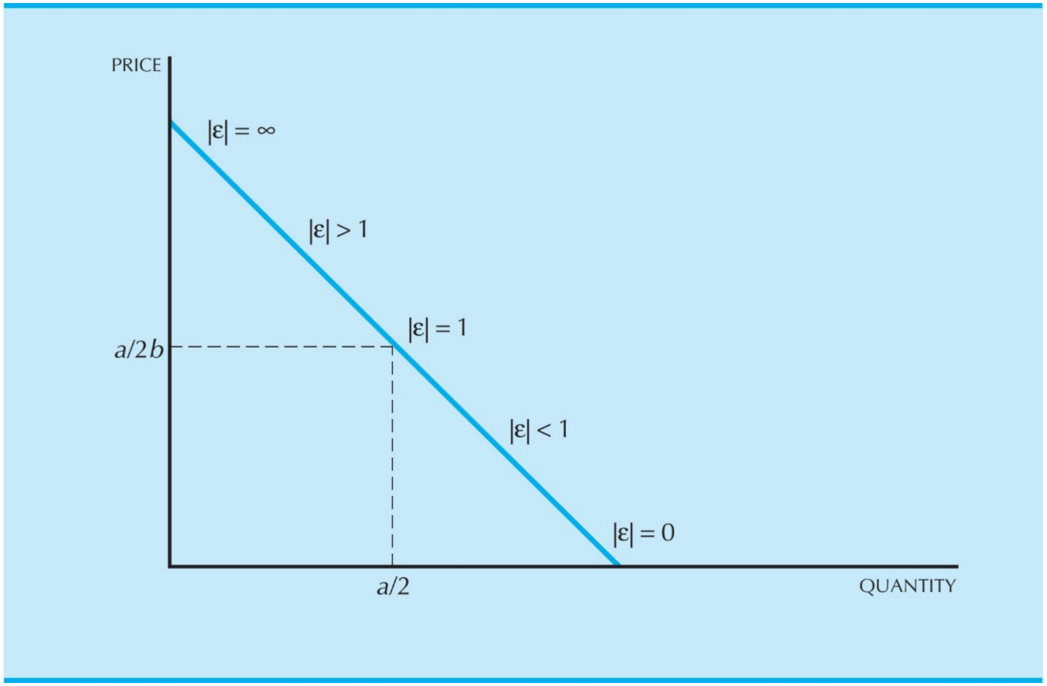
  - The sign of elasticity is generally (-)

$\rightarrow$ $|\varepsilon|$ is used.

# Elasticity

- Example: Linear demand $q = a - bp$

Figure 15.4



# Elasticity

- Elasticity and Demand

$$\begin{cases} If \ \ |\varepsilon| > 1, & \text{then } \textit{elastic} \text{ demand} \\ If \ \ |\varepsilon| = 1, & \text{then } \textit{unit elastic} \text{ demand} \\ If \ \ |\varepsilon| < 1, & \text{then } \textit{inelastic} \text{ demand} \end{cases}$$

- If a good has many close substitutes, we would expect that its demand would be elastic.

# Elasticity and Revenue

- Revenue = Price X Quantity

$$R = pq$$

- Let p $\rightarrow$ p+$\Delta$p, and then q $\rightarrow$ q+$\Delta$q
- Then, the new revenue becomes

$$\begin{aligned} R' &= (p + \Delta p)(q + \Delta q) \\ &= pq + q\Delta p + p\Delta q + \Delta p \Delta q \end{aligned}$$

- Revenue change

$$\Delta R = q\Delta p + p\Delta q + \Delta p \Delta q$$

- For small values of $\Delta$p and $\Delta$q, $\Delta p \cdot \Delta q \rightarrow 0$
- Then

$$\Delta R = q\Delta p + p\Delta q$$

# Elasticity and Revenue

- The rate of change of revenue per change in price

$$\frac{\Delta R}{\Delta p} = q + p\frac{\Delta q}{\Delta p}$$

- We can derive

$$\begin{aligned}\frac{\Delta R}{\Delta p} &= q + p\frac{\Delta q}{\Delta p} \\ &= q\left[1 + \frac{p}{q}\frac{\Delta q}{\Delta p}\right] \\ &= q\left[1 + \epsilon(p)\right] \\ &= q\left[1 - |\epsilon(p)|\right]\end{aligned}$$

# Elasticity and Revenue

- If demand is elastic, i.e. $|\varepsilon| > 1$, then $\dfrac{\Delta R}{\Delta p} < 0$
  - As price increases, so revenue will decrease and vice versa .
- If demand is inelastic, i.e. $|\varepsilon| < 1$, then $\dfrac{\Delta R}{\Delta p} > 0$
  - As price increases, so revenue will increase and vice versa .

# Elasticity and Marginal Revenue

- The change in revenue

$$\Delta R = p\Delta q + q\Delta p$$

- The marginal revenue

$$\begin{aligned} \text{MR} = \frac{\Delta R}{\Delta q} &= p + q\frac{\Delta p}{\Delta q} \\ &= p\left[1 + \frac{q\Delta p}{p\Delta q}\right] \\ &= p(q)\left[1 + \frac{1}{\epsilon(q)}\right] \\ &= p(q)\left[1 - \frac{1}{|\epsilon(q)|}\right] \end{aligned}$$

# Elasticity and Marginal Revenue

- If the demand of a good is unit elastic, then marginal revenue is zero
  - revenue dose not change when you increase output

- If the demand of a good is inelastic, then marginal revenue is negative
  - revenue will decrease when you increase output
  - If demand is not very responsive to price, then you have to cut prices a lot to increase output: so revenue goes down.

# Marginal Revenue Curves

- Linear demand curve

$$p(q) = a - bq$$

- Marginal revenue curve

$$\begin{aligned}\frac{\Delta R}{\Delta q} &= p(q) + \frac{\Delta p(q)}{\Delta q} q \\ &= p(q) - bq \\ &= a - bq - bq \\ &= a - 2bq.\end{aligned}$$

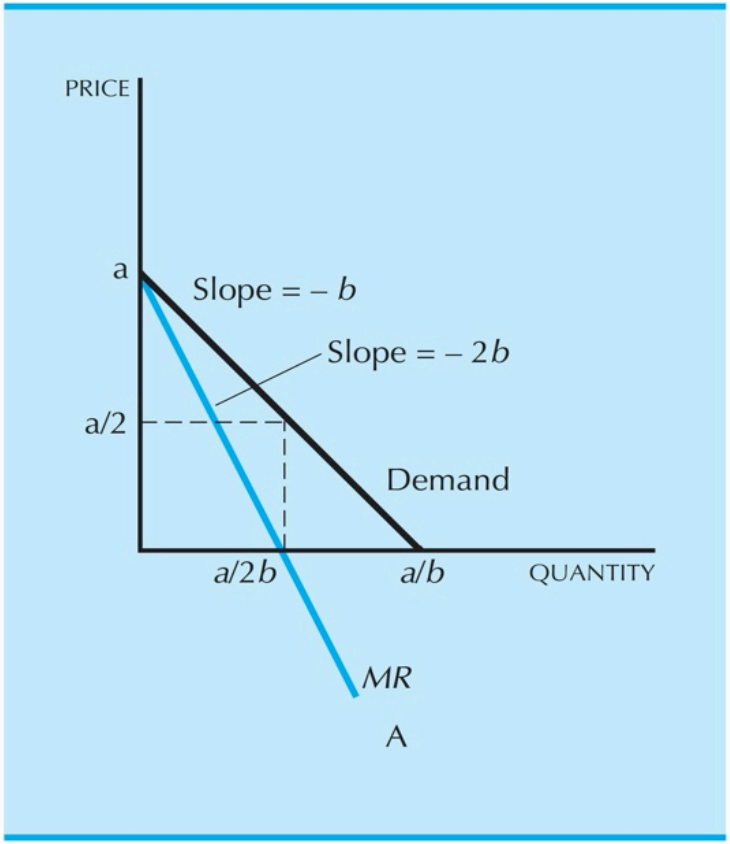
or

$$p(q) = a - bq$$

$$\text{Revenue: } R(q) = p(q) \cdot q = aq - bq^2$$

$$MR(q) = \frac{dR(q)}{dq} = a - 2bq$$

# Marginal Revenue Curves



# Constant Elasticity Demand Curve

- Demand function

$$D(p) = Ap^{\varepsilon}$$

  - Elasticity is constant

$$\frac{p}{q}\frac{dq}{dp} = \frac{p}{Ap^{\epsilon}}\epsilon A p^{\epsilon - 1} = \epsilon$$

  - Marginal revenue

$$MR = p(q)\left[1 - \frac{1}{|\epsilon|}\right]$$

constant

    – Marginal revenue curve is some constant fraction of the inverse demand curve!

# Constant Elasticity Demand Curve

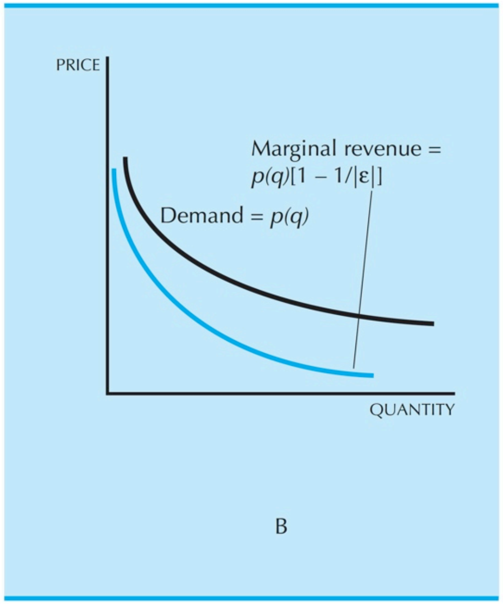
- Demand function

when $|\varepsilon| = 1$, MR=0

$|\varepsilon| > 1$, MR lies below the inverse demand curve, $p(q)$

$|\varepsilon| < 1$, MR is negarive

when $|\varepsilon| > 1$,



# Income Elasticity of Demand

- $$\text{income elasticity of demand} = \frac{\%\ \text{change in quantity}}{\%\ \text{change in income}}$$

$$\eta = \frac{\Delta q / q}{\Delta m / m} = \frac{m}{q}\frac{\Delta q}{\Delta m} = \frac{m}{q}\frac{dq}{dm}$$

- If $\eta > 0$, normal good
  If $\eta < 0$, inferior good
  If $\eta > 1$, luxary good