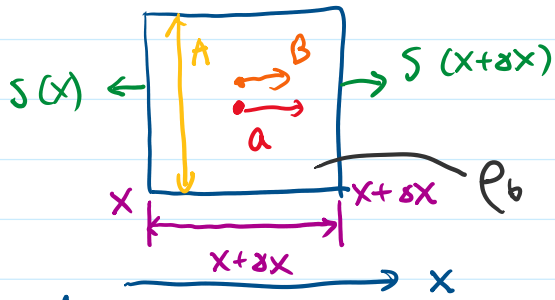
Force Equilibrium Equation

2019년 7월 15일 월요일 오후 1:42

* What is stress? See the material.

1D differential element



Δx : length

x, $x+\delta x$: locations

S : total stress

A : cross-sectional area

a : acceleration of body

B : force per unit mass of bulk rock

ρ : bulk density

Three forces are acting on the body :

$S(x+\delta x)A$, $-S(x)A$, $B\rho_b \Delta x A$

Net force acting on the body $= \rho_b \Delta x A a$

→ Equilibrium

$$S(x+\delta x)A - S(x)A + B\rho_b \Delta x A = \rho_b \Delta x A a$$

Divide by $\Delta x A$

$$\frac{S(x+\delta x)-S(x)}{\Delta x} + \rho_b B = \rho_b a$$

$\Delta x \to 0$

$$\frac{dS}{dx} + \rho_b B = \rho_b a$$

Is the rock formation moving? No. $a = 0$

$$\frac{\partial S}{\partial x} + \rho_b B = 0$$

← body force acting on unit volume

Describe this equation verbally for 1D differential element.

1차원의 매우 작은 element 에 존재하는 stress의 x 방향에 대한 변화율과 element 에 가해지는 단위 부피당 힘을

1차원의 매우 작은 element 에 대하여 stress의 x 성분에 대한 변화율과 element 에 가해지는 단위 부피당 힘을 합하면 0 이다.

What about 3D?

z-direction

$$S = \begin{bmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix}$$

x-direction: $\frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{yx}}{\partial y} + \frac{\partial S_{zx}}{\partial z} + \rho_b B_x = 0$

y-direction: $\frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial y} + \frac{\partial S_{zy}}{\partial z} + \rho_b B_y = 0$

z-direction: $\frac{\partial S_{xz}}{\partial x} + \frac{\partial S_{yz}}{\partial y} + \frac{\partial S_{zz}}{\partial z} + \rho_b B_z = 0$

$S_{xy} = S_{yx}$
$S_{yz} = S_{zy}$
$S_{zx} = S_{xz}$

S_{xy} : y-direction stress acting on the plane perpendicular to x-direction

→ S_{xy} depends on only x.

→ $\frac{\partial S_{xy}}{\partial y} = 0$, $\frac{\partial S_{xy}}{\partial z} = 0$

$$[\nabla \cdot S]_i + \rho_b B_i = 0, \quad i = x, y, z$$

If we consider only z-direction.

$B_x = B_y = 0$, $B_z = -g$