Crystal Mechanics

Lecture 8 – Plastic Deformation of Single Crystals

Ref : Texture and Related Phenomena, D. N. Lee, 2006
Continuum Theory of Plasticity, A.S. Khan and S. Huang, 1995

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Reorientation of Directions by Single Slip

$$\gamma = |\boldsymbol{u}| \,/\, (\boldsymbol{d}\cdot\boldsymbol{n}) \quad \textit{and} \quad \mathbf{u} = \gamma(\boldsymbol{d}\cdot\boldsymbol{n})\boldsymbol{b}$$

$$\mathbf{D} = \mathbf{d} + \mathbf{u} = \mathbf{d} + \gamma(\boldsymbol{d}\cdot\boldsymbol{n})\boldsymbol{b} \quad \textit{and} \quad D_i = d_i + \gamma(d_j n_j) b_i$$

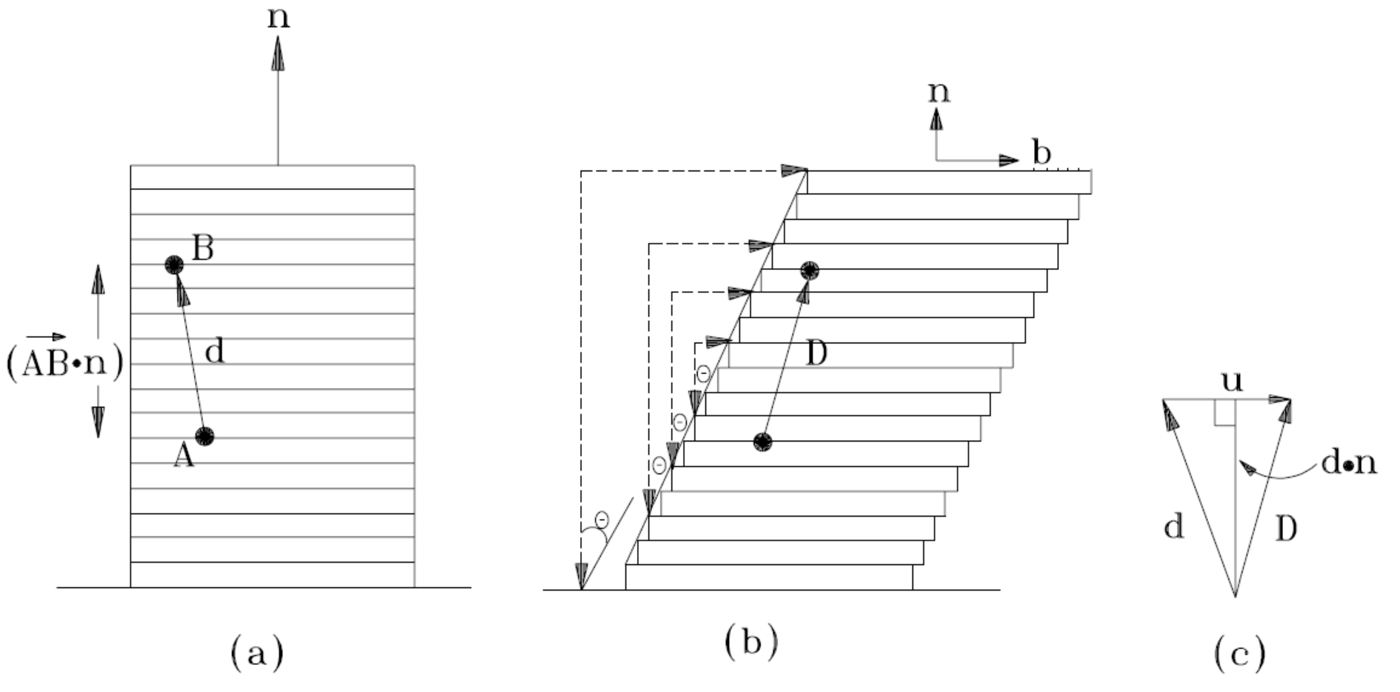


Fig.4.3.1. Reorientation of direction by single slip.

Reorientation of Planes by Single Slip

$$\mathbf{m} = \mathbf{d}\times\mathbf{d}' \quad \text{and} \quad \mathbf{M} = \mathbf{D}\times\mathbf{D}'$$

$$\begin{aligned}
\mathbf{M} &= [d + \gamma(d\cdot n)b]\times[d' + \gamma(d'\cdot n)b] \\
&= d\times d' + \gamma(d\cdot n)(b\times d') + \gamma(d'\cdot n)(d\times b) \\
&= d\times d' - \gamma\,\mathbf{b}\times[(d'\cdot n)d - (d\cdot n)d'] \\
&= d\times d' - \gamma\,\mathbf{b}\times[n\times(d\times d)] \\
&= m - \gamma b\times n\times m \\
&= m - \gamma(b\cdot m)n
\end{aligned}$$

$$M_i = m_i - \gamma(b_j m_j)n_i$$

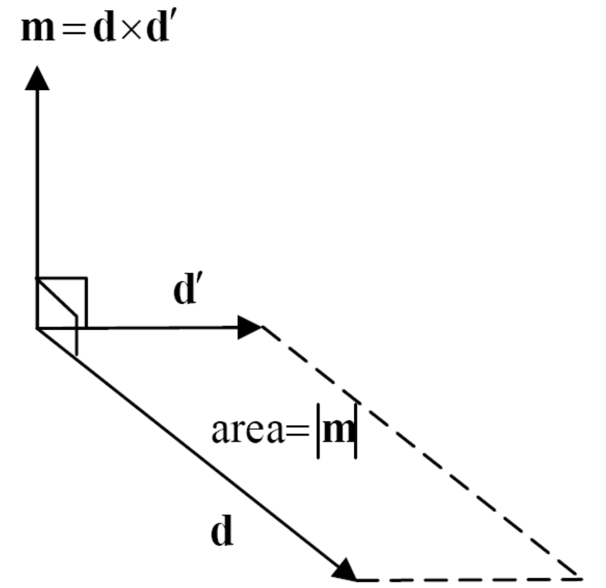
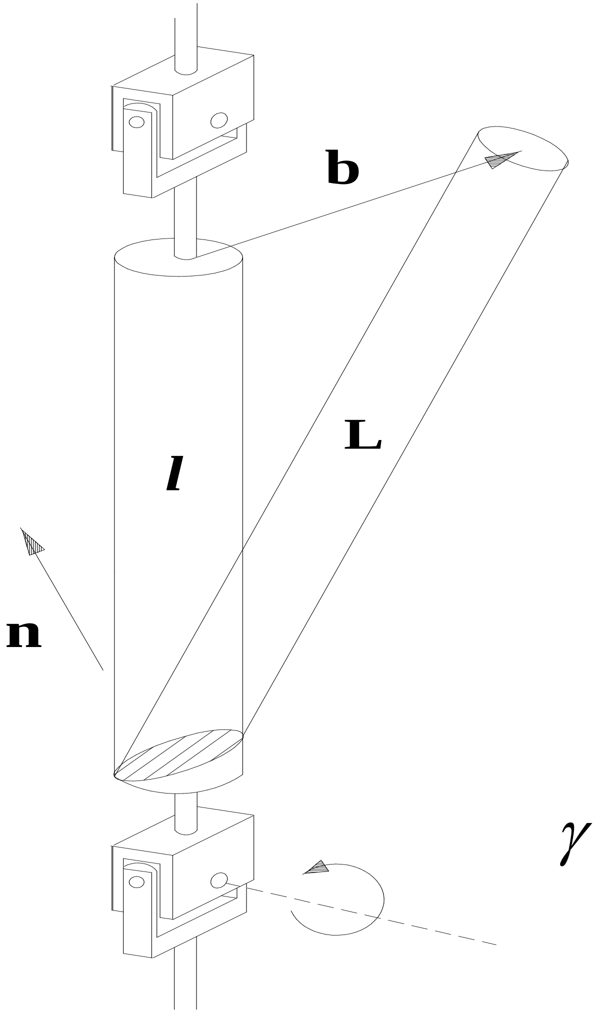
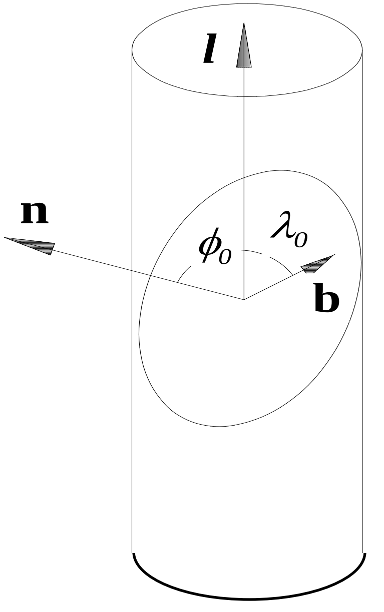


Fig.4.4.1. Definition of $\mathbf{d}\times\mathbf{d}'$.

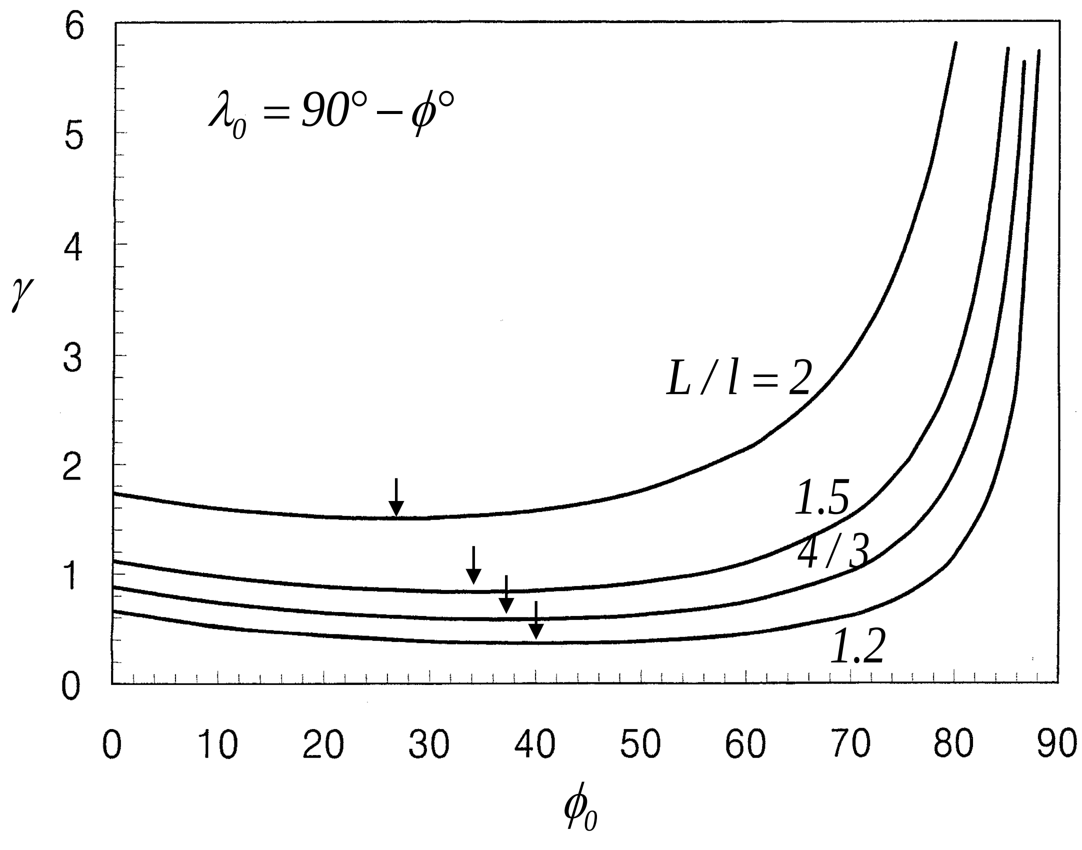
Single Slip in Response to Tensile Force





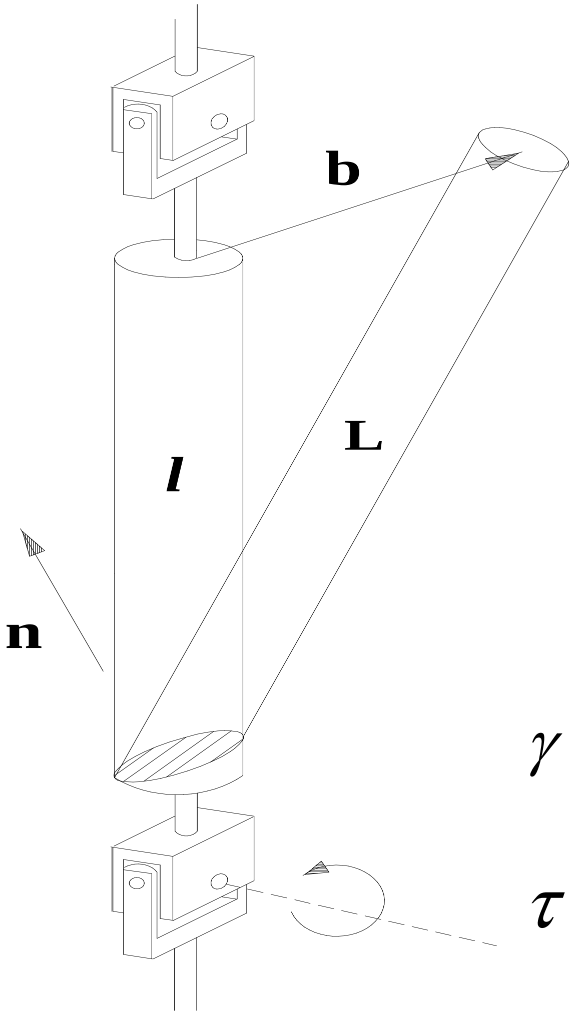
$$\gamma = [\{(L/l)^2 - \sin^2\lambda_0\}^{1/2} - \cos\lambda_0]/\cos\phi_0$$

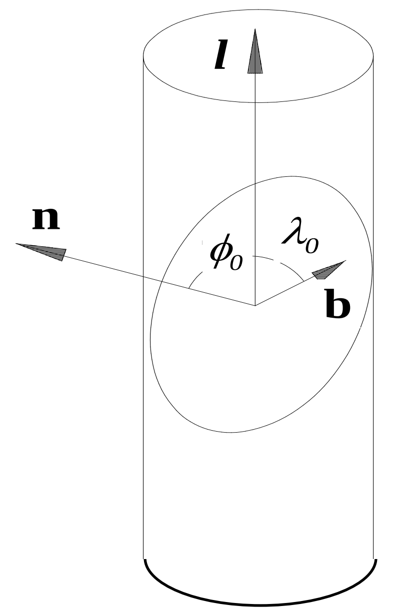
Single Slip in Response to Tensile Force



Orientation dependence of shear strain γ.

Single Slip in Response to Tensile Force





$$\gamma = [\{(L/l)^2 - \sin^2 \lambda_0\}^{1/2} - \cos \lambda_0]/\cos \phi_0$$

$$\tau = (F/A_0)\cos \phi_0 [1 - (l \sin \lambda_0 / L)^2]^{1/2}$$

Single Slip in Response to Tensile Force

$$\gamma = [\{(L/l)^2 - \sin^2\lambda_0\}^{1/2} - \cos\lambda_0]/\cos\phi_0$$

$$\tau = (F/A_0)\cos\phi_0[1-(l\sin\lambda_0/L)^2]^{1/2}$$

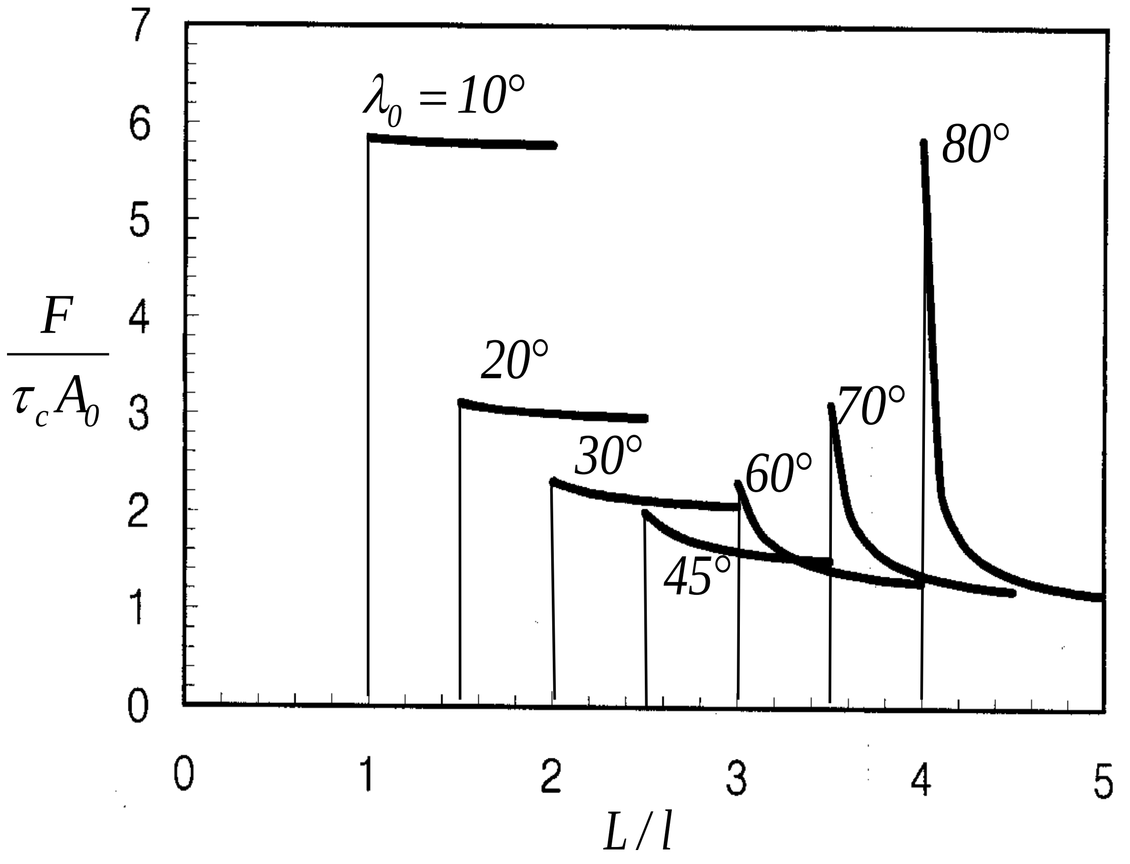
$$\Downarrow$$

$$F = \frac{\tau_c A_0}{\cos\phi_0[1-(l\sin\lambda_0/L)^2]^{1/2}}$$

F diminishes as *L* increases.

Geometrical softening

Single Slip in Response to Tensile Force

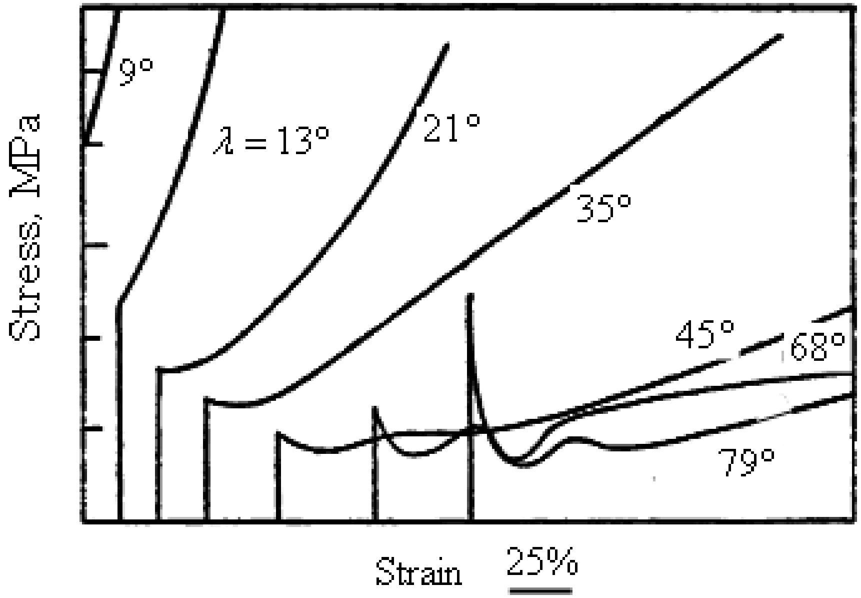


Geometrical softening

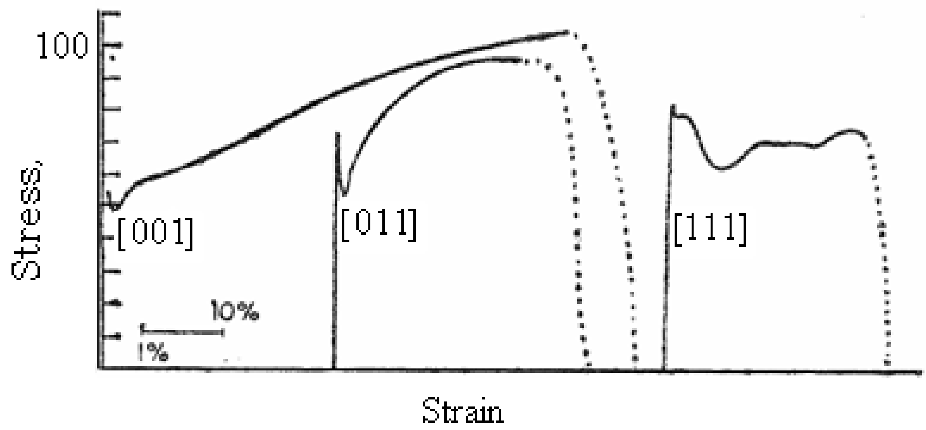
Assuming constant value of shear stress acting on slip system

Single Slip in Response to Tensile Force

Geometrical softening

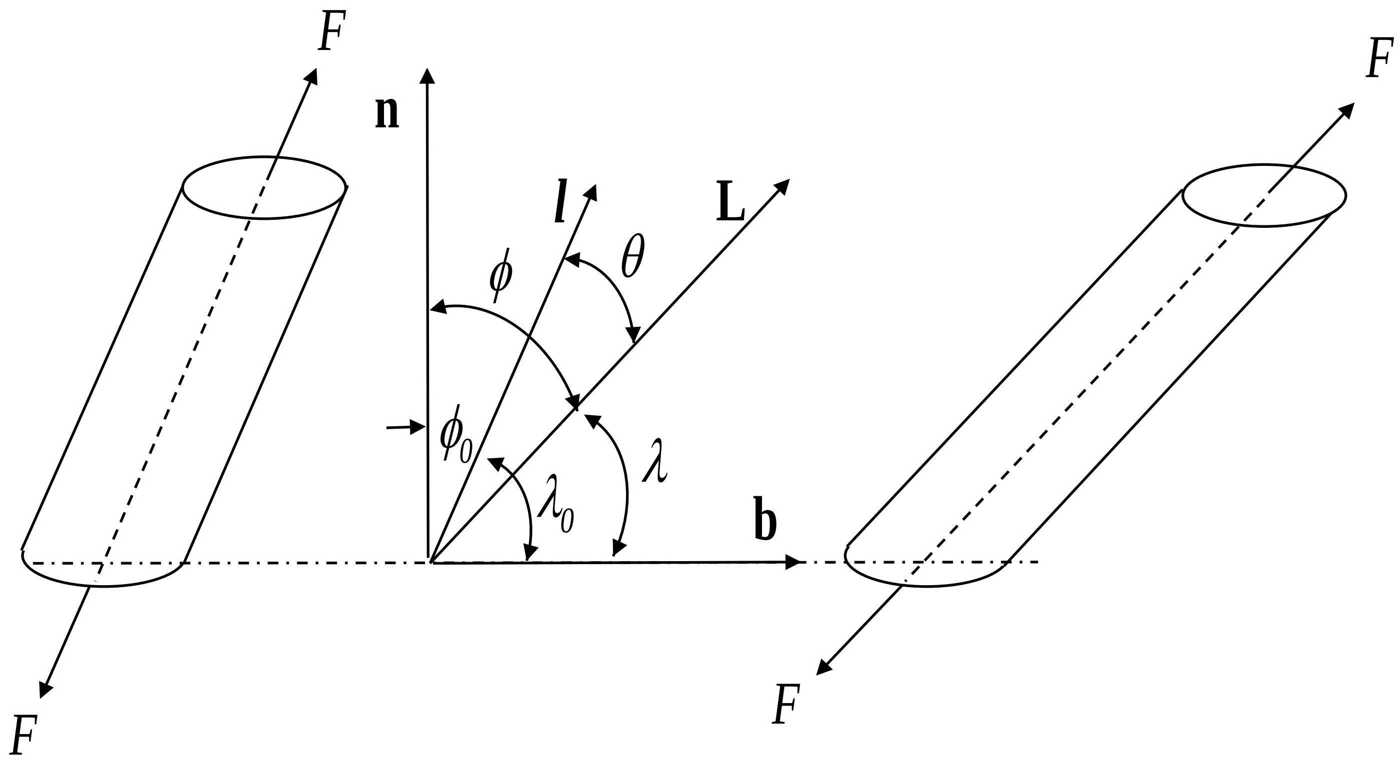


Tensile stress-strain curves measured on cadmium crystals [Boas, Schmid, 1929].



Tensile stress-strain curves for niobium crystals; initial direction of applied stress is given for each curve [Votava, 1964].

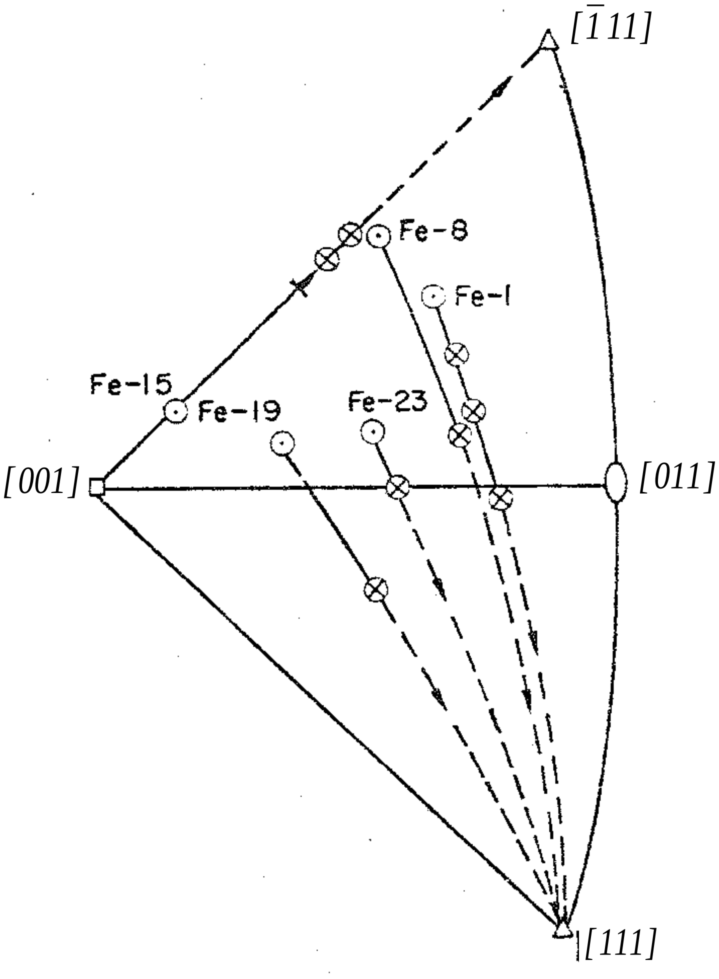
Single Slip in Response to Tensile Force



Cylindrical crystal (a) in undeformed state, and (b) after slip has occurred on system given by vectors **n** and **b**._

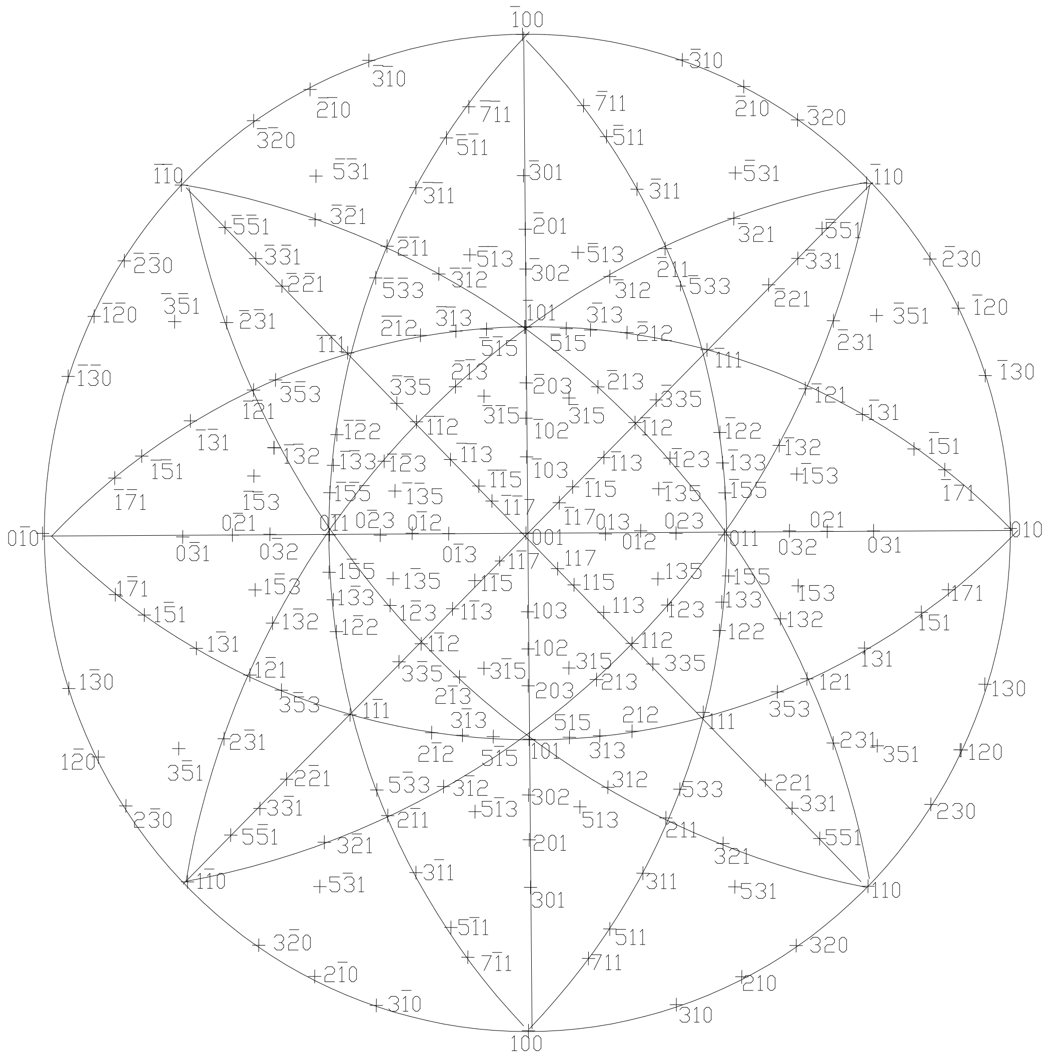
$$\mathbf{r} = \mathbf{L} \times \boldsymbol{l} = \gamma(\boldsymbol{l} \cdot \boldsymbol{n})(\boldsymbol{b} \times \boldsymbol{l})$$_

Single Slip in Response to Tensile Force



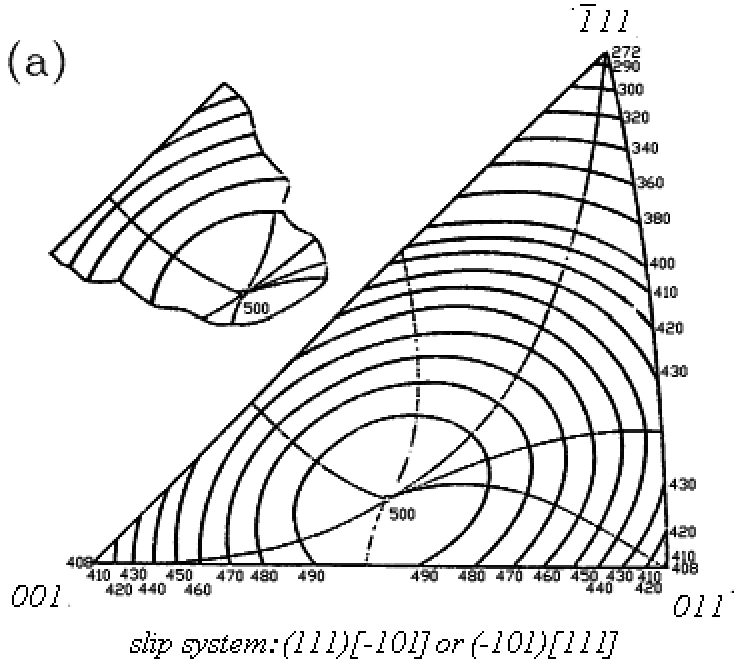
Stereographic plot of tensile axes of iron crystals.

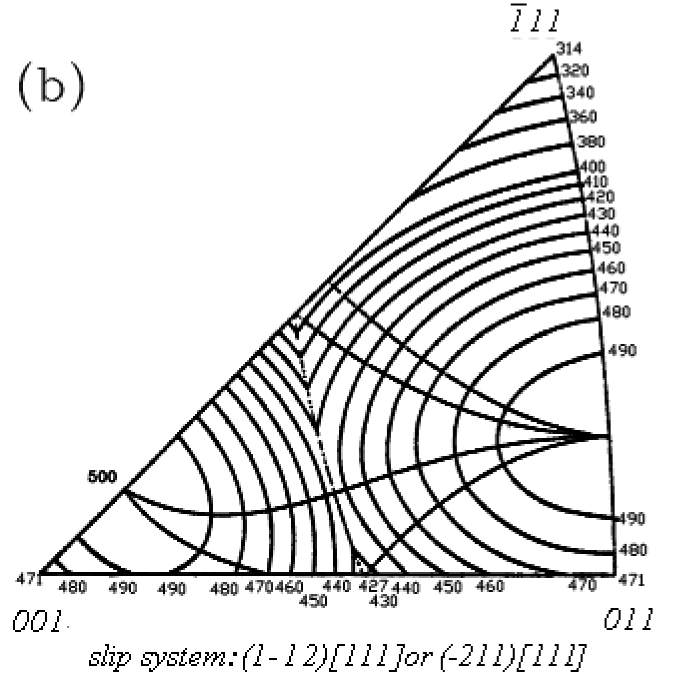
Single Slip in Crystals of Cubic Symmetry



001 standard stereographic projection of cubic crystal

Single Slip in Crystals of Cubic Symmetry



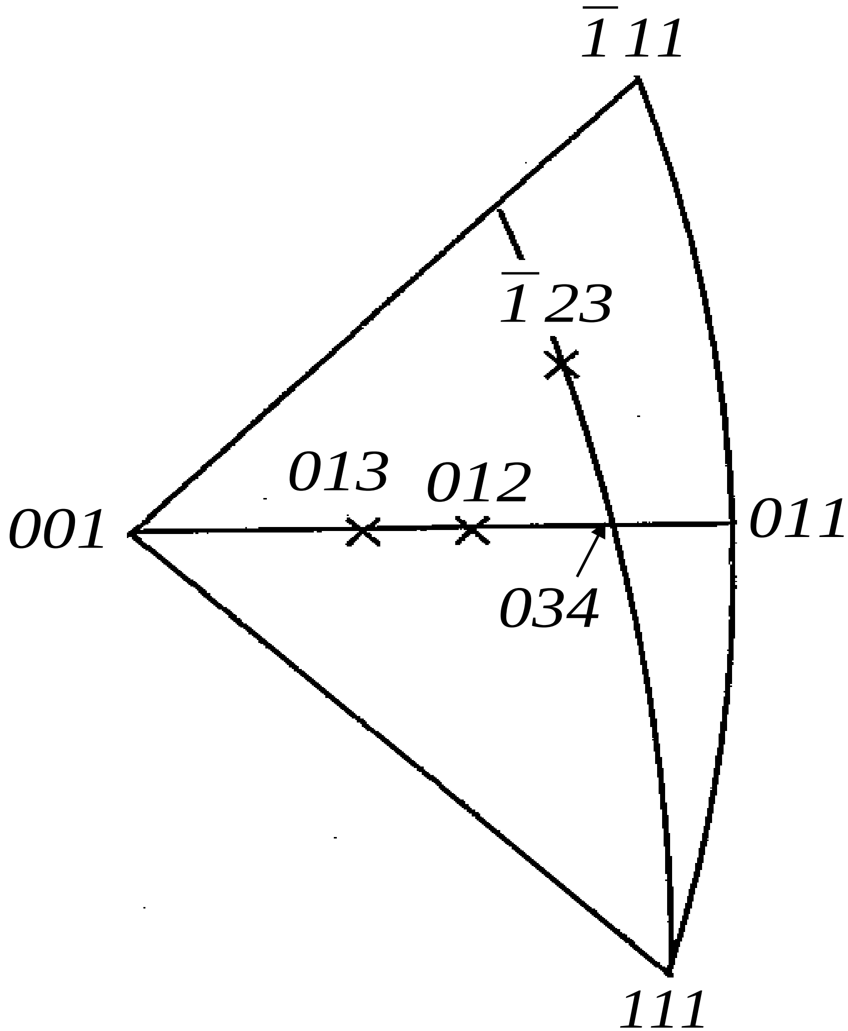


$(100\cos\phi_0\cos\lambda_0)$ plotted stereographically as a function of direction of uniaxial stress for various types of slip system in cubic crystals

Single Slip in Crystals of Cubic Symmetry

[*Example 1*] A bcc crystal slips on the (-101) [111] system in response to a tensile stress along [-123]. How much shear strain is required to rotate the longitudinal axis of the crystal to the [001]-[011] symmetry line? What is the orientation of the axis on this line?

Single Slip in Crystals of Cubic Symmetry



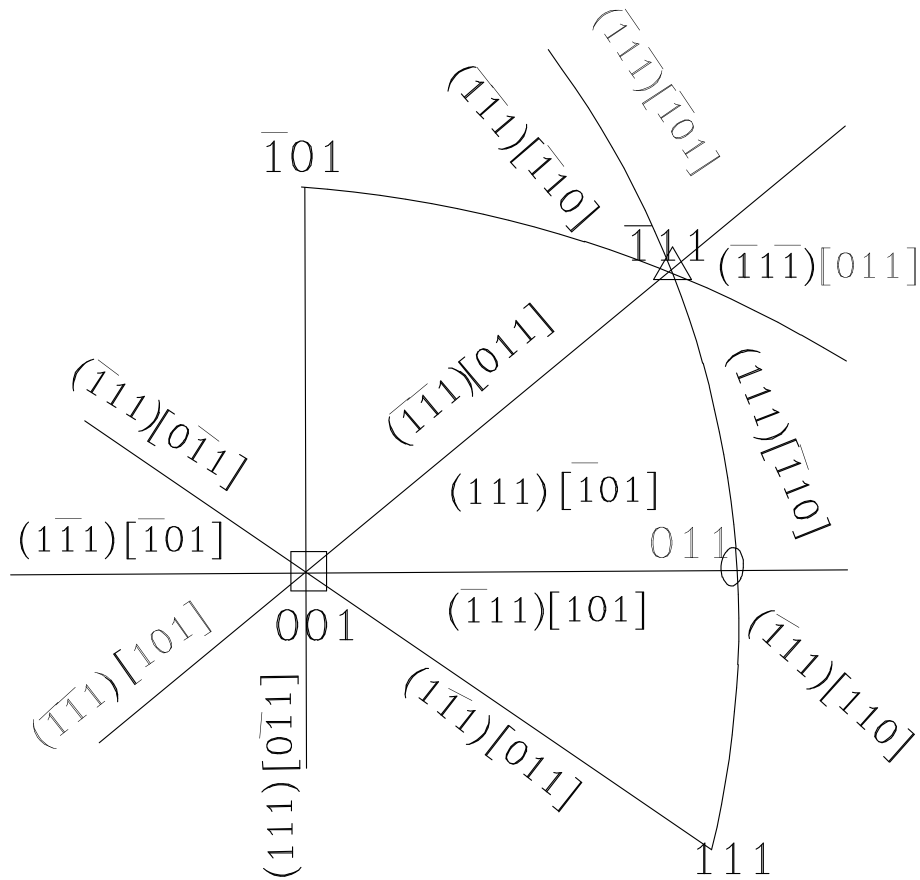
Single Slip in Crystals of Cubic Symmetry

[*Example 2*] Suppose that, in above Example 1, one side surface of the crystal is (210). What is the new orientation of this face after the crystal has undergone a shear strain of $\sqrt{6}/4$?

Single Slip in Crystals of Cubic Symmetry

[*Example 3*] Suppose that, in above Example 1, one direction of the crystal is [210]. What is the new orientation of this direction after the crystal has undergone a shear strain of $\sqrt{6}/4$?

Slip on Two Systems -Duplex Slip



Most highly stressed slip systems of {111}<110> types in terms of direction of uniaxial stress.

Slip on Two Systems -Duplex Slip

Duplex slip in tension

The vectorial change in length of the crystal

$$\boldsymbol{\Delta l} = \mathbf{L} - \boldsymbol{l} = \Delta\gamma\,(\boldsymbol{l}\cdot\mathbf{n})\,\boldsymbol{b} + \Delta\gamma\,(\boldsymbol{l}\cdot\boldsymbol{n}')\boldsymbol{b}'$$

$$\boldsymbol{\Delta l} = l\cos\phi\,\Delta\gamma\,(\boldsymbol{b} + \boldsymbol{b}')$$

Duplex Slip in Cubic Crystals

Duplex slip in tension

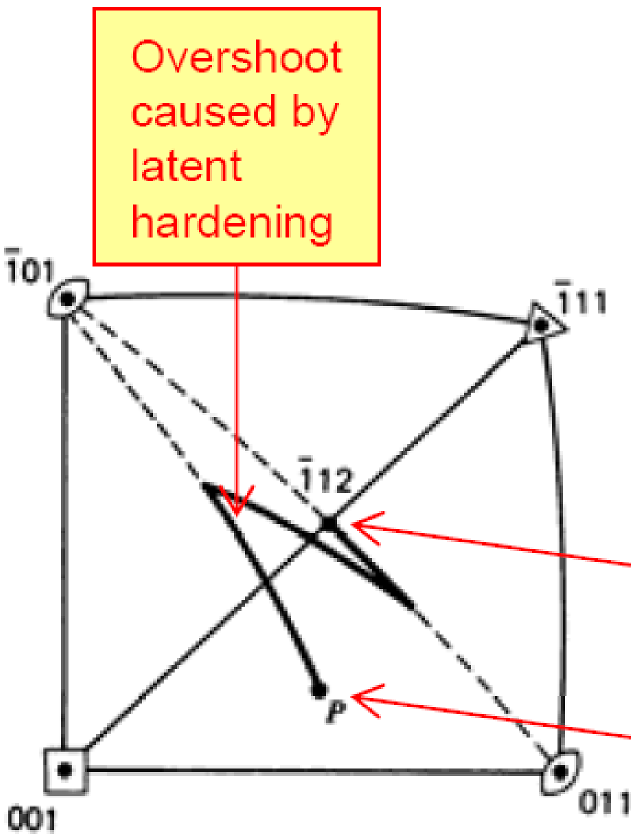
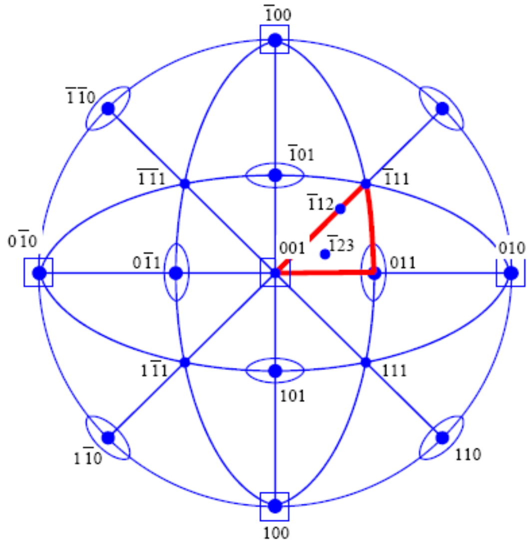


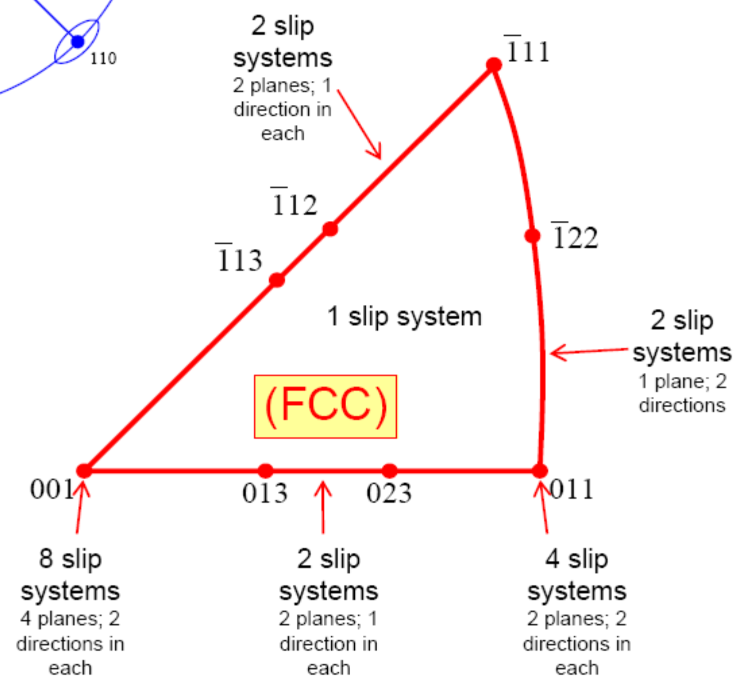
FIGURE 3.9 Lattice rotation of FCC crystal involving "overshoot" of primary and conjugate slip systems.

$$[\bar{1}12] = \sum [\bar{1}01] + [011]$$

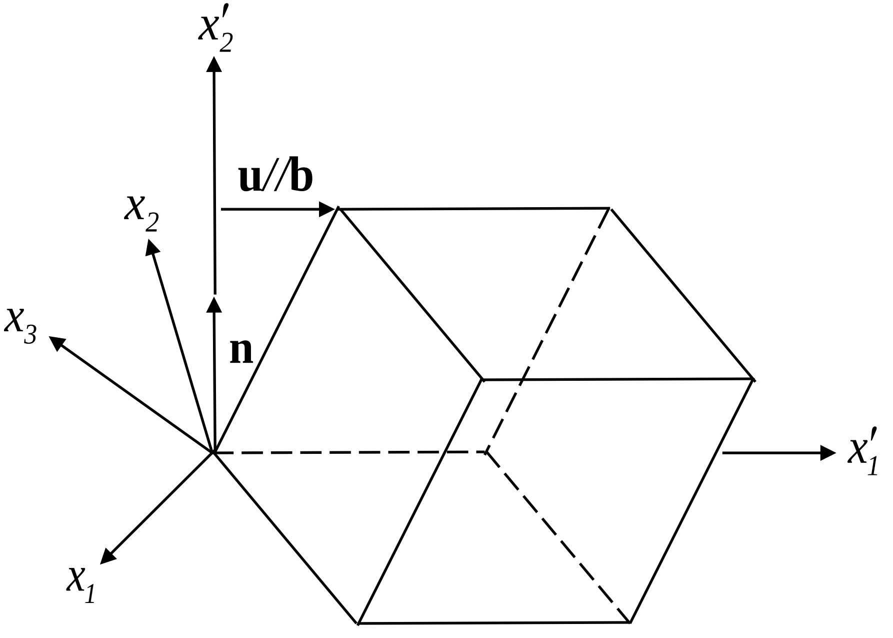
[001] stereographic projection of cubic crystal



Standard Triangle



Strains Produced by Slip



$$e'_{12} = \partial u'_1 / \partial x'_2 = \gamma$$

$$e'_{ij} = \begin{bmatrix} 0 & \gamma & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

	x_1	x_2	x_3
x_1'	a_{11}	a_{12}	a_{13}
x_2'	a_{21}	a_{22}	a_{23}

=

	x_1	x_2	x_3
x_1'	b_1	b_2	b_3
x_2'	n_1	n_2	n_3

Strains Produced by Slip

$$e_{ij} = \gamma \begin{bmatrix} a_{11}a_{21} & a_{11}a_{22} & a_{11}a_{23} \\ a_{12}a_{21} & a_{12}a_{22} & a_{12}a_{23} \\ a_{13}a_{21} & a_{13}a_{22} & a_{13}a_{23} \end{bmatrix} = \gamma \begin{bmatrix} b_1 n_1 & b_1 n_2 & b_1 n_3 \\ b_2 n_1 & b_2 n_2 & b_2 n_3 \\ b_3 n_1 & b_3 n_2 & b_3 n_3 \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} = \begin{bmatrix} b_2 n_2 \\ b_3 n_3 \\ b_2 n_3 + b_3 n_2 \\ b_1 n_3 + b_3 n_1 \\ b_1 n_2 + b_2 n_1 \end{bmatrix} \gamma$$

Independent Slip Systems

$$\begin{bmatrix}\varepsilon_2\\ \varepsilon_3\\ \varepsilon_4\\ \varepsilon_5\\ \varepsilon_6\end{bmatrix}=\begin{bmatrix} b_2^{(1)}n_2^{(1)} & b_2^{(2)}n_2^{(2)} & \cdots\cdots\cdots & b_2^{(j)}n_2^{(j)} \\ b_3^{(1)}n_3^{(1)} & b_3^{(2)}n_3^{(2)} & \cdots\cdots\cdots & b_3^{(j)}n_3^{(j)} \\ b_2^{(1)}n_3^{(1)}+b_3^{(1)}n_2^{(1)} & b_2^{(2)}n_3^{(2)}+b_3^{(2)}n_2^{(2)} & \cdots\cdots\cdots & b_2^{(j)}n_3^{(j)}+b_3^{(j)}n_2^{(j)} \\ b_1^{(1)}n_3^{(1)}+b_3^{(1)}n_1^{(1)} & b_1^{(2)}n_3^{(2)}+b_3^{(2)}n_1^{(2)} & \cdots\cdots\cdots & b_1^{(j)}n_3^{(j)}+b_3^{(j)}n_1^{(j)} \\ b_1^{(1)}n_2^{(1)}+b_2^{(1)}n_1^{(1)} & b_1^{(2)}n_2^{(2)}+b_2^{(2)}n_1^{(2)} & \cdots\cdots\cdots & b_1^{(j)}n_2^{(j)}+b_2^{(j)}n_1^{(j)} \end{bmatrix}\begin{bmatrix}\gamma^{(1)}\\ \gamma^{(2)}\\ \vdots\\ \vdots\\ \gamma^{(j)}\end{bmatrix}$$

j can be up to 12 in fcc crystal.

But, mathematically j must be 5.

$$\begin{bmatrix}\varepsilon_2\\ \varepsilon_3\\ \varepsilon_4\\ \varepsilon_5\\ \varepsilon_6\end{bmatrix}=\begin{bmatrix} b_2^{(1)}n_2^{(1)} & b_2^{(2)}n_2^{(2)} & \cdots\cdots\cdots & b_2^{(5)}n_2^{(5)} \\ b_3^{(1)}n_3^{(1)} & b_3^{(2)}n_3^{(2)} & \cdots\cdots\cdots & b_3^{(5)}n_3^{(5)} \\ b_2^{(1)}n_3^{(1)}+b_3^{(1)}n_2^{(1)} & b_2^{(2)}n_3^{(2)}+b_3^{(2)}n_2^{(2)} & \cdots\cdots\cdots & b_2^{(5)}n_3^{(5)}+b_3^{(5)}n_2^{(5)} \\ b_1^{(1)}n_3^{(1)}+b_3^{(1)}n_1^{(1)} & b_1^{(2)}n_3^{(2)}+b_3^{(2)}n_1^{(2)} & \cdots\cdots\cdots & b_1^{(5)}n_3^{(5)}+b_3^{(5)}n_1^{(5)} \\ b_1^{(1)}n_2^{(1)}+b_2^{(1)}n_1^{(1)} & b_1^{(2)}n_2^{(2)}+b_2^{(2)}n_1^{(2)} & \cdots\cdots\cdots & b_1^{(5)}n_2^{(5)}+b_2^{(5)}n_1^{(5)} \end{bmatrix}\begin{bmatrix}\gamma^{(1)}\\ \gamma^{(2)}\\ \gamma^{(3)}\\ \gamma^{(4)}\\ \gamma^{(5)}\end{bmatrix}$$

Independent Slip Systems

$$[\varepsilon_{ij}] = \sum_{k}^{5} \frac{1}{2} (b_i^{(k)} n_j^{(k)} + b_j^{(k)} n_i^{(k)}) \gamma^{(\mathrm{k})} = \sum_{k}^{5} \frac{1}{2} K_{ij}^{(k)} \gamma^{(\mathrm{k})}$$

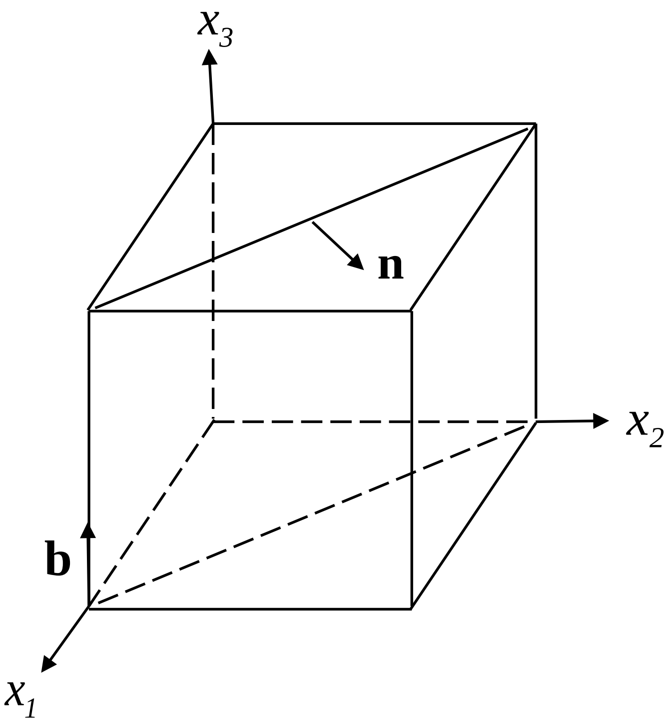
From the definition of [K]$^{-1}$, the determinant of [K] be non-zero, that means the five systems are *independent*.

We have used infinitesimal strain in the above analysis. If we have to consider large plastic strains, we can regard the (small) strains in the analysis as only increments of the total strain.

It is worth remarking that since any (small) plastic strain can be accomplished by five independent slip systems, no crystal can have more than five such systems.

Independent Slip Systems

[*Example*] Can the {110}<001> family of slip systems in a cubic crystal produce any arbitrary strain without change in volume?



$$[\mathrm{K}] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{bmatrix}$$

Table 5.3.1. Slip Systems

Crystal structure	Slip direction	Slip plane	Maximum number of independent slip systems N	Number of ways of choosing N
fcc	<110>	{111}	5	384
bcc	<111>	{110}	5	384
	<111>	{112}	5	648
hcp	<11-20>	{0001} "basal" slip	2	3
	<11-20>	{10-10} "prismatic" slip	2	3
	<11-20>	{10-11} "pyramidal" slip	4	9
	<11-21>	{11-22}	5	
NaCl	<1-10>	{110}	2	12
CsCl	<010>	{100}	3	8
	<111>	{110}	5	384
	<111>	{112}	5	648
CaF_2	<110>	{001}	3	16
	<1-10>	{110}	2	12
TiO_2	<10-1>	{101}	4	
	<001>	{110}	4	

Selection of Possible Active Systems

Which particular combination of five slip systems actually operates to achieve a given increment of strain ?

Taylor's criterion._

Bishop-Hill's criterion_

Taylor's criterion

Taylor [1938] postulated that the preferred set of slip systems will be that for which the sum of the shears on each system is a minimum.

$$dw = \tau_c \sum_k d\gamma^{(k)}$$

Taylor postulated that the work done in activating the preferred set of slip systems is less than that of all other sets of systems.

Taylor's criterion

$$[\gamma] = [K]^{-1} \ [\varepsilon]\_$$

From this equation, the Taylor criterion can be used to predict the operating slip systems to accomplish the imposed strain. [γ] is evaluated from this equation for *each* set of five independent slip systems. There are 384 of these for the fcc crystal structure.

In Talyor criterion, the minimum value of total shear was selected among these 384 cases. For the unique active slip system, various methods are suggested, for examples, the *random selection method* [Van Houtte 1984] and the *secondary work minimization model* [Renourd, Wintenberger 1976, 1981].

Bishop-Hill's criterion

Bishop and Hill [1951] showed that this Taylor criterion is equivalent to the stress criterion for yielding.

$$\sum d\gamma^{(k)} \leq \sum d\gamma^{(k)*}$$

We conclude that Taylor's criterion is that a critical resolved shear stress is required for yielding, as stated by Schmid's law.

$$dw/(\tau_c d\varepsilon_{ij}) = M$$

State of Stress for Multiple Slip

$$\begin{bmatrix}\tau^{(1)}\\ \tau^{(2)}\\ \tau^{(3)}\\ \tau^{(4)}\\ \tau^{(5)}\end{bmatrix}=\begin{bmatrix}
n_1^{(1)}b_1^{(1)} & n_2^{(1)}b_2^{(1)} & n_3^{(1)}b_3^{(1)} & n_2^{(1)}b_3^{(1)}+n_3^{(1)}b_2^{(1)} & n_1^{(1)}b_3^{(1)}+n_3^{(1)}b_1^{(1)} & n_1^{(1)}b_2^{(1)}+n_2^{(1)}b_1^{(1)}\\
n_1^{(2)}b_1^{(2)} & n_2^{(2)}b_2^{(2)} & n_3^{(2)}b_3^{(2)} & n_2^{(2)}b_3^{(2)}+n_3^{(2)}b_2^{(2)} & n_1^{(2)}b_3^{(2)}+n_3^{(2)}b_1^{(2)} & n_1^{(2)}b_2^{(2)}+n_2^{(2)}b_1^{(2)}\\
n_1^{(3)}b_1^{(3)} & n_2^{(3)}b_2^{(3)} & n_3^{(3)}b_3^{(3)} & n_2^{(3)}b_3^{(3)}+n_3^{(3)}b_2^{(3)} & n_1^{(3)}b_3^{(3)}+n_3^{(3)}b_1^{(3)} & n_1^{(3)}b_2^{(3)}+n_2^{(3)}b_1^{(3)}\\
n_1^{(4)}b_1^{(4)} & n_2^{(4)}b_2^{(4)} & n_3^{(4)}b_3^{(4)} & n_2^{(4)}b_3^{(4)}+n_3^{(4)}b_2^{(4)} & n_1^{(4)}b_3^{(4)}+n_3^{(4)}b_1^{(4)} & n_1^{(4)}b_2^{(4)}+n_2^{(4)}b_1^{(4)}\\
n_1^{(5)}b_1^{(5)} & n_2^{(5)}b_2^{(5)} & n_3^{(5)}b_3^{(5)} & n_2^{(5)}b_3^{(5)}+n_3^{(5)}b_2^{(5)} & n_1^{(5)}b_3^{(5)}+n_3^{(5)}b_1^{(5)} & n_1^{(5)}b_2^{(5)}+n_2^{(5)}b_1^{(5)}
\end{bmatrix}\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{13}\\ \sigma_{12}\end{bmatrix}$$

$A=\sigma_{22}-\sigma_{33}$ $B=\sigma_{33}-\sigma_{11}$ $C=\sigma_{11}-\sigma_{22}$ $F=\sigma_{23}$ $G=\sigma_{13}$ $G=\sigma_{12}$

$$\begin{bmatrix}\tau^{(1)}\\ \tau^{(2)}\\ \tau^{(3)}\\ \tau^{(4)}\\ \tau^{(5)}\end{bmatrix}=\begin{bmatrix}
n_2^{(1)}b_2^{(1)} & n_3^{(1)}b_3^{(1)} & n_2^{(1)}b_3^{(1)}+n_3^{(1)}b_2^{(1)} & n_1^{(1)}b_3^{(1)}+n_3^{(1)}b_1^{(1)} & n_1^{(1)}b_2^{(1)}+n_2^{(1)}b_1^{(1)}\\
n_2^{(2)}b_2^{(2)} & n_3^{(2)}b_3^{(2)} & n_2^{(2)}b_3^{(2)}+n_3^{(2)}b_2^{(2)} & n_1^{(2)}b_3^{(2)}+n_3^{(2)}b_1^{(2)} & n_1^{(2)}b_2^{(2)}+n_2^{(2)}b_1^{(2)}\\
n_2^{(3)}b_2^{(3)} & n_3^{(3)}b_3^{(3)} & n_2^{(3)}b_3^{(3)}+n_3^{(3)}b_2^{(3)} & n_1^{(3)}b_3^{(3)}+n_3^{(3)}b_1^{(3)} & n_1^{(3)}b_2^{(3)}+n_2^{(3)}b_1^{(3)}\\
n_2^{(4)}b_2^{(4)} & n_3^{(4)}b_3^{(4)} & n_2^{(4)}b_3^{(4)}+n_3^{(4)}b_2^{(4)} & n_1^{(4)}b_3^{(4)}+n_3^{(4)}b_1^{(4)} & n_1^{(4)}b_2^{(4)}+n_2^{(4)}b_1^{(4)}\\
n_2^{(5)}b_2^{(5)} & n_3^{(5)}b_3^{(5)} & n_2^{(5)}b_3^{(5)}+n_3^{(5)}b_2^{(5)} & n_1^{(5)}b_3^{(5)}+n_3^{(5)}b_1^{(5)} & n_1^{(5)}b_2^{(5)}+n_2^{(5)}b_1^{(5)}
\end{bmatrix}\begin{bmatrix}-C\\ B\\ F\\ G\\ H\end{bmatrix}$$

State of Stress for Multiple Slip

$$\begin{bmatrix} \tau^{(1)} \\ \tau^{(2)} \\ \tau^{(3)} \\ \tau^{(4)} \\ \tau^{(5)} \end{bmatrix} = \begin{bmatrix} n_2^{(1)}b_2^{(1)} & n_3^{(1)}b_3^{(1)} & n_2^{(1)}b_3^{(1)} + n_3^{(1)}b_2^{(1)} & n_1^{(1)}b_3^{(1)} + n_3^{(1)}b_1^{(1)} & n_1^{(1)}b_2^{(1)} + n_2^{(1)}b_1^{(1)} \\ n_2^{(2)}b_2^{(2)} & n_3^{(2)}b_3^{(2)} & n_2^{(2)}b_3^{(2)} + n_3^{(2)}b_2^{(2)} & n_1^{(2)}b_3^{(2)} + n_3^{(2)}b_1^{(2)} & n_1^{(2)}b_2^{(2)} + n_2^{(2)}b_1^{(2)} \\ n_2^{(3)}b_2^{(3)} & n_3^{(3)}b_3^{(3)} & n_2^{(3)}b_3^{(3)} + n_3^{(3)}b_2^{(3)} & n_1^{(3)}b_3^{(3)} + n_3^{(3)}b_1^{(3)} & n_1^{(3)}b_2^{(3)} + n_2^{(3)}b_1^{(3)} \\ n_2^{(4)}b_2^{(4)} & n_3^{(4)}b_3^{(4)} & n_2^{(4)}b_3^{(4)} + n_3^{(4)}b_2^{(4)} & n_1^{(4)}b_3^{(4)} + n_3^{(4)}b_1^{(4)} & n_1^{(4)}b_2^{(4)} + n_2^{(4)}b_1^{(4)} \\ n_2^{(5)}b_2^{(5)} & n_3^{(5)}b_3^{(5)} & n_2^{(5)}b_3^{(5)} + n_3^{(5)}b_2^{(5)} & n_1^{(5)}b_3^{(5)} + n_3^{(5)}b_1^{(5)} & n_1^{(5)}b_2^{(5)} + n_2^{(5)}b_1^{(5)} \end{bmatrix} \begin{bmatrix} -C \\ B \\ F \\ G \\ H \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} = \begin{bmatrix} b_2^{(1)}n_2^{(1)} & b_2^{(2)}n_2^{(2)} & \ldots\ldots\ldots & b_2^{(5)}n_2^{(5)} \\ b_3^{(1)}n_3^{(1)} & b_3^{(2)}n_3^{(2)} & \ldots\ldots\ldots & b_3^{(5)}n_3^{(5)} \\ b_2^{(1)}n_3^{(1)} + b_3^{(1)}n_2^{(1)} & b_2^{(2)}n_3^{(2)} + b_3^{(2)}n_2^{(2)} & \ldots\ldots\ldots & b_2^{(5)}n_3^{(5)} + b_3^{(5)}n_2^{(5)} \\ b_1^{(1)}n_3^{(1)} + b_3^{(1)}n_1^{(1)} & b_1^{(2)}n_3^{(2)} + b_3^{(2)}n_1^{(2)} & \ldots\ldots\ldots & b_1^{(5)}n_3^{(5)} + b_3^{(5)}n_1^{(5)} \\ b_1^{(1)}n_2^{(1)} + b_2^{(1)}n_1^{(1)} & b_1^{(2)}n_2^{(2)} + b_2^{(2)}n_1^{(2)} & \ldots\ldots\ldots & b_1^{(5)}n_2^{(5)} + b_2^{(5)}n_1^{(5)} \end{bmatrix} \begin{bmatrix} \gamma^{(1)} \\ \gamma^{(2)} \\ \gamma^{(3)} \\ \gamma^{(4)} \\ \gamma^{(5)} \end{bmatrix}$$

$$[\tau] = [K^T][\sigma]$$

State of Stress for Multiple Slip

$$[\sigma] = [K^T]^{-1}[\tau]$$

We conclude that combinations of five slip systems that are geometrically capable of causing an arbitrary strain are also physically capable of being activated by a feasible state of stress.

Stress states for {111}<110> or {110}<111> slip

Slip systems of fcc structure

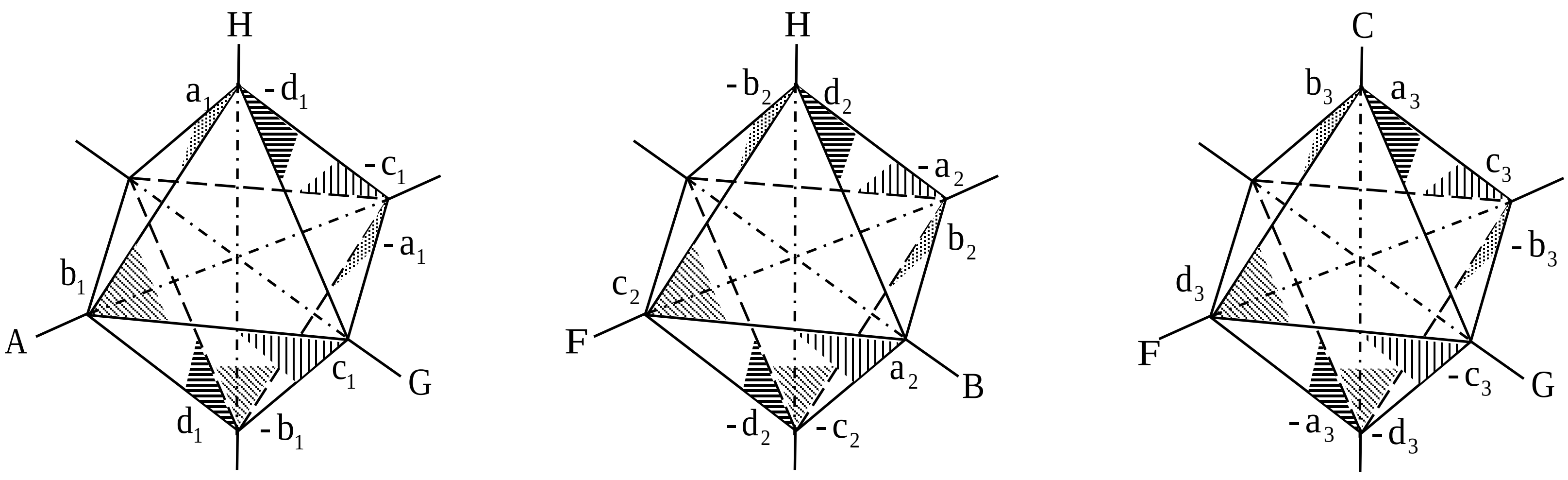
plane	111			-1-1 1			-111			1-1 1		
direction	0 1-1	-101	1-1 0	0-1-1	101	-110	01-1	101	-1-1 0	0-1-1	-101	110
slip system	a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3	d_1	d_2	d_3

On slip system a1, b1, c1, and d1_ $A \pm G \pm H = \pm\sqrt{6}\ \tau_c$

On slip system a2, b2, c2, and d2_ $B \pm F \pm H = \pm\sqrt{6}\ \tau_c$

On slip system a3, b3, c3, and d3_ $C \pm F \pm G = \pm\sqrt{6}\ \tau_c$

Stress states for {111}<110> or {110}<111> slip



Octahedral yield surface for cubic crystals that slip on {111}<110> or {110}<111> systems.

For the Cartesian spaces with axes (*A*, *G*, *H*)
A stress can be represented by a vector from the origin to the surface of the octahedron.
If such a vector ends on a face, slip occurs on one system.
If it ends on an edge of the octahedron, slip can be activated on two systems.
If the vector ends at a vertex, slip occurs on all four systems, a1, b1, c1 and d1.

Stress states for {111}<110> or {110}<111> slip

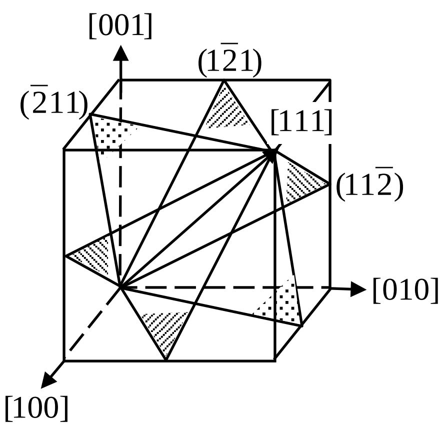
Number of stress state	A	B	C	F	G	H	a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3	d_1	d_2	d_3	Number of active slip systems
	unit: $\sqrt{6}\,\tau_c$						111			-1-1 1			-1 1 1			1-1 1			
							01-1	-101	1-10	0-1-1	101	-110	01-1	101	-1-10	0-1-1	-101	110	
1	1	-1	0	0	0	0	+	-		+	-		+	-		+	-		8
2	0	1	-1	0	0	0		+	-		+	-		+	-		+	-	8
3	-1	0	1	0	0	0	-		+	-		+	-		+	-		+	8
4	0	0	0	1	0	0		+	-		-	+		+	-		-	+	8
5	0	0	0	0	1	0	-		+	+		-	+		-	-		+	8
6	0	0	0	0	0	1	+	-		+	-		-	+		-	+		8
7	1/2	-1	1/2	0	1/2	0		-	+	+	-		+	-			-	+	8
8	1/2	-1	1/2	0	-1/2	0	+	-			-	+		-	+	+	-		8
9	-1	1/2	1/2	1/2	0	0	-	+		-		+	-	+		-		+	8
10	-1	1/2	1/2	-1/2	0	0	-		+	-	+		-		+	-	+		8
11	1/2	1/2	-1	0	0	1/2	+		-	+		-		+	-		+	-	8
12	1/2	1/2	-1	0	0	-1/2		+	-		+	-	+		-	+		-	8
13	1/2	0	-1/2	1/2	0	1/2	+		-	+	-			+	-				6
14	1/2	0	-1/2	-1/2	0	1/2	+	-		+		-					+	-	6
15	1/2	0	-1/2	1/2	0	-1/2		+	-				+		-	+	-		6
16	1/2	0	-1/2	-1/2	0	-1/2					+	-	+	-		+		-	6
17	0	-1/2	1/2	0	1/2	1/2		-	+	+	-					-		+	6
18	0	-1/2	1/2	0	-1/2	1/2	+	-			-	+	-		+				6
19	0	-1/2	1/2	0	1/2	-1/2	-		+				+	-			-	+	6
20	0	-1/2	1/2	0	-1/2	-1/2				-		+		-	+	+	-		6
21	-1/2	1/2	0	1/2	1/2	0	-	+						+	-	-		+	6
22	-1/2	1/2	0	-1/2	1/2	0	-		+		+	-				-	+		6
23	-1/2	1/2	0	1/2	-1/2	0		+	-	-		+	-	+					6
24	-1/2	1/2	0	-1/2	-1/2	0				-	+		-		+		+	-	6
25	0	0	0	1/2	1/2	-1/2	-	+					+		-		-	+	6
26	0	0	0	1/2	-1/2	1/2	+		-		-	+	-	+					6
27	0	0	0	-1/2	1/2	1/2		-	+	+		-				-	+		6
28	0	0	0	1/2	1/2	1/2				+	-			+	-	-		+	6

$A = \sigma_{22} - \sigma_{33}$, $B = \sigma_{33} - \sigma_{11}$, $C = \sigma_{11} - \sigma_{22}$, $F = \sigma_{23}$, $G = \sigma_{13}$, $H = \sigma_{12}$

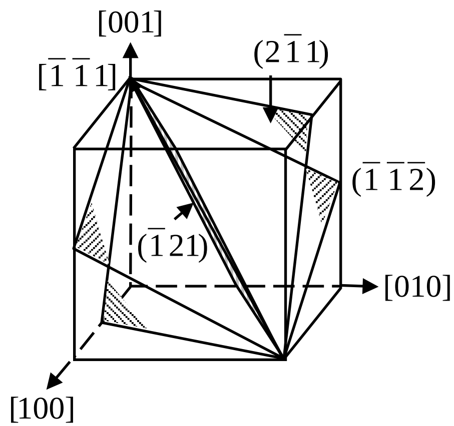
Stress states for {112}<111> slip

Table 5.6.3. The twelve {112}<111> slip systems

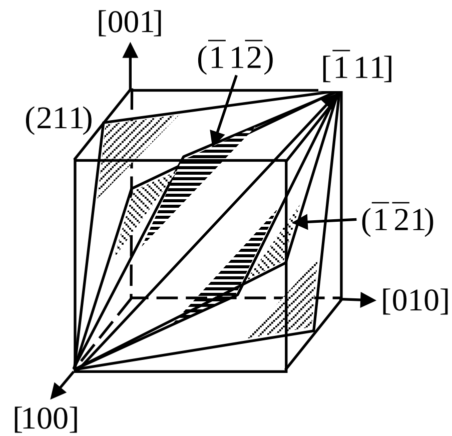
Plane	-21 1	1-2 1	1-1 2	2-1 1	-121	-1-1 2	211	-1-2 1	-1 1-2	-2-1 1	121	-1-1 2
Direction		111			-1-1 1			-111			1-1 1	
System	e_1	e_2	e_3	f_1	f_2	f_3	g_1	g_2	g_3	h_1	h_2	h_3



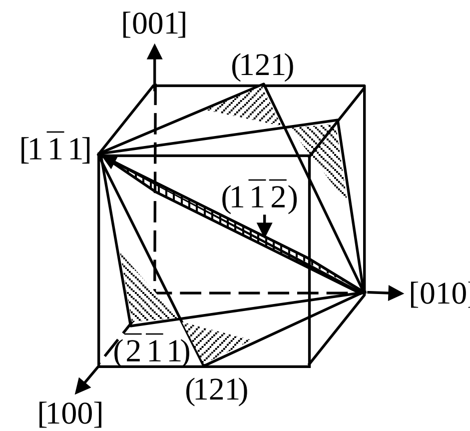
e1 , e2 , e3



f1 , f2 , f3



g1 , g2 , g3



h1 , h2 , h3

Stress states for {112}<111> slip

$$\tau_{e_1} = (B - C + 2F - G - H) / 3\sqrt{2}$$

$$\tau_{f_1} = (B - C - 2F + G - H) / 3\sqrt{2}$$

$$\tau_{g_1} = (B - C + 2F + G + H) / 3\sqrt{2}$$

$$\tau_{h_1} = (B - C - 2F - G + H) / 3\sqrt{2}$$

$$\tau_{e_2} = (C - A - F + 2G - H) / 3\sqrt{2}$$

$$\tau_{f_2} = (C - A + F - 2G - H) / 3\sqrt{2}$$

$$\tau_{g_2} = (C - A - F - 2G + H) / 3\sqrt{2}$$

$$\tau_{h_2} = (C - A + F + 2G + H) / 3\sqrt{2}$$

$$\tau_{e_3} = (A - B - F - G + 2\mathrm{H}) / 3\sqrt{2}$$

$$\tau_{f_3} = (A - B + F + G + 2H) / 3\sqrt{2}$$

$$\tau_{g_3} = (A - B - F + G - 2H) / 3\sqrt{2}$$

$$\tau_{h_3} = (A - B + F - G - 2H) / 3\sqrt{2}$$

Table 5.6.4. Stress states for {111}<112>
[Hosford, Chin 1969]

Number of stress state	A	B	C	F	G	H	Slip systems												Number of active slip systems
							e_1	e_2	e_3	f_1	f_2	f_3	g_1	g_2	g_3	h_1	h_2	h_3	
	unit : $3\sqrt{2}\,\tau_c$						111			$\bar{1}\bar{1}1$			$\bar{1}11$			$1\bar{1}1$			
							$\bar{2}11$	$1\bar{2}1$	$11\bar{2}$	$2\bar{1}\bar{1}$	$\bar{1}21$	$\bar{1}\bar{1}2$	211	$\bar{1}21$	$\bar{1}1\bar{2}$	$\bar{2}\bar{1}1$	121	$1\bar{1}\bar{2}$	
1	2/3	-1/3	-1/3	0	0	0		−	+		−	+		−	+		−	+	8
2	-1/3	2/3	-1/3	0	0	0	+		−	+		−	+		−	+		−	8
3	-1/3	-1/3	2/3	0	0	0	−	+		−	+		−	+		−	+		8
4	1/3	-1/6	-1/6	1/2	0	0	+	−		−		+	+	−		−		+	8
5	1/3	-1/6	-1/6	-1/2	0	0	−		+	+	−		−		+	+	−		8
6	-1/6	1/3	-1/6	0	1/2	0		+	−	+	−		+	−			+	−	8
7	-1/6	1/3	-1/6	0	-1/2	0	+	−			+	−		+	−	+	−		8
8	-1/6	-1/6	1/3	0	0	1/2	−		+	−		+		+	−		+	−	8
9	-1/6	-1/6	1/3	0	0	-1/2		+	−		+	−	−		+	−		+	8
10	0	-1/12	1/12	-1/12	1/3	1/3	−				−	+					+	−	5
11	0	-1/12	1/12	-1/12	-1/3	-1/3					+	−	−				−	+	5
12	0	-1/12	1/12	1/12	1/3	-1/3		+	−					−	+	−			5
13	0	-1/12	1/12	1/12	-1/3	1/3		−	+	−				+	−				5
14	1/12	0	-1/12	1/3	-1/12	1/3		−		−		+	+		−				5
15	1/12	0	-1/12	-1/3	-1/12	-1/3				+		−	−		+		−		5
16	1/12	0	-1/12	-1/3	1/12	1/3	−		+		−					+		−	5
17	1/12	0	-1/12	1/3	1/12	-1/3	+		−					−		−		+	5
18	-1/12	1/12	0	1/3	1/3	-1/12			−				+	−		−	+		5
19	-1/12	1/12	0	-1/3	-1/3	-1/12						−	−	+		+	−		5
20	-1/12	1/12	0	1/3	-1/3	1/12	+	−		−	+				−				5
21	-1/12	1/12	0	-1/3	1/3	1/12	−	+		+	−							−	5
22	0	-1/12	1/12	5/12	1/6	1/6				−		+	+			−	+		5
23	0	-1/12	1/12	5/12	-1/6	-1/6	+			−	+					−		+	5

Number of stress state	A	B	C	F	G	H	e1	e2	e3	f1	f2	f3	g1	g2	g3	h1	h2	h3	Number of active slip systems
	unit : $3\sqrt{2}\tau_c$						111			$\bar{1}\bar{1}1$			$\bar{1}11$			$1\bar{1}1$			
							$\bar{2}11$	$1\bar{2}1$	$11\bar{2}$	$2\bar{1}1$	$\bar{1}21$	$\overline{11}2$	211	$\overline{12}1$	$\bar{1}1\bar{2}$	$\overline{2}\,\overline{1}1$	121	$1\bar{1}\bar{2}$	
24	0	−1/12	1/12	−5/12	1/6	−1/6	−	+		+			−		+				5
25	0	−1/12	1/12	−5/12	−1/6	1/6	−		+				−	+		+			5
26	1/12	0	−1/12	1/6	5/12	1/6					−	+	+	−			+		5
27	1/12	0	−1/12	−1/6	5/12	−1/6		+		+	−			−	+				5
28	1/12	0	−1/12	−1/6	−5/12	1/6		−	+					+		+	−		5
29	1/12	0	−1/12	1/6	−5/12	−1/6	+	−			+						−	+	5
30	−1/12	1/12	0	1/6	1/6	5/12						+	+		−		+	−	5
31	−1/12	1/12	0	−1/6	−1/6	5/12			+					+	−	+		−	5
32	−1/12	1/12	0	1/6	−1/6	−5/12	+		−		+	−						+	5
33	−1/12	1/12	0	−1/6	1/6	−5/12		+	−	+		−			+				5
34	0	−5/12	5/12	1/12	1/6	1/6	−			−		+				−	+		5
35	0	−5/12	5/12	1/12	−1/6	−1/6				−	+		−			−		+	5
36	0	−5/12	5/12	−1/12	1/6	−1/6	−	+					−		+	−			5
37	0	−5/12	5/12	−1/12	−1/6	1/6	−		+	−			−	+					5
38	5/12	0	−5/12	1/6	1/12	1/6		−			−	+	+	−					5
39	5/12	0	−5/12	−1/6	1/12	−1/6				+	−			−	+		−		5
40	5/12	0	−5/12	−1/6	−1/12	1/6		−	+		−					+	−		5
41	5/12	0	−5/12	1/6	−1/12	−1/6	+	−						−			−	+	5
42	−5/12	5/12	0	1/6	1/6	1/12			−				+		−		+	−	5
43	−5/12	5/12	0	−1/6	−1/6	1/12						−		+	−	+		−	5
44	−5/12	5/12	0	1/6	−1/6	−1/12	+		−		+	−			−				5
45	−5/12	5/12	0	−1/6	1/6	−1/12		+	−	+		−						−	5

$A = \sigma_{22} - \sigma_{33}$, $B = \sigma_{33} - \sigma_{11}$, $C = \sigma_{11} - \sigma_{22}$, $F = \sigma_{23}$, $G = \sigma_{13}$, $H = \sigma_{12}$

Stress states for {123}<111> Slip

Twenty four {123}<111> slip systems

Slip direction	111						$\bar{1}\bar{1}1$					
Slip plane	$\bar{3}12$	$2\bar{3}1$	$12\bar{3}$	$\bar{3}21$	$1\bar{3}2$	$21\bar{3}$	$3\bar{1}2$	$\bar{2}31$	$\bar{1}\bar{2}\bar{3}$	$3\bar{2}1$	$\bar{1}32$	$\bar{2}\bar{1}\bar{3}$
Slip system	j_1	j_2	j_3	j_4	j_5	j_6	k_1	k_2	k_3	k_4	k_5	k_6
Slip direction	$\bar{1}11$						$1\bar{1}1$					
Slip plane	312	$\bar{2}\bar{3}1$	$\bar{1}2\bar{3}$	321	$\bar{1}\bar{3}2$	$\bar{2}1\bar{3}$	$\bar{3}\bar{1}2$	231	$1\bar{2}\bar{3}$	$\bar{3}\bar{2}1$	132	$\bar{2}\bar{1}\bar{3}$
Slip system	m_1	m_2	m_3	m_4	m_5	m_6	n_1	n_2	n_3	n_4	n_5	n_6

Stress states for {123}<111> Slip

$\tau_{j_1} = 2B - C + 3F - G - 2H$

$\tau_{k_1} = 2B - C - 3F + G - 2H$

$\tau_{m_1} = 2B - C + 3F + G + 2H$

$\tau_{n_1} = 2B - C + 3F - G + 2H$

$\tau_{j_2} = 2C - A - 2F + 3G - H$

$\tau_{k_2} = 2C - A + 2F - 3G - H$

$\tau_{m_2} = 2C - A - 2F - 3G + H$

$\tau_{n_2} = 2C - A + 2F + 3G + H$

$\tau_{j_3} = 2A - B - F - 2G + 3H$

$\tau_{k_3} = 2A - B + F + 2G + 3H$

$\tau_{m_3} = 2A - B - F + 2G - 3H$

$\tau_{n_3} = 2A - B + F - 2G - 3H$

$\tau_{j_4} = B - 2C + 3F - 2G - H$

$\tau_{k_4} = B - 2C - 3F + 2G - H$

$\tau_{m_4} = B - 2C + 3F + 2G + H$

$\tau_{n_4} = B - 2C - 3F - 2G + H$

$\tau_{j_5} = C - 2A - F + 3G - 2H$

$\tau_{k_5} = C - 2A + F - 3G - 2H$

$\tau_{m_5} = C - 2A - F - 3G + 2H$

$\tau_{n_5} = C - 2A + F + 3G + 2H$

$\tau_{j_6} = A - 2B - 2F - G + 3H$

$\tau_{k_6} = A - 2B + 2F + G + 3H$

$\tau_{m_6} = A - 2B - 2F + G - 3H$

$\tau_{n_6} = A - 2B + 2F - G - 3H$

Stress states for {123}<111> Slip

Table 5.6.6. Stress states for {123}<111>
[Chin, Wonsiewics, 1970]

Number of stress state	A	B	C	F	G	H	Slip systems																							
							j1	j2	j3	j4	j5	j6	k1	k2	k3	k4	k5	k6	m1	m2	m3	m4	m5	m6	n1	n2	n3	n4	n5	n6
	unit : $\sqrt{42}\,\tau_c$						111						$\bar{1}11$						$\bar{1}11$						$1\bar{1}1$					
							$\bar{3}12$	$2\bar{3}1$	$12\bar{3}$	$\bar{3}21$	$1\bar{3}2$	$21\bar{3}$	$3\bar{1}2$	$\bar{2}31$	$\bar{1}\bar{2}3$	$3\bar{2}1$	$\bar{1}32$	$\bar{2}\bar{1}3$	312	$\bar{2}\bar{3}1$	$\bar{1}2$	321	$\bar{1}\bar{3}2$	$\bar{2}1$	$\bar{3}\bar{1}2$	231	$1\bar{2}\bar{3}$	$\bar{3}\bar{2}1$	132	$2\bar{1}\bar{3}$
1	1/3	−1/3	0	0	0	0			+			+			+			+			+			+			+			+
2	0	1/3	−1/3	0	0	0	+			+			+			+			+			+			+			+		
3	−1/3	0	1/3	0	0	0		+			+			+			+			+			+			+			+	
4	0	0	0	1/3	0	0	+			+			−			−			+			+			−			−		
5	0	0	0	0	1/3	0		+			+			−			−			−			−			+			+	
6	0	0	0	0	0	1/3			+			+			+			+			−			−			−			−
7	1/5	−1/10	−1/10	3/10	0	0		−		+			−					+		−		+			−					+
8	1/5	−1/10	−1/10	−3/10	0	0	−					+		−		+			−					+		−		+		
9	−1/10	1/5	−1/10	0	3/10	0			−		+			−		+				−		+					−		+	
10	−1/10	1/5	−1/10	0	−3/10	0		−		+					−		+				−		+			−		+		
11	−1/10	−1/10	1/5	0	0	3/10	−					+	−					+			−		+				−		+	
12	−1/10	−1/10	1/5	0	0	−3/10			−		+				−		+		−					+	−					+
13	1/3	−1/6	−1/6	1/6	0	0		−			−				+			+		−			−				+			+
14	1/3	−1/6	−1/6	−1/6	0	0			+			+		−			−				+			+		−			−	
15	−1/6	1/3	−1/6	0	1/6	0			−			−	+			+			+			+					−			−
16	−1/6	1/3	−1/6	0	−1/6	0	+			+					−			−			−			−	+			+		
17	−1/6	−1/6	1/3	0	0	1/6	−			−			−			−				+			+			+			+	
18	−1/6	−1/6	1/3	0	0	−1/6		+			+			+			+		−			−			−			−		
19	0	−4/15	4/15	1/30	1/10	1/10	−			−			−					+								+		−		
20	0	−4/15	4/15	−1/30	−1/10	1/10	−					+	−			−				+		−								
21	0	−4/15	4/15	1/30	−1/10	−1/10								+		−			−			−			−					+

Stress states for {123}<111> Slip

Total number is 106.

Principle of Maximum Work

This was proposed by Bishop and Hill (1951), and it states that *in the deformation of a single crystal the actual stress corresponding to a given strain does not do less work than any other stress that satisfies the yielding conditions*.

We can put this another way: *The state of stress required to cause a given increment of strain is the one that maximizes the work done on the material*.

Principle of Maximum Work

Which particular stress state will be preferred in enforcing a given increment of strain ?

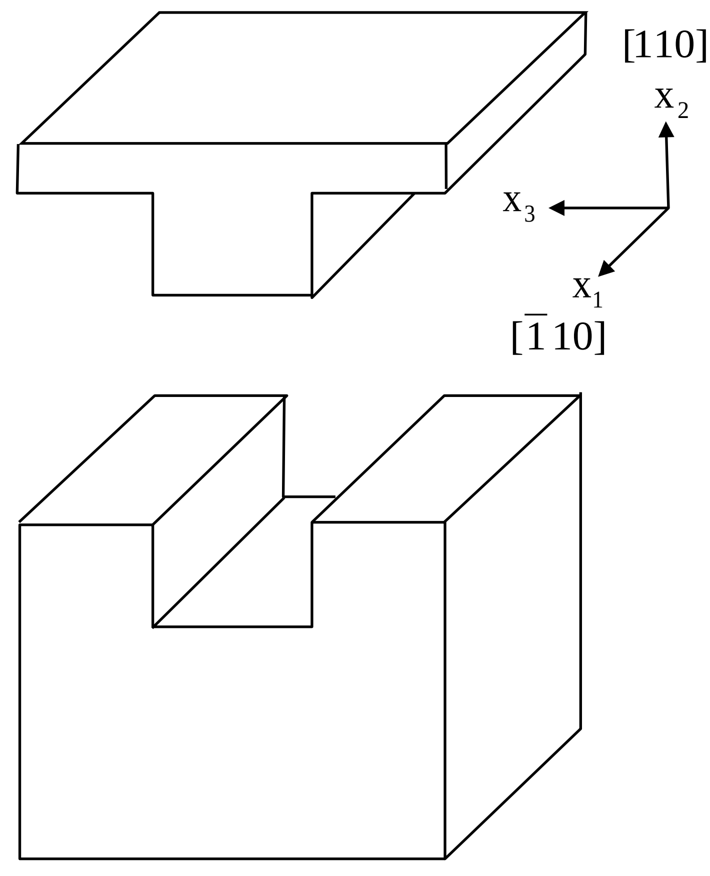
Which slip systems are active in accomplishing the strain ?

The Taylor approach gives the same results as *Bishop and Hill's Principle of Maximum Work.*

When considering {111}<110> slip, the Bishop-Hill method has the great advantage that the maximum of only fifty-six numbers is sought whereas the minimum of 384 numbers is required in the Taylor approach.

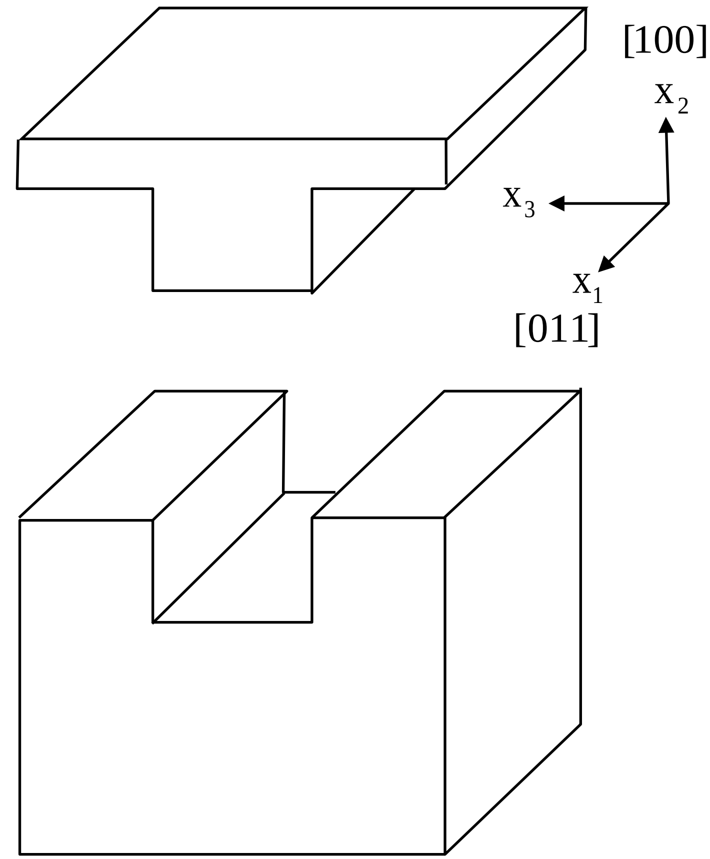
Principle of Maximum Work

[*Example*] A compression test is carried out on a rectangular single crystal, preventing it from expanding in one transverse direction (a plane-strain compression test). This is conveniently achieved by placing the crystal in a channel. Which slip systems will operate, and what will the compressive yield strength be? Suppose that the direction of compression is x_2= [110] and the channel lies along x_1=[-110] (permitting expansion along this direction), and that slip occurs on the {110} <111> systems.



Principle of Maximum Work

[*Example*] A plane strain compression test is carried out on a rectangular fcc single crystal, in which the compression direction is = [100] and the elongation direction is x_1 = [011]. Which slip systems will operate and what will the compressive yield strength be?



Principle of Maximum Work

No. of stress state	$=A/2 + B + F$
1	1/2 + (-1) + 0 = -1/2
2	0 + 1 + 0 = 1
3	-1/2 + 0 + 0 = -1/2
4	0 + 0 + 1 = 1
5	0 + 0 + 0 = 0
6	0 + 0 + 0 = 0
7	1/4 + (-1) + 0 = -3/4
8	1/4 - 1 + 0 = 3/4
9	-1/2 + 1/2 + 1/2 = 1/2
10	-1/2 + 1/2 - 1/2 = -1/2
11	1/4 + 1/2 + 0 = 3/4
12	1/4 + 1/2 + 0 = 3/4
13	1/4 + 0 + 1/2 = 3/4
14	1/4 + 0 - 1/2 = -1/4
15	1/4 + 0 + 1/2 = 3/4
16	1/4 + 0 + (-1/2) = -1/4
17	0 - 1/2 + 0 = -1/2
18	0 - 1/2 + 0 = -1/2
19	0 - 1/2 + 0 = -1/2
20	0 - 1/2 + 0 = -1/2
21	-1/4 + 1/2 + 1/2 = 3/4
22	-1/4 + 1/2 - 1/2 = -1/4
23	-1/4 + 1/2 + 1/2 = 3/4
24	-1/4 + 1/2 - 1/2 = -1/4
25	0 + 0 + 1/2 = 1/2
26	0 + 0 + 1/2 = 1/2
27	0 + 0 - 1/2 = -1/2
28	0 + 0 + 1/2 = 1/2

Principle of Maximum Work

Note that in certain orientations, more than two stress states maximize the work done. In such cases, the operative slip systems will be those that are common to both stress states, and there is no ambiguity about the choice of active systems.

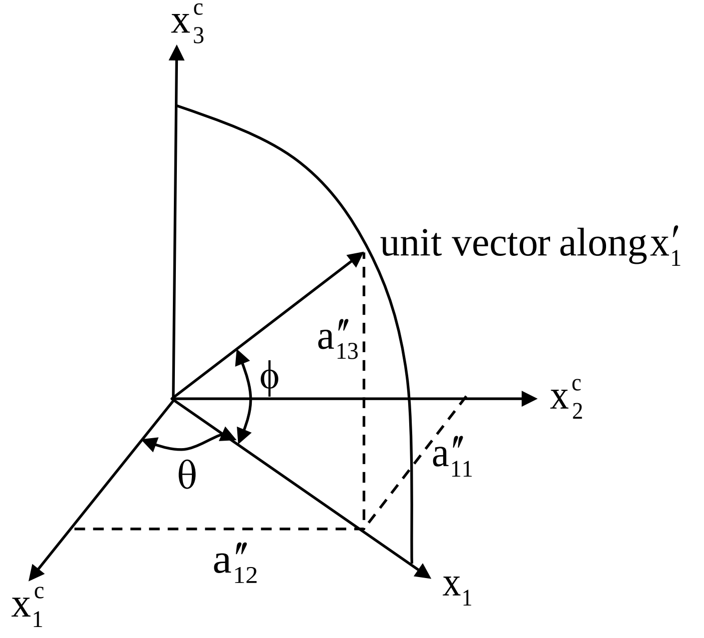
Principle of Maximum Work

Values of the relative strength parameter M for cubic crystals of various orientations. Subjected to plane strain compression. It is assumed that slip occurs on systems {111}<110> or {110}<111>.

Compress direction	Enlong. Direction	$M/\sqrt{6}$	Stress State	Active slip system
1 0 0	0 0 1	1	-1, 2, -7, -8	a_2 , b_2 ,c_2 , d_2
1 0 0	0 1 1	1	2, 4,	a_2 , $-a_3$, c_2 , $-c_3$
1 1 0	0 0 1	1	-1, -6	$-a_1$, a_2 , $-b_1$, b_2
1 1 0	-1 1 0	2	-6	$-a_1$, a_2 , $-b_1$, b_2 , c_1 , $-c_2$, d_1 , $-d_2$
1 1 0	-1 1 2	4 / 3	-6, -27	a_2 , $-b_1$, d_1 , $-d_2$, $-d_1$, d_2
1–1 0	1 1 1	5 / 3	6	$-a_1$, a_2 , $-b_1$, b_2 , c_1 , $-c_2$, d_1 , $-d_2$
1 1 1	-1 1 0	5 / 3	-6	a_1 , $-a_2$, b_1 , $-b_2$, $-c_1$, c_2 , $-d_1$, d_2
1 1 1	1 1-2	3 / 2	24, -28	$-b_1$, b_2 , c_3 , $-d_3$
1 1-2	1 1 1	3 / 2	-24, 28	b_1 , $-b_2$, $-c_3$, d_3
1 1 2	1 1-1	3 / 2	-21, -25	a_1 , $-a_2$, c_3 , $-d_3$
0 0 1	1 1 0	1	1, 6	a_1 , $-a_2$, b_1 , $-b_2$
1 1 2	1 1- 1	3 / 2	-21, -25	a_1 , $-a_2$, c_3 , $-d_3$

Principle of Maximum Work

[*Example*] Let us consider the case of an fcc or bcc crystal undergoing axisymmetrical deformation about an arbitrary direction. This type of deformation occurs, for example, when a cylindrical crystal is pulled through a round wire-drawing die. Calculate the *M* value and the active slip systems for a given crystal orientation.



$$a_{11}'' : a_{12}'' : a_{13}'' = h : k : l = \cos\phi\cos\theta : \cos\phi\sin\theta : \sin\phi$$

Principle of Maximum Work

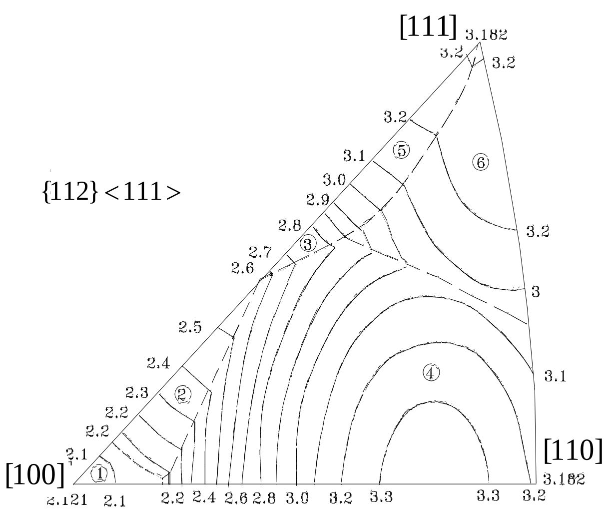
$$[d\varepsilon^c] = \begin{bmatrix} \cos\phi\cos\theta & -\sin\theta & -\sin\phi\cos\phi \\ \cos\phi\sin\theta & \cos\theta & -\sin\phi\sin\theta \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} d\varepsilon & 0 & 0 \\ 0 & -d\varepsilon/2 & 0 \\ 0 & 0 & -d\varepsilon/2 \end{bmatrix}$$

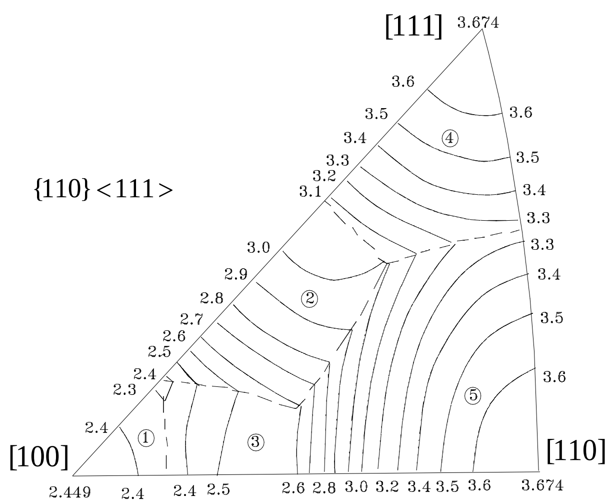
$$\begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta & \sin\phi \\ -\sin\theta & \cos\theta & 0 \\ -\sin\phi\cos\phi & -\sin\phi\sin\theta & \cos\phi \end{bmatrix}$$

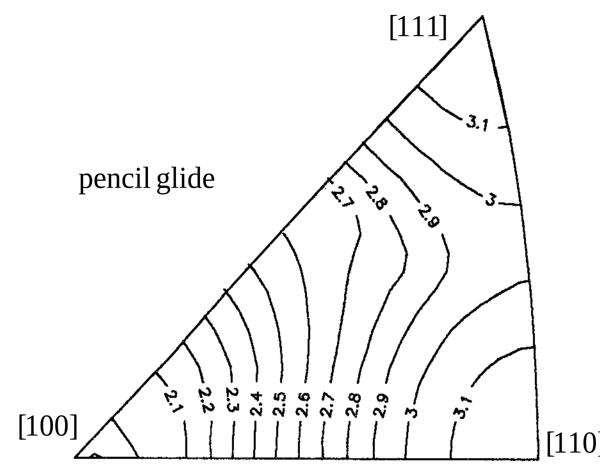
$$[d\varepsilon^c] = (1/2)d\varepsilon \begin{bmatrix} 3\cos^2\phi\cos^2\theta - 1 & 3\cos^2\phi\sin\theta\cos\theta & 3\sin\phi\cos\phi\cos\theta \\ 3\cos^2\phi\sin\theta\cos\theta & 3\cos^2\phi\sin^2\theta - 1 & 3\sin\phi\cos\phi\sin\theta \\ 3\sin\phi\cos\phi\cos\theta & 3\sin\phi\cos\phi\sin\theta & 3\sin^2\phi - 1 \end{bmatrix}$$

Principle of Maximum Work

Stereographic plot of parameter *M* as a function of direction of axisymmetric deformation for {110}<111> and {112}<111> slip systems [Chin, Mammel, 1967] and pencil glide [Kwon, 1995] in cubic crystals.







Rigid Body Rotation during Slip

$$e_{ij}^{c}=\sum_{k=1}^{k}(d\gamma b_i^{(k)}n_j^{(k)}) \qquad d\omega_{ij}^{c}=(1/2)\sum_{k=1}^{k}[d\gamma^{(k)}(b_i^{(k)}n_j^{(k)}-b_j^{(k)}n_i^{(k)})]$$

$$|d\omega|=[(d\omega_{12})^2+(d\omega_{23})^2+(d\omega_{31})^2]^{1/2}$$

$$r_1=d\omega_{23}/|\mathrm{d}\omega|,\ \ r_2=d\omega_{31}/|\mathrm{d}\omega|,\ \ r_3=d\omega_{12}/|\mathrm{d}\omega|$$

When the orientation T ([hkl] or [uvw]) is rotated to T'([$h'k'l'$] or [$u'v'w'$]) through angle θ, the relation between T and T' can be obtained like this.

$$\begin{bmatrix} h' \\ k' \\ l' \end{bmatrix}=\begin{bmatrix} (1-r_1^2)\cos\theta+r_1^2 & r_1r_2(1-\cos\theta)-r_3\sin\omega & r_1r_3(1-\cos\theta)+r_2\sin\theta \\ r_1r_2(1-\cos\theta)+r_3\sin\theta & (1-r_2^2)\cos\theta+r_2^2 & r_2r_3(1-\cos\theta)-r_1\sin\theta \\ r_1r_3(1-\cos\theta)-r_2\sin\theta & r_2r_3(1-\cos\theta)+r_1\sin\theta & (1-r_3^2)\cos\theta+r_3^2 \end{bmatrix}\begin{bmatrix} h \\ k \\ l \end{bmatrix}$$