Chapter 17.

MOSFETs - An Introduction

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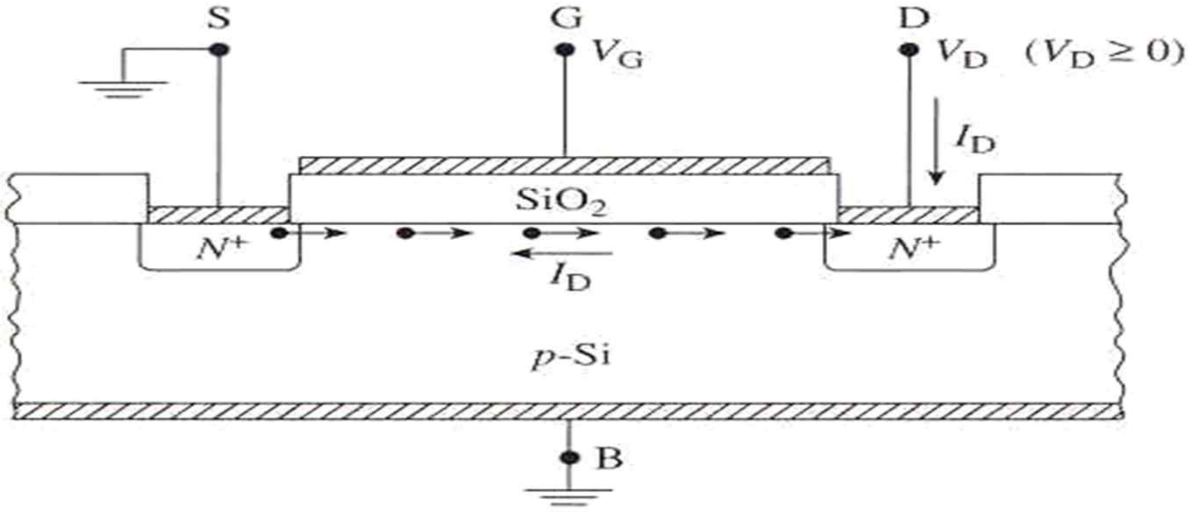
CONTENTS

- **Qualitative Theory of Operation**
- **Quantitative I_D - V_D Relationships**
- **Subthreshold Swing**
- **ac Response**

Qualitative Theory of Operation

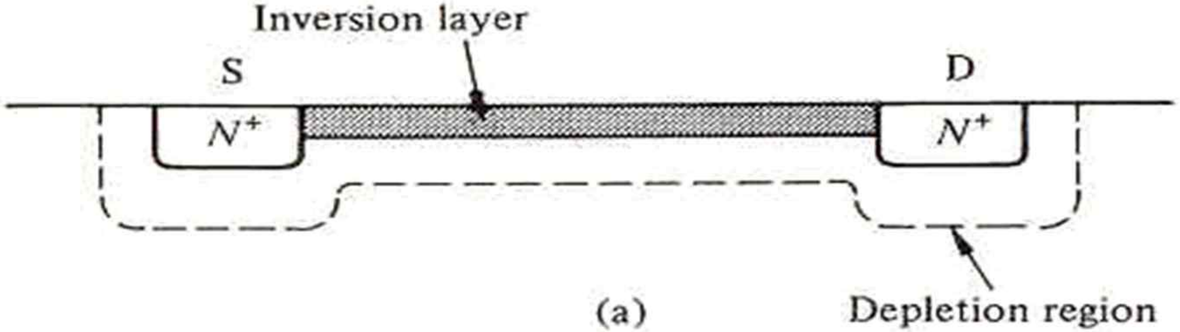
- **Assumption**
  - **Ideal Structure**
  - **Long Channel Enhancement-Mode**
  - **MOSFET=MOS-Capacitor + 2 pn junctions**
  - **n - channel (p-type substrate)**



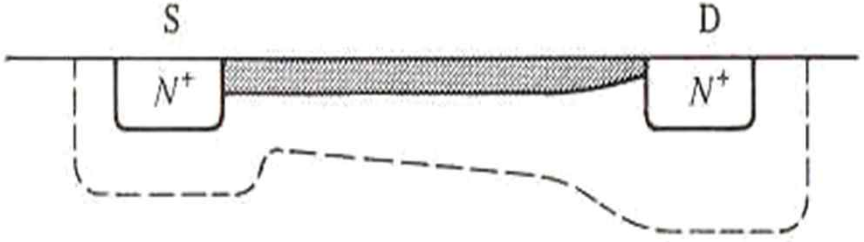
(b)

$V_D = 0$ Case

- **When $V_G \leq V_T$, very few electrons in the channel. an open circuit between the n+ region**
- **When $V_G > V_T$,**
  - **Inversion layer is formed**
  - **The conducting channel (induced "n-type" region, inversion layer) connects the D & S**
  - **V_G↑ the pile up of electrons↑ conductance↑**
  - **∴ V_G determines the maximum conductance**
  - **Thermal equilibrium prevails, and I_D=0**



- **The V_D is increased in small steps starting from $V_D = 0$**
  - **The channel acts like a simple resistor**
  - **$I_D \propto V_D$**
  - **The reverse bias junction current is negligible**
  - **voltage drop from the drain to the source starts to negate the inverting effect of the gate**
  - **V_D↑ Depletion of the channel↑ # of carriers↓ conductance↓ slope-over in the *IV***



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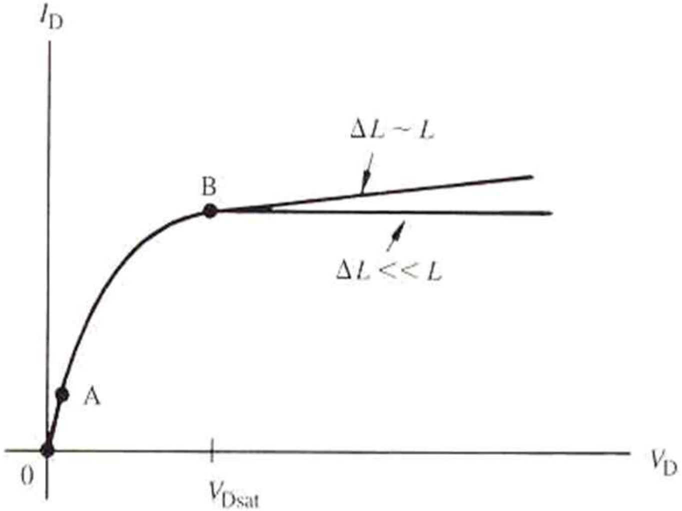
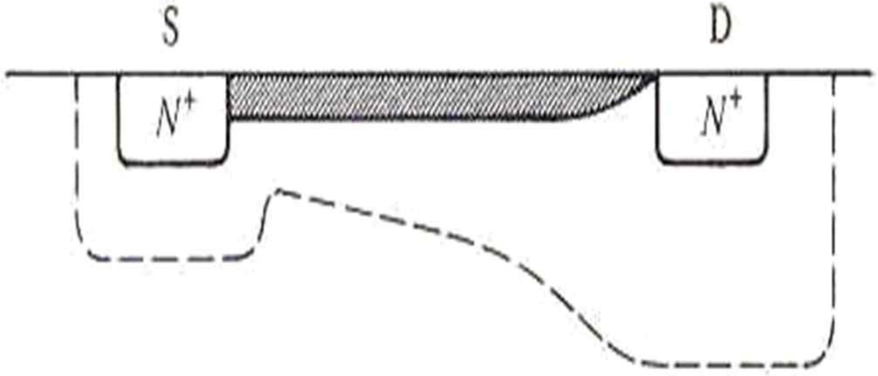
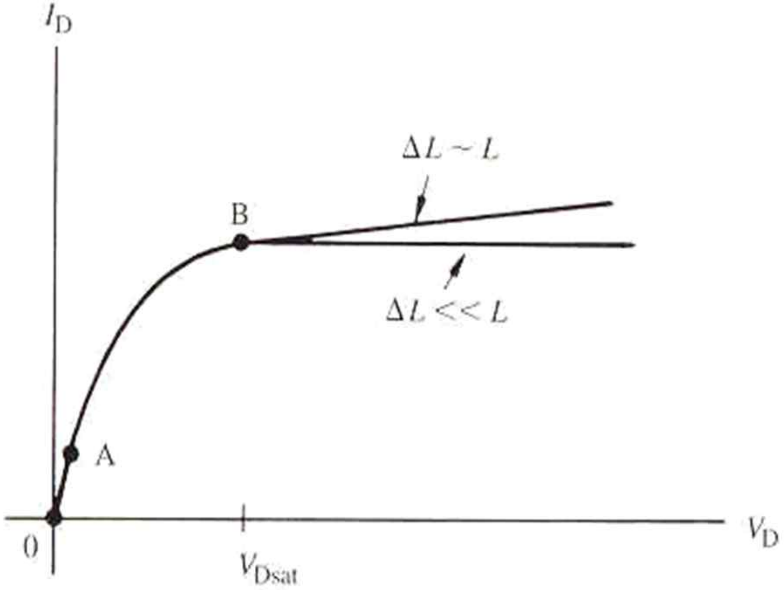
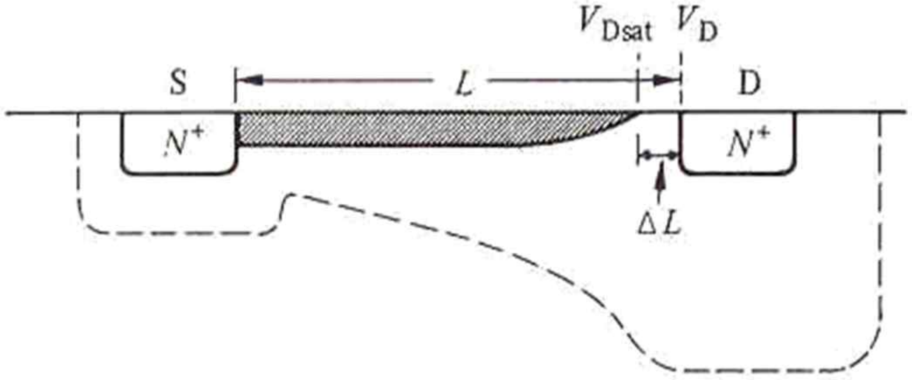


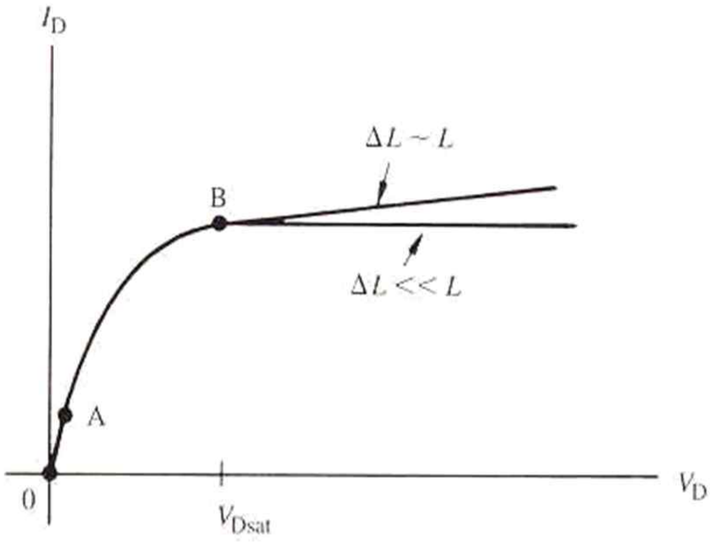
- **Pinch - off ($V_D = V_{Dsat}$)**
  - **Disappearance of the channel adjacent to the drain**
    - **The slope of the I_D-V_D becomes approximately zero(Point B)**





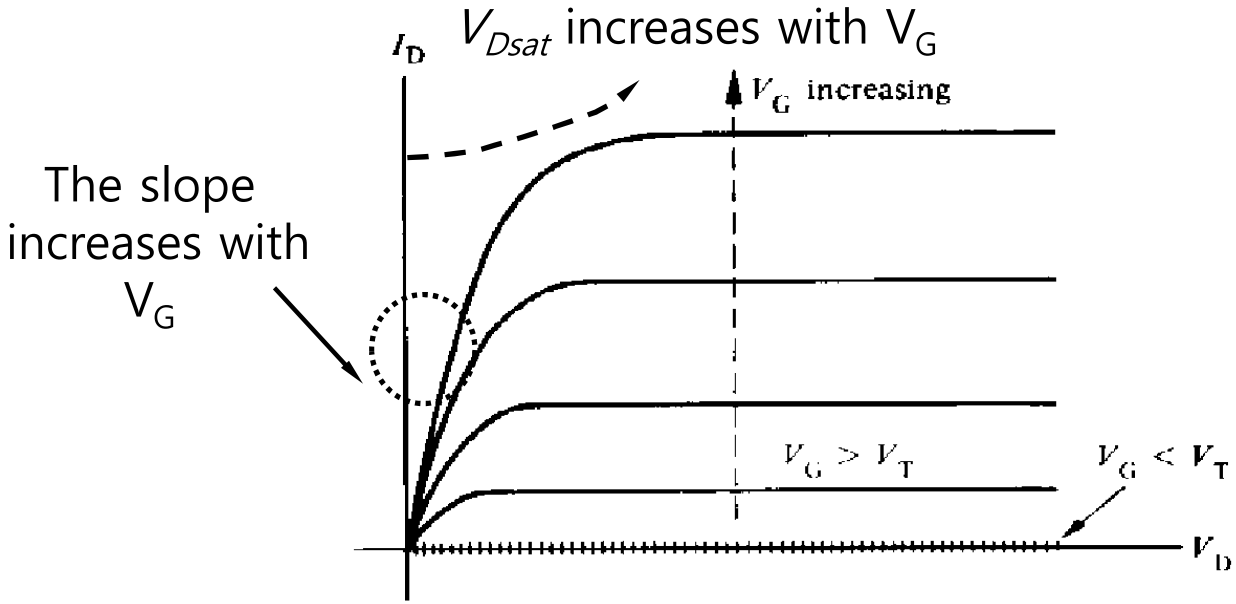
- **Post-pinch-off ($V_D > V_{Dsat}$)**
  - **The pinched-off portion widens from just a point into a depleted channel section ΔL**
  - **The pinched-off section absorbs most of the voltage drop in excess of V_{Dsat}**
  - **For $\Delta L << L$, the shape of the conducting region and the potential across the region do not change Constant I_D**
  - **For $\Delta L \sim L$, I_D will increase with $V_D > V_{Dsat}$**





I_D - V_D Characteristics

- **General form of the I_D - V_D ($\Delta L << L$)**
  - **For $V_G \le V_T, I_D \approx 0$**
  - **For $V_G > V_T$, transistor action: I_D is modulated by V_G**
  - **$V_D > V_{Dsat}$: saturation region**
  - **$V_D < V_{Dsat}$: linear(triode) region**



V_{Dsat} increases with V_G

The slope increases with V_G

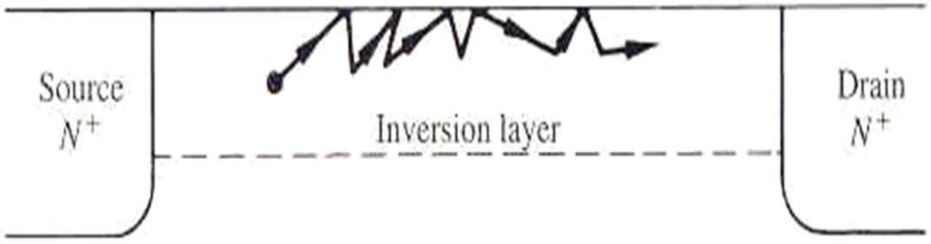
Quantitative I_D - V_D Relationships

- **Effective Mobility**

: Impurity Scattering

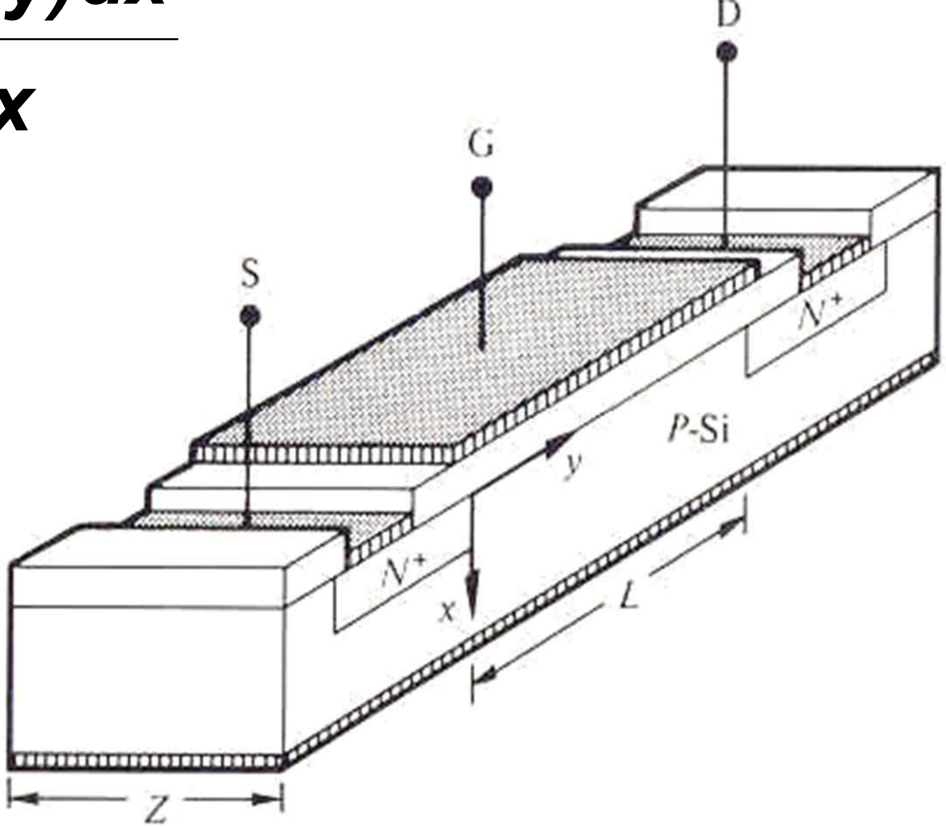
+ Lattice Scattering

+ Surface Scattering

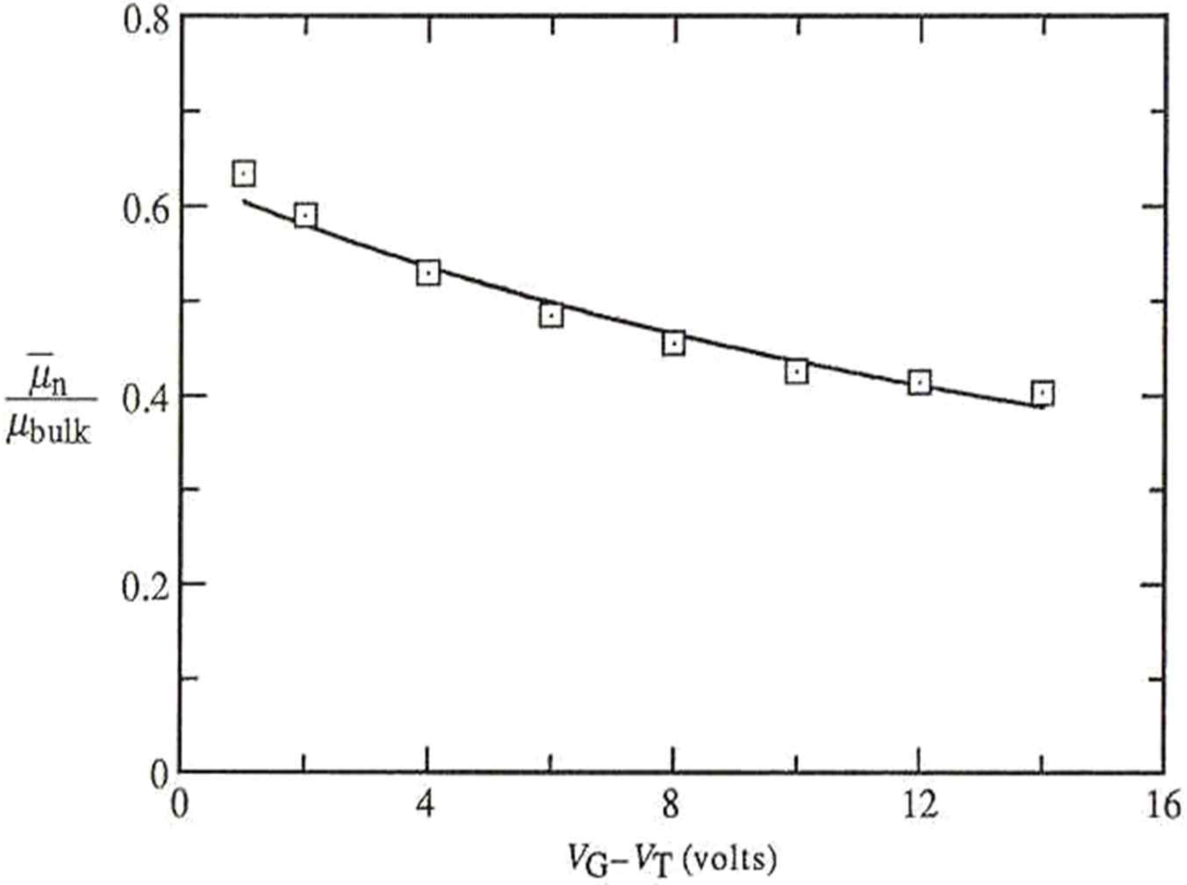


- **Effective Mobility**

$$\bar{\mu}_n = \frac{\int_0^{x_c(y)} \mu_n(x,y)n(x,y)dx}{\int_0^{x_c(y)} n(x,y)dx}$$



—$V_G \uparrow \rightarrow \left\{ \begin{array}{l} \text{more carriers closer to the interface higher} \\ \text{electric field} \end{array} \right\} \begin{array}{l} \rightarrow \text{surface scattering} \uparrow \\ \rightarrow \bar{\mu}_n \downarrow \end{array}$



- **Square - Law Theory**
  - $V_D < V_{Dsat}$
    - A drift current is dominant

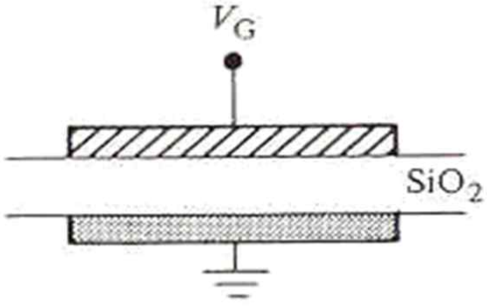
$$Q_{gate} = -Q_{semi} \cong -Q_N$$

$$Q_N \cong -C_o(V_G - V_T)$$

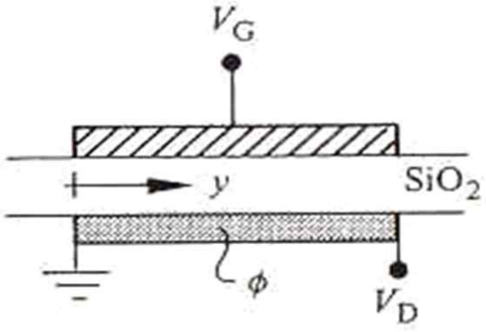
At an arbitrary point y

$$Q_N(y) \cong -C_o(V_G - V_T - \phi) \rightarrow I_D = -WQ_N(y)\bar{\mu}_n E(y)$$

$$= W\bar{\mu}_n C_o(V_G - V_T - \phi)\frac{d\phi}{dy}$$



MOS-C



MOSFET

Integrating

$$\int_0^L I_D dy = W\bar{\mu}_n C_o \int_0^{V_D} (V_G - V_T - \phi) d\phi$$

$$\rightarrow I_D = \frac{W\bar{\mu}_n C_o}{L}\left[(V_G - V_T)V_D - \frac{V_D{}^2}{2}\right]$$

- $V_D \geq V_{Dsat}$
  - **I_D is approximately constant**

$$I_{Dsat} = \frac{W\bar{\mu}_n C_o}{L}\left[(V_G - V_T)V_{Dsat} - \frac{V_{Dsat}{}^2}{2}\right]$$

$$Q_N(L) = -C_o(V_G - V_T - V_{Dsat}) = 0$$

$$\boxed{V_{Dsat} = V_G - V_T}$$

$$\boxed{I_{Dsat}} = \frac{W\bar{\mu}_n C_o}{2L}\left(V_G - V_T\right)^2$$

• Subthreshold transfer characteristics

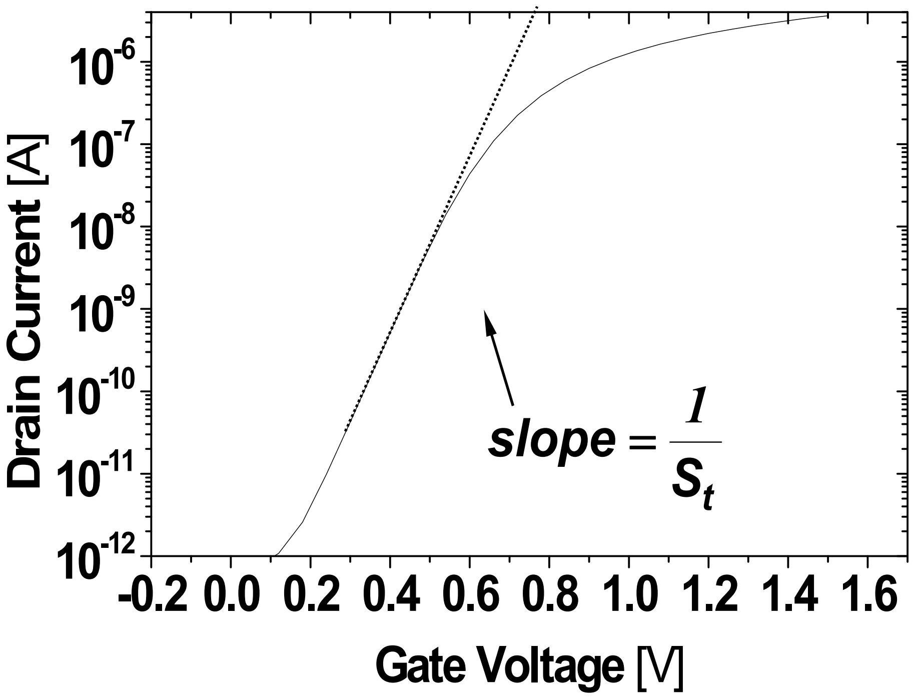
– **At weak inversion, diffusion current dominates**

$$I \approx I_{diff} \propto \frac{Q_S\text{-}Q_D}{L} \propto \frac{Q_S}{L}\left[1-\exp(-\frac{qV_D}{kT})\right]$$

where $$Q_D = Q_S \exp\left(-\frac{qV_D}{kT}\right)$$

where $\boldsymbol{Q_S}$ is exponential function of $\boldsymbol{V_G}$

– **In long channel MOSFETS the subthreshold current varies exponentially with V_G and is independent of V_D provided V_D > a few kT/q**



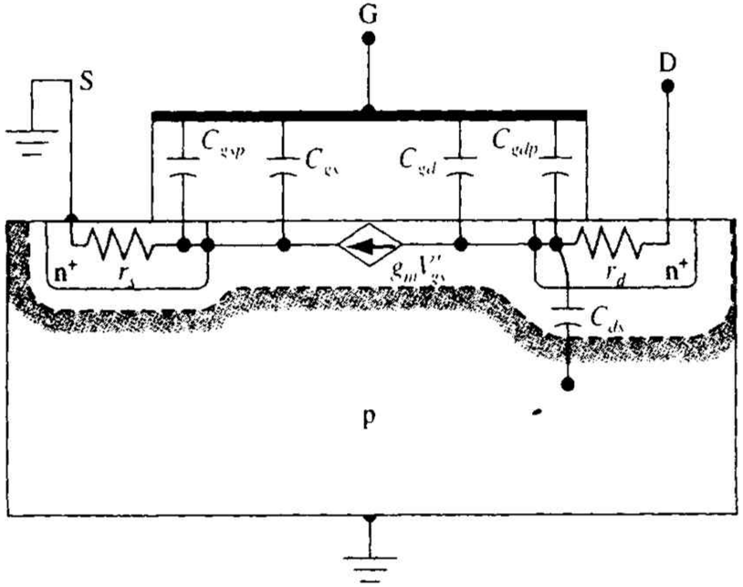
Subthreshold swing

$$S_t = \frac{1}{\frac{d \log I_D}{dV_G}} \ [mV/dec]$$

$= $ change in V_G

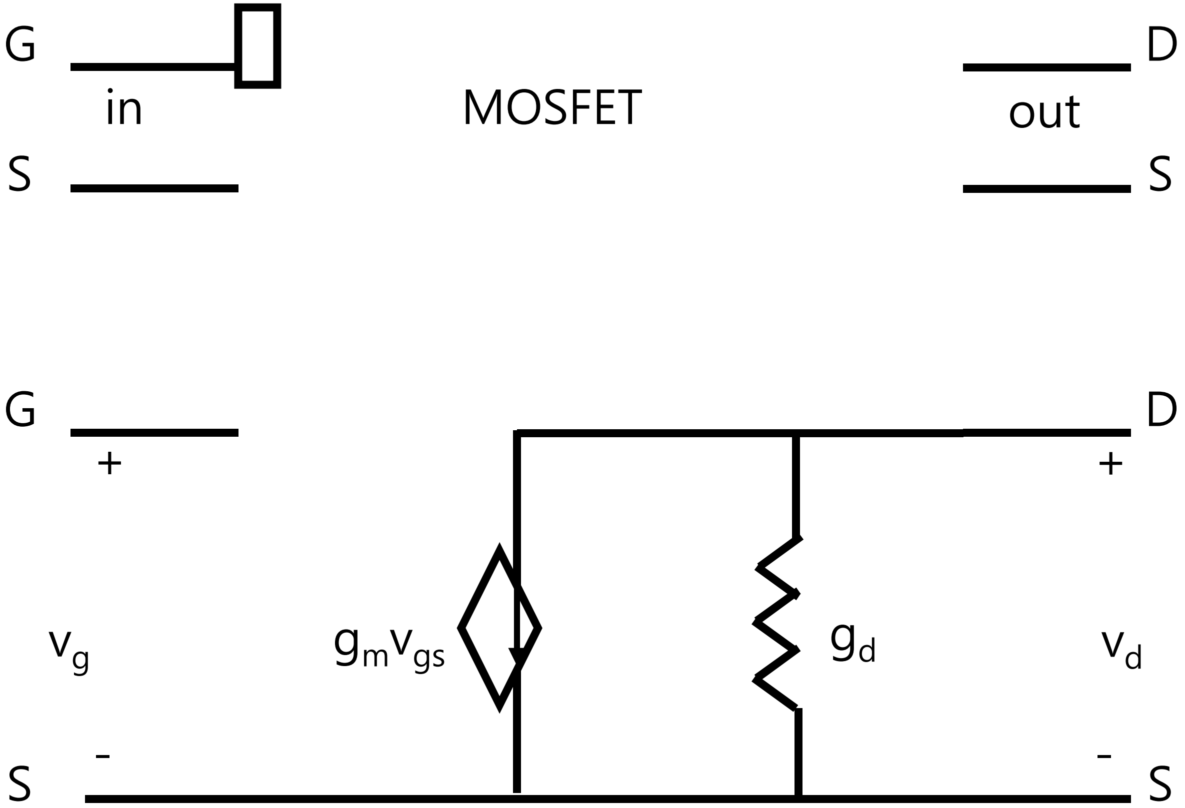
for a decade change in I_D

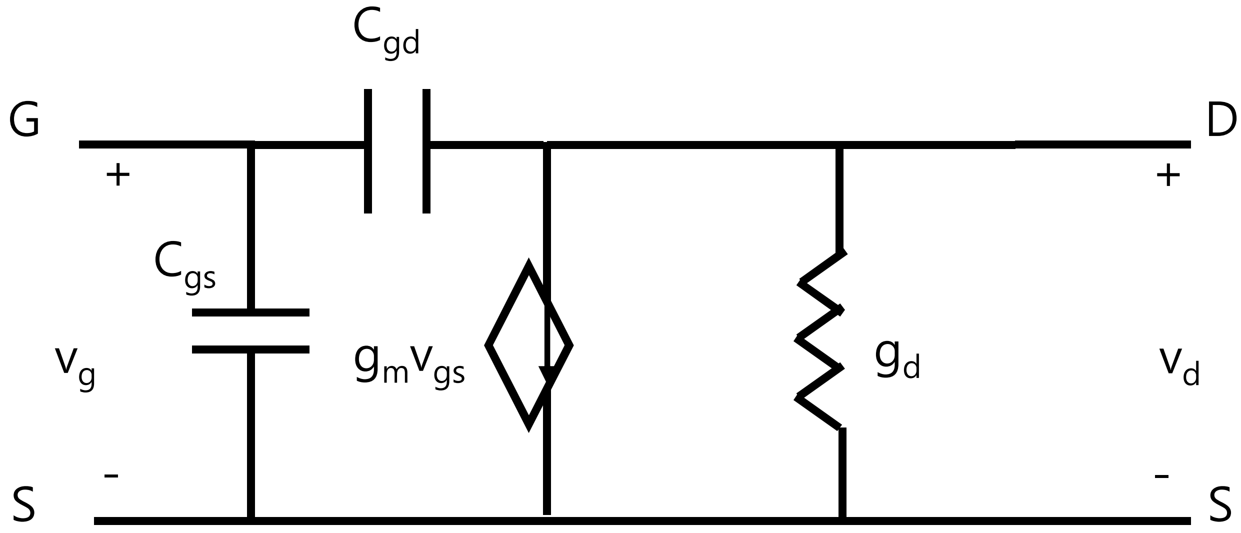
17.3 a.c. Response(skip)



- C_{gs}(C_{gd}): The interaction between G and the channel charge near S(D)
- C_{gsp},C_{gdp}: parasitic or overlap capacitances

- **Small-signal equivalent circuit**





- A capacitor behaves like an open circuit at low frequencies.

- $I_D(V_D,V_G) + i_d = I_D(V_D + v_d , V_G + v_g)$
- $i_d = I_D(V_D + v_d , V_G + v_g) - I_D(V_D,V_G)$

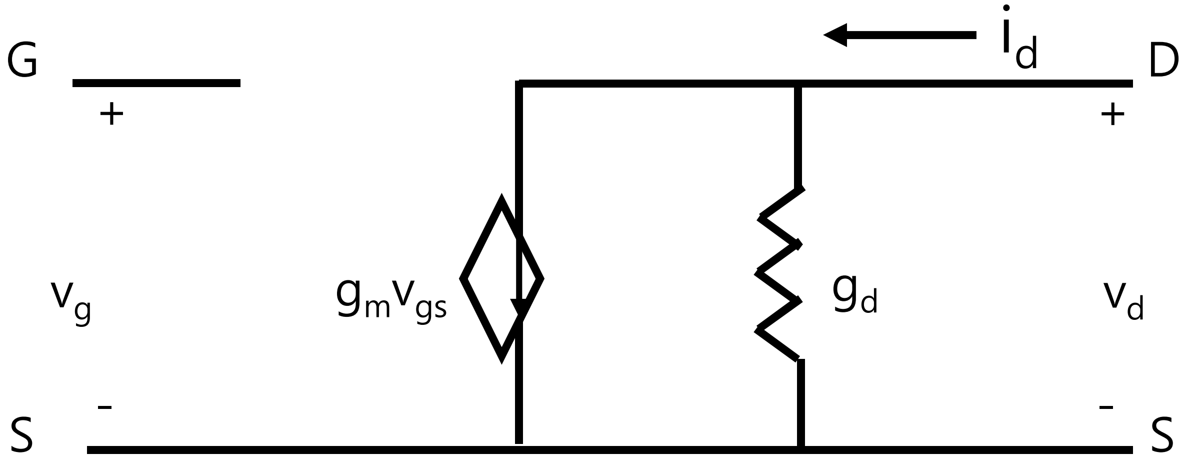
$$I_D\left(V_D + v_d, V_G + v_g\right) = I_D\left(V_D, V_G\right) + \left.\frac{\partial I_D}{\partial V_D}\right|_{V_G} v_d + \left.\frac{\partial I_D}{\partial V_G}\right|_{V_D} v_g$$

$$\therefore i_d = \left.\frac{\partial I_D}{\partial V_D}\right|_{V_G} v_d + \left.\frac{\partial I_D}{\partial V_G}\right|_{V_D} v_g$$

$$g_d = \left.\frac{\partial I_D}{\partial V_D}\right|_{V_G = \text{constant}} \quad \cdots \textit{the drain or channel conductance}$$

$$g_m = \left.\frac{\partial I_D}{\partial V_G}\right|_{V_D = \text{constant}} \quad \cdots \textit{transconductance or mutual conductance}$$

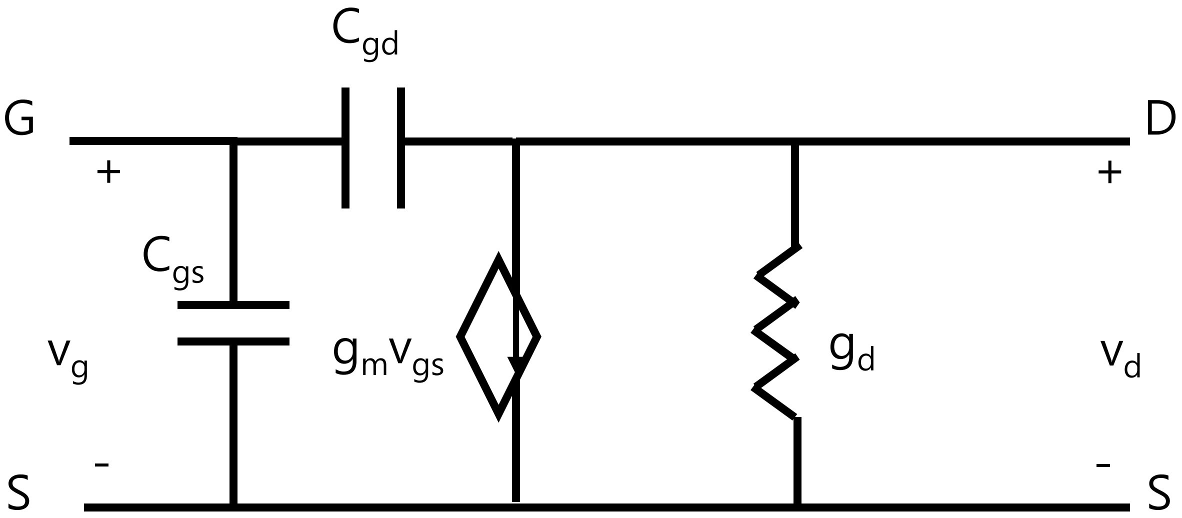
$$\therefore i_d = g_d v_d + g_m v_g$$



- Table 17.1

Below pinch-off ($V_D \le V_{Dsat}$)	Above pinch-off ($V_D > V_{Dsat}$)
$g_d = \frac{Z\overline{\mu_n}C_O}{L}(V_G - V_T - V_D)$	$g_d = 0$
$g_m = \frac{Z\overline{\mu_n}C_O}{L}V_D$	$g_m = \frac{Z\overline{\mu_n}C_O}{L}(V_G - V_T)$

- At the higher operational frequencies encountered in practical applications, the circuit must be modified to take into account capacitive coupling between the device terminals.
- The overlap capacitance is minimized by forming a thicker oxide in the overlap region or preferably through the use of self aligned gate procedures. ➔ C_{gd} is typically negligible.



• 17.3.2. Cutoff Frequency

The f_T be defined as the frequency where the MOSFET is no longer amplifying the input signal under optimum conditions.
→ Value of the output current to input current ratio is unity

$$i_{in} = jw(C_{gs} + C_{gd})v_g \cong j(2\pi f)C_o v_g$$

$$i_{out} \approx g_m v_g$$

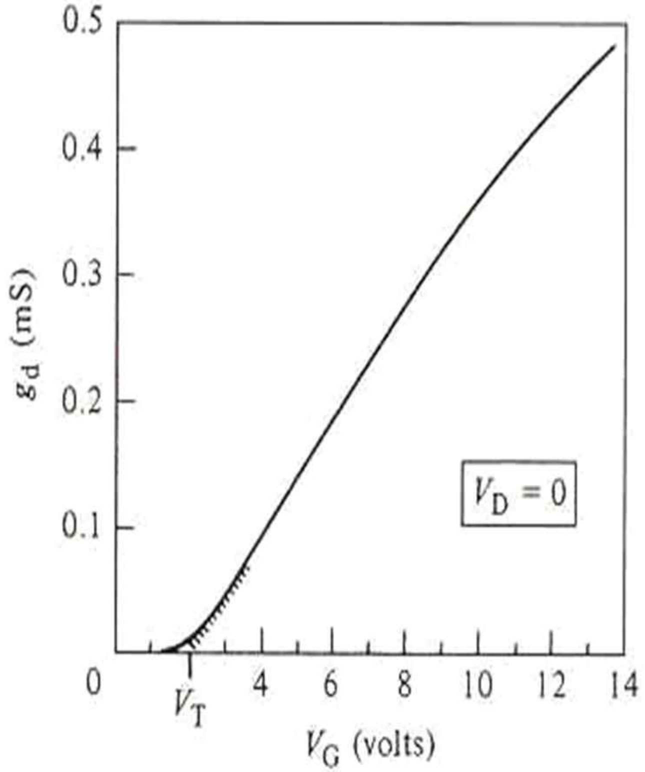
$$\left| i_{out} / i_{in} \right| = 1$$

$$\therefore f_T = \frac{g_m}{2\pi C_o} = \frac{\overline{\mu_n} V_D}{2\pi L^2} \qquad if\ V_D \leq V_{Dsat}$$

Small Signal Characteristics

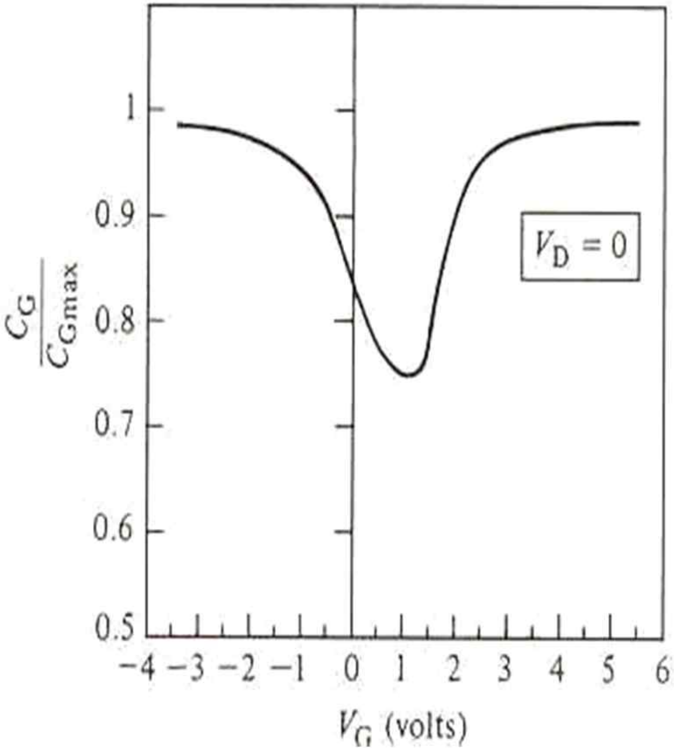
- g_d vs. $V_G(V_D=0)$
  - Extrapolating the linear portion of the g_d-V_G characteristics into the V_G axis and equating the voltage intercept to V_T
  - Deduce the effective mobility from the slope

$$g_d = \frac{W\bar{\mu}_n C_o}{L}(V_G - V_T) \quad (V_D=0)$$



• Gate Capacitance vs. V_G (V_D=0)

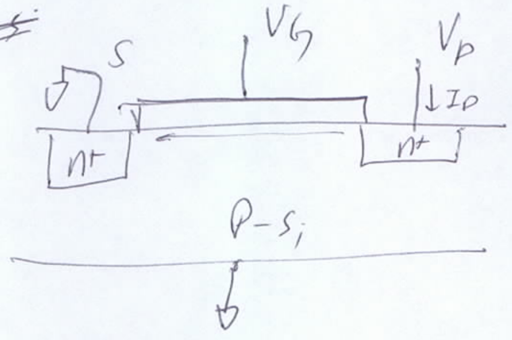
- **Diagnostic purposes in much the same manner as the MOS-C *C-V_G* characteristics**
- **Unlike the MOS-C , a low-frequency characteristics is observed even for frequencies exceeding 1MHz**
  - **Because the source and drain islands supply the minority carries**



ch17 11/18/10 수요일. 아빠가바 1

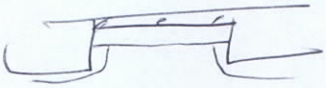
MOSFET

~~As~~



$V_G < V_T$ ~~[illegible]~~

(inversion layer)
$V_G > V_T$ inversion, conducty chanl.

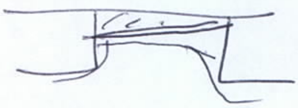


$V_G \uparrow$ → Conductance ↑

simple Resistn ~~up to certain point.~~

But as $V_D \uparrow$ V_D쪽의 chanl은 $(V_G - \phi)$만큼의 gate 영향을 덜게 된다.

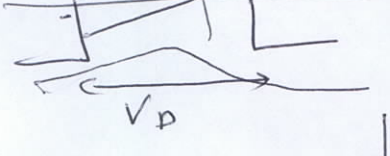
→ charge가 줄고 → chanl conductn ↓
(Slope over



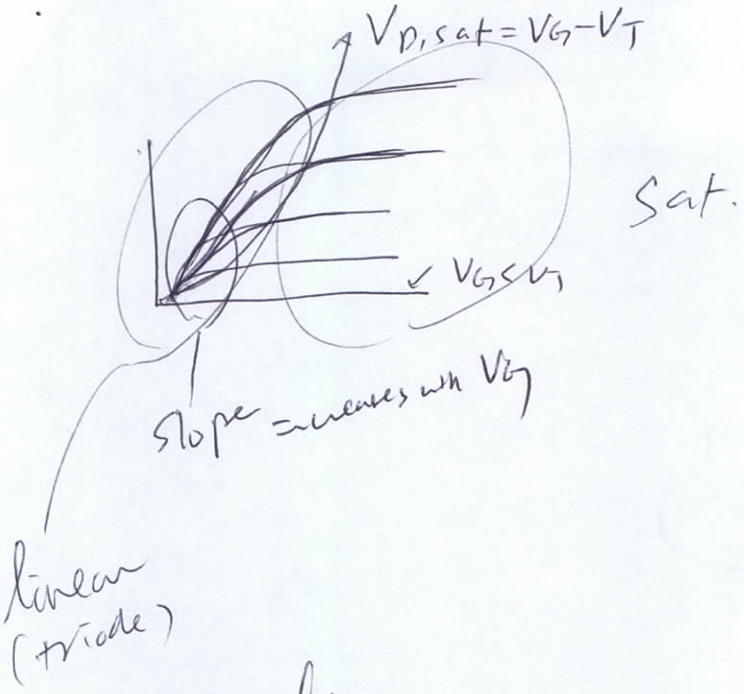
pinch off (of chanl) where $V_G - V_{D,sat} = V_T$ $V_{D,sat} = V_G - V_T$

post pinchoff $V_D > V_{Dsat}$

The pinched-off rogin widens & absorb $V_D - V_{D,sat}$
So the channel sees only $V_{D,sat}$



I_D saturates



수식으로, Quantitatively.

Square-Law Theory => 기본

$V_D < V_{D\,sat}$

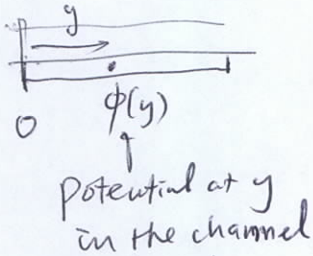
Drift current, ~~$Q_{gate} = -Q_N$~~

channel charge at arbitrary port y $Q_N(y) = -C_o(V_G - V_T - \phi(y))$

effective mobility

$$\rightarrow I_D(y) = -W Q_N(y) \cdot \bar{\mu}_n E(y)$$

$$= W \bar{\mu}_n C_o (V_G - V_T - \phi) \frac{d\phi}{dy}$$



potential at y in the channel

$0 \le \phi \le V_D$

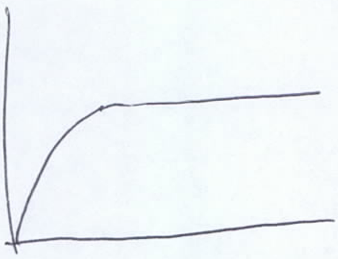
$$\int_0^L I_D(y) dy = W \bar{\mu}_n C_o \int_0^{V_D} (V_G - V_T - \phi) d\phi$$

$$\therefore I_D = \frac{W \bar{\mu}_n C_o}{L} \left[(V_G - V_T) V_D - \frac{V_D^2}{2} \right]$$

$V_D \geq V_{D,sat}$.

$$I_{D,sat} = I_D \Big|_{V_{D,sat} = V_G - V_T}$$

$$= \frac{W \mu_n C_o}{2L} (V_G - V_T)^2$$



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when $\Delta L \ll L$, Long channel

$\Delta L \sim L$, Short channel

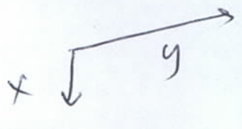
the reduced L increases current

short channel Effect !

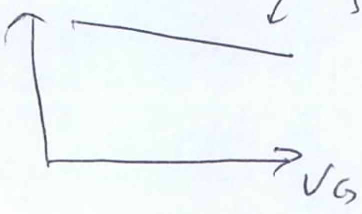
$\bar{\mu}$ due to surface scattering

in addition to lattice & impurity scattering

$$\bar{\mu} = \frac{\int_0^{x(y)} \mu_n(x,y)\, n(x,y)\, dx}{\int_0^{x(y)} n(x,y)\, dx}$$



V_G ↑
More carriers at surface.
due to surface scattering.



Modification

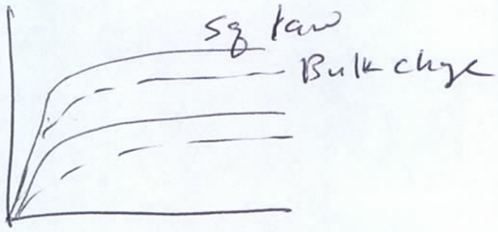
Is $Q_G = -Q_N$?
Not exactly.

① Bulk charge theory

Some gate vtg is used to widen the depletion width & there is going to be some charge there ~~to~~ to go into the eq

$$Q_{gate} = -(Q_N + Q_W)$$

So actual current will be less



② Subthreshold current

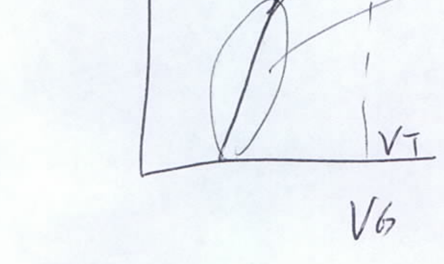
there is still current when $V_G < V_T$ ~~&~~ This is diffusion current.

weak inversion

The charge is $Q_S \propto \exp\left(\frac{qV_G}{nkT}\right)$

$$Q_D = Q_S \exp\left(-\frac{qV_{DS}}{kT}\right)$$

$$I_{Diff} \propto \frac{Q_S - Q_D}{L} \propto \frac{Q_S}{L}\left[1 - \exp\left(-\frac{qV_D}{kT}\right)\right]$$



$$\text{slope} = \frac{1}{S_t} = \frac{d \log I_D}{d V_G}$$

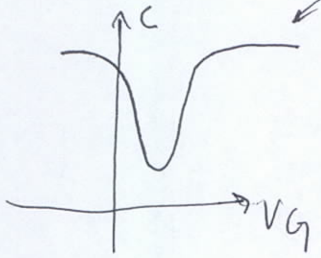
$I_{Diff} \propto Q_S$

$I_{Diff} \propto \left(\exp \frac{qV_G}{nkT}\right)$

$\ln I_{Diff} \propto V_G$

11/23/10. ch 17 계속

MOSFET C-V.



MOS-C의
Lo freq curve 처럼 MOSFET인 경우
high freq 에서도 Cox 가 됨
inversion cap은
∵ source supplies carriers

< small signal ~~equivalent~~ ckt >

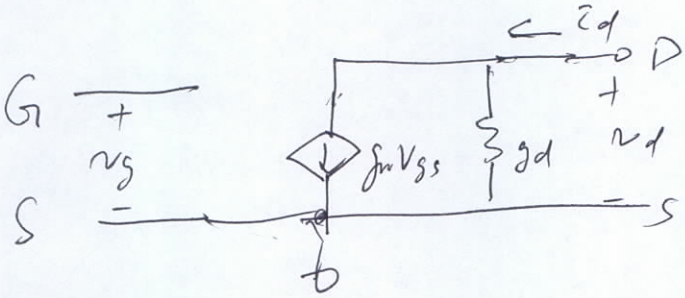
~~I_D~~ = $i_d(v_d, v_g)$

$$i_d = \underbrace{\left.\frac{\partial I_D}{\partial V_D}\right|_{V_G}}_{g_d} v_d + \underbrace{\left.\frac{\partial I_D}{\partial V_G}\right|_{V_D}}_{g_m} v_g \equiv g_d \cdot v_d + g_m v_g$$

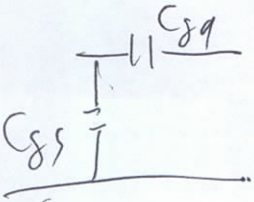
(V_D, V_G) Bias point 에서의 값

gd (channel conductance)

gm (transconductance)



Then add cap



→ small, But Miller cap

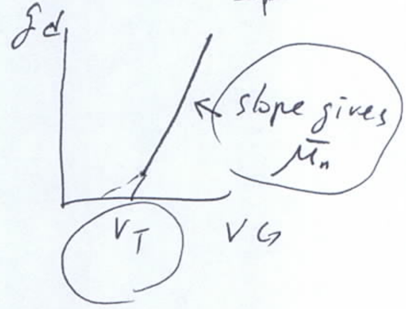
↑ usually larger

~~Linea~~ Triode Region

$$I_D = \frac{W \bar{\mu}_n C_o}{L}\left[(V_G - V_T)V_D - \frac{V_D^2}{2}\right]$$

$$g_d = \frac{\partial I_D}{\partial V_D}\Big|_{V_G} = \frac{W \bar{\mu}_n C_o}{L}\left[V_G - V_T - V_D\right] \longrightarrow$$

at $V_D = 0$



$$g_m = \frac{\partial I_D}{\partial V_G}\Big|_{V_D} = \frac{W \bar{\mu}_n C_o}{L}\left[V_D\right]$$

Sat. Region

$$I_D = \frac{W \bar{\mu}_n C_o}{2L}(V_G - V_T)^2$$

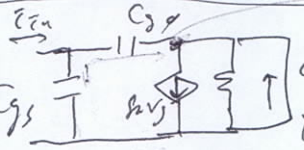
$$g_d = 0$$

$$g_m = \frac{W \bar{\mu}_n C_o}{L}(V_G - V_T)$$

Cut off freq

defined as max. freq where $\left|\frac{i_{out}}{i_{in}}\right| = 1$ with output short circuited.

$I_{in} \neq g_m v_g - i_{out}$ $g_m v_g >> I_{in}$ $\rightarrow g_m v_g = i_{out}$



← "output short ckted"

$$i_{in} = j\omega(\underbrace{C_{gs}}_{\sim C_o} + \underbrace{C_{gd}}_{small})v_g \simeq j(2\pi f)C_o v_g$$

$$i_{out} \simeq g_m v_g$$

$\left|\frac{i_{out}}{i_{in}}\right| = 1$ 이되는 freq?

$\therefore f_T = \boxed{\frac{g_m}{2\pi C_o}} =$ ~~$\frac{\mu_n V_D}{2\pi L^2}$ if $V_D \le V_{Dsat}$~~.

$$\frac{g_m v_g}{2\pi f C_o v_g} = 1 \rightarrow f = \frac{g_m}{2\pi C_o}$$

g_m ← transconductance ; → g_m/mm 가 중요!

$2\pi C_o$ → [illegible]..