Chapter 7 2nd law analysis for a Control Volume

As was done with the first law analysis, we now consider more general application of entropy.

Recall we had

$$dS = \frac{\delta Q}{T} + \frac{\delta LW}{T}$$

During a time interval δt

$$\frac{S_2 - S_1}{\delta t} = \frac{1}{\delta t}\left(\frac{\delta Q}{T}\right) + \frac{1}{\delta t}\left(\frac{\delta LW}{T}\right) \qquad \text{(i)}$$

where

$$S_2 - S_1 = (S_{t+\delta t} - S_t) + \underbrace{s_e \delta m_e - s_i \delta m_i}_{net\ flow\ of\ entropy\ out\ of\ c\forall}$$

Then we may rewrite (i) as

$$\frac{S_{t+\delta t} - S_t}{\delta t} + \frac{s_e \delta m_e}{\delta t} - \frac{s_i \delta m_i}{\delta t} = \frac{1}{\delta t}\sum_{cv}\left(\frac{\delta Q}{T}\right)_{c.v.} + \frac{1}{\delta t}\sum_{cv}\left(\frac{\delta LW}{T}\right)_{c.v.}$$

In terms of a rate equation

$$\frac{dS_{cv}}{dt} + \sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \sum_{cv} \frac{\dot{Q}_{cv}}{T} + \sum_{cv} \frac{L\dot{W}_{cv}}{T}$$

Or using inequality

$$\frac{dS_{cv}}{dt} + \sum \dot{m}_e s_e - \sum \dot{m}_i s_i \geq \sum_{cv} \frac{\dot{Q}_{cv}}{T}$$

For steady state, we know that there is no change with time of the entropy per unit mass at any point within the control volume,

$$\frac{dS_{cv}}{dt} = 0 \qquad \text{(between state 1 \& 2)}$$

So that the second law becomes

$$\sum \dot{m}_e s_e - \sum \dot{m}_i s_i \geq \sum_{cv} \frac{\dot{Q}_{cv}}{T}$$

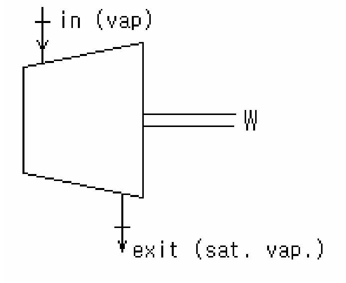
If there is one exit, one inlet, then

$$\dot{m}_e s_e - \dot{m}_i s_i \geq \sum_{cv} \frac{\dot{Q}_{cv}}{T}$$

- **Example-1 (a steady state turbine)**

  Steam enters a steam turbine at a pressure of 1MPa, T=300°C, velocity of 50m/s. The steam leaves the turbine at a pressure of 150kPa and a velocity of 200m/s. Determine the work per kilogram of steam flowing through the turbine, assuming the process to be reversible and adiabatic.

$$m = 1kg$$
$$p_i = 1MPa$$
$$T_i = 300°C$$
$$V_i = 50m/s$$
$$p_e = 150kPa$$
$$V_e = 200m/s$$



Continuity

$$\frac{\cancel{dm_{cv}}}{\cancel{dt}} + \dot{m}_e - \dot{m}_i = 0$$

$$\dot{m}_e = \dot{m}_i = \dot{m}$$

Energy (1st Law)

$$\dot{Q}_{cv} + \dot{m}(h_i + \frac{V_i^2}{2}) = \dot{m}(h_e + \frac{V_e^2}{2}) + \frac{\cancel{d}}{\cancel{dt}}[\ ] + \dot{m}w_{cv}$$

$$h_i + \frac{V_i^2}{2} = h_e + \frac{V_e^2}{2} + w_{cv}$$

Entropy (2nd Law)

$$\underbrace{\frac{dS_{cv}}{dt}}_{S_e = S_i} + \dot{m}s_e - \dot{m}s_i = \sum_{cv} \frac{\cancel{\dot{Q}_{cv}}}{\cancel{T}} + \sum_{cv} \frac{\cancel{L\dot{W}_{cv}}}{\cancel{T}}$$

From the steam table, (superheated vapor!)

$@\ p_i = 1MPa, T_i = 300°C$

$$h_i = 3051.2\,{}^{kJ}\!/\!_{kg},\ s_i = 7.1229\,{}^{kJ}\!/\!_{kg.K}$$

$@\ p_e = 150kPa$

$s_e = s_i = 7.1229$ (saturated steam exits the turbine)

$$s_e = (1-x)s_f + xs_g$$

$$7.1229 = (1-x)1.4336 + x\,7.2233$$

$$5.6893 = 5.7897x$$

$$x = 0.98265$$

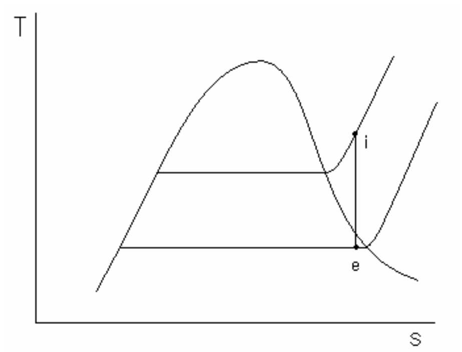
$$\therefore h_e = (1-x)h_f + xh_g$$

$$= 2655.0\,{}^{kJ}\!/\!_{kg}$$

Plot the T-s diagram!

∴The work per kilogram of steam for this isentropic process is (from 1st law)

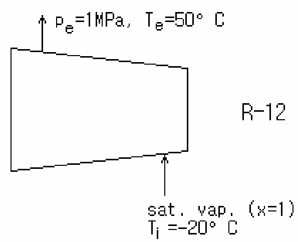
$$w_{cv} = 377.5\,{}^{kJ}\!/\!_{kg}$$

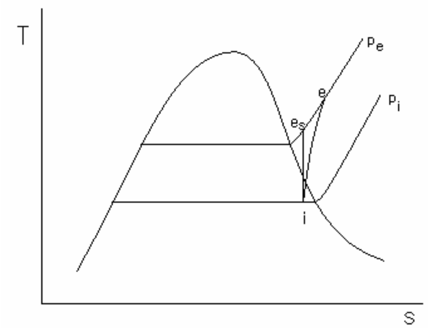


Steady State Turbine

- Example-2

  An inventor reports that he has a refrigeration compressor that receives saturated R-12 vapor at −20°C and delivers the vapor at 1Mpa, 50°C. The compression process is adiabatic. Does the process described violate the second law? (a Steady State process)





Q: 그림과 일치하나?
(entropy increase?)

Steady state

$$\dot{m}(s_e - s_i) \geq \frac{\overbrace{\dot{Q}_{cv}}^{0}}{T}$$

That is, evaluate $S_e - S_i > 0$

$T_i = -20°C,\ x = 1,\ s_i = 0.7082$ (Sat Vapor)

$T_e = 50°C,\ p_e = 1MPa,\ s_e = 0.7021$ (Super heat)

$\therefore S_e < S_i$ 2nd Law is violated!!!

- **The Reversible Steady-State, Steady Flow Process**

We consider the 1st law,

$$q + h_i + \frac{V_i^2}{2} + gz_i = h_e + \frac{V_e^2}{2} + gz_e + w \qquad \text{(iii)}$$

and the 2nd law is

$$\dot{m}(s_e - s_i) \geq \sum_{cv} \frac{\dot{Q}_{cv}}{T}$$

- Consider Reversible and adiabatic, the 2nd law becomes $S_e = S_i$

Recall the property relation $T\,ds = dh - v\,dp = 0$ (*)

or $h_e - h_i = \int_i^e v\,dp$

Sub this into (iii), (w/ q=0)

$$w = (h_i - h_e) + \frac{V_i^2 - V_e^2}{2} + g(z_i - z_e) \qquad (**)$$

$$= -\int_i^e v\,dp + \frac{V_i^2 - V_e^2}{2} + g(z_i - z_e)$$

i.e. Isentropic (Rev + Adiabatic)

- If the process is Reversible + isothermal, 2nd law reduces to

$$\dot{m}(s_e - s_i) = \frac{1}{T}\sum_{cv}\dot{Q}_{cv} = \frac{\dot{Q}_{cv}}{T}$$

Or $T(s_e - s_i) = \frac{\dot{Q}_{cv}}{\dot{m}} = q$ (iv)

and the property relation (*) can be integrated to give

$$T(s_e - s_i) = (h_e - h_i) - \int_i^e v\,dp \qquad \text{(v)}$$

Upon sub. (iii) 1st law

$$w = -\int_i^e v\,dp + \frac{V_i^2 - V_e^2}{2} + g(z_i - z_e)$$

Same as (**).
In other words, (**) is valid for both

Rev+Adiabatic
Rev+Isothermal

- **Nozzle (**for nearly liquid fluid, left of the dome, i.e. dv=0, w=0)))

We can integrate (**) w/ assumption $v = const.$ (incompressible), and

$w = 0$

$$v(p_e - p_i) + \frac{V_e^2 - V_i^2}{2} + g(z_e - z_i) = 0$$

"Bernoulli equation" in fluid mechanics or (i.e. first law)

$$\frac{p_i}{\rho} + \frac{V_i^2}{2} + gz_i = \frac{p_e}{\rho} + \frac{V_e^2}{2} + gz_e$$

- **Turbine Compressor** (majority is vapor, dv =non zero)

$KE = PE = 0$ (**) becomes

$$w = -\int_i^e v\,dp$$

Note:

Since the pump handles liquid, which has a very small specific volume compared to that of the vapor that flows through the turbine, the power input to the pump is much less than the power output of the turbine.