# Lecture 8. Special Topics: Design Trade-Offs in LC-Tuned Oscillators

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### **Tuned Oscillator Basic**

- LC resonant tank resonates at  $\omega_0 = 1/sqrt(LC)$
- Oscillation is sustained if –Gm = 1/R





## **Review: Performance Metrics for LC Osc.**

- Performance Metrics
  - □ Phase noise/jitter
  - □ Frequency tuning range
  - Power dissipation
  - □ Supply/substrate noise sensitivity
  - □ Output swing
  - □ ...





#### **Phase Noise**

A clock signal v(t) can be expressed by its amplitude A, nominal frequency  $f_0$  and its phase noise  $\phi(t)$ :

 $v(t) = A \cdot \sin(2\pi f_0 t + \phi(t))$ 

Phase noise  $S_{\phi}(f)$  is a power-spectral density (PSD) of the instantaneous phase  $\phi(t)$ :

 $\hfill\square$  Describes how close v(t) is to a perfectly periodic waveform

In measurements,  $S_{\phi}(f)$  is approximated by the PSD of the signal v(t):

$$\mathcal{L}(\Delta f) := 4S_v(f_0 + f_m)/A^2$$



## Lecture Outline

- Understanding the origins of phase noise/jitter
  - □ LTI phase noise theory (high-Q on-chip resonators)
  - □ LTV phase noise theory (1/f noise up-conversion)
  - NTV phase noise theory (voltage-limiting; AM-to-PM conversion)
- Optional topics regarding optimal LC oscillator designs
  - □ Wide tuning range
  - □ Multiphase clock generation
  - Optimization



## LTI Phase Noise Theory

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# LTI Phase Noise Theory (1)

Noise sources in an LC oscillator: R and –Gm

$$\frac{i_{n,total}^2}{\Delta f} = \frac{i_{n,R}^2 + i_{n,Gm}^2}{\Delta f} = \frac{4kT}{R}(1+\gamma) \quad (\because -Gm = R)$$

- □ Thermal noise where  $\gamma$  is the excess noise factor for the active devices
- $\square$   $\gamma$  for a MOS device is 2/3 for long-channel, 1~4 for shortchannel devices





# LTI Phase Noise Theory (2)

• Impedance Z of the parallel RLC near  $\omega_0$ :

$$Z(\omega_0 + \Delta\omega) = \frac{\omega_0 R}{2Q\Delta\omega}$$

• Phase noise at offset  $\Delta \omega$ :

$$\mathcal{L}(\Delta\omega) = 10 \cdot \log\left(\frac{i_n^2}{\Delta f} \cdot |Z(\omega_0 + \Delta\omega)|^2 \cdot \frac{1}{2P_{sig}}\right)$$
$$= 10 \cdot \log\left(\frac{2kT(1+\gamma)}{P_{sig}} \cdot \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right)$$

- White noise translates to 1/f<sup>2</sup> phase noise
- High Q and large P<sub>sig</sub> desired for low phase noise

#### Leeson's Formula

$$\mathcal{L}\{\Delta\omega\} = 10 \cdot \log\left[\frac{2FkT}{P_{sig}}\left(1 + \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right)\left(1 + \frac{\Delta\omega_{1/f^3}}{|\Delta\omega|}\right)\right]$$



- In addition to thermal noise contribution (1/f<sup>2</sup>), the formula also includes:
  - 1/f-noise contribution
     (1/f<sup>3</sup>)
  - □ Buffer noise



#### Phase Noise vs. Power

- Shunt N identical LC VCOs in parallel
  - □ Power is doubled; phase noise is halved
  - □ R is halved; L is halved; C is doubled
  - $\square$  Q,  $\omega_{o}$ , and output swing remain unchanged
- But, scaling I<sub>bias</sub> only doesn't always improve your VCO phase noise
  - □ In fact, it can get worse (voltage-limiting)



# Figure-of-Merit (FOM)

•  $\mathcal{L}{\Delta\omega}$  normalized to  $\Delta\omega/\omega_o$  and  $P_{sig}$ 

FOM = 
$$-\mathcal{L}\{\Delta\omega\} - 10 \cdot \log\left(\left(\frac{\Delta\omega}{\omega_0}\right)^2 \cdot \frac{P_{sig}}{kT}\right)$$
  
=  $10 \cdot \log\left(\frac{2Q^2}{1+\gamma}\right)$ 

- Indicates the qualities of the tank (Q) and the energy restorer (γ)
- Typical ranges of FOM: 0 ~ 20dB
   Q/sqrt(1+γ) of 0.7 ~ 7



# **Clock Jitter Definitions**

- Suppose {t<sub>n</sub>} is a sequence of rising transition times from a clock with the nominal period of T (=1/f<sub>0</sub>)
- Absolute jitter (a.k.a. aperture jitter):

$$\{t_n - nT\}$$

Absolute jitter in radians:  $\{\theta_n := 2\pi f_0 \cdot (t_n - nT)\}$ 

Relation with phase noise:  $\sigma_{abs}^{2} = \frac{1}{2\pi f_{0}^{2}} \int_{-f_{0}/2}^{f_{0}/2} S_{\theta}(f) df$ where  $S_{\theta}(f) \approx S_{\phi}(f)$   $T+j_{1} T+j_{2} T+j_{3}$   $T+j_{1} T+j_{2} T+j_{3}$ 



D. Lee, "Analysis of Jitter in Phase-Locked Loops," TCASII, Nov. 2002

# Clock Jitter Definitions (2)

Period jitter (a.k.a. cycle-to-cycle jitter):

$$\{j_n := t_{n+1} - t_n - T\}$$

Jitter over k periods (a.k.a. long-term jitter):

$$\{j_n(kT) := t_{n+k} - t_n - kT\}$$

#### Relation with phase noise:





D. Lee, "Analysis of Jitter in Phase-Locked Loops," TCASII, Nov. 2002

## Long-Term Jitter of an Oscillator

Another way of defining the long-term jitter is by the standard deviation of phase difference between time τ:

$$\sigma_{\tau}^{2} = \sigma_{J}^{2}(\tau) = \mathbb{E}\left[\left(\phi(t+\tau) - \phi(t)\right)^{2}\right] / \omega_{0}^{2}$$

Assuming thermal noise only (1/f<sup>2</sup> noise):

$$\sigma_\tau = \kappa \cdot \sqrt{\tau}$$

where

$$\kappa = \Delta \omega / \omega_0 \cdot 10^{\mathcal{L}\{\Delta \omega\}/20}$$



# **Typical CMOS LC Oscillator**

2.5GHz, 1mW LC VCO with FOM=17dB

 $\kappa = \Delta \omega / \omega_0 \cdot 10^{\mathcal{L}\{\Delta \omega\}/20}$  $= \sqrt{FOM \cdot \frac{P_{sig}}{kT}}$ 

- Cycle-to-cycle jitter (τ=T<sub>period</sub>): 5.8fs,rms
- PLL with BW=1MHz ( $\tau$ =1/2 $\pi$ BW): 115fs,rms



# Clock Jitter Definitions (3)

Adjacent period jitter (a.k.a. cycle-to-cycle jitter):

$$\{\Delta j_n := j_{n+1} - j_n = (t_{n+2} - t_{n+1}) - (t_{n+1} - t_n)\}\$$

Adjacent period jitter over k periods:

$$\{\Delta j_n(kT) := t_{n+2k} - 2t_{n+k} - t_n\}$$

#### Relation with phase noise:



D. Lee, "Analysis of Jitter in Phase-Locked Loops," TCASII, Nov. 2002

# **Relationship Among Clock Jitters**

Relation between period jitter and adjacent period jitter:

$$\sigma_{\Delta J}^2(kT) = 4\sigma_J^2(kT) - \sigma_J^2(2kT)$$

And some known inequalities:

 $\sigma_J(kT) \leqslant 2 \cdot \sigma_A$  $\sigma_{\Delta J}(kT) \leqslant 4 \cdot \sigma_A$  $\sigma_{\Delta J}(kT) \leqslant 2 \cdot \sigma_J(kT)$  $\sigma_J(2kT) \leqslant 2 \cdot \sigma_J(kT)$ 



D. Lee, "Analysis of Jitter in Phase-Locked Loops," TCASII, Nov. 2002

# **Quality Factor**

General definition of the Q-factor:

 $Q = 2\pi \cdot \frac{energy \ stored}{energy \ loss \ in \ one \ oscillation \ cycle}$ 

#### • For a parallel RLC tank:

$$Q = 2\pi \cdot \frac{average \ magnetic \ energy + average \ electric \ energy}{energy \ loss \ in \ one \ oscillation \ cycle} \Big|_{\omega=\omega_0}$$

$$= 2\pi \cdot \frac{peak \ magnetic \ energy}{energy \ loss \ in \ one \ oscillation \ cycle} \Big|_{\omega=\omega_0} = \frac{R}{\omega_0 L}$$

$$= 2\pi \cdot \frac{peak \ electric \ energy}{energy \ loss \ in \ one \ oscillation \ cycle} \Big|_{\omega=\omega_0} = \omega_0 RC$$

$$= \frac{R}{\sqrt{L/C}}$$
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# **Quality Factor (2)**

• Q of inductor: 
$$Q_L = \omega_0 L_S / R_S$$

$$L_{P} = L_{S} \cdot (Q_{L}^{2}+1)/Q_{L}^{2}, \quad R_{P} = R_{S} \cdot (Q_{L}^{2}+1)$$

• Q of capacitor: 
$$Q_C = 1/\omega_0 R_S C_S$$
  
 $C_P = C_S \cdot Q_C^2/(Q_C^2+1), R_P = R_S \cdot (Q_C^2+1)$ 

• Q of RLC:  $Q_L \parallel Q_C$ 



## **Start-Up Condition**

- Oscillation starts if -Gm > 1/R<sub>tank</sub>
- Assuming that inductor loss (R<sub>s</sub>) dominates:  $1/R_{tank} \approx 1/R_s \cdot 1/((\omega_o L_s/R_s)^2 + 1) \approx R_s/(\omega_o L_s)^2$
- Worst-case start-up is at the lowest ∞₀!
   □ Don't reset your VCO to the lowest Vctrl



## **Spiral Inductor**

- Minimize R<sub>s</sub> for highest Q (dc, skin-effect)
- Use top metal for low C<sub>ox</sub> and high f<sub>self-resonant</sub>
- PGS removes substrate uncertainties







### **Spiral Inductor Q**

•  $Q = \omega_o L_s / R_s \cdot (\text{substrate loss factor}) \cdot (\text{self-resonance factor})$ 





# Patterned-Ground Shield (PGS)

- Shields the electric field of the inductor from the lossy substrate
  - □ Can't shield the magnetic field
- Patterned in order to prevent eddy current loss
  - High resistance along the induction loop
- Alleviates uncertainty in substrate modeling





#### **Transmission Lines**

- Distributed element with  $\gamma = \alpha + j\beta$
- Short-circuited T/L:  $Z_{in,SC} \approx jZ_o \cdot tan(\beta L)$ □ Inductive for L <  $\lambda/4$
- Open-circuited T/L:  $Z_{in,OC} \approx -jZ_o \cdot \cot(\beta L)$ □ Inductive for  $\lambda/4 < L < \lambda/2$



# Spiral L vs. Transmission Line

- Spiral inductors:
  - □ High L per area
  - Open-field structure
- Transmission lines:
  - □ Closed-field structure; less coupling, higher Q
  - □ Higher accuracy in L
  - □ Lengthy at low frequencies



## CPW vs. Microstrip

- CPW: Higher  $Z_o \Rightarrow$  higher Q and large L
  - Dimensions are set by lithography not by oxide thickness; less process sensitive
- Microstrip: inherent shield to the substrate
  - □ Insensitive to substrate parameters



# Traveling vs. Standing Wave

- Both are equivalent to  $\lambda/4$  short-circuited T/L
- [Wood01], [O'Mahony03], [Ham04]



## LTV Phase Noise Theory



# LTV Phase Noise Theory

- VCO response depends on when noise hits
- Hajimiri ISF: linear, but time-varying response  $h_{\phi}(t,\tau) = \Gamma(\omega_{o}\tau)/q_{max} \cdot u(t-\tau)$





### **Periodic ISF: Noise Folding**

- $\phi(t) = \int h_{\phi}(t,\tau) \cdot i(\tau) d\tau = \int \Gamma(\omega_{o}\tau)/q_{max} \cdot i(\tau) d\tau (*)$
- $\Gamma(\cdot)$  is periodic; expanded to Fourier series:

$$\Gamma(\omega_{o}\tau) = c_{0}/2 + \sum c_{n} \cdot \cos(n\omega_{o}\tau)$$

Noise at  $n \cdot \omega_{\lambda} + \Delta \omega$  folds down to  $\Delta \omega$ 



#### **Thermal Noise Contribution**

- Sum of noise power  $\approx \sum c_i^2 = 2 \cdot \Gamma_{rms}^2$
- $\mathcal{L}(\Delta \omega) = 10 \cdot \log(i_n^2 / \Delta f \cdot \Gamma_{\rm rms}^2 / (2q_{\rm max}^2 \Delta \omega^2))$
- Minimize  $\Gamma_{\rm rms}$  for low 1/f<sup>2</sup> phase noise



### **Flicker Noise Contribution**

- Flicker noise:  $i_{n,1/f}^2 = i_n^2 \cdot \omega_{1/f} / \Delta \omega$
- Only  $c_0$  term counts:  $c_0 = \Gamma_{dc}$
- $\mathcal{L}(\Delta \omega) = 10 \cdot \log(i_n^2 / \Delta f \cdot \omega_{1/f} \cdot \Gamma_{dc}^2 / (8q_{max}^2 \Delta \omega^3))$
- Minimize  $\Gamma_{dc}$  for low 1/f<sup>3</sup> phase noise



## CMOS vs. NMOS/PMOS-only

- Better 1/f<sup>3</sup>: symmetric ISF lowers  $\Gamma_{dc}$  [Hajimiri99]
- Better  $1/f^2$ : twice  $G_m$  and  $V_{swing}$  for same  $I_{bias}$

 $\Rightarrow$  6-dB less phase noise [Andreani06]



## NMOS vs. PMOS

- [Jerng05]: PMOS has less thermal noise than NMOS, because it's less velocity-saturated
- PMOS also has lower flicker noise





# **NMF: Cyclostationary Noise**

- Noise profile changes with the circuit state
   Changes periodically; cyclostationary noise
- Use  $\Gamma eff = \Gamma \cdot NMF$ 0.8 Minimize 0.4  $\Gamma$ eff,rms and 0.0 Γeff,dc --ISF NMF -0.4 Eff. ISF Radians <CMOS LC> π/2 3π/2 0 2π Π Slide 35

## **Colpitts Oscillator**

- Colpitts adds noise when osc is least sensitive
- But Colpitts generates more noise [Andreani05]



## Loop Delay

- Loop delay adds to phase shift causing  $\Delta f$
- Effective Q and ISF degrade [Shaeffer03]





# **Delay Compensation via Coupling**

- VCOs coupled in quadrature pull each other
- Pulling cancels the delay





## Tail Current 1/f Noise Up-Conversion

- Tail node fluctuates at  $2\omega_{o}$ ; ISF periodic with  $2\omega_{o}$
- DC-noise converts into AM only
- 2ω<sub>o</sub>-noise converts into AM+PM [Hegazi01]



## **Tail Current Source**

- Sets I<sub>bias</sub> and provides high Z to diff pair
- Without CS, the device in triode adds noise





### **Noise Filter**

- High Z is required for even harmonics only
- $L_F$  provides high Z at  $2\omega_o$



## **Inductor Tail**

- Removes the current source device
- $L_{\rm S}$  provides high Z at  $2\omega_{\rm o}$







## **NTV Phase Noise Theory**

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# **NTV Phase Noise Theory**

- In LTV, frequency translation occurs only due to the periodic ISF and NMF
- Nonlinearity in VCO can also cause frequency translation, thus noise folding
   Nonlinear Gm of switches
   Nonlinear C/V of varactors
- AM-noise gets converted to PM-noise
  - □ Tail bias 1/f noise
  - □ Vdd noise



# **Voltage Limiting**

- Current-limited:  $V_{swing} \approx I_{bias} \cdot R_{tank}$
- Voltage-limited: V<sub>swing</sub> becomes limited by Vdd
- Phase noise may become worse!
   Devices operate more in linear region (noisy)



# Switch Nonlinearity Adds Phase Noise

- Hard switching creates 2nd and 3rd harmonics of current in response to sine voltage
  - □ Nonlinear Gm; dependency on amplitude
- The harmonics flow into C causing imbalance
   Imbalance will result in phase shift
- Thus, amplitude noise will turn into phase noise
  - □ Tail current noise including flicker
  - □ Vdd noise



# Switch Device Sizing

- In LTV, excess noise factor γ doesn't depend on switch device sizes [Rael00]
- But switch nonlinearity suggests that smaller device with higher
   Vgs-Vt yields lower
   [Jerng05]





#### **AM-PM due to Varactor Nonlinearity**

- Nonlinear C-V of varactor excites harmonics
   Single-tone voltage in 
   Multi-tone current out
- Another mechanism of noise folding
  - □ Up-conversion of tail bias 1/f noise



# $A_0$ -Dependency of $f_{OSC}$

- Due to large swing  $A_0$ , varactor C v
- Ceff =  $C_0 C_2/2$

C<sub>0</sub>: time average of C C<sub>2</sub>: second-order Fourier coeff [Hegazi03]

■  $f_{OSC}$  varies with A<sub>0</sub> ≠ 1/2π√LC<sub>nom</sub>





# A<sub>0</sub>-Dependency of Tuning Range

1st-order approximation of tuning range:

 $\mathsf{TR} = \mathsf{Kvco} \cdot \Delta \mathsf{V}_{\mathsf{TUNE}} = (\omega_{\mathsf{H}} \text{-} \omega_{\mathsf{L}}) / \mathsf{A}_{\mathsf{0}} \cdot \Delta \mathsf{V}_{\mathsf{TUNE}}$ 

**TR** also depends on  $A_0$  and narrower than  $\omega_H:\omega_L$ 





## Vdd Noise Sensitivity

- CMOS: output V<sub>CM</sub> varies with Vdd
  V across varactor and switches also vary with
  - V across varactor and switches also vary with Vdd
- PMOS: output V<sub>CM</sub> is fixed at gnd
  - □ By 1st order, Vswing and Vgs-Vt are fixed by Ibias





# Selected Topics in LC Oscillator Design

- Design for Wide Tuning Range
- Multiphase Clock Generation
- VCO Design Optimization



## **Issues for Wide Tuning Range**

- MOS varactor is preferred for large dC/dV
- But nonlinear C/V of varactor degrades PN
- Use small varactor + switched array of fixed C





## Switched Capacitor Array

- Switch size affects Q and tuning range
- ~2:1 range is feasible; but be aware of the broad-range design issues [Berny05]





### **Switched Inductor**

- Switch L to mitigate trade-offs in varying C only
  - □ If  $Q_L$  dominates,  $R_{tank} \approx (\omega_o L_S)^2 / R_S$
  - □ Waste of power occurs at high frequency



# Multi-Phase Clock Generation (1)

Coupled oscillators [JJKim00]





## Multi-Phase Clock Generation (2)

LC-delay ring oscillators [Rogers02]



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## Multi-Phase Clock Generation (3)

Rotary-wave oscillators [Wood01]





#### **Optimization of LC Oscillator Design**





## **VCO Design Constraints**

Power

: I<sub>bias</sub>

- Start-up
- Osc freq
- Tuning range :  $C_{varactor}/C_{tank}$
- :  $g_{active} \ge \alpha_{min} \cdot g_{tank} (\alpha_{min} = 3 \sim 5)$
- $: \omega_0^2 = 1/L_{tank}C_{tank}$



### **VCO Optimization via Iteration**

Find transmission line geometry that maximizes Q and satisfies  $\omega_o = \omega_{desired}$ 

Calculate C<sub>tank</sub> (Size C<sub>varactor</sub>) Extract

**g**tank



### **Graphical Interpretation**





#### **Iteration Minimizes Phase Noise**



# Summary

- LTI theory gives the main guideline:
  - □ Maximize Q of the tank
  - **u** Fundamental limit of  $\gamma$  :  $\gamma_{MOS}$
- LTV/NTV theory explains noise mechanisms that determine excess noise factor γ
  - $\hfill \square$  Major factor is still Q and  $\gamma_{MOS}$
  - □ Not everyone agrees what the next dominant one is



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