Lecture 15. Writing Circuit Equations

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Outlines

- Readings
  - Willy M. C. Sansen, “Analog Design Essentials,” Ch. 2
- Overview
  - Despite the advance in circuit simulators and optimizers, tools can never replace designers’ intuition and expertise. The best way to cultivate such expertise is to exercise writing as many equations as possible, since analytical equations can tell how the performance metrics will change with the design parameters while each simulation only tells the point-wise information.
  - However, most students find writing equations difficult and necessary. It is often because they try to model everything – let’s focus only on the design intent (that is everything is ideal in the way you want). Then you will find it much easier.

g_m/I_D Methodology

- Characterizes the operating region of a saturated MOS device by the ratio Gm/Ids
- According to the square law model, Gm/Id is an equivalent measure to the gate overdrive (Vgs-Vth):

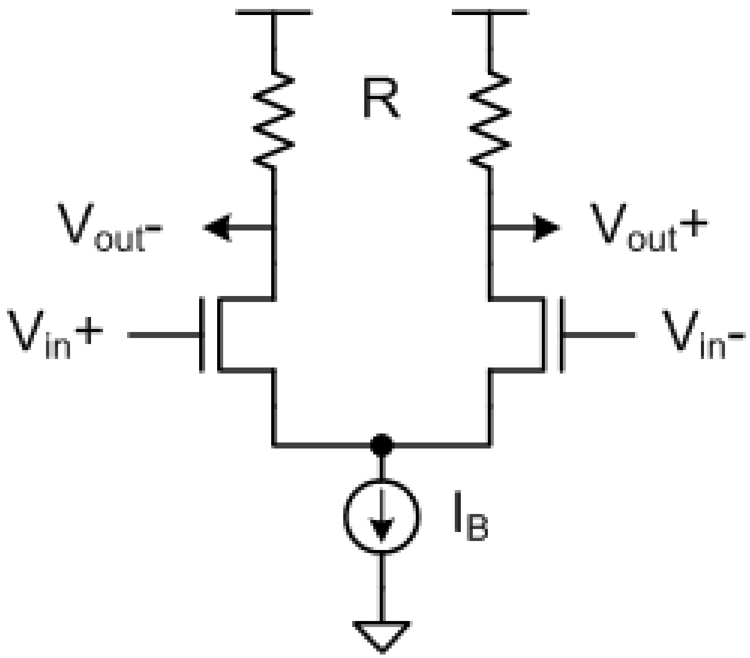
$$I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{GS} - V_{th})^2$$

$$\frac{g_m}{I_D} = \frac{1}{I_D}\sqrt{2 I_D \mu C_{ox}\frac{W}{L}} = \frac{2}{V_{GS} - V_{th}} = \frac{2}{V_{OV}}$$

- If so, why bother using Gm/Id?

g_m/I_D Methodology

- Often, we need to increase Gm/Id ratio:



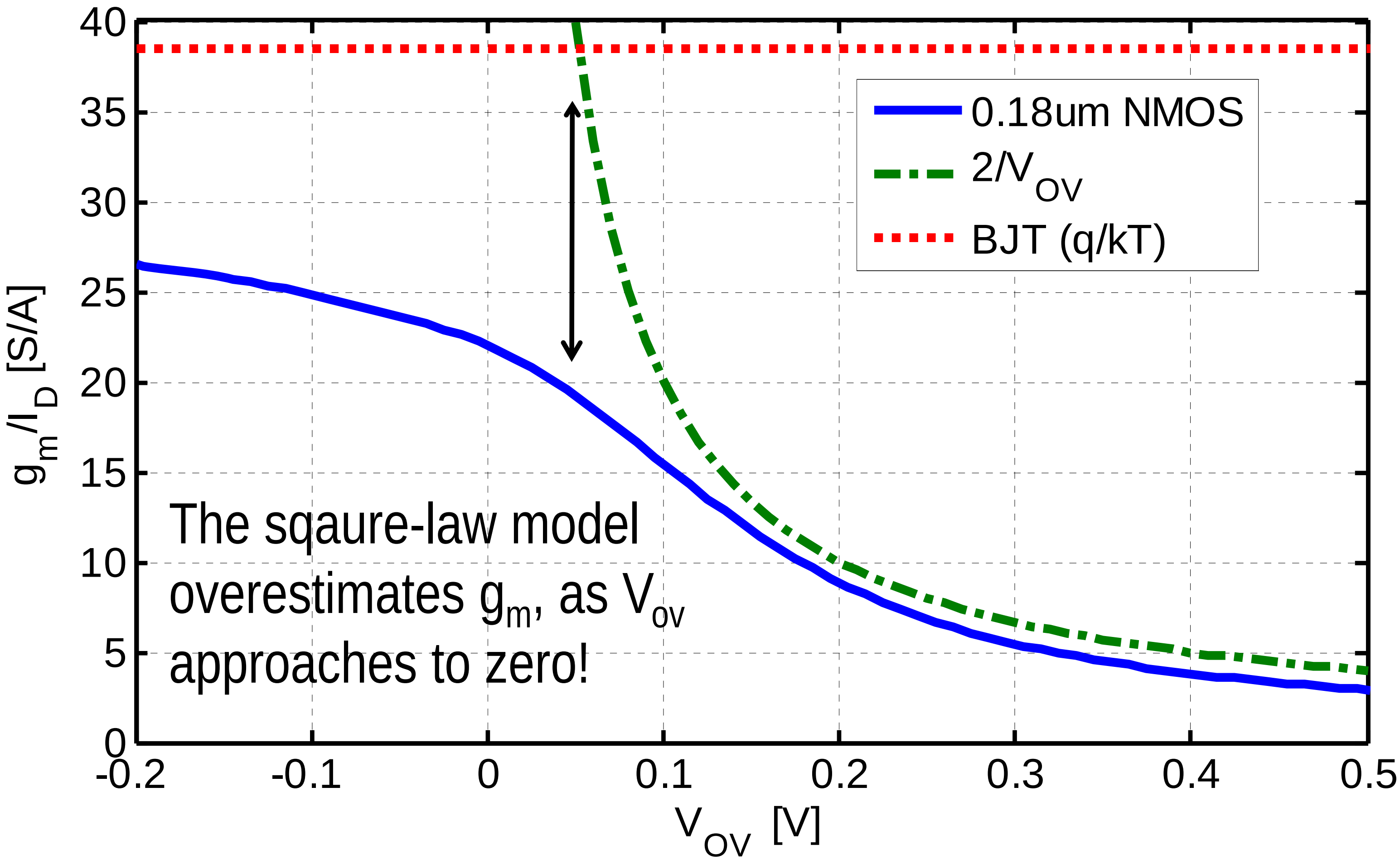
- For a fixed I_B and R, the largest g_m maximizes the gain

  $Av = g_m \cdot R$

- The largest g_m also minimizes the input-referred noise:

$$\overline{v_{n,in}}^2 = \frac{4kTg_m\gamma}{g_m^2} = \frac{4kT\gamma}{g_m}$$

Why use g_m/I_D?



- Using g_m/I_D directly sustains the accuracy into the W.I.

Basic Figures of Merit

Square Law

- Current efficiency
  - Want large g_m, for as little current as possible

$$\boxed{\frac{g_m}{I_D}} \quad = \frac{2}{V_{OV}}$$

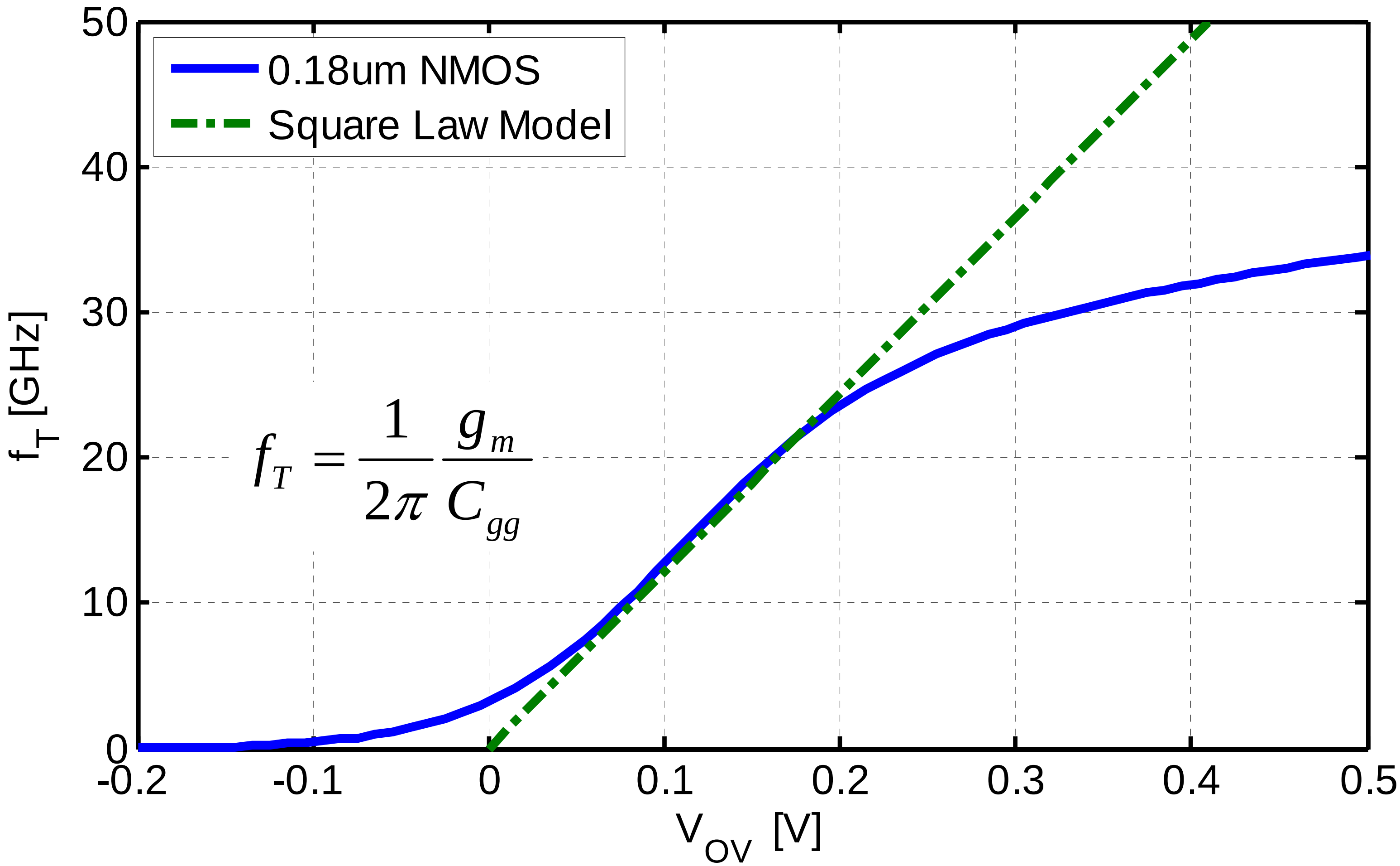
- Transit frequency
  - Want large g_m, without large C_{gg}

$$\boxed{\frac{g_m}{C_{gg}}} \quad = \frac{3}{2}\frac{\mu V_{OV}}{L^2}$$

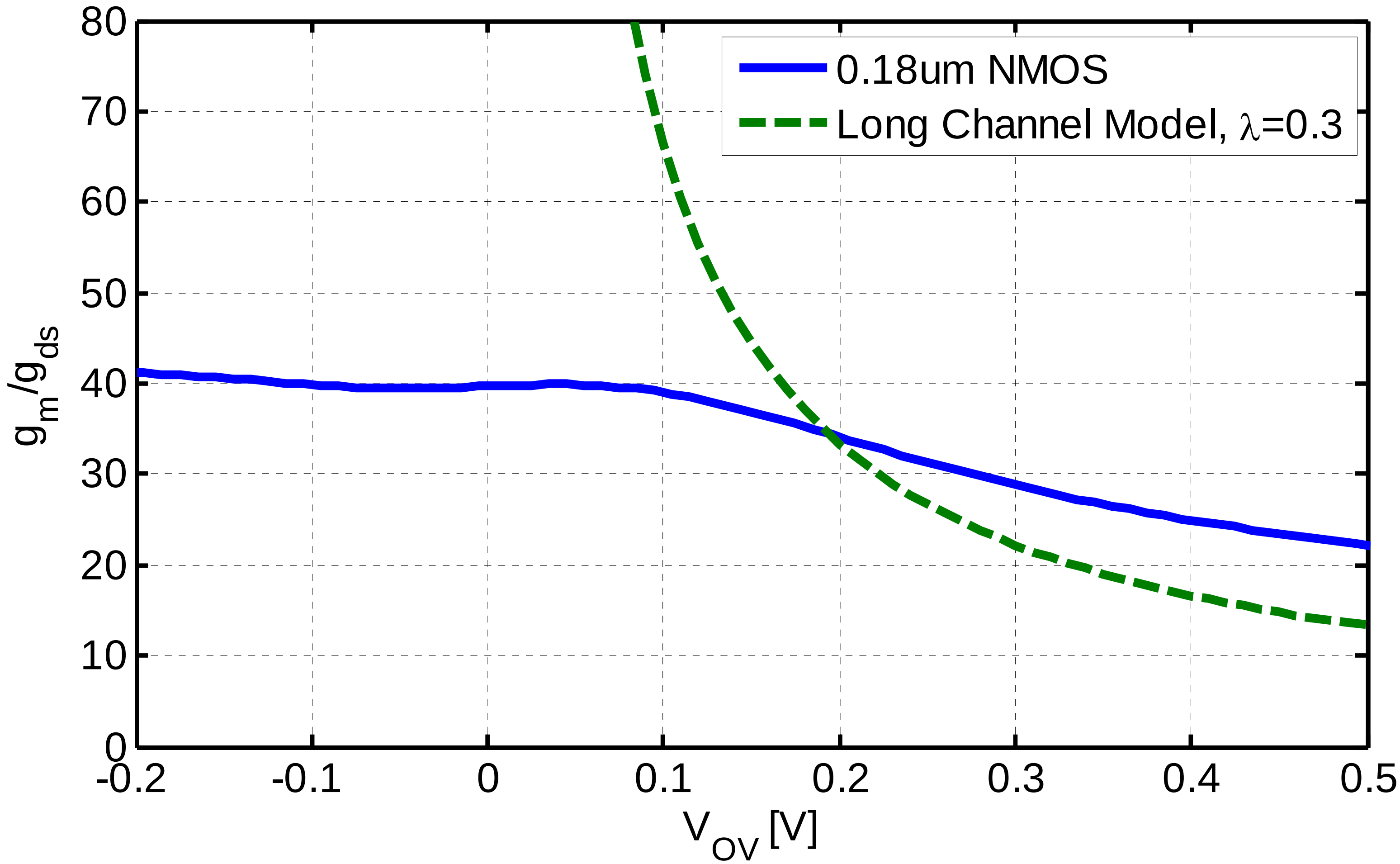
- Intrinsic gain
  - Want large g_m, but no g_{ds}

$$\boxed{\frac{g_m}{g_{ds}}} \quad \cong \frac{2}{\lambda V_{OV}}$$

Transit Frequency Plot



Intrinsic Gain Plot

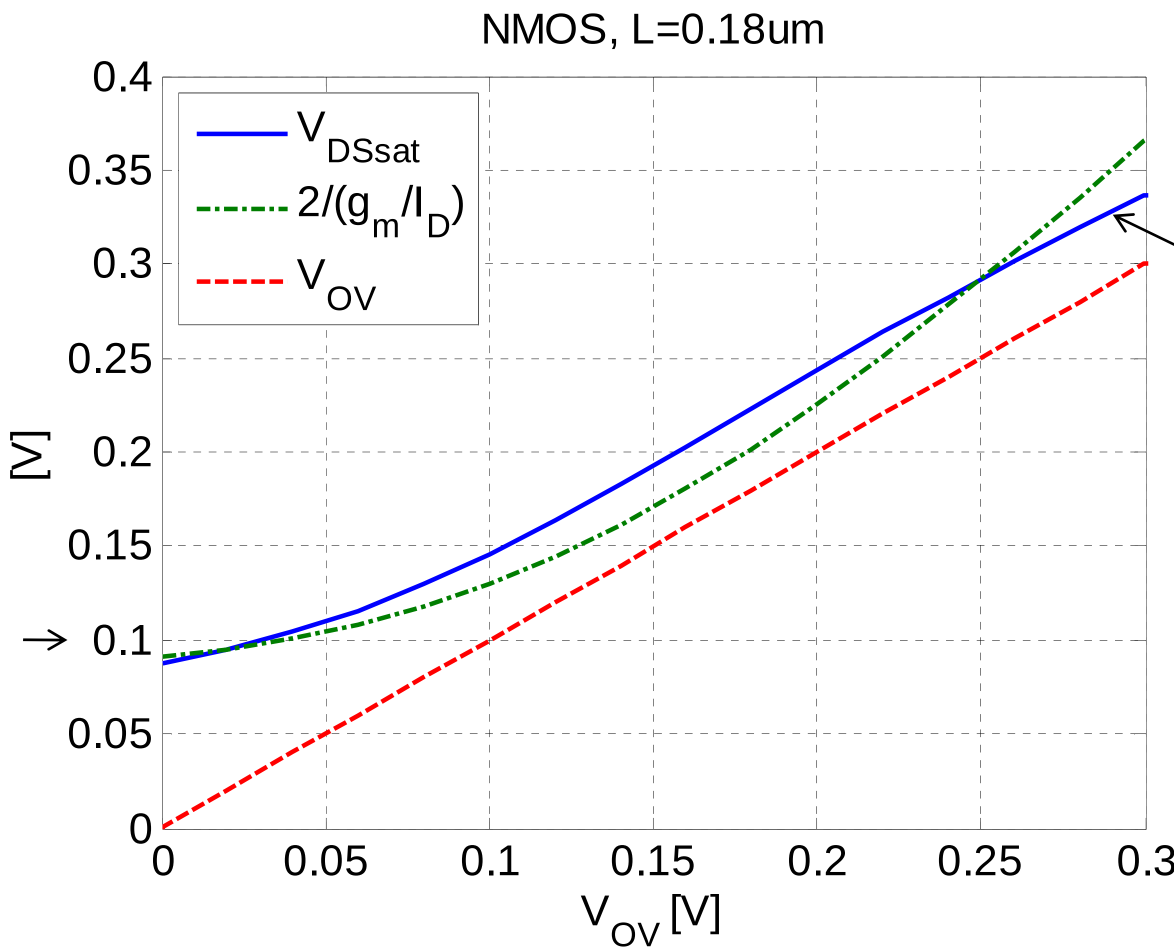


$V_{DS,SAT}$

- So far, we assumed the MOS device operated in saturated region (hence considered its g_m)
  - We need to make sure that $V_{DS} > V_{DS,SAT}$
  - $V_{DS,SAT} = V_{GS}-V_{TH} = V_{OV}$ according to the square-law model

- But, what if we use g_m/I_D, f_T, and intrinsic gain to describe the device characteristics instead of V_{OV}?
  - Need a way to estimate VDS,SAT with these metrics
  - Often use $V_{DS,SAT} \cong 2/(g_m/I_D)$
  - This is a conservative estimate, esp. in the velocity saturation region

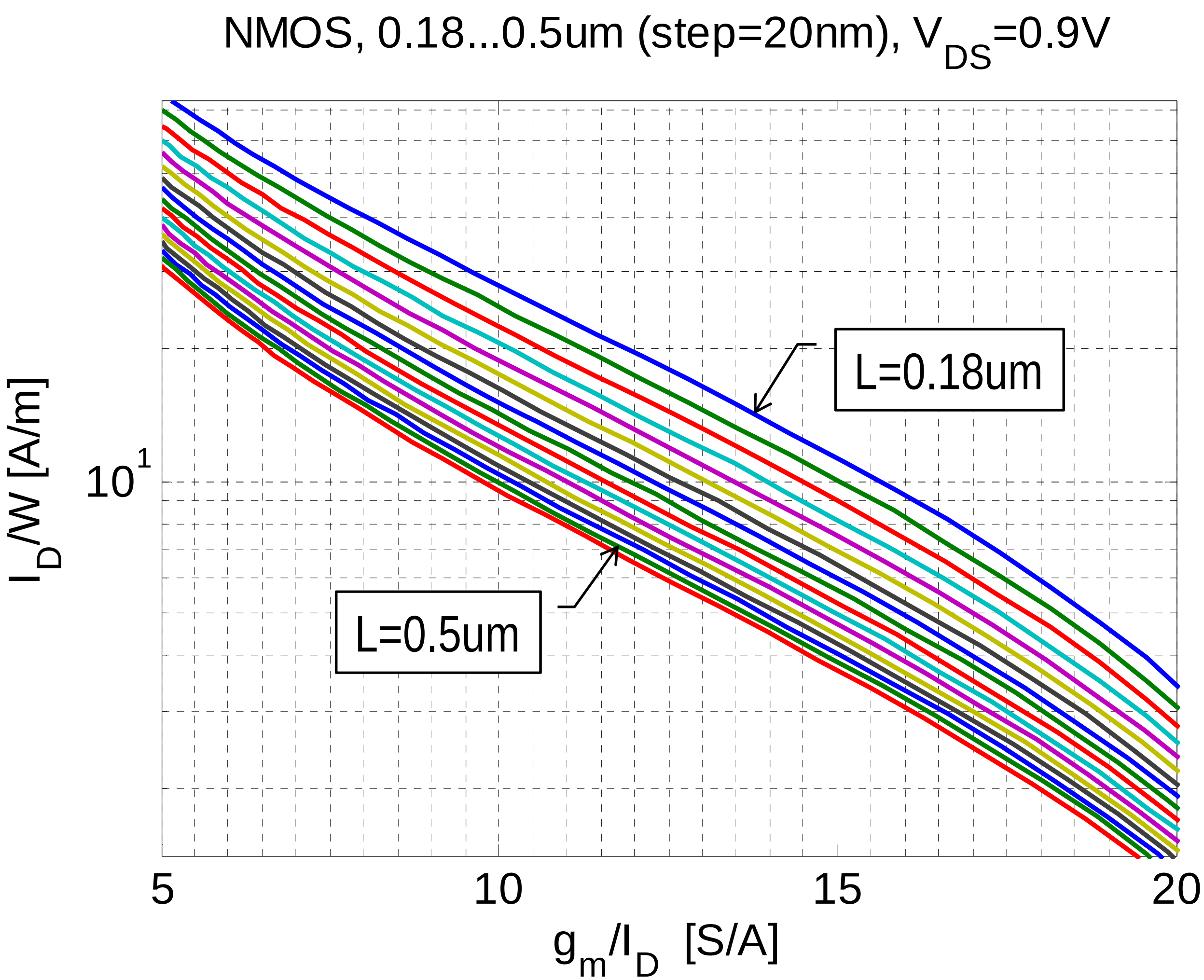
V_{DSsat} Estimate Based on g_m/I_D



"V_{DSsat}" defined (arbitrarily) as V_{DS} at which $1/g_{ds}$ is equal to ½ of the value at $V_{DS} = V_{DD}/2$

- $2/(g_m/I_D)$ is a reasonable estimate of "V_{DSsat}"

Sizing with g_m/I_D

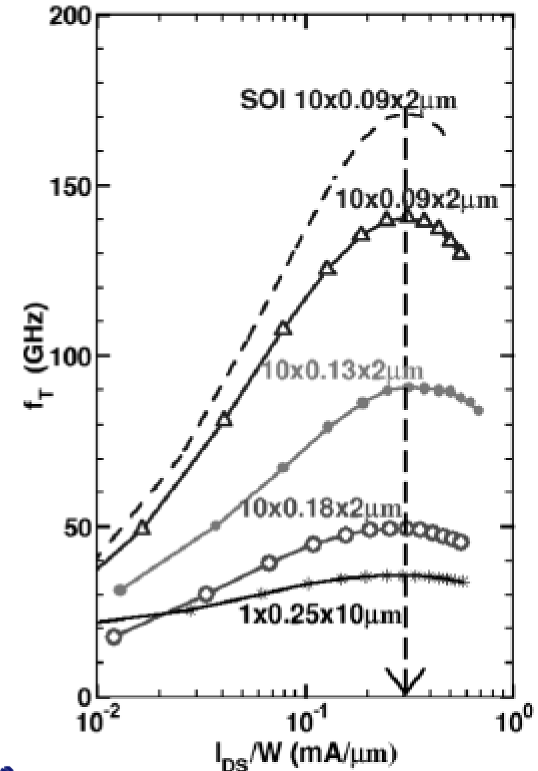


- For the chosen g_m/I_D and L, one can determine W from this "sizing chart" – current density vs. g_m/I_D

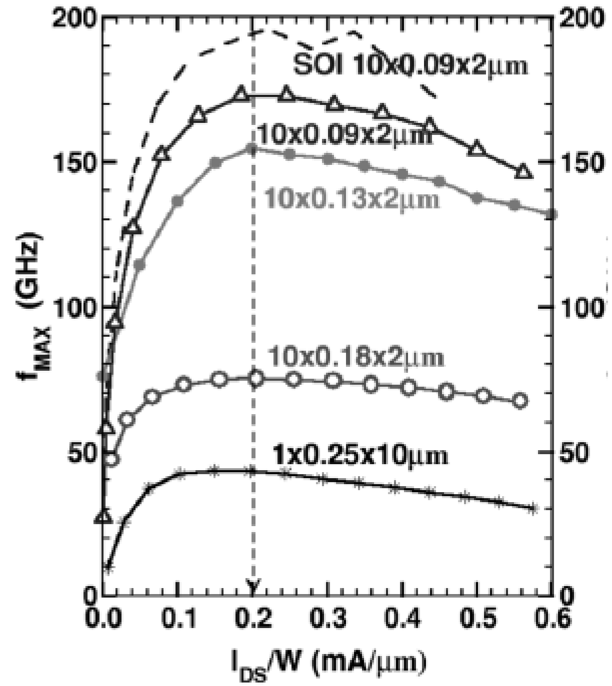
Other Viewpoint: Max f_T and f_{MAX}

- Dickson, et al., "The Invariance of Characteristic Current Densities in Nanoscale MOSFETs…," JSSC 08/2006.

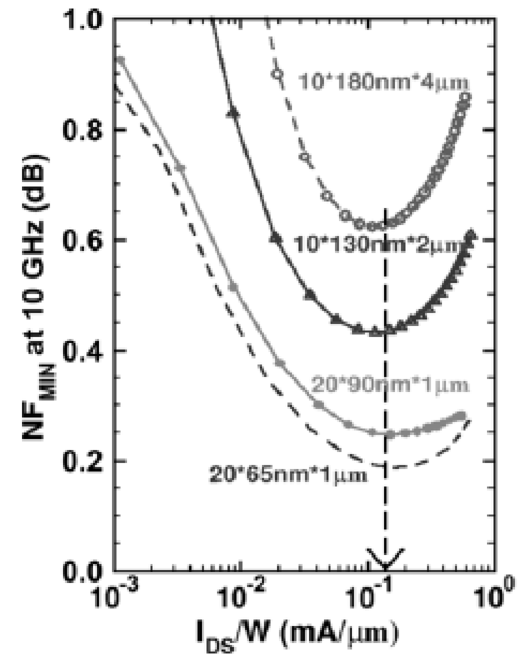
max f_T @ 0.3mA/um



max f_{max} @ 0.2mA/um

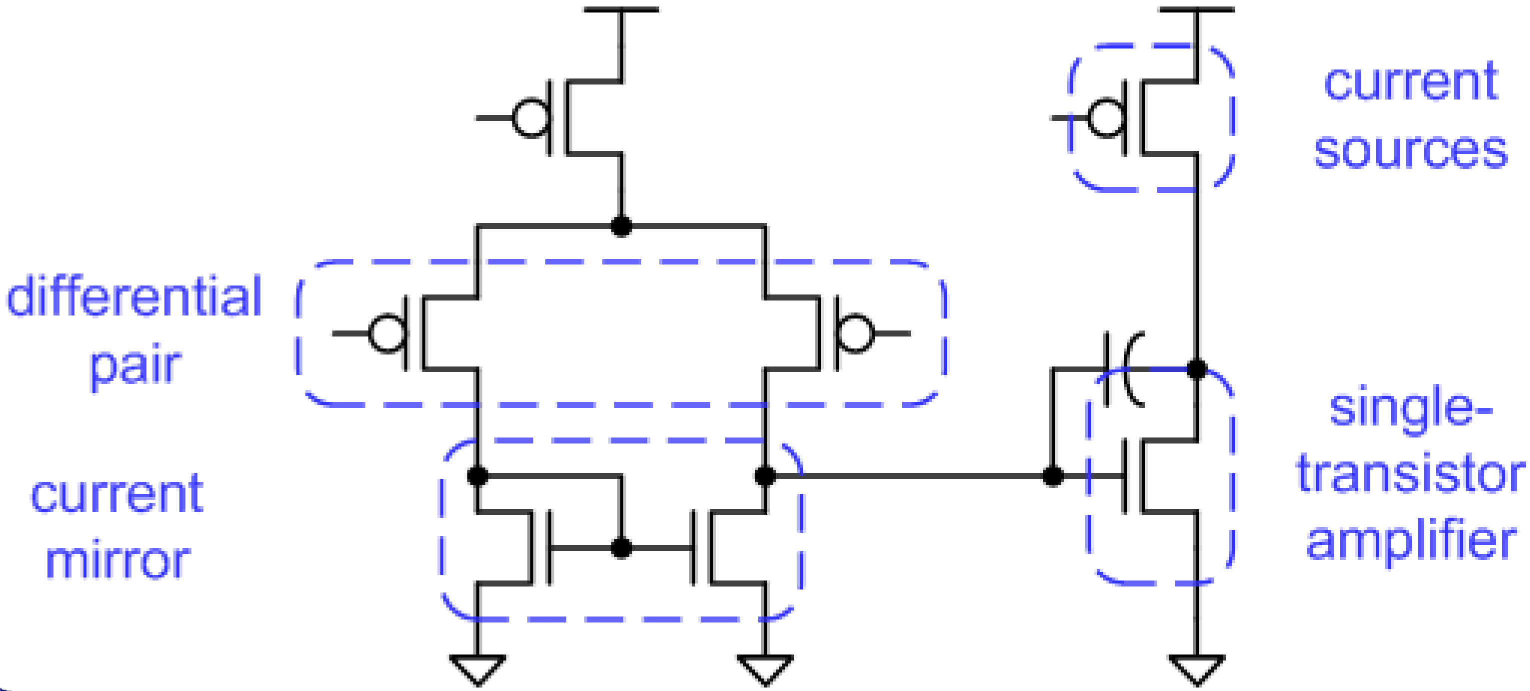


min NF@ 0.15mA/um



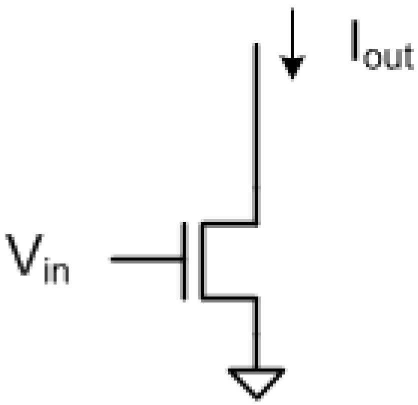
Writing Design Equations for Analog

- Analog building blocks:
  - CS/CD/CG stages, differential pair, current mirror, …
- Let's review their characteristics focusing on their intents



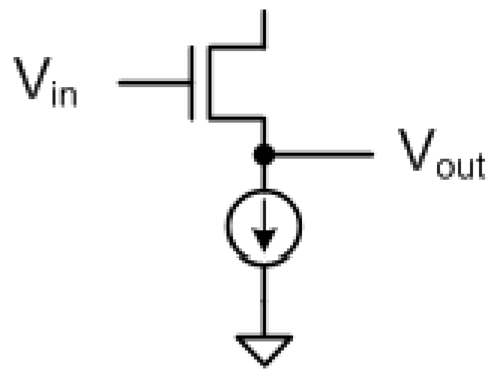
Review: Single-Stage Configurations

common source (amplifier)



$$i_{out} = g_m v_{in}$$

common drain (source follower)

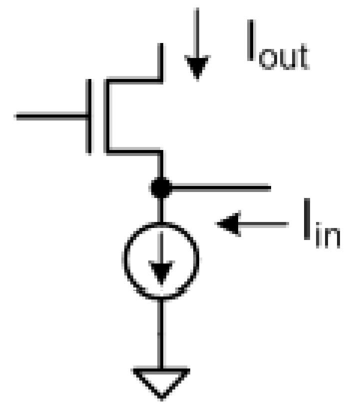


$$v_{out} = v_{in}$$

$$R_{out} \approx 1/g_m$$

voltage buffer

common gate (cascode)

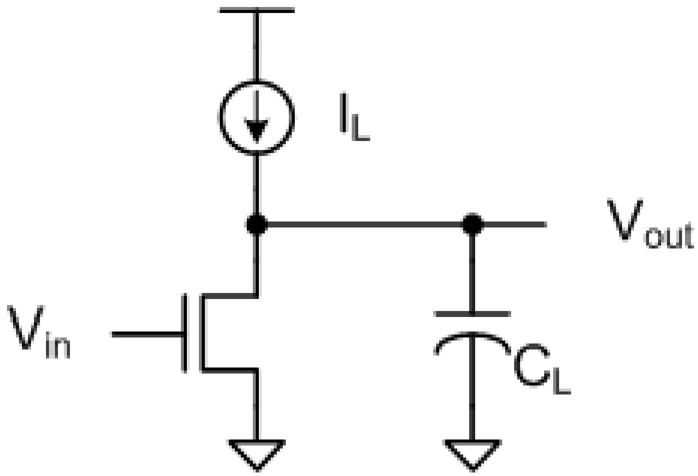


$$i_{out} = i_{in}$$

$$R_{in} \approx 1/g_m$$

current buffer

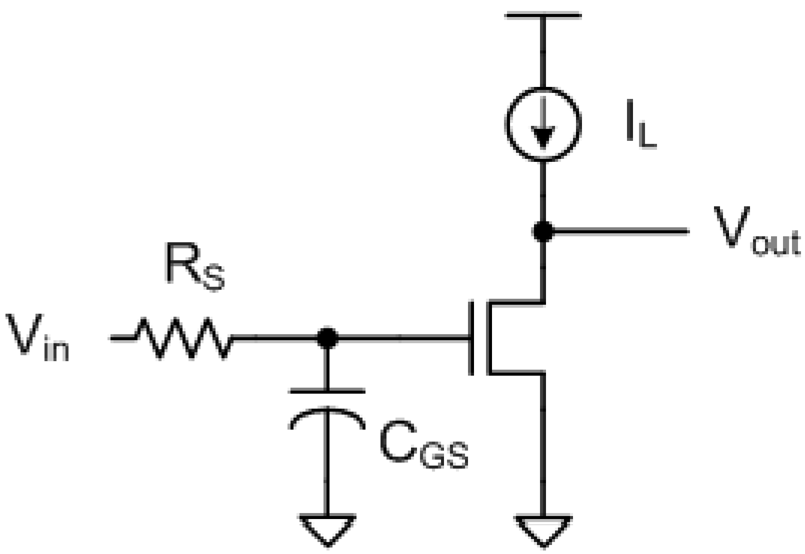
Common-Source Amplifier



- Gain = $g_m \cdot r_{ds}$
- BW = $\dfrac{1}{2\pi \ r_{ds} \ C_L}$
- GBW = $\dfrac{g_m}{2\pi C_L}$

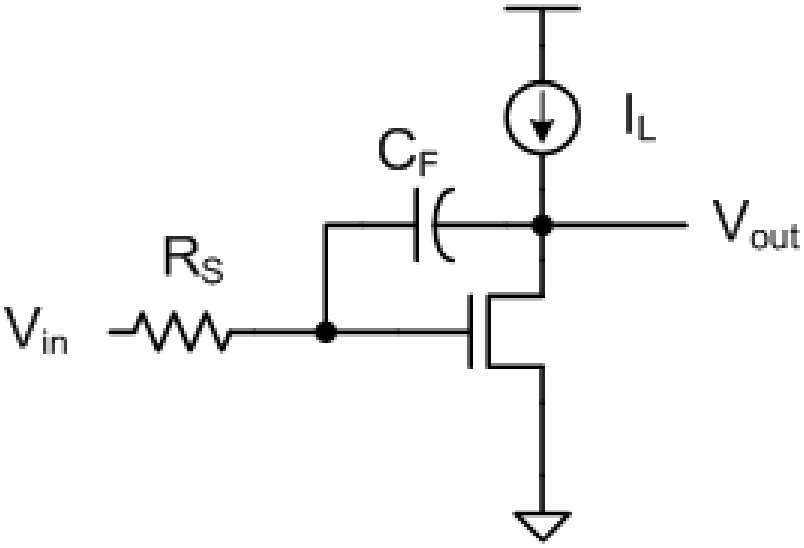
- Exercise: size transistor for GBW = 100MHz when C_L=3pF, I_L = 0.2mA
  - The required gm : 2mS → g_m/I_D = 10
  - Find I_D/W and thus W from the sizing chart

CS Amplifier with Large R_S and C_{GS}



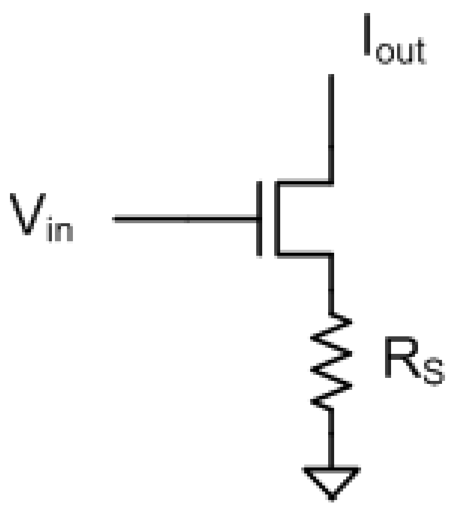
- Gain = $g_m \cdot r_{ds}$
- BW = $\dfrac{1}{2\pi\ R_S\ C_{GS}}$
- GBW = $\dfrac{g_m}{2\pi C_{GS}} \cdot \dfrac{r_{ds}}{R_S} = f_T \dfrac{r_{ds}}{R_S}$

CS Amplifier with Feedback Capacitance



- Gain = $g_m \cdot r_{ds}$
- BW = $\dfrac{1}{2\pi \ R_S \ (1 + g_m r_{ds}) C_F}$
- GBW $\cong \dfrac{1}{2\pi R_S C_F}$

CS Amplifier with Source Degeneration

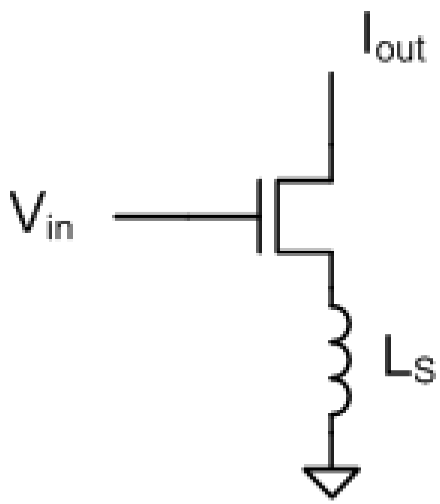


- $g_{m,eff} = \dfrac{g_m}{1 + g_m R_S}$

- $R_{out} = r_{ds} \cdot (1 + g_m R_S)$
  $\approx (g_m r_{ds}) R_S$

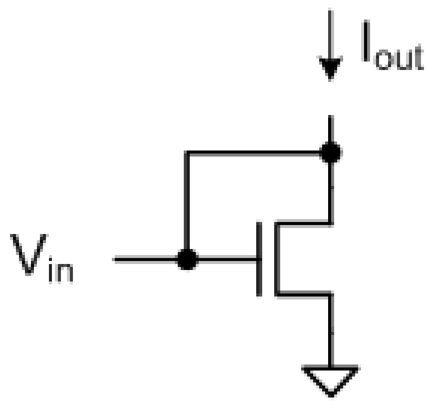
- $C_{in} = \dfrac{C_{GS}}{1 + g_m R_S}$

CS Amplifier with Source Degeneration (2)

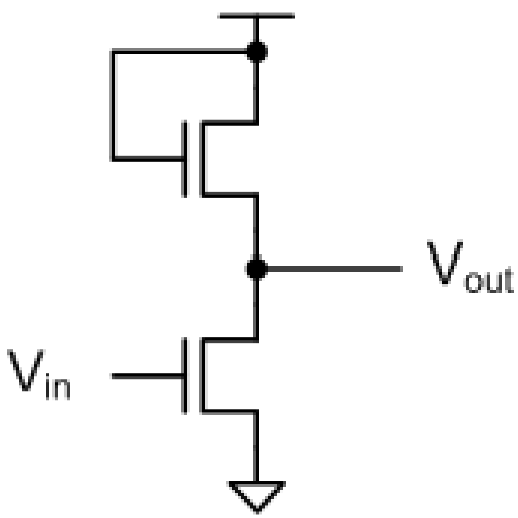


- $g_{m,eff} = \dfrac{g_m}{1 + g_m L_S \cdot s}$
- $R_{out} = r_{ds} \cdot (1 + g_m L_S \cdot s)$
- $Z_{in} = \dfrac{1}{sC_{GS}} \cdot (1 + g_m L_S \cdot s) + sL_S$
  $= L_S \omega_T + sL_S + \dfrac{1}{sC_{GS}}$

CS Amplifier with Diode-Connected Load

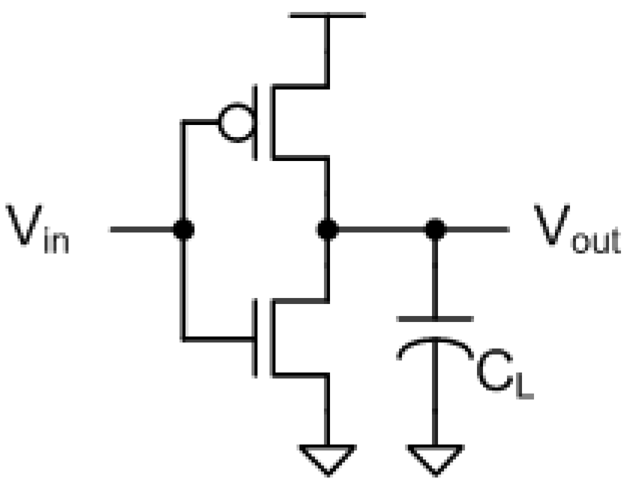


- R_{out} = $1/g_m$



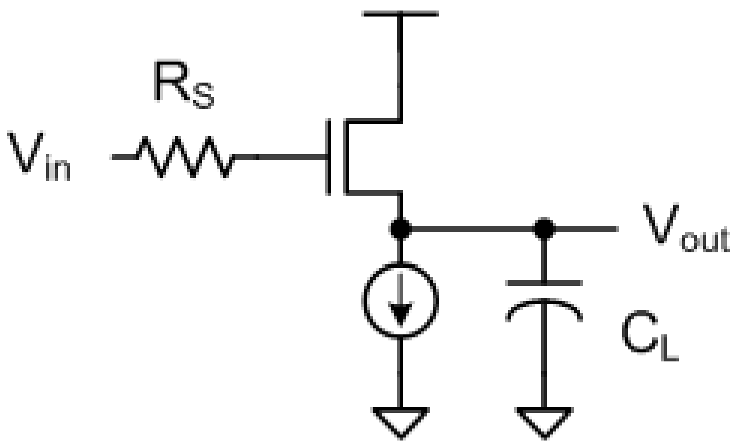
- Gain = $\dfrac{g_{m1}}{g_{m2}} = \dfrac{V_{OV1}}{V_{OV2}}$

Push-Pull Amplifier



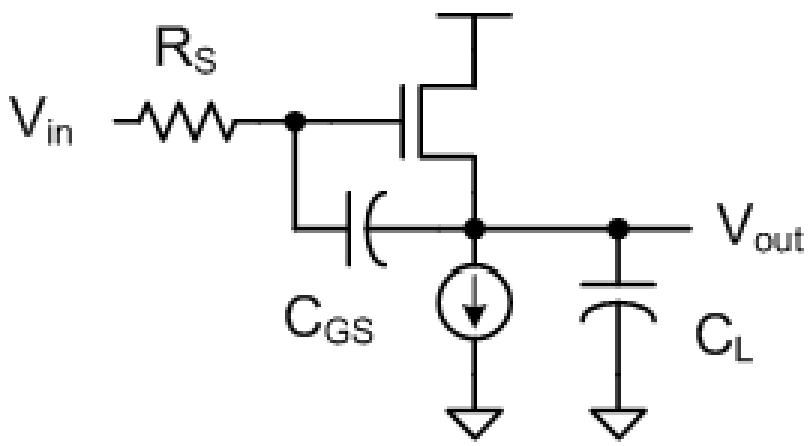
- Rout = $r_{ds,p} || r_{ds,n} \approx r_{ds}/2$
- Gain = $(g_{m,p} + g_{m,n}) \cdot R_{out}$
  $\approx 2 g_m R_{out}$
- BW = $\dfrac{1}{2\pi \; R_{out} \; C_L}$
- GBW = $\dfrac{2 g_m}{2\pi C_L}$

Source Follower



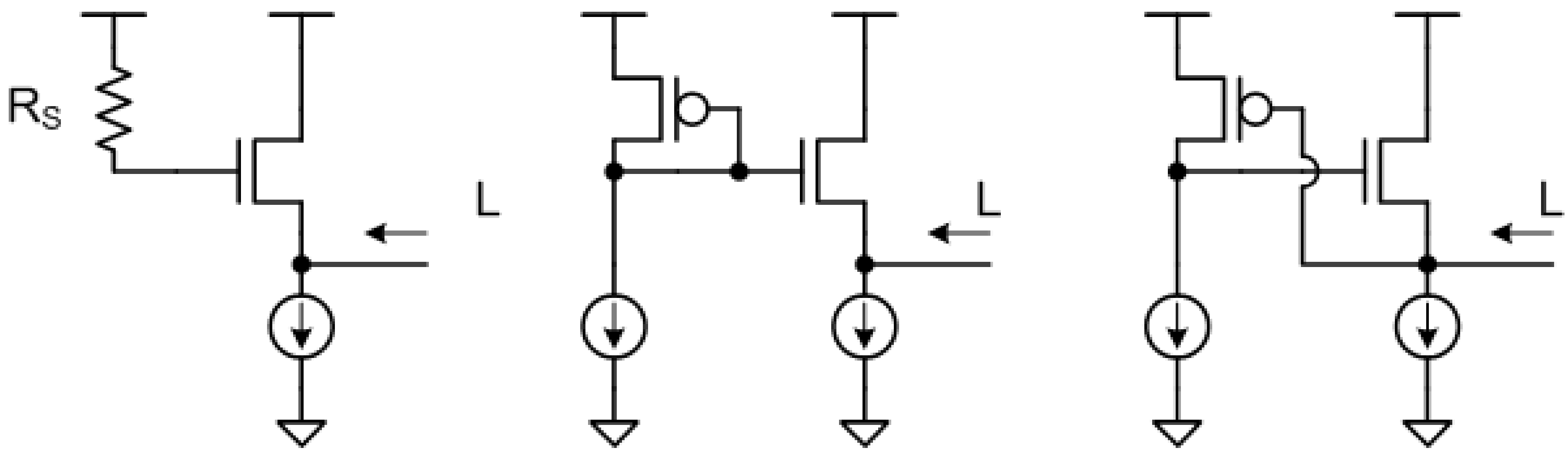
- Gain = 1
- Rout = $1/g_m$

Active Inductor



- Gain = 1
- Zout = $\frac{1}{g_m}(1 + sR_S C_{GS})$
- Lout = $\frac{R_S}{2\pi f_T}$

Active Inductor Loads



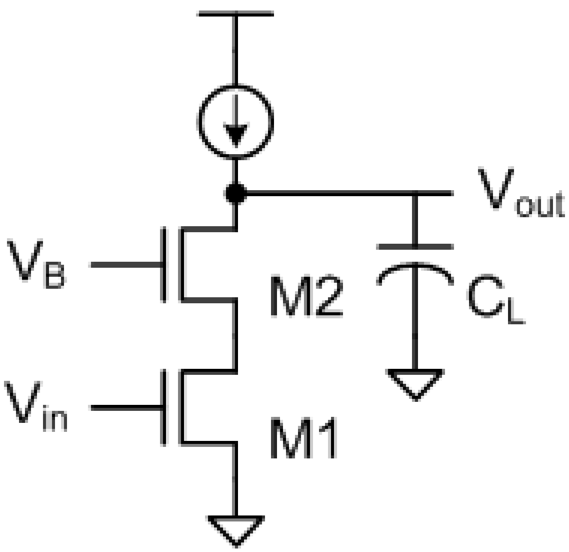
$$L \approx \frac{R_S}{2\pi f_T}$$

$$L \approx \frac{1/g_{m,p}}{2\pi f_T}$$

$$L \approx \frac{1/g_{m,p}}{2\pi f_T}$$

- Compare their output bias voltages

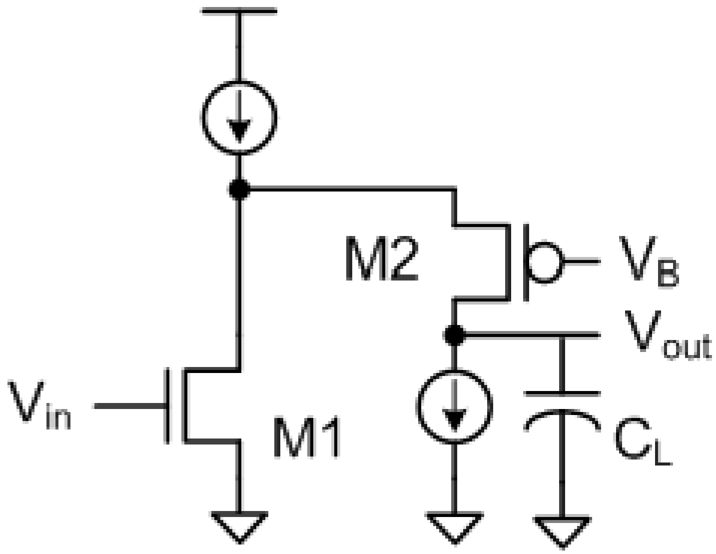
Cascode



- Gain = $(g_m r_{ds})_1 \cdot (g_m r_{ds})_2$
- Rout = $r_{ds,1} \cdot (g_m r_{ds})_2$
- BW = $\dfrac{1}{2\pi R_{out} C_L}$
- GBW = $\dfrac{g_{m,1}}{2\pi C_L}$

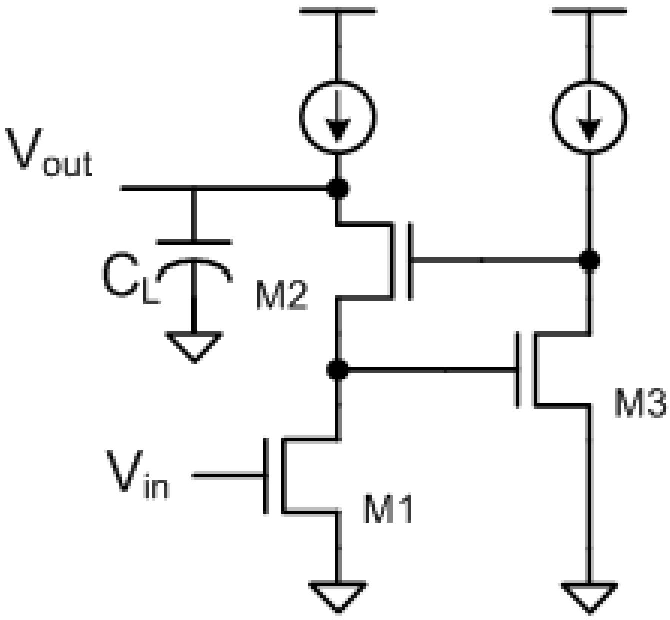
The GBW remains unchanged!

Folded Cascode



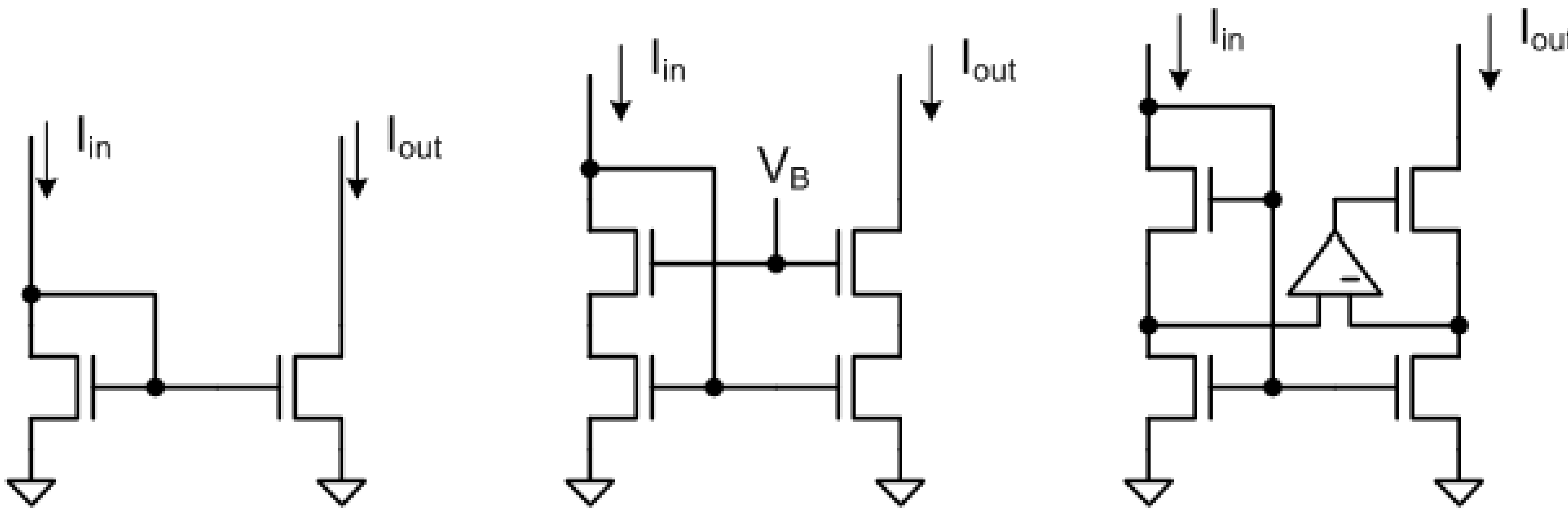
- Gain = $(g_m r_{ds})_1 \cdot (g_m r_{ds})_2$
- Rout = $r_{ds,1} \cdot (g_m r_{ds})_2$
- BW = $\dfrac{1}{2\pi R_{out} C_L}$
- GBW = $\dfrac{g_{m,1}}{2\pi C_L}$

Regulated Cascode



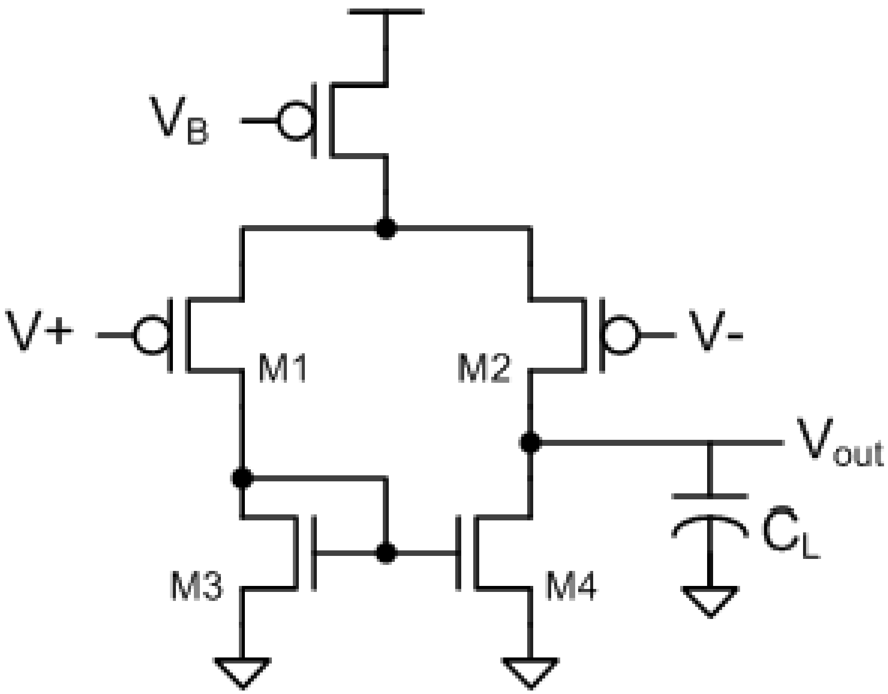
- Gain = $(g_m r_{ds})_1 \cdot (g_m r_{ds})_2 \cdot (g_m r_{ds})_3$
- Rout = $r_{ds,1} \cdot (g_m r_{ds})_2 \cdot (g_m r_{ds})_3$
- BW = $\frac{1}{2\pi R_{out} C_L}$
- GBW = $\frac{g_{m,1}}{2\pi C_L}$

Current Mirrors



- Compare their output resistances and minimum Vout's

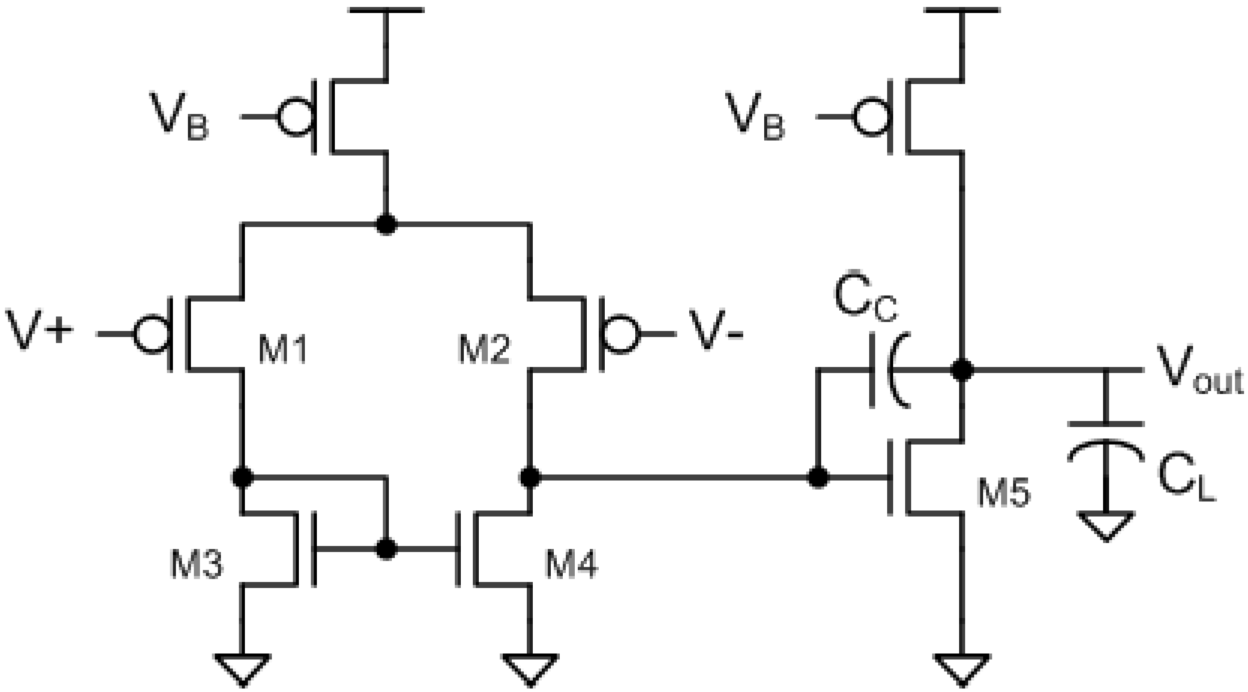
Single-Stage OTA



- Gain = $g_{m,1} \cdot R_{out}$
- Rout = $r_{ds,2} || r_{ds,4}$
- BW = $\frac{1}{2\pi R_{out} C_L}$
- GBW = $\frac{g_{m,1}}{2\pi C_L}$

MI
CS

Two-Stage OTA



- Gain = $A_{v,1} \cdot A_{v,2}$

$$A_{v,1} = g_{m,1} r_{o,24}$$
$$A_{v,2} = g_{m,5} r_{o,56}$$

- BW ≅

$$\frac{1}{2\pi r_{o,24} C_C A_{v,2}}$$

- GBW ≅

$$\frac{g_{m,1}}{2\pi C_C}$$