

Optical solitons

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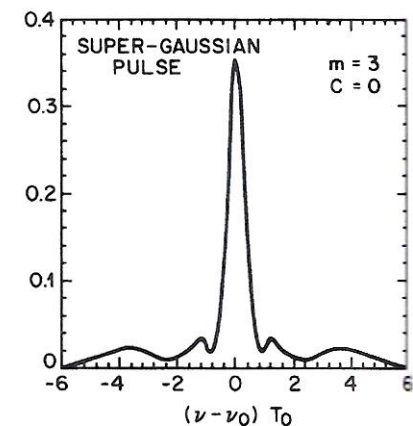
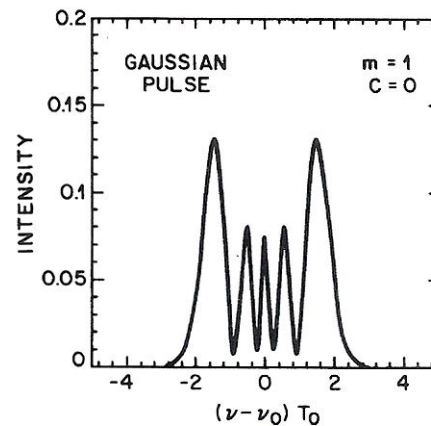
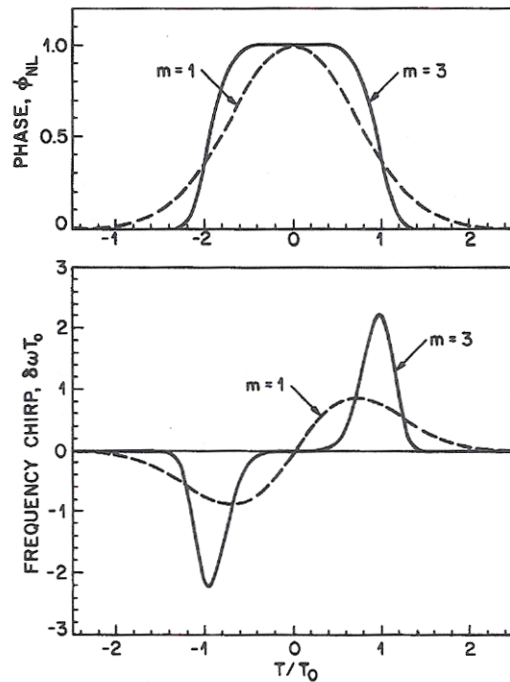
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Self-phase modulation (SPM)

For Gaussian and super-Gaussian pulses:



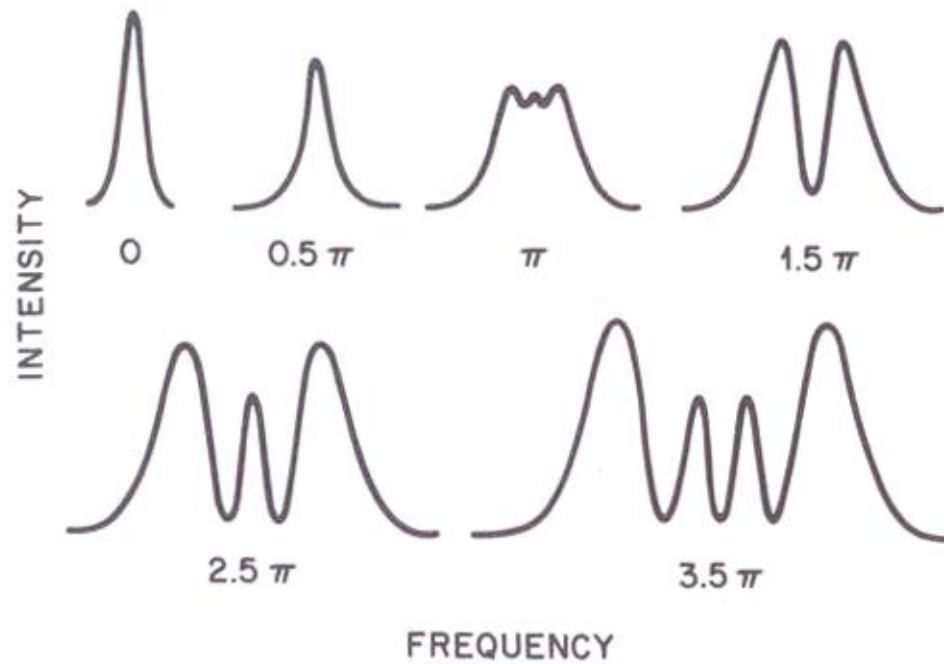
$$\phi_{NL, \max} = 4.5\pi$$

Frequency chirp incurred

Source: Nonlinear Fiber Optics, G. P. Agrawal

Self-phase modulation (SPM)

For Gaussian pulses:



Source: Nonlinear Fiber Optics, G. P. Agrawal

Nonlinear pulse propagation

NLSE:

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2} A = i\gamma |A|^2 A \quad \text{for } \exp(-i\omega t)$$

$$\rightarrow \frac{\partial A}{\partial z} + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial T^2} + \frac{\alpha}{2} A = i\gamma |A|^2 A,$$

where $T = t - z/v_g = t - \beta_1 z$

Modulation instability

If fibre losses are ignored:

$$i \frac{\partial A}{\partial z} = \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - \gamma |A|^2 A$$

Steady-state solution:

$$\bar{A} = \sqrt{P_0} \exp(i\phi_{NL}),$$

where $\phi_{NL} = \gamma P_0 z$.

With a small perturbation:

$$A = (\sqrt{P_0} + a) \exp(i\phi_{NL})$$

$$\rightarrow i \frac{\partial a}{\partial z} = \frac{\beta_2}{2} \frac{\partial^2 a}{\partial T^2} - \gamma P_0 (a + a^*)$$

With a perturbation in the amplitude, i.e. $A = (\sqrt{P_0} + a) \exp(i\phi_{NL})$

$$\begin{aligned}
 i \frac{\partial A}{\partial z} &= i \frac{\partial}{\partial z} [(\sqrt{P_0} + a) \exp(i\phi_{NL})] \\
 &= i \frac{\partial a}{\partial z} \exp(i\phi_{NL}) + i(\sqrt{P_0} + a) \cdot i \frac{\partial \phi_{NL}}{\partial z} \exp(i\phi_{NL}) \\
 &= i \frac{\partial a}{\partial z} \exp(i\phi_{NL}) - (\sqrt{P_0} + a) \gamma P_0 \exp(i\phi_{NL})
 \end{aligned}$$

$$\begin{aligned}
 -\gamma |A|^2 A &= -\gamma (\sqrt{P_0} + a) \exp(i\phi_{NL}) \cdot (\sqrt{P_0} + a^*) \exp(-i\phi_{NL}) \cdot (\sqrt{P_0} + a) \exp(i\phi_{NL}) \\
 &= -\gamma [P_0(\sqrt{P_0} + a) + P_0(a + a^*) + \sqrt{P_0}(a^2 + 2a^*a) + a^*a^2] \exp(i\phi_{NL}) \quad \text{(as } a \rightarrow O^2\text{)} \\
 &\approx -\gamma P_0(\sqrt{P_0} + a) \exp(i\phi_{NL}) - \gamma P_0(a + a^*)
 \end{aligned}$$

In result (w/o loss of generality):

$$i \frac{\partial a}{\partial z} = \frac{\beta_2}{2} \frac{\partial^2 a}{\partial T^2} - \gamma P_0(a + a^*)$$

Modulation instability

Consider its solution in the form:

$$a(z, T) = a_1 \exp[i(Kz - \Omega T)] + a_2 \exp([-i(Kz - \Omega T)])$$

Solution with the dispersion relation:

$$K = \pm \frac{1}{2} |\beta_2 \Omega| [\Omega^2 + \text{sgn}(\beta_2) \Omega_c^2]^{1/2},$$

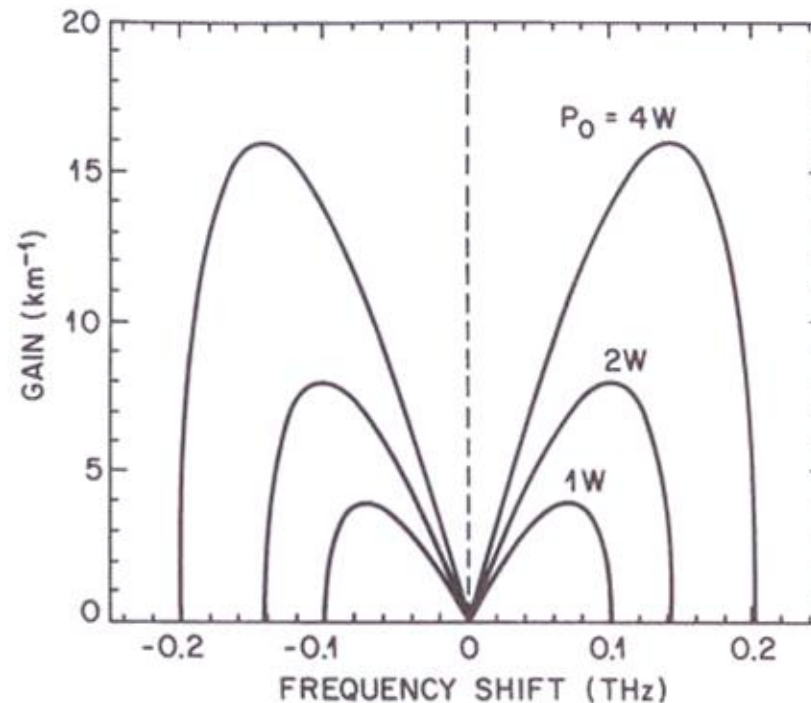
where $\text{sgn}(\beta_2) = \pm 1$ depending on the sign of β_2 ,

$$\Omega_c^2 = \frac{4\gamma P_0}{|\beta_2|} = \frac{4}{|\beta_2| L_{NL}}.$$

The sign of β_2 (GVD) determines the stability against the perturbation.

Modulation instability

Gain spectra: $g(\Omega) = 2 \text{Im}(K) = |\beta_2 \Omega| (\Omega_c^2 - \Omega^2)^{1/2}$



Source: Nonlinear Fiber Optics, G. P. Agrawal

$$\beta_2 = -20 \text{ps}^2 / \text{km}, \quad \gamma = 2 \text{W}^{-1} / \text{km}$$

Fibre solitons

Recall NLSE:

$$i \frac{\partial A}{\partial z} = \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - \gamma |A|^2 A$$

Introduce three dimensionless variables:

$$U = \frac{A}{\sqrt{P_0}}, \quad \xi = \frac{z}{L_D}, \quad \tau = \frac{T}{T_0}$$

$$\text{where } L_D = \frac{T_0^2}{|\beta_2|}.$$

NLSE:

$$i \frac{\partial U}{\partial \xi} = \text{sgn}(\beta_2) \frac{1}{2} \frac{\partial^2 U}{\partial \tau^2} - N^2 |U|U,$$

$$\text{where } N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|}.$$

$$\rightarrow i \frac{\partial u}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} - |u|u = 0,$$

$$\text{where } u = NU = \sqrt{\gamma L_D} A \text{ and } \text{sgn}(\beta_2) = -1.$$

Fundamental solitons

For $N = 1$:

$$i \frac{\partial u}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} - |u|^2 u = 0$$

Shape-preserving solution:

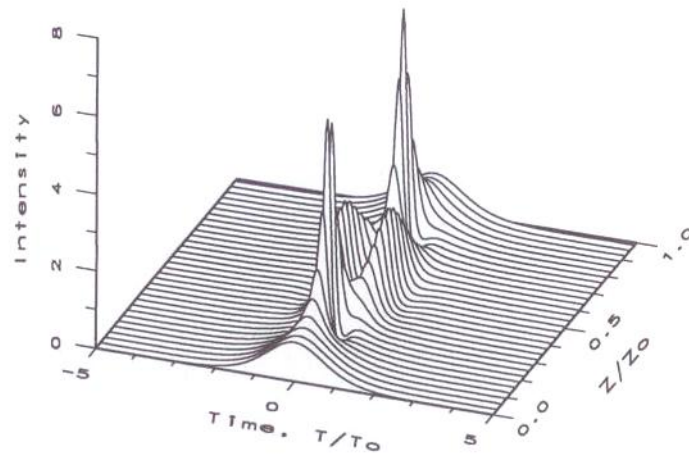
$$u(\xi, \tau) = \operatorname{sech}(\tau) \exp(i\xi / 2)$$

How to solve?

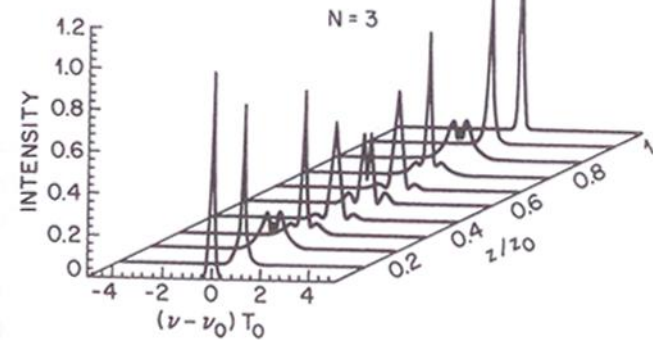
Higher-order solitons

e.g. $N = 3$:

Temporal evolution



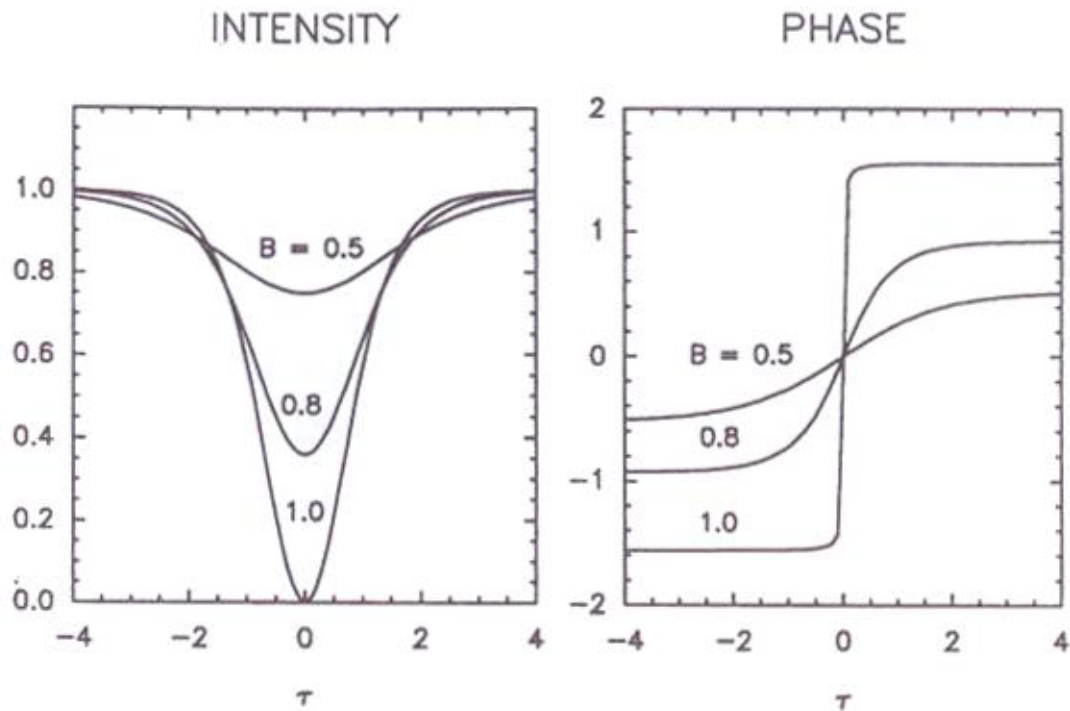
Spectral evolution



Source: Nonlinear Fiber Optics, G. P. Agrawal

Dark solitons

What if $\beta_2 > 0$?

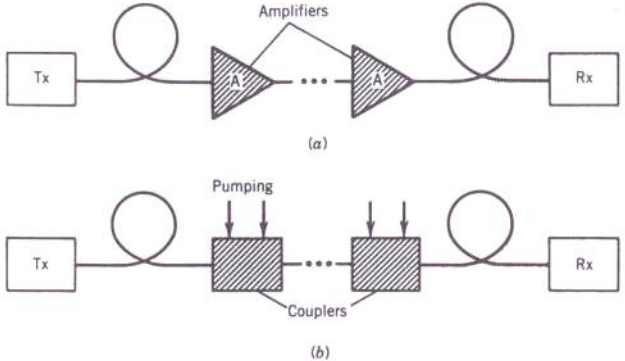


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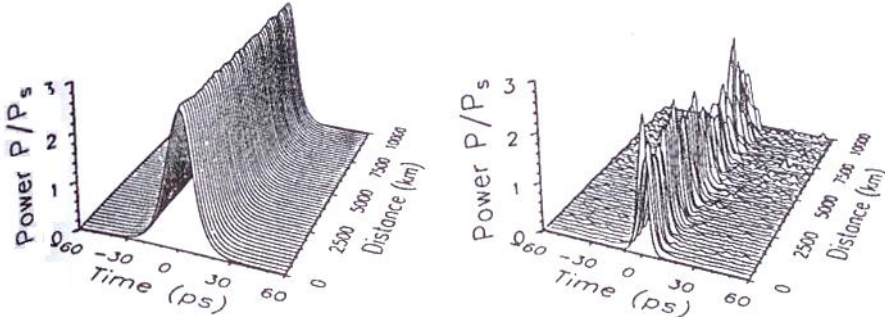
Soliton transmission

Fibre losses?

Lumped or distributed amplification:



Experimental results:



$$L_A < L_D$$

$$L_A > L_D$$

Source: Nonlinear Fiber Optics, G. P. Agrawal