Four-wave mixing II

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Tensor notation for the third-order susceptibility

Constitutive relations for E-field:

$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_o\mathbf{E} + \mathbf{P},$$

$$\mathbf{P} = \varepsilon_o\chi\mathbf{E} = \varepsilon_o\left(\chi^{(1)}\mathbf{E} + \chi^{(2)}\mathbf{EE} + \underline{\chi^{(3)}}\mathbf{EEE} + \cdots\right)$$

Tensor notation for $\chi^{(3)}$:

$$\chi^{(3)} \rightarrow \chi_{ijkl}^{(3)}(\omega_4 = \omega_1 + \omega_2 + \omega_3)$$

$$P_{i,NL} \leftarrow \sum_{j,k,l}\chi_{ijkl}^{(3)}E_jE_kE_l$$

Note that we normally omit summation notation.

Tensor nature of the third-order susceptibility

How many elements for $\chi_{ijkl}^{(3)}$?   *81 elements*

For isotropic media:

$$\chi_{1111}^{(3)} = \chi_{2222}^{(3)} = \chi_{3333}^{(3)},$$

$$\chi_{1122}^{(3)} = \chi_{1133}^{(3)} = \chi_{2211}^{(3)} = \chi_{2233}^{(3)} = \chi_{3311}^{(3)} = \chi_{3322}^{(3)},$$

$$\chi_{1212}^{(3)} = \chi_{1313}^{(3)} = \chi_{2323}^{(3)} = \chi_{2121}^{(3)} = \chi_{3131}^{(3)} = \chi_{3232}^{(3)},$$

$$\chi_{1221}^{(3)} = \chi_{1331}^{(3)} = \chi_{2112}^{(3)} = \chi_{2332}^{(3)} = \chi_{3113}^{(3)} = \chi_{3223}^{(3)}.$$ $\rightarrow$ *21 nonzero elements*

In addition:

$$\chi_{1111}^{(3)} = \chi_{1122}^{(3)} + \chi_{1212}^{(3)} + \chi_{1221}^{(3)}$$ $\rightarrow$ *Why?*

$\rightarrow$ In the compact form: $\chi_{ijkl}^{(3)} = \chi_{1122}^{(3)}\delta_{ij}\delta_{kl} + \chi_{1212}^{(3)}\delta_{ik}\delta_{jl} + \chi_{1221}^{(3)}\delta_{il}\delta_{jk}$

Proof
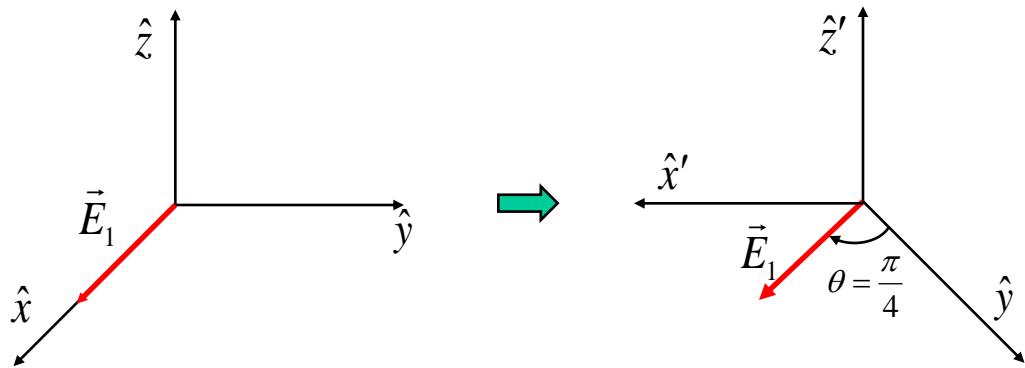
$$\vec{P}_{NL} = \hat{x}\frac{3}{4}\varepsilon_0\chi^{(3)}_{xxxx}\left|E_1^2\right|E_1$$

$$\vec{E}_1' = \hat{x}'\frac{1}{\sqrt{2}}E_1 + \hat{y}'\frac{1}{\sqrt{2}}E_1 = \hat{x}'E_{1,x}' + \hat{y}'E_{1,y}'$$

Rotation invariant:

$$\boxed{\begin{aligned} P_{NL} &= P_{NL}' \\ \to \chi^{(3)}_{xxxx} &= \chi^{(3)}_{xxyy} + \chi^{(3)}_{xyxy} + \chi^{(3)}_{xyyx} \end{aligned}}$$

$$P_{NL,x}' = \frac{3}{4}\varepsilon_0(\chi^{(3)}_{xxxx}\left|E_{1,x}'^2\right|E_{1,x} + \chi^{(3)}_{xxyy}\left|E_{1,y}'^2\right|E_{1,x} + \chi^{(3)}_{xyxy}\left|E_{1,y}'^2\right|E_{1,x} + \chi^{(3)}_{xyyx}\left|E_{1,y}'^2\right|E_{1,x})$$

$$= \frac{3}{4}\varepsilon_0\left|E_1\right|^2 E_1 \frac{1}{2\sqrt{2}}(\chi^{(3)}_{xxxx} + \chi^{(3)}_{xxyy} + \chi^{(3)}_{xyxy} + \chi^{(3)}_{xyyx})$$

$$P_{NL,y}' = \frac{3}{4}\varepsilon_0(\chi^{(3)}_{yyyy}\left|E_{1,y}'^2\right|E_{1,y} + \chi^{(3)}_{yyxx}\left|E_{1,x}'^2\right|E_{1,y} + \chi^{(3)}_{yxyx}\left|E_{1,x}'^2\right|E_{1,y} + \chi^{(3)}_{yxxy}\left|E_{1,x}'^2\right|E_{1,y})$$

$$= \frac{3}{4}\varepsilon_0\left|E_1\right|^2 E_1 \frac{1}{2\sqrt{2}}(\chi^{(3)}_{xxxx} + \chi^{(3)}_{xxyy} + \chi^{(3)}_{xyxy} + \chi^{(3)}_{xyyx})$$

$$\vec{P}_{NL}' = \hat{x}'P_{NL,x}' + \hat{y}'P_{NL,y}'$$

$$P_{NL}' = \sqrt{2}P_{NL,x}' = \frac{3}{4}\varepsilon_0\left|E_1\right|^2 E_1 \frac{1}{2}(\chi^{(3)}_{xxxx} + \chi^{(3)}_{xxyy} + \chi^{(3)}_{xyxy} + \chi^{(3)}_{xyyx})$$

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Four-wave mixing (FWM)

Consider four optical waves oscillating at ω_1, ω_2, ω_3 and ω_4 :

$$\mathbf{E}(r,t) = \frac{1}{2}\hat{x}\sum_{j=1}^{4}E_j \exp[i(k_j z - \omega_j t)] + c.c.$$

Induced nonlinear polarisation:

$$\mathbf{P}_{NL} = \varepsilon_0 \chi^{(3)}\mathbf{EEE} = \frac{1}{2}\hat{x}\sum_{j=1}^{4}P_j \exp[i(k_j z - i\omega_j t)] + c.c$$

$$+\cdots$$

For ω_4 :

$$P_4 = \frac{3\varepsilon_0 \chi_{xxxx}^{(3)}}{4}[|E_4|^2 E_4 + 2(|E_1|^2 + |E_2|^2 + |E_3|^2)E_4$$

$$+ 2E_1 E_2 E_3 \exp(i\theta_+) + 2E_1 E_2 E_3^* \exp(i\theta_-) + \cdots],$$

where

$$\theta_+ = (k_1 + k_2 + k_3 - k_4)z - (\omega_1 + \omega_2 + \omega_3 - \omega_4)t,$$

$$\theta_- = (k_1 + k_2 - k_3 - k_4)z - (\omega_1 + \omega_2 - \omega_3 - \omega_4)t.$$

Easy for phase-matching

Phase-matching condition

For energy conservation:

$$\omega_3 + \omega_4 - \omega_1 - \omega_2 = 0$$

$$\omega_3 + \omega_4 - 2\omega_1 = 0, \quad if \ \omega_1 = \omega_2$$

For momentum conservation:

$$\Delta k = k_3 + k_4 - k_1 - k_2$$
$$= (n_3\omega_3 + n_4\omega_4 - n_1\omega_1 - n_2\omega_2)/c = 0$$
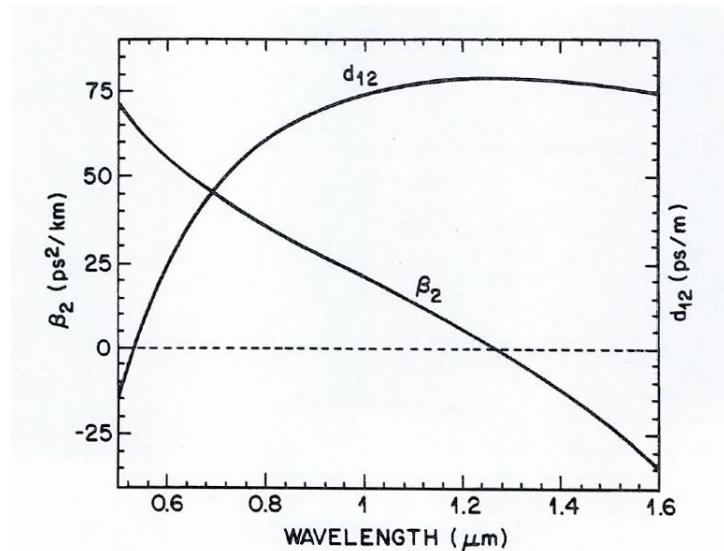
Source: Nonlinear Fiber Optics, G. P. Agrawal

$$\Delta k = k_3 + k_4 - 2k_1$$
$$= (n_3\omega_3 + n_4\omega_4 - 2n_1\omega_1)/c = 0, \quad if \ \omega_1 = \omega_2 \qquad \textit{Is this it?}$$

Recall: $\ \beta(\omega) = n(\omega)\dfrac{\omega}{c} = \beta_0 + \beta_1(\omega - \omega_0) + \dfrac{1}{2}\beta_2(\omega - \omega_0)^2 + \cdots,$

$$\textit{where} \quad \beta_m = \left(\frac{d^m\beta}{d\omega_m}\right)_{\omega = \omega_0} \quad (m = 1, 2, 3, \ldots)$$

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Coupled wave equations

Assuming: $E_j(\mathbf{r}) = F_j(x, y) A_j(z),$

$$\frac{dA_1}{dz} = \frac{in_2'\omega_1}{c}[(f_{11}|A_1|^2 + 2\sum_{j\neq 1}|A_j|^2)A_1 + 2f_{1234}A_2^*A_3A_4e^{i\Delta kz},$$

$$\frac{dA_2}{dz} = \frac{in_2'\omega_2}{c}[(f_{22}|A_2|^2 + 2\sum_{j\neq 2}|A_j|^2)A_2 + 2f_{2134}A_1^*A_3A_4e^{i\Delta kz},$$

$$\frac{dA_3}{dz} = \frac{in_2'\omega_3}{c}[(f_{33}|A_3|^2 + 2\sum_{j\neq 3}|A_j|^2)A_3 + 2f_{3412}A_1A_2A_4^*e^{-i\Delta kz},$$

$$\frac{dA_4}{dz} = \frac{in_2'\omega_4}{c}[(f_{44}|A_4|^2 + 2\sum_{j\neq 4}|A_j|^2)A_4 + 2f_{4312}A_1A_2A_3^*e^{-i\Delta kz},$$

where $f_{jk} = \dfrac{\langle|F_j|^2|F_k|^2\rangle}{\langle|F_j|^2\rangle\langle|F_k|^2\rangle},$

Effective phase-matching condition:

$$\kappa = \Delta k + 2\gamma P_0 = 0$$

where $P_0 = P_1 + P_2$

$$f_{ijkl} = \frac{\langle F_i^* F_j^* F_k F_l\rangle}{[\langle|F_i|^2\rangle\langle|F_j|^2\rangle\langle|F_k|^2\rangle\langle|F_l|^2\rangle]^{1/2}}.$$

Four-wave mixing in fibre

Fibre Parameters

Structure: 7-point core defect, 7½ ring structure.
Hole-to-hole spacing: 3.6 μm
Hole diameter relative size (d/Λ): 0.95
Air filling factor: ~87%
Core diameter: 11.6 μm.
Fundamental bandgap: 1570 nm.
Higher order: 458, 505, 560 nm
*(Fabricated from the resources outside the project)*


Secondary cores: 430 nm in diameter

Mag = 35.00 K X   2μm   Detector = SE1
EHT = 20.00 kV   Date :25 Aug 2005
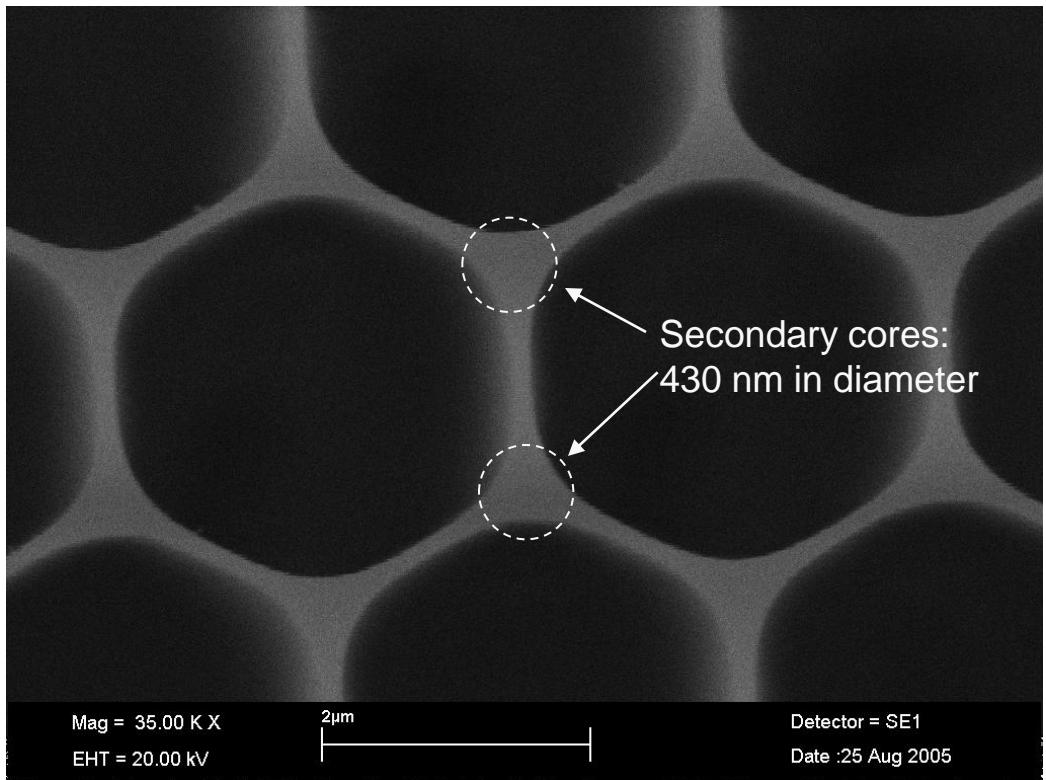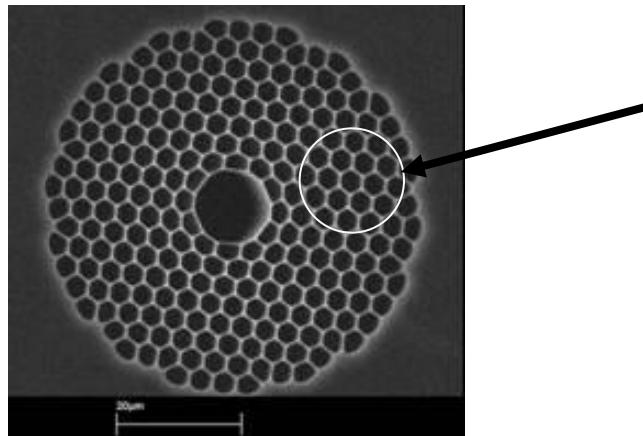
Secondary cores to be investigated for four-wave mixing!

RGB by four-wave mixing in fibre

Green pump at 530 nm
@32 MHz & 90 ps

Green pump at 530 nm
@32 MHz & 90 ps

Diffracted output image
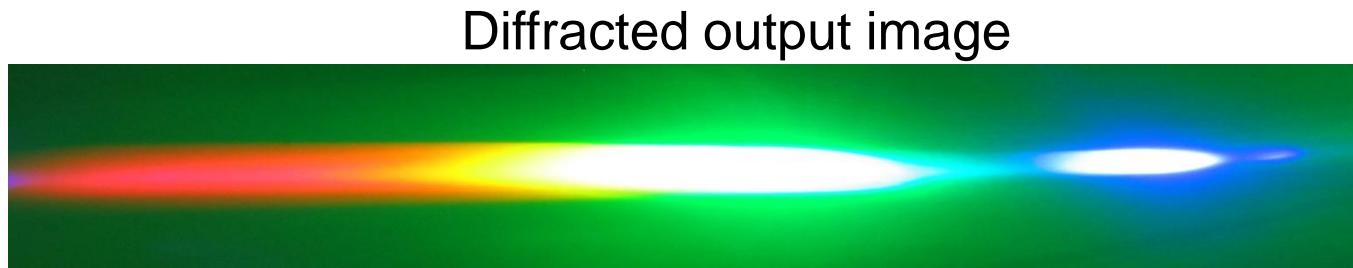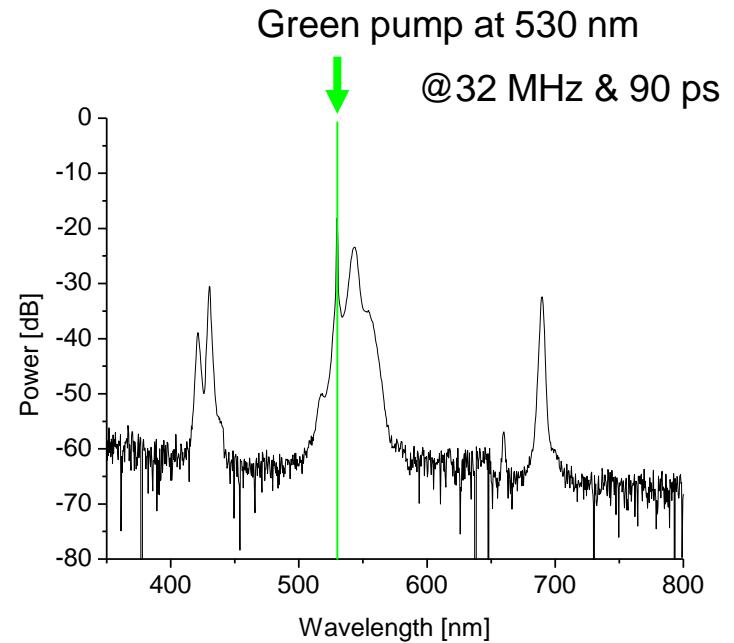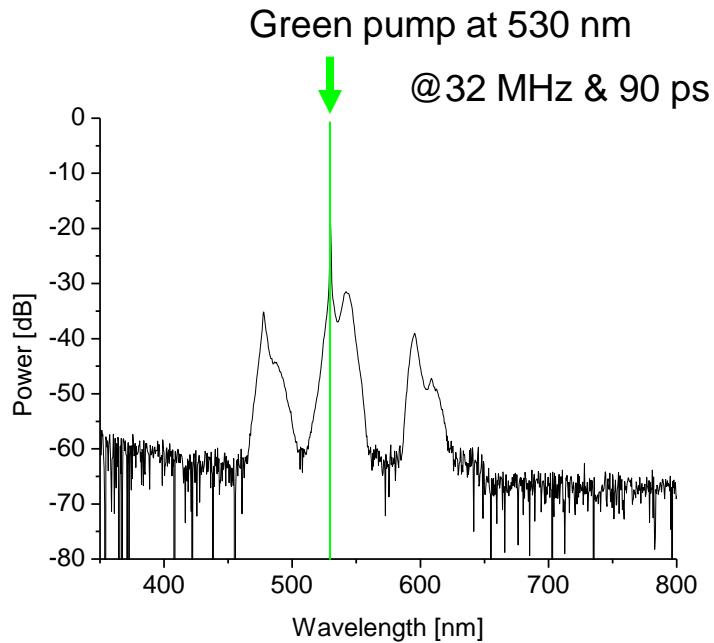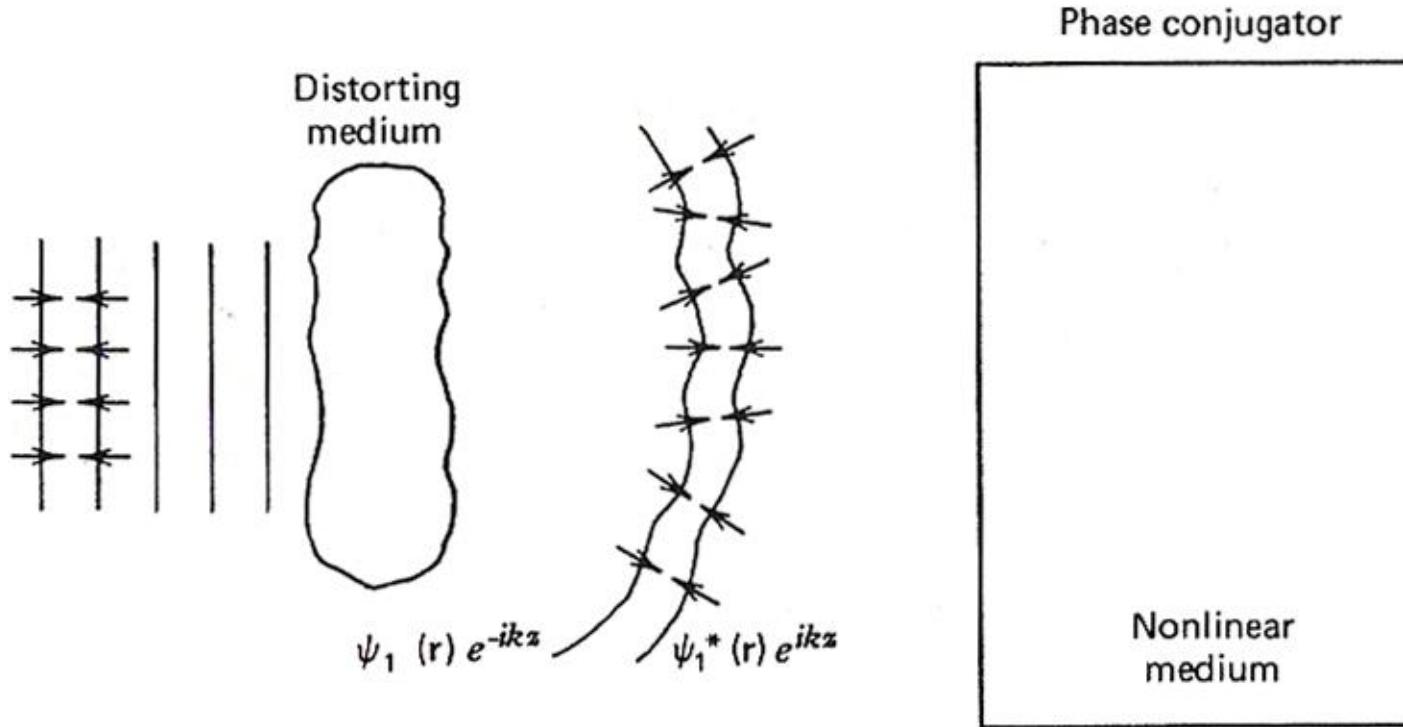
Phase conjugation

Source: Optical Waves in Crystals, A. Yariv and P. Yeh

Complex conjugation of an input field via FWM

Self-phase modulation (SPM)?

For Gaussian pulses:
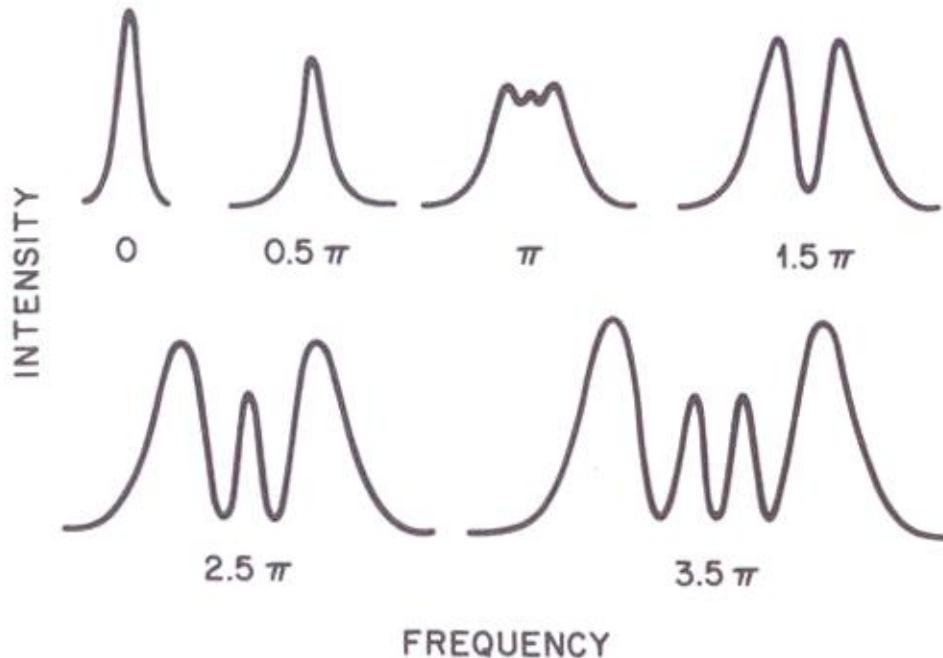
Source: Nonlinear Fiber Optics, G. P. Agrawal

SPM or XPM?

$$P_1 = \frac{3\varepsilon_0 \chi_{xxxx}^{(3)}}{4} [|E_1|^2 E_1 + 2(|E_2|^2 + |E_3|^2 + |E_4|^2)E_1] \text{ ?}$$