Chap. 4 Analysis of Nonlinear Circuits

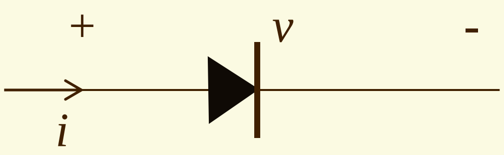
Diode: Nonlinear Element

Analytical Solutions

Graphical Solutions

Piecewise Linear Analysis

Diode: Nonlinear resistor

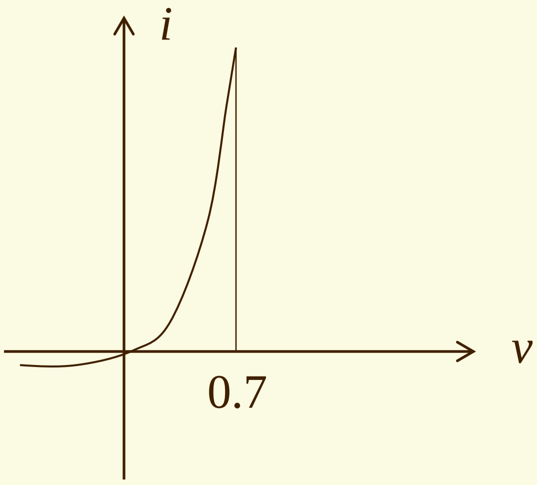


(a) Diode circuit symbol

$$i = I_S\left(e^{\frac{v_D}{V_{TH}}} - 1\right)$$

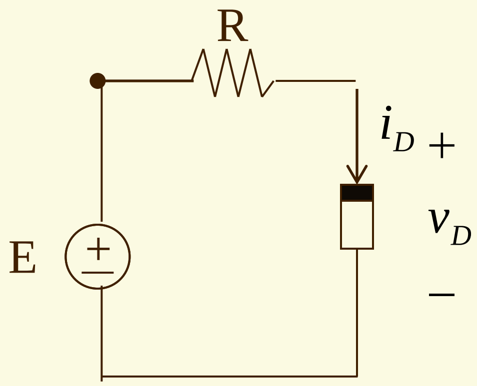
(b) characteristic equation

$$I_S \sim 10^{-12}, \quad V_{TH} = 0.025V$$



(c) i-v characteristic

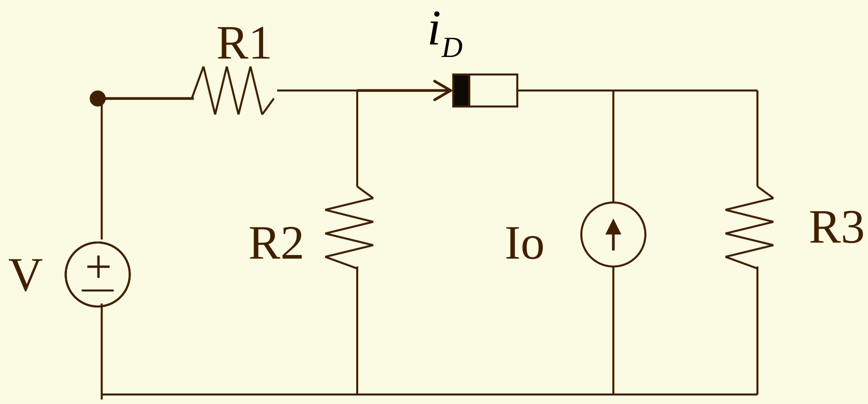
Circuit Example: Analytical Method

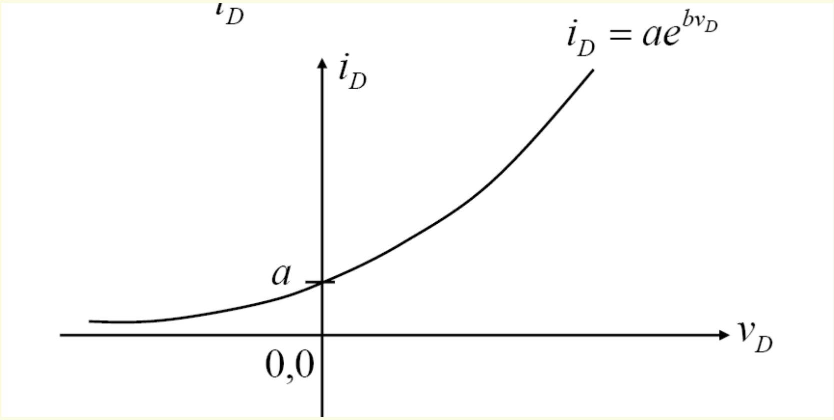


$$i_D = \begin{cases} K v_D^{\;2} & for\ v_D > 0 \\ 0 & for\ v_D \le 0 \end{cases}$$

- **Node method**
  - Solve the equation by
    - Trial and error, or
    - Numerical method

Another Example

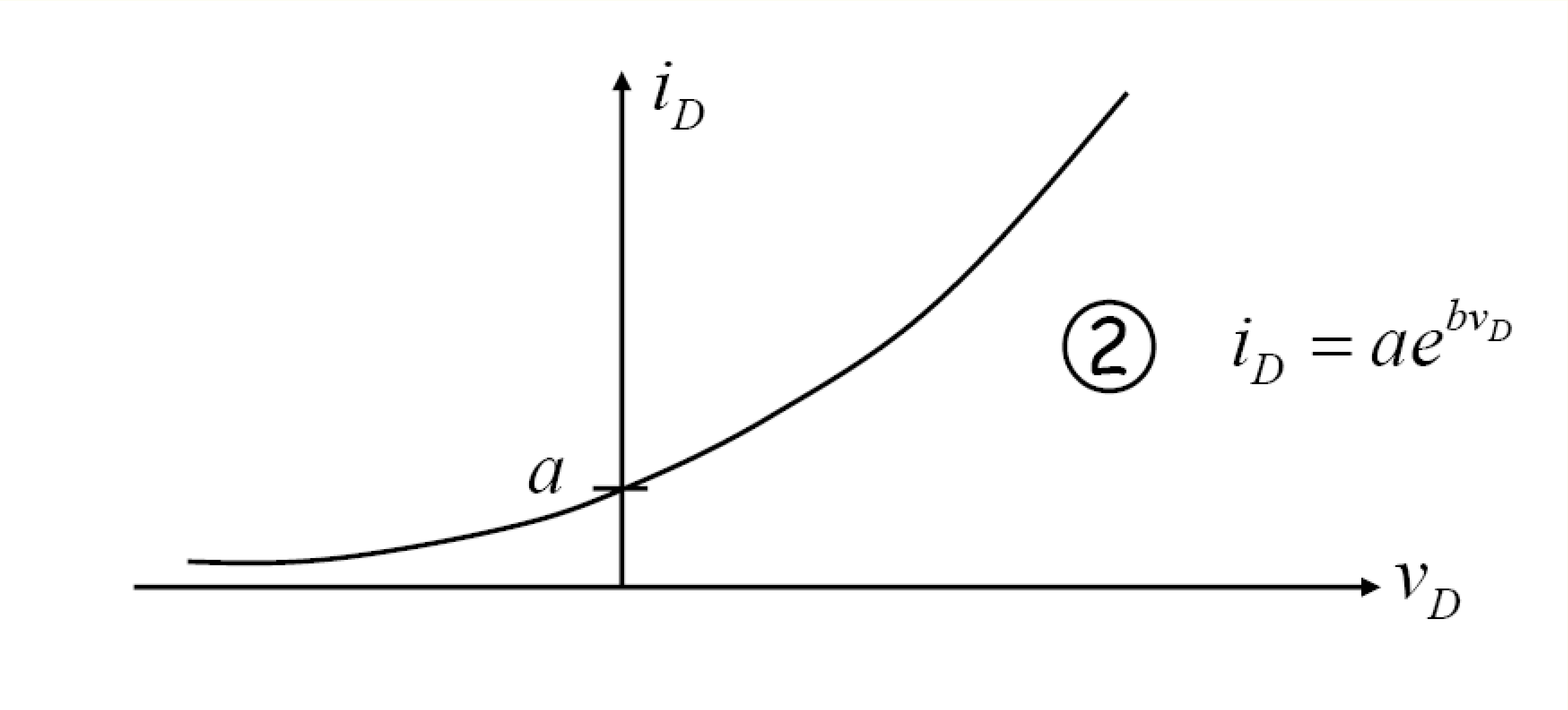




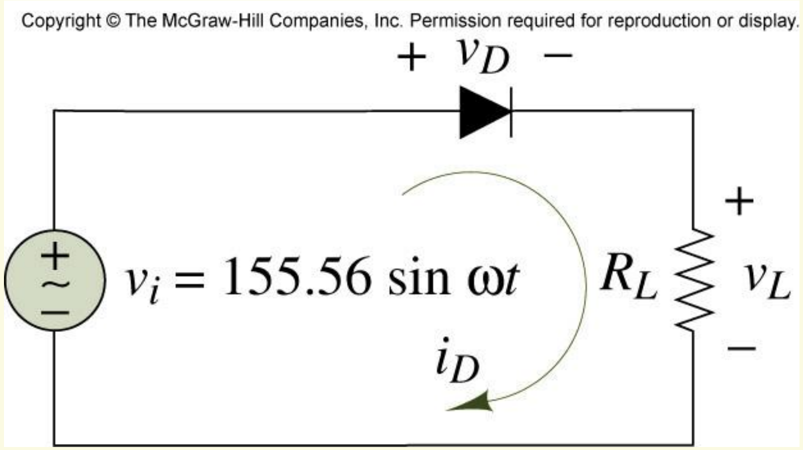
- Step 1: Simplify the circuit using Thevenin's theorem
- Step 2: Solve for the nonlinear element
- Step 3: Solve for the other elements

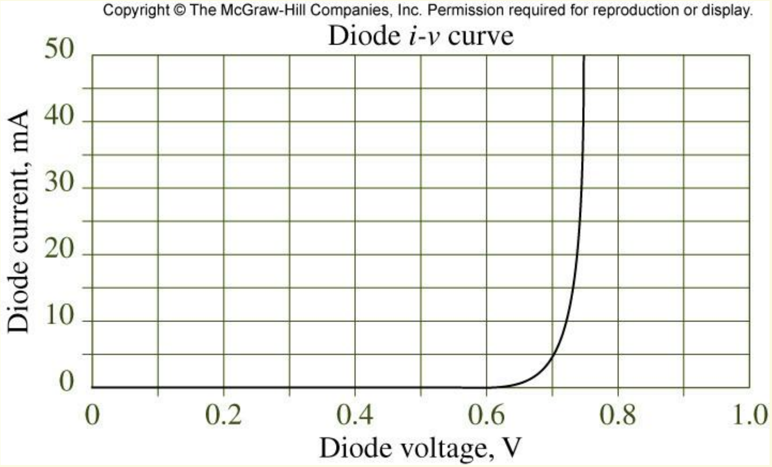
Graphical Method

- Load line:



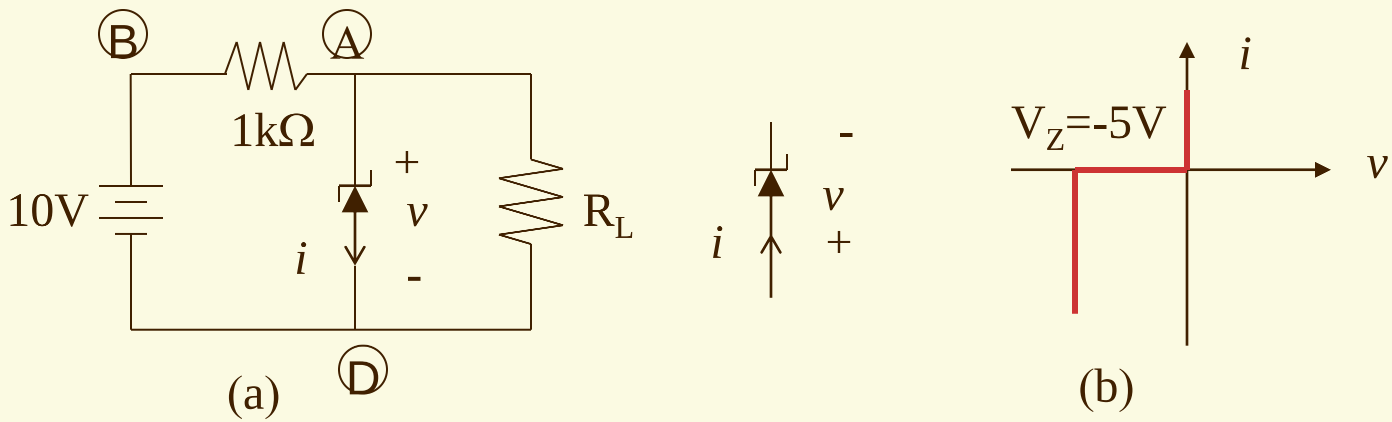
Example 4.10 Diode Rectifier





Exercise

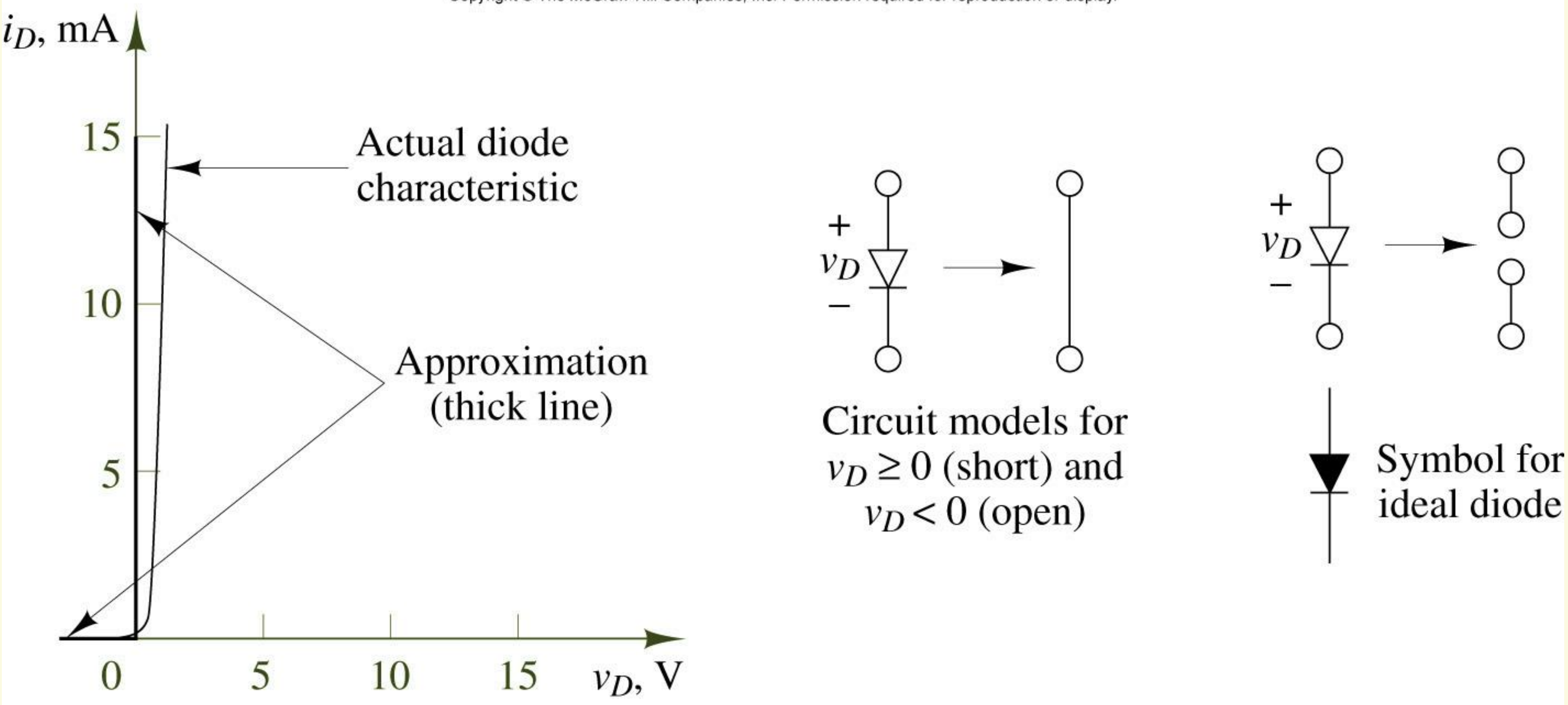
This is a non-linear circuit with a zenor diode. The i–v characteristic of the zenor diode is shown below. Find the minimum R to make the output voltage equal to 5V.



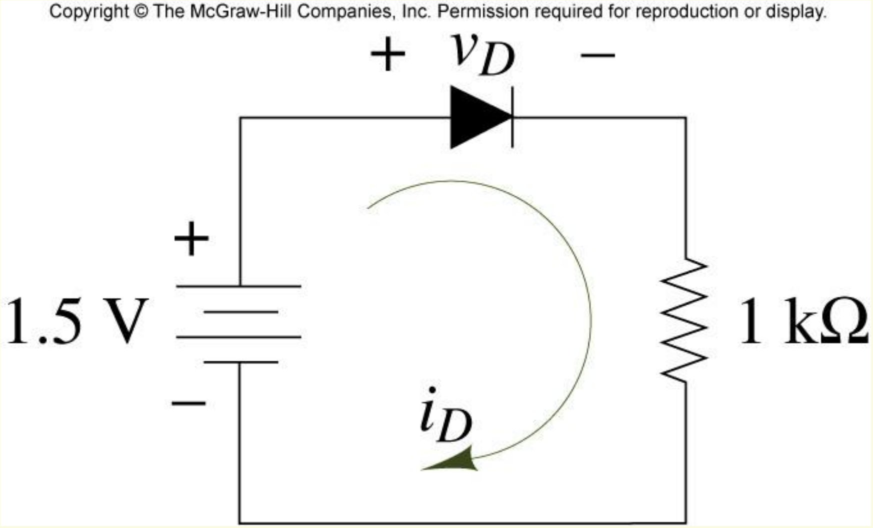
Piecewise Linear Analysis

- (1) Idea Diode Model

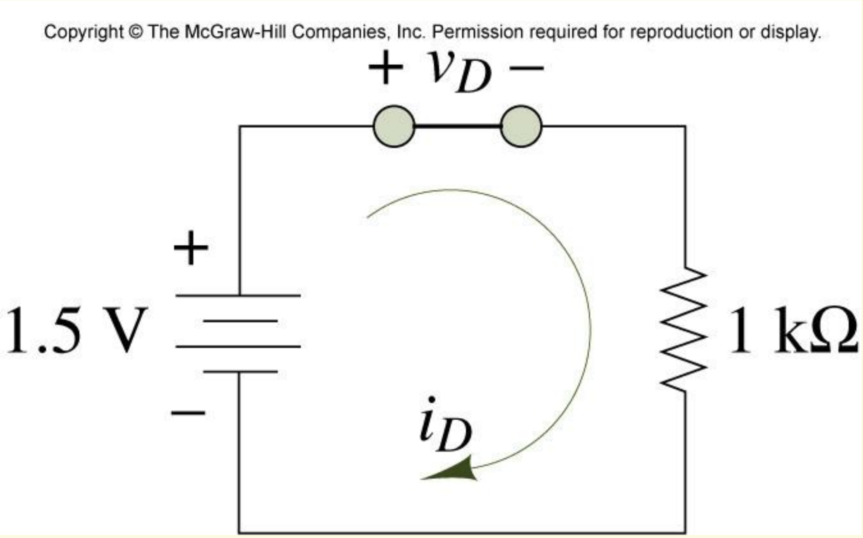
Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display.

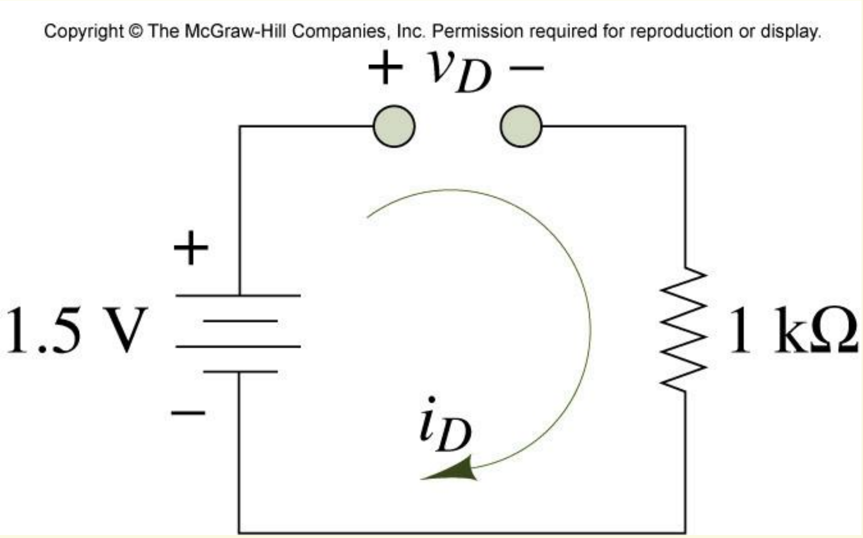


Circuit containing ideal diode

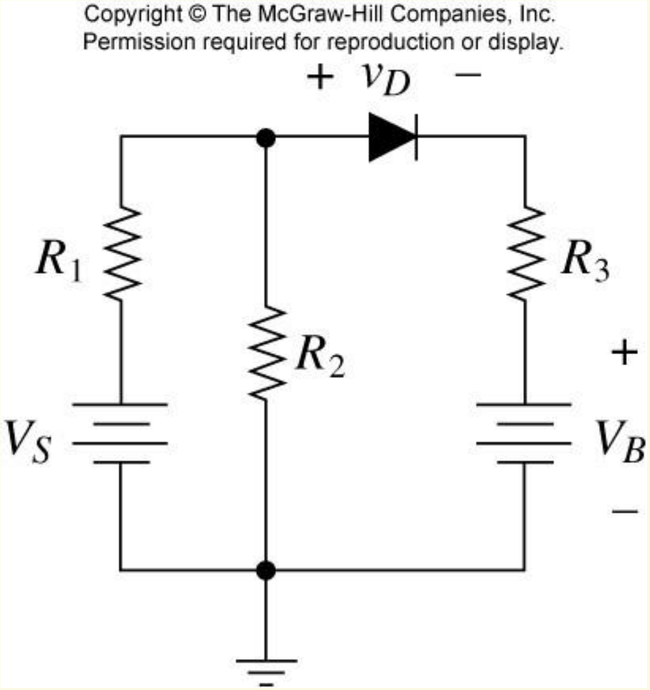


Determine the conduction state





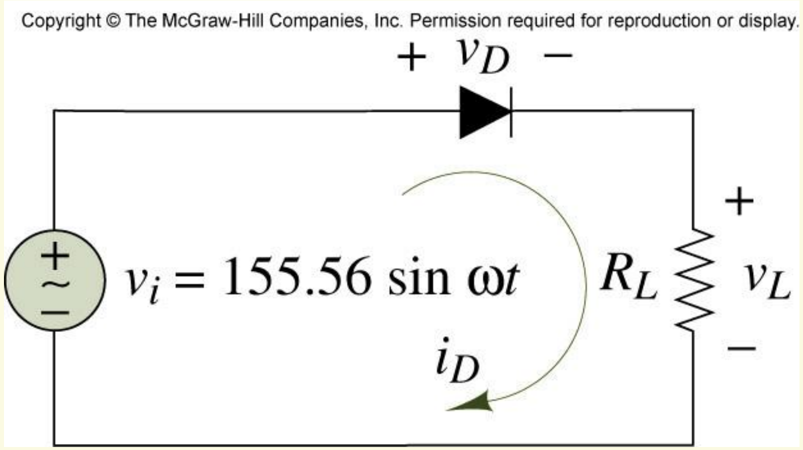
Example



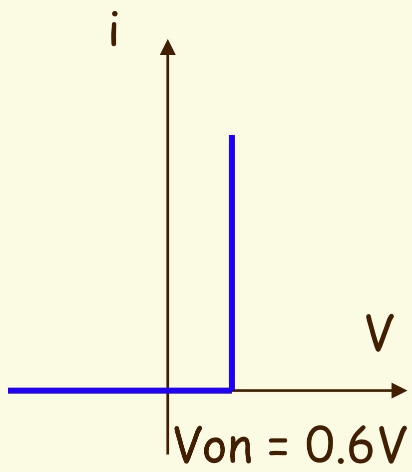
$$V_S = 12V, V_B = 11V,$$

$$R_1 = 5\Omega, R_2 = 10\Omega, R_3 = 10\Omega$$

Revisit: Halfwave Rectifier

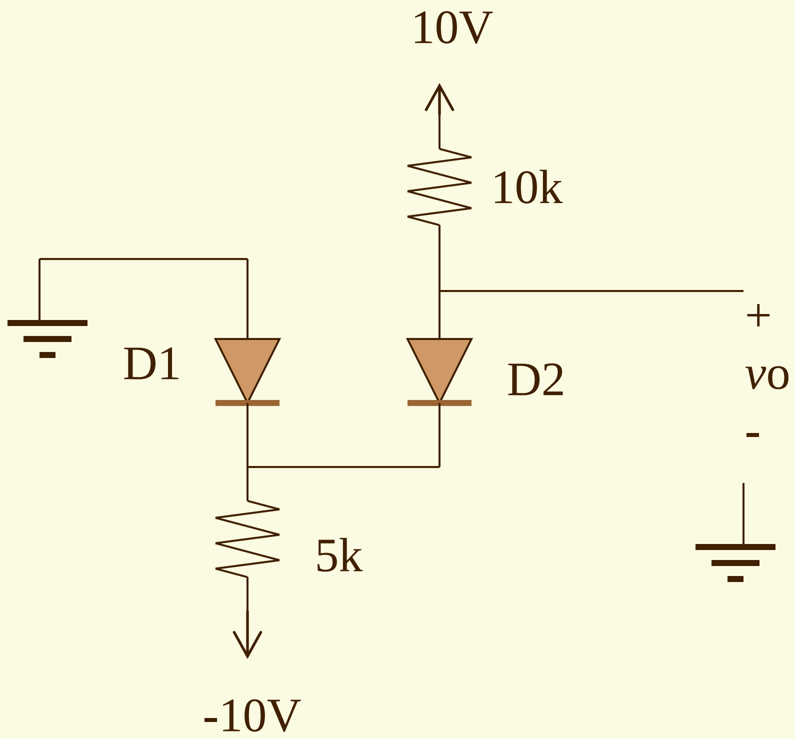


Offset Diode Model



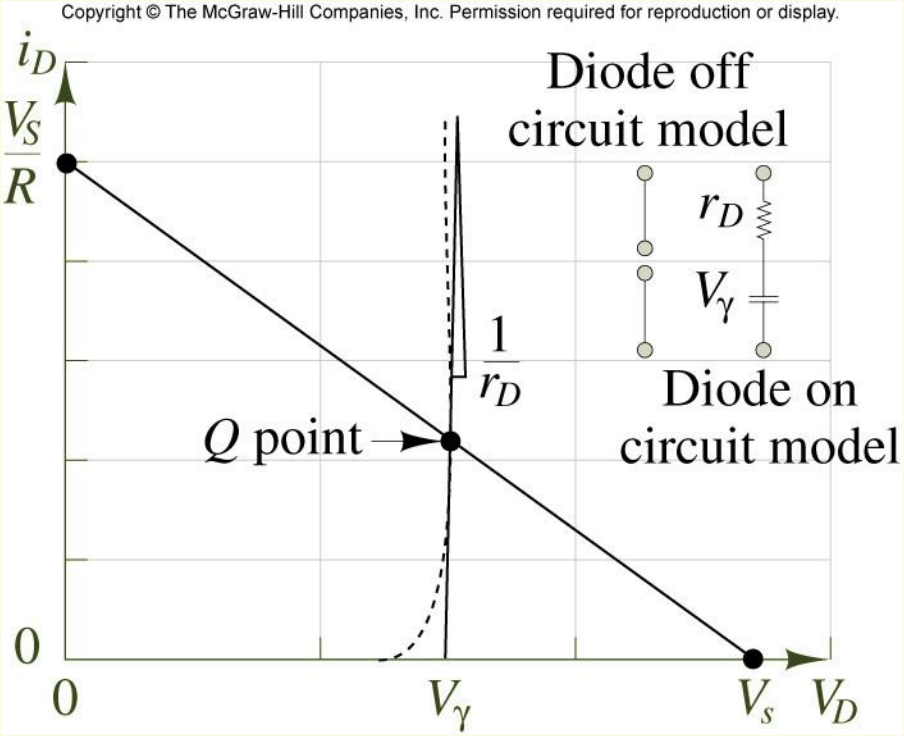
How will the Rectifier output change with this model?

Exercise



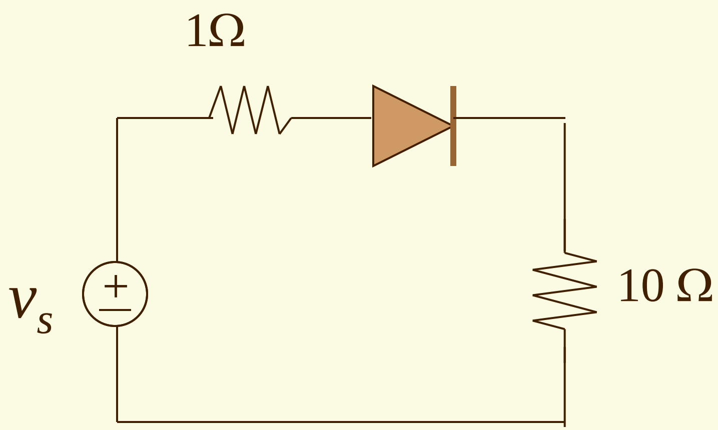
What would happen if we switch two resistor values?

Piecewise linear diode model

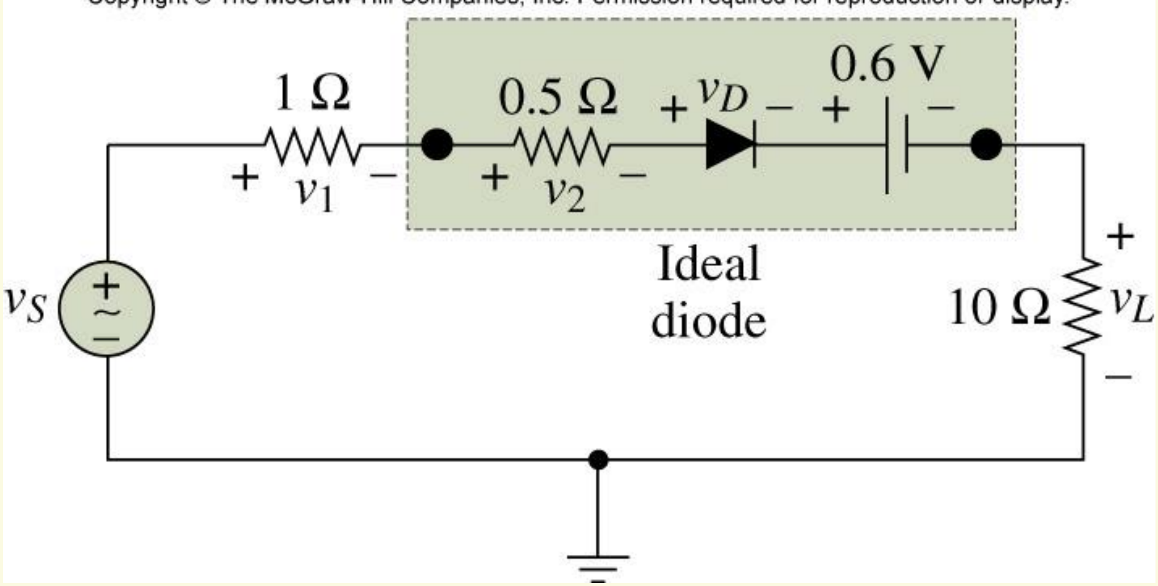


$$\frac{1}{r_d} = \frac{\partial i_D}{\partial v_D} = \frac{I}{V_T}$$

Example

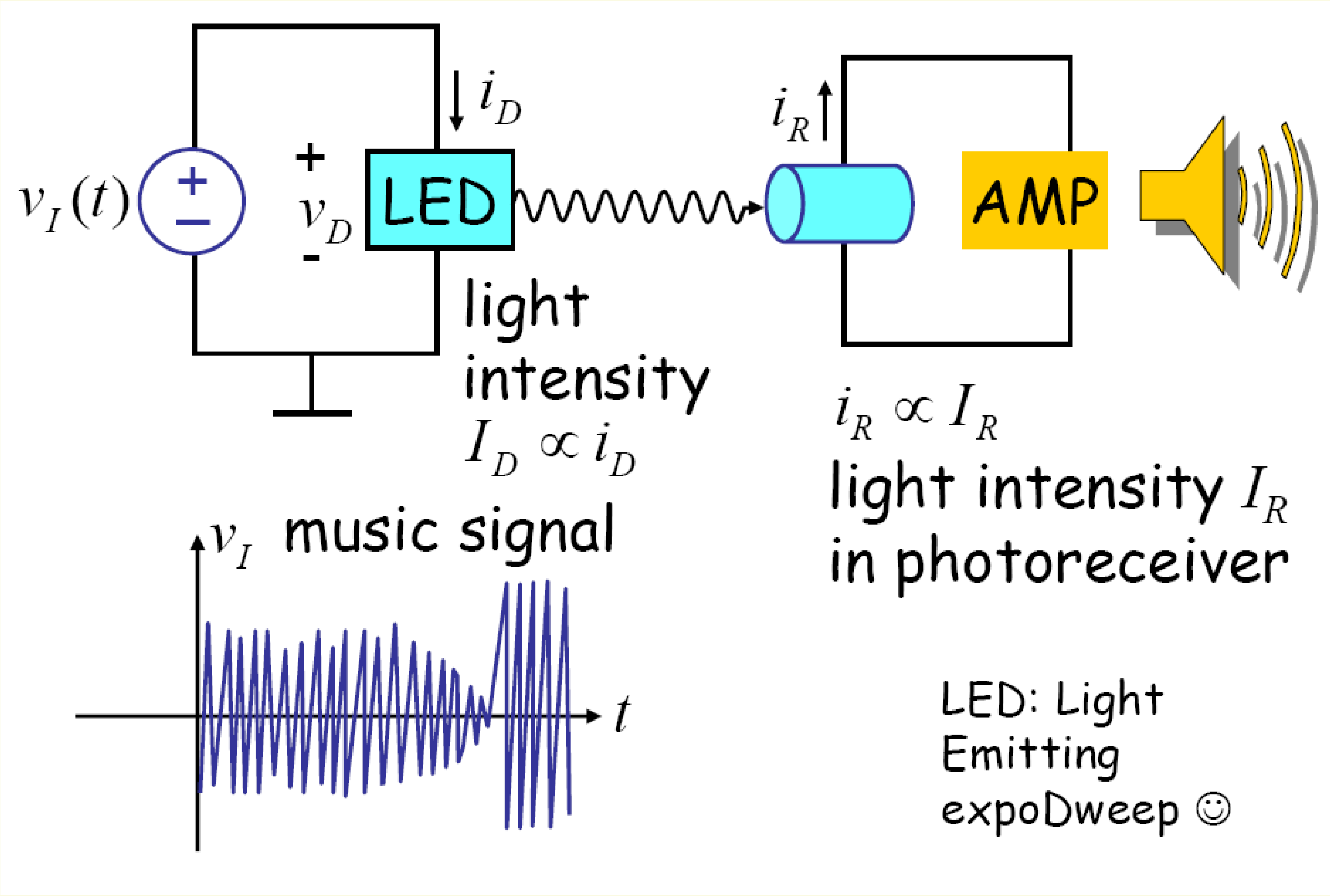


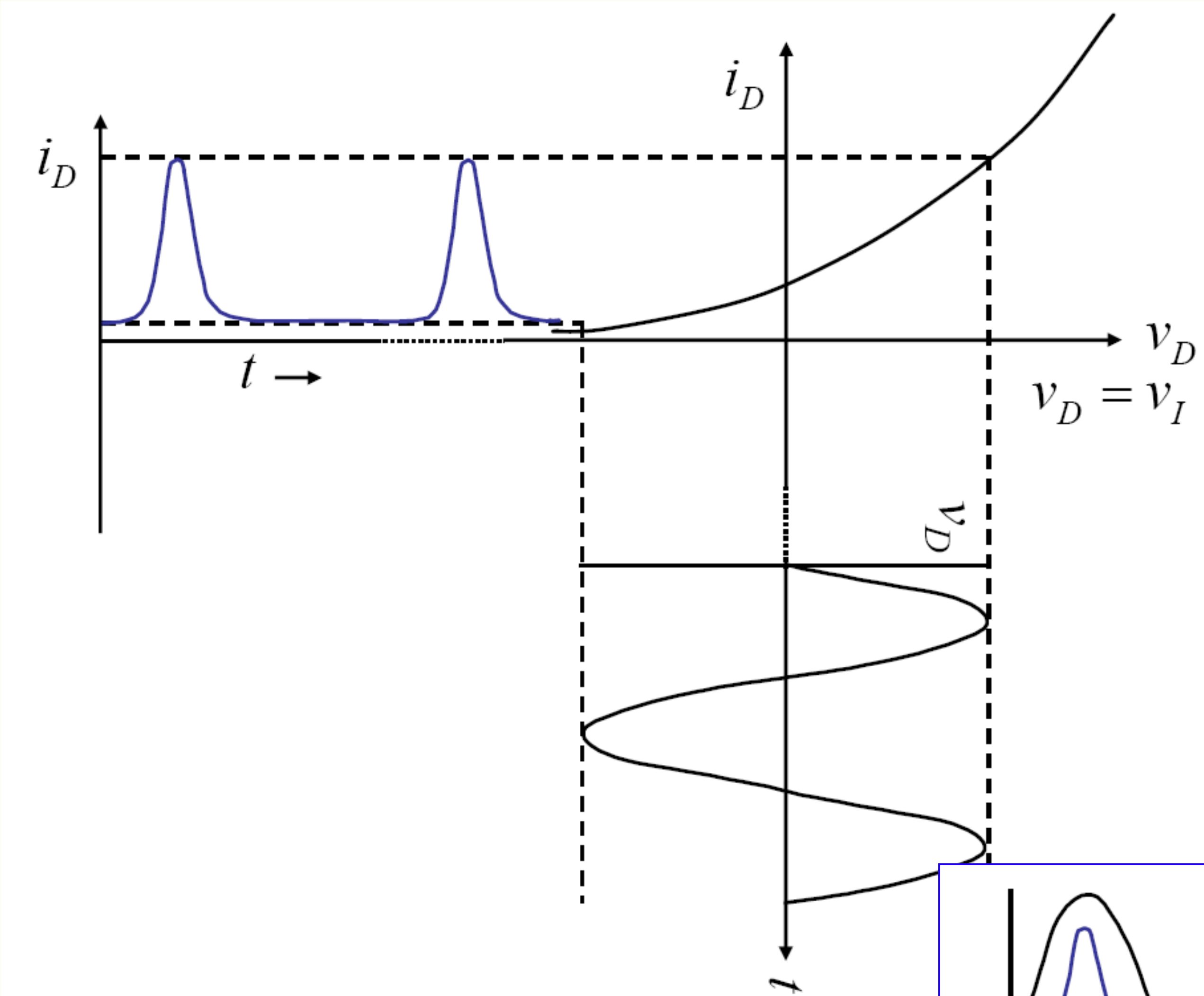
Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display.

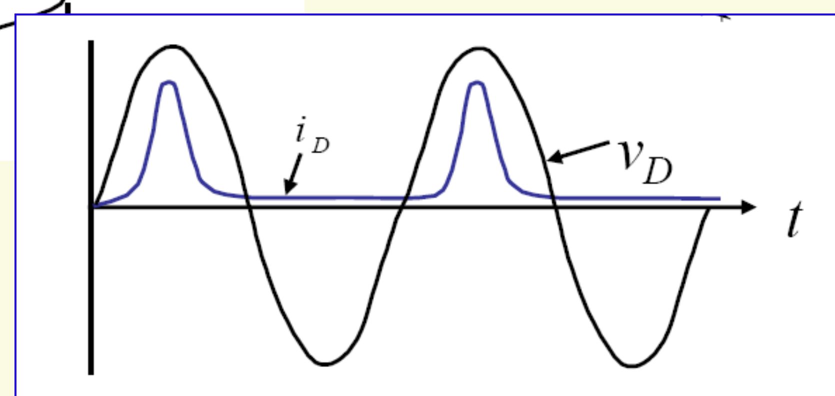


Incremental Analysis

- Motivational Example

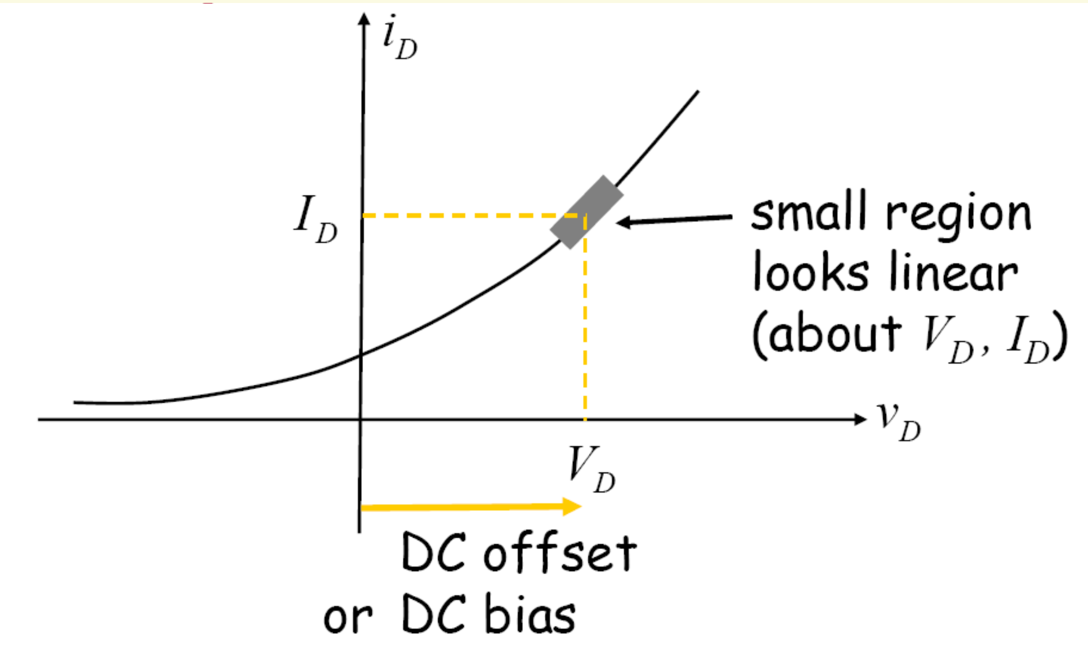



i_D
i_D
v_D
$t \rightarrow$
$v_D = v_I$
v_D
t

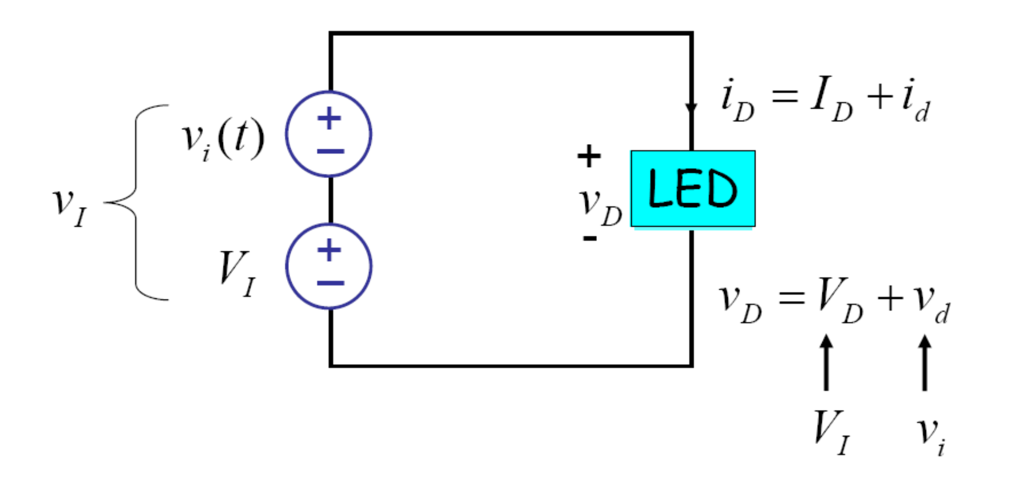

i_D
v_D
t

How to solve this problem?

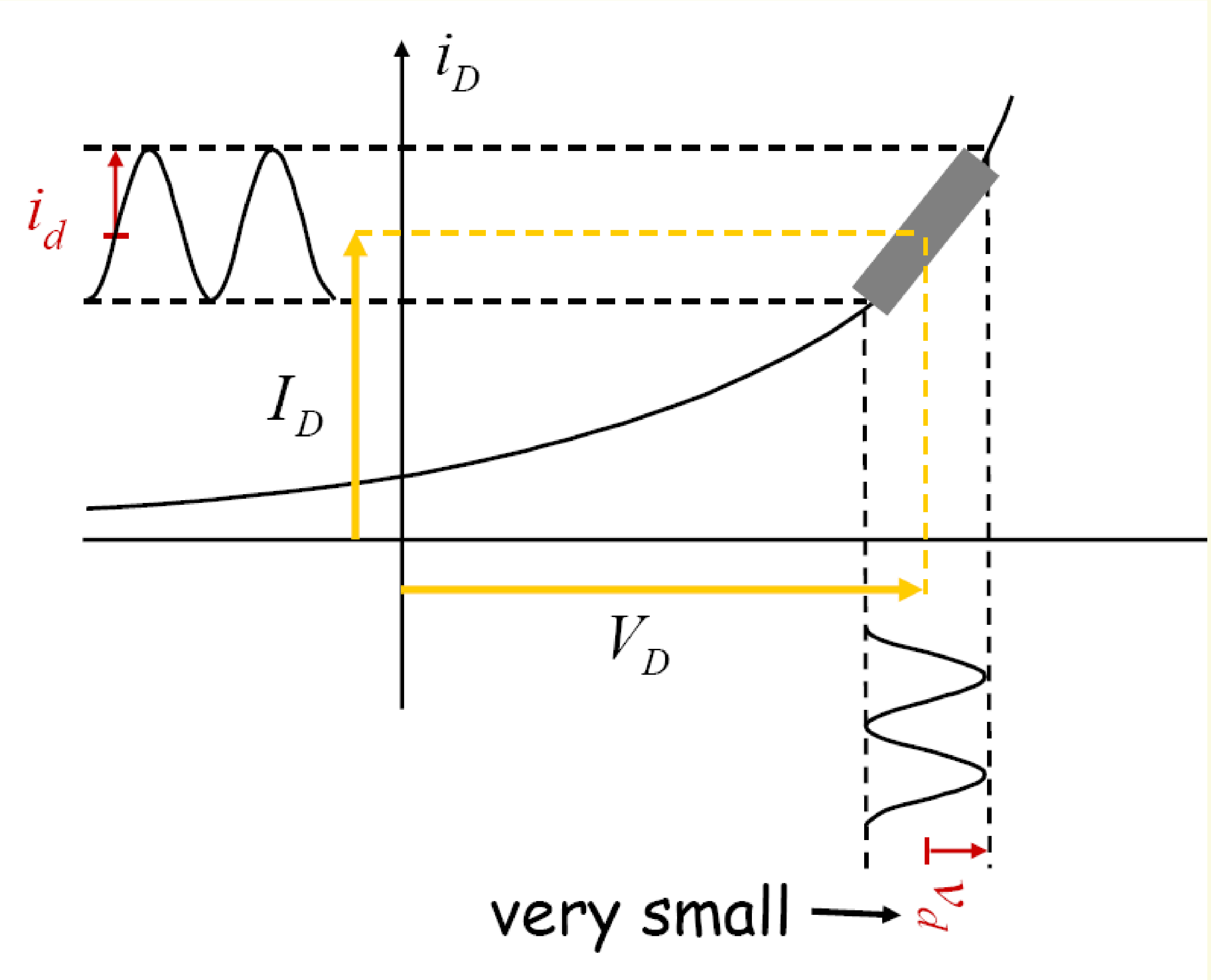
- Insight



- Solution: biasing



Solution



Incremental Method: small signal method

- Operate at some DC offset: bias point (V_D, I_D)
- Superimpose small signal on top of DC offset
- Solve the small signal circuit
  response to small signal is (approximately) linear

$$i_D = f(v_D), \quad v_D = V_D + \Delta v_D$$

$$i_D = I_D + \Delta i_D$$

In our example

$$i_D = a\, e^{b v_D}$$

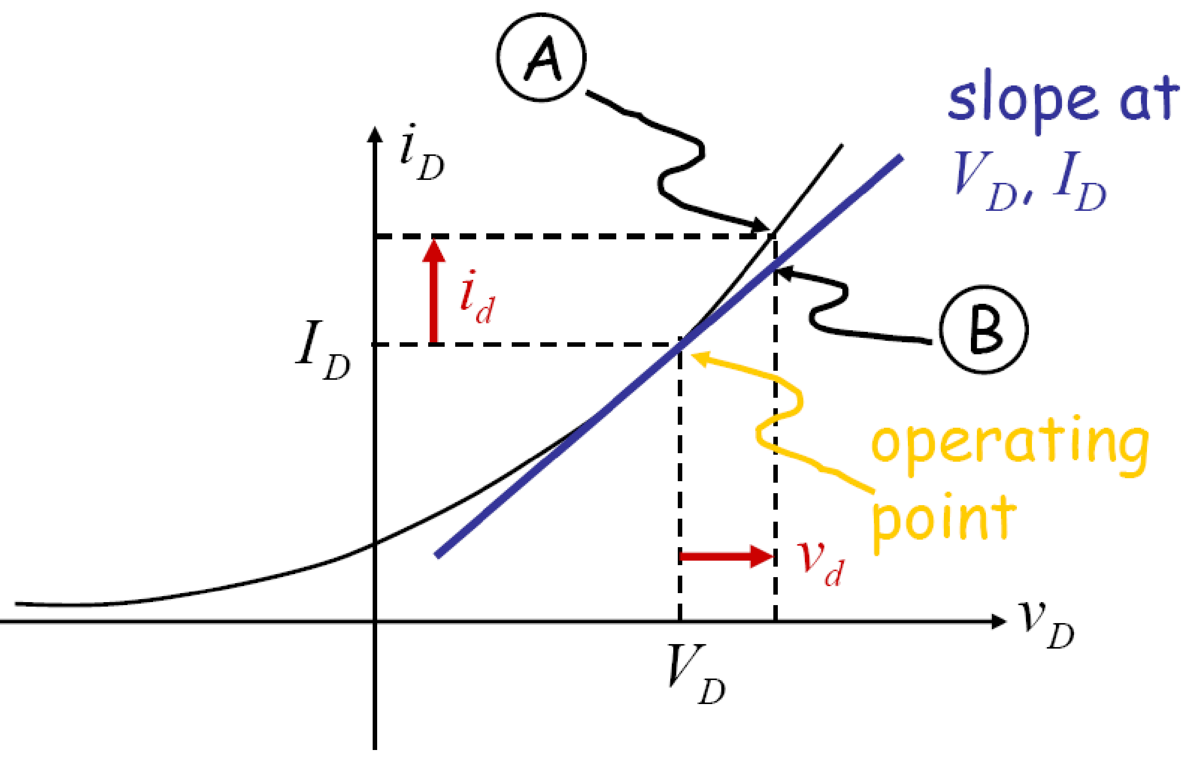
$$I_D + i_d \approx a\, e^{b V_D} + a\, e^{b V_D} \cdot b \cdot v_d$$

- Operating point

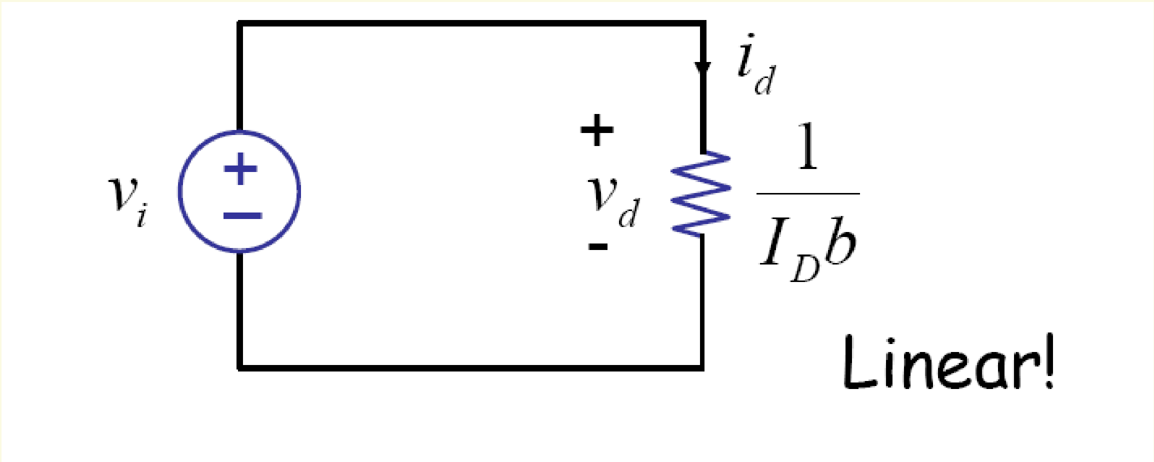
$$\boxed{I_D = a\, e^{b V_D}}$$

- Small signal

$$i_d = I_D \cdot b \cdot v_d$$

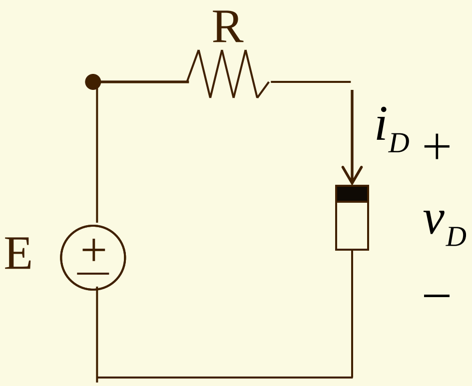


Small Signal Circuit



Example

- Draw the small signal circuit



$$i_D = \begin{cases} K v_D^{\,2} & \text{for } v_D > 0 \\ 0 & \text{for } v_D \le 0 \end{cases}$$

$$K = 1\, mA/V^2$$

$$V_D = 1V$$

Conclusion

- Nonlinear element: Diode is an example
- Solution methods
  - Analytical method
  - Graphical method
  - Piecewise linear method
- Incremental analysis: small signal analysis