Fusion Reactor Technology I
(459.760, 3 Credits)

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Week 14. Divertor and Plasma-Wall Interaction

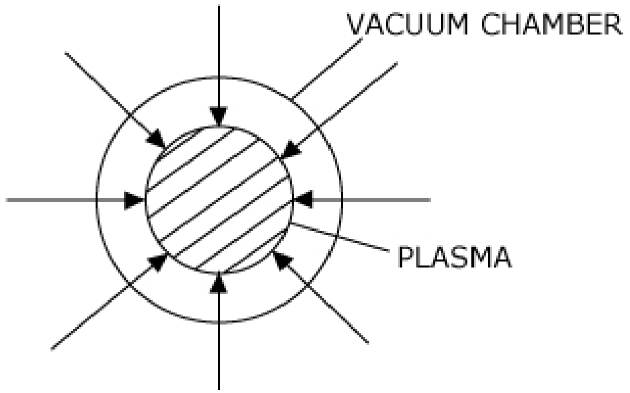
Poloidal Field Coils

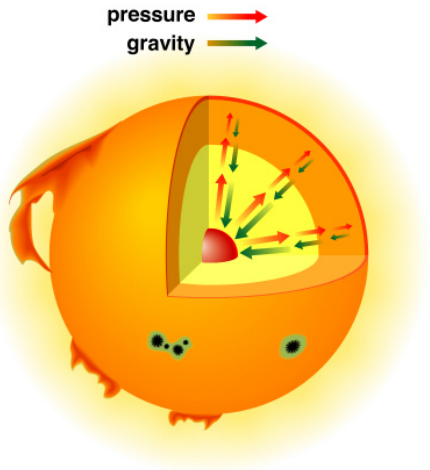
- Functions

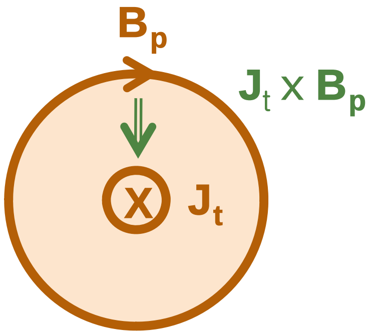
- To produce the magnetising flux
- To produce the main equilibrium field, shaping fields including divertor configuration
- Fast position feedback

Tokamak Equilibrium

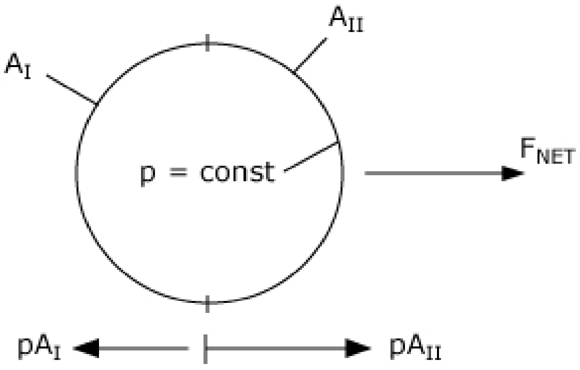
- **Basic Forces Acting on Tokamak Plasmas**
  - Radial pressure force







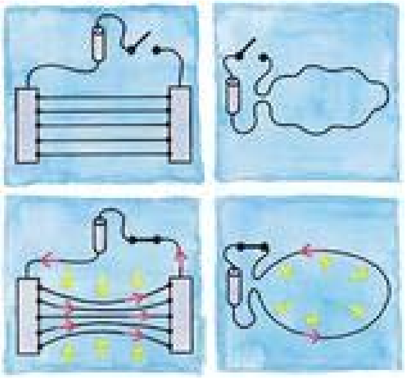
  - Tire tube force

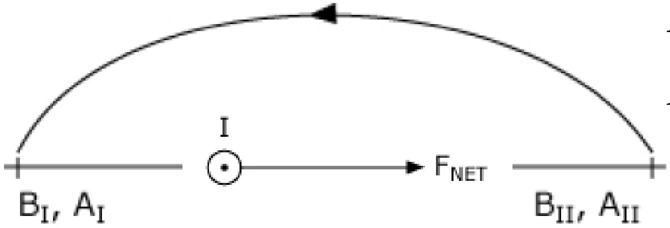


$$F_{NET} \sim -e_R(pA_I - pA_{II})$$

Tokamak Equilibrium

- **Basic Forces Acting on Tokamak Plasmas**
  - Hoop force

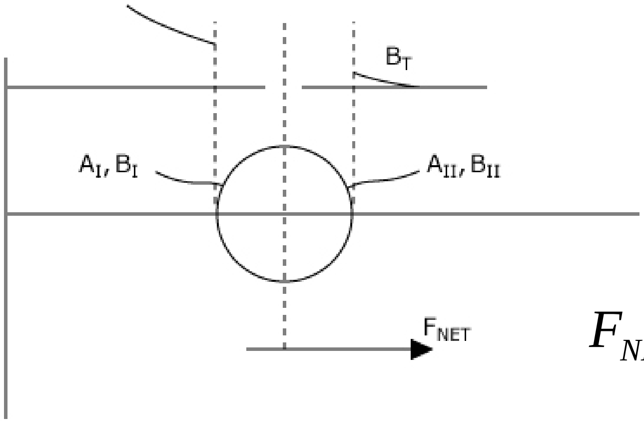




$$\phi_I = \phi_{II}$$
$$B_I > B_{II}, \quad A_I < A_{II}$$
$$B_I^2 A_I > B_{II}^2 A_{II}$$

$$F_{NET} \sim e_R (B_I^2 A_I - B_{II}^2 A_{II}) / 2\mu_0$$

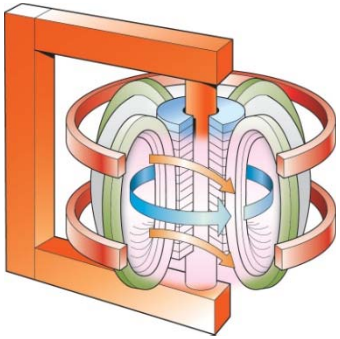
  - 1/*R* force



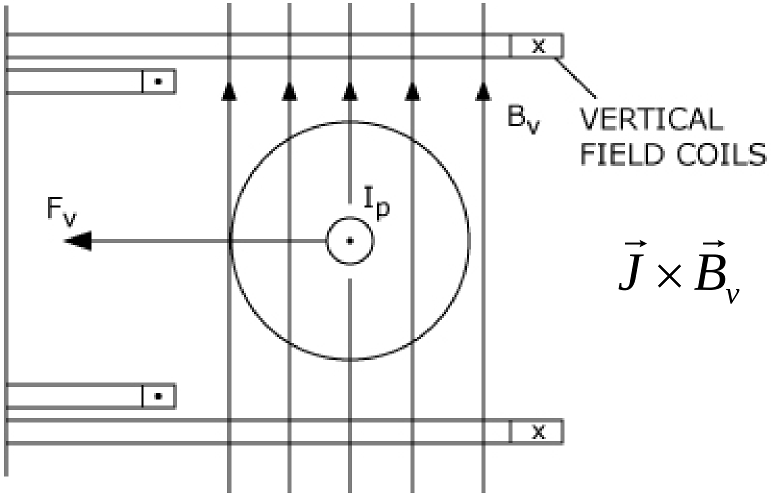
$$\phi_I = \phi_{II}$$
$$B_I > B_{II}, \quad A_I < A_{II}$$
$$B_I^2 A_I > B_{II}^2 A_{II}$$

$$F_{NET} = 2\pi^2 a^2 \frac{B^2}{2\mu_0}$$

Tokamak Equilibrium



- **Basic Forces Acting on Tokamak Plasmas**
  - External coils required to provide the force balance

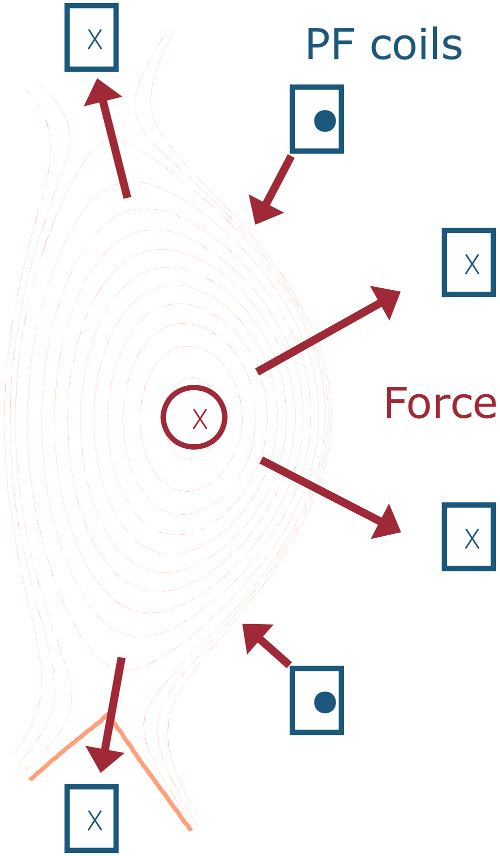


$$\vec{J} \times \vec{B}_v$$

$$F_v = BIL = 2\pi R_0 I_p B_v$$

$$B_\phi > B_\theta > B_v$$

$$B_\phi : B_\theta : B_v \approx 1 : \frac{\varepsilon}{q} : \frac{\varepsilon^2}{q}$$



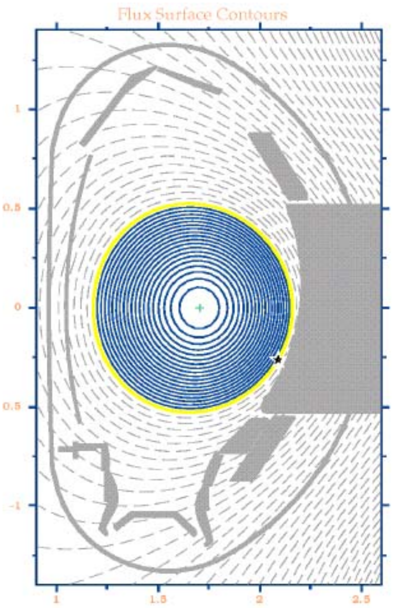
Tokamak Equilibrium

- **The Shafranov Shift**

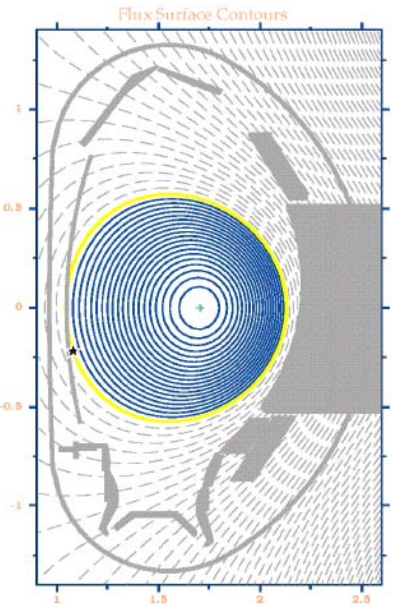
- outward shift of the flux surfaces
- consequences of toroidicity

ASDEX Upgrade

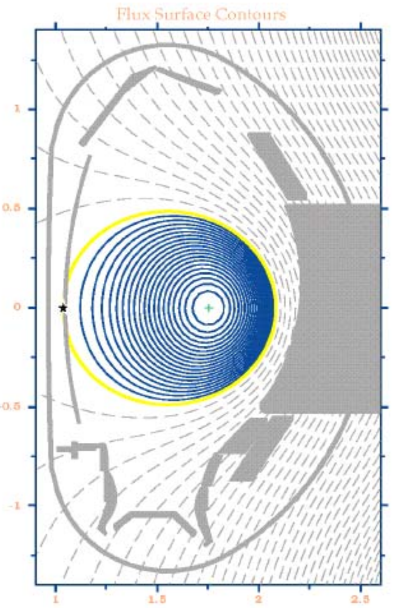
KSTAR



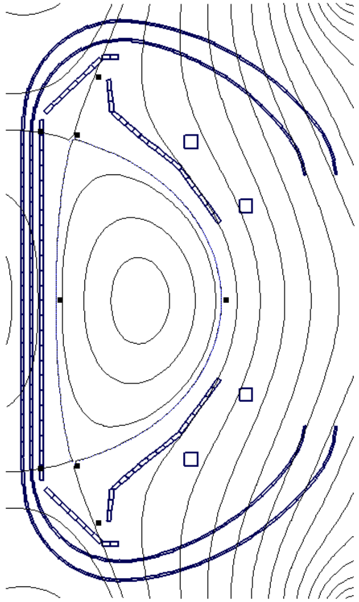
$\beta_p = 0.2$



$\beta_p = 1.2$



$\beta_p = 3.2$

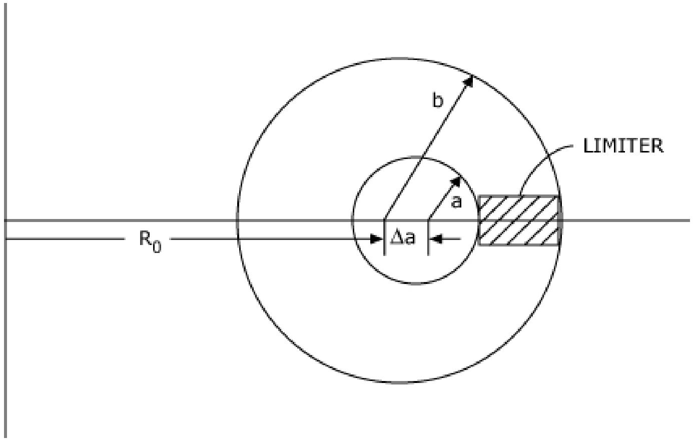


Tokamak Equilibrium

- **The Shafranov Shift**

- outward shift of the flux surfaces
- consequences of toroidicity

$$\frac{\Delta_a}{b} = \frac{b}{2R_0}\left\{\left(\beta_p + \frac{l_i}{2} - \frac{1}{2}\right)\left(1 - \frac{a^2}{b^2}\right) + \ln\frac{b}{a}\right\}$$



$$\frac{\Delta_a}{b} \sim \frac{b}{R_0} \sim \varepsilon << 1$$ small shift

$$\frac{\Delta_a^{(1)}}{b} \propto \beta_p$$ outward shift due to the tire tube force and the 1/*R*

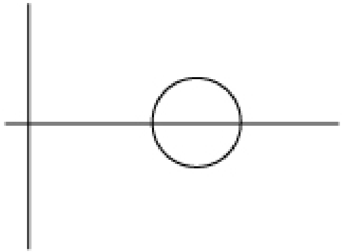
$$\frac{\Delta_a^{(2)}}{b} \propto \frac{l_i}{2}\left(1 - \frac{a^2}{b^2}\right) + \ln\frac{b}{a} - \frac{1}{2}\left(1 - \frac{a^2}{b^2}\right)$$ outward shift due to the hoop force

internal field   external field

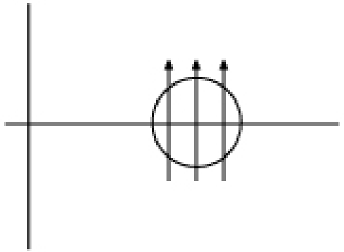
hoop force

Tokamak Equilibrium

- **Toroidal Force Balance by Means of a Vertical Field**
  - the new boundary condition including the vertical field



OLD: $\psi(b,\theta) = \text{CONST}$



NEW: $\psi(b,\theta) = \text{CONST} + R_0 B_V\, b\cos\theta$

  - new shafranov shift

$$\frac{\Delta_a}{b} = \frac{b}{2R_0}\left\{\left(\beta_p + \frac{l_i}{2} - \frac{1}{2}\right)\left(1 - \frac{a^2}{b^2}\right) + \ln\frac{b}{a}\right\} - \frac{B_v}{B_\theta(b)}$$

  - How much vertical field do we need to keep the plasma centered?

$$B_v = \frac{\mu_0 I}{4\pi R_0}\left\{\beta_p + \frac{l_i}{2} - \frac{3}{2} + \ln\frac{8R_0}{a}\right\}$$

Poloidal Field Coils

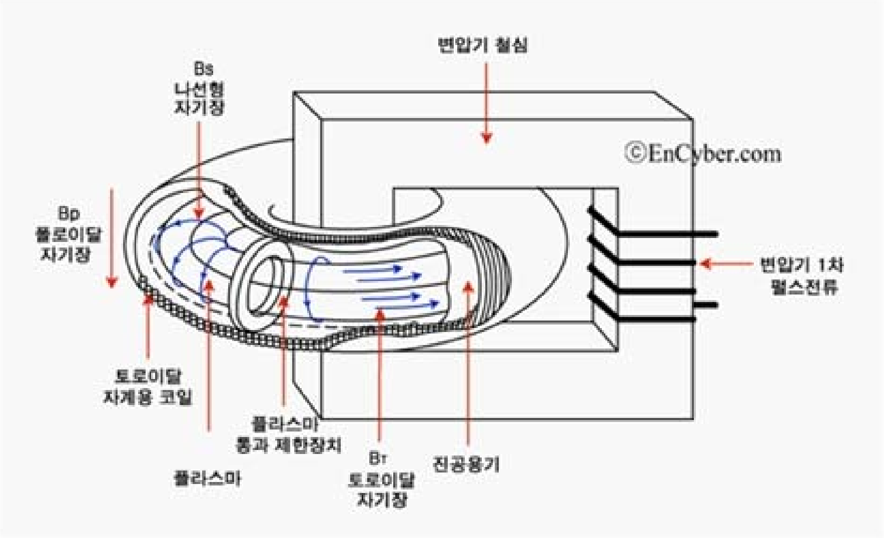
- Functions

- To produce the magnetising flux
- To produce the main equilibrium field, shaping fields including divertor configuration
- Fast position feedback

- Iron core

Discussion Time (5 min)

- Simpler magnetising system
- Difficult stray fields and equilibrium modelling



Poloidal Field Coils

- Acting forces

- Principal forces: self- (hoop) and vertical and radial forces arising from other PF coils and the plasma current
- Most highly stressed coil: OH solenoid due to the requirement of the large volt-seconds swing

- Magnetising winding

- Providing the flux swing necessary to produce and sustain the plasma current for the desired pulse duration
- The volt-second consumption (empirical)

$\Delta\Phi \approx 1.5IL + t_{pulse}V_{loop}$ for small machines

$\Delta\Phi \approx 2 + IL + t_{pulse}V_{loop}$ for large machines

- Total inductance of the plasma loop (*L*)
  Internal inductance + External inductance to the plasma CX

$$L = L_{\mathrm{int}} + L_{ext} \approx \mu_0 R\left[l_i / 2 + \ln\left(1.3R / a\kappa^{1/2}\right)\right] \approx 2R$$

li: normalised internal inductance (~ 0.8-1.6)

Poloidal Field Coils

- Flux swing with an iron-cored transformer

- Simple transformer design:
  very small net ampere-turns required for magnetising the iron
  good coupling between the primary and the plasma
  (total primary ampere-turns ~ plasma current)
- The primary windings can be placed almost anywhere.
- Soft iron saturates at around 2 T
  → required cross-sectional area of the core ≈ $\Delta\Phi/4$ (m^2),
  assuming a bidirectional flux swing

- Introducing toroidal asymmetries including RMPs
- Loss of equilibrium when saturated and sharp increase of the stray fields

Poloidal Field Coils

- Flux swing with an air-cored transformer

- Overcoming disadvantages of iron-cored transformer
- Extremely poor coupling generally → large primary ampere-turns, and strong constraints on the primary winding distribution to avoid the generation of stray fields in the plasma

- The volt-seconds produced by a simple long solenoid (bidirectional swing)

$$\Delta\Phi = 2B_{\max}\pi r_{sol}^2 = 8\pi^2 J r_{sol}^2 \delta r_{sol} f_{sol}$$

r_{sol}: radius
δr_{sol}: thickness
f_{sol}: packing fraction

- The average hoop stress

$$\sigma = 20\pi J^2 r_{sol} \delta r_{sol} f_{sol}$$

σ_{max} ≈ 30 MPa for OFHC Cu, 200 MPa for special alloys due to the fatigue-failure limit for the envisaged life

- The stress-limited flux swing

$$\Delta\Phi(\sigma) \approx 10\left(r_{sol}^3 \delta r_{sol} f_{sol} \sigma_{\max}\right)^{1/2} \longleftarrow J = \left(\sigma / 20\pi r_{sol} \delta r_{sol} f_{sol}\right)^{1/2}$$

Poloidal Field Coils

Heating rate for an epoxy-resin insulated OFHC copper solenoid

$$\int J^2 dt \leq 133\,\mathrm{kA^2 cm^{-4}} s$$

$$\int J^2 dt = J_{\max}^2 t_{pulse} / 3$$

For a triangular current waveform

$$J_{\max}(\theta) \leq 20 / t_{pulse}^{1/2}$$

$$\Delta\Phi(\theta) \approx 1600 r_{sol}^2 \delta r_{sol} f_{sol} / t_{pulse}^{1/2} \longleftarrow$$

$$\sigma = 20\pi J^2 r_{sol} \delta r_{sol} f_{sol}$$

$$\Delta\Phi(\sigma) \approx 10\left(r_{sol}^3 \delta r_{sol} f_{sol} \sigma_{\max}\right)^{1/2}$$

- Juggling with the r_{sol} and δr_{sol} to obtain the desired volt-seconds swing without breaking or overheating the magnetising solenoid

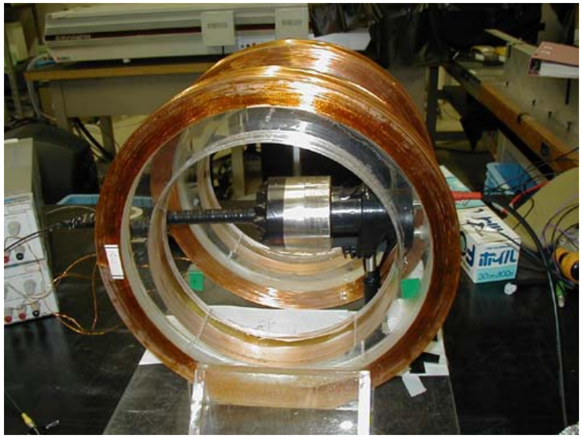
Poloidal Field Coils

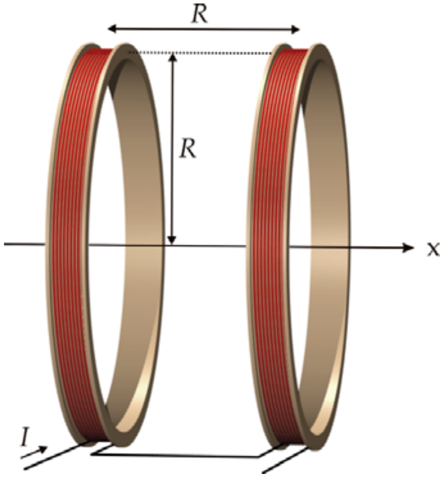
- Vertical field requirement

$$B_v \approx \frac{I}{10R_0}\left[\ln(8R_0/a)+\beta_p+l_i/2-3/2\right]\approx\frac{I}{10R_0}\ln(6R_0/a)$$

$B_v \approx I_{coil}/1.1R_{coil}$ Vertical field produced by a Helmholtz coil pair

What is a Helmholtz coil pair?





a device for producing a region of nearly uniform magnetic field.
It is named in honor of the German physicist Hermann von Helmholtz.
http://en.wikipedia.org/wiki/Helmholtz_coil

Poloidal Field Coils

- Vertical field requirement

$$B_v \approx \frac{I}{10R_0}\left[\ln\left(8R_0/a\right)+\beta_p+l_i/2-3/2\right]\approx\frac{I}{10R_0}\ln\left(6R_0/a\right)$$

$B_v \approx I_{coil}/1.1R_{coil}$ Vertical field produced by a Helmholtz coil pair

$$I_{coil}\approx 0.11\frac{R_{coil}}{R_0}I\ln\left(6R_0/a\right) \longleftarrow J_{\max}(\theta)\le 20/t_{pulse}^{1/2}$$

\- Minimum cross-sectional area for this coil if constructed with epoxy-resin insulated copper

$$A_{coil}\ge 10t_{pulse}^{1/2}R_{coil}I\ln\left(6R_0/a\right)/R_0 f_{coil}$$

Poloidal Field Coils

- Flux swing with an iron-cored transformer

- The primary voltage

$$V_{pri} = N_t V_{loop} + I_{pri} R_{pri}$$

N_t: number of primary turns

$$I_{pri} = I / N_t$$

$$V_{loop} = V_{resistive} + \left(\dot{I}L\right) \approx \left(1 \sim 2\right) + 2\dot{I}R_0$$

- Power supply

$$L_{tot} \approx 1.5 \times 3.9 N_{sol}^2 r_{sol}^2 l_{sol}$$

$$P_{resistive(sol)} \approx 5\pi r_{sol} f_{sol} \delta r_{sol} l_{sol} J^2$$

$$P_{resistive}\left(B_v\right) \approx J^2 f_{coil} A_{coil} R_{coil} / 400$$

for flattop

$$P_{resistive}\left(B_v\right) \approx 3 I_{coil} \dot{I}_{coil} R_{coil} \ln\left(\frac{1.1 R_{coil}}{a_{coil}}\right)$$

: including the ramp-up phase

- Feedback systems: relatively lower power but fast, typically based on thyrister choppers or linear amplifiers of 10-500 kW

Poloidal Field Coils

- Alignment

- The PFCs have to be circular and well aligned to the TFC to avoid producing RMPs and possible islands.
- The PFCs also have to be positioned in radius and height so as to minimise stray perpendicular fields.
- The required positional tolerance is ~ 10^{-3} of the major radius of the machine for each type of error.

Support Structure

- Need to accommodate the toppling forces on the TFCs and the vertical forces on the PFCs
- Responsible for maintaining the alignment of all the TFCs and PFCs
- Stainless steel commonly used to obtain high strength with low magnetic permeability

  Cf. Permeability of stainless steel increases where worked, cut or welded, and so one (sometimes even after heat treatment).

  → generating nonaxisymmetric and potentially RMPs
- Any volume of unsaturated magnetic material creates a disturbance in the field breaking symmetry and likely to generate islands.

Ex. ITER TBM

- Critical volume to generate 10^{-4} of the PF at the plasma edge

$$V_c \leq 300 R^6 / R_0^2 a(\mu_R - 1)$$

R: range of the offending volume from the machine centre

- Critical volume to generate 10^{-4} of the PF at the machine centre

$$V_c \leq 250 IRR_0^2 / aB_0(\mu_R - 1)$$