Fusion Reactor Technology I
(459.760, 3 Credits)

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Contents

Contents

Fusion Reactor Energetics

- **Fundamental requirement of a fusion reactor system**

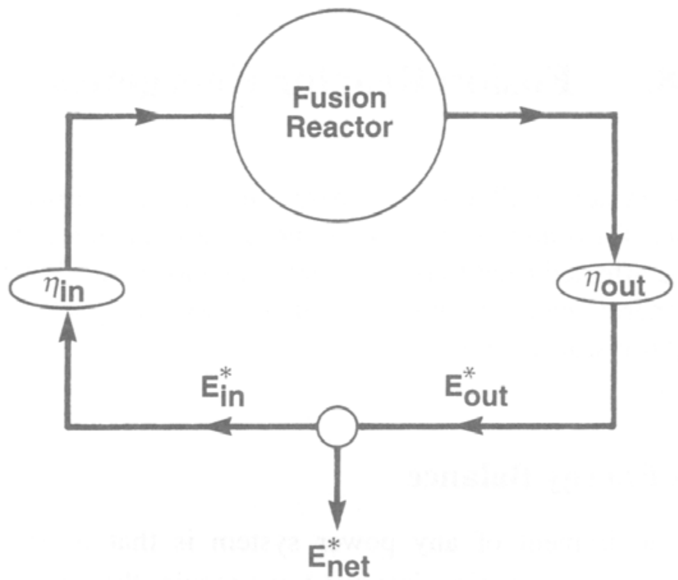
$$E^*_{net} = E^*_{out} - E^*_{in} > 0$$

Considering the time variations of power

$$\int_o^\tau \left(\frac{dE^*}{dt}\right)_{net} dt = \int_o^\tau \left(\frac{dE^*}{dt}\right)_{out} dt - \int_o^\tau \left(\frac{dE^*}{dt}\right)_{in} dt > 0$$

- **Fusion Plasma Energy Balance**

τ_b: burning time



$$\int_o^{\tau_b} \frac{dE^*_{th}}{dt} dt = E^*_{aux} + E^*_{fu} - E^*_n - E^*_{rad} - \int_o^{\tau_b} \frac{E^*_{th}}{\tau^*_E} dt$$

$$E^*_{aux} = \eta_{in} E^*_{in}$$

$$\frac{E^*_n}{E^*_{fu}} = 1 - f_c \qquad E^*_{fu} - E^*_n = f_c E^*_{fu}$$

Fusion Reactor Energetics

$$Q_p = \frac{E^*_{fu}}{E^*_{aux}} = \frac{E^*_{fu}}{\eta_{in} E^*_{in}}$$

Plasma *Q*-value (fusion multiplication factor): measure for how efficiently an energy input to the plasma is converted into fusion energy

$$\int_o^{\tau_b} \frac{dE^*_{th}}{dt} dt = E^*_{aux} + E^*_{fu} - E^*_n - E^*_{rad} - \int_o^{\tau_b} \frac{E^*_{th}}{\tau^*_E} dt \rightarrow \int_o^{\tau_b} \frac{dE^*_{th}}{dt} dt = \left(f_c + \frac{1}{Q_p} \right) E^*_{fu} - E^*_{rad} - \int_o^{\tau_b} \frac{E^*_{th}}{\tau^*_E} dt$$

$$\int_o^{\tau_b} dE^*_{th} dt = E^*_{th}(\tau_b) - E^*_{th}(0)$$

If, steady state

$$E^*_{fu} = \frac{E^*_{rad} + \int_o^{\tau_b} \frac{E^*_{th}}{\tau^*_E} dt}{f_c + \frac{1}{Q_p}}$$

- if, $Q_p \rightarrow \infty$, the fusion energy delivered to the plasma via the charged reaction products is seen to balance the total energy loss from the plasma.

Fusion Reactor Energetics

$$f_c E_{fu}^* = E_{rad}^* + \int_o^{\tau_b} \frac{E_{th}^*}{\tau_E^*} dt$$

Considering a D-T plasma with $Q_p \to \infty$ (ignition),

$$f_{c,dt} \int_V d^3r \int_o^{\tau_b} R_{dt}(\vec{r},t) Q_{dt} dt = \int_V d^3r \left[\int_o^{\tau_b} (P_{br} + P_{cyc}^{net}) dt + \int_o^{\tau_b} \frac{E_{th}(\vec{r},t)}{\tau_E(\vec{r},t)} dt \right]$$

$$P_{br} = A_{br} n_i n_e Z^2 \sqrt{kT_e} \qquad A_{br} \approx 1.6 \times 10^{-38} \left(\frac{m^3 J}{\sqrt{eV} s} \right)$$

$$P_{cyc}^{net} = A_{cyc} n_e B^2 k T_e \psi \qquad A_{cyc} \approx 6.3 \times 10^{-20} (JeV^{-1} T^{-2} s^{-1})$$

$$\int_V d^3r \frac{E_{th}(\vec{r},t)}{\tau_E(\vec{r},t)} = \frac{E_{th}^*(t)}{\tau_{E*}(t)}$$

volume integrated

In a steady state, homogeneous plasma, local D-T fusion ignition condition

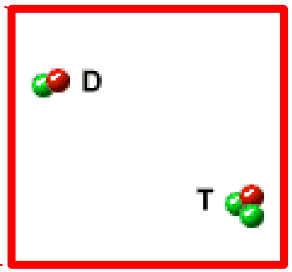
$$f_{c,dt} P_{dt}(n_i, T_i) = P_{br}(n_i, n_e, T_e) + P_{cyc}^{net}(n_e, T_e) + \frac{3}{2} \frac{(n_i T_i + n_e T_e)}{\tau_E}$$

$$E_{th.j} = \frac{3}{2} n_j T_j , \quad j = i, e$$

- complex interrelation between the plasma density and its temperature as required for ignition

Fusion Reactor Energetics

What is required to light a fire in a stove?



- Fuel: D, T
- Amount/density: n
- Heat insulation: τ
- Ignition temperature: T

Deuterium

Tritium

$\geq ?$

Lawson Criterion

Fusion Reactor Energetics

- **Ignition**

$$\frac{E^*_{fu}}{\eta_{in} E^*_{in}} = Q_p \rightarrow \infty$$

Energy viability of the fusion plasma:
actual self-sustaining engineering reactor condition with no heating power

$$f_{c,dt} P_{dt}(n_i, T_i) = P_{br}(n_i, n_e, T_e) + P^{net}_{cyc}(n_e, T_e) + \frac{3}{2}\frac{(n_i T_i + n_e T_e)}{\tau_E}$$

- Charged particle self-heating power > loss powers (radiation+plasma)

$$f_{c,dt} P_{dt} \tau_{E*} \geq (P_{br} + P^{net}_{cyc}) \tau_{E*} + 3nT$$

$$(n\tau_{E*})_{dt} \geq \frac{3T}{f_{c,dt} Q_{dt} \frac{<\sigma v>_{dt}(T)}{4} - A_{br}\sqrt{T} - \frac{A_{cyc} B^2 \psi T}{n}}$$

no energy conversion efficiency contained

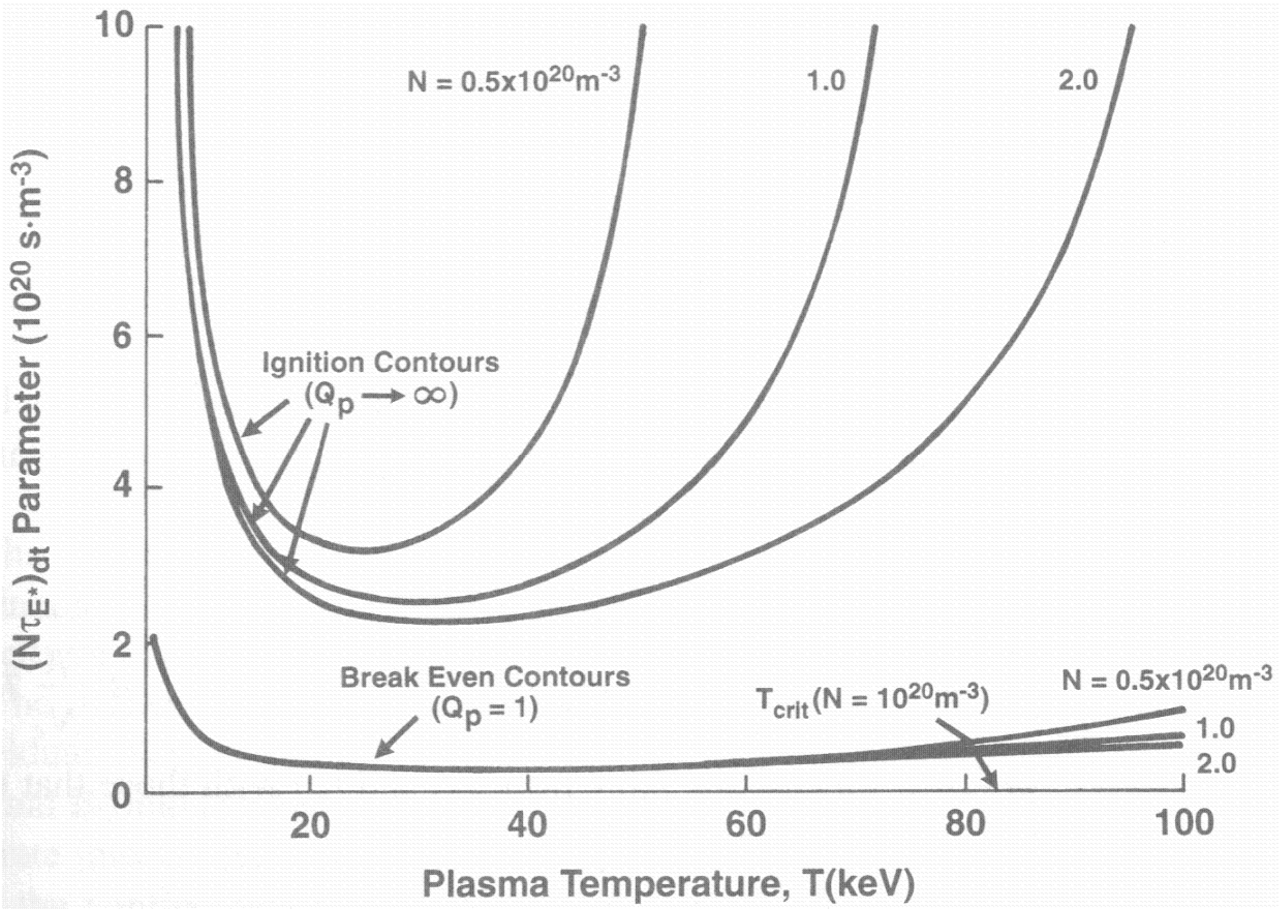
$<\sigma v>_{dt} \propto T^2$ at 10-20 keV $\rightarrow n\tau_{E*} T$

- **Break-even (scientific)**

The total fusion energy production amounts to a magnitude equal to the effective plasma energy input.

$$\frac{E^*_{fu}}{\eta_{in} E^*_{in}} = Q_p = 1$$

Status of the Tokamak Research



- n = 10^{20} m^{-3}: T ~ 30 keV, $n\tau_{E*}$ ~ 2.7×10^{20} $m^{-3}s$, τ_{E*} ~ 2.7 s
- Ignition contours tend towards infinity as T approaches T_{crit} due to high cyclotron radiation.

Fusion Reactor Energetics

- **Lawson criterion**
  - reactor criterion: energy viability of the entire plant

Some Criteria for a Power Producing Thermonuclear Reactor

By J. D. LAWSON

Atomic Energy Research Establishment, Harwell, Berks.

Communicated by D. W. Fry; MS. received 2nd November 1956

Abstract. Calculations of the power balance in thermonuclear reactors operating under various idealized conditions are given. Two classes of reactor are considered: first, self-sustaining systems in which the charged reaction products are trapped and, secondly, pulsed systems in which all the reaction products escape so that energy must be supplied continuously during the pulse. It is found that not only must the temperature be sufficiently high, but also the reaction must be sustained long enough for a definite fraction of the fuel to be burnt.

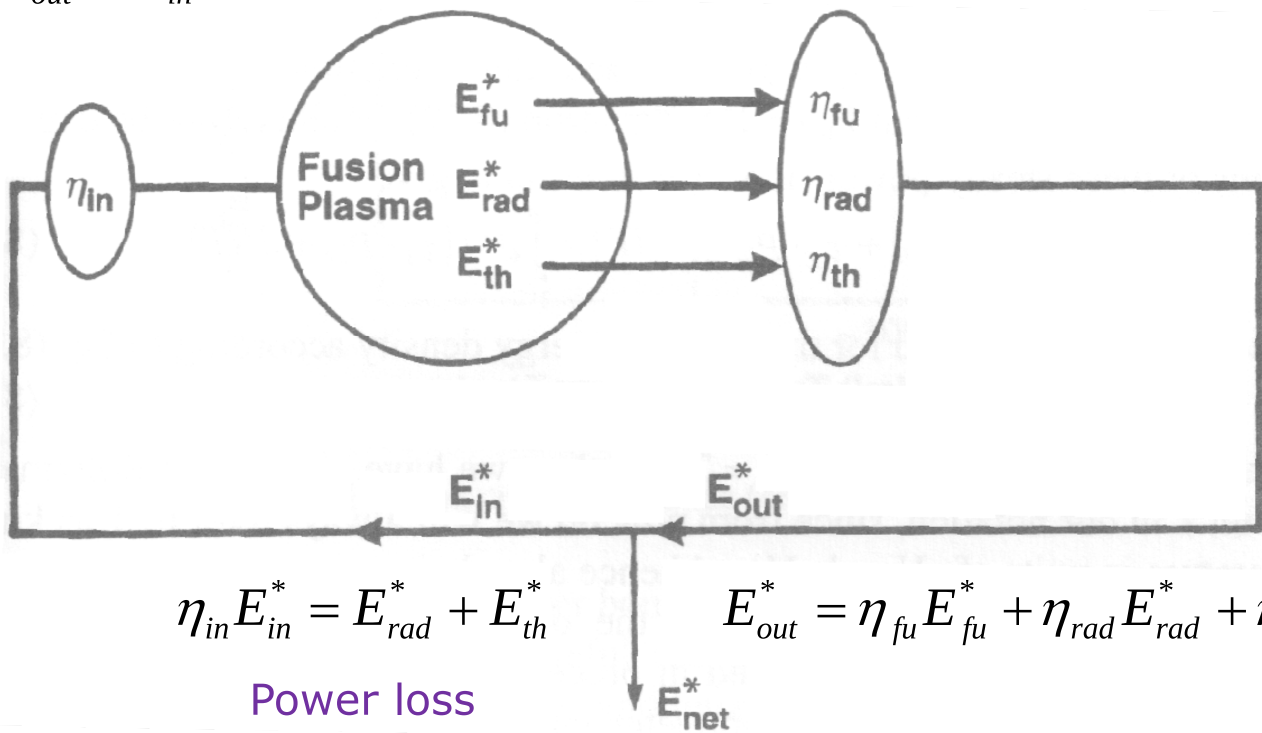
*Proceedings of the Physical Society (London), **B70**, 6 (1957)*

Fusion Reactor Energetics

- **Lawson criterion**

  - The recoverable energy from a fusion reactor must exceed the energy which is supplied to sustain the fusion reaction.

$$E^*_{out} > E^*_{in}$$



$$\eta_{in} E^*_{in} = E^*_{rad} + E^*_{th}$$

$$E^*_{out} = \eta_{fu} E^*_{fu} + \eta_{rad} E^*_{rad} + \eta_{th} E^*_{th}$$

Power loss

Fusion Reactor Energetics

- **Lawson criterion**

  - output electric energy > required input energy

$$\eta_{fu} E^*_{fu} + \eta_{rad} E^*_{rad} + \eta_{th} E^*_{th} > \frac{E^*_{rad} + E^*_{th}}{\eta_{in}}$$

$$\eta_{in} \eta_{out} (E^*_{fu} + E^*_{rad} + E^*_{th}) > E^*_{rad} + E^*_{th}$$

$$\eta_{out} = \frac{\sum_l \eta_l E_l}{\sum_l E_l}, \quad l = fu, rad, th$$

average conversion efficiency

$$E^*_l = \tau_{E*} \int_V P_l(\vec{r}) d^3 r$$

global energy terms

Fusion Reactor Energetics

Assuming, Bremsstrahlung only

$$\eta_{in}\eta_{out}\int_V d^3r(\tau_{E*}P_{fu}+\tau_{E*}P_{br}+3nT)>\int_V d^3r(\tau_{E*}P_{br}+3nT)$$

$$E_{th}(\vec{r})=\frac{3}{2}(n_iT_i+n_eT_e)=3nT$$

Assuming, homogeneity throughout the plasma volume *V*

$$\eta_{in}\eta_{out}\left(\frac{n_a n_b}{1+\delta_{ab}}<\sigma v>_{ab}Q_{ab}\tau_{E*}+A_{br}n^2\sqrt{T}\tau_{E*}+3nT\right)>A_{br}n^2\sqrt{T}\tau_{E*}+3nT$$

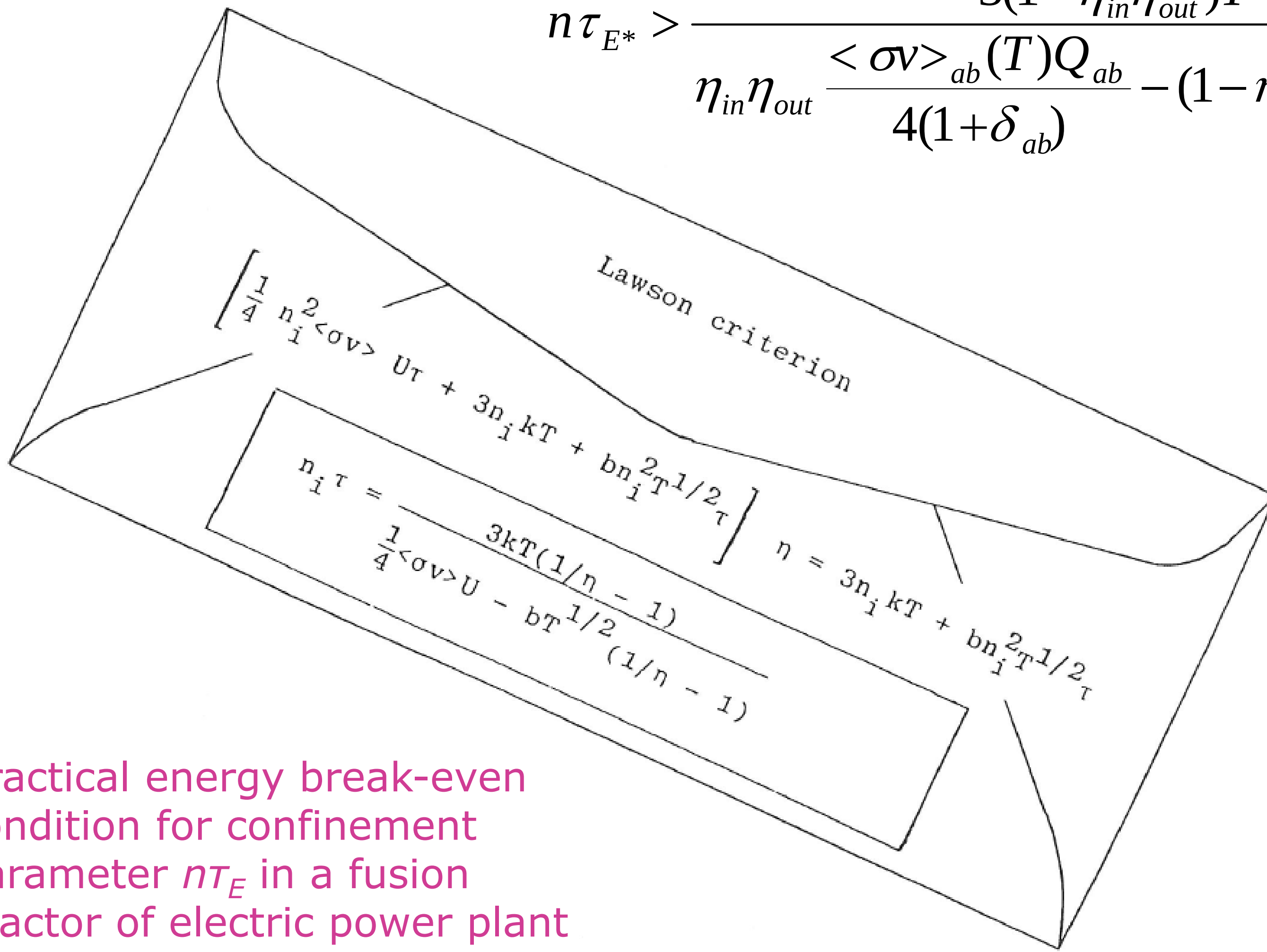
Kronecker-δ introduced to account for the case of indistinguishable reactants

$$n\tau_{E*}>\frac{3(1-\eta_{in}\eta_{out})T}{\eta_{in}\eta_{out}\frac{<\sigma v>_{ab}(T)Q_{ab}}{4(1+\delta_{ab})}-(1-\eta_{in}\eta_{out})A_{br}\sqrt{T}} \qquad \eta_{in}\eta_{out}\approx 1/3$$

Fusion Reactor Energetics

- **Lawson criterion**

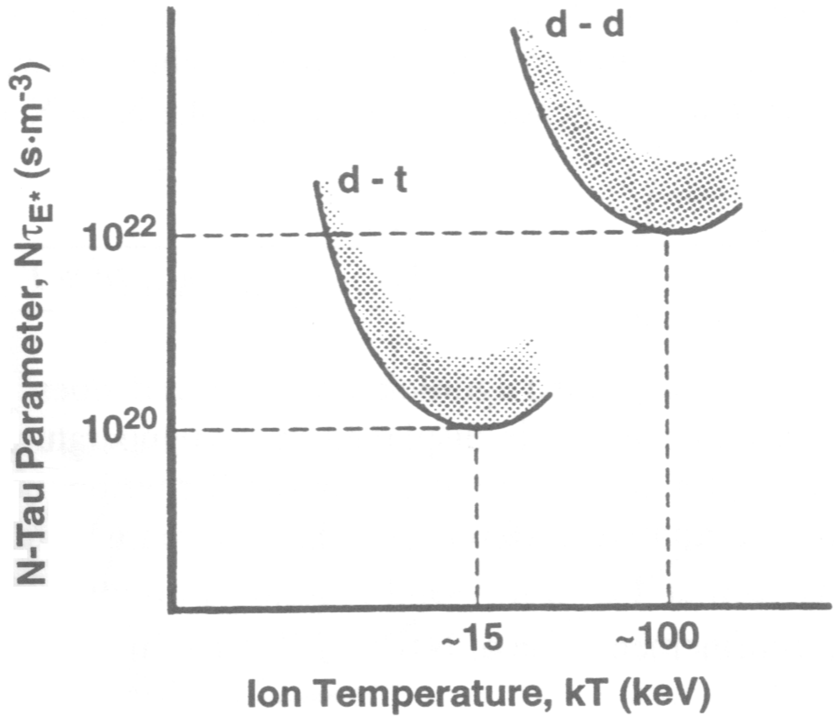
$$n\tau_{E*} > \frac{3(1-\eta_{in}\eta_{out})T}{\eta_{in}\eta_{out}\frac{<\sigma v>_{ab}(T)Q_{ab}}{4(1+\delta_{ab})} - (1-\eta_{in}\eta_{out})A_{br}\sqrt{T}}$$



- Practical energy break-even condition for confinement parameter $n\tau_E$ in a fusion reactor of electric power plant

Fusion Reactor Energetics

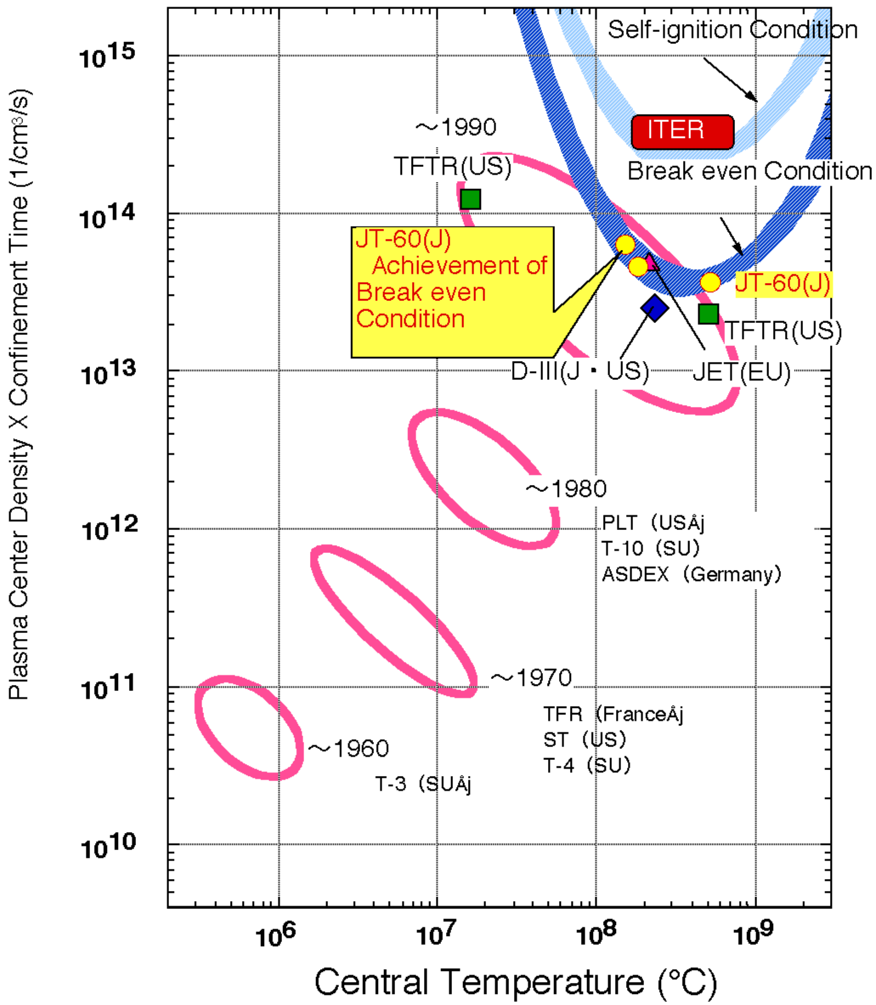
- **Lawson criterion**

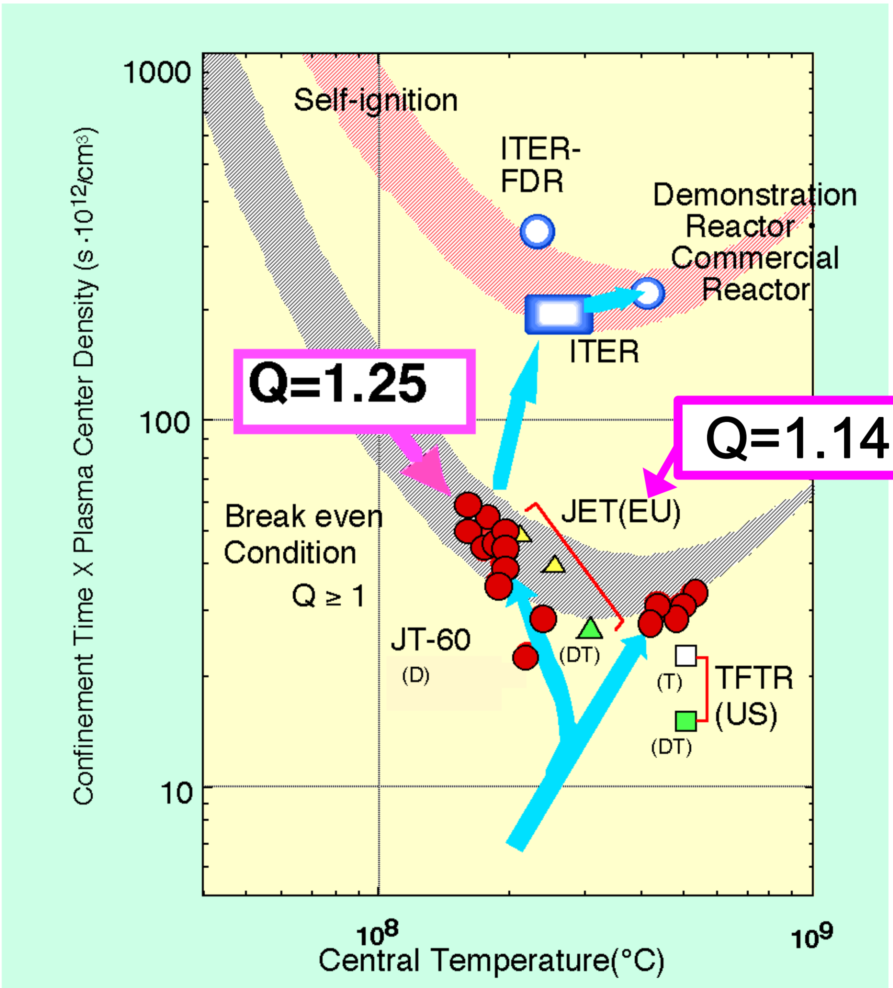


Homework:
Problems 8.6

- No particular fusion design was necessary in the derivation of this criterion.
- Although it does not contain all relevant processes such as cyclotron radiation, it is a useful and widely employed criterion.
- For commercial power applications, it would be necessary to exceed the minimum Lawson limit by perhaps a factor of ten or better.

Status of the Tokamak Research





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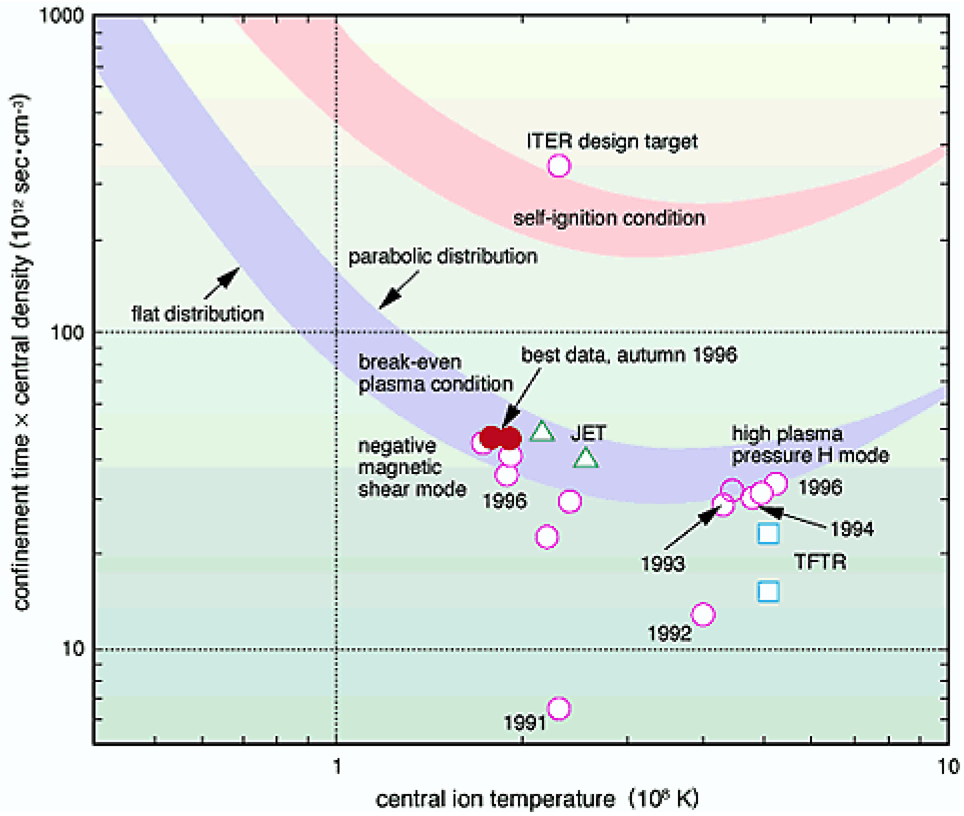

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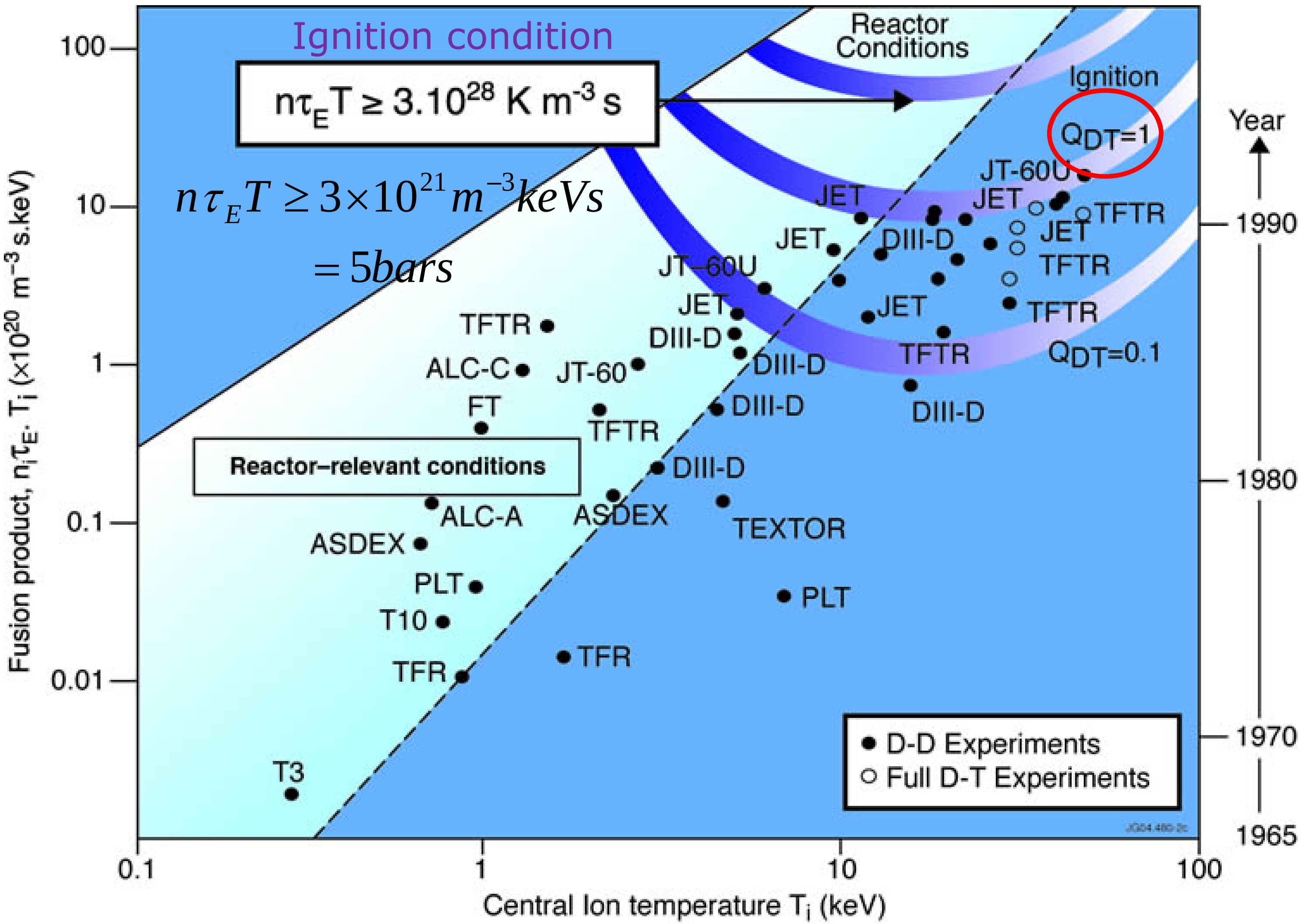

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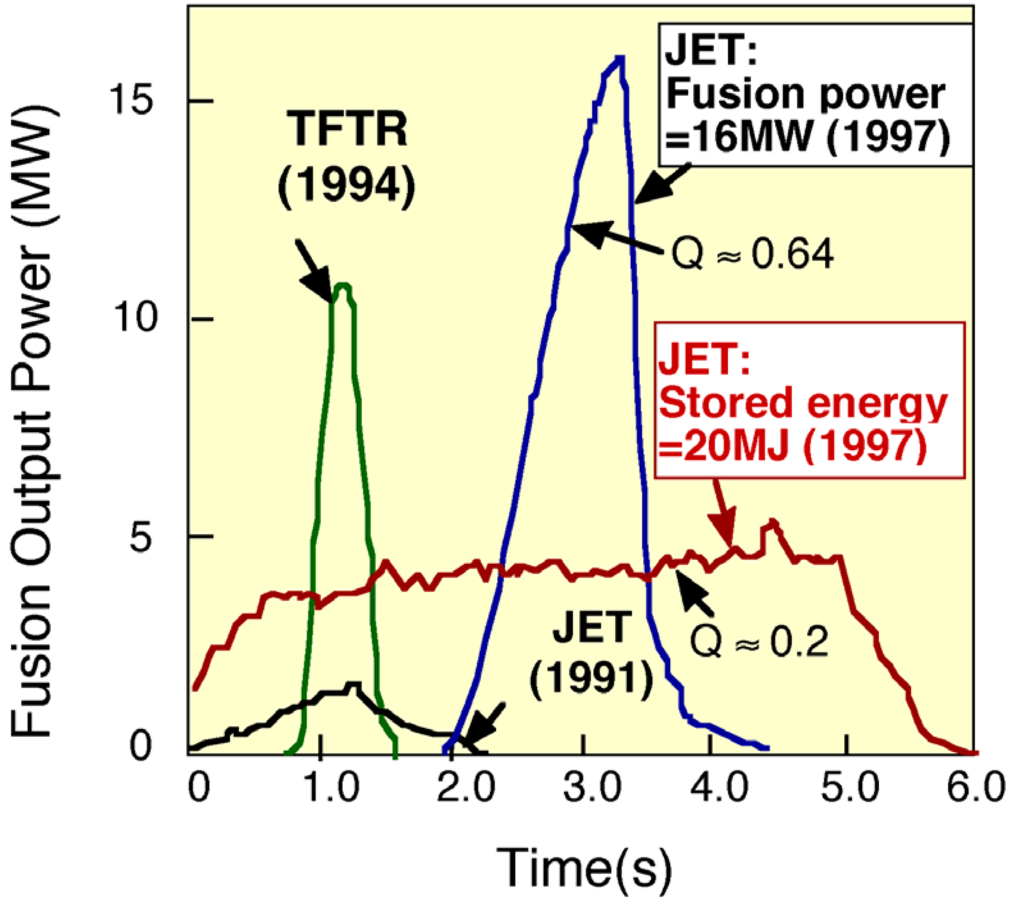
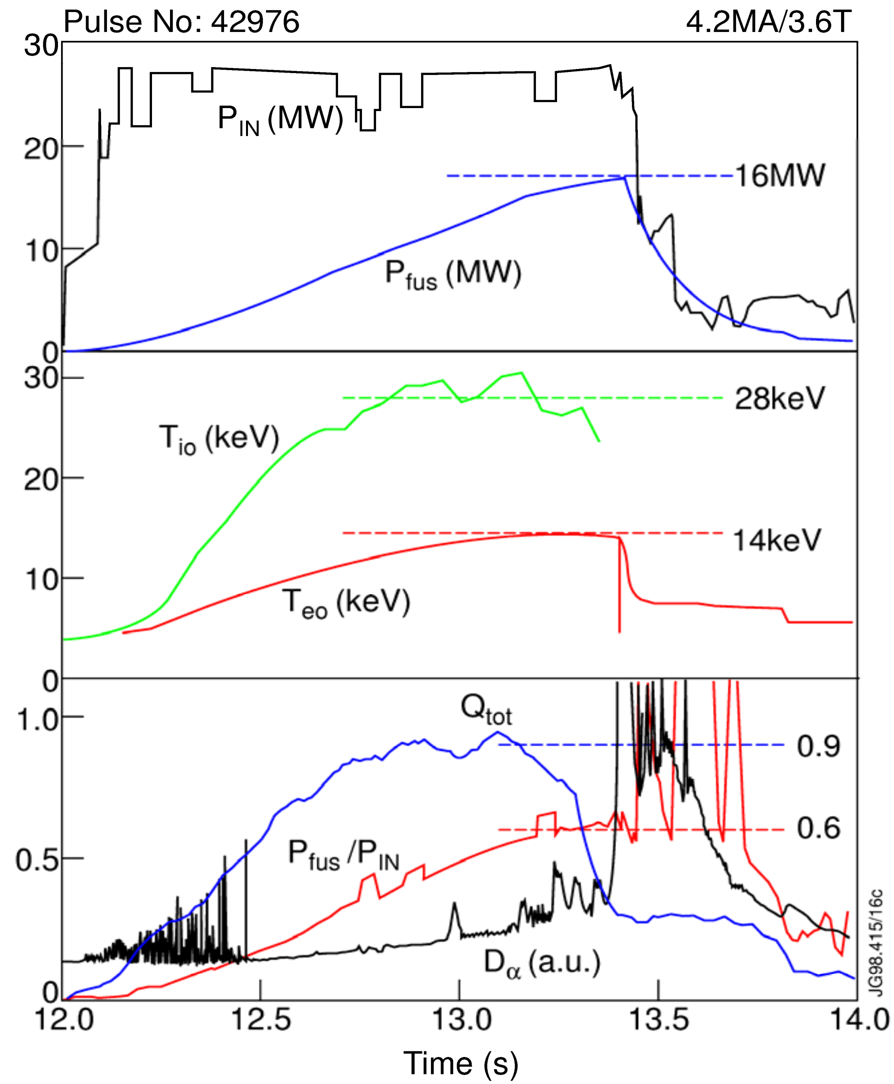


- Present machines produce significant fusion power:
  - TFTR (USA) ~10 MW in 1994
  - JET (EU) 16 MW (Q=0.64) in 1997

Status of the Tokamak Research

- DT-Experiments only in
- JET
- TFTR

- with world records in JET:
- P_{fusion} = 16 MW
- Q = 0.65



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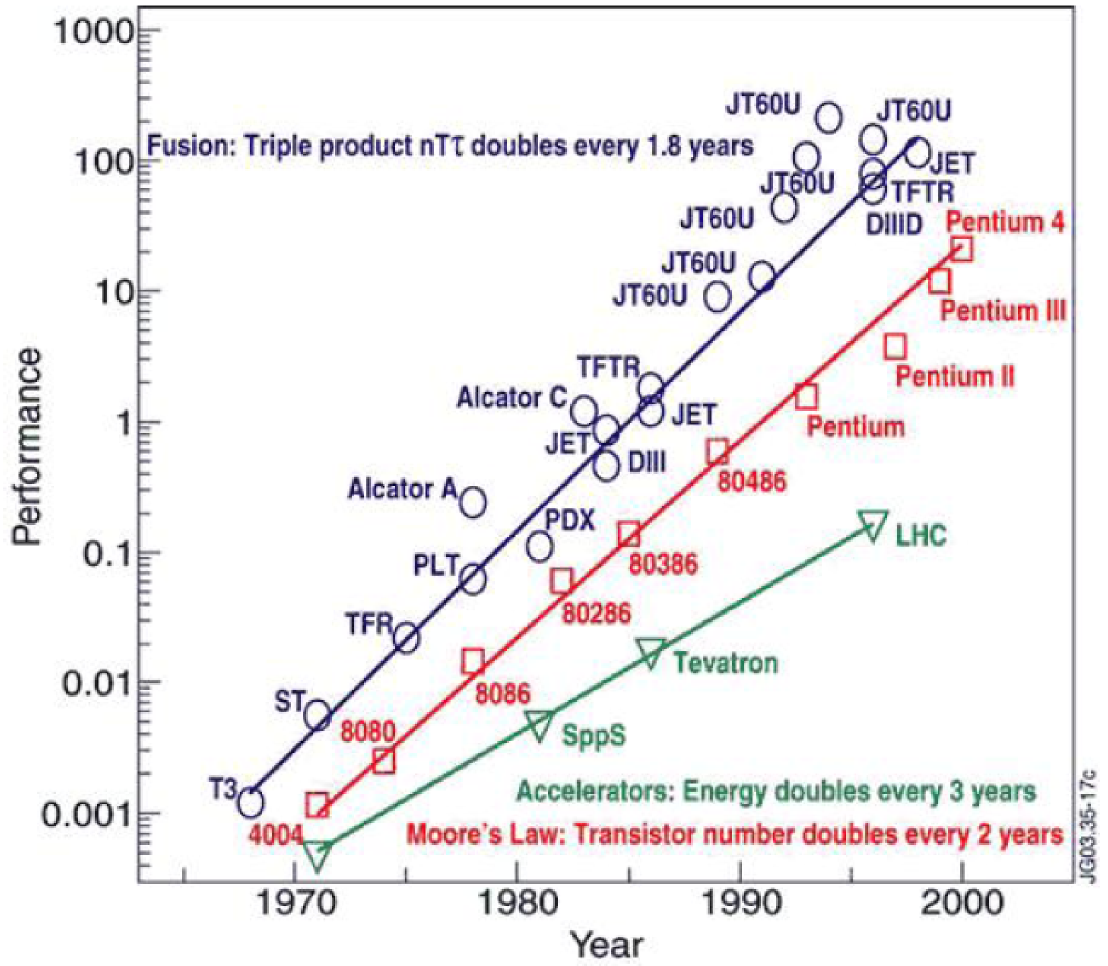

- Progress in fusion can be compared with the computing power and particle physics accelerator energy.