Fusion Reactor Technology I
(459.760, 3 Credits)

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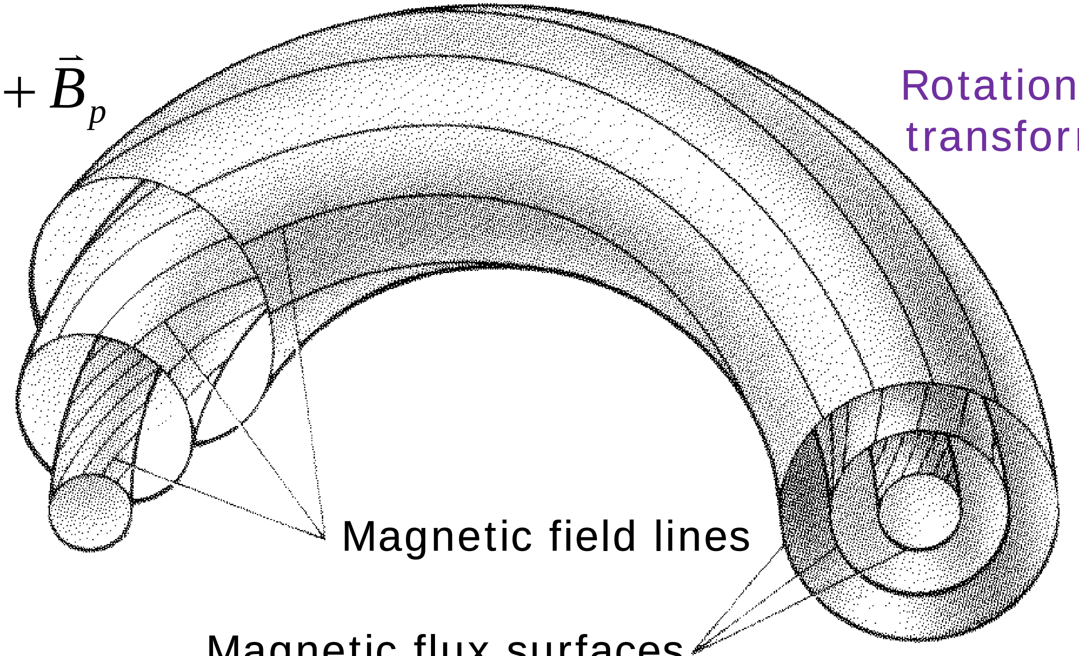
Basic Tokamak Variables

- **Safety factor *q* = number of toroidal orbits per poloidal orbit**

$$\vec{B} = \vec{B}_\phi + \vec{B}_p$$



Rotational transform $\iota = \dfrac{\dfrac{\Delta\theta}{2\pi}}{\Delta\phi} = \dfrac{\dfrac{B_\theta}{r}}{\dfrac{B_\phi}{2\pi R}}$

Δθ ? when *ΔΦ* = 2π

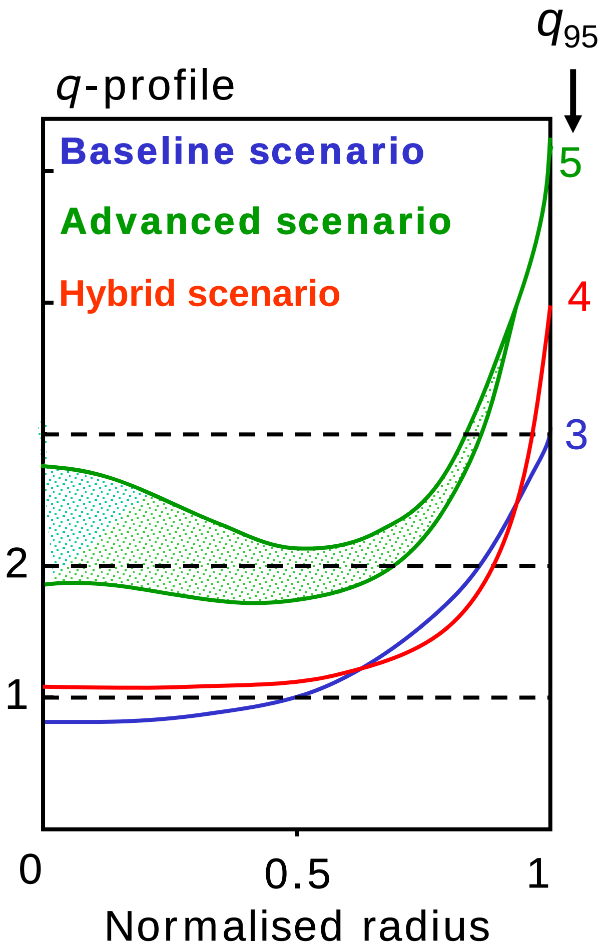
$$\frac{Rd\phi}{B_\phi} = \frac{rd\theta}{B_\theta}$$

- The effect of the twisted magnetic field lines–each of which completely traces out a magnetic flux surface by its revolutions around the toroidal and poloidal axes–is to create a system of nested toroidal flux surfaces which guide ion motion.

$$q = \frac{\text{number of toroidal windings}}{\text{number of poloidal windings}} = \frac{2\pi}{\iota} = \frac{r}{R}\frac{B_\phi}{B_\theta}$$

Basic Tokamak Variables

- **Safety factor *q* = number of toroidal orbits per poloidal orbit**



Basic Tokamak Variables

- **Safety factor *q* = number of toroidal orbits per poloidal orbit**

Large aspect ratio tokamak with a circular CX

$$q(r) = \frac{rB_\varphi}{R_0 B_\theta} = \frac{\varepsilon}{s}$$

$$\varepsilon \equiv \frac{r}{R_0}, \quad s \equiv \frac{B_\theta}{B_\varphi} = \frac{\mu_0}{rB_\varphi}\int_0^r j_\varphi(r')r'dr'$$

$$q_a = \frac{aB_\varphi}{R_0 B_{\theta a}} = \frac{2\pi a^2 B_\varphi}{\mu_0 I_p R_0}, \quad \langle j_\varphi \rangle = \frac{I_p}{\pi a^2}$$

$$\mu_0 \langle j_\varphi \rangle = \frac{2B_\varphi}{R_0 q_a}, \quad q_0 = \frac{2B_\varphi}{\mu_0 j_{\varphi 0} R_0}$$

Derive this!

Homework

$$\frac{q_a}{q_0} = \frac{j_{\varphi 0}}{\langle j_\varphi \rangle}$$

Current profile peakedness

Basic Tokamak Variables

- **Safety factor *q* = number of toroidal orbits per poloidal orbit**
  - General definition

$$q = \oint \frac{B_\varphi}{R_0 B_\theta} ds$$

Integral is along a closed path enclosing the minor axis and lying on a specific magnetic surface; thus *q* is a surface quantity.

$$q_{95} = \frac{5a^2 B_T}{R I_{MA}} f$$

f : describing the role of plasma shape

$$f = \frac{1+\kappa^2\left(1+2\delta_{95}^2 - 1.2\delta_{95}^3\right)}{2} \frac{\left(1.17 - 0.65A^{-1}\right)}{\left(1-A^{-2}\right)^2}$$

A: aspect ratio

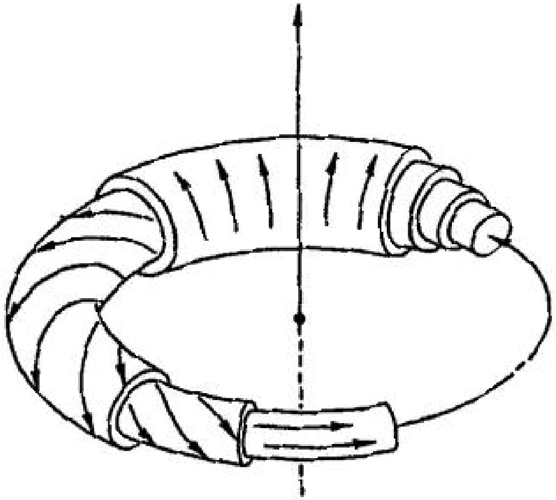
ITER Physics Basis, Nucl. Fusion ***39*** *2169 (1999)*

Basic Tokamak Variables

- **Magnetic Shear**
  - Measuring the change in pitch angle of a magnetic field line from one flux surface to the next
  - Playing an important role in stabilizing MHD instabilities, particularly those driven by the pressure gradient:

  A perturbation aligned with **B**(*r*) will, at a point with increased minor radial distance *r*+*dr*, encounter field lines at a different angle which again will vary as the perturbation grows to another distance *r*+*dr*'. Any helically resonant instabilities are thus radially localised.

$$s(V) \equiv 2\frac{V}{q}\frac{dq}{dV}$$



Basic Tokamak Variables

- ***Z*-effective: a convenient measure of the extent to which the plasma contaminated**

$$n_e Z_{eff} = \sum_s n_s Z_s^2 \,, \quad n_e = \sum_s n_s Z_s$$

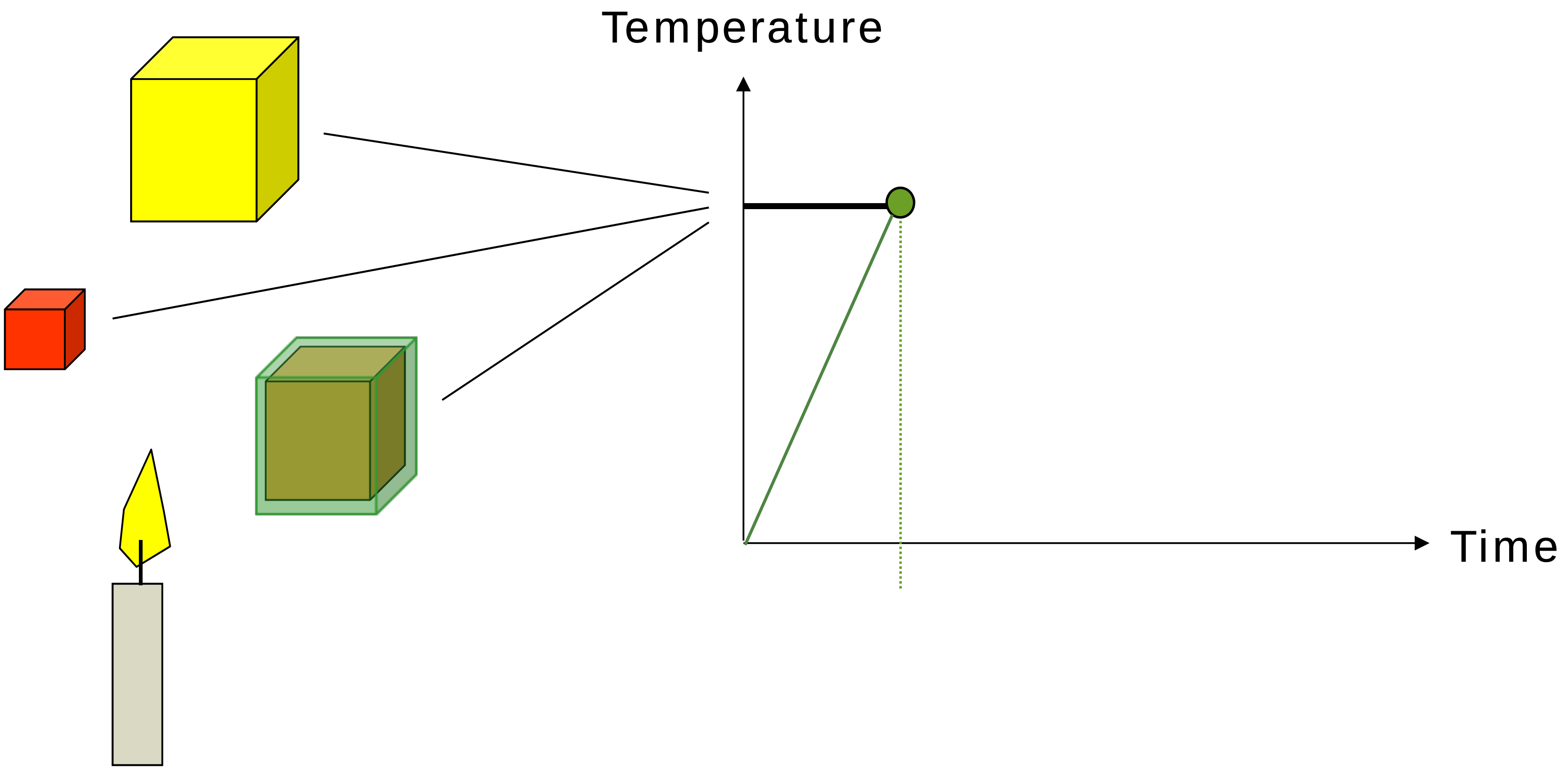
Z_s: charge number for the s-type ion

- Z_{eff} = 1 in a pure hydrogen plasma
- Method to determine Z_{eff}

- Impurity concentration determined by analyzing resonance line intensities in the vacuum UV, supplemented by measurements of soft X-ray spectra; this data, coupled with a theory for ionization rates
- Visible Bremsstrahlung radiation
- Spitzer's formula for the parallel resistivity

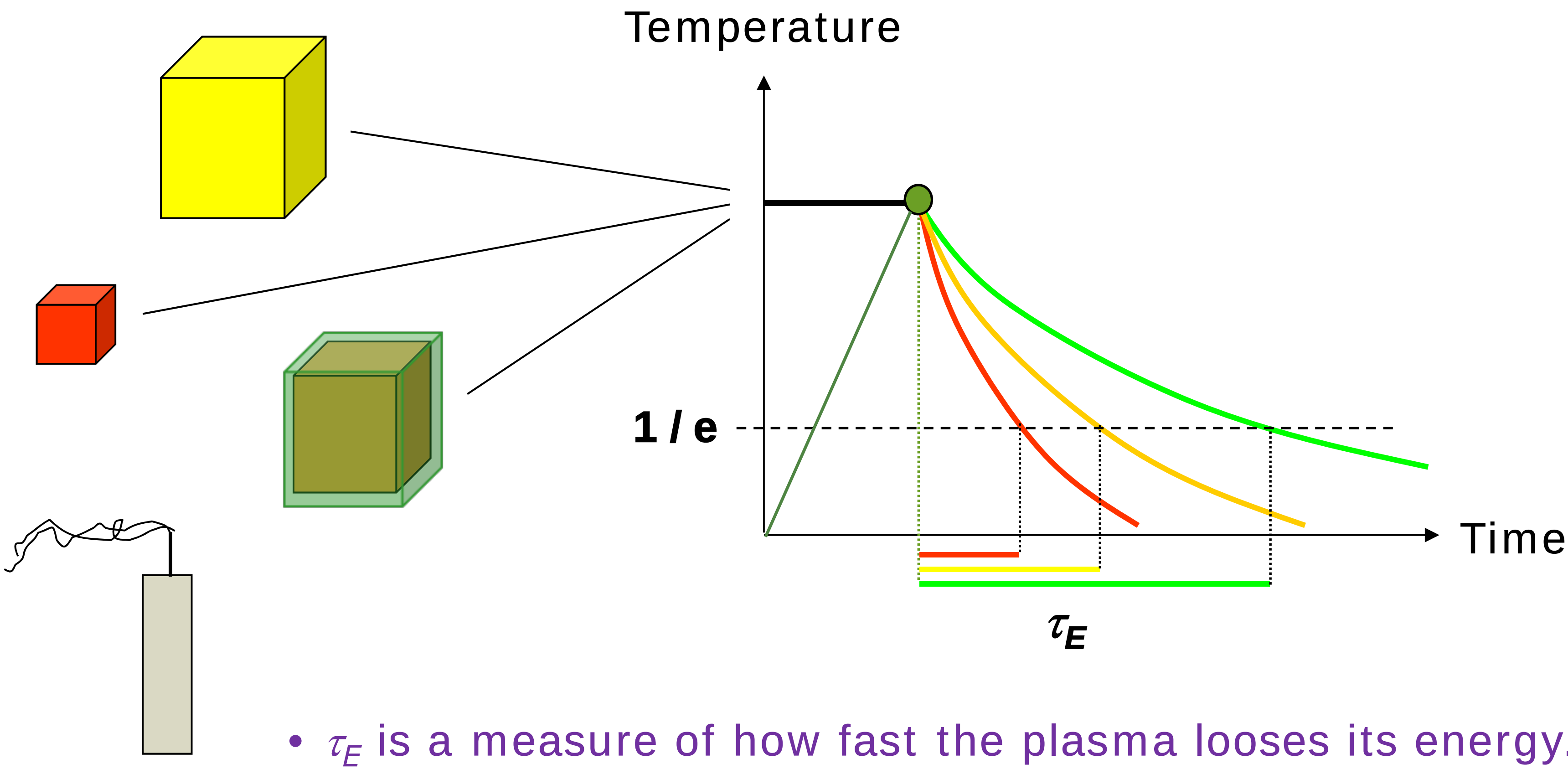
Basic Tokamak Variables

- **Energy confinement time**



Basic Tokamak Variables

- **Energy confinement time**



- τ_E is a measure of how fast the plasma looses its energy.
- The loss rate is smallest, τ_E largest
  if the fusion plasma is big and well insulated.

Basic Tokamak Variables

- **Boltzmann Equation**

$$\frac{\partial f_\alpha}{\partial t} + \vec{u}\cdot\nabla f_\alpha + \frac{q_\alpha}{m_\alpha}(\vec{E}+\vec{u}\times\vec{B})\cdot\nabla_u f_\alpha = \left(\frac{\partial f_\alpha}{\partial t}\right)_c$$

- **Fluid Equations**

$$\int Q_i\left[\frac{df_\alpha}{dt} - \left(\frac{\partial f_\alpha}{\partial t}\right)_c\right]d\vec{u} = 0$$

$Q_1 = 1$ mass

$Q_2 = m_\alpha \vec{u}$ momentum

$Q_3 = m_\alpha u^2/2$ energy

$$\frac{\partial n_j}{\partial t} + n_j\nabla\cdot\vec{u}_j = S_{nj}$$

$$m_j n_j \frac{d\vec{u}_j}{dt} + \nabla\cdot\overleftrightarrow{P}_j - q_j n_j(\vec{E}+\vec{u}_j\times\vec{B}) = \sum_k^l \vec{R}_{jk} - m_j\vec{u}_j S_{nj}$$

$$\frac{3}{2}n_j\frac{dT_j}{dt} + \overleftrightarrow{P}_j : \nabla\vec{u}_j + \nabla\cdot\vec{h}_j = \sum_k^l Q_{jk} + S_{Ej} + \left(\frac{m_j u_j^2}{2} - \frac{3}{2}T_j\right)S_{nj}$$

Basic Tokamak Variables

- **Energy confinement time**

$$W = \frac{1}{2\pi}\int \frac{3}{2} p dS = \int_0^a \frac{3}{2} k_B (n_e T_e + n_i T_i) r dr$$ ~ total thermal energy in the torus

total heat flux radiation energy loss rate

$$\frac{\partial(\rho u)}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\{r(\rho h v_D + Q_r)\} = j_\varphi E_\varphi - L \quad (\rho u = \frac{3}{2} p, \quad \rho h = \frac{5}{2} p)$$

internal energy enthalpy density

$$\frac{\partial}{\partial t}(\ln W) + \frac{1}{\tau_E} = \frac{1}{\tau_E^*} - \frac{1}{\tau_E^R}$$

$$\tau_E \equiv \frac{W}{\left[r\left(\frac{5}{2} p v_D + Q_r \right) \right]_{r=a}}, \quad \tau_E^* \equiv \frac{W}{\int_0^a j_\varphi E_\varphi r dr}, \quad \tau_E^R \equiv \frac{W}{\int_0^a L r dr}$$

Energy confinement time — Energy replacement time — Radiation loss time

Basic Tokamak Variables

- **Energy confinement time**

$$\tau_E = \frac{W}{\frac{W}{\tau_E^*} - \frac{\partial W}{\partial t}} = \frac{W}{P_{in} - \frac{\partial W}{\partial t}} \approx \frac{W}{P_{in}} = \frac{\text{stored energy}}{\text{applied heating power}}$$

In steady conditions, neglecting radiation loss, Ohmic heating replaced by total input power

- To predict the performance of future devices, the energy confinement time is one of the most important parameter.
- Since tokamak transport is anomalous, empirical scaling laws for energy confinement are necessary.
- ***Empirical scaling laws***: regression analysis from available experimental database.

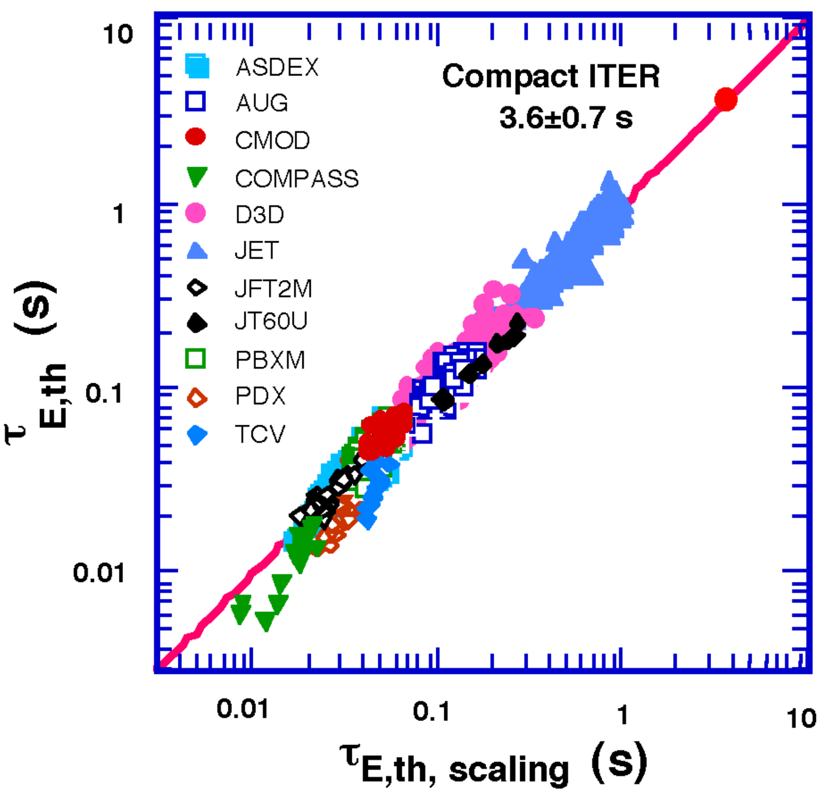
$$\tau_{th,E}^{fit} = CI^{\alpha I} B^{\alpha B} P^{\alpha P} n^{\alpha n} M^{\alpha M} R^{\alpha R} \varepsilon^{\alpha \varepsilon} \kappa^{\alpha \kappa}$$

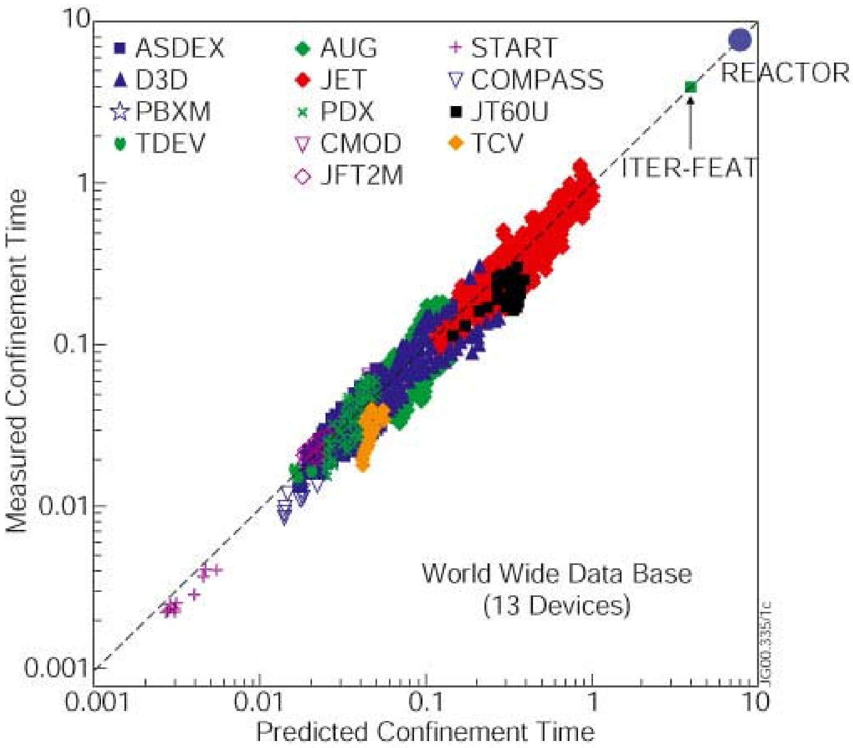
in engineering variables

Basic Tokamak Variables

- **Energy confinement time**

$$\tau_{th,E}^{IPB98(y,2)} = 0.0562\, I^{0.93} B^{0.15} P^{-0.69} n^{0.41} M^{0.19} R^{1.97} \varepsilon^{0.58} \kappa_a^{0.78}$$





τ_E in KSTAR and ITER?

Why should ITER be large?

Basic Tokamak Variables

- **Particle confinement time**

$$\frac{\partial n_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r n_e v_D) = S_e(r)$$ electron number density source

$$\tau_p = \tau_p^*$$ In steady state

$$\tau_p \equiv \frac{\int_0^a n_e r\,dr}{\left[r n_e v_D\right]_{r=a}}, \quad \tau_p^* \equiv \frac{\int_0^a n_e r\,dr}{\int_0^a S_e r\,dr}$$

particle confinement time

particle replacement time

Basic Tokamak Variables

- **Momentum confinement time**

$$\frac{\partial}{\partial t}(\rho v_\varphi) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r v_\varphi) + \nabla \cdot \Pi \cdot \hat{\varphi} = F_b \cdot \hat{\varphi}$$

Momentum equation having the toroidal component

$$\tau_\varphi = \tau_\varphi^*$$

In steady state

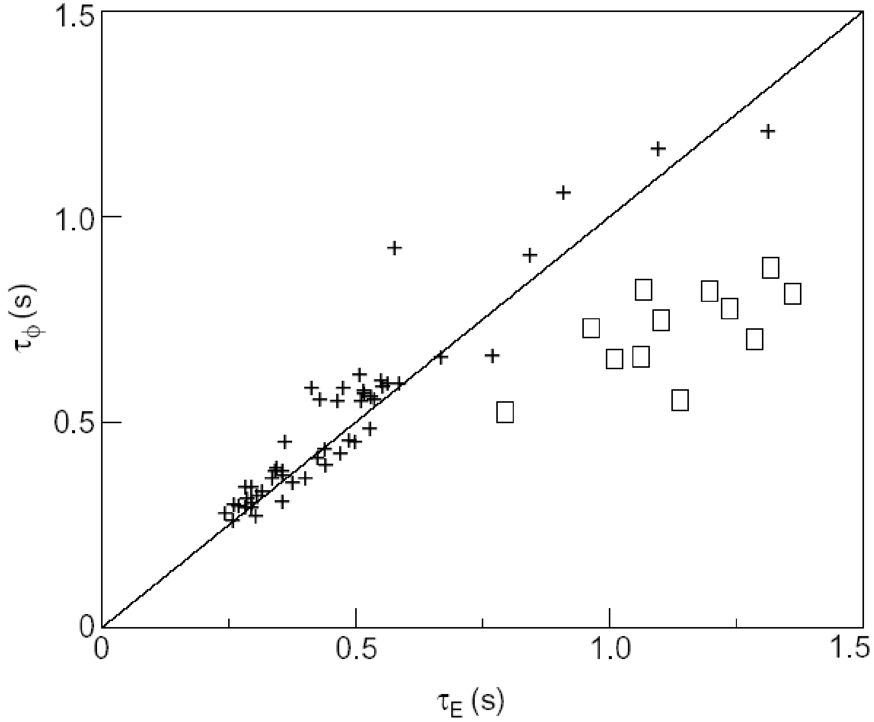
$$\tau_\varphi \equiv \frac{H_\varphi}{\int_0^a \nabla \cdot \Pi \cdot \hat{\varphi} r dr + \left[r\rho v_r v_\varphi \right]_{r=a}}, \quad \tau_\varphi^* \equiv \frac{H_\varphi}{\int_0^a F_b \cdot \hat{\varphi} r dr} = \frac{2\pi^2 R_0^2 a^2 H_\varphi}{\text{Beam torque}}, \quad H_\varphi \equiv \int_0^a \rho v_\varphi r dr$$

Toroidal momentum confinement time

Toroidal momentum replacement time

Basic Tokamak Variables

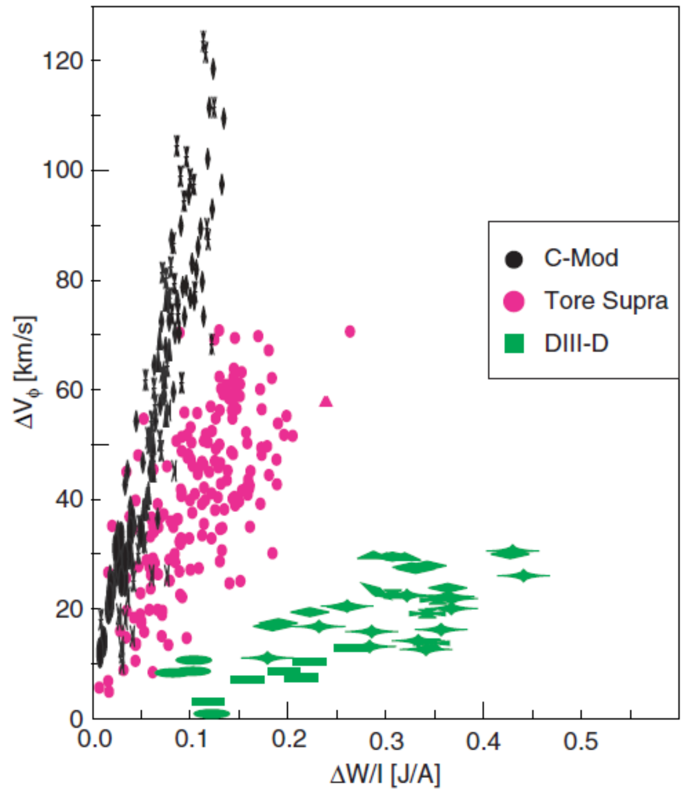
- **Momentum confinement time**



- Toroidal angular momentum confinement time of thermal particles during NBI versus simultaneously measured energy confinement time for steady state L-mode and ELMy H-mode discharges (crosses), and for transient ELM free phase of hot ion H-mode discharges (squares) in JET

Basic Tokamak Variables

- **Intrinsic rotation**

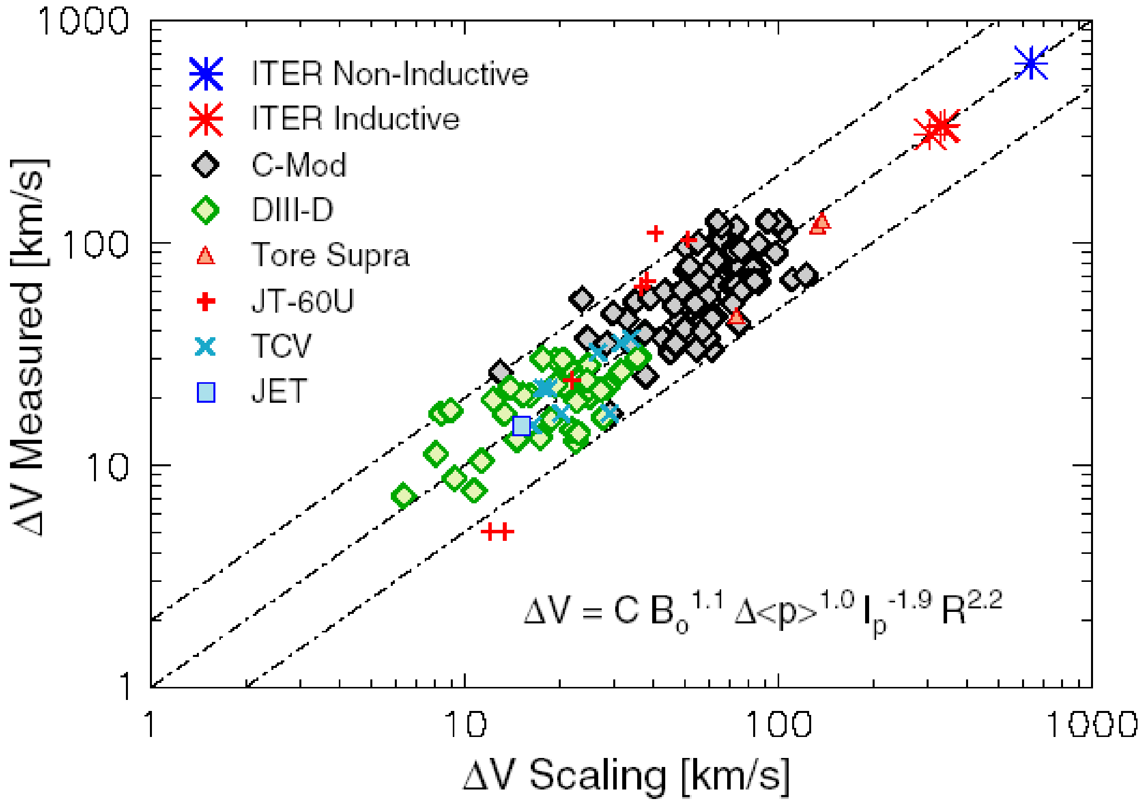


- The intrinsic rotation velocity (the difference between the L-mode velocity and the enhanced confinement value) as a function of the change in the stored energy normalized to the plasma current

*J. E. Rice et al, Nucl. Fusion **47** 1618 (2007)*

Basic Tokamak Variables

- **Intrinsic rotation**



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