Fusion Reactor Technology I
(459.760, 3 Credits)

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Contents

Contents

Dimensional Analysis of Tokamaks

- **Dimensional Analysis of Energy Confinement**

- Complicated mechanisms of energy losses from tokamak plasmas

  several channels of energy leakage

  anomalous thermal conductivities

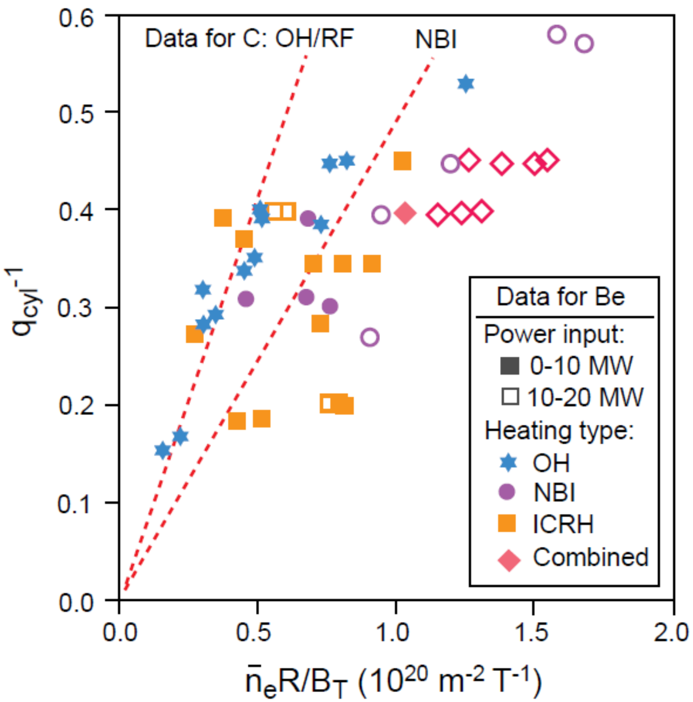
- One may try to apply methods of dimensional analysis to find out the main non-dimensional parameters which control transport

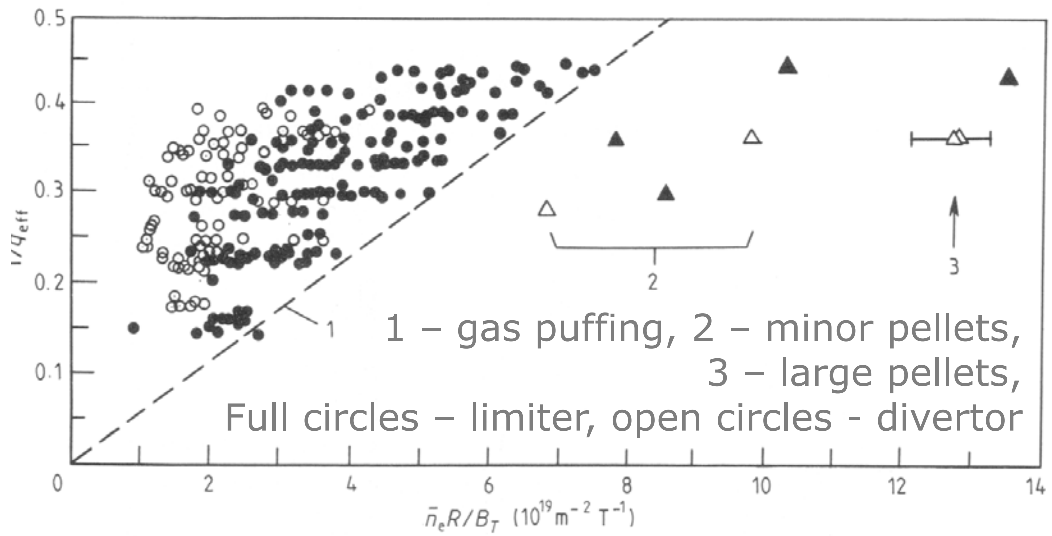
- Although it cannot replace real theory, it can help to shed light upon the most important physical mechanisms of transport.

Dimensional Analysis of Tokamaks

- **Hugill diagram**

- Attaining high densities by using beryllium coating of the chamber wall in JET and with the help of pellet injection in JT-60U





- The two dashed lines illustrate the density limits in earlier OH/ICRF and NBI experiments with a mainly carbon 1st wall.
- When heating power is increased, the Hugill limit shifts towards higher densities.

ITER Physics Basis, Nuclear Fusion 39 2261 (1999)

Dimensional Analysis of Tokamaks

- **Energy Confinement in Ohmic Discharges**

- Neo-ALCATOR scaling when the densities are not high enough

$$\tau_E = 7\times10^{-2} naR^2 q_a$$

- Restoring dimensionality

$$\tau_E = 0.4\frac{\Lambda^{-2}}{v_0} aR^2 q_a \qquad \Lambda^{-2} = 4\pi e^2 \overline{n} / m_e c^2, \quad v_0 = 2\times10^7 m/s$$

Λ: internal characteristic length

mean thermal velocity at 1 keV ($v_e = v_0 T^{1/2}$)

- Neo-ALCATOR scaling is valid only up to some critical density (n_s): above n_s, saturation and sometimes slight decrease of τ_E with density observed
- Shimomura relation describing the critical density at which the transition from one dependence to another occur

$$n_s \cong B_T \sqrt{A_i/2} / q_a R \;\rightarrow\; H_s = \sqrt{A_i/2}$$

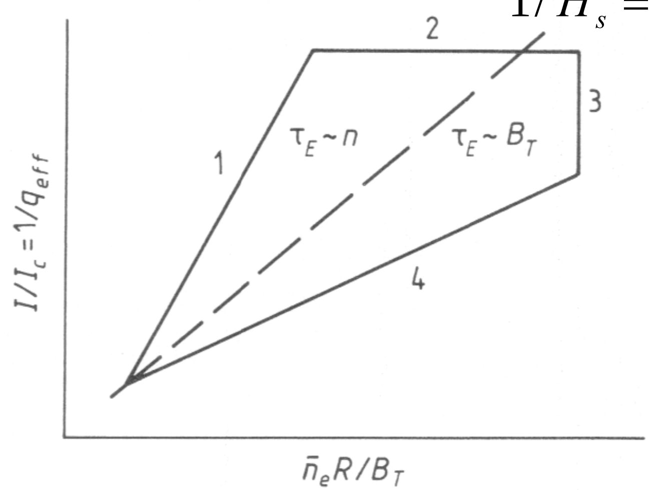
Dimensional Analysis of Tokamaks

- **Energy Confinement in Ohmic Discharges**

- Above n_s, τ_E may be considered as neo-ALCATOR at $n = n_s$

$$\tau_E = 7\times10^{-2} n_s a R^2 q_a = aRB_T\sqrt{A_i/2}$$

$$1/H_s = 1/\sqrt{A_i/2}$$



- Transition from linear ALCATOR scaling to saturation in plasmas with OH occurs at a fixed Hugill number: related to atomic processes

Why is there critical density?

- Critical density related to ballooning instability?

$$H_c = \frac{7aB_T}{T_0} \longleftarrow \beta_c = gI_N = g\frac{I}{I_c}\frac{5b}{R}$$

Hugill number at $\beta = \beta c$ (Troyon limit)

$$T = T_0(1-r^2/a^2)$$
$$n = n_0(1-r^2/a^2)$$
$$g = 2.8$$

Dimensional Analysis of Tokamaks

- Transport is related to plasma edge phenomena (A_i included in n_s)

• **Energy Confinement with Auxiliary Heating (L-mode)**

- Goldston scaling: A_i included, related to plasma edge phenomena

$$\tau_E = 3.7 \times 10^{-2} I R^{1.75} a^{-0.37} K^{0.5} P^{-0.5} \left(A_i / 0.5\right)^{0.5}$$

Cf. A_i = 2.5 for D and T mixture

- For simplicity, K =1 (circular plasma), A_i = 2

$$\tau_E \cong 6 \times 10^{-2} I R^{1.5} a^{-0.5} P^{-0.5} \qquad P = W_T / \tau_E, \quad W_T = 2\pi^2 a^2 R \langle 3nT \rangle$$

$$\tau_E \cong 7 \times 10^{-2} \frac{R^2}{a\beta_p}$$

- Restoring dimensionality

$$\tau_E = 1.4 \times 10^6 \frac{R^2}{v_0 a \beta_p}$$

Dimensional Analysis of Tokamaks

- **Energy Confinement with Auxiliary Heating (L-mode)**

- Transition from neo-ALCATOR scaling to additional heating scaling 1

$$\tau_E = 0.4\frac{\Lambda^{-2}}{v_0}aR^2 q_a \quad > \quad \tau_E = 1.4\times 10^6 \frac{R^2}{v_0 a \beta_p}$$

$$\Lambda^2 < 3\times 10^{-7} a^2 \beta_p q_a$$

- Introducing an average Larmor electron radius in the poloidal field

$$\rho_\theta = \sqrt{2Tm_e / e^2 B_p^2} = \sqrt{\frac{c^2 m_e \varepsilon_0}{ne^2}\frac{2nT}{2B_p^2/2\mu_0}} = \Lambda\sqrt{\beta_p/2}$$

$\Lambda < 4\times 10^{-2}\sqrt{a\rho_\theta}$ For $q \sim 4$

- Transition from neo-ALCATOR scaling to additional heating scaling 2

$$\tau_E = 7\times 10^{-2} naR^2 q_a \quad > \quad \tau_E \cong 7\times 10^{-2}\frac{R^2}{a\beta_p} \longrightarrow \bar{n}a^2\beta_p q_a > 1$$

Dimensional Analysis of Tokamaks

- **Energy Confinement with Auxiliary Heating (L-mode)**

- Transition from neo-ALCATOR scaling to additional heating scaling 3

$$\tau_E = 7\times10^{-2} n_s a R^2 q_a = aRB_T\sqrt{A_i/2} \quad > \quad \tau_E' = 6\times10^{-2} I^{0.5} R^{1.5} a^{-0.5}$$

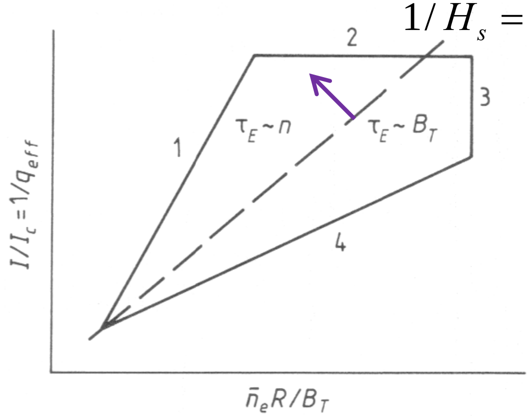
$$\leftarrow \quad \tau_E \cong 6\times10^{-2} I R^{1.5} a^{-0.5} P^{-0.5}$$

$$aB_T > \sqrt{IR/aA_i} \qquad\qquad P = P_\Omega = IV = I$$

in facilities with ohmically heated plasmas at high magnetic field

- The transition point from linear dependence upon saturation density will be shifted with power increase to lower density.

$$1/H_s = 1/\sqrt{A_i/2}$$



$$\tau_E = 3.7\times10^{-2} I R^{1.75} a^{-0.37} K^{0.5} P^{-0.5} \left(A_i/0.5\right)^{0.5}$$

- Goldston scaling with modifications considered in large machines.
- Neo-ALCATOR valid only for low densities

Non-linear Plasma Activity

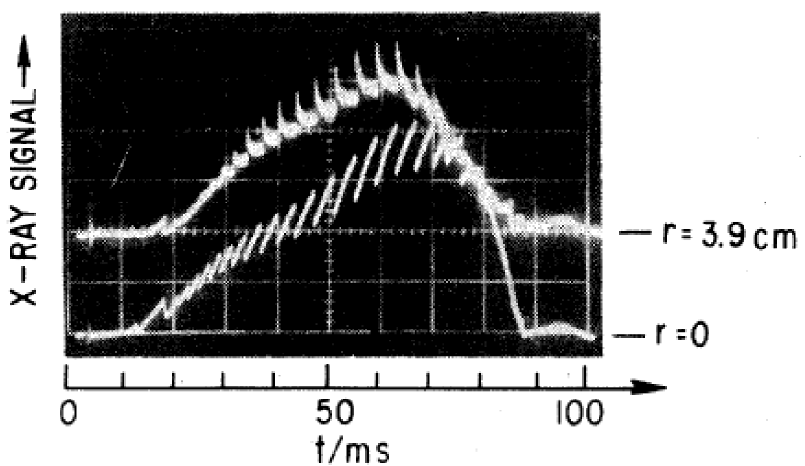
- **Sawtooth**

VOLUME 33, NUMBER 20 PHYSICAL REVIEW LETTERS 11 NOVEMBER 1974

Studies of Internal Disruptions and $m = 1$ Oscillations in Tokamak Discharges with Soft–X-Ray Techniques*

S. von Goeler, W. Stodiek, and N. Sauthoff
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08540
(Received 11 July 1974)

Fluctuations in x-ray intensity from the ST tokamak show a characteristic sawtooth behavior. This behavior is identified as an internal disruption. The internal disruptions are preceded by growing sinusoidal $m=1$, $n=1$ oscillations. The properties of these oscillations are compared with predictions for the $m=1$ internal kink mode.



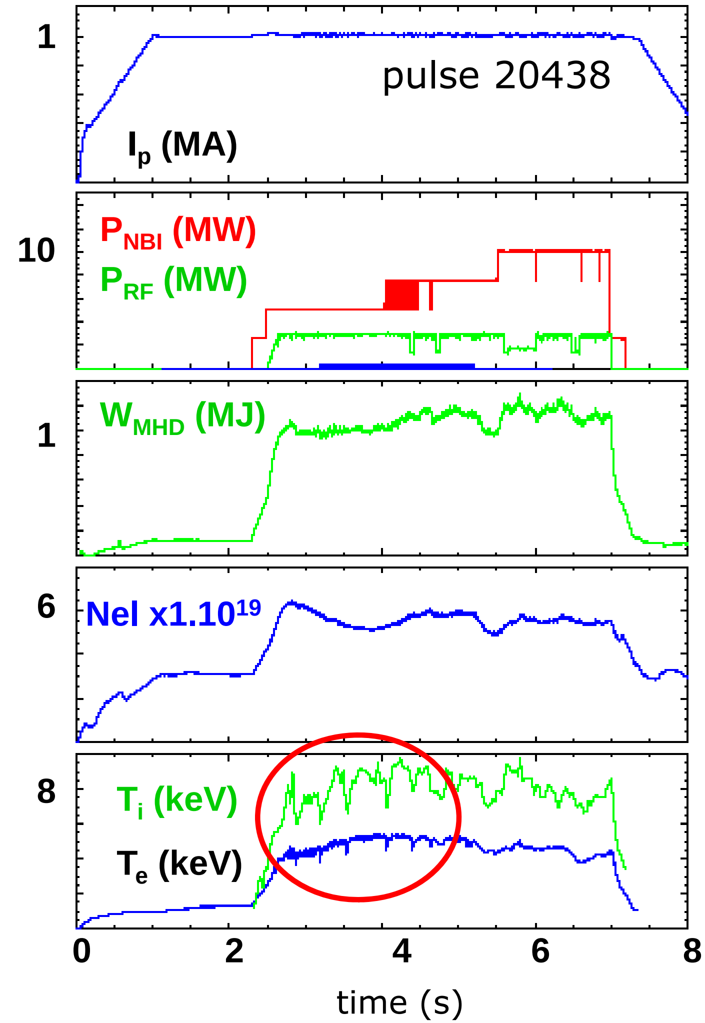
of the plasma electrons and consists predominantly of the recombination-radiation continuum of the partly ionized oxygen and iron impurities.[1] The radiation intensity is therefore a function of the electron density and temperature and of the impurity concentration. The x-ray fluctuations are caused by a fluctuation in either of these quantities, but predominantly by temperature fluctuations.

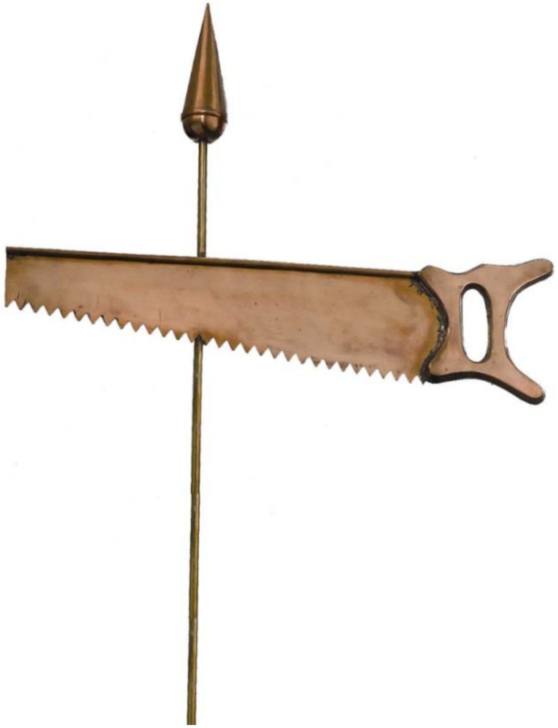
The oscillograms of the x-ray emissions, shown in Fig. 1, are typical for high-density discharges in the ST tokamak. The traces exhibit a "sawtoothlike" oscillation. The sawtooth is "inverted," showing a fast rise and a slow exponential drop,

Non-linear Plasma Activity

ASDEX Upgrade

- **Sawtooth**





- nonlinear low-*n* internal mode
- internal (minor) disruption
- enhanced energy transport in the plasma centre

Non-linear Plasma Activity

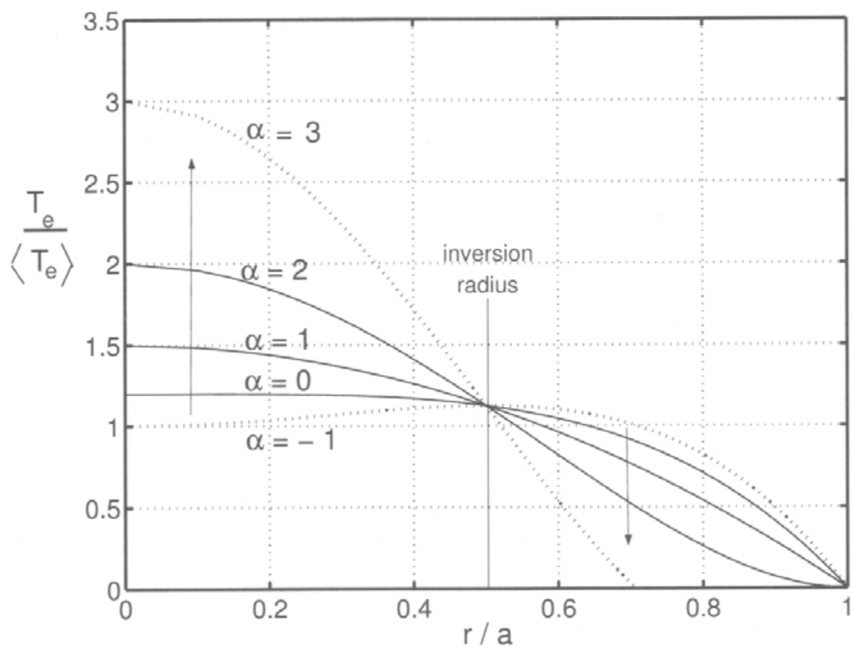
- **Sawtooth**

- It occurs so commonly that its presence is accepted as a signal that the tokamak is operating normally.
- Important type of plasma non-linear activity
  - Decreasing the thermal insulation
  - Key to understanding the disruptive instability
- Consisting of periodically repeated phases of
  - slow temperature rise at the centre of the plasma column
  - fast drop ($m = 1$, $n = 1$ oscillatory MHD modes oscillation precursors observed before the drop)

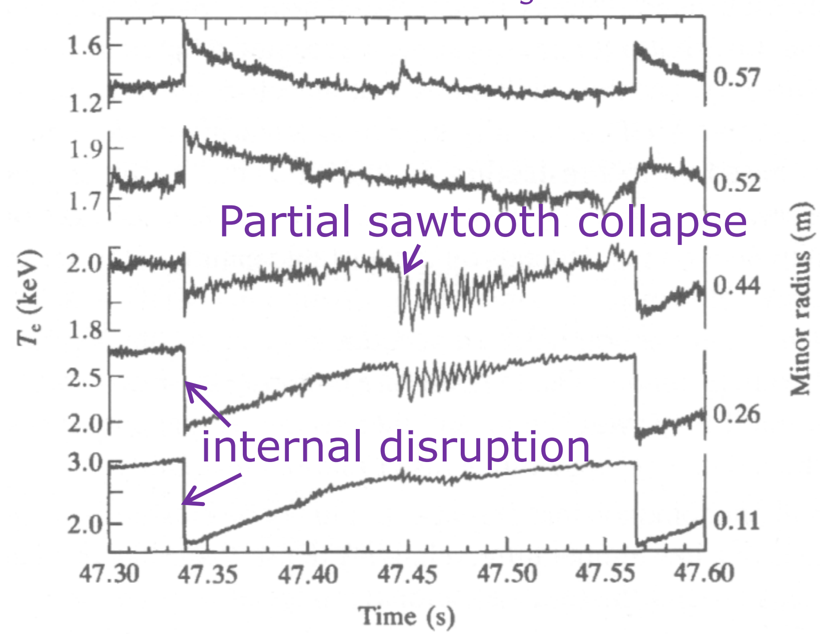
Non-linear Plasma Activity

- **Sawtooth**

- Inversion radius (r_s): when central temperature drops and flattens, the temperature decreases inside the radius and increases directly beyond it.



$r_s \sim 0.5a$



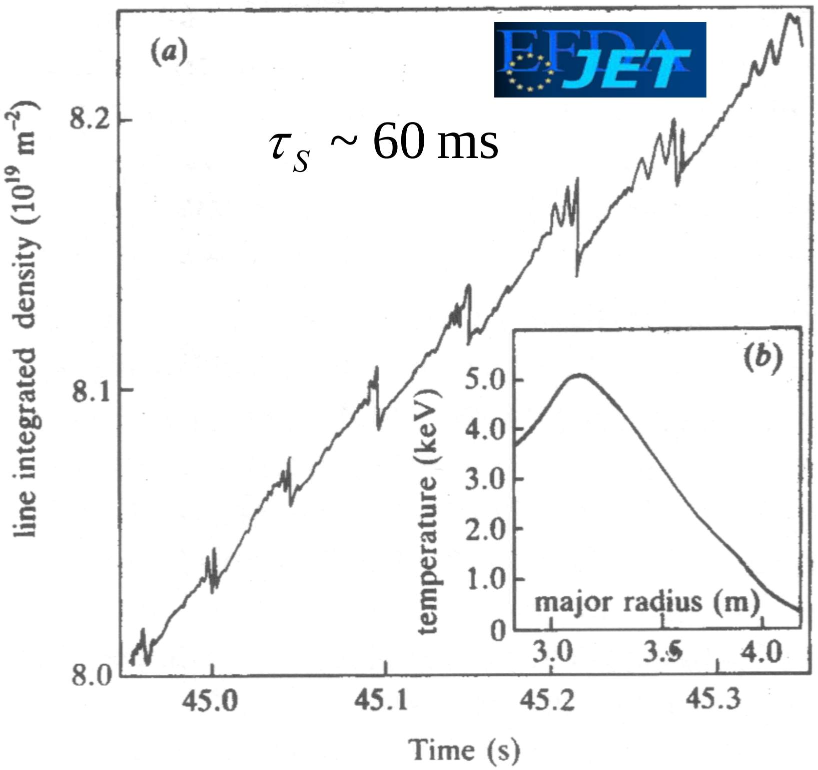
$$T_e = T_{e0}\left[1-\alpha y-(1-\alpha)y^2\right],\quad T_{e0}=6\langle T_e\rangle/(5-\alpha)$$

Non-linear Plasma Activity

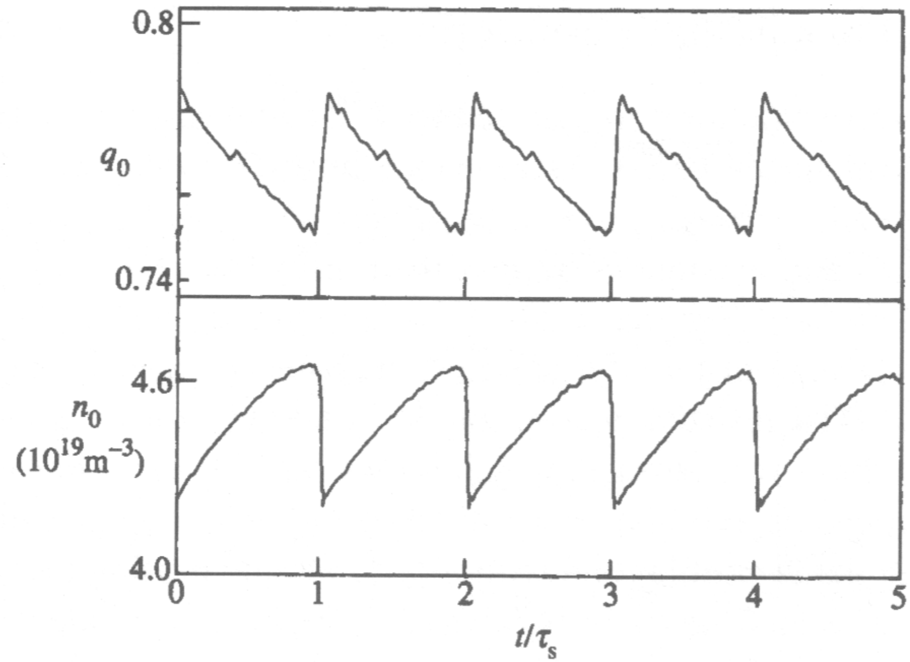
- **Sawtooth**

- $\Delta T_e \sim 40\%$, $\Delta q_0 \sim 4\%$, $\Delta n_0 \sim 9\%$

$q_0 < 1$ during all the sawtooth period
(*F. M. Levinton et al, PRL* ***63*** *2060 (1*



TEXTOR
(H. Soltwisch et *al, APS (1987)*)



- Simple semi-empirical scaling for the period of sawtooth oscillations

$$\tau_S \approx 10^{-2} R^2 T_e^{3/2} / Z_{eff}$$

Non-linear Plasma Activity

- **Sawtooth**

- Why the sawtooth oscillation should occur at all has not yet been explained.
- Two instabilities are required to drive the process

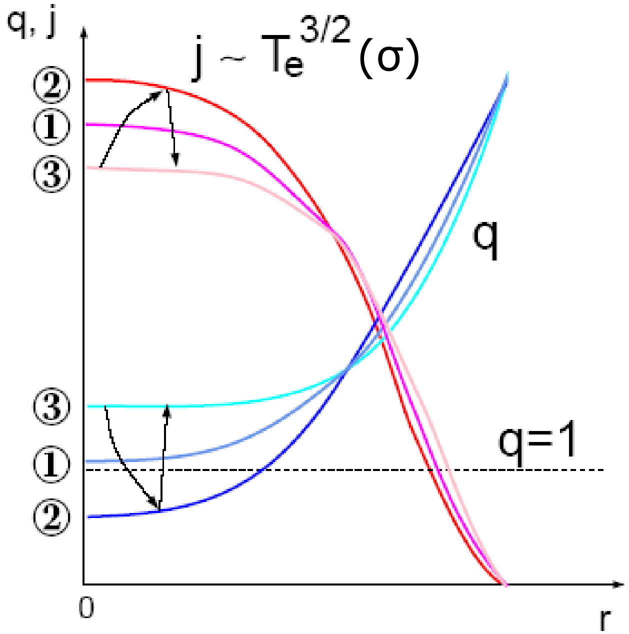
    abrupt collapse

    ramp phase

Non-linear Plasma Activity

- **Sawtooth**

- Kadomtsev model

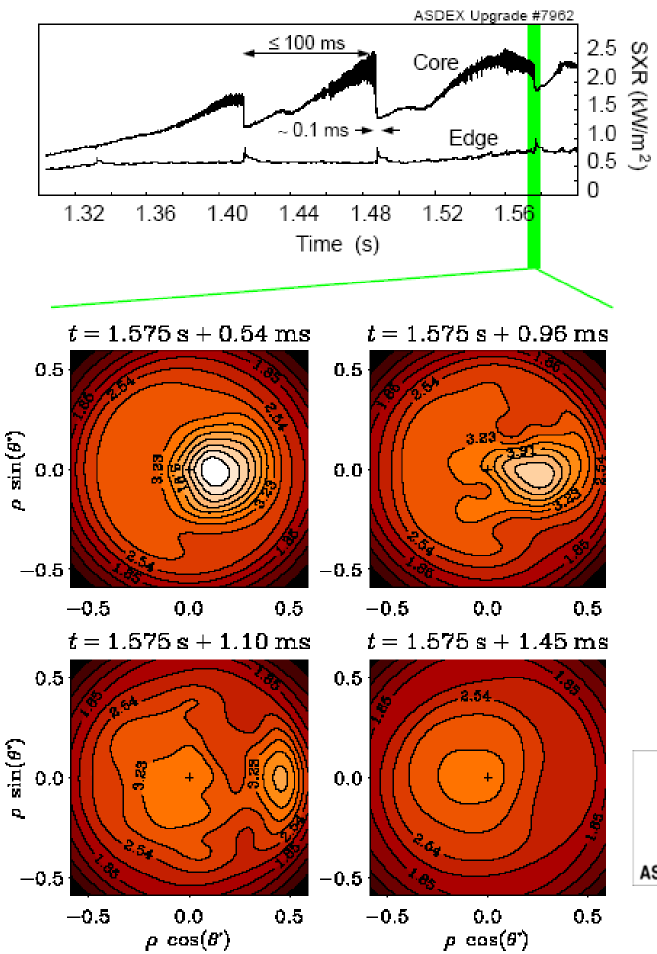


1. $T(0)$ and $j(0)$ rise due to ohmic heating (slower phase, resistive time scale)
2. $q(0)$ falls below 1, $q(r_s) = 1$
   → kink instability ($m/n=1/1$) grows
3. Fast reconnection event:
   T, n flattened inside $q = 1$ surface
   $q(0)$ rises slightly above 1
   kink stable

Non-linear Plasma Activity

- **Sawtooth**

- Kadomtsev model



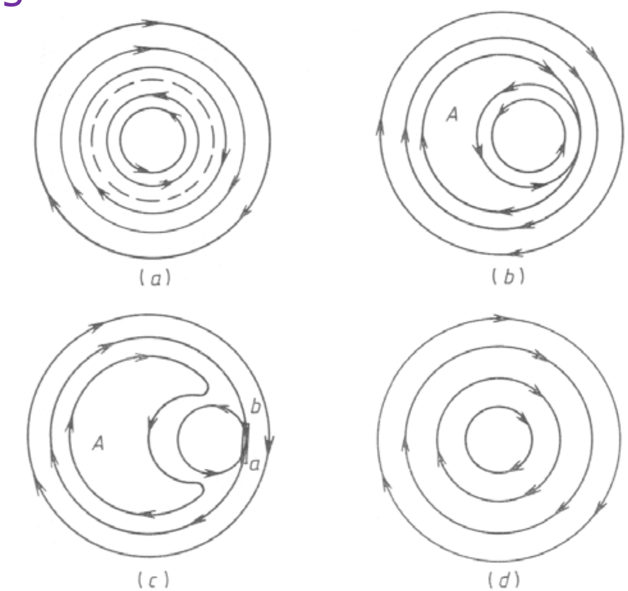
ASDEX Upgrade

(a) auxiliary transverse field $B_*=B_\theta-(r/R)B_T$ (different direction of magnetic lines relative to the surface with $B_* = 0$)

(b) contact of surfaces with oppositely directed fields B_*

(c) reconnection of the current layer *ab* due to finite plasma conductivity. A moon-like island *A* formed due to the reconnection

(d) final result of reconnections: auxiliary magnetic field is unidirectional



Non-linear Plasma Activity

- **Sawtooth**

- Kadomtsev model

Shortcomings 1: collapse time for the disruption orders of magnitude longer than observed

$\tau_c = \omega^2 / \xi_\perp$ collapse time (ω: island width)

$$\xi_\perp = \frac{\eta}{\mu_0} = \frac{m_e}{\mu_0 e^2 n_e \tau_e} = 1.025\times 10^8 \ln\Lambda / T_e^{3/2} \approx 4.4\times 10^{-2} T_e^{3/2}$$ magnetic diffusivity

Ex) τ_c ~ 10 ms at ω = 1 cm, T_e = 3 keV

JET: τ_c = 50-200 μs but Kadomtsev model gives τ_c ≥ 10 ms

→ If the collapse is associated with a magnetic rearrangement an explanation of its rapidity was required

Non-linear Plasma Activity

- **Sawtooth**

- Kadomtsev model

Shortcomings 2: no precise specification for the occurrence of a disruption

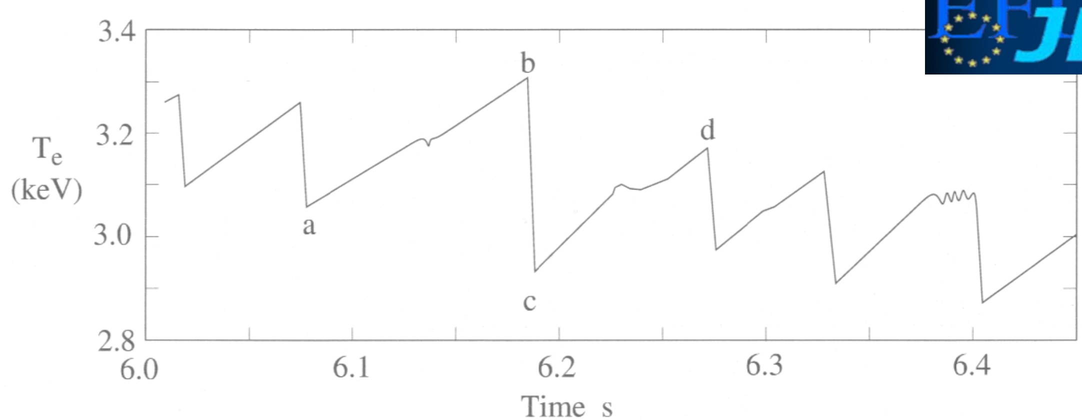
Shortcomings 3: sometimes precursors are absent or lacking in experiments as is the case with the large amplitude oscillations known as ‘giant’ or ‘compound’ sawteeth

Non-linear Plasma Activity

- **Sawtooth**

- Kadomtsev model

Shortcomings 4: existence of 'double' sawteeth with a longer and sometimes erratic period and a larger amplitude – requiring a hollow current profile with two q = 1 surfaces



Shortcomings 5: could not reproduce the sharp spikes on the disruption profile and that at the outset of the disruption the island size was much too small

Non-linear Plasma Activity

- **Sawtooth**

- Reconnection of the magnetic field lines: Sweet-Parker model

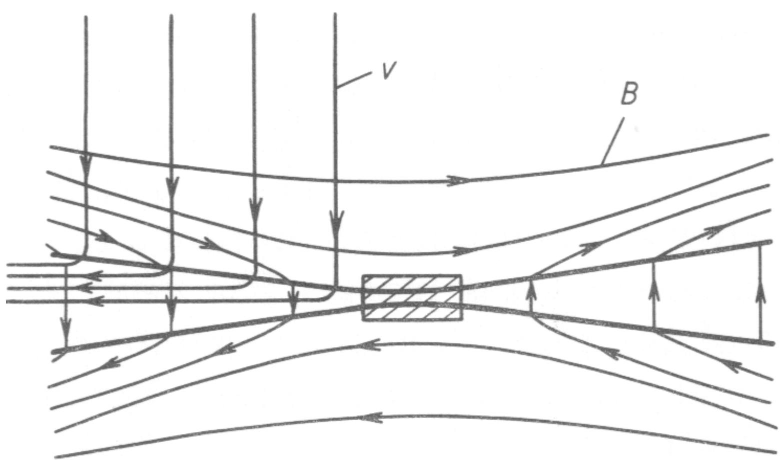
1. Magnetic fields are pushed together by flows into a narrow region. In the flow regions the resistivity is low and hence the magnetic field is frozen in the flow. The two regions are separated by a current sheet (the reversal of the magnetic field requires a current to flow in the thin layer separating them). Within this layer resistive diffusion plays a key role.
2. As the two regions come together the plasma is squeezed out along the field lines allowing the fields to get closer and closer to the neutral sheet.
3. At some stage the field lines break and reconnect in a new configuration at a magnetic null-point, X. The large stresses in the acutely bent field lines in the vicinity of the null-point result in a double-action magnetic 'catapult' that ejects plasma in both directions, with velocity of $O(v_A)$. This in turn allows plasma to flow into the reconnection zone from the sides.

Non-linear Plasma Activity

- **Sawtooth**

- Reconnection of the magnetic field lines: Sweet-Parker model

1. The field diffuses into plasma and magnetic lines reconnect.
2. A kind of 'catapult' of strained magnetic lines is formed.
3. It throws out the plasma from the layer into the moon-like region *A* of the magnetic island (b)

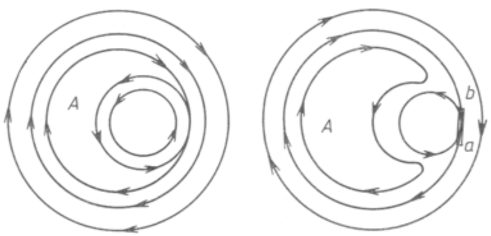


Non-linear Plasma Activity

- **Sawtooth**

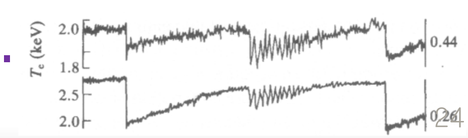
- Phase of the sharp temperature profile flattening (internal disruption)

  1. What is the trigger of the internal disruption (type of instability)?
  2. How does the disruption develop?
  3. What is the time of disruption?



(b) (c)

- Internal $m = 1/n = 1$ snake
- In some cases, instability when β_p inside r_s exceeds a certain critical value
- Every force tube 'catapulting' into A may drastically perturb plasma and create MHD-turbulence. If a turbulent zone is formed in A, then the B_* mean value may disappear due to mixing of magnetic lines. Then there is no force that would 'press' the internal core to the magnetic surface with the inverse magnetic field.
  → partial (incomplete) reconnection



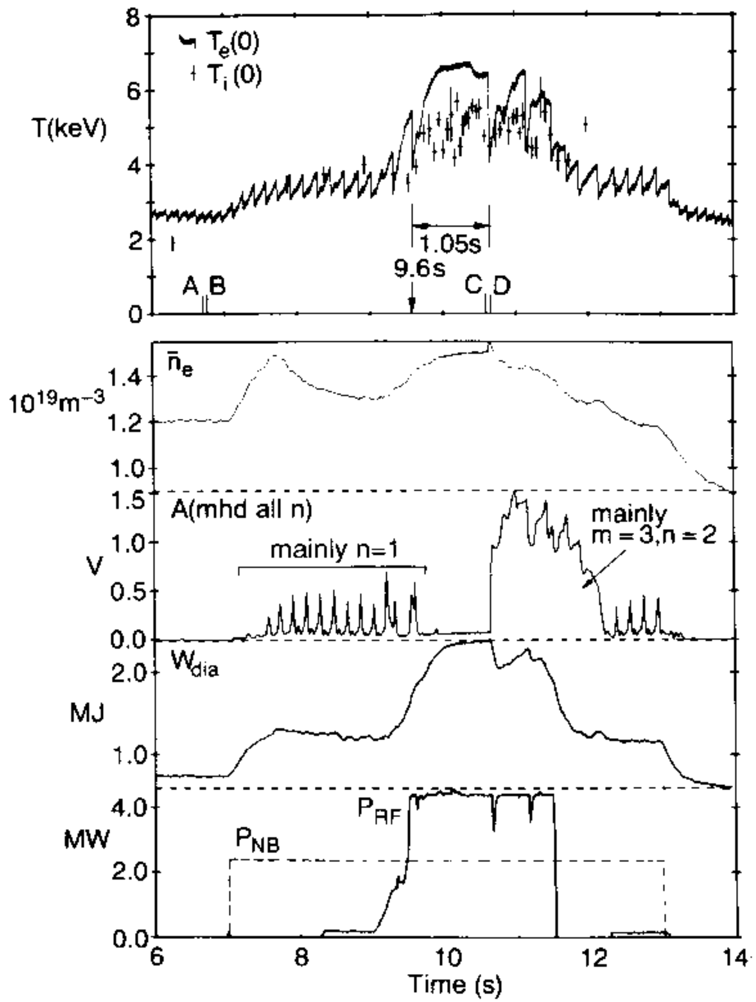
Non-linear Plasma Activity

- **Sawtooth**

- Stochasticity of the magnetic field lines may appear due to the toroidicity which violates the ideal helical symmetry.
  → change significantly the resistivity value inside the current layer
  → electron does not return back to the same point if after crossing the current layer, an anomalous skin-layer can develop.
  → significantly increasing the reconnection rate and makes it close to the observed one at the fastest internal disruptions.

Non-linear Plasma Activity

- **Monster Sawtooth**

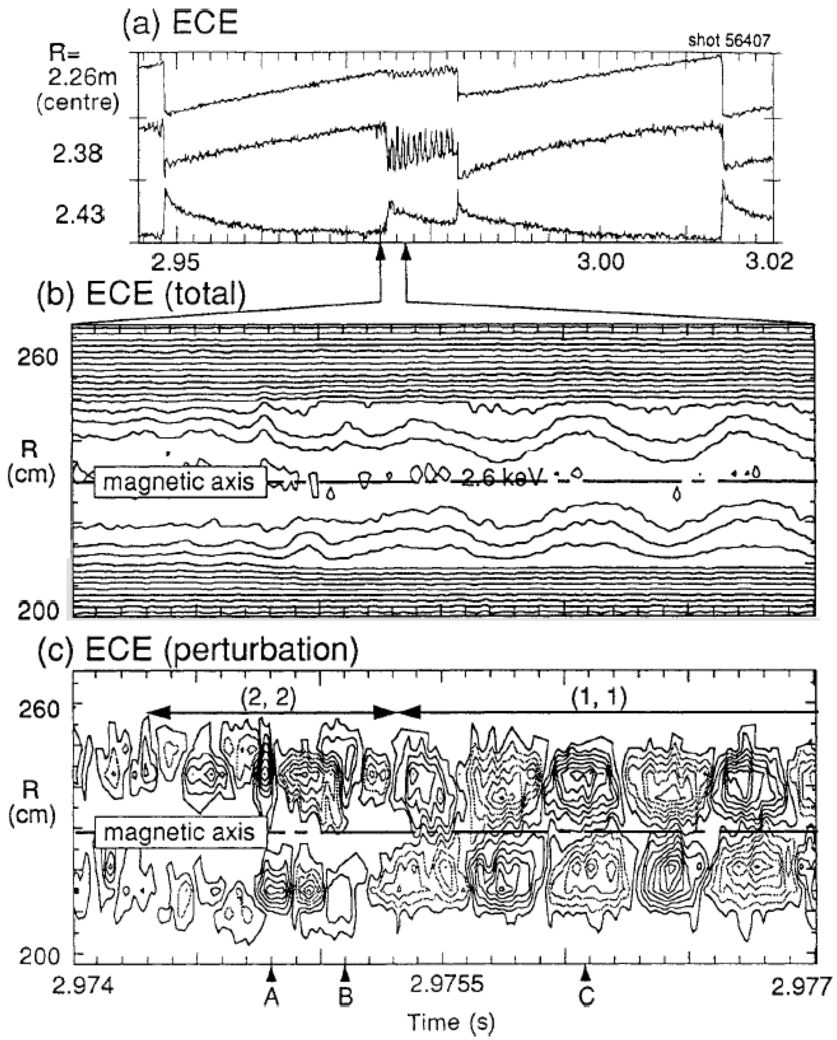


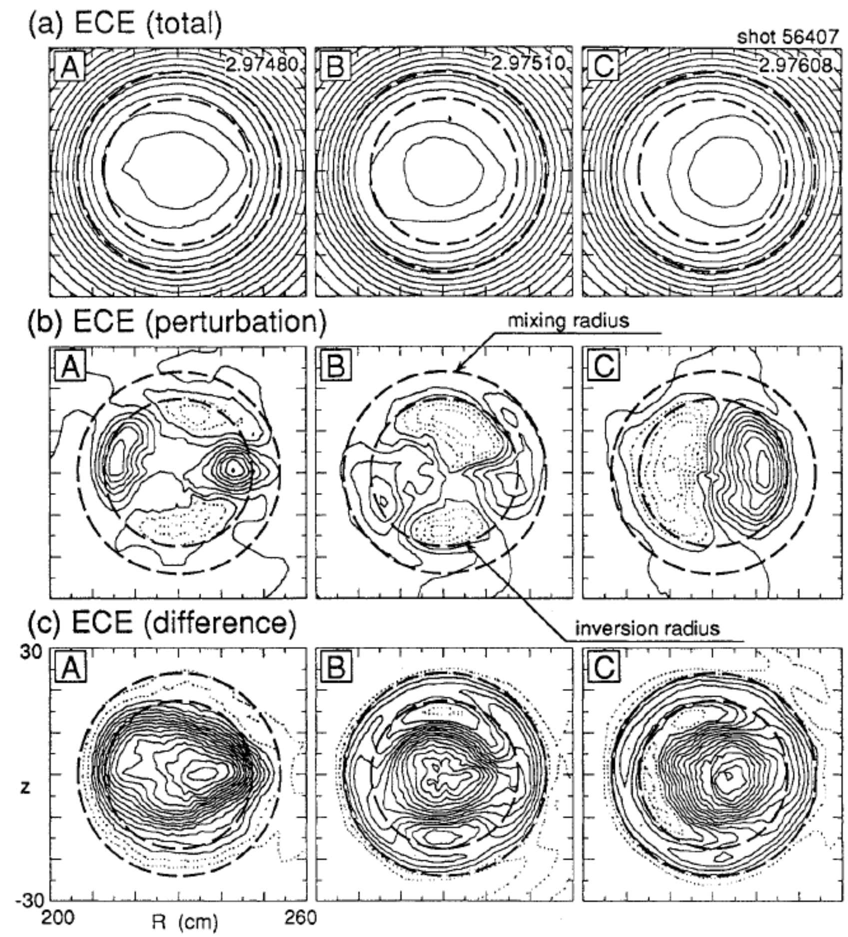
- No low (m,n) number coherent MHD activity observed during the temperature saturation phase
- ICRH and/or NBI above 5 MW
- Possibly due to stabilisation of the m = 1 instability by fast ions

*D. J. Campbell et al, PRL **60** 2148 (1988)*

Non-linear Plasma Activity

- **Partial crash by higher modes**

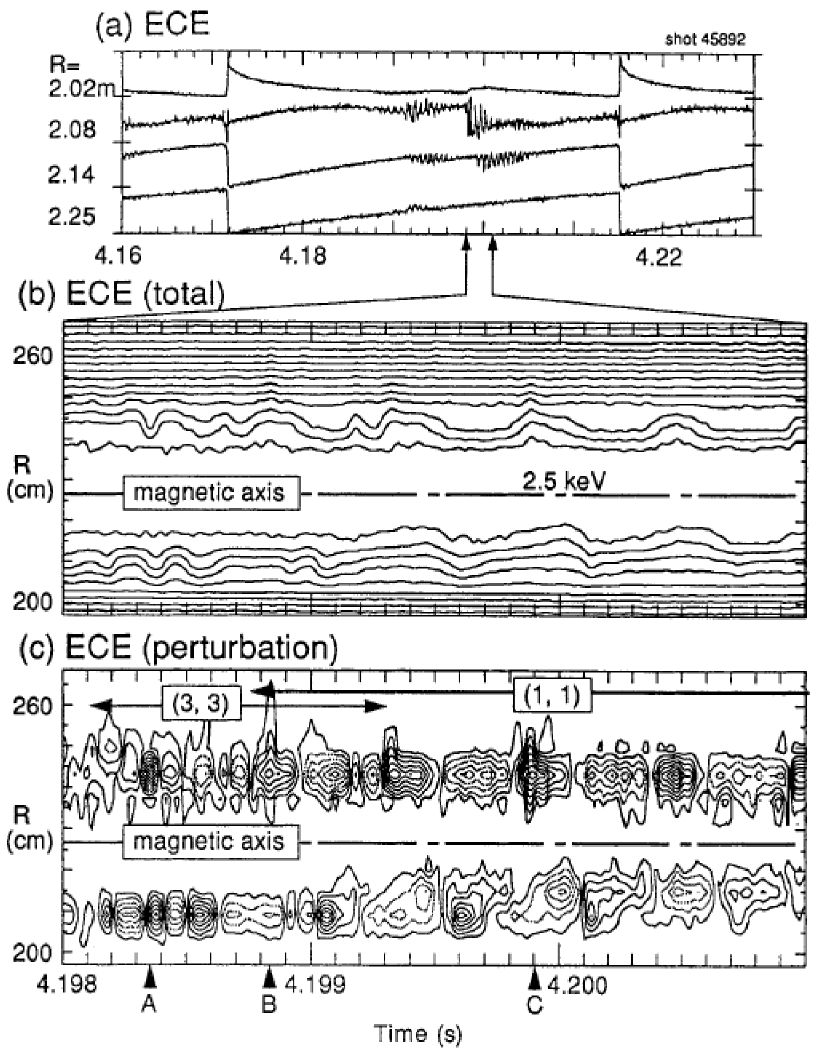


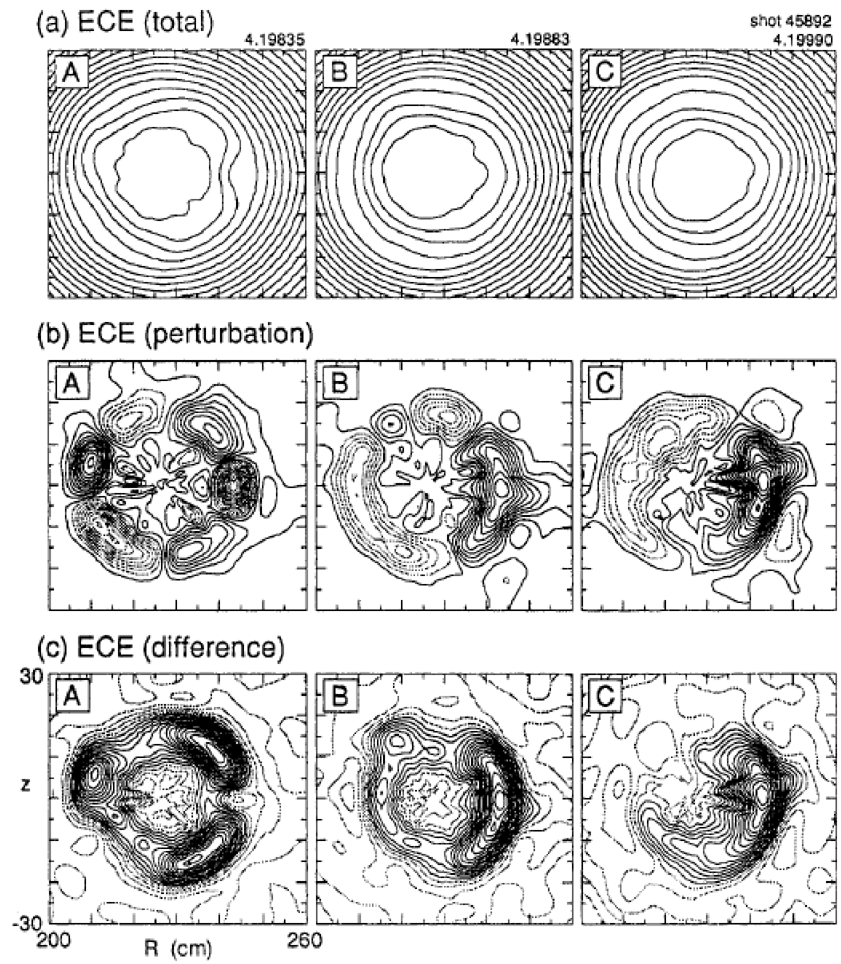


Y. Nagayama et al, NF ***36*** *521 (1996)*

Non-linear Plasma Activity

- **Partial crash by higher modes**



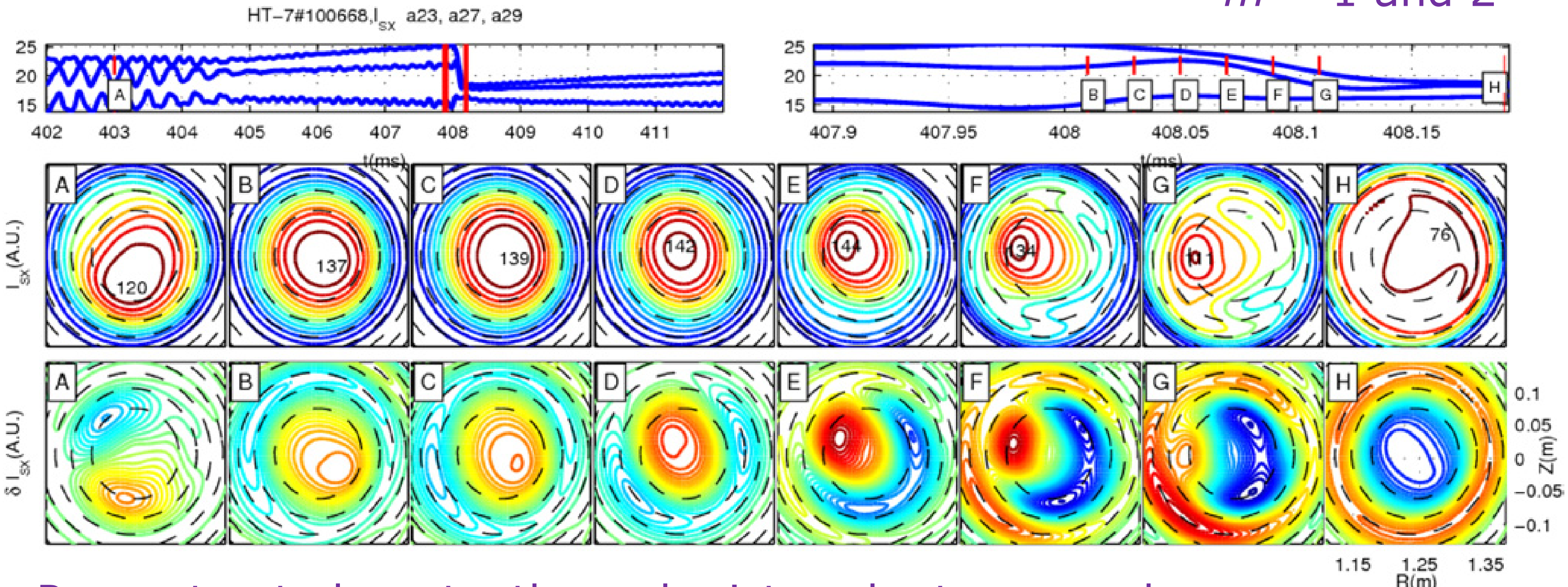


*Y. Nagayama et al, NF **36** 521 (1996)*

Non-linear Plasma Activity

- **Partial crash by higher modes**

m = 1 and 2



- Reconstructed sawtooth crash picture by tomography:

line-integrated soft-x-ray signals at 3 chords,

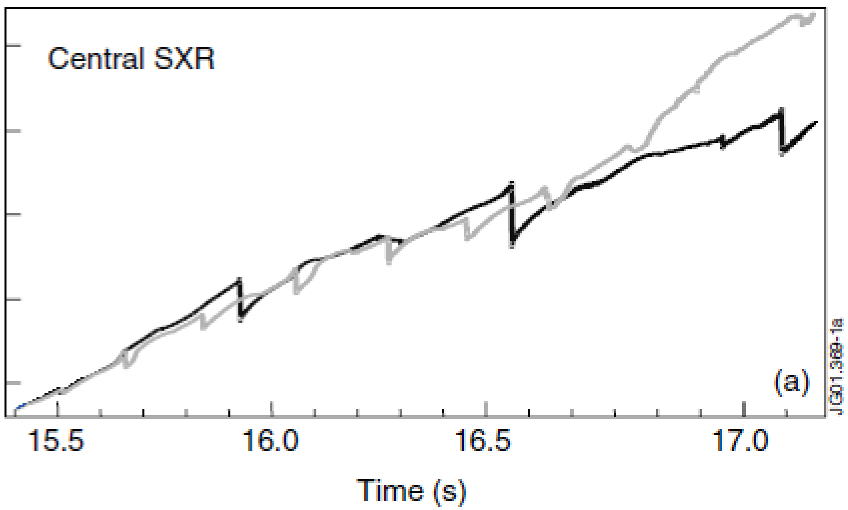
the contour plot of the reconstructed local emission intensities profile from the total signals,

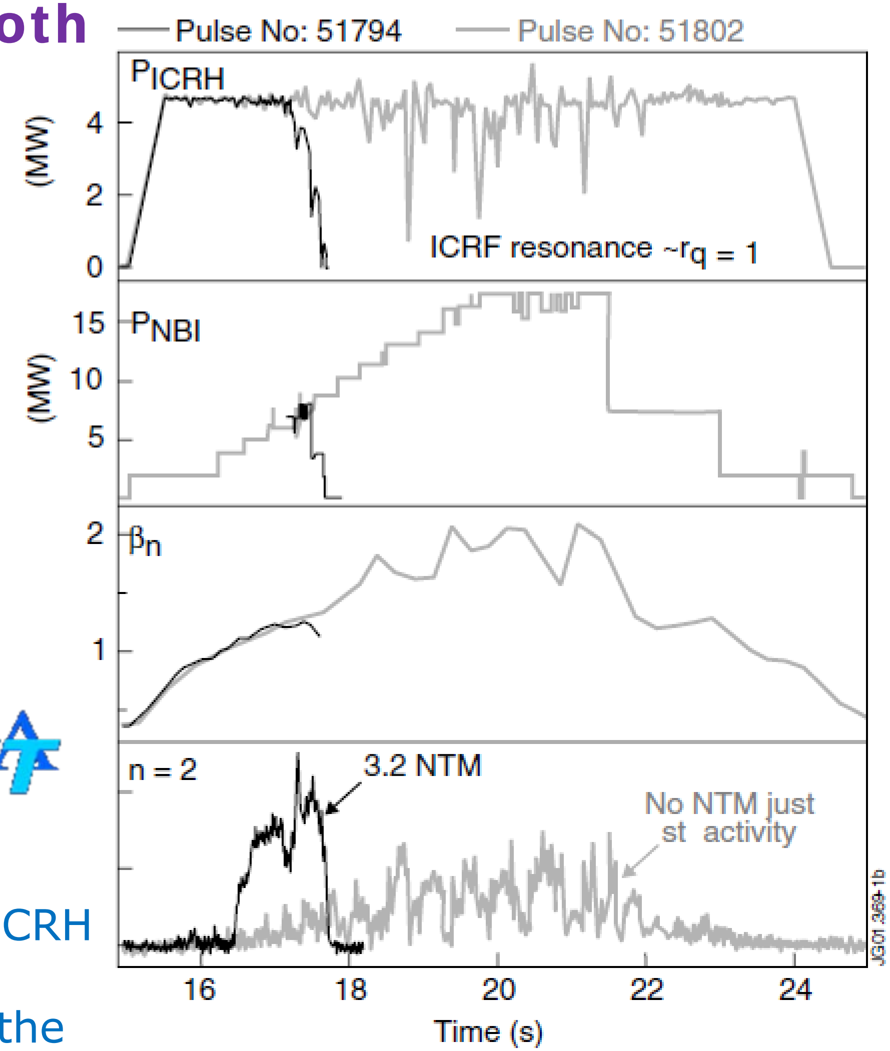
the contour plot of the reconstructed perturbation of the local emission intensities from the perturbation signals extracted by the SVD method

*Youwen Sun et al, PPCF **51** 065001 (2009)*

Non-linear Plasma Activity

- **NTM triggered by Sawtooth**





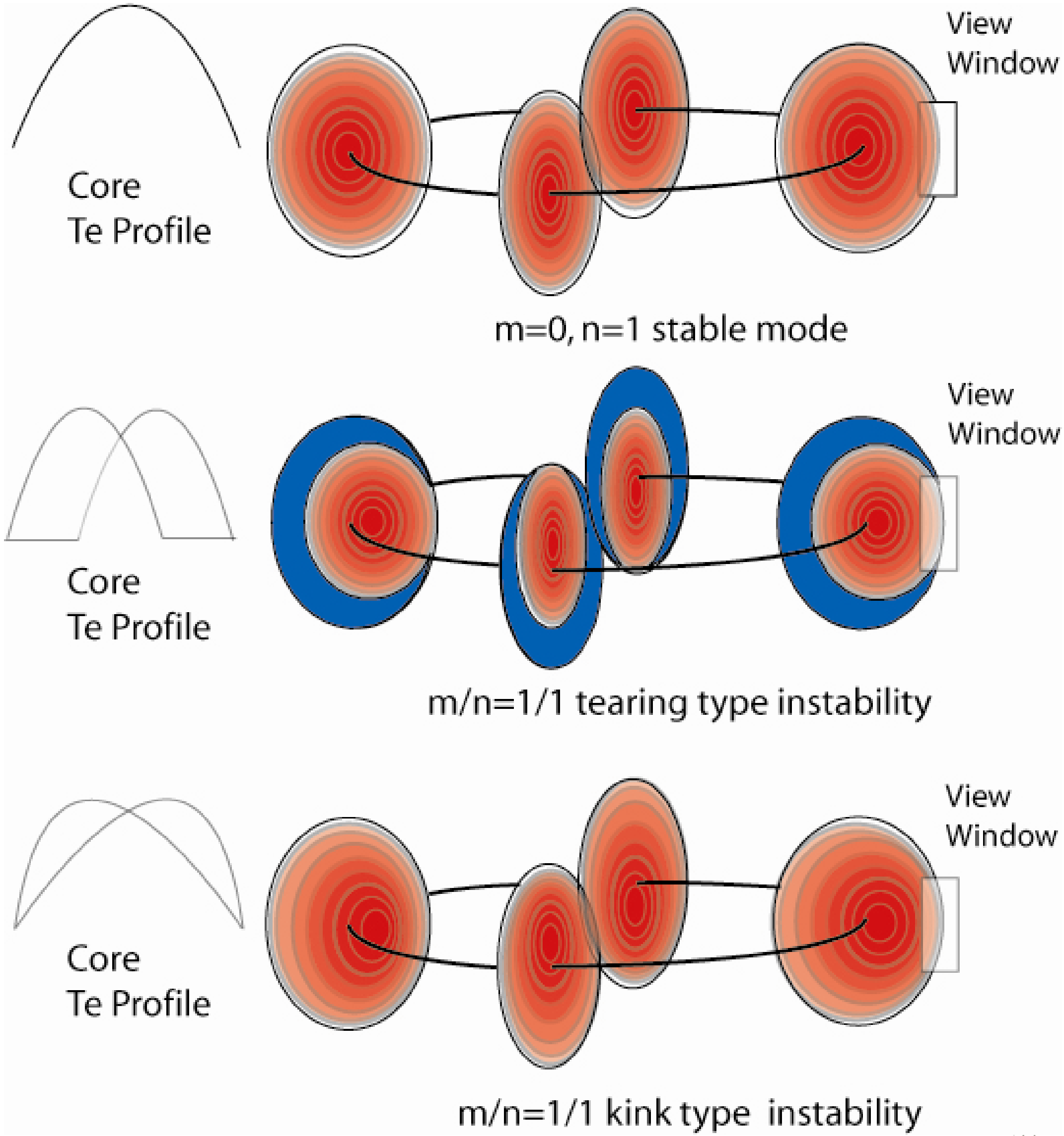
- Increased sawtooth period due to stabilisation by fast ions produced by ICRH leads to the triggering of n = 2 NTM activity which causes a termination of the discharge.

Sawtooth

- Stable *m*/*n*=0/1 mode in the initial stage

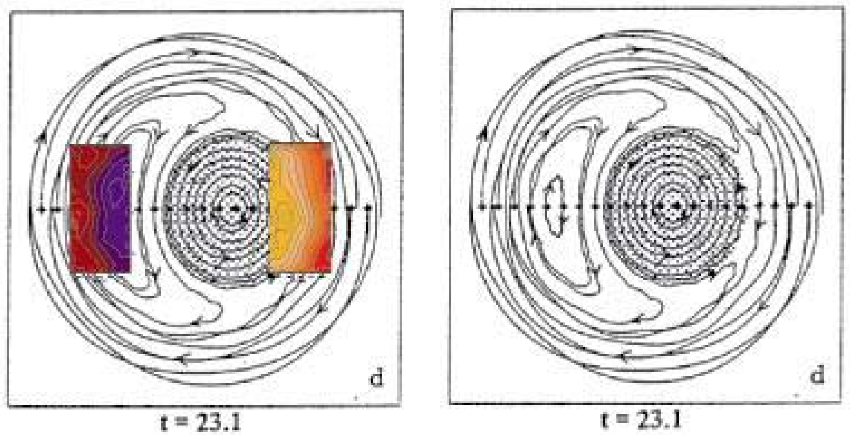
- *m*/*n*=1/1 mode develops as the instability grows (kink or tearing instability) and reconnection occurs
  - Tearing mode instability (slow evolution of the island/hot spot)
  - Kink mode instability (sudden crash)

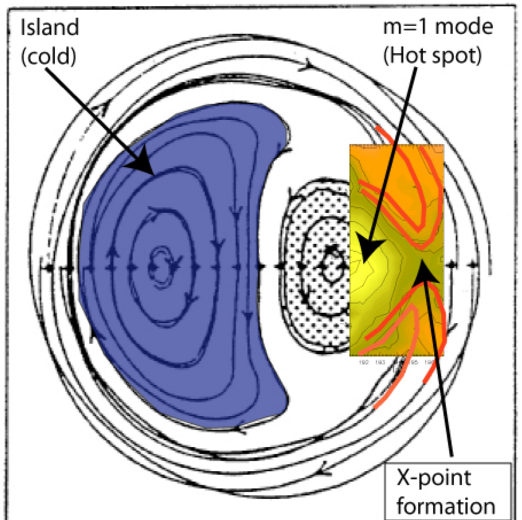
- Reconnection time scale is any different in these two types?



Sawtooth

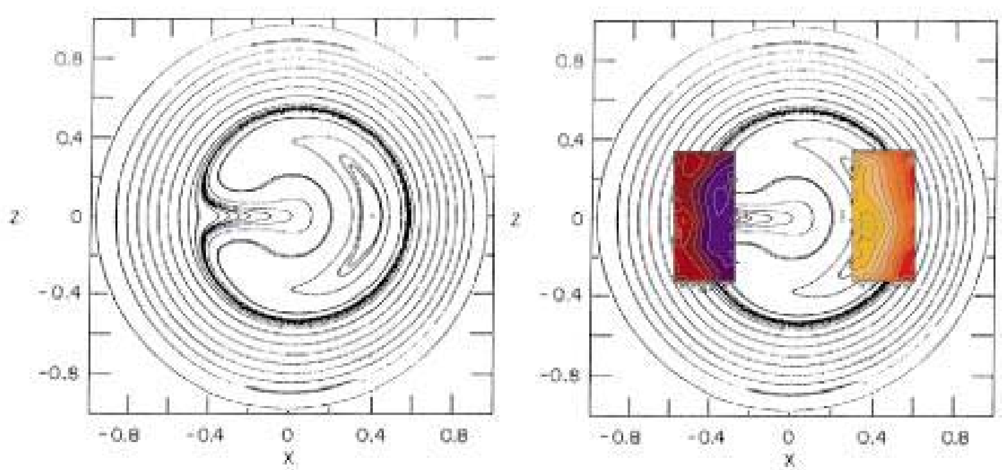
- Comparison with the full reconnection model





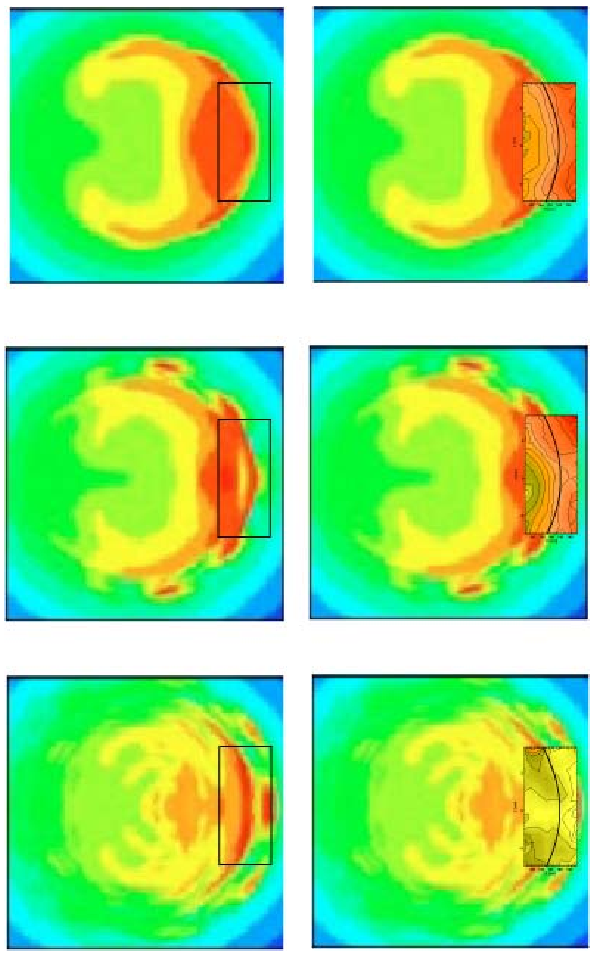
Comparison with the full reconnection model simulation result at the low field side

- Comparison with the quasi-interchange model

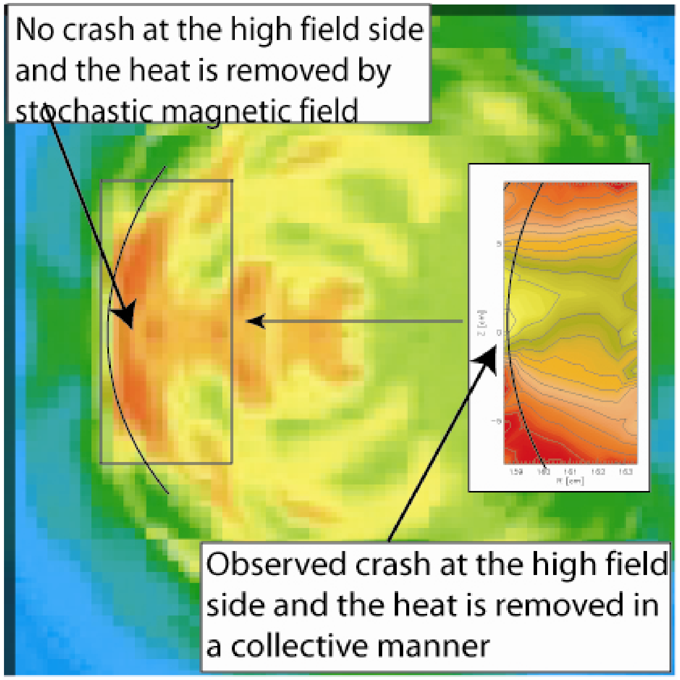


Sawtooth

- Comparison with the ballooning mode model



Low field side



Comparison with the Ballooning model simulation result at the high field side

References

- *Wolfgang Suttrop, "Experimental Results from Tokamaks", IPP Summer School, IPP Garching, September, 2001*