Fusion Reactor Technology I
(459.760, 3 Credits)

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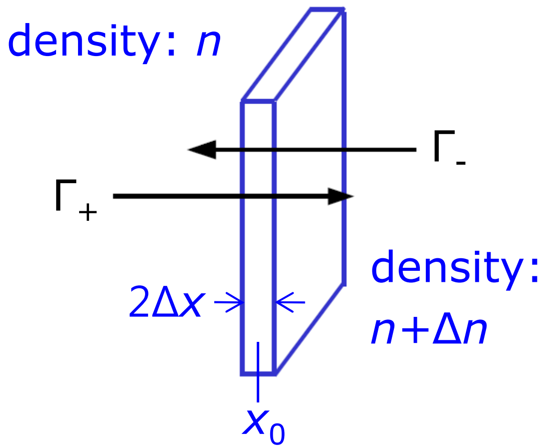
Plasma Transport

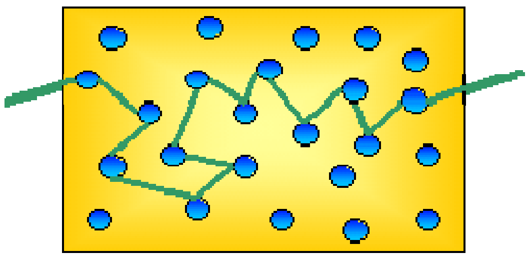
- **Classical Transport**
  - Particle transport



$$n(x) = n(x_0) + \left.\frac{\partial n}{\partial x}\right|_{x=x_0} (x - x_0)$$

$$\Gamma_+ = \frac{1}{2}\int_{x_0-\Delta x}^{x_0} \frac{1}{\tau} n(x)dx = \frac{1}{2}\left[n(x_0) - \frac{\partial n}{\partial x}\frac{\Delta x}{2}\right]\frac{\Delta x}{\tau}$$

$$\Gamma_- = \frac{1}{2}\int_{x_0}^{x_0+\Delta x} \frac{1}{\tau} n(x)dx = \frac{1}{2}\left[n(x_0) + \frac{\partial n}{\partial x}\frac{\Delta x}{2}\right]\frac{\Delta x}{\tau}$$



$$\Gamma = \Gamma_+ - \Gamma_- = -\frac{(\Delta x)^2}{2\tau}\frac{\partial n}{\partial x} = -D\frac{\partial n}{\partial x}$$: Particle flux- Fick's law

$$D = \frac{(\Delta x)^2}{2\tau}$$: diffusion coefficient (m^2/s)

The heat and momentum fluxes can be estimated in similar fashion.

Plasma Transport

- **Classical Diffusion**
  - Momentum transport

    Momentum flux

$$\pi_{\alpha\beta} = -\eta \frac{\partial v_y}{\partial x}$$

$$\eta \sim \frac{mn(\Delta x)^2}{\tau} \sim mnD$$: viscosity coefficient

  - Heat transport

    Heat flux

$$q = -\kappa \frac{\partial T}{\partial x}$$

$$\kappa \sim \frac{n(\Delta x)^2}{\tau} \sim nD$$: thermal conductivity

Plasma Transport

- **Braginskii Equations**

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) = 0$$

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = 0$$

$$m_e n_e \frac{dv_{e\alpha}}{dt} = -\frac{\partial p_e}{\partial x_\alpha} - \frac{\partial \pi_{e\alpha\beta}}{\partial x_\beta} - en_e(E + [\mathbf{v}_e \times \mathbf{B}]_\alpha) + R_\alpha$$

$$m_i n_i \frac{dv_{i\alpha}}{dt} = -\frac{\partial p_i}{\partial x_\alpha} - \frac{\partial \pi_{i\alpha\beta}}{\partial x_\beta} - Zen_i(E + [\mathbf{v}_i \times \mathbf{B}]_\alpha) - R_\alpha$$

$$\frac{3}{2} n_e \frac{dT_e}{dt} + p_e \nabla \cdot \mathbf{v}_e = -\nabla \cdot \mathbf{q}_e - \pi_{e\alpha\beta} \frac{\partial v_{e\alpha}}{\partial x_\beta} + Q_e$$

$$\frac{3}{2} n_i \frac{dT_i}{dt} + p_i \nabla \cdot \mathbf{v}_i = -\nabla \cdot \mathbf{q}_i - \pi_{i\alpha\beta} \frac{\partial v_{i\alpha}}{\partial x_\beta} + Q_i$$

$$p_e = n_e T_e, \quad p_i = n_i T_i$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{v}\nabla)$$

$$n = n_e = Zn_i$$

assume $Z = 1$

R, $\pi_{\alpha\beta}$, q , Q ?

Plasma Transport

- **Braginskii Equations**
  - Transfer of momentum from ions to electrons by collisions

$$\mathbf{R} = \mathbf{R}_u + \mathbf{R}_T$$

R$_u$: force of friction due to the existence of a relative velocity

u=v$_e$-v$_i$

R$_T$: thermal force which arises by virtue of a gradient in the electron temperature

$$\mathbf{R}_u = -\frac{m_e n_e}{\tau_e}(0.51\mathbf{u}_\parallel + \mathbf{u}_\perp) = en\left(\frac{\mathbf{j}_\parallel}{\sigma_\parallel} + \frac{\mathbf{j}_\perp}{\sigma_\perp}\right)$$

$$\mathbf{R}_T = -0.71 n_e \nabla_\parallel T_e - \frac{3}{2}\frac{n_e}{\omega_e \tau_e}\left(\frac{\mathbf{B}}{B} \times \nabla T_e\right)$$

$$\sigma_\perp = \frac{e^2 n_e \tau_e}{m_e} = \sigma_1 T_e^{3/2}$$

$$\sigma_\parallel = 1.96\sigma_\perp = 1.96\sigma_1 T_e^{3/2}$$

$$\sigma_1 = \frac{0.9\times10^{13}}{(\lambda/10)Z} \quad \left[\mathrm{s}^{-1}\cdot\mathrm{eV}^{-3/2}\right]$$

Plasma Transport

- **Braginskii Equations**
  - Heat flux

$$\mathbf{q}_e = \mathbf{q}_u^e + \mathbf{q}_T^e$$

$$\mathbf{q}_u^e = 0.71 n_e T_e \mathbf{u}_\parallel + \frac{3}{2}\frac{n_e T_e}{\omega_e \tau_e}\left(\frac{\mathbf{B}}{B}\times\mathbf{u}\right)$$

$$\mathbf{q}_T^e = -\kappa_\parallel^e \nabla_\parallel T_e - \kappa_\perp^e \nabla_\perp T_e - \frac{5}{2}\frac{n_e T_e}{eB}\left(\frac{\mathbf{B}}{B}\times\nabla T_e\right)$$

$$\kappa_\parallel^e = 3.16\frac{n_e T_e \tau_e}{m_e}, \quad \kappa_\perp^e = 4.66\frac{n_e T_e}{m_e \omega_e^2 \tau_e}$$

$$\mathbf{q}_i = -\kappa_\parallel^i \nabla_\parallel T_i - \kappa_\perp^i \nabla_\perp T_i + \frac{5}{2}\frac{n_i T_i}{ZeB}\left(\frac{\mathbf{B}}{B}\times\nabla T_i\right) \qquad \omega_i \tau_i >> 1$$

$$\kappa_\parallel^i = 3.9\frac{n_i T_i \tau_i}{m_i}, \quad \kappa_\perp^i = 2\frac{n_i T_i}{m_i \omega_i^2 \tau_i}$$

Plasma Transport

- **Braginskii Equations**
  - Heat generated as a consequence of collisions

$$Q_i = Q_\Delta = \frac{3m_e}{m_i}\frac{n_e}{\tau_e}(T_e - T_e)$$

$$Q_e = -\mathbf{Ru} - Q_\Delta = \frac{j_\parallel^2}{\sigma_\parallel} + \frac{j_\perp^2}{\sigma_\perp} + \frac{1}{en_e}\mathbf{jR}_T - \frac{3m_e}{m_i}\frac{n_e}{\tau_e}(T_e - T_e)$$

  - Stress tensor in the absence of a magnetic field

$$\pi_{\alpha\beta} = nm\left\langle v'_\alpha v'_\beta - (v'^2/3)\delta_{\alpha\beta}\right\rangle = -\eta_0 W_{\alpha\beta}$$

viscosity coefficient

Rate of strain tensor

$$W_{\alpha\beta} = \frac{\partial v_\alpha}{\partial x_\beta} + \frac{\partial v_\beta}{\partial x_\alpha} - \frac{2}{3}\delta_{\alpha\beta}\nabla\cdot\mathbf{v}$$

Plasma Transport

- **Braginskii Equations**

In a strong magnetic field $\omega\tau >> 1$

$$\pi_{zz} = -\eta_0 W_{zz}$$

$$\pi_{xx} = -\eta_0 \frac{1}{2}\left(W_{xx} + W_{yy}\right) - \eta_1 \frac{1}{2}\left(W_{xx} - W_{yy}\right) - \eta_3 W_{xy}$$

$$\pi_{yy} = -\eta_0 \frac{1}{2}\left(W_{xx} + W_{yy}\right) - \eta_1 \frac{1}{2}\left(W_{yy} - W_{xx}\right) + \eta_3 W_{xy}$$

$$\pi_{xy} = \pi_{yx} = -\eta_1 W_{xy} + \eta_3 \frac{1}{2}\left(W_{xx} - W_{yy}\right)$$

$$\pi_{xz} = \pi_{zx} = -\eta_2 W_{xz} - \eta_4 W_{yz}$$

$$\pi_{yz} = \pi_{zy} = -\eta_2 W_{yz} + \eta_4 W_{xz}$$

$$\eta_0^i = 0.96 n_i T_i \tau_i$$

$$\eta_1^i = \frac{3}{10}\frac{n_i T_i}{\omega_i^2 \tau_i}, \quad \eta_2^i = 4\eta_1^i$$

$$\eta_3^i = \frac{1}{2}\frac{n_i T_i}{\omega_i}, \quad \eta_4^i = 2\eta_3^i$$

viscosity coefficients

$$\eta_0^e = 0.73 n_e T_e \tau_e$$

$$\eta_1^e = 0.51 \frac{n_e T_e}{\omega_e^2 \tau_e}, \quad \eta_2^e = 4\eta_1^e$$

$$\eta_3^e = -\frac{1}{2}\frac{n_e T_e}{\omega_e}, \quad \eta_4^i = 2\eta_3^e$$

Plasma Transport

- **Braginskii Equations**
  - Heat generated as a result of viscosity

$$Q_{vis} = -\pi_{\alpha\beta}\frac{\partial v_\alpha}{\partial x_\beta} = -\frac{1}{2}\pi_{\alpha\beta}W_{\alpha\beta}$$

Individual Charge Trajectories

- **Invariant of Motion**

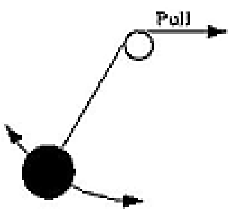
$$\frac{d}{dt}E_0 = \frac{d}{dt}\left(\frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2\right) = 0 \qquad \mu = \frac{mv_\perp^2/2}{B}$$

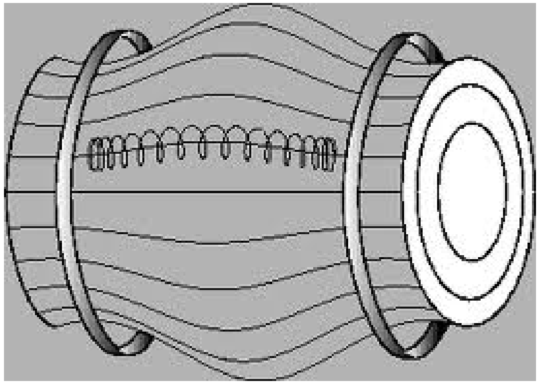
$$\mathbf{F}_\parallel = m\frac{d\mathbf{v}_\parallel}{dt} = -\mu\nabla_\parallel \mathbf{B} = -\mu\frac{\partial \mathbf{B}}{\partial s} = -\mu\frac{\partial \mathbf{B}}{\partial s}\cdot\frac{ds}{dt}\cdot\frac{1}{v_\parallel} = -\frac{\mu}{v_\parallel}\frac{d\mathbf{B}}{dt} \longrightarrow \frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2\right) = -\mu\frac{dB}{dt}$$

$$\longrightarrow \frac{d}{dt}\left(\frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2\right) = \frac{d}{dt}(\mu B) + \left(-\mu\frac{dB}{dt}\right) = 0$$

$$\longrightarrow \frac{d}{dt}(\mu) = 0$$: adiabatic invariant

- If B is constant
- $\frac{r_L}{B}\nabla_\parallel B << 1$
- $\frac{1}{\omega_c B}\frac{dB}{dt} << 1$



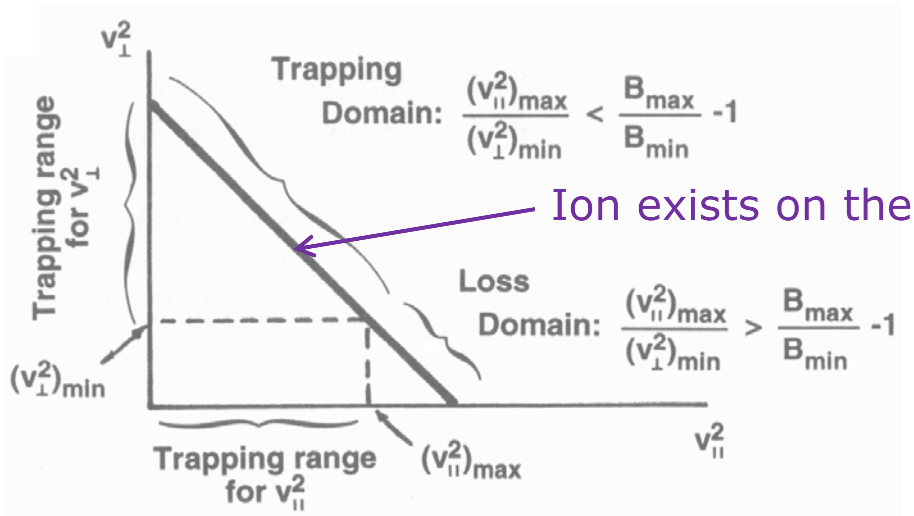


Magnetic Mirror

- **Condition for Trapping of Particles**

$$E_0 = \frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2 = \frac{1}{2}mv_\parallel^2 + \mu B = \frac{1}{2}m\left(v_\parallel^2\right)_{\max} + \mu B_{\min}$$

$$\mathbf{F}_\parallel = -\frac{1}{2}\frac{mv_\perp^2}{B}\nabla_\parallel B = -\mu\nabla_\parallel B$$



Ion exists on the E_0=constant line.

$$v_\perp^2 = \frac{2E_0}{m} - v_\parallel^2$$

Condition for trapping of particles

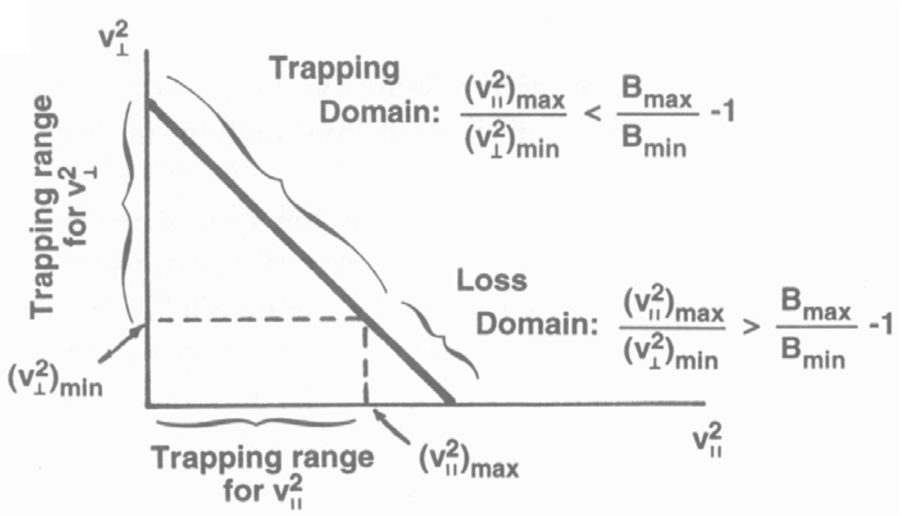
$$v_\parallel\Big|_{B\le B_{\max}} = 0 \longrightarrow E_0 = \frac{1}{2}m\left(v_\parallel^2\right)_{\max} + \mu B_{\min} \le 0 + \mu B_{\max}$$

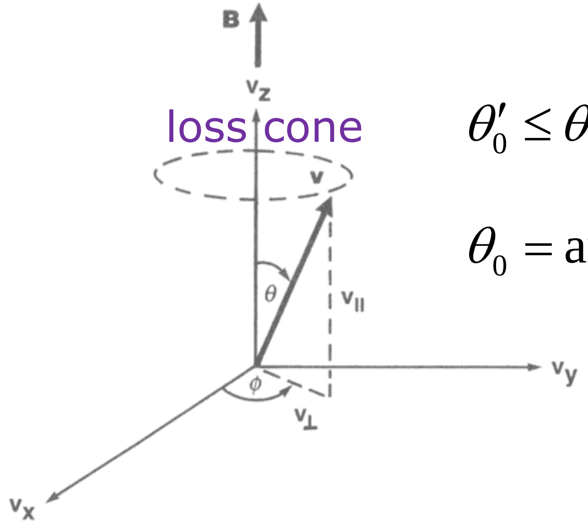
Magnetic Mirror

- **Condition for Trapping of Particles**

$$\frac{\frac{1}{2}m\left(v_\parallel^2\right)_{\max}}{\mu B_{\min}} \le \frac{B_{\max}}{B_{\min}} - 1$$

$$\frac{\left(v_\parallel^2\right)_{\max}}{\left(v_\perp^2\right)_{\min}} = \left(\frac{v_\parallel^2}{v_\perp^2}\right)_{mid-plane} \le \frac{B_{\max}}{B_{\min}} - 1$$





$$\theta_0' \le \theta \le \pi - \theta_0'$$

$$\theta_0 = \arcsin\sqrt{\frac{B_{\min}}{B_{\max}}}$$

$$\frac{v_\parallel^2}{v_\perp^2} = \left(\frac{\cos\theta}{\sin\theta}\right)^2 = \frac{1-\sin^2\theta}{\sin^2\theta} \le \frac{B_{\max}}{B_{\min}} - 1$$

$$\sin\theta \ge \sqrt{\frac{B_{\min}}{B_{\max}}}$$

Magnetic Mirror

- **Mirror Ratio**

$$f_{loss} = \frac{\int\limits_{\text{double cone}} f(\vec{v})d^3v}{\int\limits_0^\infty f(\vec{v})d^3v} = \frac{\int\limits_0^{2\pi} d\phi\left[\int\limits_0^{\theta_0'}\sin\theta d\theta + \int\limits_{\pi-\theta_0'}^{\pi}\sin\theta d\theta\right]\int\limits_0^\infty \frac{f(\vec{v})}{4\pi v^2}v^2d^3v}{\int\limits_0^{2\pi} d\phi\int\limits_0^{\pi}\sin\theta d\theta\int\limits_0^\infty f(\vec{v})d^3v}$$

$$= 1 - \cos\theta_0'$$

$$f_{trap} = \cos\theta_0' = \sqrt{1-\frac{B_{\min}}{B_{\max}}}$$

$$\frac{B_{\min}}{B_{\max}}$$

mirror ratio:
Determining the effectiveness of confinement

Why are particles reflected in the increased field of the mirrors?

$$\mathbf{F}_{\parallel} = -\frac{1}{2}\frac{mv_\perp^2}{B}\nabla_{\parallel}B = -\mu\nabla_{\parallel}B$$

Tokamak Transport

- **Neoclassical theory of transport**

- A. A. Galeev and R. Z. Sagdeev
  "Transport phenomena in a collisionless plasma in a toroidal magnetic system", Zhurnal Experimentalnoi i Teoreticheskoi Fiziki **53** 348 (1967)

Tokamak Transport

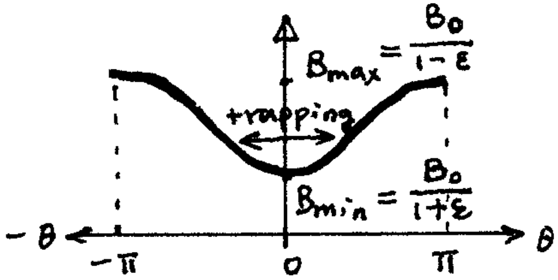
- **Particle Trapping**

$$\nabla \cdot B = 0$$

$$\Rightarrow \quad \frac{1}{1+\varepsilon\cos\theta}\left\{\frac{1}{r}\frac{\partial}{\partial r}(rB_r)+\frac{1}{r}\frac{\partial}{\partial\theta}\left[(1+\varepsilon\cos\theta)B_\theta\right]+\frac{1}{R_0}\frac{\partial B_\phi}{\partial\phi}\right\}=0$$

$$\Rightarrow \quad B_\theta(r,\theta)=\frac{B_\theta^0(\theta=0)}{1+\varepsilon\cos\theta}$$

$$B(r,\theta)=B_\theta(r,\theta)\hat{\theta}+B_\phi(r,\theta)\hat{\phi}=\frac{B_0}{1+\varepsilon\cos\theta}$$



Condition for trapping of particles

$$\frac{\left(v_\parallel^2\right)_{max}}{\left(v_\perp^2\right)_{min}}=\left(\frac{v_\parallel^2}{v_\perp^2}\right)_{mid-plane}\leq\frac{B_{max}}{B_{min}}-1=\frac{\dfrac{B_0}{1-\varepsilon}}{\dfrac{B_0}{1+\varepsilon}}-1=\frac{2\varepsilon}{1-\varepsilon}\sim 2\varepsilon$$

$$\Rightarrow \quad v_\parallel^2 \leq 2\varepsilon v_\perp^2$$

Tokamak Transport

- **Particle Trapping**

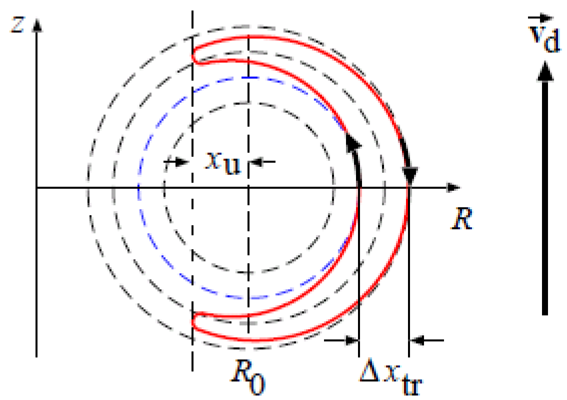
- Particle trapping by magnetic mirrors
  trapped particles with banana orbits
  untrapped (transit or passing) particles with circular orbits

- Trapped fraction: $f_{trap} = \sqrt{1 - \frac{1}{R_m}} = \sqrt{1 - \frac{B_{\min}}{B_{\max}}} = \sqrt{1 - \frac{1-\varepsilon}{1+\varepsilon}} = \sqrt{\frac{2\varepsilon}{1+\varepsilon}} \sim \sqrt{\varepsilon}$

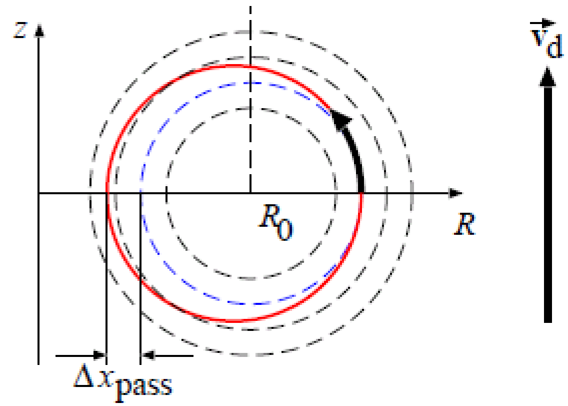
  for a typical tokamak, $\varepsilon \sim 1/3 \rightarrow f_{trap} \sim 70\%$

Tokamak Transport

- **Particle Trapping**



trapped particles



passing particles

Expelling force of diamagnetic Larmor motion

$$F_D = -\mu \nabla_\perp B = m v_\perp^2 / 2R$$

$$\mathbf{v}_{d,\nabla B} = \pm \frac{v_\perp^2}{2\omega_c} \frac{\mathbf{B} \times \nabla B}{B^2} = \pm \frac{1}{2} v_\perp r_L \frac{\mathbf{B} \times \nabla B}{B^2}$$

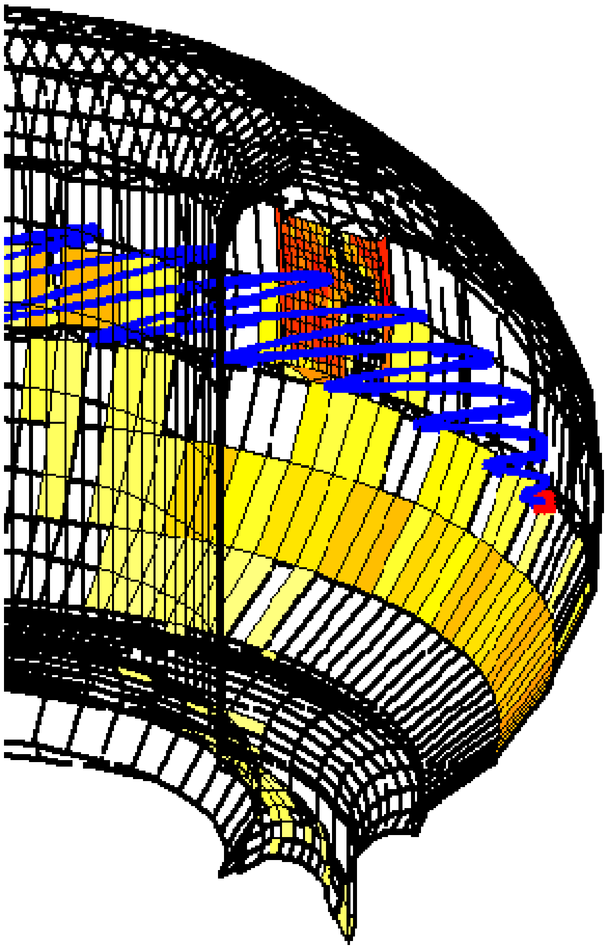
Centrifugal force

$$F_C = m v_\parallel^2 / R$$

$$\mathbf{v}_{d,R} = \frac{m v_\parallel^2}{q B_0^2} \frac{\mathbf{R}_0 \times \mathbf{B}_0}{R^2}$$

Tokamak Transport

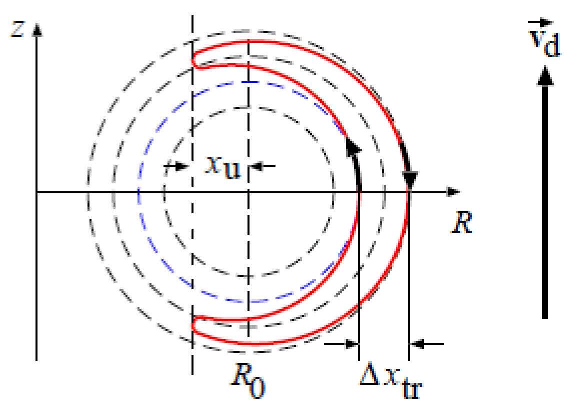
- **Particle Trapping**



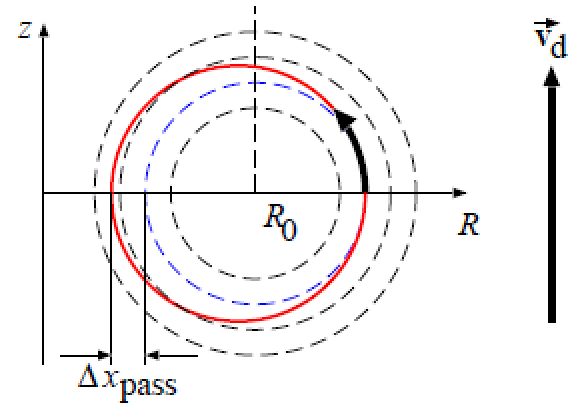
HOMEWORK:
The real particle trajectory is as shown. Why?

Tokamak Transport

- **Particle Trapping**



trapped particles



passing particles

Banana width: $\Delta x_{tr} \approx v_d t \approx q\rho/\sqrt{\varepsilon}$

$$v_d = \frac{v_\parallel^2 + v_\perp^2/2}{\omega_c R}, \quad v_\parallel \sim v_\perp \sqrt{\varepsilon}$$

t: transit time of one half of the banana

$$q = \frac{rB_T}{RB_\theta}, \quad \rho = \frac{v_T}{\omega_b}, \quad v_T = \sqrt{2T/m}$$

Displacement of transit particles:

$\Delta x_{pass} \approx q\rho/\sqrt{\varepsilon}$ for particles which have just become transit ones $v_\parallel \sim v_\perp \sqrt{\varepsilon}$

$\Delta x_{pass} \approx q\rho$ for a typical particle $v_\parallel \sim v_\perp$

Tokamak Transport

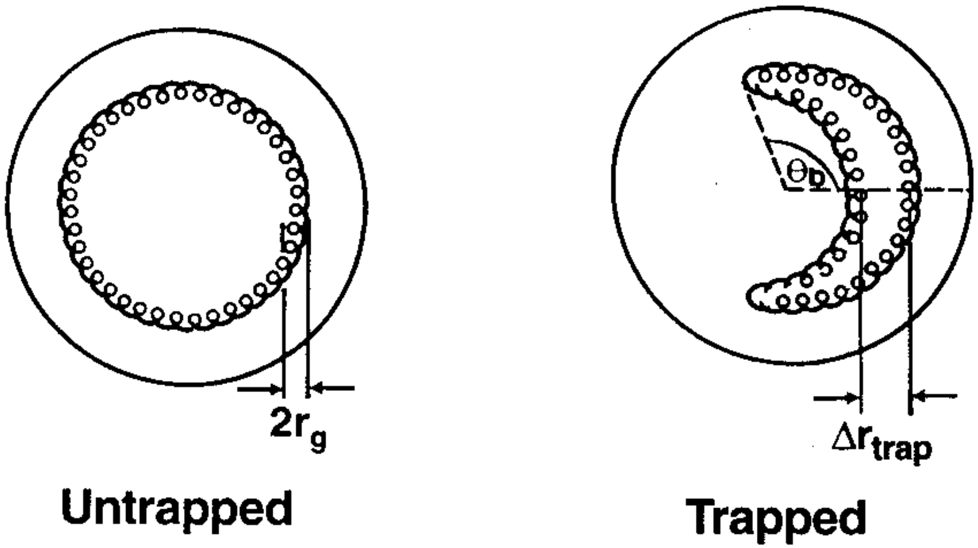
- **Particle Trapping**

- Collisional excursion across flux surfaces

  untrapped particles: $2r_g = 2r_{Li}$

  trapped particles: $\Delta r_{trap} >> 2r_g$

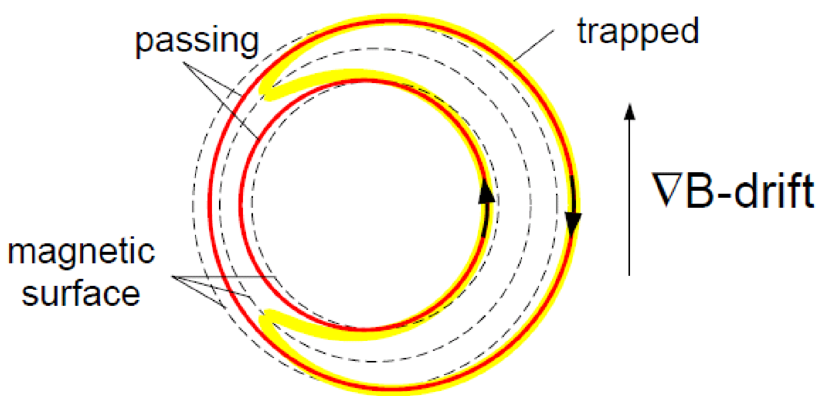
  – enhanced radial diffusion across the confining magnetic field

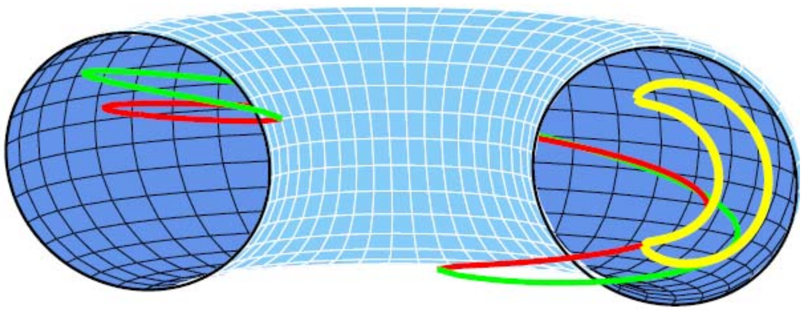


- If the fraction of trapped particle is large, this leakage enhancement constitutes a substantial problem in tokamak confinement.

Tokamak Transport

- **Particle Trapping**





- With known particle trajectories it is possible to find corresponding kinetic coefficients by solving the kinetic equations with Coulomb collisions.
- Rough estimation of transport coefficients: $\delta^2 \nu_{eff}$
  δ: particle displacement between collisions
  ν_{eff} : appropriate frequency of collisions

Tokamak Transport

- **Neoclassical Transports**

- Rarefied plasma at high temperature:
trapped particles are the main contributors to transport.
Diffusion and thermal conductivity are dominated by the collisions which correspond to transferring the particles from being trapped to transit ones and vice versa.

$$\lambda_{eff} = v_T / \nu_{eff} >> qR, \quad \Delta \approx \Delta x_{tr} \approx q\rho / \sqrt{\varepsilon}$$

- Effective collision frequency:

$$\nu_{eff} \approx \left(v_\perp / v_\parallel\right)^2 \nu \approx \nu / \varepsilon \quad \longleftarrow \quad v_\parallel \sim v_\perp \sqrt{\varepsilon}$$

- The banana diffusion region is limited by the condition:

$$\sqrt{\varepsilon}\lambda_{eff} = \sqrt{\varepsilon} v_T / \nu_{eff} = \varepsilon^{3/2} v_T / \nu >> qR$$

$$\nu^* = \nu \varepsilon^{-3/2} qR / v_T << 1$$

- Transport coefficients:

$$\sqrt{\varepsilon}\left(\Delta x_{tr}\right)^2 \nu_{eff} = \varepsilon^{-3/2} \nu q^2 \rho^2 \quad \longleftarrow$$ Trapped particle fraction = $\varepsilon^{1/2}$

Tokamak Transport

- **Neoclassical Transports**

- In the plateau region, $1 < \nu^* < \varepsilon^{-3/2}$
  ($\varepsilon^{3/2} < \nu Rq/v_T < 1$ or $\varepsilon^{3/2} v_T/Rq < \nu < v_T/Rq$)
- The average collision frequency is less than the mean bounce frequency → only slow-transit particles contribute to the transport
- The relative number of slow-transit particles: ν/v_T
- Displacement: $\Delta \approx q\rho v_T / \nu$
- Effective collision frequency: $\nu_{eff} \approx \nu v_T^2 / \nu^2$
- Transport coefficients:

$$\Delta^2 \nu_{eff} \nu / v_T \sim q^2 \rho^2 v_T / qR$$ ⟵ Slow-transit particle fraction = ν/v_T

Tokamak Transport

- **Neoclassical Transports**

D. Pfirsch and A. Schlueter, Der Einfluss der elecktrischen Leitfaehigkeit auf das Gleichgewichtsverhalten von Plasmen niedrigen Drucks in Stellaratoren, Max-Planck-Institut, Report MPI/PA/7/62 (1962)

- In the Pfirsch-Schlueter region,

$$j_\perp = -B^{-1} dp/dr$$

$$e\eta_\perp j_\perp = evB$$ Friction force = Lorentz force

- Diffusion flux in a uniform field

$$nv = -n\frac{1}{B^2}\eta_\perp \frac{dp}{dr}$$

- Modified diffusion flux by the additional flux due to longitudinal current, so-called, the Pfirsch-Schlueter current owing to the toroidal effect

$$nv = -n\frac{1}{B^2}\eta_\perp \frac{dp}{dr}\left(1+\frac{2\eta_\parallel}{\eta_\perp}q^2\right)$$ $\eta_\perp \approx 2\eta_\parallel$ in H and D plasmas

- Compared with a uniform magnetic field, the flux in toroidal plasma is enhanced by a factor $(1+q^2)$.

Tokamak Transport

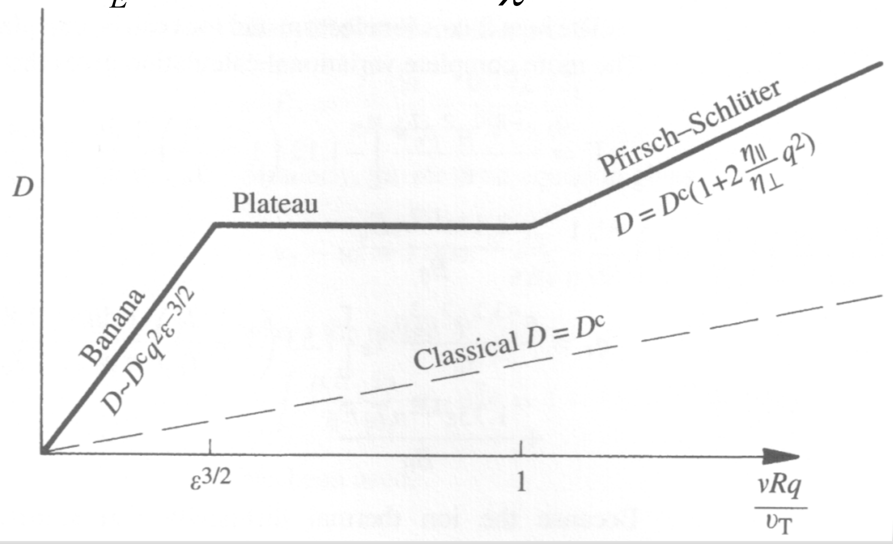
- **Neoclassical Transports**

$$\Gamma = -D\nabla n \approx -\frac{(\Delta r)^2}{\tau}\nabla n$$: Fick's law

$$q = -\kappa\nabla T \approx -\frac{(\Delta r)^2 n}{\tau_E}\nabla T$$: Fourier's law

Thermal diffusivity

$$\chi \equiv \frac{\kappa}{n} \approx \frac{(\Delta r)^2}{\tau_E} \approx D \;\rightarrow\; \tau_E \approx \frac{a^2}{\chi}$$



J. Wesson, Tokamaks (2004)

Tokamak Transport

- **Ware pinch**

VOLUME 25, NUMBER 1 PHYSICAL REVIEW LETTERS 6 JULY 1970

PINCH EFFECT FOR TRAPPED PARTICLES IN A TOKAMAK

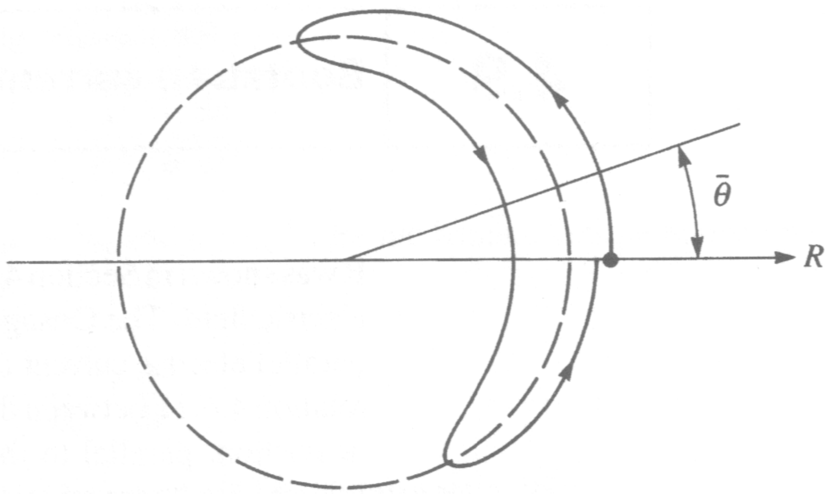
A. A. Ware
University of Texas, Austin, Texas 78712
(Received 11 May 1970)

Conservation of canonical angular momentum is shown to require that all trapped particles drift towards the magnetic axis with velocity cE_φ/B_θ (E_φ is the toroidal electric field; B_θ the poloidal magnetic field). This property, plus an amplification process for the number of trapped particles, will explain the relaxation oscillations which occur for $q<3$. In addition, there is experimental evidence that it is an important contribution to the good containment when $q>3$.

- Inward particle transport due to the toroidal electric field

Tokamak Transport

- **Ware pinch**



$$v_W \approx -\frac{cE}{B_\theta}\sqrt{\varepsilon}$$

- The drift velocity is controlled by the balance of two forces, the electrical field force and the Lorentz force.
- v_W ~ 0.2 m/s for E = 0.1 V/m, B_θ = 0.5 T
- The effect is much larger ($1/\varepsilon^2$) for trapped particles than that experienced by passing particles

Tokamak Transport

- **Bootstrap current**

- Named after the reported ability of Baron von Munchausen to lift himself by his bootstraps (Raspe, 1785)
- Suggested with ‘Alice in Wonderland’ in mind where the heroine managed to support herself in the air by her shoelaces.

http://en.wikipedia.org/wiki/Bootstrapping

Tokamak Transport

- **Bootstrap**

MEANING:
verb tr.: To help oneself with one's own initiative and no outside help.
noun: Unaided efforts.
adjective: Reliant on one's own efforts.

Baron Münchausen lifting himself up from the swamp by his own hair
Illustrator: Theodor Hosemann

ETYMOLOGY:
While pulling on bootstraps may help with putting on one's boots, it's impossible to lift oneself up like that. Nonetheless the fanciful idea is a great visual and it gave birth to the idiom "to pull oneself up by one's (own) bootstraps", meaning to better oneself with one's own efforts, with little outside help. It probably originated from the tall tales of Baron Münchausen who claimed to have lifted himself (and his horse) up from the swamp by pulling on his own hair.
In computing, booting or bootstrapping is to load a fixed sequence of instructions in a computer to initiate the operating system.
Earliest documented use: 1891.1

http://wordsmith.org/words/bootstrap.html

Tokamak Transport

- **Bootstrap current**

Diffusion Driven Plasma Currents and Bootstrap Tokamak

by

R. J.

UKAEA Res

the usual toroidal coordinates. Then in the regime of low collision frequency and in the absence of any driving electric field, steady state diffusion is accompanied by a toroidal current density of magnitude

$$j = -A\left(\frac{r}{R}\right)^{1/2} \frac{1}{B_\theta} \frac{\mathrm{d}p}{\mathrm{d}r} \qquad (1)$$

where A is a coefficient whose value depends on the exact collision operator but is of order unity, and p is the plasma pressure.

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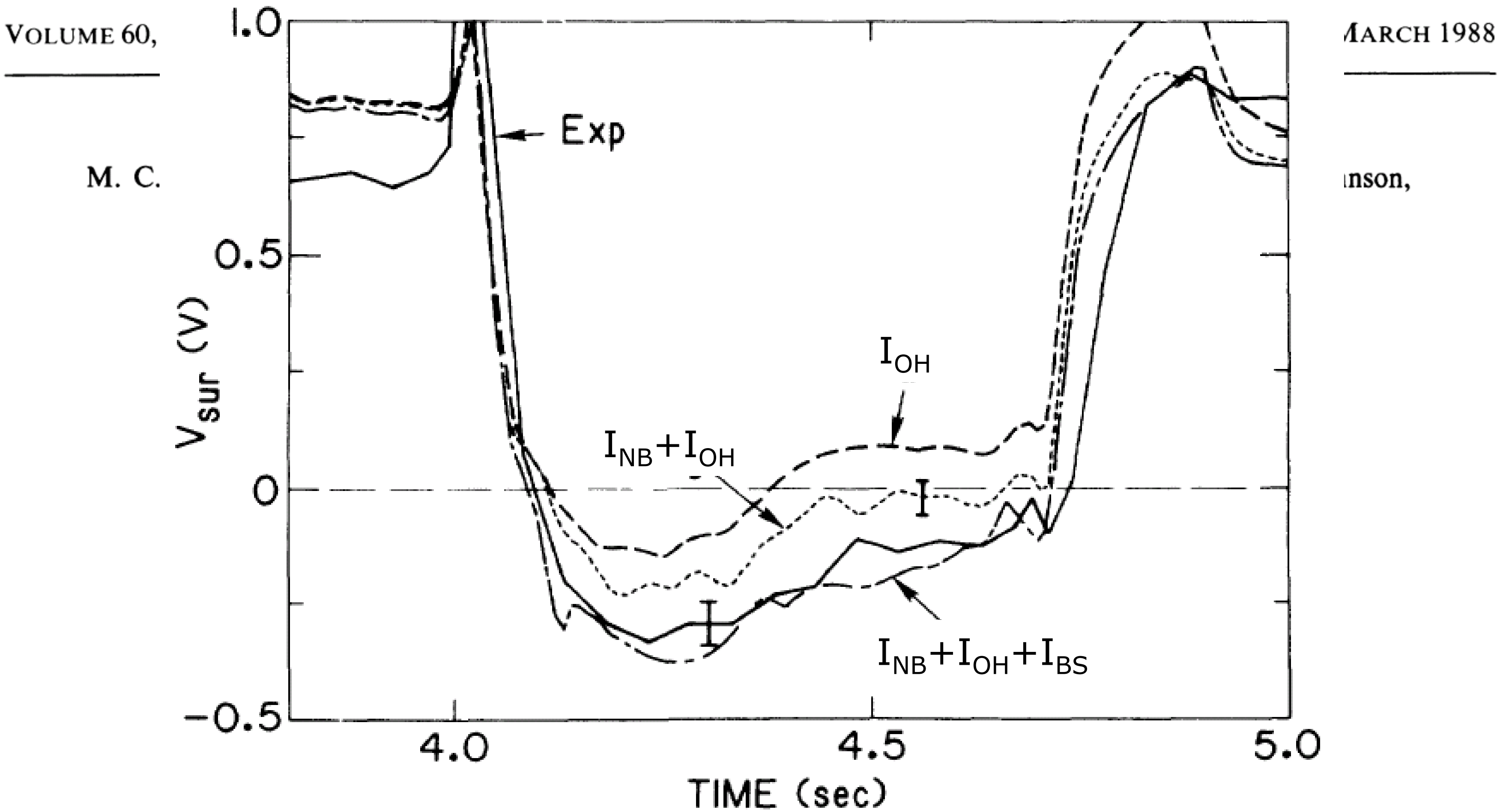
of Tokamak machine which operates in a steady state, unlike present pulsed designs.

Nature Physical Science ***229*** *110 (1971)*

Tokamak Transport

- **Bootstrap current**



Tokamak Transport

Assumption:
- Uniform temperature
- Infinitely massive ions

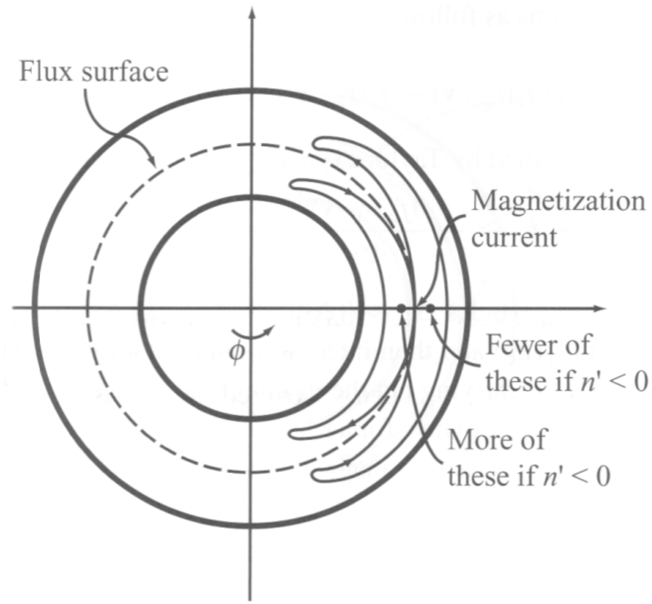
- **Bootstrap current**

- The trapped electron magnetization current

$$J^+ + J^- = -e\left[f_e\left(r_g^+, \mathbf{v}\right) - f_e\left(r_g^-, \mathbf{v}\right)\right]v_\parallel d\mathbf{v}, \quad v_\parallel > 0$$

$$\approx -e\frac{\partial f_e\left(r_0, \mathbf{v}\right)}{\partial r_0}\Delta r v_\parallel d\mathbf{v}, \quad v_\parallel > 0$$

$$\Delta r \approx \left\langle \left(\Delta r\right)^2 \right\rangle^{1/2} \approx q r_L / \varepsilon^{1/2}$$



$$J_t = -\frac{m_e q}{B_0}\varepsilon^{-1/2}\int \frac{\partial F_M}{\partial r_0} v_\perp v_\parallel d\mathbf{v}, \quad v_\parallel > 0$$

$$= -\frac{3}{2} q \varepsilon^{-1/2}\frac{T}{B_0}\frac{\partial n}{\partial r}\int_{\theta_c}^{\pi/2} \sin^2\theta \cos\theta d\theta \longleftarrow \cos\theta_c = \sqrt{2\varepsilon}$$

$$\approx -q\varepsilon^{1/2}\frac{T}{B_0}\frac{\partial n}{\partial r}$$

not depending on collisions, generated solely because of the density (or temperature) gradient of the guiding centers

Tokamak Transport

- **Bootstrap current**

- The passing electron magnetization current

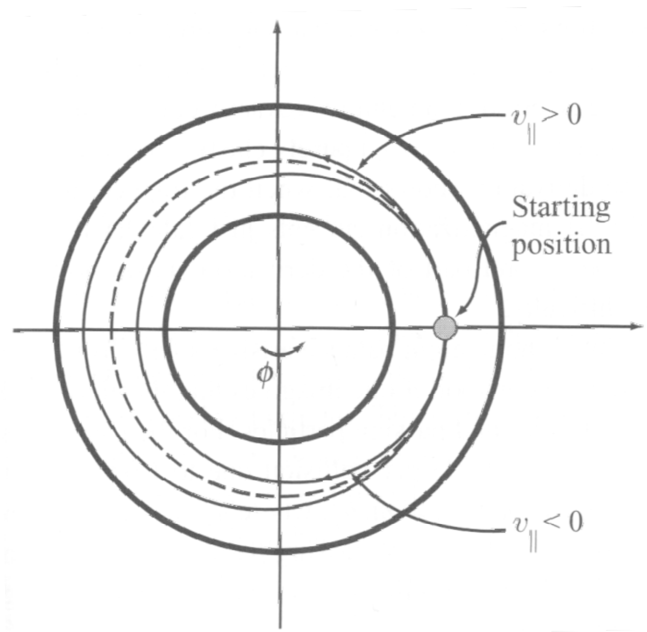
$$J_p \approx -e \frac{\partial f_e(r_0, \mathbf{v})}{\partial r_0} \Delta r v_\parallel d\mathbf{v}, \quad v_\parallel > 0$$

$$\Delta r \approx q r_L$$

$$J_t = -\frac{m_e q}{B_0} \int \frac{\partial F_M}{\partial r_0} v_\perp v_\parallel d\mathbf{v}, \quad v_\parallel > 0$$

$$= -\frac{3}{2} q \frac{T}{B_0} \frac{\partial n}{\partial r} \int_{\theta_c}^{\pi/2} \sin^2\theta \cos\theta d\theta$$

$$\approx -q \frac{T}{B_0} \frac{\partial n}{\partial r}$$



Tokamak Transport

- **Bootstrap current**

- The collision-driven bootstrap current

$$\left(\Delta P_{\parallel}\right)_t = m_e u_t n_t \bar{\nu}_{tp} = m_e\left(-\frac{J_t}{e n_t}\right) n_t \bar{\nu}_{tp} \approx -\frac{m_e}{e} J_t \frac{\bar{\nu}_{ee}}{\varepsilon} \approx qT\varepsilon^{-1/2}\frac{\partial n}{\partial r}\frac{\bar{\nu}_{ee}}{\left|\omega_{ce}\right|}$$

$$\left(\Delta P_{\parallel}\right)_p = m_e u_p n_p \bar{\nu}_{pt} = m_e\left(-\frac{J_p}{e n_p}\right) n_p \bar{\nu}_{pt} \approx -\frac{m_e}{e} J_p \bar{\nu}_{ee} \approx qT\frac{\partial n}{\partial r}\frac{\bar{\nu}_{ee}}{\left|\omega_{ce}\right|}$$

$$\left(\Delta P_{\parallel}\right)_p \neq \left(\Delta P_{\parallel}\right)_t$$ Collisional momentum balance violated $$\left(\Delta P_{\parallel}\right)_p = \varepsilon^{1/2}\left(\Delta P_{\parallel}\right)_t$$

$$f_p\left(r_g, \mathbf{v}\right) = \frac{n\left(r_g\right)}{\pi^{3/2} v_T^3}\exp\left(-\frac{v_\perp^2 + v_\parallel^2}{v_\perp^2}\right) \rightarrow \frac{n\left(r_g\right)}{\pi^{3/2} v_T^3}\exp\left[-\frac{v_\perp^2 + \left(v_\parallel - u_B\right)^2}{v_\perp^2}\right]$$ Shifted Maxwellian

$$\approx \frac{n\left(r_g\right)}{\pi^{3/2} v_T^3}\left[1 + \frac{v_\parallel}{\left|v_\parallel\right|}\left(\frac{1}{n}\frac{\partial n}{\partial r}\right)\left(\Delta r\right)_p + 2\frac{v_\parallel u_B}{v_T^2}\right]\exp\left(-\frac{v_\perp^2 + v_\parallel^2}{v_\perp^2}\right)$$

- The shift must be in the passing particles since the trapped particles are "trapped" and thus are not allowed to drift toroidally.

Tokamak Transport

the usual toroidal coordinates. Then in the regime of low collision frequency and in the absence of any driving electric field, steady state diffusion is accompanied by a toroidal current density of magnitude

$$j = -A\left(\frac{r}{R}\right)^{1/2} \frac{1}{B_\theta} \frac{\mathrm{d}p}{\mathrm{d}r} \qquad (1)$$

where A is a coefficient whose value depends on the exact collision operator but is of order unity, and p is the plasma pressure.

- **Bootstrap current**

$$\left(\Delta P_{\parallel}\right)_p \approx m_e\left(-\frac{J_p}{e} + n_p u_B\right)\bar{\nu}_{ee}$$

$$\left(\Delta P_{\parallel}\right)_p = \left(\Delta P_{\parallel}\right)_t \qquad qT\frac{\partial n}{\partial r}\frac{\bar{\nu}_{ee}}{|\omega_{ce}|} + m_e u_B n_p \bar{\nu}_{ee} = qT\varepsilon^{-1/2}\frac{\partial n}{\partial r}\frac{\bar{\nu}_{ee}}{|\omega_{ce}|}$$

$$J_B = -en_p u_B \approx -q\varepsilon^{-1/2}\frac{T}{B_0}\frac{\partial n}{\partial r}$$

$1/\varepsilon$ and $1/\varepsilon^{1/2}$ larger than the trapped and passing particle magnetization current, respectively

$$J_B = -4.71q\varepsilon^{-1/2}\frac{T}{B_0}\left[\frac{\partial n}{\partial r} + 0.04\frac{n}{T}\frac{\partial T}{\partial r}\right]$$

Large aspect ratio
Circular CX
Non-massive ions
Non-uniform temperature

- A transport driven toroidal plasma current carried by the passing electrons generated by collisional friction with the trapped electron magnetization current

Tokamak Transport

- **Bootstrap current**

- Bootstrap current fraction

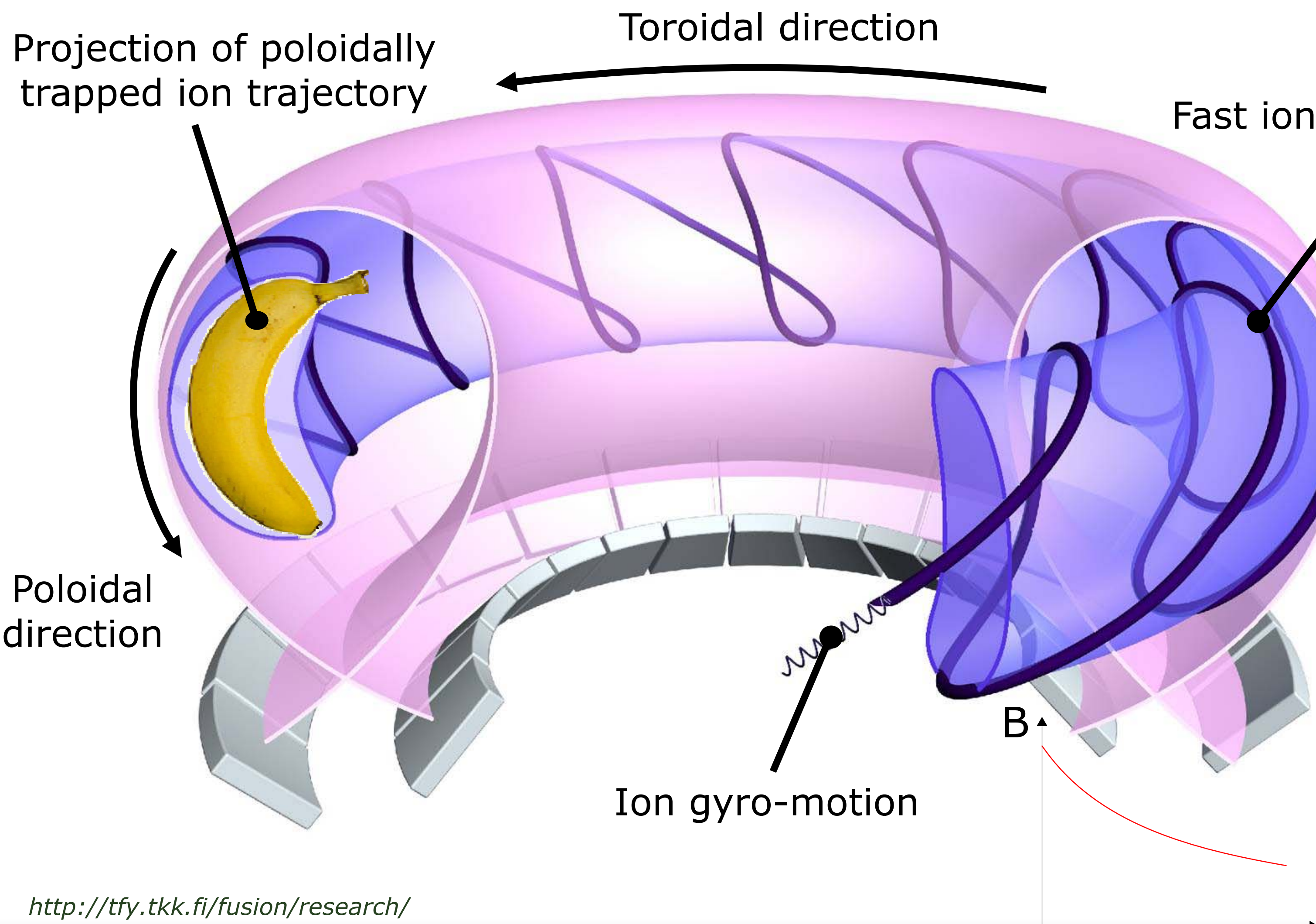
$$f_B(r) \equiv \frac{J_B}{J_\phi} \approx -1.18 G \varepsilon^{1/2} \beta_P \sim \varepsilon^{1/2} \beta_P$$

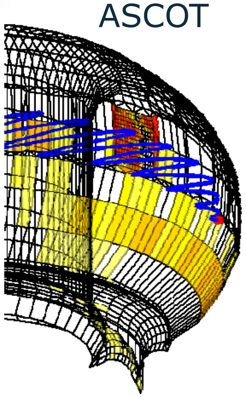
$$G(r) = (\ln n + 0.04 \ln T)' / (\ln r B_\theta)'$$

- In high-β tokamak, $\beta_p \sim 1/\varepsilon$, implying that $f_B \sim 1/\varepsilon^{1/2} >> 1$: The bootstrap current can theoretically overdrive the total current
- No obvious "anomalous" degradation of J_B due to micro-turbulence
- The bootstrap current is capable of being maintained in steady state without the need of an Ohmic transformer or external current drive. This is indeed a favourable result as it opens up the possibility of steady state operation without the need for excessive amounts of external current drive power.
- This is critical since bootstrap current fractions on the order of $f_B > 0.7$ are probably required for economic viability of fusion reactors.

Tokamak Transport

- **Neoclassical Bootstrap current**





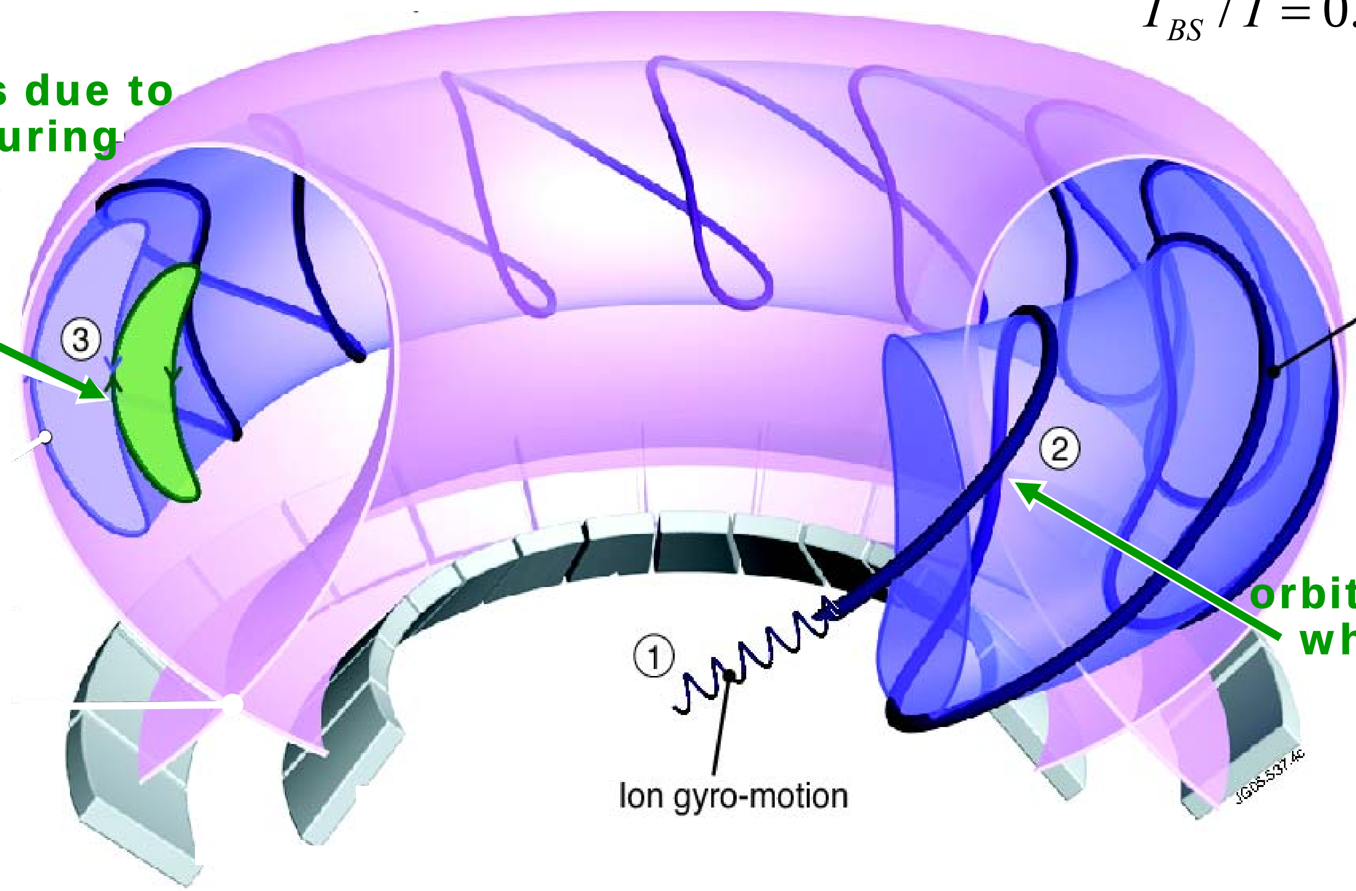
Tokamak Transport

$$J_{BS} \approx -\frac{\varepsilon^{1/2}}{B_\theta}\frac{dp}{dr}$$

$$I_{BS}/I = 0.67\beta_p\sqrt{\varepsilon}$$

- **Neoclassical Bootstrap current**

Currents due to neighbouring bananas largely cancel



orbits tighter where field stronger

- More & faster particles on orbits nearer the core (green .vs. blue) lead to a net “banana current”.
- This is transferred to a helical bootstrap current via collisions.

References

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- Jeffrey P. Freidberg, "Plasma Physics and Fusion Energy", Cambridge University Press (2007)
- Tim Hender, "Neoclassical Tearing Modes in Tokamaks", 2009 Korean Physical Society/ Division of Plasma Physics (KPS/DPP) in Daejun, Korea, 24 April 2009