2.4 Pore Pressure Developed during Undrained Loading

(1) Undrained Loading

- Loading effects on soils can be divided into two phases;
  i) Undrained loading : Pore pressure is developed but there is no flow of pore fluid.
  ii) Dissipation : Total load remains constant and fluid flow occurs to dissipate the developed pore pressure.

  ⇒ Can be distinctly created in the lab by i) loading with closed drainage line + ii) opening drainage line in triaxial test.

  ⇒ Can be realized in the field whenever loading interval is very short compared to the dissipation time of pore pressure. ⇒ Frequently this condition occurs with clays.

- Field situation of loading + dissipation (Fig. 2-36)

  - Prior to the loading { Hydrostatic pore pressure. No variation of total head (⇒ no flow of water).

  - Right after the loading.
    - ⇒ Increase of pore pressure and total head within clay layer.
    - ⇒ No response of pore pressure within coarse material layers.
    - ⇒ Gradient at top and bottom boundaries of clay initiates <u>the flow</u>.
      - → Transient (or unsteady) flow because total head is changing with time.

  - The strength and compressibility at any given points of subsoils at any given time.
    - ⇒ Must know the effective stress (or pore pressure) at the point at that time.
      - i) The initial pore pressure (undrained loading).
      - ii) The final equilibrium pressure (drained loading).
      - iii) The pore pressure during the intervening transient condition (consolidation).

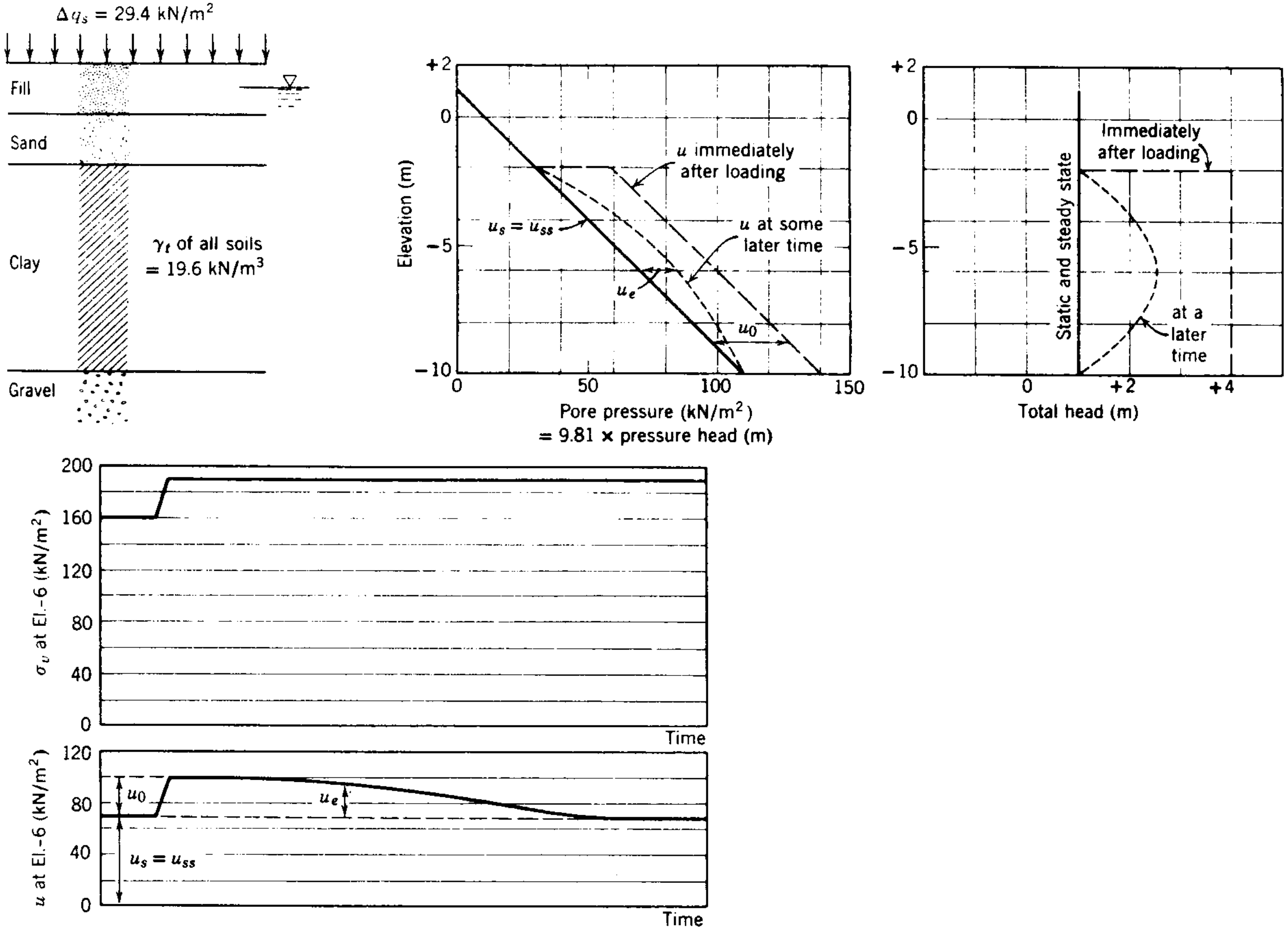


Fig. 2-36 Undrained surface loading

- Pore pressure parameters
  - Spring analogy of soil-water system in the oedometer;

    Spring ⇒ soil skeleton.

    Water ⇒ pore fluid.

  - For undrained loading (W),

    Intuitively, nearly all of W ⇒ the water
    very little amount of W ⇒ the spring } In spring analogy

    Similarly, most of $\Delta\sigma_1$ ⇒ carried by Δu . → In real soil-water system.

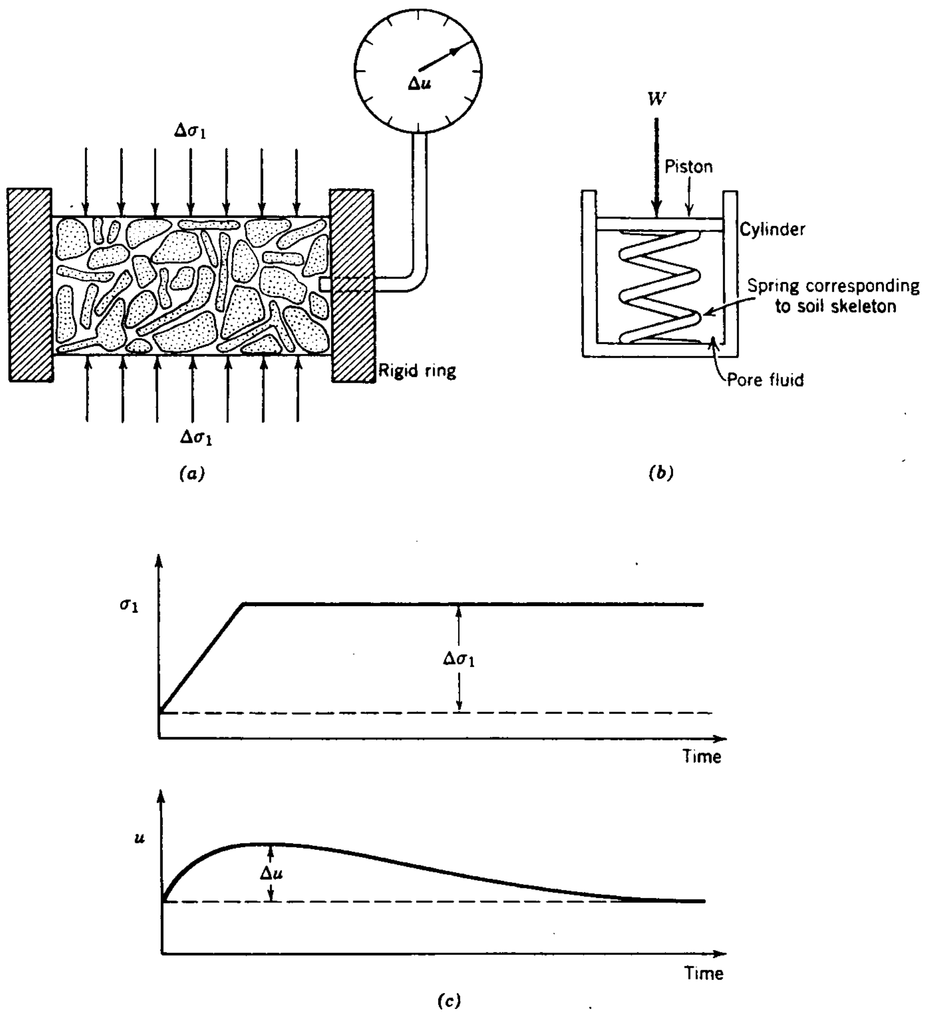


Fig. 2-37 Loading in oedometer (a) Soil-water system (b) spring analogy

- Pore pressure parameter; $\Delta u / \Delta\sigma$

  A ratio of pore pressure increment to the total stress increment.

- For oedometer test, pore pressure parameter C is equal to $\Delta u / \Delta\sigma_1$.

  (Fig. 2-38 shows that slope of $\Delta\sigma_1 - \Delta u$ plot is C (=1).)

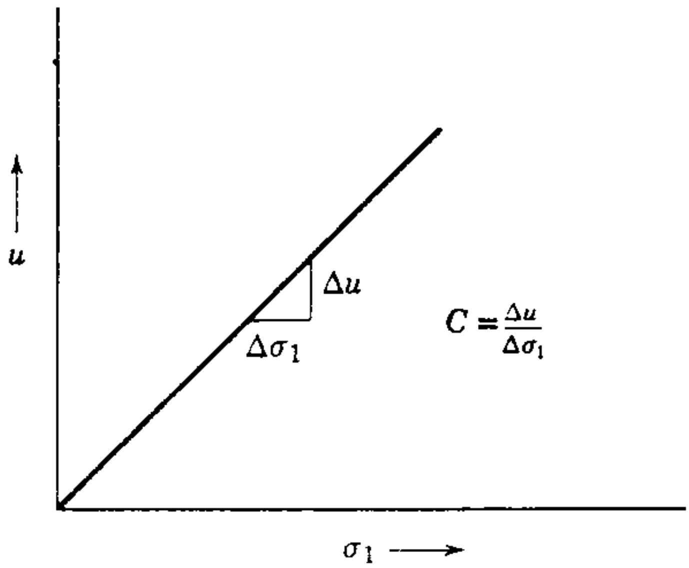


Fig. 2-38 Results of loading in oedometer.

- Pore pressure parameters based on types of stress system

  i) Oedometer loading : C = $\Delta u / \Delta\sigma_1$

  ii) Isotropic loading : B = $\Delta u / \Delta\sigma$

  iii) Uniaxial loading : D = $\Delta u / \Delta\sigma_1$

  iv) Triaxial loading : Increment of pore pressure can be determined by combining ii) and iii).

- Other situations causing transient flow.
  - Given conditions ;
    i) Clay layer between two permeable layers.
    ii) Lowering the water table in the strata above the clay while the piezometric level in the underlying gravel remains constant.
    ⇒ No change in total load but transient flow occurs.

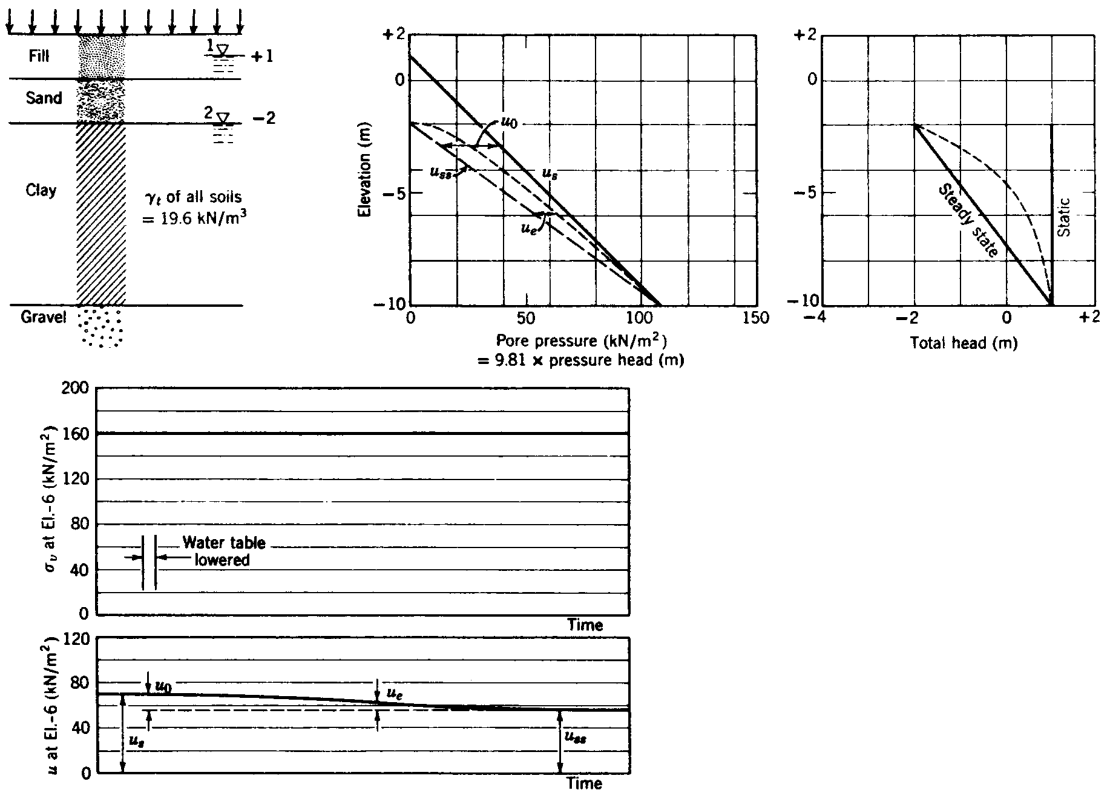


Fig. 2-39 Water lowering from +1 to -2

- Initial excess pore pressure ;

$$u_0 = u_s - u_{ss}$$

- The excess pore pressure at any time ;

$$u_e = u - u_{ss}$$

⇒ u_e's are referenced to the final steady-state pore pressure.

- Final equilibrium condition; total head variation across the clay and upward flow of water from the gravel to the sand.

(2) Pore Pressure Developed in the Oedometer Test

- Apportioning total stress increment to a soil specimen.
  ⇒ Combining individual compression response of soil skeleton and pore fluid.

- One-dimensional compressibility of the soil skeleton with free escape of pore fluid, (assuming drained compressibility is equal to undrained compressibility based on effective stress.)

$$C_{c1} = +\frac{\Delta V}{V_0} \cdot \frac{1}{\Delta \overline{\sigma}_1} = -\frac{\Delta e}{1+e_0} \cdot \frac{1}{\Delta \overline{\sigma}_1} \quad \text{----------------------------} \quad (1)$$

- Compressibility of the pore fluid.

$$C_w = +\frac{\Delta V}{V_0} \cdot \frac{1}{\Delta u} \quad \text{---} \quad (2)$$

- Derivation of pore pressure parameter C.

$$\Delta V_{sk} = \Delta V_p \quad \text{(assuming solid is incompressible.)} \quad \text{---------} \quad (3)$$

Based on coefficient of compressibility,

$$\Delta V_{sk} = +V_0 C_{c1} \Delta \overline{\sigma}_1$$

$$\Delta V_p = +n V_0 C_w \Delta u$$

From (3), $\cancel{V_0} C_{c1} \Delta \overline{\sigma}_1 = n \cancel{V_0} C_w \Delta u$

$$C_{c1}(\Delta \sigma_1 - \Delta u) = n C_w \Delta u$$

Therefore, $C = \frac{\Delta u}{\Delta \sigma_1} = \frac{1}{1 + n\left(C_w / C_{c1}\right)}$ ------------------------------------- (4)

- Table 2-6 lists values of C computed from measured values of C_w and C_{c1}.
- For saturated soils, C is essentially unity (C_{c1} >> C_w).

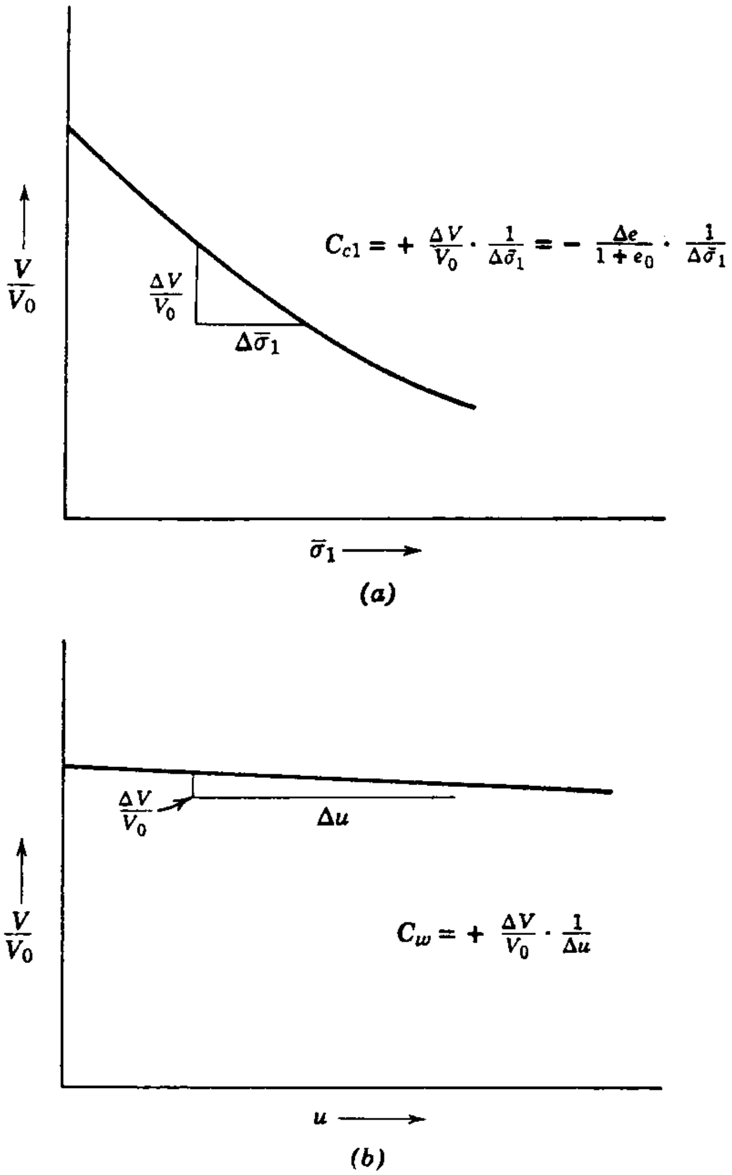


Fig. 2-40 One-dimensional deformation (a) Soil skeleton (b) Water

Table 2-6 Values of Parameter C

VALUES OF PARAMETER C		
Material (S = 100%)	C	Reference
Vicksburg buckshot clay slurry	0.99983	M.I.T. Test
Lagunillas soft clay	0.99957	M.I.T. Test
Lagunillas sandy silt	0.99718	M.I.T. Test

(3) Pore Pressure Developed by an Increment of Isotropic Stress

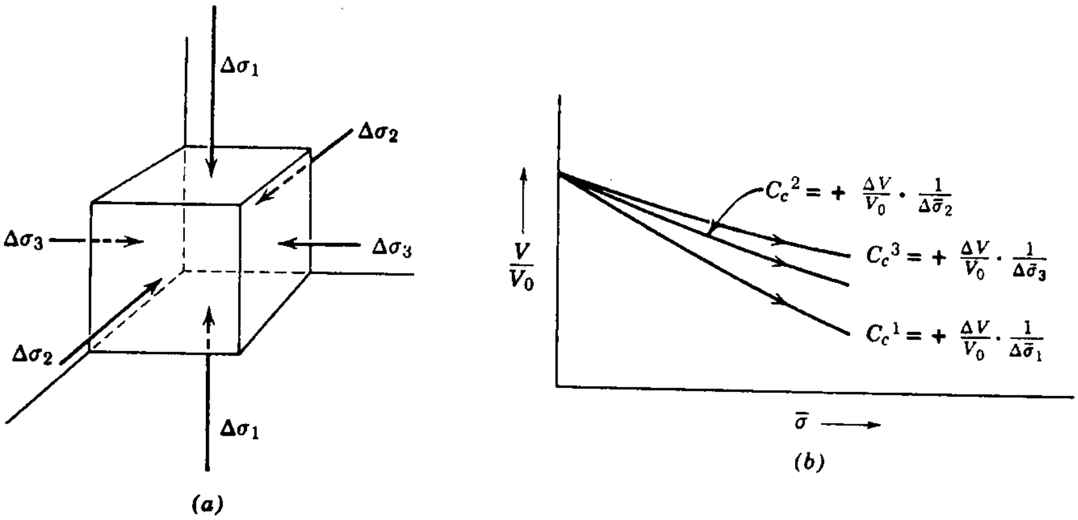


Fig. 2-41 Three-dimensional loading (a) Loading (b) Deformation

For the three-dimensional loading, the total change in volume of the soil skeleton is

$$\Delta V_{sk} = V_0 C_c^1 \Delta\bar{\sigma}_1 + V_0 C_c^2 \Delta\bar{\sigma}_2 + V_0 C_c^3 \Delta\bar{\sigma}_3$$

and

$$\Delta V_p = nV_0 C_w \Delta u$$

For the isotropic stress application,

$$\Delta\sigma_1 = \Delta\sigma_2 = \Delta\sigma_3 = \Delta\sigma$$

and

$$\Delta\bar{\sigma}_1 = \Delta\bar{\sigma}_2 = \Delta\bar{\sigma}_3 = \Delta\bar{\sigma} = \Delta\sigma - \Delta u$$

So, with $\Delta V_{sk} = \Delta V_p$.

$$nV_0 C_w \Delta u = V_0 (\Delta\sigma - \Delta u)(C_c^1 + C_c^2 + C_c^3)$$

$$\frac{\Delta u}{\Delta\sigma} = \frac{C_c^1 + C_c^2 + C_c^3}{nC_w + C_c^1 + C_c^2 + C_c^3}$$

If the soil element is isotropic,

$$B = \frac{\Delta u}{\Delta \sigma} = \frac{1}{1 + n(C_w / C_{c3})}$$

where $C_{c3}(= 3C_c^3) = +\frac{\Delta V}{V_0} \cdot \frac{3}{\Delta \bar{\sigma}}$

- In most soils, $C_{c1} \approx C_{c3} \Rightarrow C \approx B$.

Table 2-7 Values of Parameter B

VALUES OF PARAMETER *B*

Material	*S*(%)	*B*	Reference
Sandstone	100	0.286	
Granite	100	0.342	
Marble	100	0.550	Computed from
Concrete	100	0.582	compressibi-
Dense sand	100	0.9921	lities given by
Loose sand	100	0.9984	Skempton (1961)
London clay (OC)	100	0.9981	
Gosport clay (NC)	100	0.9998	
Vicksburg buckshot clay	100	0.9990	M.I.T.
Kawasaki clay	100	0.9988 to 0.9996	M.I.T.
Boulder clay	93	0.69	Measured by
	87	0.33	Skempton
	76	0.10	(1954)

(4) Pore Pressure Developed by an Increment of Uniaxial Stress

- Uniaxial loading with free lateral strain.

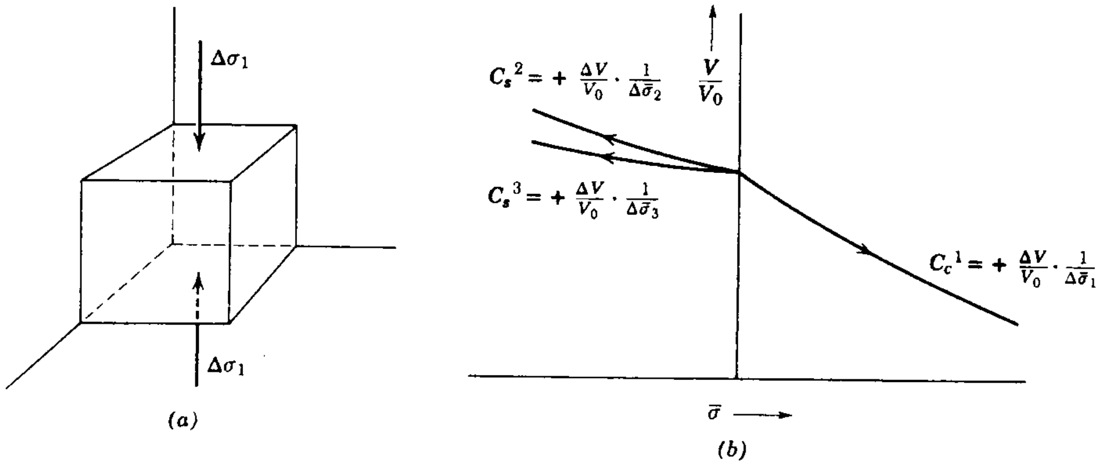


Fig. 2-42 Uniaxial loading (a) Loading (b) Deformations

(Compression curve for C_c^1 by increasing $\overline{\sigma_1}$ with constant $\overline{\sigma_2}$ and $\overline{\sigma_3}$.)

(Expansion curves for C_s^2 or C_s^3 by decreasing $\overline{\sigma_2}$ and $\overline{\sigma_3}$ with constant $\overline{\sigma_1}$.)

- An increment of $\Delta\sigma_1$.

$\rightarrow$ $\Delta\overline{\sigma_1} = \Delta\sigma_1 - \Delta u$.

$\rightarrow$ $\Delta\overline{\sigma_2} = \Delta\overline{\sigma_3} = -\Delta u$.

Since $\Delta V_p = \Delta V_{sk}$,

$$nV_0C_w\Delta u = V_0C_c^1(\Delta\sigma_1 - \Delta u) + V_0C_s^2(-\Delta u) + V_0C_s^3(-\Delta u)$$

$$\frac{\Delta u}{\Delta\sigma_1} = \frac{1}{1 + n(C_w/C_c^1) + C_s^2/C_c^1 + C_s^3/C_c^1}$$

If $C_s^2 = C_s^3$,

$$D = \frac{\Delta u}{\Delta \sigma_1} = \frac{1}{1 + n(C_w / C_c^1) + 2C_s^3 / C_c^1}$$

If the soil element is elastic ($C_c = C_s$) and isotropic,

$$D = \frac{\Delta u}{\Delta \sigma_1} = \frac{1}{1 + n(C_w / C_c^1) + 2}$$

For a saturated soil ($C_w / C^1{}_c = 0$),

$$D = \frac{1}{3}$$

Thus, $\Delta\sigma_1 = 3$ $\Delta u = 1$ $\Delta\overline{\sigma_1} = 2$

$\Delta\sigma_2 = 0$ $\Delta\overline{\sigma_2} = -1$

$\Delta\sigma_3 = 0$ $\Delta\overline{\sigma_3} = -1$

$\Rightarrow$ No change in effective isotropic stress.

$\rightarrow$ Gives correct result if no volume change is occurred.

(5) Pore Pressure Developed by Triaxial Stress

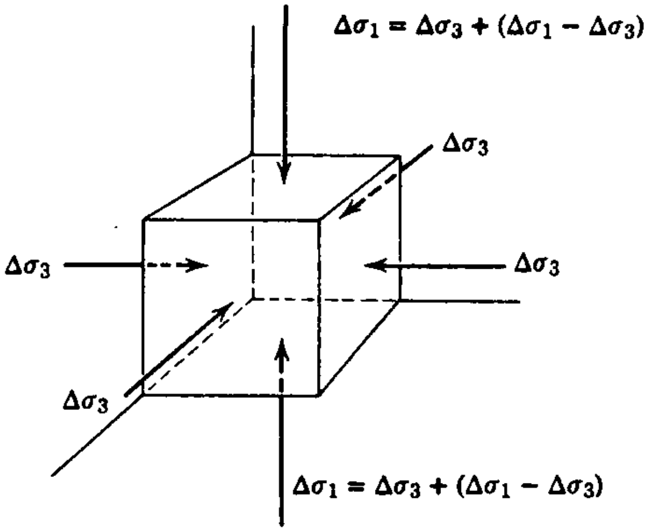


Fig. 2-43 Isotropic loading followed by uniaxial loading

- Triaxial loading can be considered to be made up of $\Delta\sigma_3 + (\Delta\sigma_1 - \Delta\sigma_3)$.

  $\rightarrow$ Initially, $\Delta\sigma_3$ and then failed under an increase $(\Delta\sigma_1 - \Delta\sigma_3)$.

$$\rightarrow \quad \Delta u = \frac{\Delta\sigma_3}{1+n(C_w/C_{c3})} + \frac{\Delta\sigma_1 - \Delta\sigma_3}{1+n(C_w/C_c^1)+2(C_s^3/C_c^1)}$$
$$= B\Delta\sigma_3 + D(\Delta\sigma_1 - \Delta\sigma_3)$$

For an saturated soils,

$$\Delta u = \Delta\sigma_3 + \frac{\Delta\sigma_1 - \Delta\sigma_3}{1+2C_s^3/C_c^1}$$
$$= \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)$$

where $A = \dfrac{1}{1+2(C_s^3/C_c^1)} = \dfrac{1}{1+C_{s2}/C_c^1}$

$$(C_{s2} = C_s^2 + C_s^3 = 2C_s^3)$$

For an isotropic and elastic soil mass saturated with an incompressible pore fluid,

$$\Delta u = \Delta\sigma_3 + \frac{1}{3}(\Delta\sigma_1 - \Delta\sigma_3)$$

(6) The Pore Pressure Parameter A

$$A = \frac{\Delta u - \Delta\sigma_3}{\Delta\sigma_1 - \Delta\sigma_3}$$

For the usual undrained triaxial test ($\Delta\sigma_3 = 0$),

$$A = \frac{\Delta u}{\Delta\sigma_1}$$

- Pore pressure parameter associated with effective stress path on CIU TXC test.

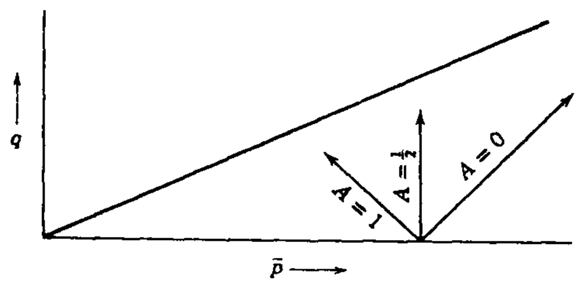


Fig. 2-44 Special values of A

- A > 1 ⇒ Soil with a loose structure, which collapses upon load application.
- A < 0 ⇒ A heavily OC clay or a very dense sand.

- The pore pressure parameter A is not a constant soil property.
  ⇒ Factors influencing A.
    i) Strain.
    ii) Initial stress system.
    iii) Stress history.
    iv) Total stress path.

Table 2-8 Values of Parameter A

Values of Parameter A

Material (S = 100%)	A (at failure)	Reference
Very loose fine sand	2 to 3	Typical values given by Bjerrum
Sensitive clay	1.5 to 2.5	
Normally consolidated clay	0.7 to 1.3	
Lightly overconsolidated clay	0.3 to 0.7	
Heavily overconsolidated clay	−0.5 to 0	

Material (S = 100%)	A (for foundation settlement)	Reference
Very sensitive soft clays	>1	From Skempton and Bjerrum (1957)
Normally consolidated clays	$\frac{1}{2}$ to 1	
Overconsolidated clays	$\frac{1}{4}$ to $\frac{1}{2}$	
Heavily overconsolidated sandy clays	0 to $\frac{1}{4}$	

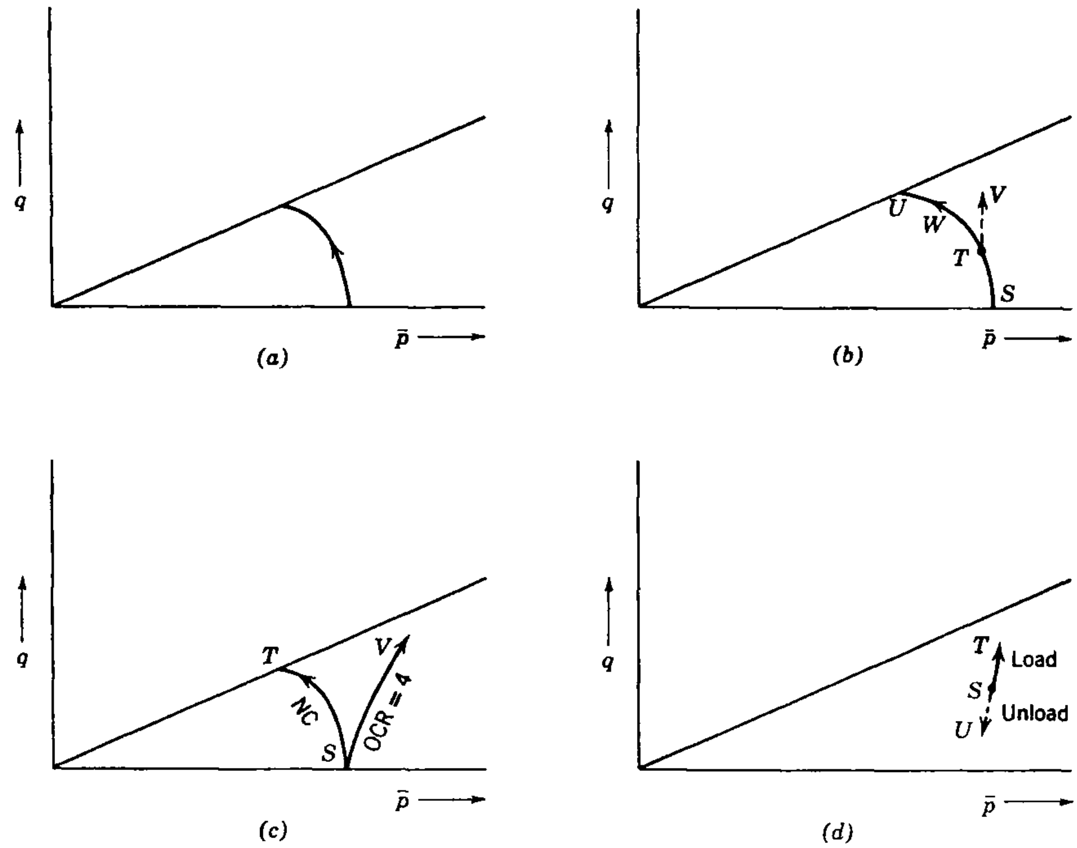


Fig. 2-45 Factors influencing A (a) Strain (b) Initial stress system (c) stress history (d) Type of stress change

(7) The Estimation of Pore Pressure in the Field

- Pore pressure parameters

  $\Rightarrow$ Estimating the initial pore pressure accompanied by total stress change.

i) Loading Example

- Comparison of the developed pore pressures of a foundation soil caused by a heavy preload.
  - $\rightarrow$ Measured value by piezometer.
  - $\rightarrow$ Estimated value based on pore pressure parameter determined by lab test.

- Point under consideration;

  At elev. of -9.45 m under center of load (P21)

  $\rightarrow$ Initial stresses,

$$\overline{\sigma_v} = 67.4\,kPa\,, \quad u_s = 64.3\,kPa$$

  $\rightarrow$ Stress increments by a preload (elastic theory),

$$\Delta\sigma_3 = 85.2\,kPa\,, \quad \Delta\sigma_1 = 195.8\,kPa$$

- Estimation of the developed excess pore pressure

  $\rightarrow$ Performing CIU TXC test,

$$\overline{\sigma_{1c}} = \overline{\sigma_{3c}} = 74\,kPa \quad > \overline{\sigma_v}(= 67.4\,kPa)$$

  (isotropic consolidation)

  $\rightarrow$ A = 0.85 at strain greater than 3%.

$$(\Delta(\sigma_1 - \sigma_3)_{peak} \approx 110\,kPa \approx (\Delta\sigma_1 - \Delta\sigma_3)_{at\ field})$$

  $\Rightarrow$ It means that clay layer almost reaches to the failure state. (But partial drainage during loading would prevent failure.)

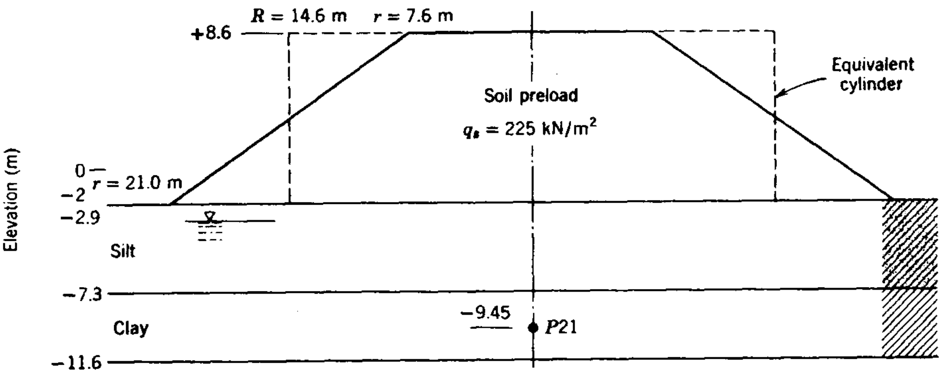
$$\rightarrow \quad \Delta u = \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)$$

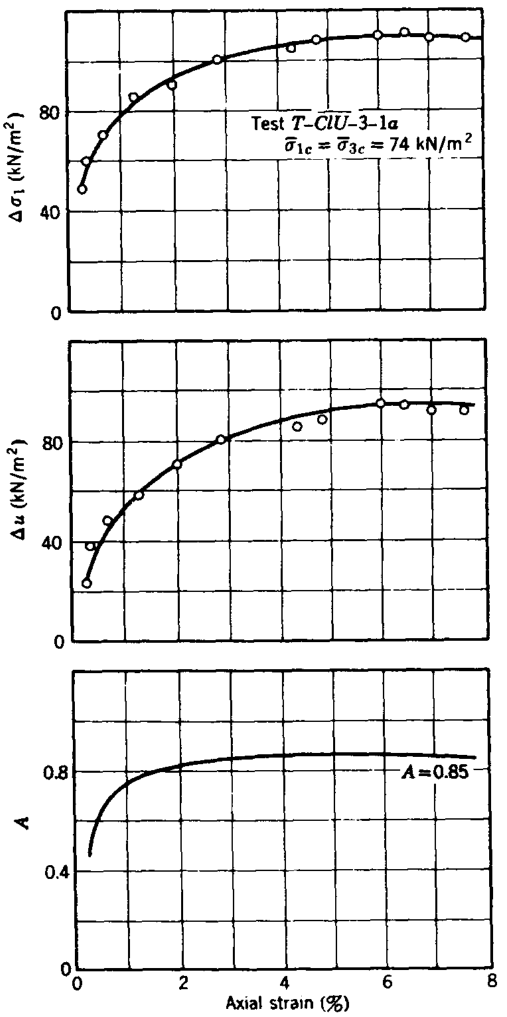
$$= 179.2\ kN/m^2$$

$$= 18.3\ m \text{ water .}$$

- Measured piezometer reading

$$\Delta h_p = 17.7\ m$$





STRESS AT $P21$

Initial:

$$\bar{\sigma}_v = 67.4 \text{ kN/m}^2$$

$$u_s = 64.3 \text{ kN/m}^2$$

Increments from preload:

$$\Delta\sigma_3 = 85.2 \text{ kN/m}^2$$

$$\Delta\sigma_1 = 195.8 \text{ kN/m}^2$$

Pore pressure parameter A from lab test,

$$A = 0.85$$

Excess pore pressure
Calculated:

$$\Delta u = \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)$$

$$\Delta u = 85.2 + 0.85(110.6) = 179.2 \text{ kN/m}^2$$

$$\Delta u = 18.3 \text{ m water}$$

Measured on Field piezometer $P21$

$$\Delta u = 17.7 \text{ m}$$

Fig. 2-45 Determination of Δu_i

ii) Unloading Example

- Unloading by an excavation (43.3 m by 71.6 m in plan) for a building

  Stage 1 : from elev. +6.86 m to elev. +4.88 m (1.98 m excavation).

  Stage 2 : from elev. +4.88 m to elev. +2.29 m (2.59 m excavation).

- Points under consideration ;

  At elev. -14.51 m (P3) and elev. -18.81 m (P4) at the approximate center at the excavation.

  $\rightarrow$ Stress paths for the unloading at P4 are shown in Fig. 2-46
  - AB , BC $\rightarrow$ total stress path.
  - Point B and C were found by computing the p-q values with decrements in σ_1 and σ_3 at P4 for the 1st and 2nd stages of excavation.

  $\rightarrow$ Performing stress path tests with undrained condition, following exact stress states at initial and following excavation conditions.
  - It gives the location of $\overline{B}$ and $\overline{C}$.
    ($\overline{AB}$, $\overline{BC}$ $\rightarrow$ effective stress path)

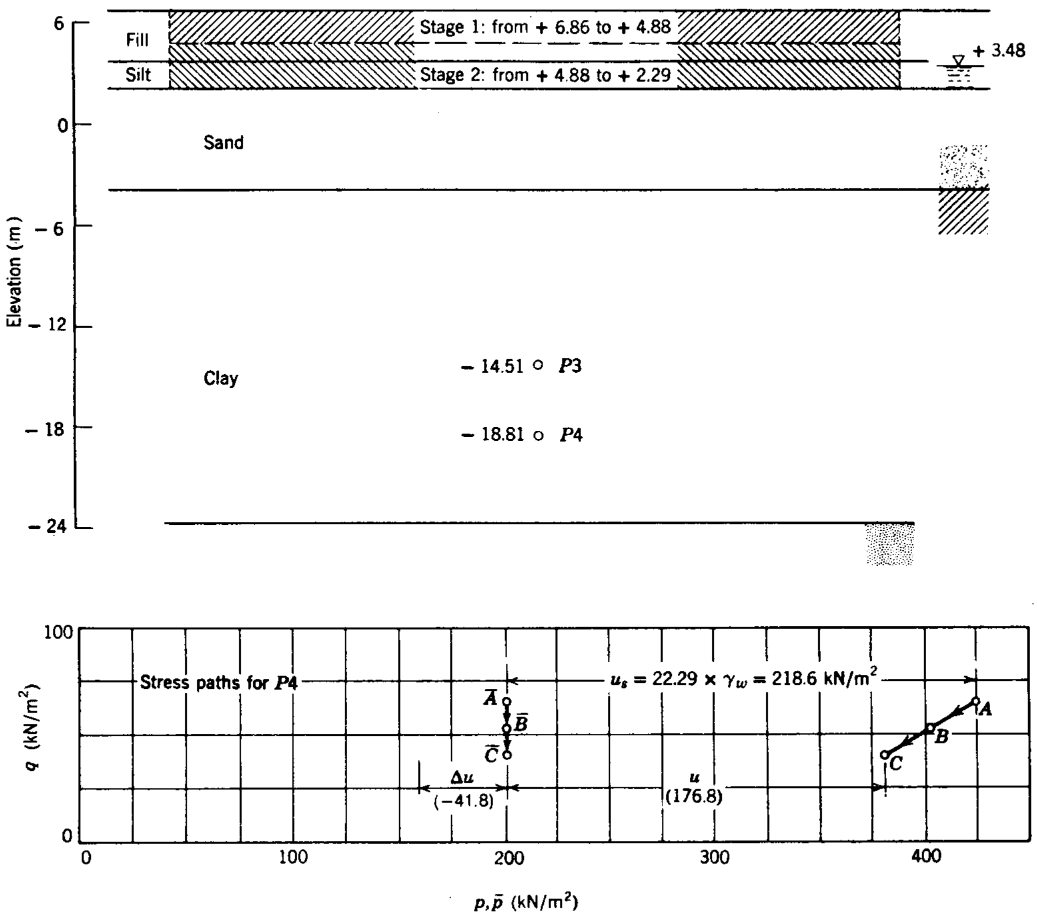
  $\rightarrow$ $u_e = -41.8\,kPa$ (pore pressure reduced from 218.6 kPa to 176.8 kPa)

  $\rightarrow$ From effective stress path, we can find out that A is approximately equal to 0.5.
  $\rightarrow$ Based on effective and total stress paths.

→ Determination of Δu_i for unloading.

Piezometer Number	Stage	Δu_i (kPa)	
		Calculated	Measured
P3	1	-21.5	-21.1
P4	1	-18.7	-22.0
P3	2	-47.9	-61.8
P4	2	-41.8	-54.1

measured directly from lab test.



Piezometer Number	Stage	Δu_i (kN/m²)	
		Calculated	Measured
*P*3	1	21.5	21.1
*P*4	1	18.7	22.0
*P*3	2	47.9	61.8
*P*4	2	41.8	54.1

Fig. 2-46 Determination of Δu_i for unloading

(8) Pore Pressure in Soil not Saturated with Water

- Pore pressure parameters of soils filled with air or partly with air and partly with water.
  → Highly compressible in pore phase relative to the compressibility of soil skeleton.
  → Gives very small pore pressure parameter.

$$B = \frac{1}{1 + n\dfrac{C_{pore\ phase}}{C_{skeleton}}}$$

For a coarse, well-graded sand,

S=0% → B=0.00380

S=50% → B=0.00967 → The air has a very significant effect on the pore pressure parameters.