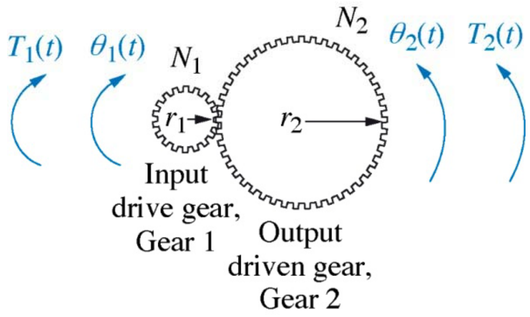
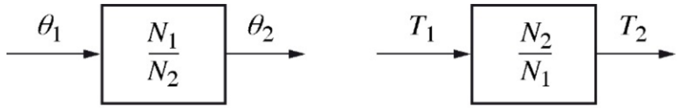
Mechanical Systems II

Transfer function for systems with Gears

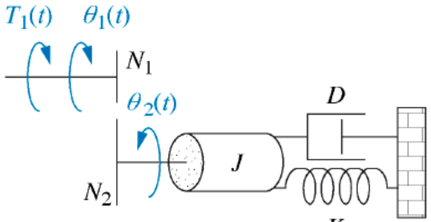


$$\frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2}$$

Torque relationship? $$\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{N_1}{N_2}$$

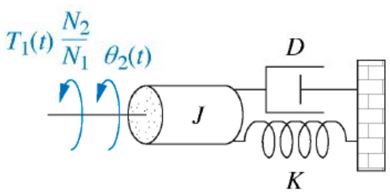


(*a*) (*b*)



(*a*)

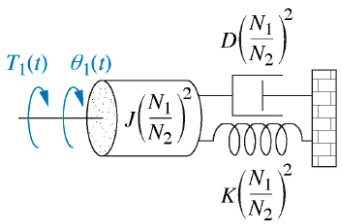
Represent the
system with gears
without gears.



(*b*)

$$(Js^2 + Ds + K)\theta_2(s) = T_1(s)\frac{N_2}{N_1}$$

Transfer function for systems with Gears



(c)

Impedances are reflected from the output to the input, therby eliminating the gears.

$$(Js^2 + Ds + K)\frac{N_1}{N_2}\theta_1(s) = T_1(s)\frac{N_2}{N_1}$$

$$(J\left(\frac{N_1}{N_2}\right)^2 s^2 + D\left(\frac{N_1}{N_2}\right)^2 s + K\left(\frac{N_1}{N_2}\right)^2)\theta_1(s) = T_1(s)$$

Rotational mechanical impedances can be reflected through gear trains by multiplying the mechanical impedance by the ratio,

$$\left(\frac{\theta_2}{\theta_1}\right)^2 = \left(\frac{N_1}{N_2}\right)^2 = \left(\frac{\text{Number of teeth of gear on } \textbf{destination} \text{ shaft}}{\text{Number of teeth of gear on } \textbf{source} \text{ shaft}}\right)^2$$

Work , Energy, and Power

- Mechanical work : $W = F \cdot x$ [N·m] = [Joule]

  $= \text{Force} \times \text{displacement}$

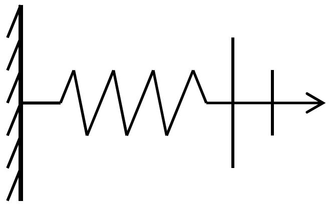
- Energy : capacity or ability to do work. Electrical, Chemical, Mechanical, etc.

- Mechanical energy : Potential energy – position

  Kinetic energy – velocity

Potential Energy

ex1)

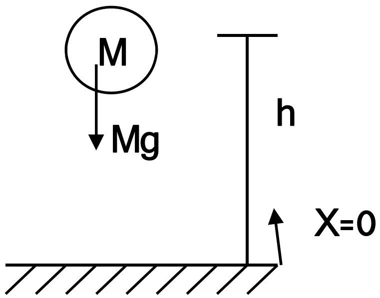


$$F = kx$$

$$dw = F \cdot dx = kx \cdot dx$$

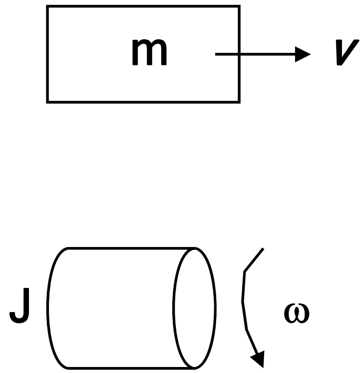
$$\int_0^{x1} dw = \int_0^{x1} kx\, dx = \frac{1}{2} kx^2$$

ex2)



$E_1 = mgx$ (Potential Energy)

Kinetic Energy



v : $\frac{1}{2}mv^2$

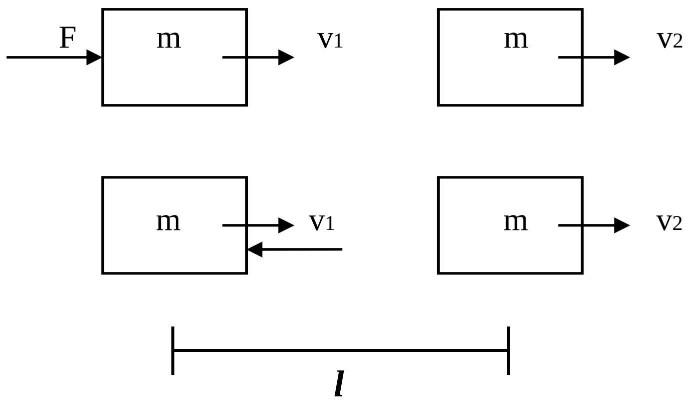
ω : $\frac{1}{2}J\omega^2$

Work and Energy

System + External Work = E_2

Energy E_1 W

$$E_1 + W = E_2$$



$$E_1 + W = E_2$$

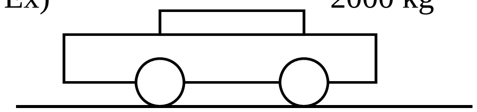
$$\frac{1}{2}mv_1^2 + Fl = \frac{1}{2}mv_2^2$$

$$\frac{1}{2}mv_1^2 - Fl = \frac{1}{2}mv_2^2$$

Power

Power : time rate & doing work $P = \dfrac{dw}{dt} \left[\dfrac{Nm}{\sec}\right] = [Watt]$

Ex) 2000 kg



$V = 72\text{km/h} = 20\text{m/s}$ (in 10sec)

$V_0 = 0$

$$\frac{1}{2}m\cancel{v_0^2} + W = \frac{1}{2}mv^2$$

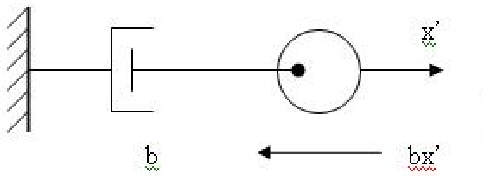
$$W = \frac{1}{2}(2000)(20)^2 = 400\times1000\ Nm = 400\ kNm[kJ]$$

$$P = \frac{W}{t} = \frac{400\times1000}{10}\ \frac{Nm}{\sec}$$

$$= 40\times1000\ Nm/s = 40\times1000\ W = 40W$$

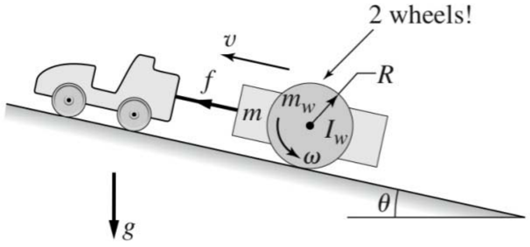
$1hp = 745.7\ W$ $\therefore P = 54\ hp$

Power dissipated in a damper



$$P = Fv = b\dot{x}\cdot\dot{x} = b\dot{x}^2$$

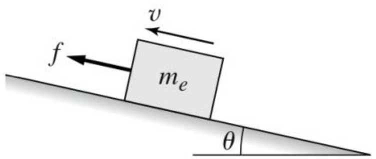
Energy Method for Deriving Equivalent Mass and Inertia



(a)

- **Kinetic Energy of the total System**

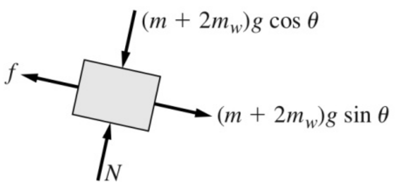
$$KE = \frac{1}{2}mv^2 + \frac{1}{2}(2m_w v^2) + \frac{1}{2}I_w\omega^2$$



(b)

- **Kinetic Energy represented with a single variable**

$$KE = \frac{1}{2}\left(m + 2m_w + 2\frac{I_w}{R^2} \right) v^2$$



(c)

- **Equation of motion using equivalent mass**

$$m_e = \left(m + 2m_w + 2\frac{I_w}{R^2} \right)$$

Energy Method for Deriving Equations of Motion

- Conservative system : No energy dissipation

$$E_1 + W = E_2$$
$$E_2 - E_1 = W$$

- Kinetic Energy T

- Potential Energy U

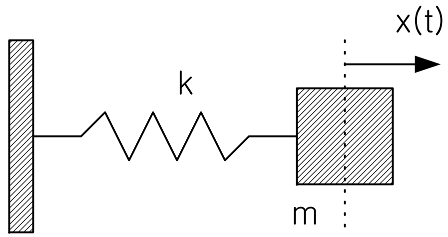
$$\Delta(T+U) = \Delta W$$

(the change in the total energy)
= (the net work done on the system by the external force)

no external force ; $\Delta W = 0$
$\Delta(T+U) = 0$ T+U = constant

Examples of Energy Method

ex1)



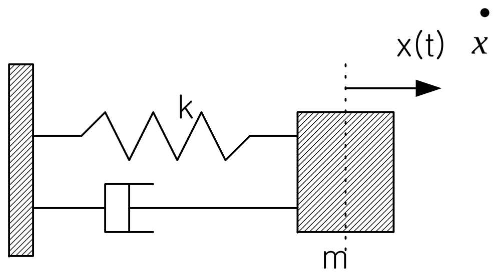
$$T=\frac{1}{2}m\dot{x}^2,\quad U=\frac{1}{2}kx^2,\quad T+U=C$$

$$\rightarrow \quad \frac{1}{2}m\dot{x}^2+\frac{1}{2}kx^2=0$$

$$\frac{d}{dt}(T+U)=0,\quad m\dot{x}\ddot{x}+kx\dot{x}=(m\ddot{x}+kx)\dot{x}=0$$

$$\dot{x}\neq 0,\quad \therefore (m\ddot{x}+kx)=0$$

ex2)



$$T=\frac{1}{2}m\dot{x}^2,\quad U=\frac{1}{2}kx^2$$

$$\frac{d}{dt}(T+U)=-b\dot{x}^2,\quad m\dot{x}\ddot{x}+kx\dot{x}=-b\dot{x}\dot{x}=0$$

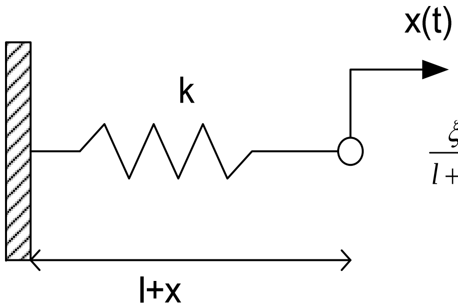
$$(m\ddot{x}+kx+b\dot{x})\dot{x}=0$$

$$\dot{x}\neq 0,\quad \Rightarrow (m\ddot{x}+kx+b\dot{x})=0$$

Examples of Energy Method

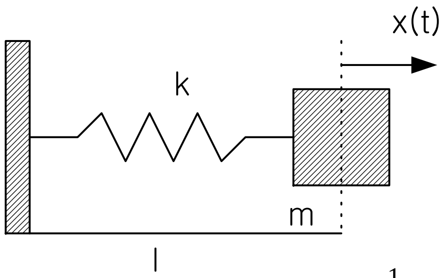
ex3)

Spring with mass



$$\frac{\xi}{l+x}\dot{x} \qquad \frac{d\xi}{l+x}\bullet m_s$$

$$dT = \frac{1}{2}m_s \cdot \frac{d\xi}{l+x} \cdot \left(\frac{\xi}{l+x}\dot{x}\right)^2$$



$$m\ddot{x} + kx = 0$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$\int_0^{l+x} dT = \int_0^{l+x} \frac{1}{2}m_s \frac{1}{(l+x)^3}\dot{x}^2\xi^2 d\xi$$

$$= \frac{1}{2}m_s\dot{x}^2 \frac{1}{(l+x)^3}\frac{1}{3}(l+x)^3$$

$$= \frac{1}{2}\left(\frac{1}{3}m_s\right)\dot{x}^2$$

Potential E $\quad U = \frac{1}{2}kx^2$

Kinetic E $\quad T = \frac{1}{2}m\dot{x}^2$

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\left(\frac{1}{3}m_s\right)\dot{x}^2 = \frac{1}{2}\left(m + \frac{1}{3}m_s\right)\dot{x}^2 \qquad \left(m + \frac{1}{3}m_s\right)\ddot{x} + kx = 0 \qquad \omega_n = \sqrt{\frac{k}{m + \frac{1}{3}m_s}}$$