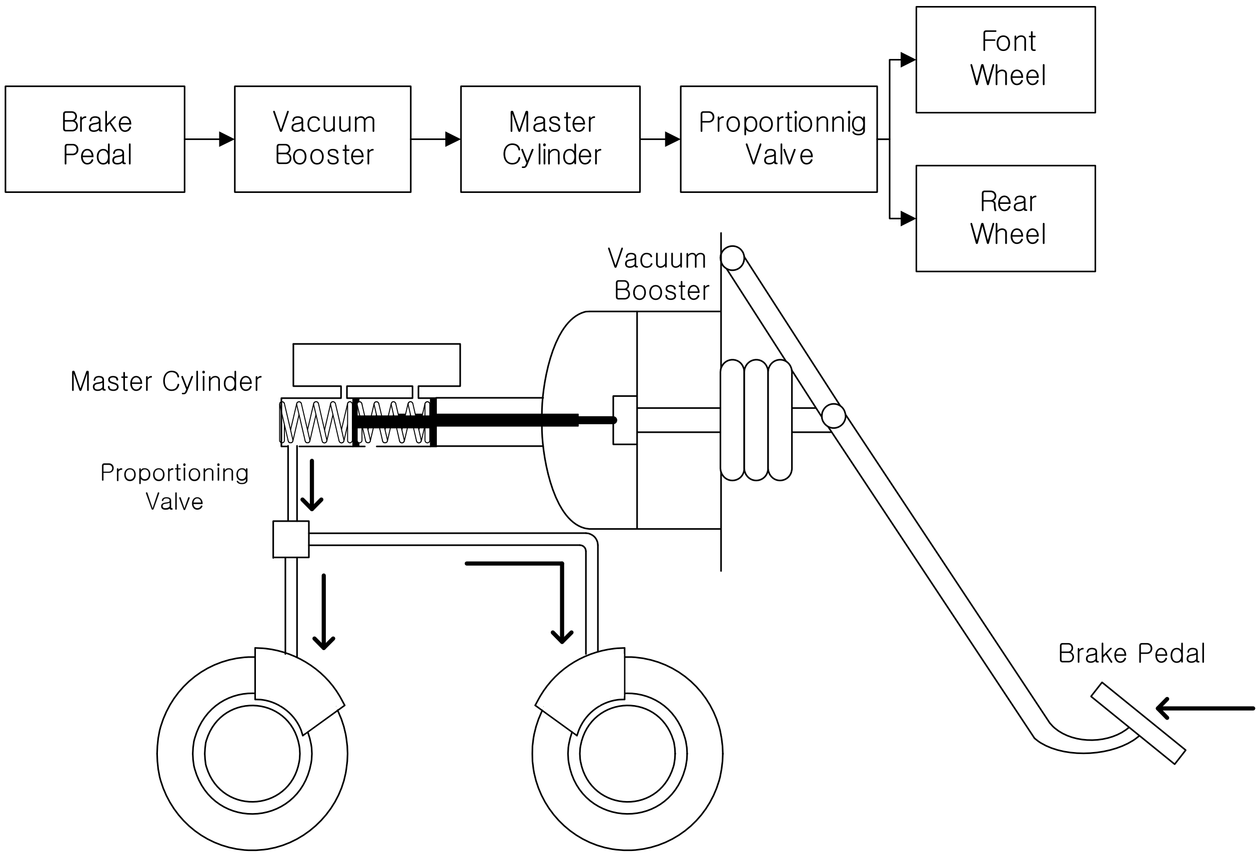
Hydraulic Systems

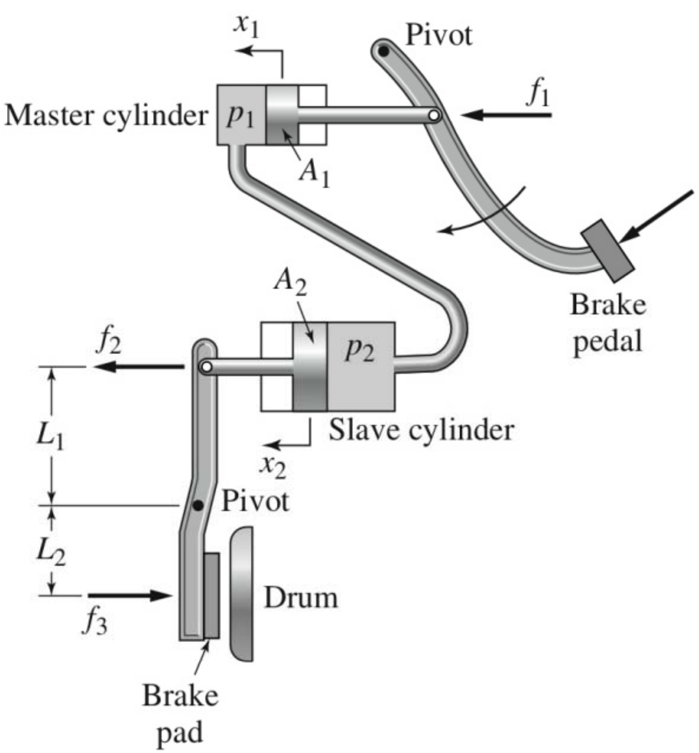
Brake Model



Fundamental structure of a hydraulic brake

Conservation of Mass, Force and Pressure

$$\dot{m} = q_{mi} - q_{mo}$$



$$i)\ Volume \quad A_1 dx_1 = A_2 dx_2$$

$$\rightarrow dx_2 = \frac{A_1}{A_2} dx_1$$

$$ii)\ \text{Pr}essue \quad p_2 = p_1 + \rho g h$$

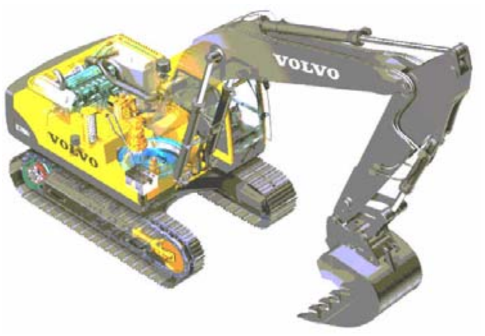
$$iii)\ \textbf{Force} \quad f_1 = p_1 A_1$$

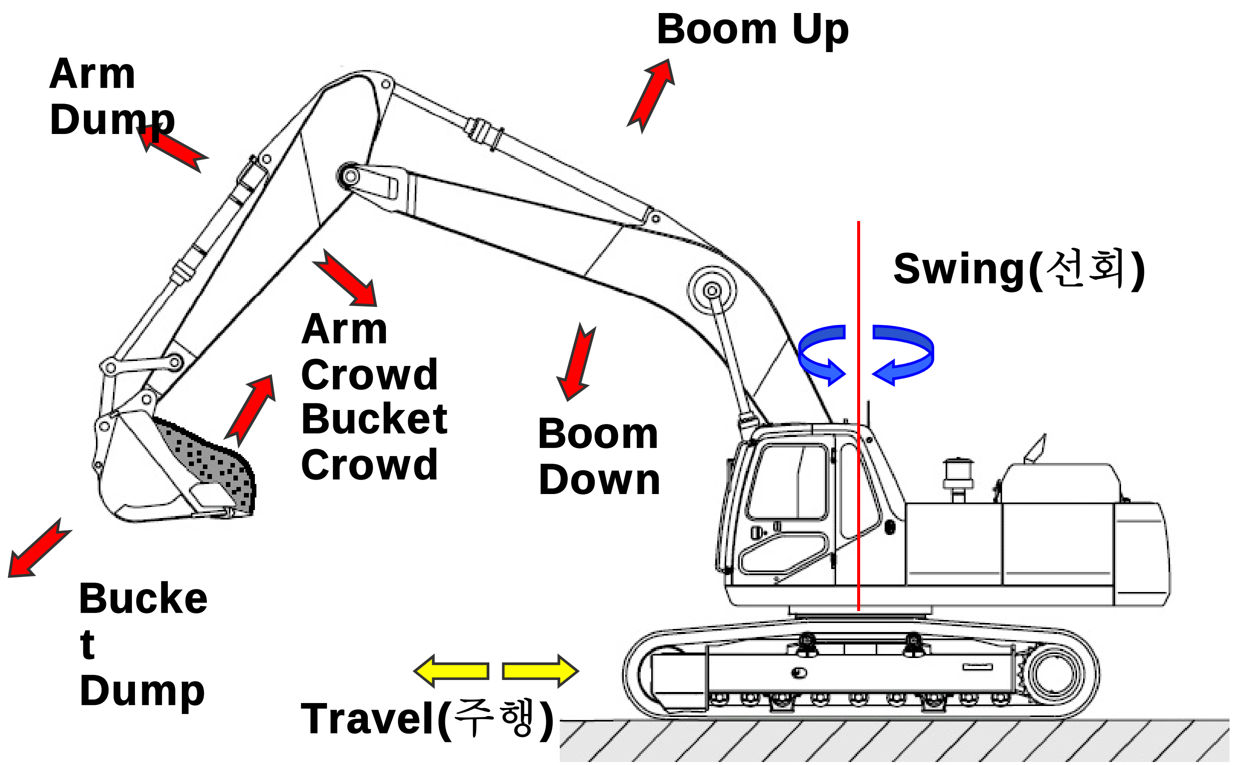
$$f_2 = p_2 A_2$$

$$f_2 = \frac{A_2}{A_1} f_1 \qquad (P_1 \approx P_2)$$

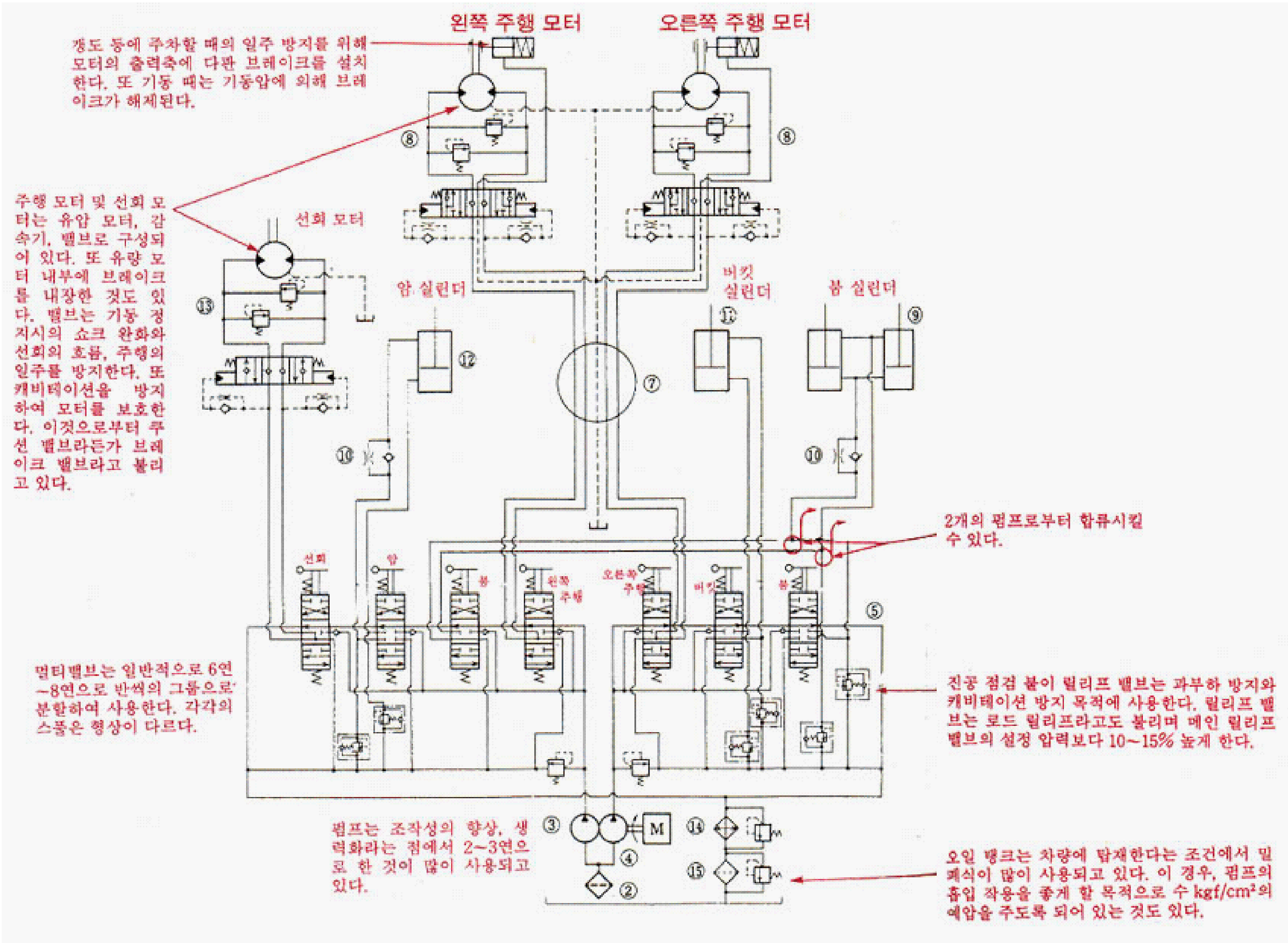
$$f_3 = \frac{L_1}{L_2} f_2 = \frac{L_1}{L_2} \frac{A_2}{A_1} f_1$$

Hydraulic Excavator



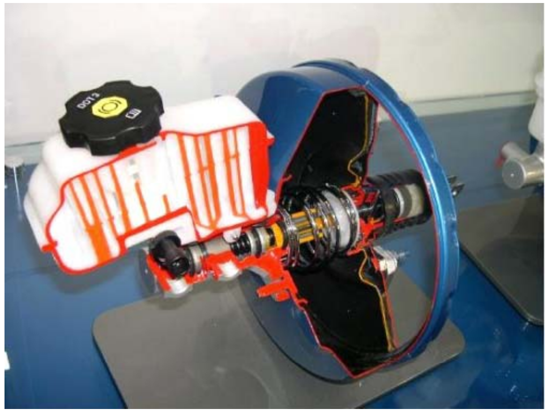


유압굴삭기 회로도

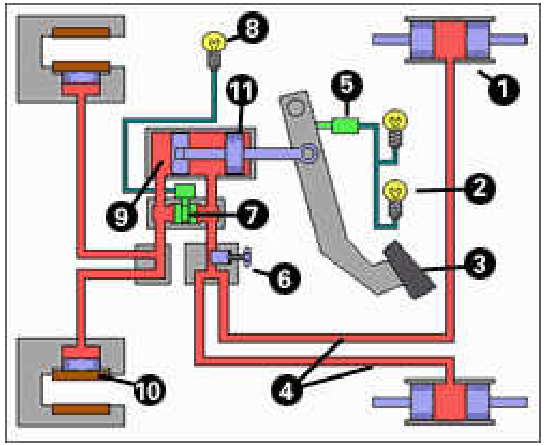


왼쪽 주행 모터
오른쪽 주행 모터
갱도 등에 주차할 때의 일주 방지를 위해 모터의 출력축에 다판 브레이크를 설치한다. 또 기동 때는 기동압에 의해 브레이크가 해제된다.
주행 모터 및 선회 모터는 유압 모터, 감속기, 밸브로 구성되어 있다. 또 유량 모터 내부에 브레이크를 내장한 것도 있다. 밸브는 기동 정지시의 쇼크 완화와 선회의 흐름, 주행의 일주를 방지한다. 또 캐비테이션을 방지하여 모터를 보호한다. 이것으로부터 주선 밸브라든가 브레이크 밸브라고 불리고 있다.
선회 모터
암 실린더
버킷 실린더
붐 실린더
2개의 펌프로부터 합류시킬 수 있다.
선회
암
붐
왼쪽 주행
오른쪽 주행
버킷
멀티밸브는 일반적으로 6연~8연으로 반씩의 그룹으로 분할하여 사용한다. 각각의 스풀은 형상이 다르다.
진공 점검 붙이 릴리프 밸브는 과부하 방지와 캐비테이션 방지 목적에 사용한다. 릴리프 밸브는 로드 릴리프라고도 불리며 메인 릴리프 밸브의 설정 압력보다 10~15% 높게 한다.
펌프는 조작성의 향상, 생력화라는 점에서 2~3연으로 한 것이 많이 사용되고 있다.
M
오일 탱크는 차량에 탑재한다는 조건에서 밀폐식이 많이 사용되고 있다. 이 경우, 펌프의 흡입 작용을 좋게 할 목적으로 수 kgf/cm²의 예압을 주도록 되어 있는 것도 있다.

Hydraulic Brake Systems

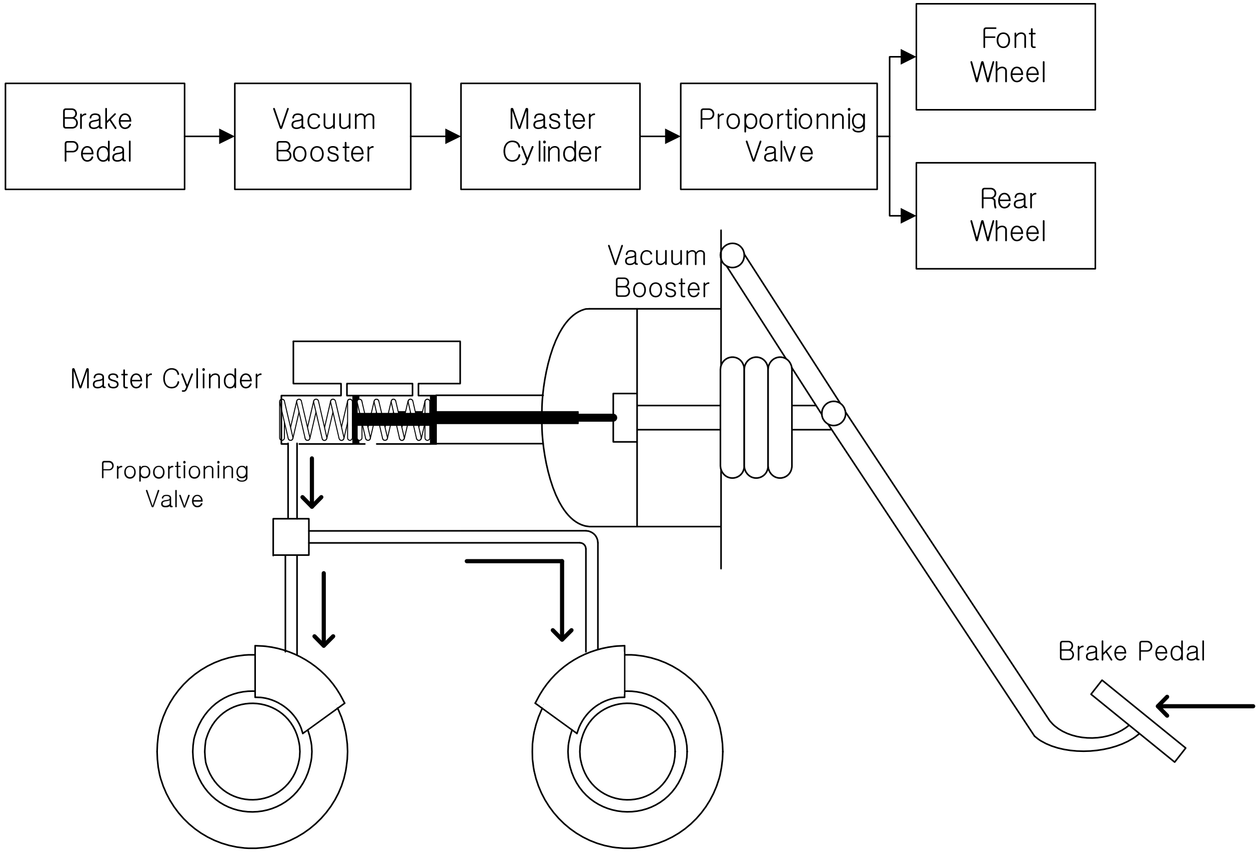


Brake System



1) Wheel Cylinder
2) Brake Light
3) Brake Pedal
4) Rear Brake Lines
5) Stop Light Switch (Mechanical)
6) Front/Rear Balance Valve
7) Pressure Differentiavl Valve
8) Brake Warning Lamp
9) Brake Fluid
10) Brake Pad
11) Master Cylinder

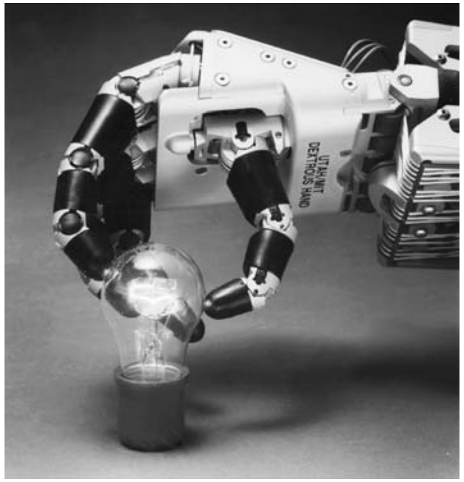
Brake Model



Fundamental structure of a hydraulic brake

Applications of Fluid Power

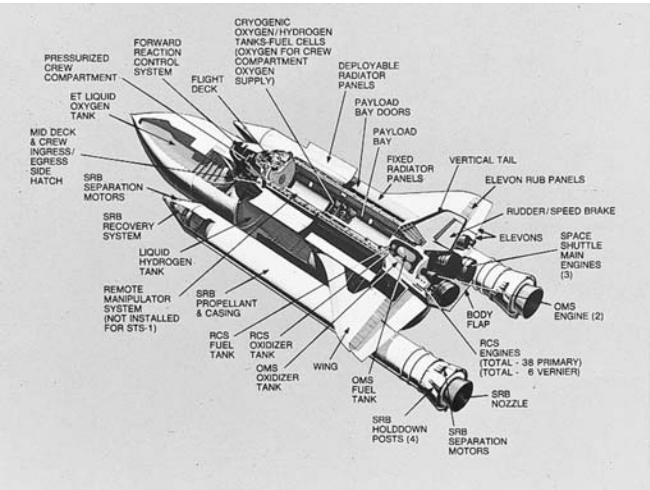
- Pneumatically controlled dexterous hand



- Hydraulically powered dexterous

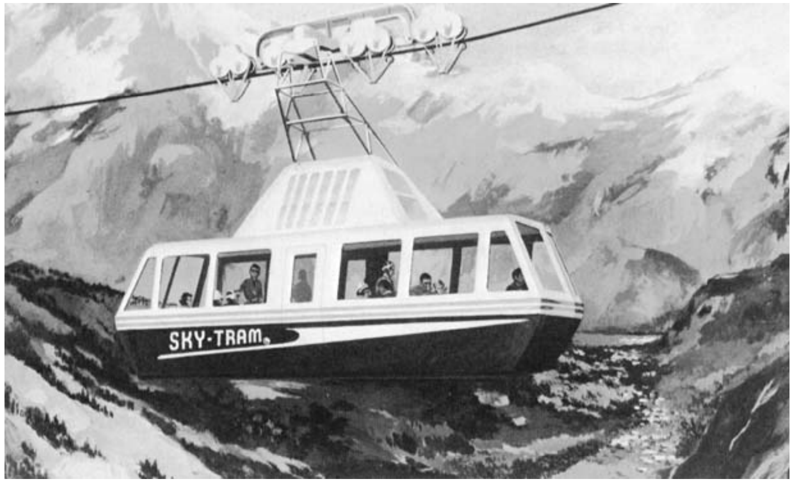
- Space shuttle Columbia

- Space shuttle vehicle



Applications of Fluid Power

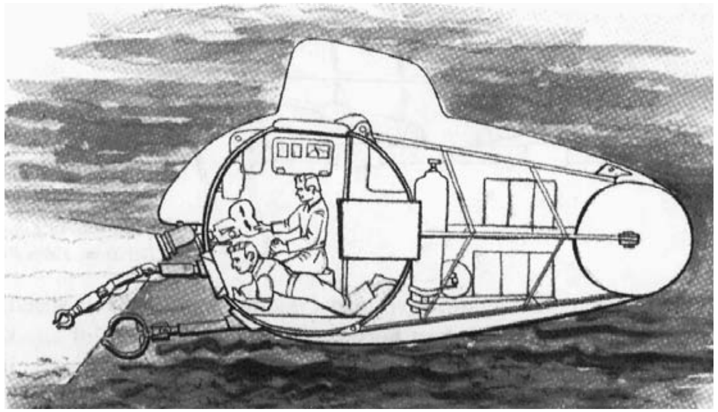
- Hydraulically powered Sky-tram



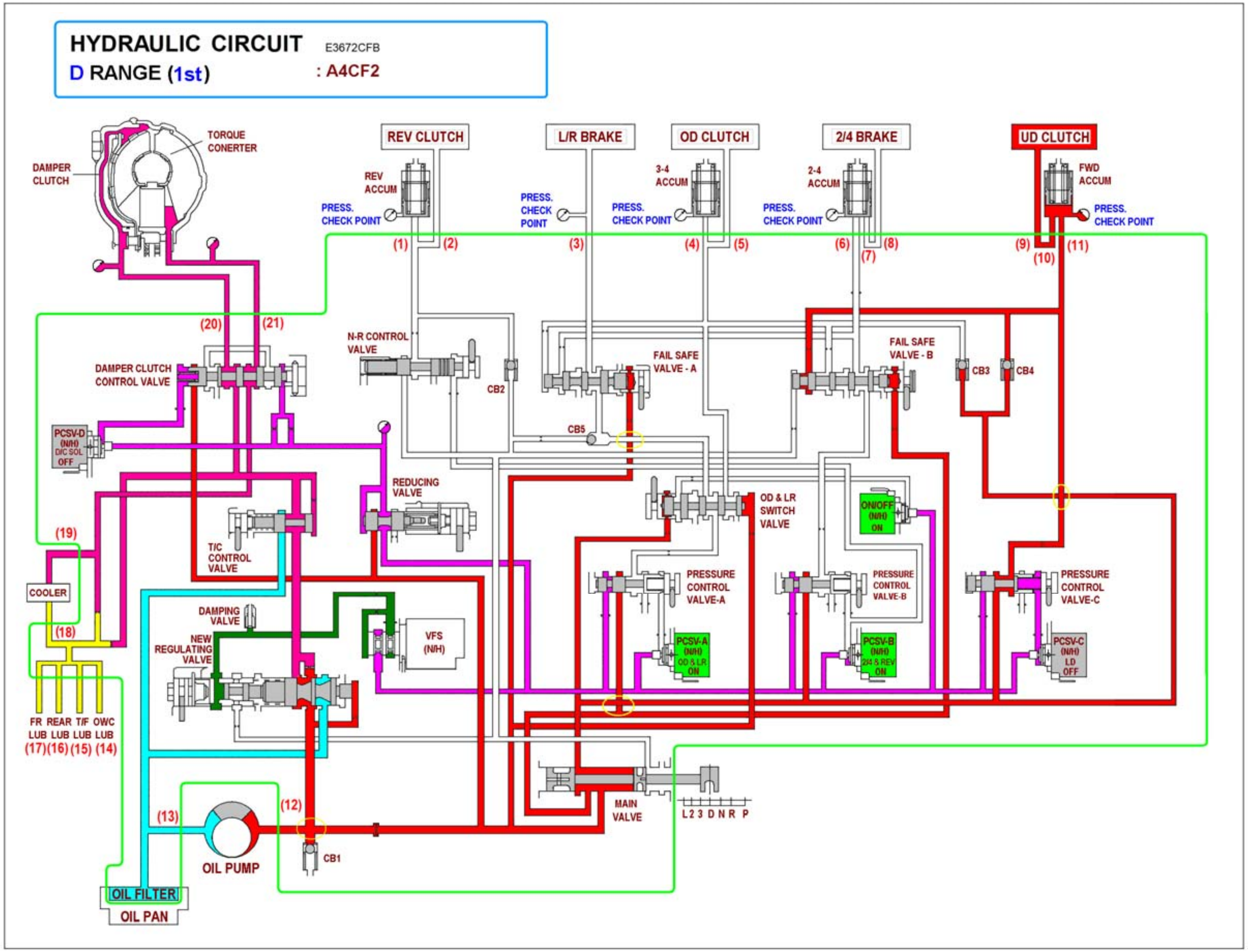
- Hydraulically driven turntable

- Hydraulic power brush drive

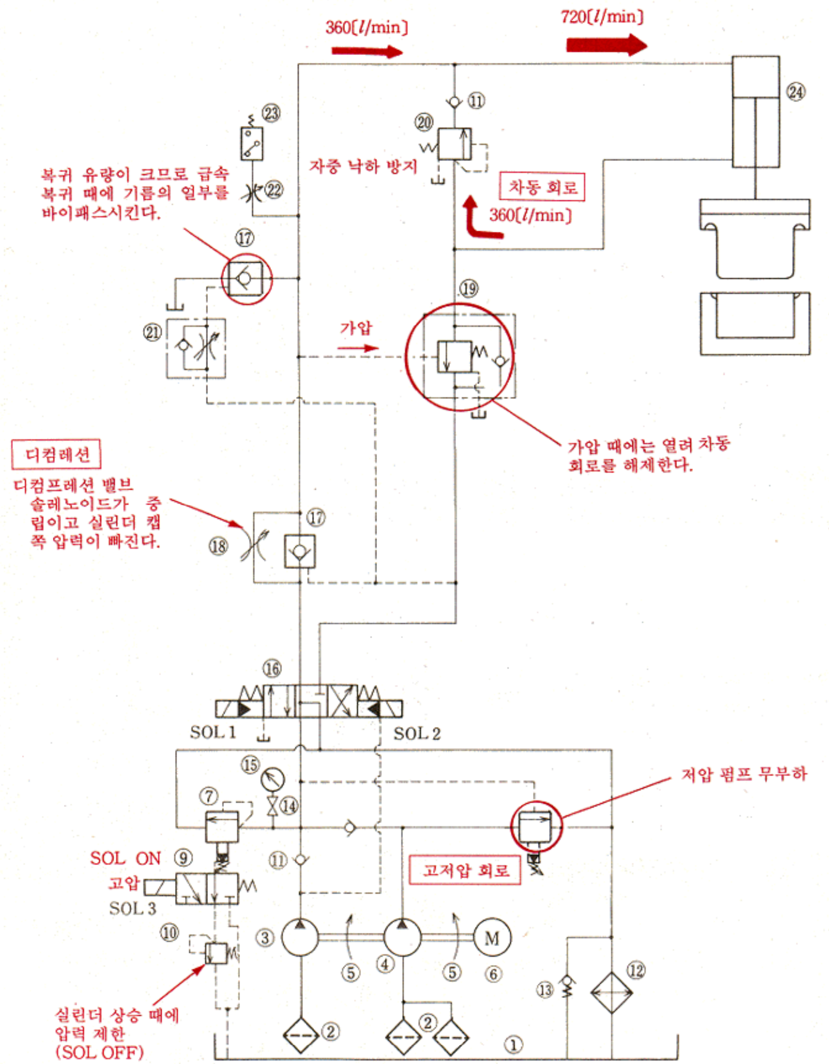
- Oceanography



유압기술의 응용 – 자동변속기 유압제어



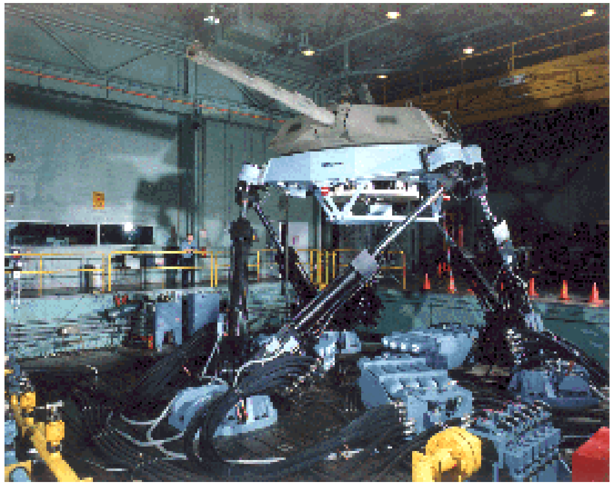
유압기술의 응용 - 판금 프레스



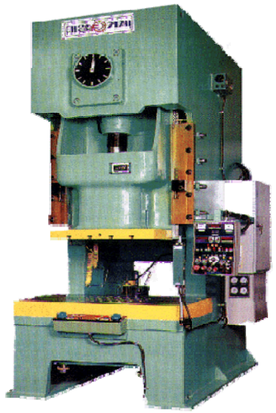
선형 액츄에이터(Linear Actuators)의 응용

지게차

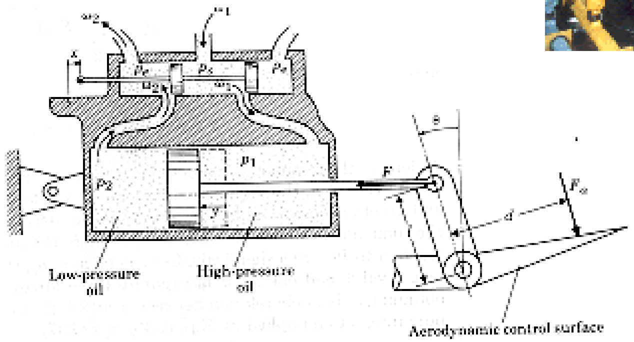
Motion Simulator



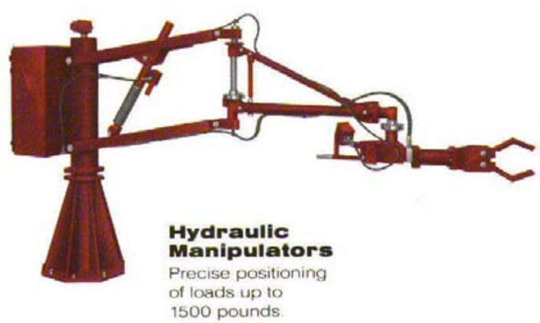
Press



비행기 조종익

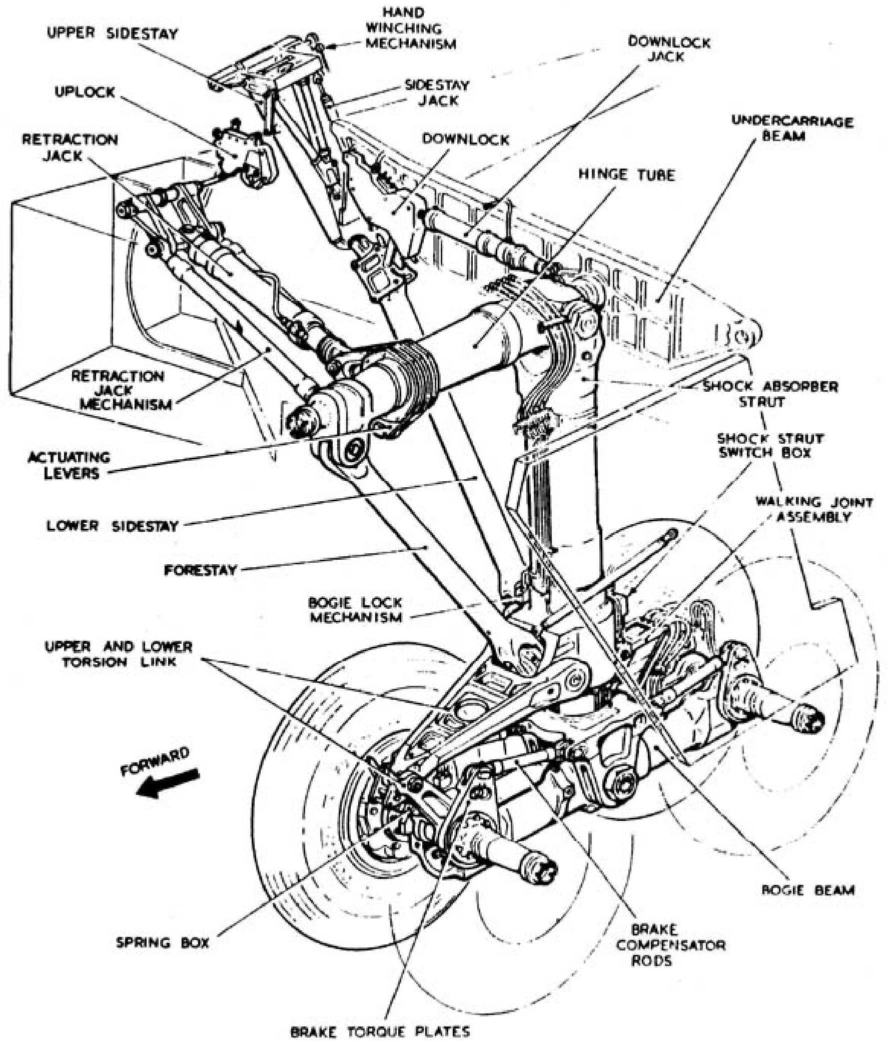


Robot



Hydraulic Systems : Landing Gear System

Landing gear system of AIRBUS A330



Why hydraulic ?

Internal combustion Engine

Turbine

Electric motor

Hydraulic actuator

……

Why hydraulic ?

1. smaller and lighter

   horsepower to weight ratio > 2 hp/lb

2. heat/lubrication – long component life
3. no saturation and losses
   - saturation and losses in magnetic materials of electrical machine
   - torque limit only by safe stress levels
4. high natural frequency/high speed of response/high loop gains
   - electrical motors, a simple lag device from applied voltage to speed
5. dynamic breaking with relief valve without damage

Disadvantages

1. not so readily available

2. small allowable tolerances result in high costs

3. hydraulic fluids imposes upper temperature limit.

4. fluid contamination: dirt and contamination

5. basic design procedures are lacking and difficult, complexity of hydraulic control analysis

6. not so flexible, linear, accurate, and inexpensive as electronic and/or electromechanical devices

Hydraulic Systems

장점

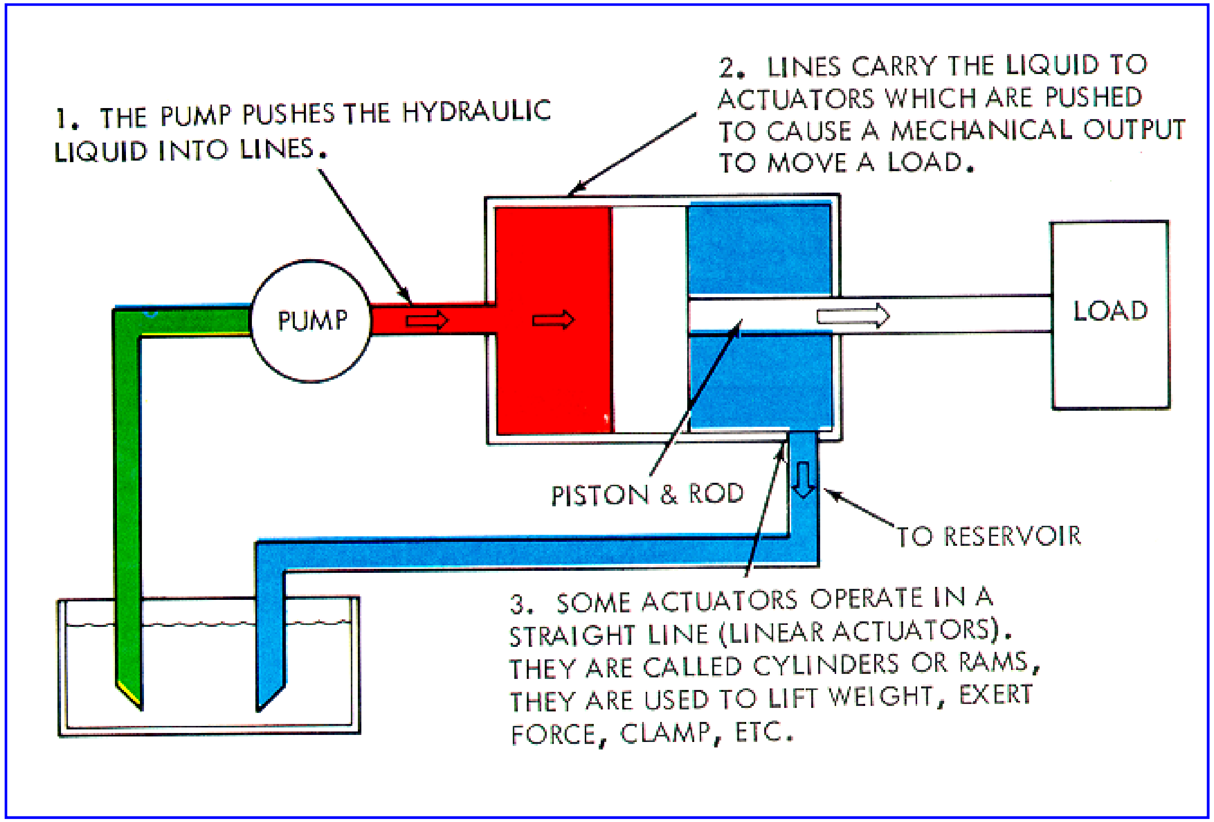
- 무게당 동력의 크기가 크다. (고압 사용, 시스템의 크기가 작다.)
- 제어의 용이성과 정확도 (허용오차: 1/10,000인치)
- 응답이 빠르다.
- 윤활성, 방청성 우수, 보수 용이
- 내부 발생 열 제거 용이, 내열성 우수

단점

- 오일의 유속에 제한 --> 액츄에이터의 속도 한계
- 누유(leakage)로 인해 시스템이 불결
- 작동유에 기포가 흡입되면 (aeration) 압축성이 커져 작동 불량
- 캐비테이션(cavitation)이 발생하면 기기 파손, 소음 발생, 고장

Primary Functions of a Hydraulic Fluid

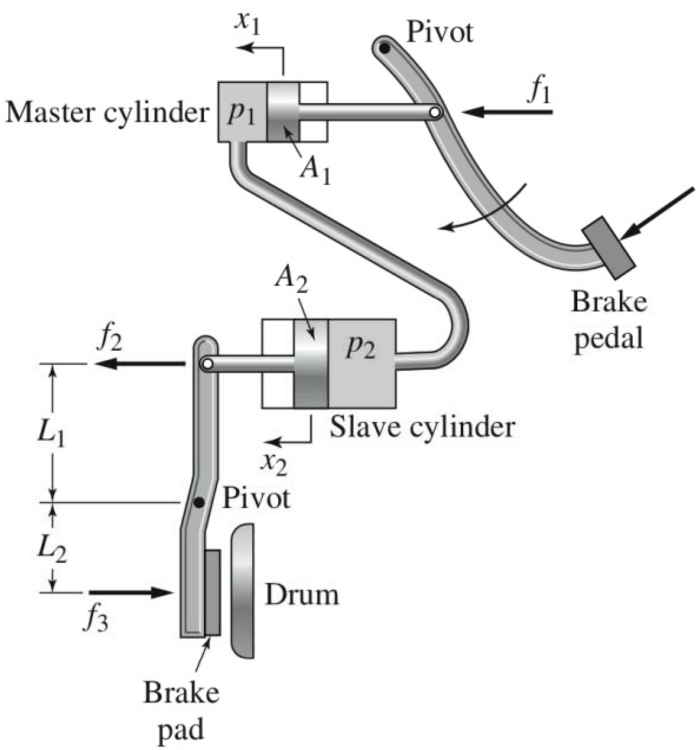
유압 동력 전달



유압동력의 전달과정

Conservation of Mass, Force and Pressure

$$\dot{m} = q_{mi} - q_{mo}$$



$i)$ $Volume \quad A_1 dx_1 = A_2 dx_2$

$$\rightarrow dx_2 = \frac{A_1}{A_2} dx_1$$

$ii)$ Pr$essue \quad p_2 = p_1 + \rho g h$

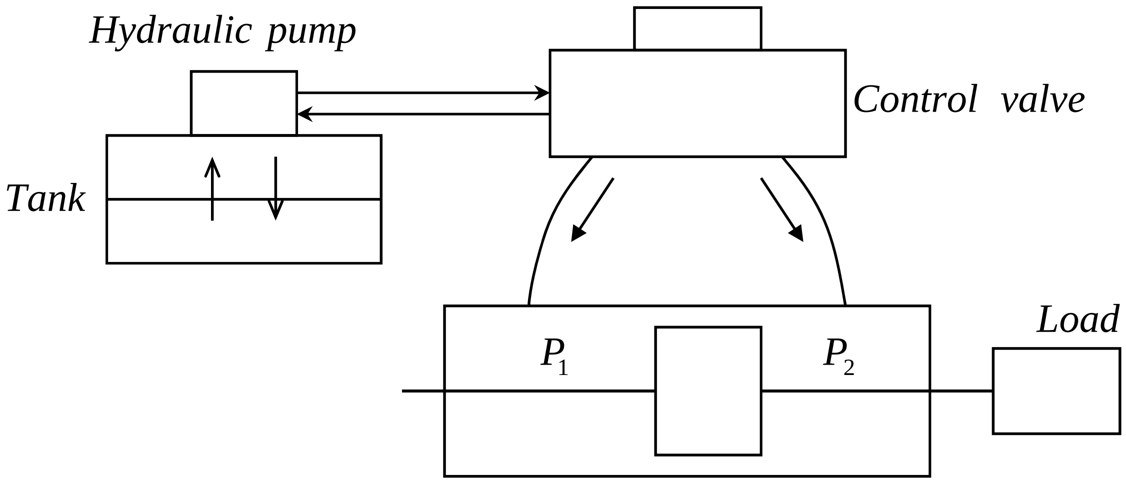
$iii)$ **Force** $\quad f_1 = p_1 A_1$

$$f_2 = p_2 A_2$$

$$f_2 = \frac{A_2}{A_1} f_1 \qquad (P_1 \approx P_2)$$

$$f_3 = \frac{L_1}{L_2} f_2 = \frac{L_1}{L_2} \frac{A_2}{A_1} f_1$$

Hydraulic Systems

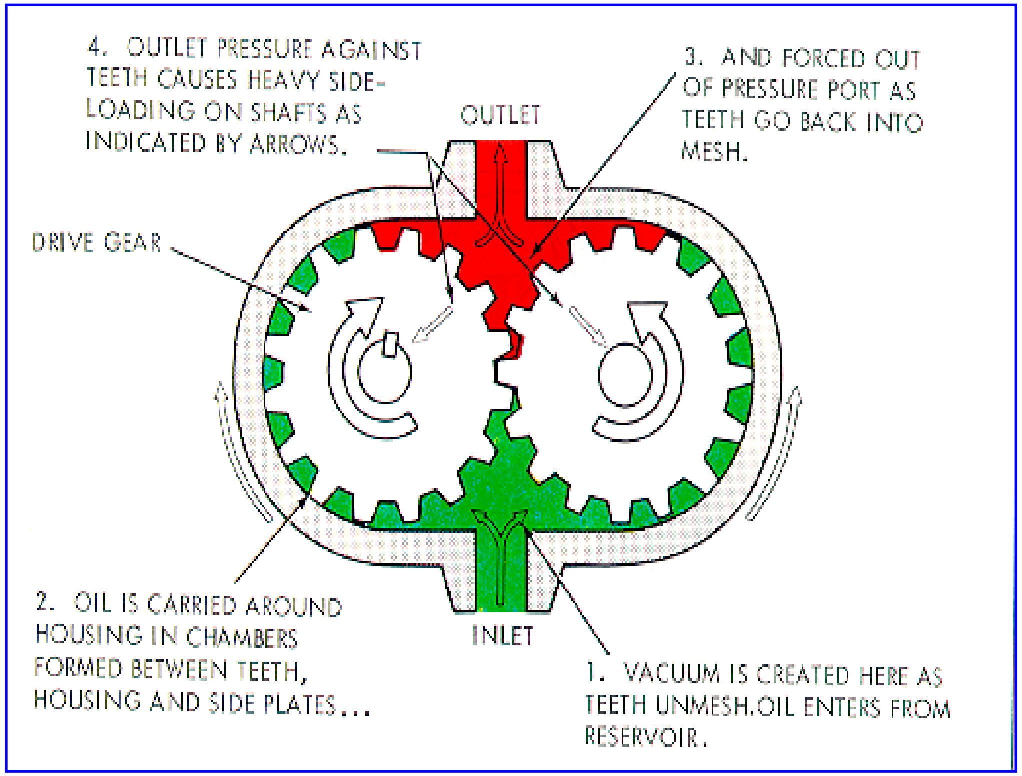


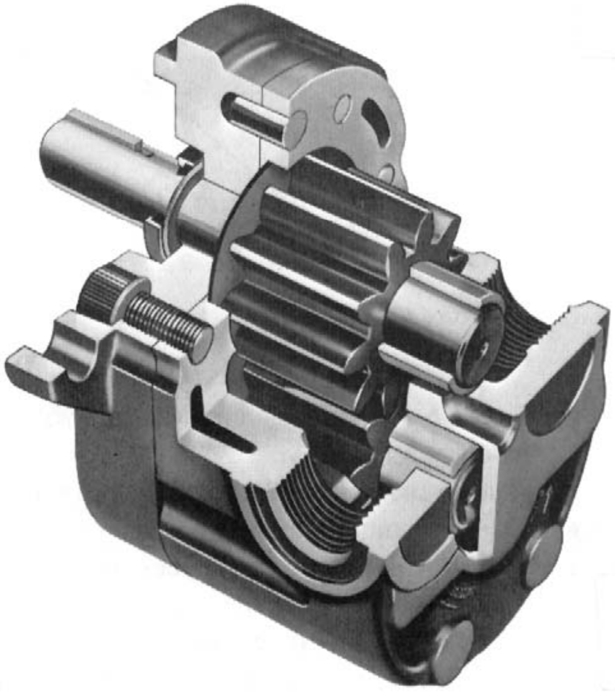
$$Hydraulic\ \ actuator\ \ (70 \sim 210bar)$$

$$F = A_P \cdot (P_1 - P_2)$$

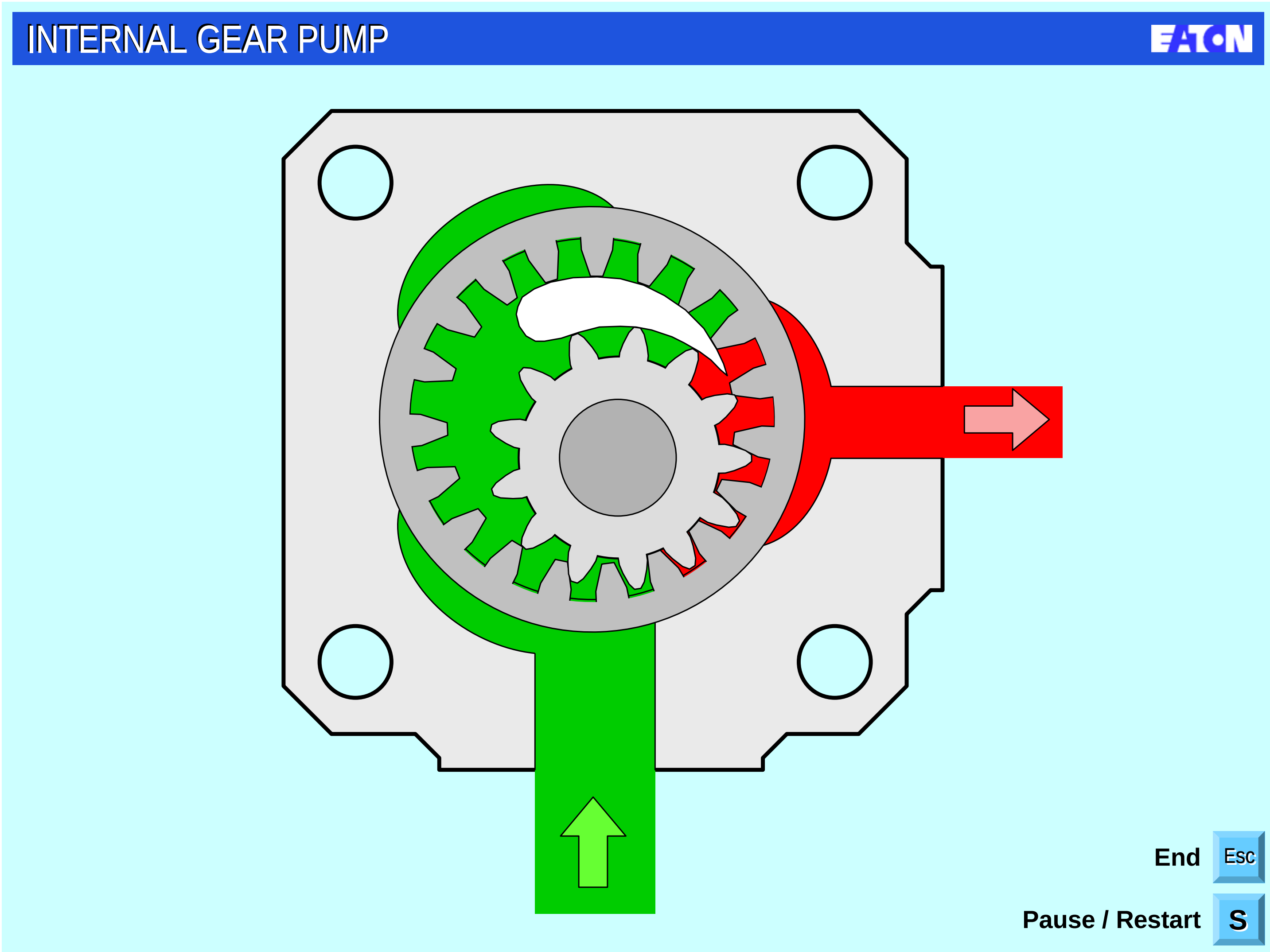
Gear Pumps (External)

- fixed displacement pump
- uses spur gear (teeth are parallel to the axis of the gear)
- noisy at relatively high speeds

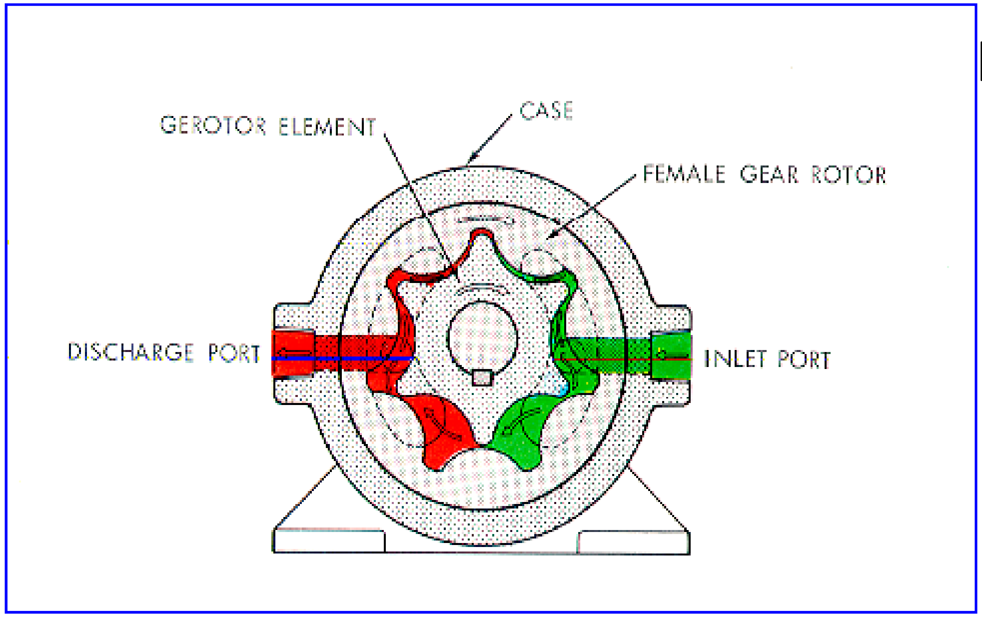




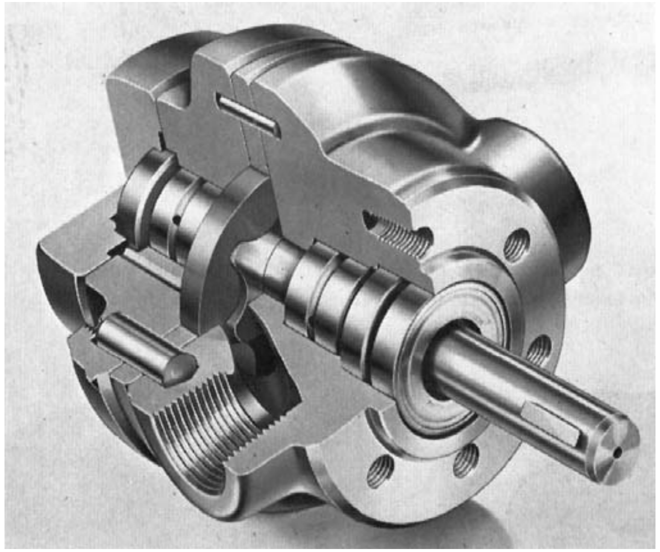
Internal Gear Pump



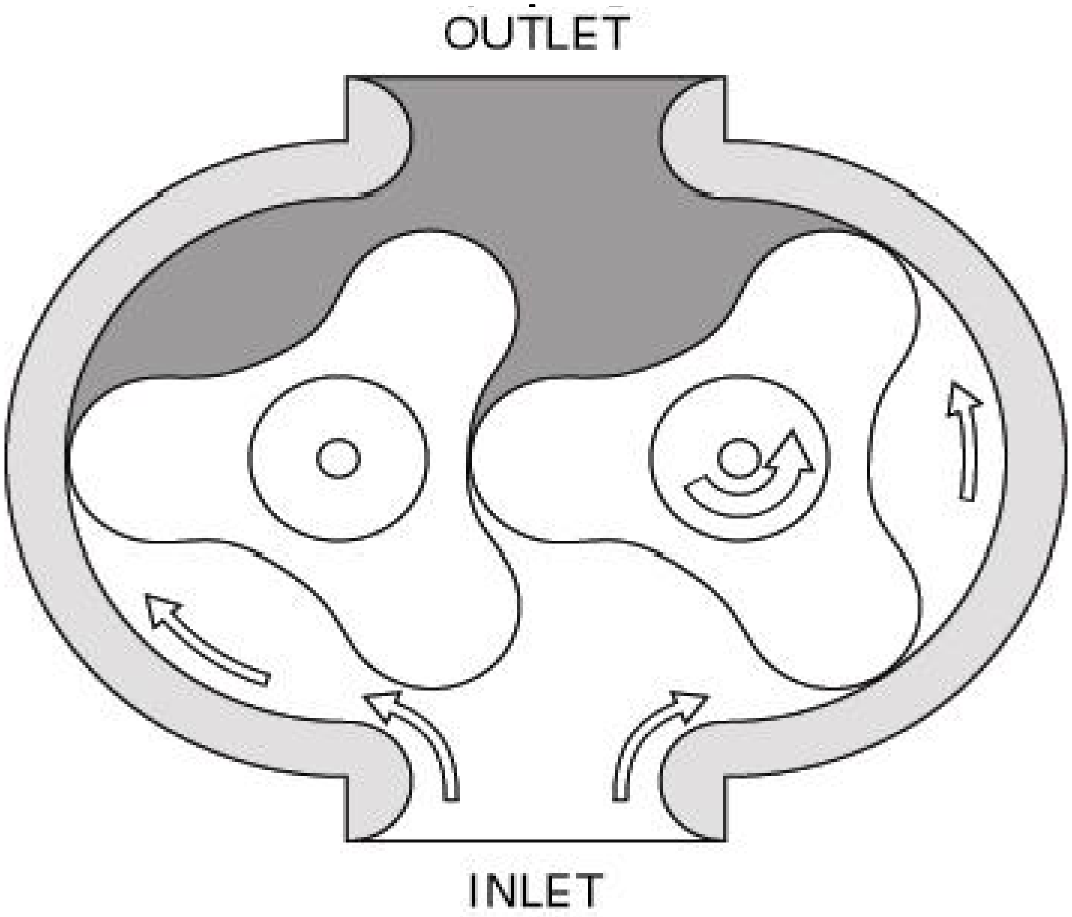
Internal Gear Pump



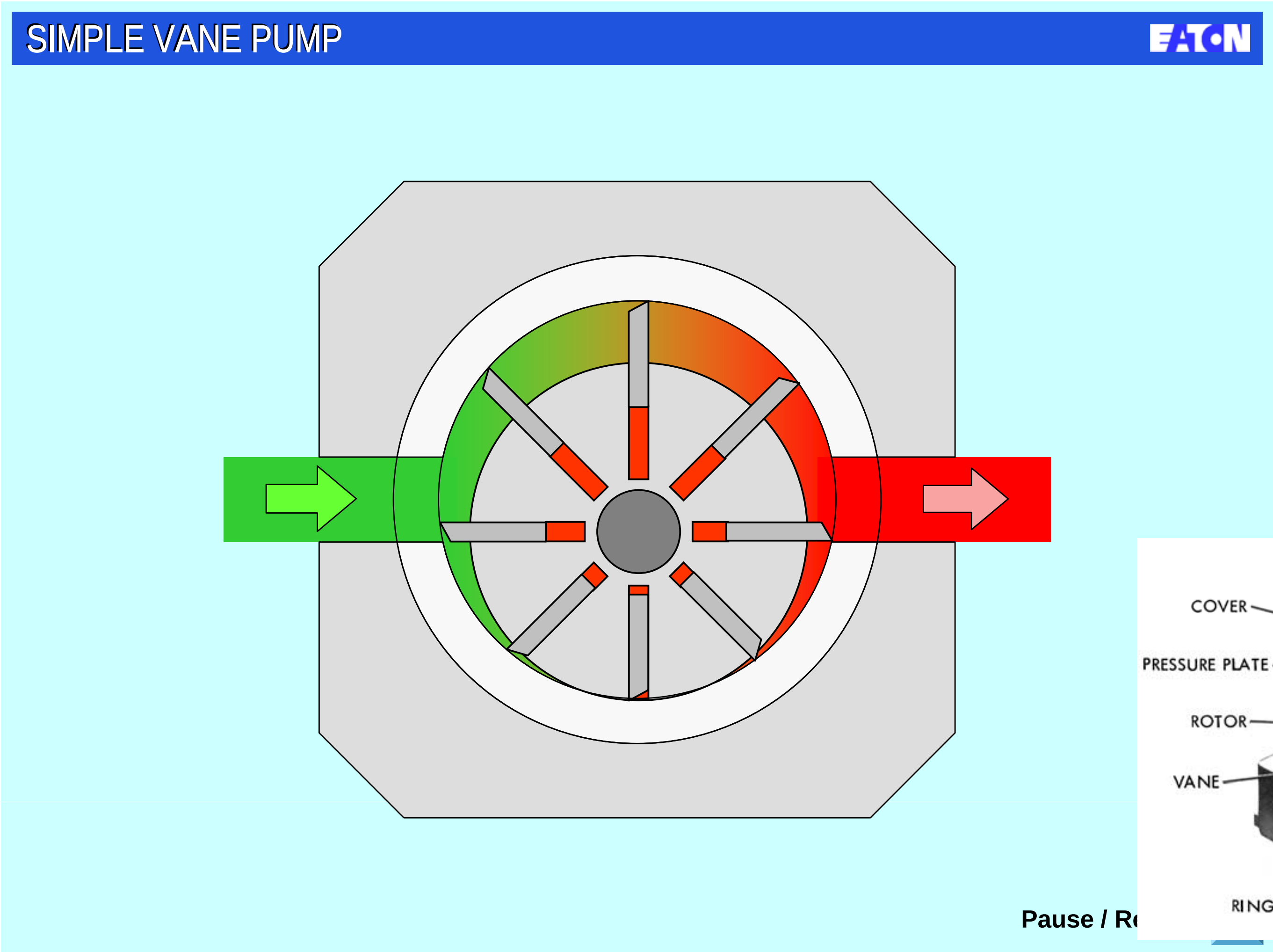
p

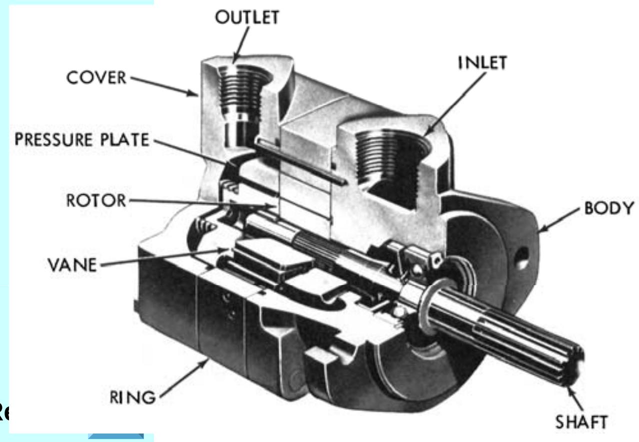


Internal Gear Pump

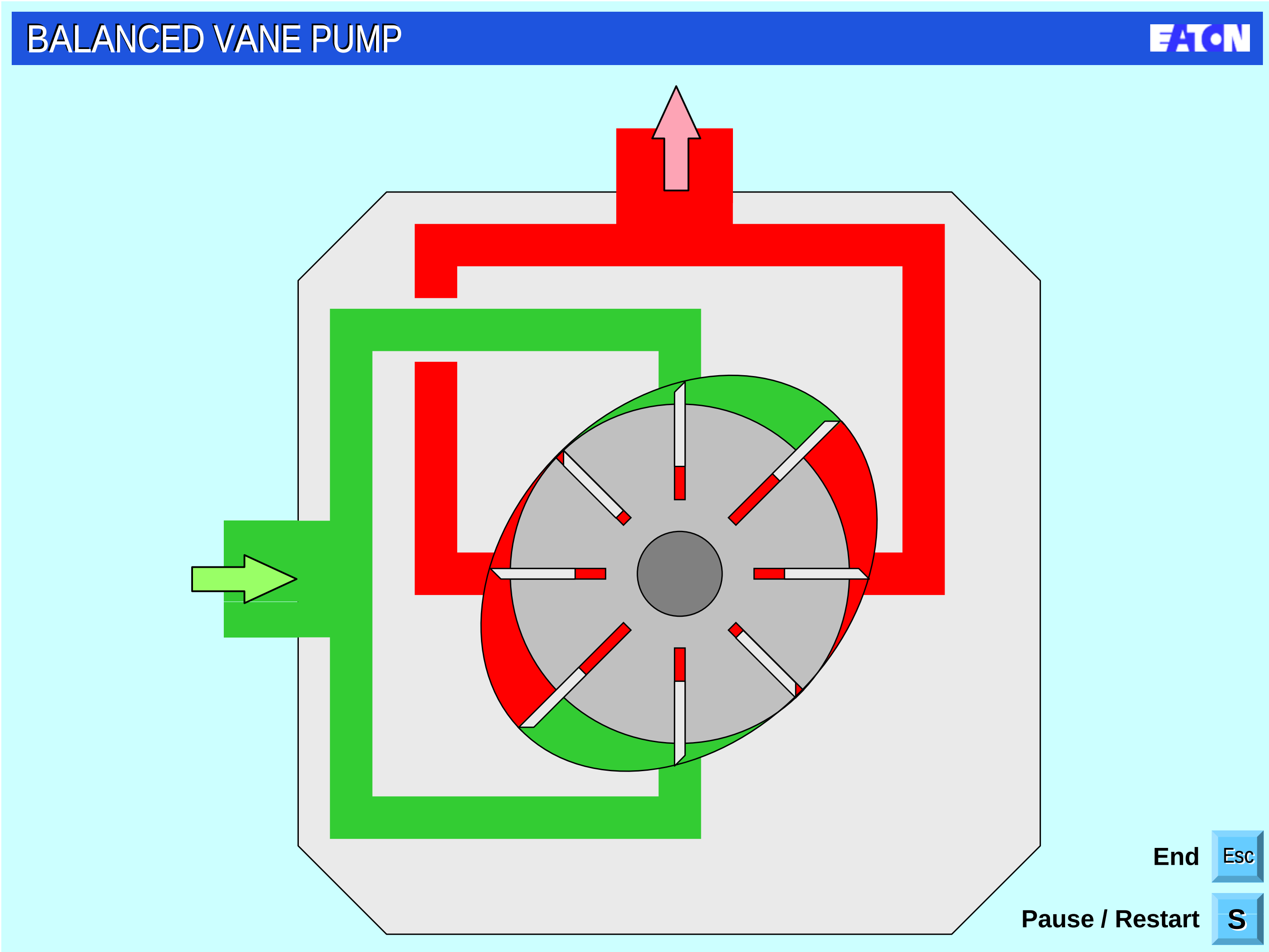


Simple Vane Pump

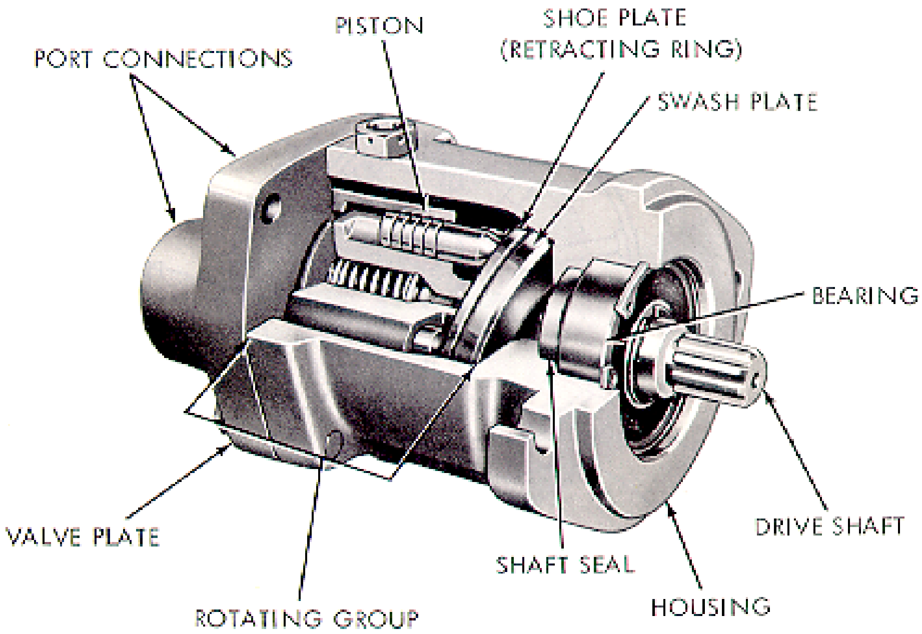


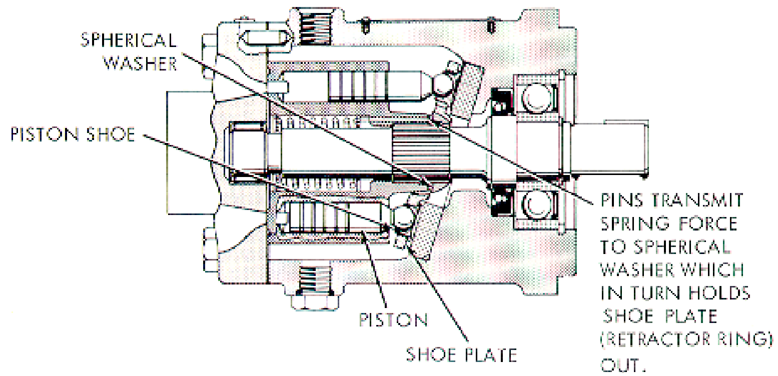


Balanced Vane Pump

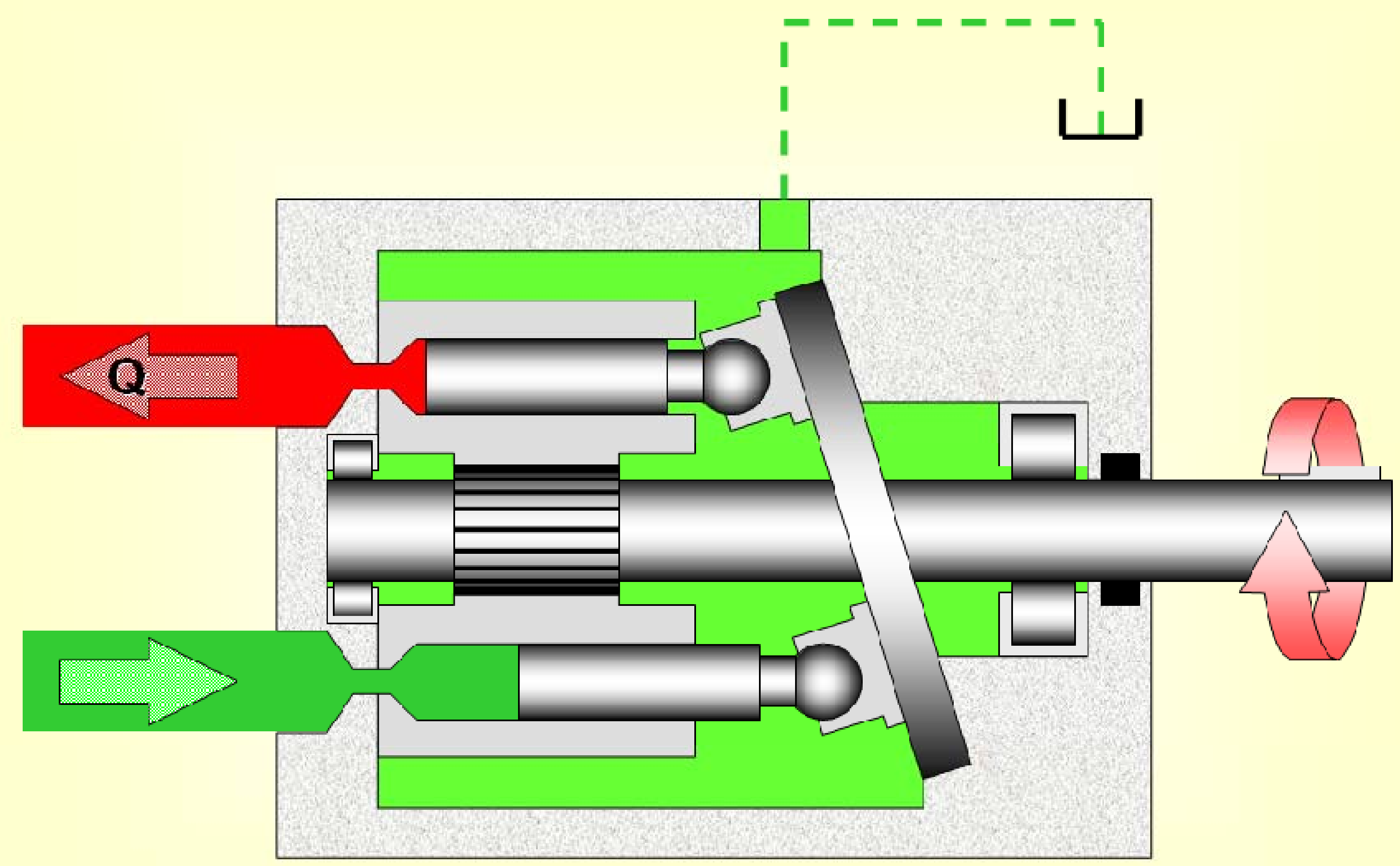


Piston Pump (Swash Plate Type)





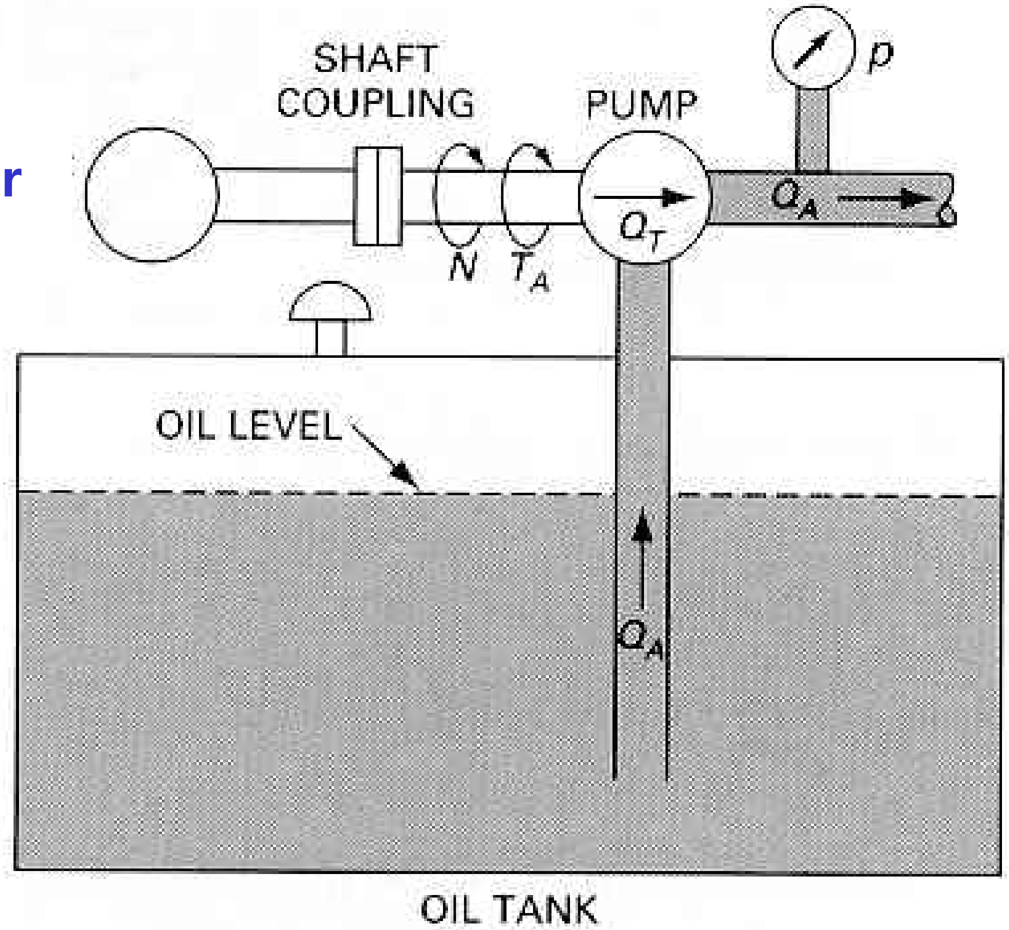
Piston Pump



Q = (No. of Pistons) x (Piston Size) x (Piston Stroke) x (Drive Speed)

Pump

Electric
Motor or
Engine



Hydraulic Pump

- **the heart of a hydraulic system**
- **converts mechanical energy into hydraulic energy**
- **유압공학에서 다루는 펌프는 hydrostatic pump**
- **밀어내기식, 용적식 (positive displacement) 펌프**
- **작동 cycle**
  - 흡입(빨아들임), 공간차단, 압축 및 토출(밀어냄), 공간차단
- **Displacement (배제용적)**
  - the amount of fluid ejected per revolution
  - unit: cm^3/rev, cc/rev, cm^3/rad, cc/rad

Pump 구동에 필요한 Torque

- **Mechanical power supplied to pump**

$$H_m = T\omega$$

- **Hydraulic power delivered by pump**

$$H_p = PQ$$

$P : pressure\ rise\ across\ the\ pump$

$Q : delivery\ rate$

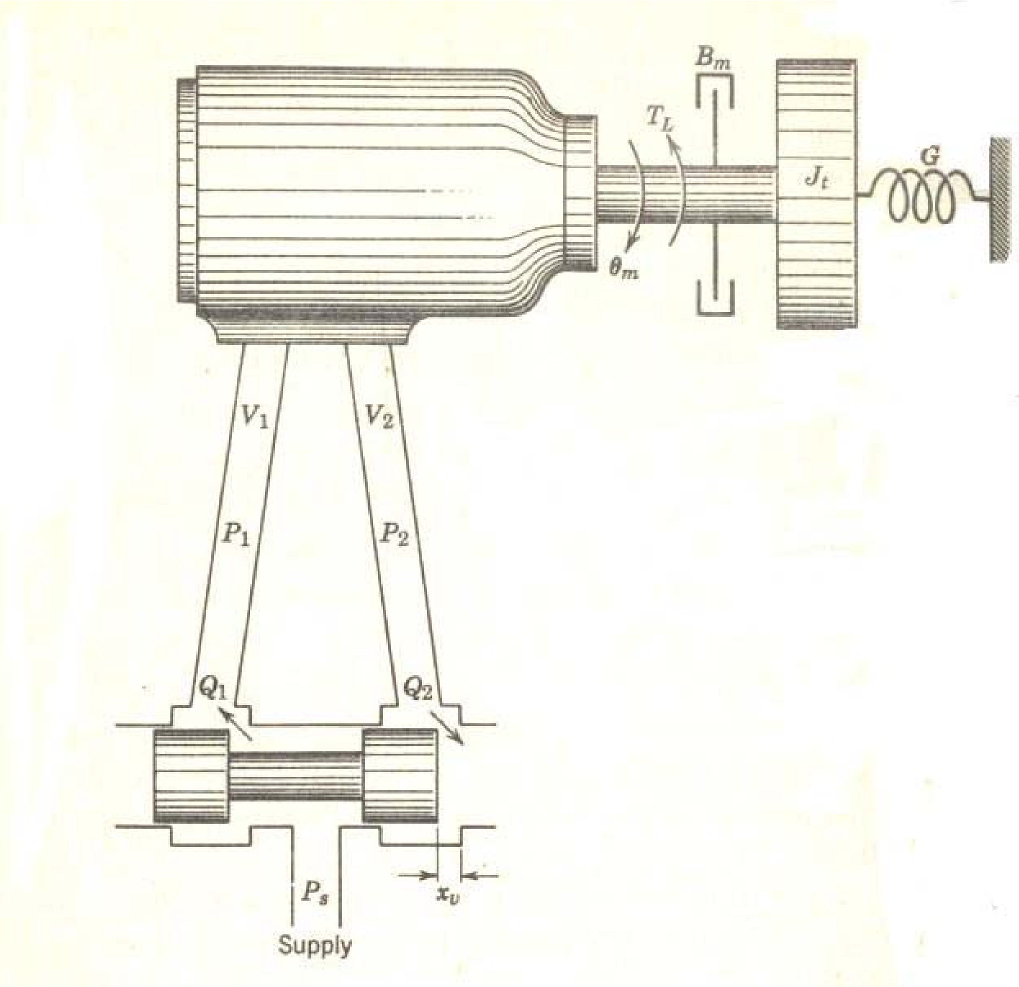
- **두 관계식으로부터**

$$T_{th}\omega = PQ_{th} = P\omega D_p$$

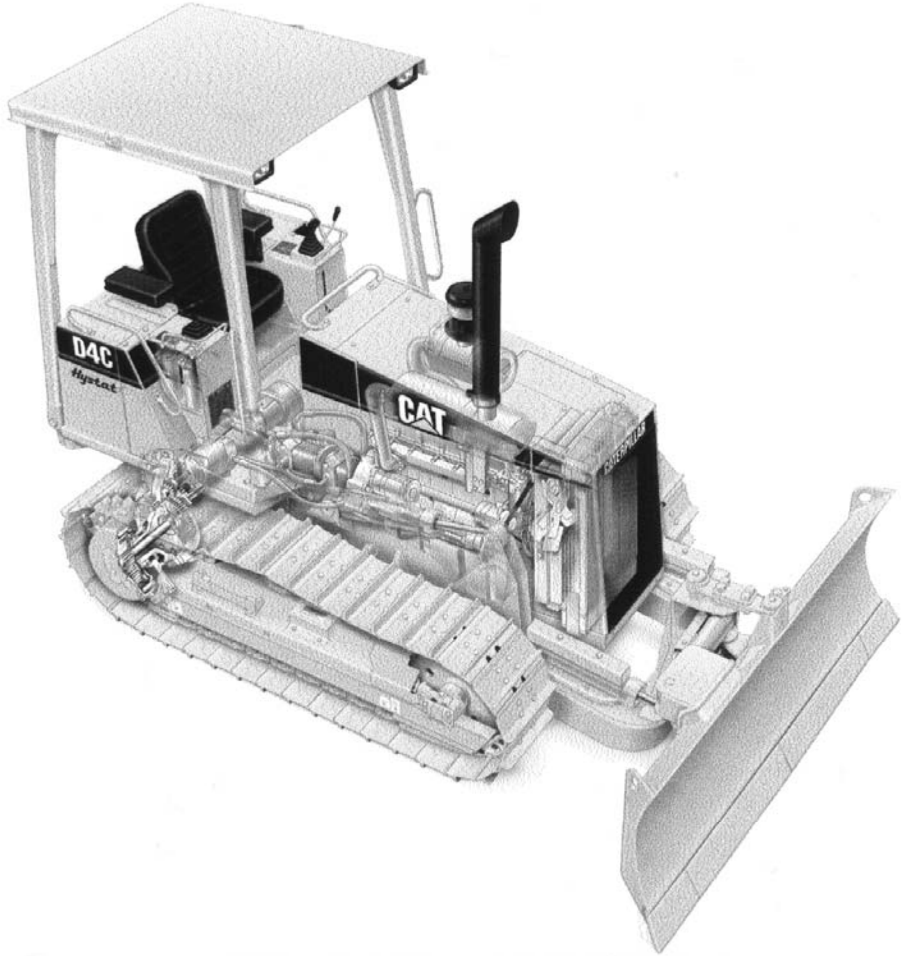
$$T_{th} = PD_p \qquad \text{여기서} D_p : \text{펌프배제용적}[m3/rad]$$

Hydraulic Motors and Actuators

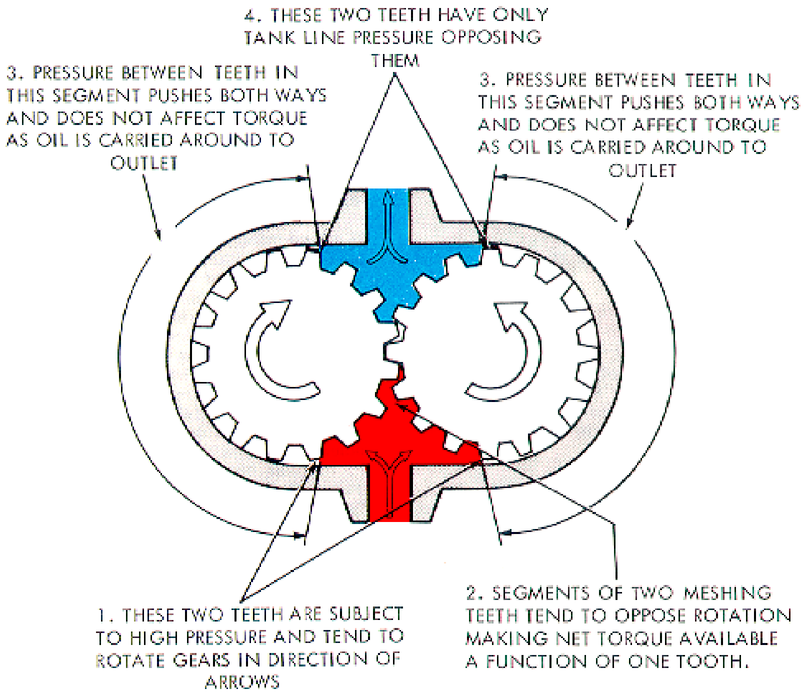
Hydraulic Systems : Valve-motor Combination

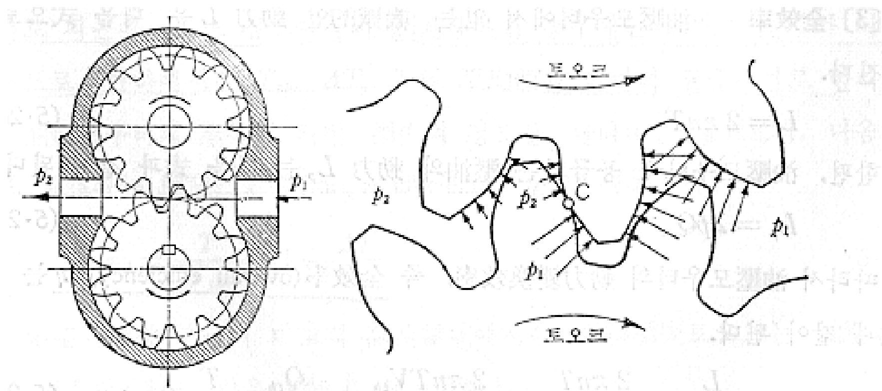


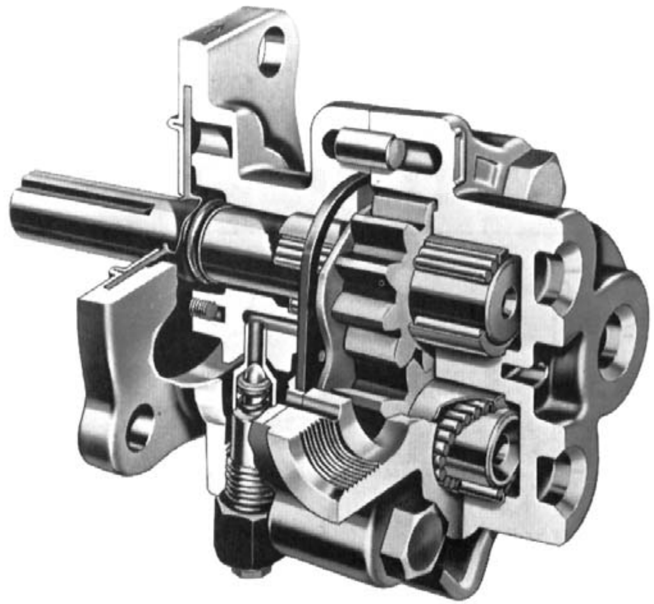
Application of Hydraulic Motors



Gear Motors


4. THESE TWO TEETH HAVE ONLY TANK LINE PRESSURE OPPOSING THEM
3. PRESSURE BETWEEN TEETH IN THIS SEGMENT PUSHES BOTH WAYS AND DOES NOT AFFECT TORQUE AS OIL IS CARRIED AROUND TO OUTLET
3. PRESSURE BETWEEN TEETH IN THIS SEGMENT PUSHES BOTH WAYS AND DOES NOT AFFECT TORQUE AS OIL IS CARRIED AROUND TO OUTLET
1. THESE TWO TEETH ARE SUBJECT TO HIGH PRESSURE AND TEND TO ROTATE GEARS IN DIRECTION OF ARROWS
2. SEGMENTS OF TWO MESHING TEETH TEND TO OPPOSE ROTATION MAKING NET TORQUE AVAILABLE A FUNCTION OF ONE TOOTH.


p_2
p_1
토오크
p_2
p_2
C
p_1
p_1
토오크

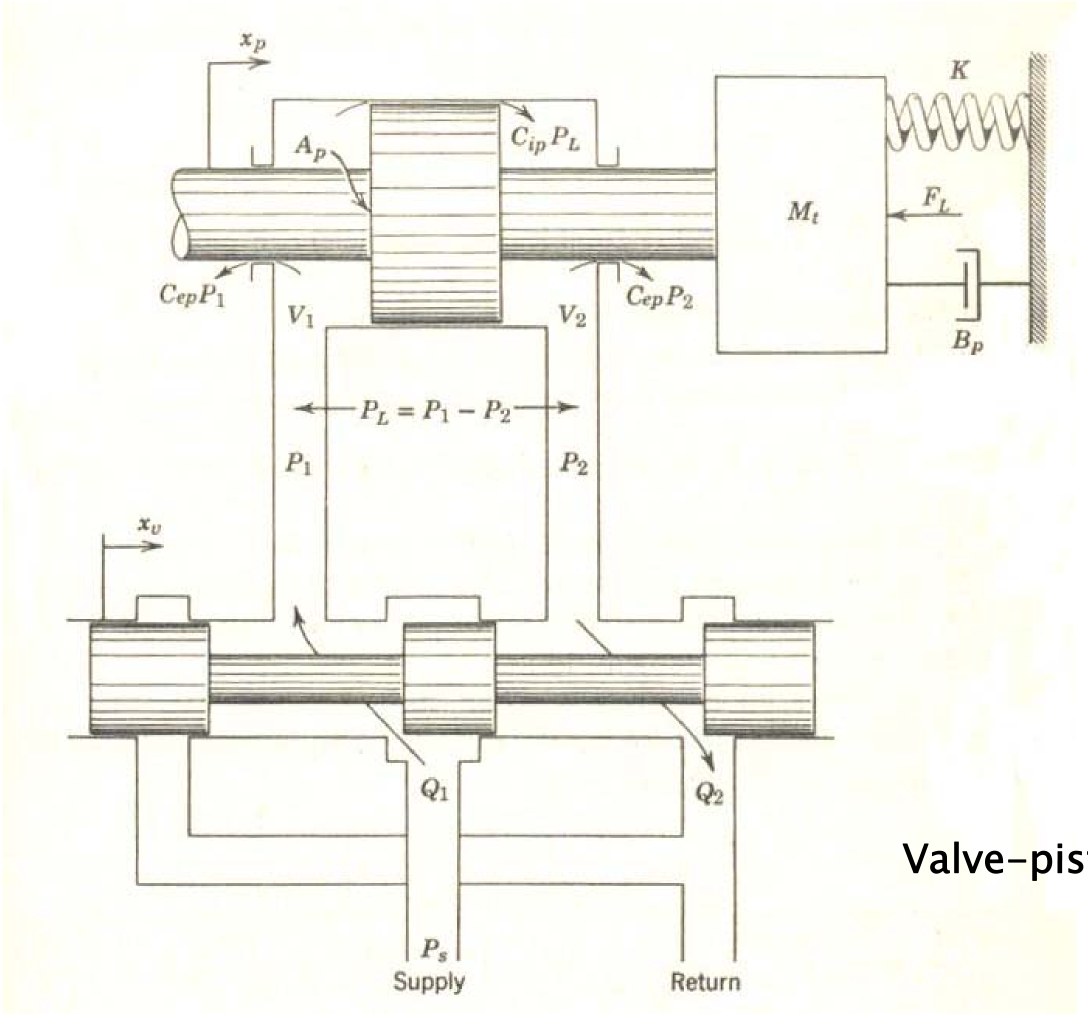


Gear Motors

- 구조는 기어 펌프와 거의 동일
- 탱크로 연결되는 드레인(drain) 라인이 있다.

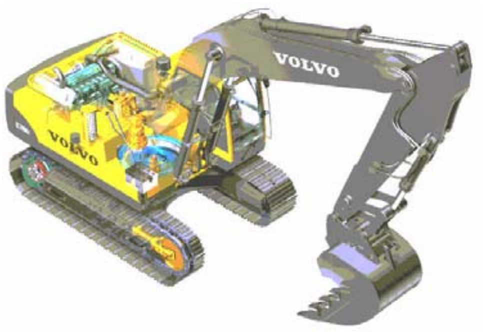
- 장점
  - 구조 간단
  - 값이 싸다
  - 유압유 중의 이물질에 의한 고장이 생기기 어렵다.
  - 가혹한 운전 조건에 비교적 잘 견딜 수 있다.
- 단점
  - 누설 유량이 많다.
  - 토크 변동이 크다.
  - 베어링 하중이 크므로 수명이 좀 짧다.

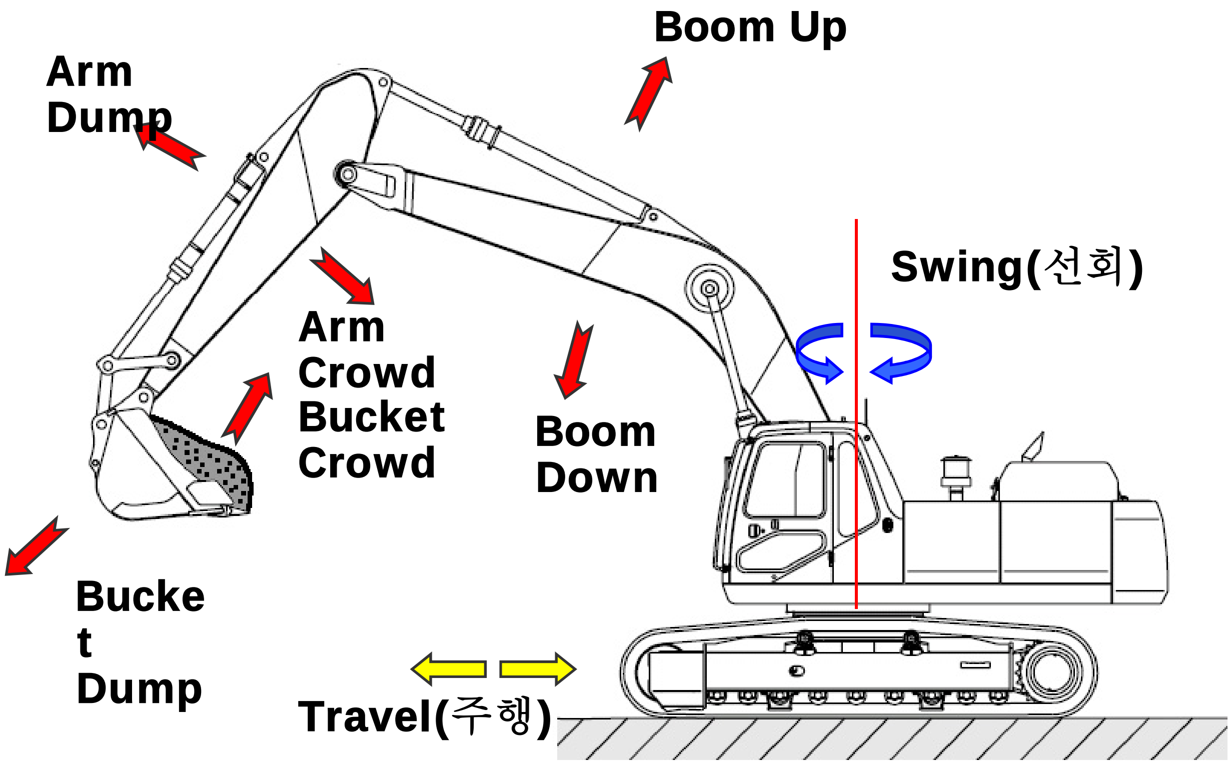
Hydraulic Systems : Valve-piston Combination



Valve-piston combination

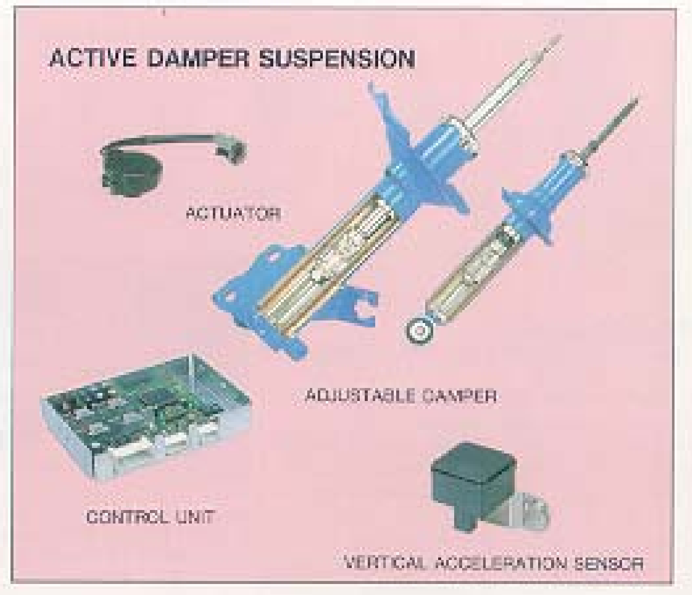
Hydraulic Excavator

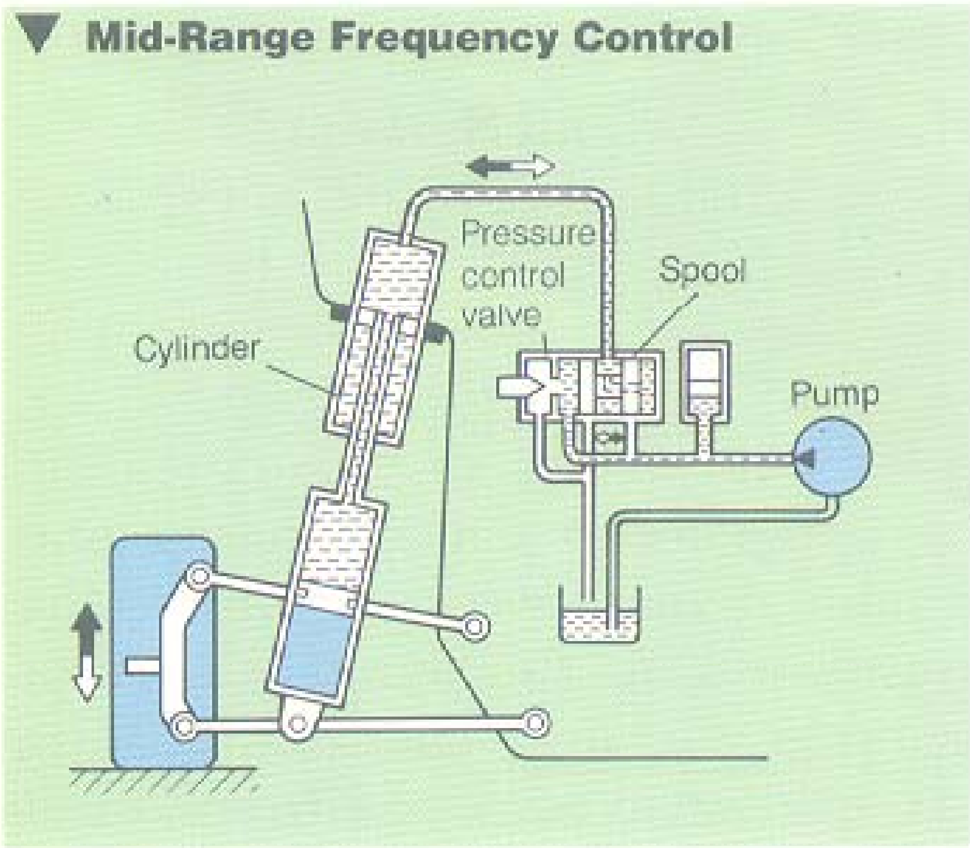




Hydraulic Excavator

Automotive Application



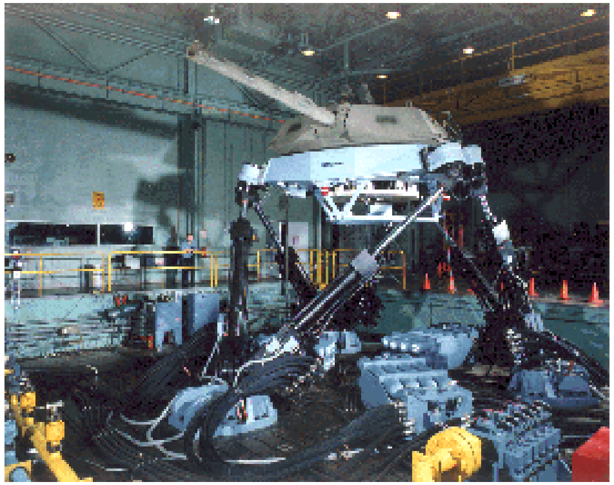


Active Suspension

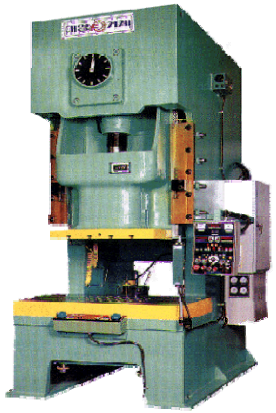
선형 액츄에이터(Linear Actuators)의 응용

지게차

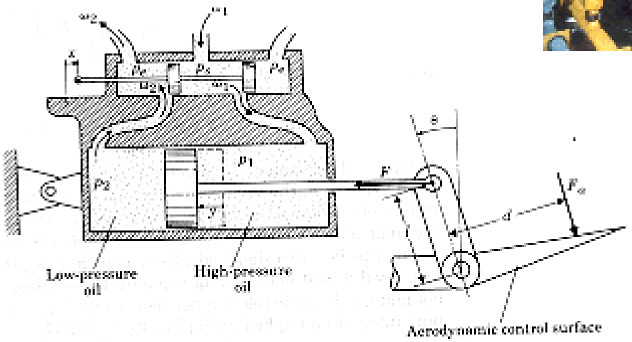
Motion Simulator



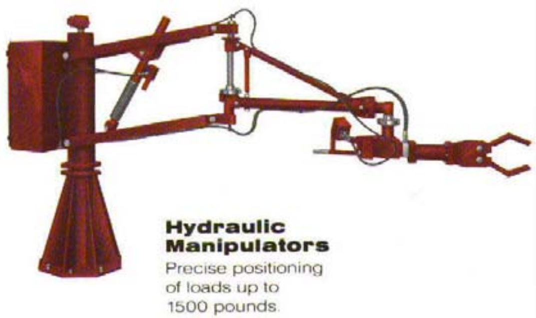
Press



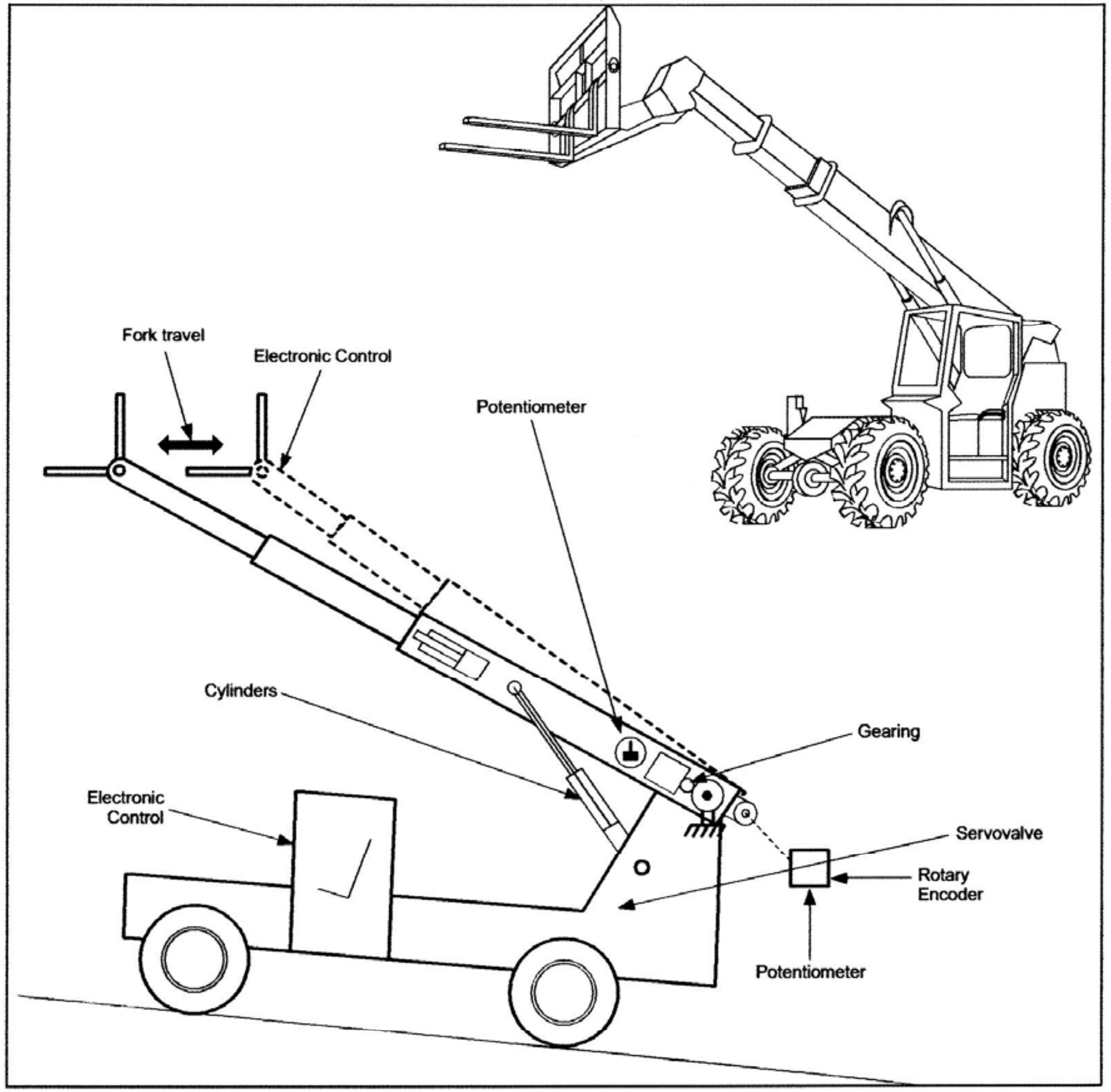
비행기 조종익



Robot



Rough terrain forklift driven by hydraulic cylinders



유압 실린더의 종류

- 단동형(Single acting)
  복동형(Double acting)

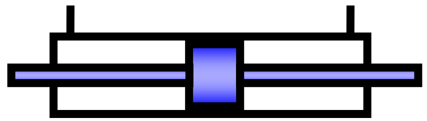
❖ 플런저 램형 (단동식)

❖ 피스톤형 (단동식)

- 단로드형(Single rod)
  양로드형(Double rod)

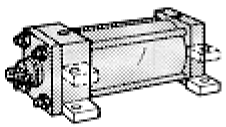
❖ 단로드형 (복동식)

❖ 양로드형 (복동식)

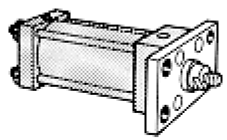


•속도는 일반적으로 10m/min 이내, 5m/min 이상이면 쿠션장치 필요

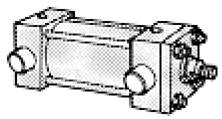
- 축심고정형/ 축심회전형
- foot type
- flange type
- trunnion type
- clevis type



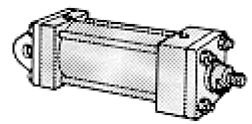
FOOT AND CENTERLINE LUG MOUNTS



RECTANGULAR FLANGE MOUNT

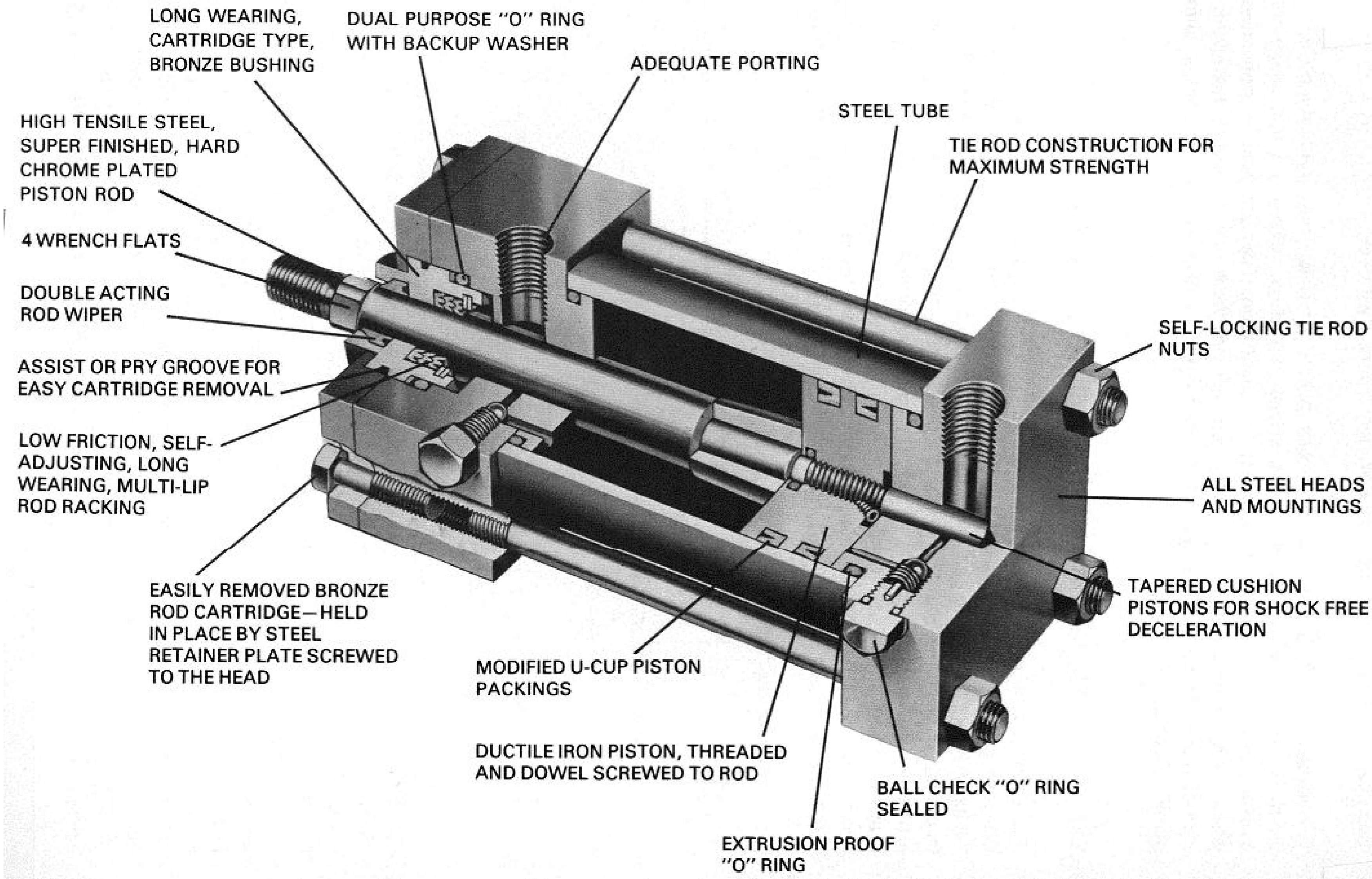


TRUNNION MOUNT

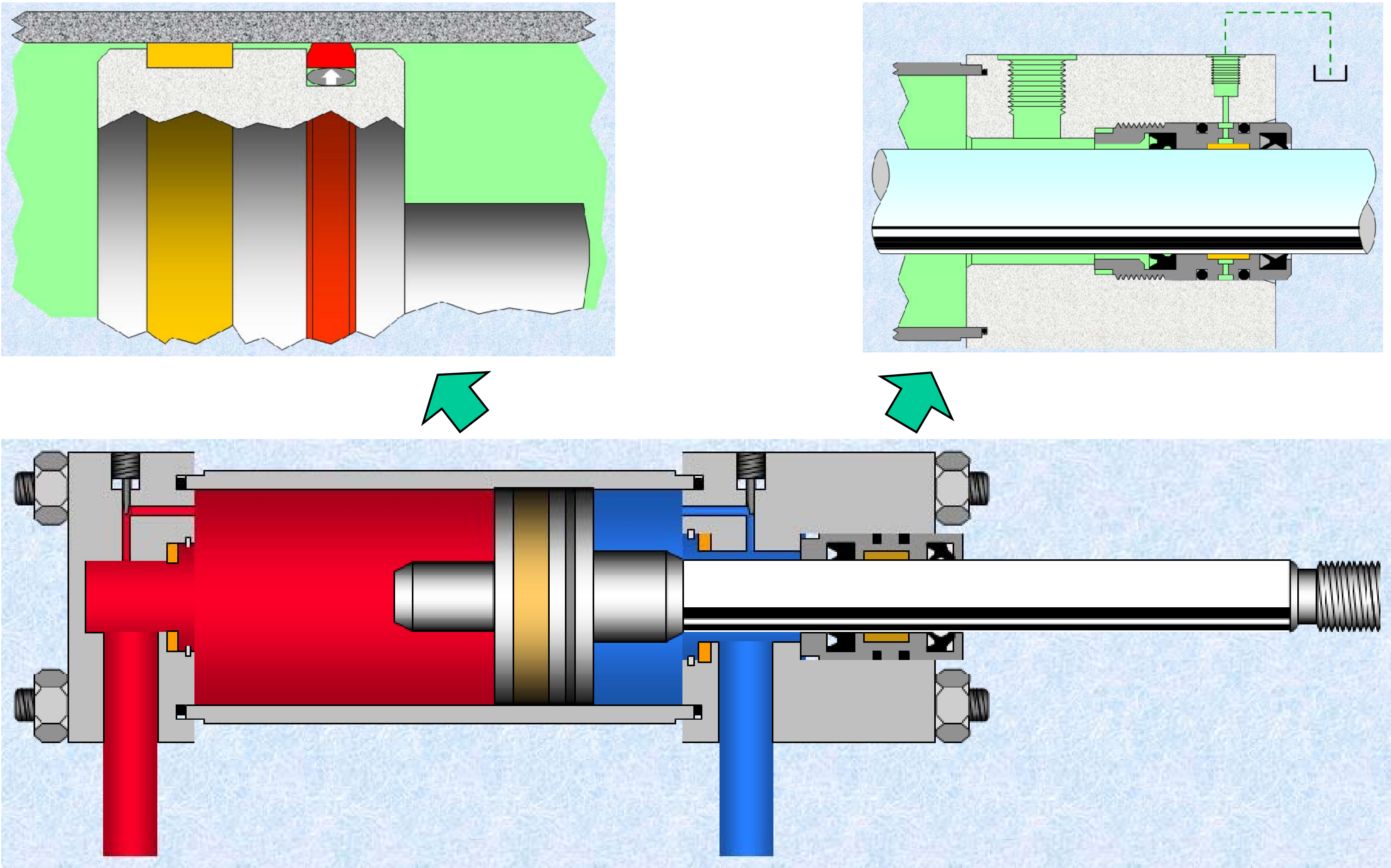


CLEVIS MOUNT

Double-acting Cylinder Design

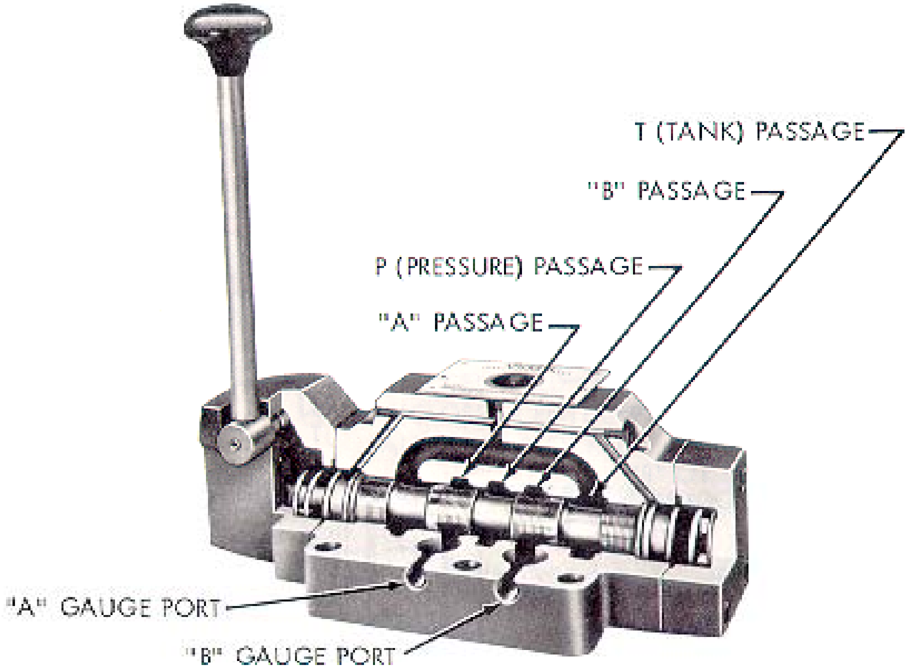


Cylinder Construction

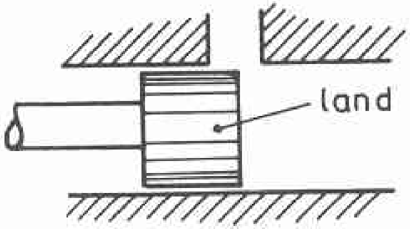


Hydraulic Valves

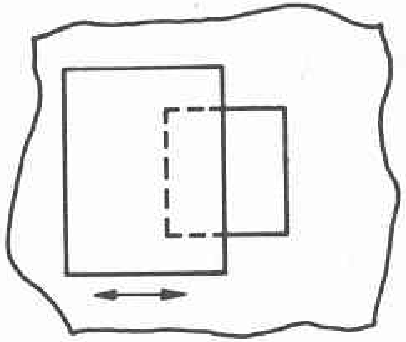
- 밸브의 정의
  - 유체동력원(fluid power source)의 흐름 방향, 유량, 압력을 제어하기 위하여 기계적인 운동(mechanical motion)을 사용하는 장치



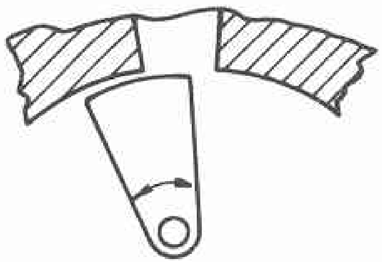
Types of Valves: shearing elements



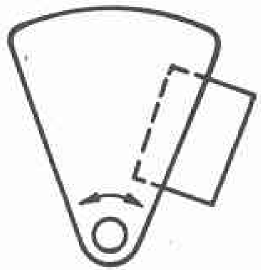
(a) Spool



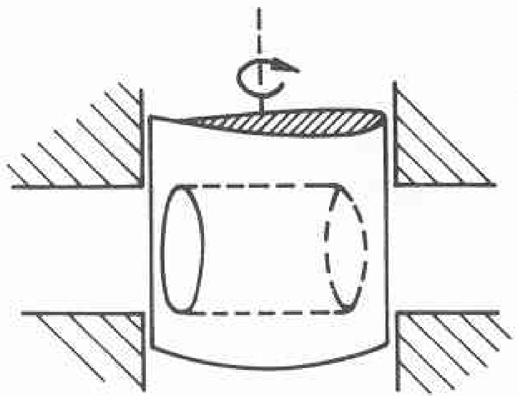
(b) Sliding Plate



(c) Rotary Spool

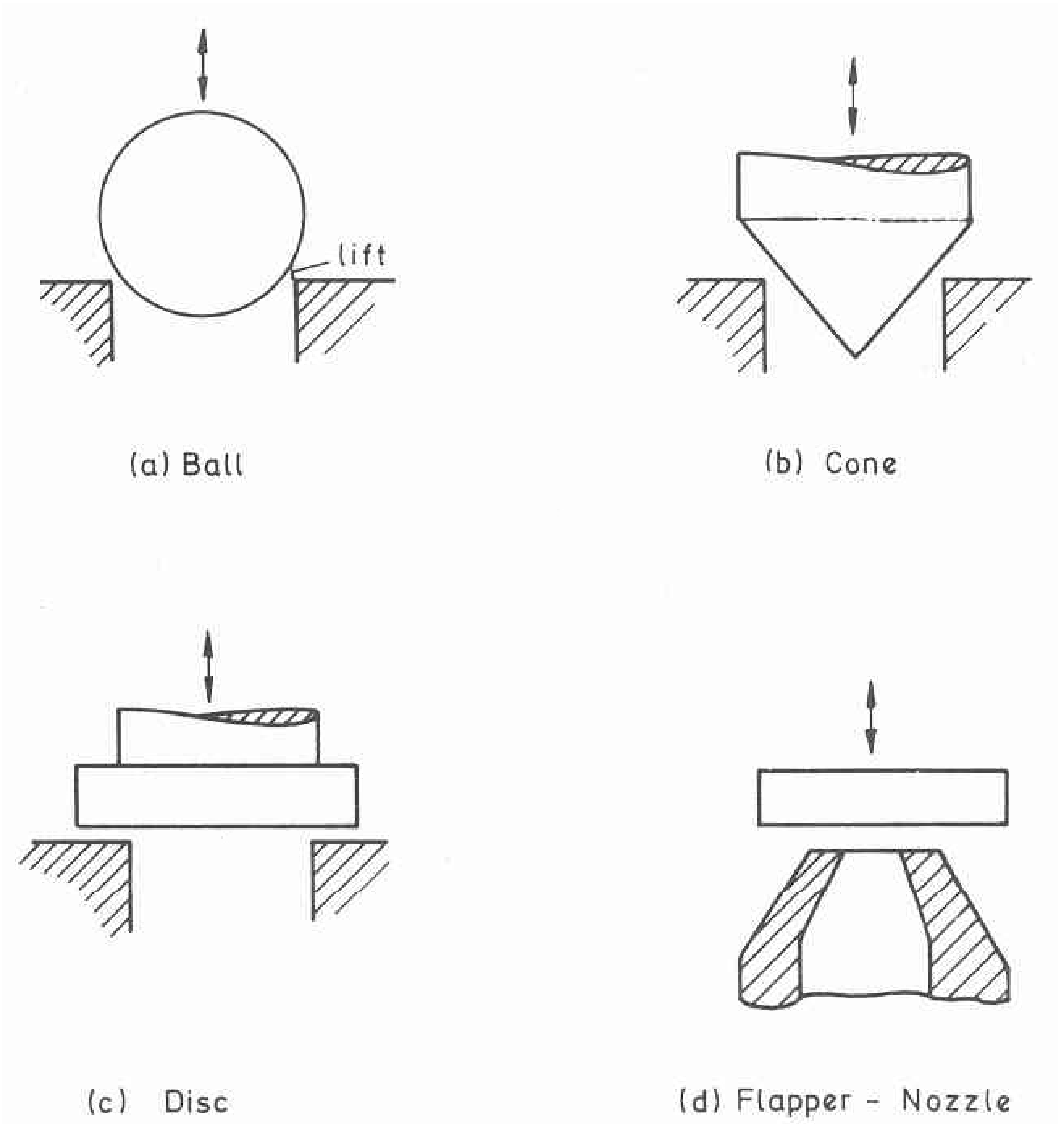


(d) Rotary Plate



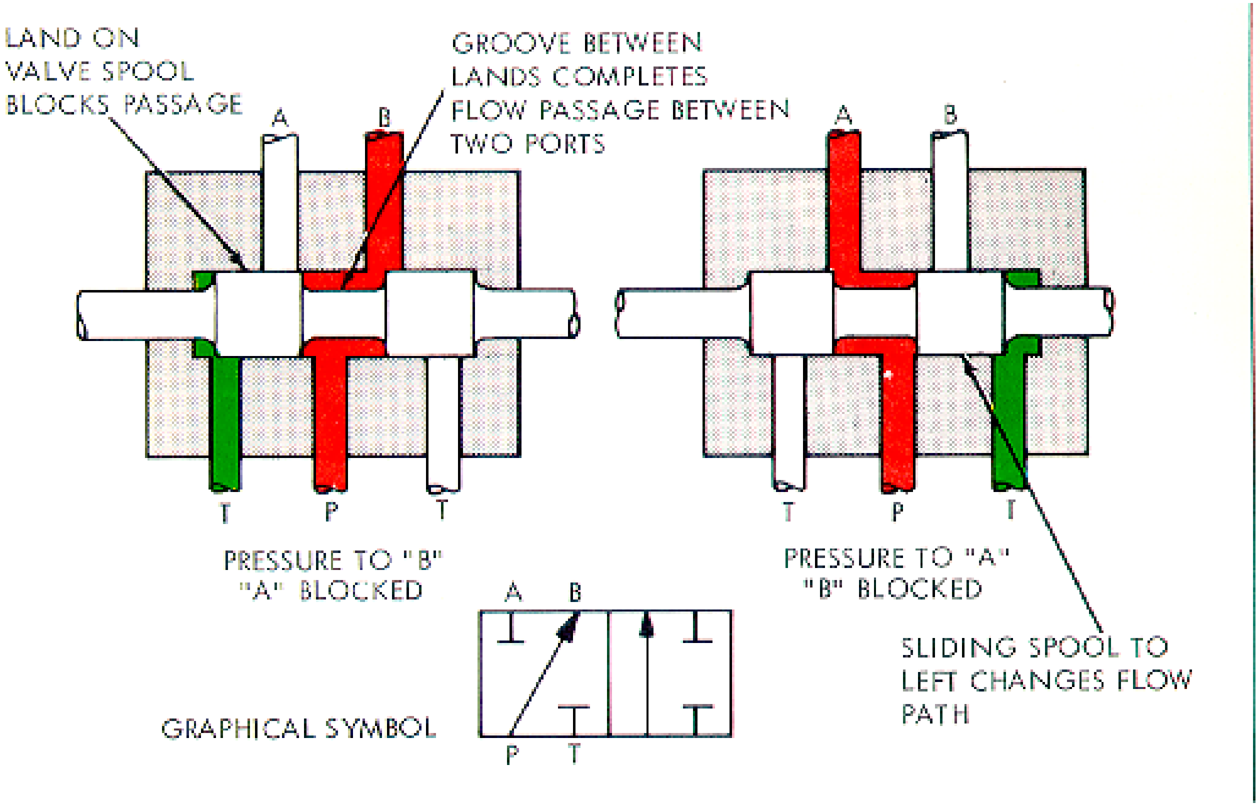
(e) Rotary Plug

Types of Valves: seating elements

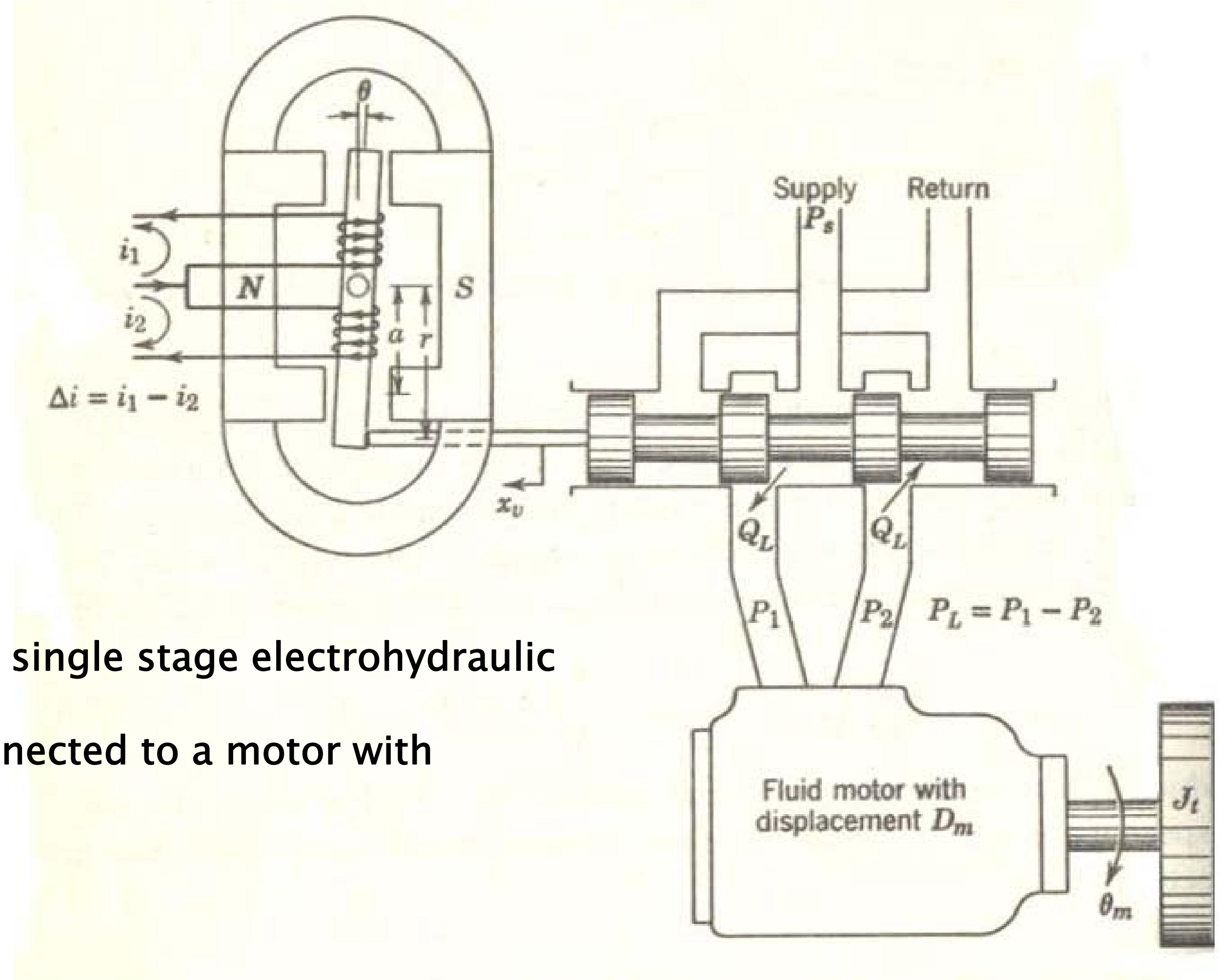


Directional Control Valve

- 방향제어 밸브 (Directional control valve)
  - 회로 내에서 작동신호에 따라 유체의 경로 결정

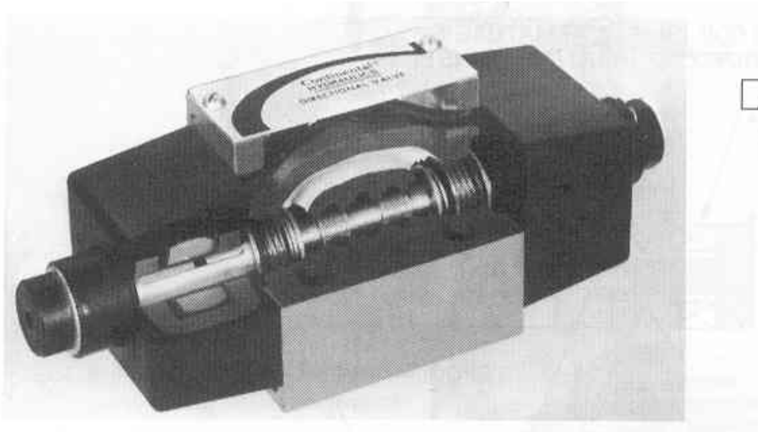


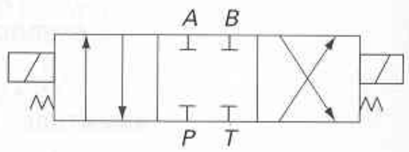
Hydraulic Systems : Single Stage Electrohydraulic Servovalve



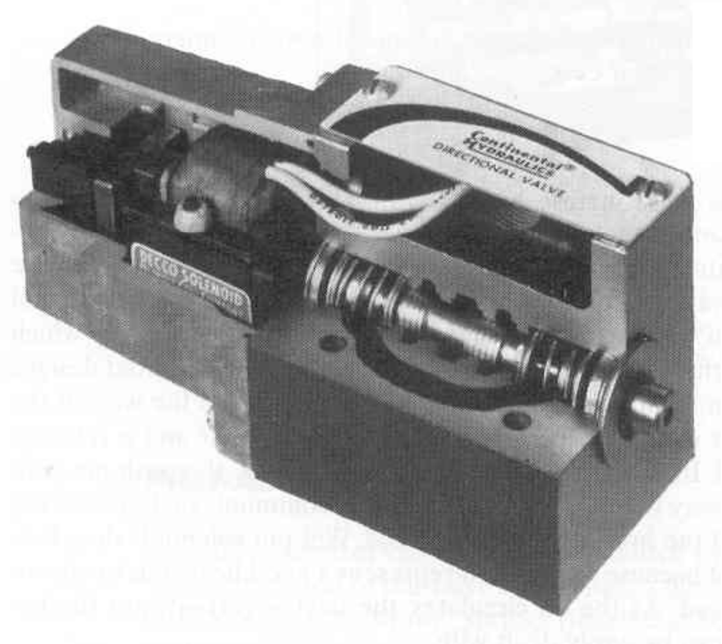
Schematic of a single stage electrohydraulic servovalve connected to a motor with inertia load

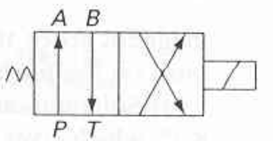
Solenoid-Actuated Valves





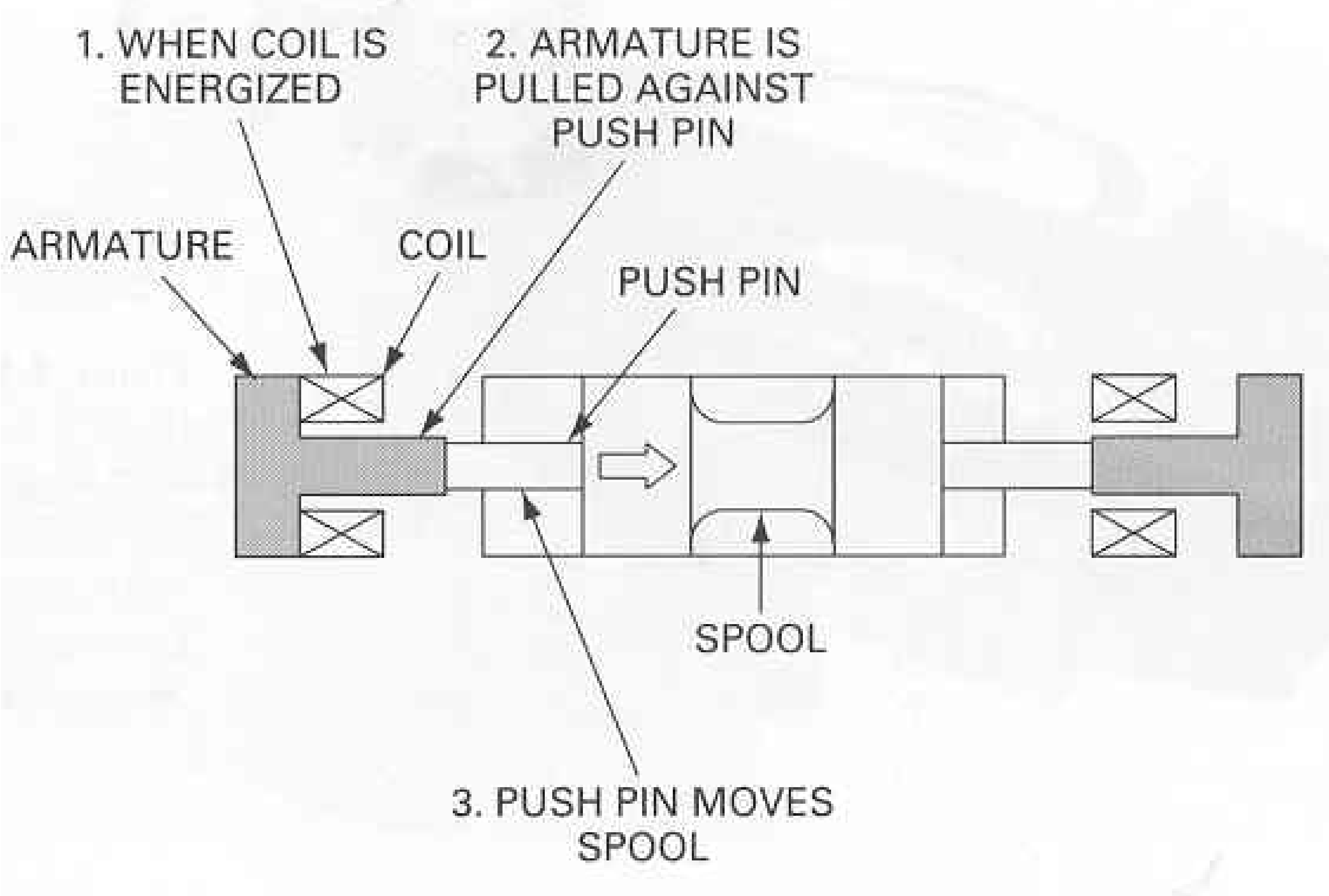
❖ Solenoid-actuated, three-position, spring-centered, four-way, directional control valve



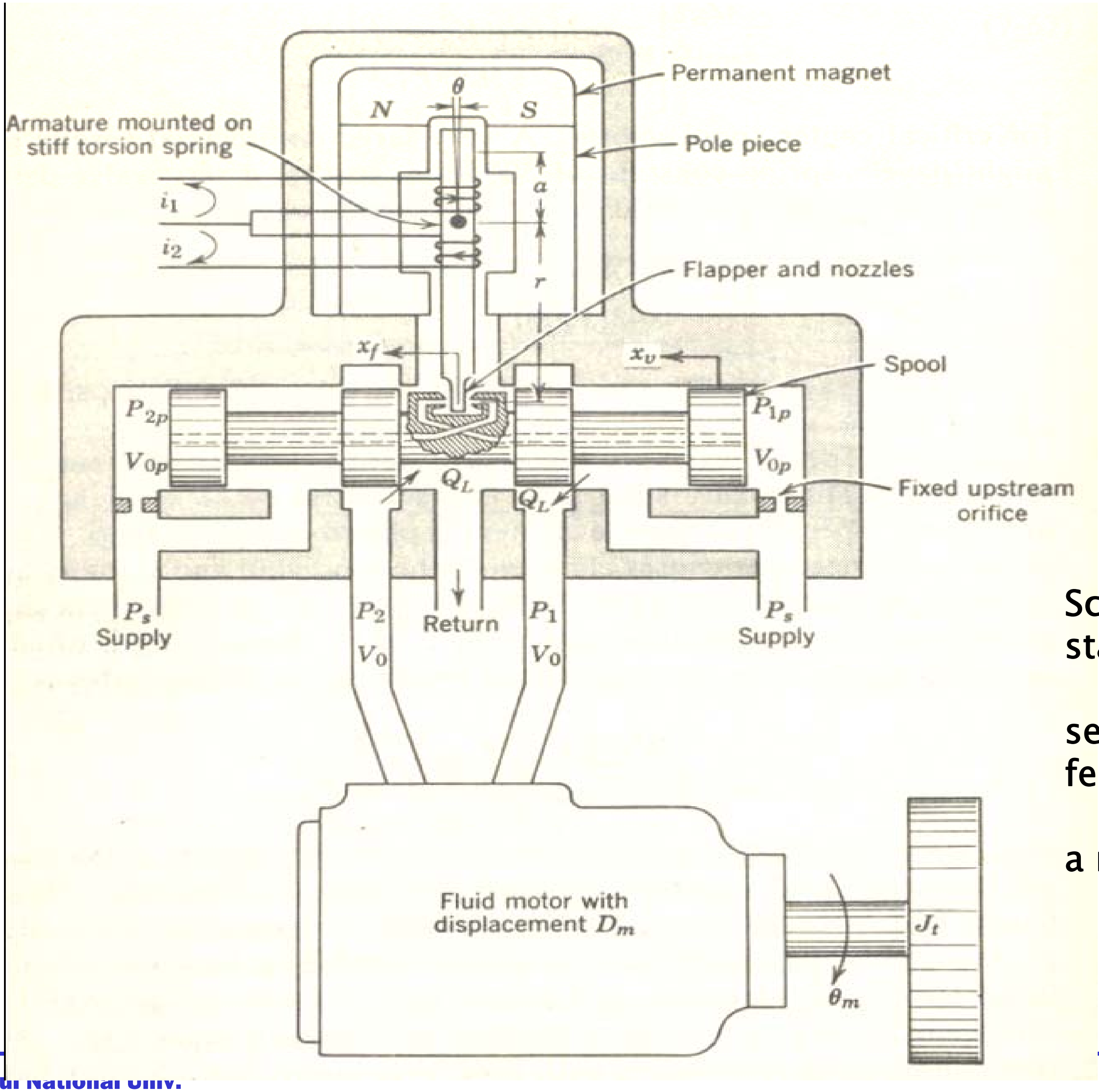


❖ Single solenoid-actuated, two-position, spring-offset, four-way, directional control valve

Operation of Solenoid to Shift of Valve



Hydraulic Systems : Two-stage Electrohydraulic Servovalve

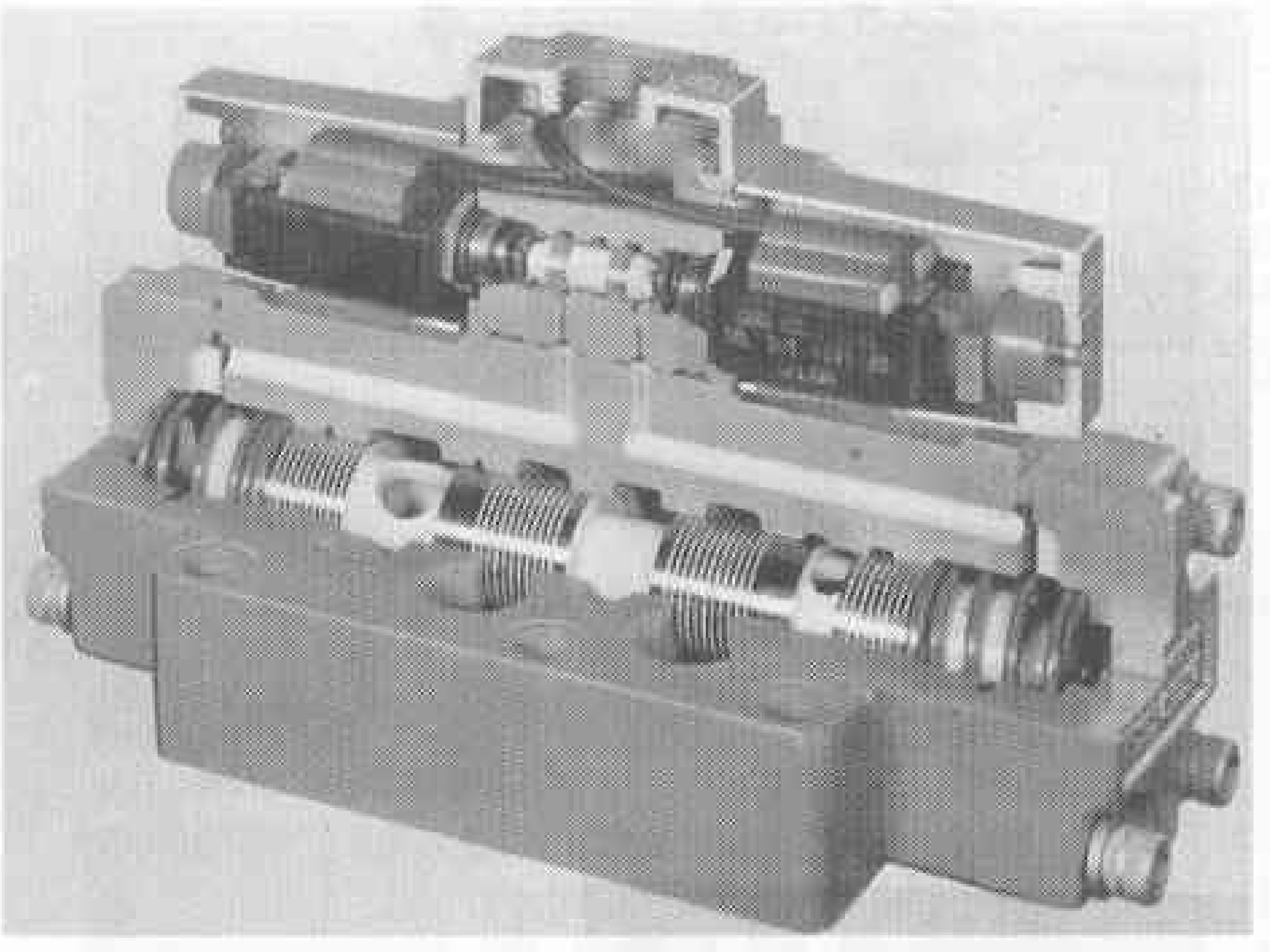


Schematic of a two-stage electrohydraulic

servovalve with force feedback controlling

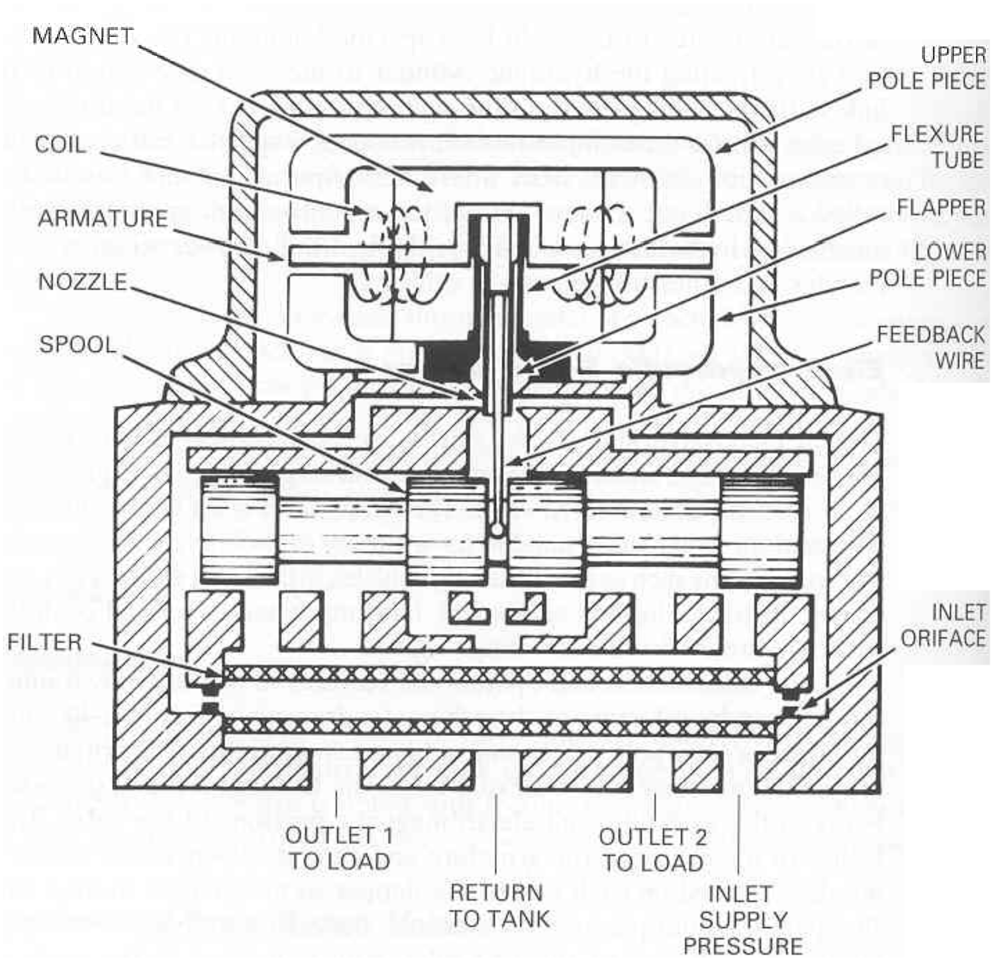
a motor with inertia load

Solenoid-controlled, Pilot-operated Valve

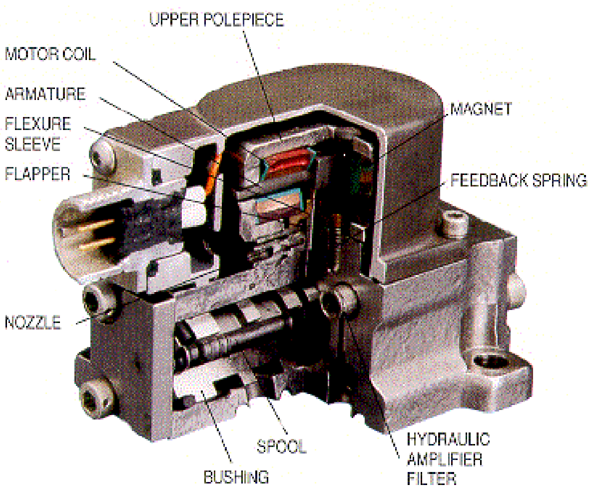


Servo Valve Structure

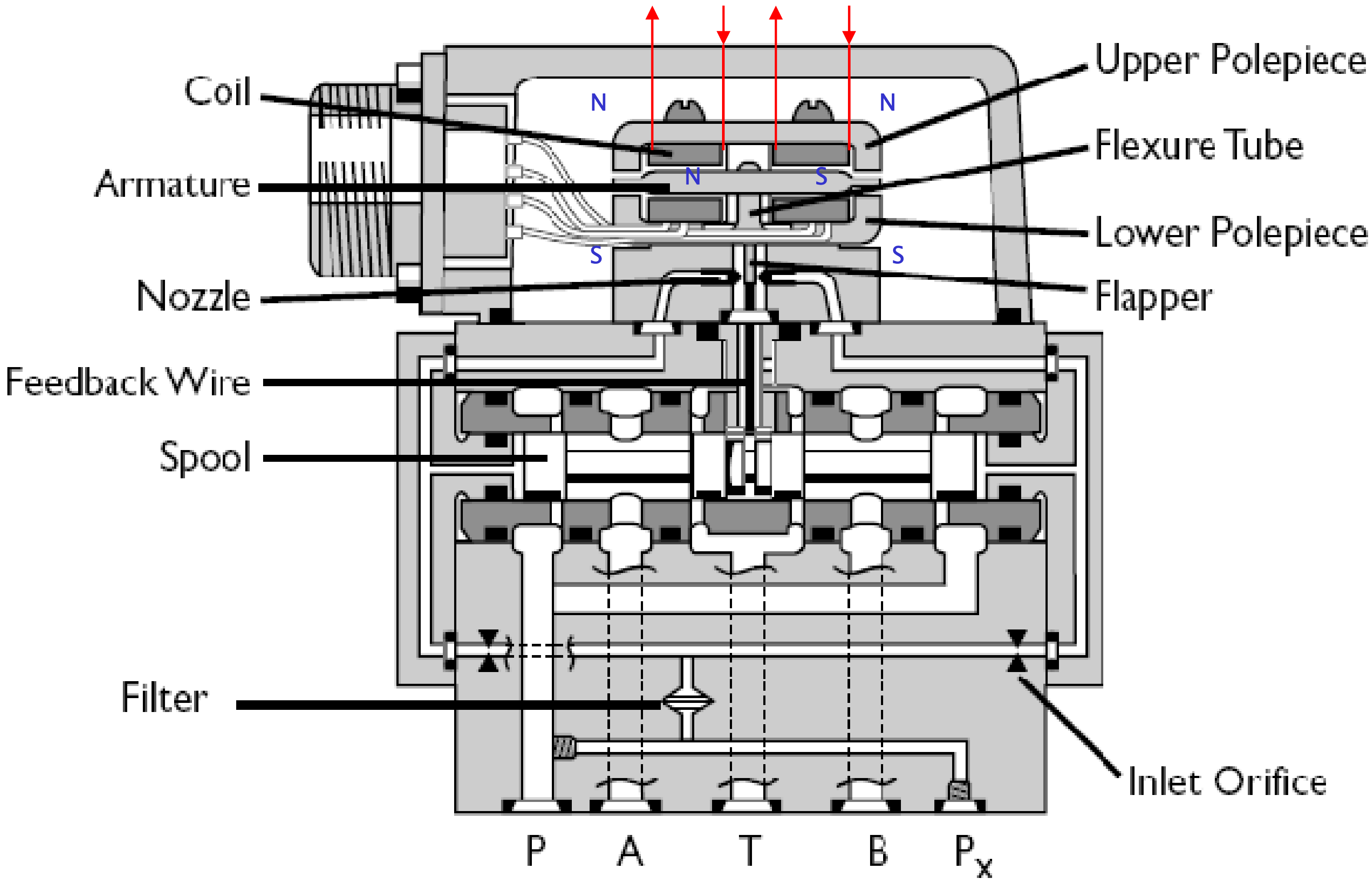
❖Moog 760 Series



❖Moog 30 Series

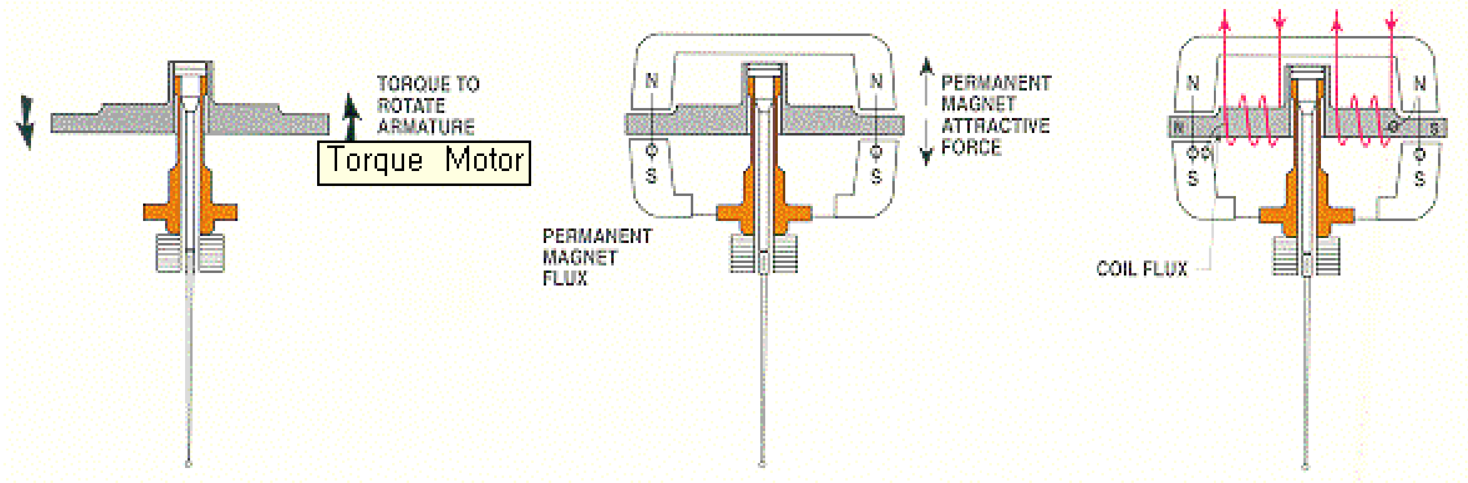


Operation of Servo Valve

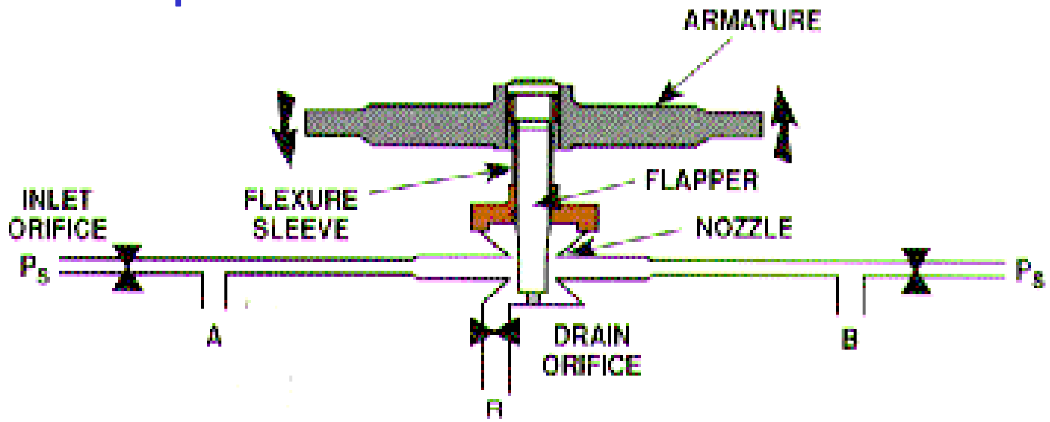


Operation of Servo Valve: Torque Motor

❖Torque Motor

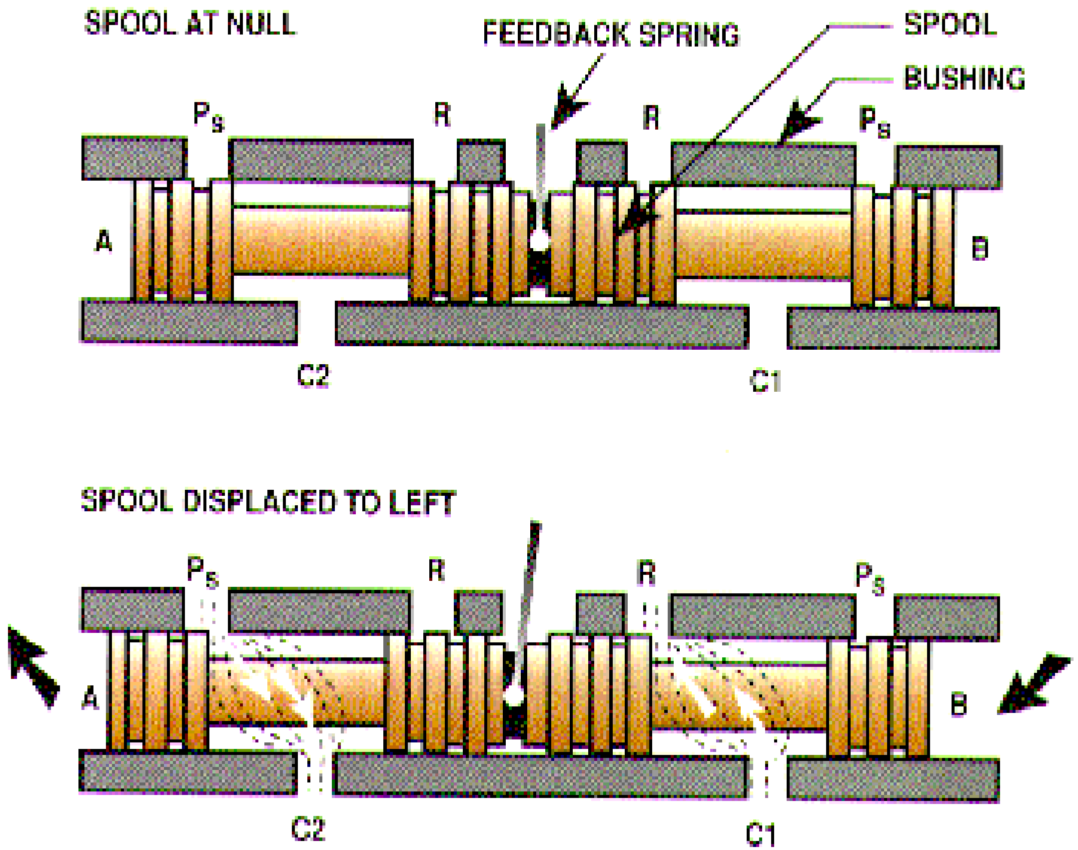


❖Hydraulic Amplifier



Operation of Servo Valve: Valve Spool

❖Valve Spool



Valve Spool Schematic

Basic Modeling of Dynamic Cylinder

- **Generalized Flow - Continuity equation**

$$Q_1 - 0 = \frac{dV_1}{dt} + \frac{V_1}{\beta_e}\frac{dP_1}{dt}$$

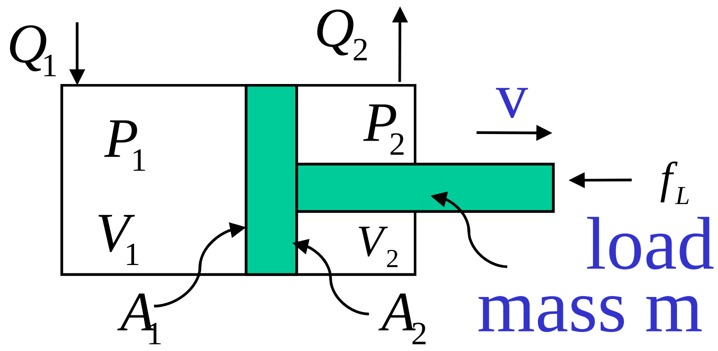
$$0 - Q_2 = \frac{dV_2}{dt} + \frac{V_2}{\beta_e}\frac{dP_2}{dt}$$

+ 부호를 그림과 같이 설정하고, 위의 식을 단순화함.

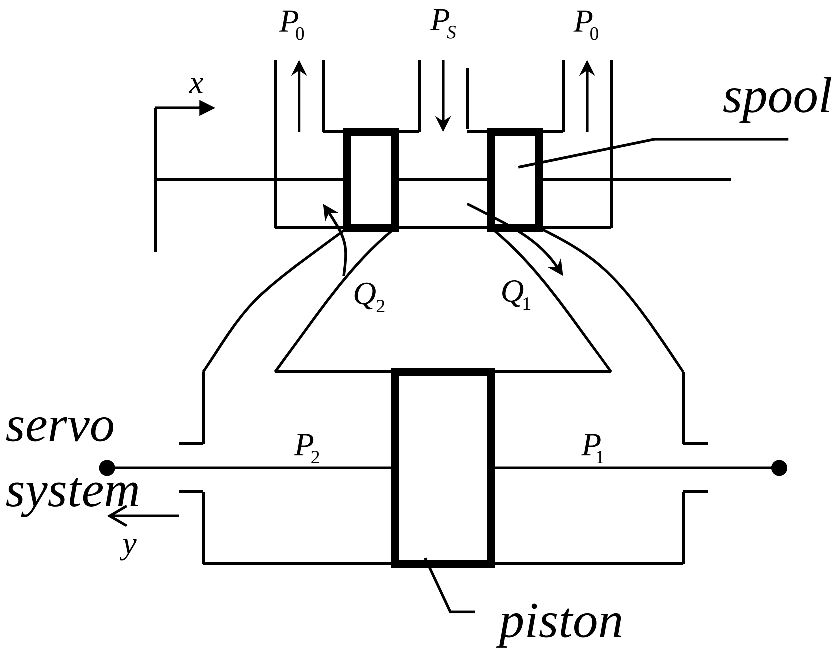
$$Q_1 = A_1 u + \frac{V_1}{\beta_e}\frac{dP_1}{dt} \qquad -Q_2 = -A_2 u + \frac{V_2}{\beta_e}\frac{dP_2}{dt}$$

- **Equation of motion**

$$P_1 A_1 - P_2 A_2 = m\frac{dv}{dt} + bv + f_L$$



Hydraulic Servo System



P_S : supply pressure

$$Q_1 = C_d \cdot a \cdot x \sqrt{\frac{2}{\rho}(P_S - P_1)} \qquad \left[m^2/s\right]$$

a : area gradient, x : displacement

ρ : density, C_d : discharge coefficient

$$Q_2 = C_d \cdot a \cdot x \sqrt{\frac{2}{\rho}(P_2 - P_0)}$$

$$= C_d \cdot a \cdot x \sqrt{\frac{2}{\rho} P_2} \qquad (P_0 \approx 0)$$

Hydraulic Servo System

no leakage, no compressibility

$$Q_1 = Q_2 \quad \rightarrow \quad P_s - P_1 = P_2 \quad \rightarrow \quad P_s = P_1 + P_2$$

$$P_L = \Delta P = P_1 - P_2 \quad \rightarrow \quad P_S + P_L = 2P_1, \quad P_S - P_L = 2P_2$$

$$\rightarrow \quad P_1 = \frac{P_S + P_L}{2}, \quad P_2 = \frac{P_S - P_L}{2}$$

$$Q = Q_1 = Q_2 = C_d \cdot a \cdot x \sqrt{\frac{2}{\rho} \frac{P_S - P_L}{2}} = C \cdot x \sqrt{P_S - P_L}$$

$$Q = A_P \cdot \frac{dy}{dt} = C \cdot x \sqrt{P_S - P_L}$$

$$\frac{dy}{dt} = C \cdot x \sqrt{P_S - P_L}$$

Hydraulic Servo System

$$\frac{d\bar{y}}{dt} = C \cdot \bar{x}\sqrt{P_S - \overline{P_L}}, \qquad y = \bar{y} + \Delta y, \qquad x = \bar{x} + \Delta x, \qquad P_L = \overline{P_L} + \Delta P_L$$

$$\frac{dy}{dt} = f(\bar{x}, \overline{P_L}) + \frac{\partial f}{\partial x}\Big|_{\bar{x}\ \overline{P_L}} \cdot (x - \bar{x}) + \frac{\partial f}{\partial P_L}\Big|_{\bar{x}\ \overline{P_L}} \cdot (P_L - \overline{P_L})$$

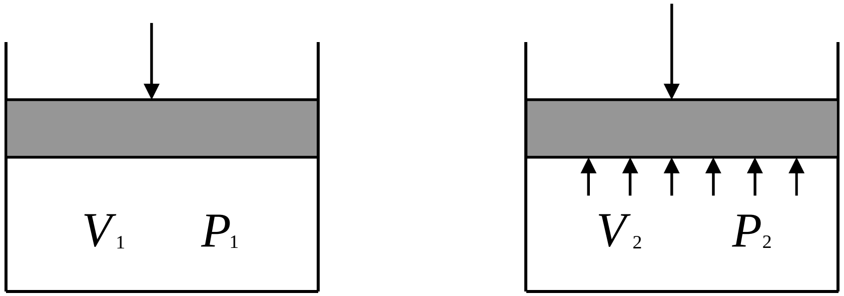
$$= \frac{d\bar{y}}{dt} + C\sqrt{P_S - \overline{P_L}} \cdot (x - \bar{x}) + \left(-\frac{1}{2}C\bar{x}\frac{1}{\sqrt{P_S - \overline{P_L}}}\right) \cdot (P_L - \overline{P_L})$$

$$if \quad \bar{x} = 0, \qquad \overline{P_L} = 0, \qquad \frac{d\bar{y}}{dt} = 0$$

$$\frac{dy}{dt} = C\sqrt{P_S} \cdot x = K_1 \cdot x$$

$$\therefore\ T.F = \frac{Y(s)}{X(s)} = \frac{K_1}{S}$$

Hydraulic Servo System : Compressibility



$$\frac{\Delta V}{V_1} = \beta \, \Delta P$$

$where, \quad \beta = Compressibility$

$$PV = mRT$$

$$\frac{1}{\beta} = \frac{\Delta P}{\Delta V / V}$$

$$\Delta P = dP \ , \qquad \Delta V = V_1 - V_2 = -(V_2 - V_1) = -dV$$

$$\frac{1}{\beta} = -V \frac{dP}{dV} = K_B \qquad ; \ Bulk \ \ modulus$$

$$dP = -\frac{1}{\beta} \cdot \frac{1}{V} dV = -K_B \frac{1}{V} dV$$

$$\frac{dP}{dt} = -K_B \cdot \frac{1}{V} \cdot \frac{dV}{dt}$$

Bulk Modulus

체적탄성계수

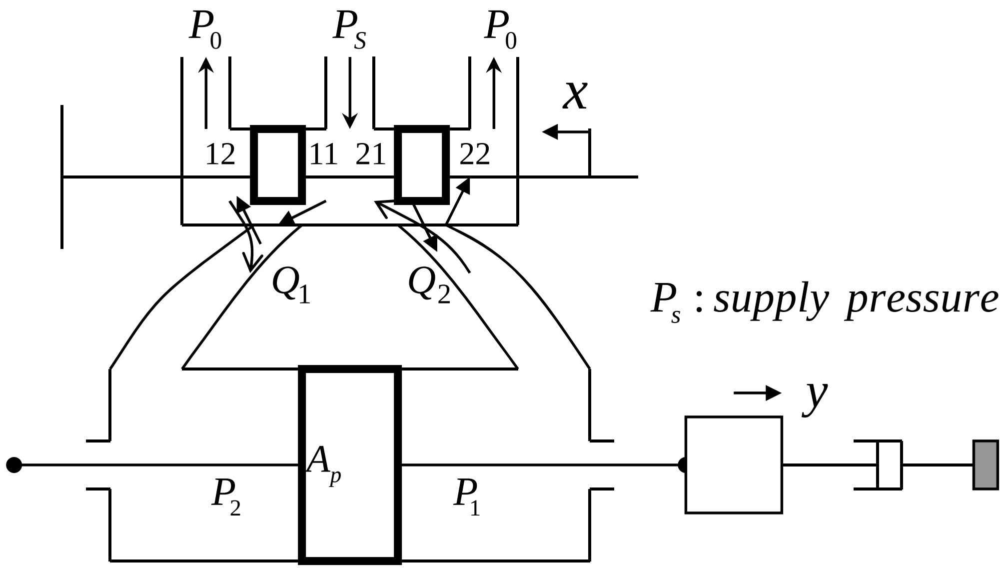
- 비 압축성의 척도
- 체적탄성계수가 클수록 유체는 더 큰 강성을 갖고 있어 압축되기가 어렵다.

$$\beta = -\frac{\Delta P}{\Delta V / V} = -\frac{\Delta P}{\Delta V} V$$

- 전형적인 유압유의 체적탄성계수: 250,000psi (1,720MPa)

종류	압축률 [cm^2/kgf]	체적탄성계수 [kgf/cm^2]	비고
석유계작동유	6.0×10^{-5} $5.2 \sim 7.2 \times 10^{-5}$	1.7×10^{4} $1.4 \sim 1.9 \times 10^{4}$	
항공작동유 (MIL H 5606A)	5.0×10^{-5}	2.0×10^{4}	압력 $40 \sim 60 kgf/cm^2$
각종 연료유	5.0×10^{-5}	2.0×10^{4}	
수-글리콜 W/O 형 에멀젼 인산에스테르	2.9×10^{-5} 4.4×10^{-5} 3.3×10^{-5}	3.5×10^{4} 2.3×10^{4} 3.0×10^{4}	$20^{o}C$, $700 kgf/cm^2$

Hydraulic Servo System



$x = 0\,, \qquad A = A_0$

Flow equations :

$$Q_{11} = C_d\,(A_0 + ax)\sqrt{\frac{2}{\rho}(P_S - P_1)}$$

$$Q_{12} = C_d\,(A_0 - ax)\sqrt{\frac{2}{\rho}(P_1 - 0)}$$

$$Q_1 = Q_{11} - Q_{12}$$

$$Q_{21} = C_d\,(A_0 - ax)\sqrt{\frac{2}{\rho}(P_S - P_2)}$$

$$Q_{22} = C_d\,(A_0 + ax)\sqrt{\frac{2}{\rho}(P_2 - 0)}$$

$$Q_2 = Q_{22} - Q_{21}$$

Hydraulic Servo System

Assume no leakage $Q_1 = Q_2$

$$\dot{y} = 0,$$

$$\frac{dP}{dt} = -\frac{1}{\beta} \cdot \frac{1}{V} \cdot \frac{dV}{dt}$$

$$= -\frac{1}{\beta}\frac{1}{V_1}(-Q_1)$$

$$\dot{y} \neq 0,$$

$$\frac{dp_1}{dt} = \frac{1}{\beta}\frac{1}{V_1}(Q_1 - A_p\dot{y}), \qquad \frac{dp_2}{dt} = \frac{1}{\beta}\frac{1}{V_2}(-Q_2 + A_P\dot{y})$$

Equation of motion : $m\ddot{y} = A_p(p_1 - p_2) - b\dot{y}$

$$m\ddot{y} + b\dot{y} = A_p(p_1 - p_2)$$

Hydraulic Servo System Model

$$m\ddot{y} + b\dot{y} = A_p(p_1 - p_2)$$

$$\frac{dp_1}{dt} = \frac{1}{\beta}\frac{1}{V_1}(Q_1 - A_p\dot{y})$$

$$\frac{dp_2}{dt} = \frac{1}{\beta}\frac{1}{V_2}(-Q_2 + A_P\dot{y})$$

$$Q_1 = Q_{11} - Q_{12} = \left\{C_d\,(A_0 + ax)\sqrt{\frac{2}{\rho}(P_S - P_1)}\right\} - \left\{C_d\,(A_0 - ax)\sqrt{\frac{2}{\rho}(P_1 - 0)}\right\}$$

$$Q_2 = Q_{22} - Q_{21} = \left\{C_d\,(A_0 + ax)\sqrt{\frac{2}{\rho}(P_2 - 0)}\right\} - \left\{C_d\,(A_0 - ax)\sqrt{\frac{2}{\rho}(P_S - P_2)}\right\}$$

$$Q_1 = Q_2$$

Hydraulic Servo System : Linearization

$$Q_1 = Q_2 \qquad \Rightarrow \qquad Q_1 - Q_2 = 0$$

$$C_d(A_0 + ax)\left\{\sqrt{\frac{2}{\rho}(P_S - P_1)} - \sqrt{\frac{2}{\rho}P_2}\right\} - C_d(A_0 - ax)\left\{\sqrt{\frac{2}{\rho}P_1} - \sqrt{\frac{2}{\rho}(P_S - P_2)}\right\}$$

$$when \quad x = 0,$$

$$C_d ax\left\{\sqrt{\frac{2}{\rho}(P_S - P_1)} - \sqrt{\frac{2}{\rho}P_2} + \sqrt{\frac{2}{\rho}P_1} - \sqrt{\frac{2}{\rho}(P_S - P_2)}\right\}$$

$$+ C_d A_0\left\{\sqrt{\frac{2}{\rho}(P_S - P_1)} - \sqrt{\frac{2}{\rho}P_2} - \sqrt{\frac{2}{\rho}P_1} + \sqrt{\frac{2}{\rho}(P_S - P_2)}\right\} = 0$$

To make an identical equation, $P_s - P_1 = P_2, \quad P_1 = P_s - P_2 \quad \Rightarrow \quad P_s = P_1 + P_2$

let $P_L = P_1 + P_2, \qquad \Rightarrow \qquad P_1 = \dfrac{P_s + P_L}{2}, \qquad P_2 = \dfrac{P_s - P_L}{2}$

Hydraulic Servo System : Linearization

$$Q_1 = C_d(ax + A_0)\sqrt{\frac{1}{\rho}(P_s - P_L)} - C_d(A_0 - ax)\sqrt{\frac{1}{\rho}(P_s + P_L)}$$

$$Q_L = Q_L(x, P_L)$$

Operating point : $x = 0, \quad p_L = 0$

$$Q_1(x, p_L) = Q_1(0,0) + \frac{\partial Q_1}{\partial x}\Big|_{x=0,\, p_L=0}(x-0) + \frac{\partial Q_1}{\partial p_L}\Big|_{x=0,\, p_L=0}(p_L - 0) + \cdots$$

$$\frac{\partial Q_1}{\partial x}\Big|_{x=0,\, p_L=0} = 2C_d \cdot a\sqrt{\frac{1}{\rho}p_S} = K_1$$

$$\frac{\partial Q_1}{\partial p_L}\Big|_{x=0,\, p_L=0} = -C_d \cdot A_0 \frac{1}{\sqrt{\rho \cdot p_S}} = -K_2$$

$$Q = K_1 x - K_2 p_L = Q_2 = Q_L$$

$$p_L = p_1 - p_2, \qquad \frac{dp_L}{dt} = \frac{dp_1}{dt} - \frac{dp_2}{dt}$$

Hydraulic Servo System

$$m\ddot{y} + b\dot{y} = A_p p_L$$

$$\frac{dp_1}{dt} = \frac{1}{\beta}\frac{1}{V_1}(Q_L - A_p\dot{y}) = \frac{1}{\beta}\frac{1}{V_1}(K_1 x - K_2 p_L - A_p\dot{y})$$

$$\frac{dp_2}{dt} = \frac{1}{\beta}\frac{1}{V_2}\left\{-(K_1 x - K_2 p_L) + A_p\dot{y}\right\}$$

$$let \;\; V_1 = V_2$$

$$\frac{dp_L}{dt} = \frac{1}{\beta}\frac{1}{V}(2K_1 x - 2K_2 p_L - 2A_p\dot{y})$$

$$\therefore \frac{Y(s)}{X(s)} = \frac{(\;\;)}{(\;\;)} \qquad (\;\;) : cubic\ equation\ \ form$$

Hydraulic Servo System

Simplification : No compressibility, No leakage

$$m\ddot{y} + b\dot{y} = p_L A_p$$

$$Q_L = K_1 x - K_2 p_L = A_p \dot{y} \qquad \Rightarrow \qquad p_L = \frac{1}{K_2}(K_1 x - A_p \dot{y})$$

$$\Rightarrow \quad m\ddot{y} + \left(b + \frac{A_p{}^2}{K_2} \right) \dot{y} = A_p \frac{K_1}{K_2} x$$

$$\therefore \frac{Y(s)}{X(s)} = \frac{K}{s(Ts+1)}$$

$$K = \frac{K_1 A_p}{K_2 b + A_p{}^2}, \quad T = \frac{m K_2}{K_2 b + A_p{}^2}$$

Appendix

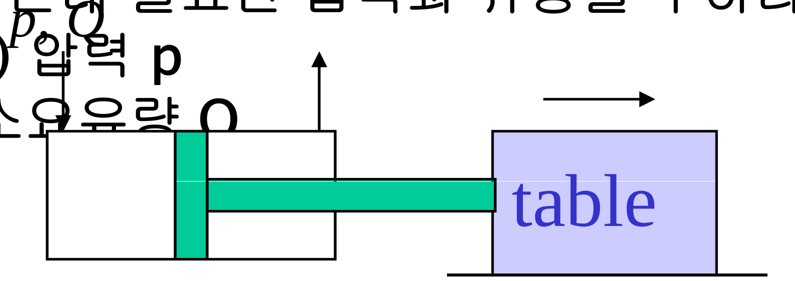
예제 1: 실린더 구동 시스템

- 단로드형 복동실린더를 이용하여 테이블을 구동하고자 한다.
  - 피스톤 직경은 **150mm**, 로드 직경은 **85mm**이다.
  - 피스톤 속도 **12m/min** , 가속시간은 **0.4s** 이다 .
  - 실린더 배압은 **5** kgf/cm^2 이다.
  - 피스톤행정은 **1000mm**이다
  - 테이블의 중량은 **1 ton**, 마찰계수는 **0.22** 이다.
  - 실린더의 압력효율은 **90%**이다.

위의 조건에서 테이블을 전진시키는데 필요한 압력과 유량을 구하라.

(1) 압력 p

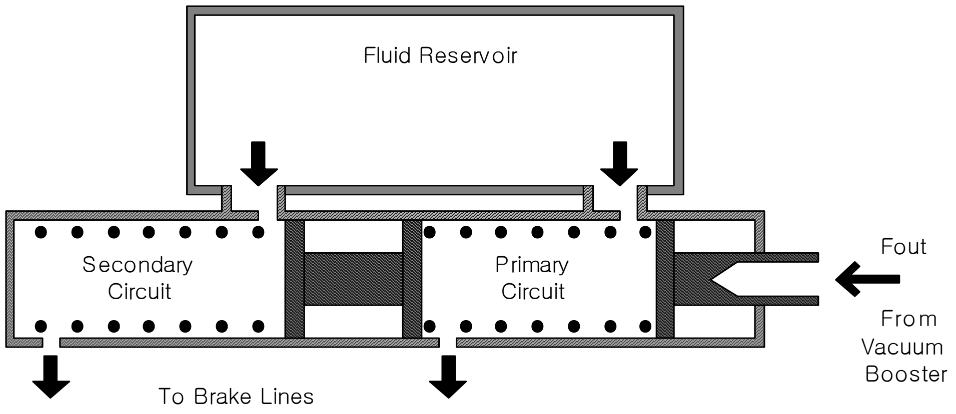
(2) 소요유량 Q



[답] (1) 5.48 kgf/cm^2

(2) 211.9 l/min

Master cylinder

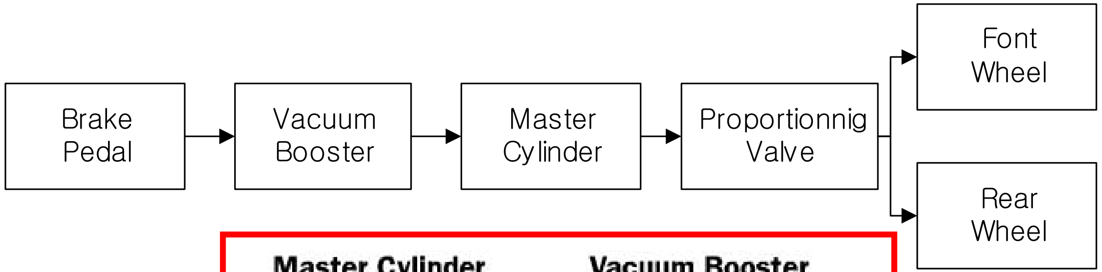


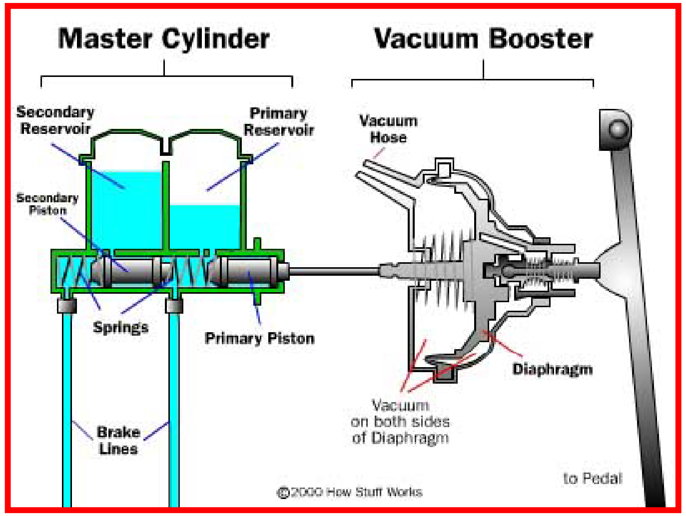
- Equation of motion of master cylinder piston :

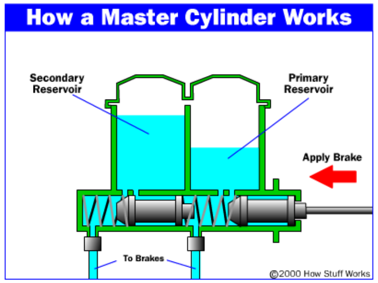
$$m_{mc}\ddot{x}_{mc} = -b_{mc}\dot{x}_{mc} - F_{cs} - A_{mc}P_{mc} + F_{out} - sign(\dot{x}_{mc})F_{loss}$$

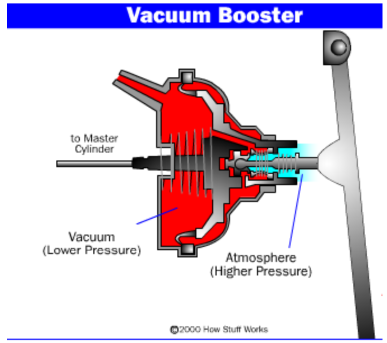
$$x_{mc} = x_{pp}$$

Brake System

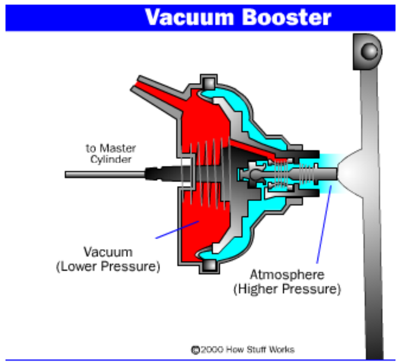






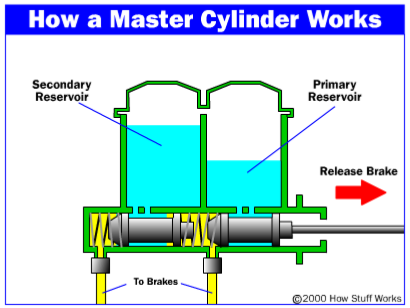


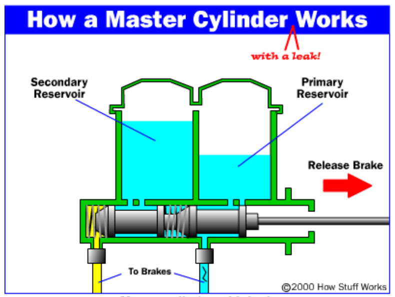
Apply brake



w/o leak

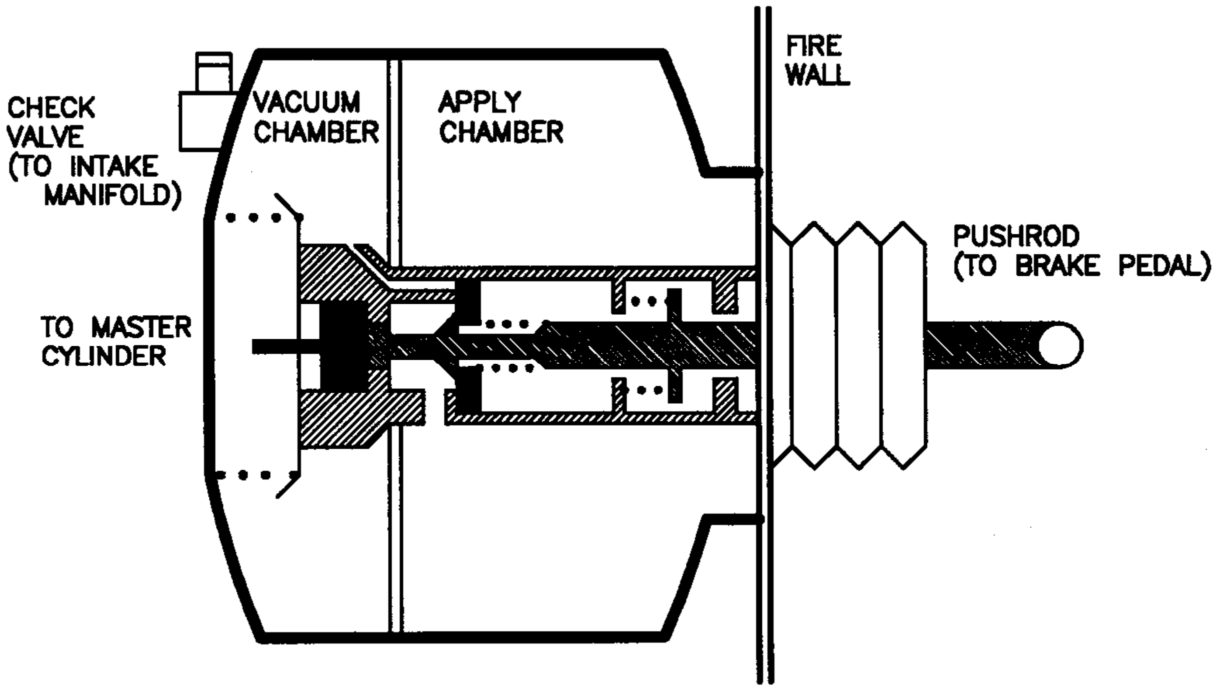
with leak





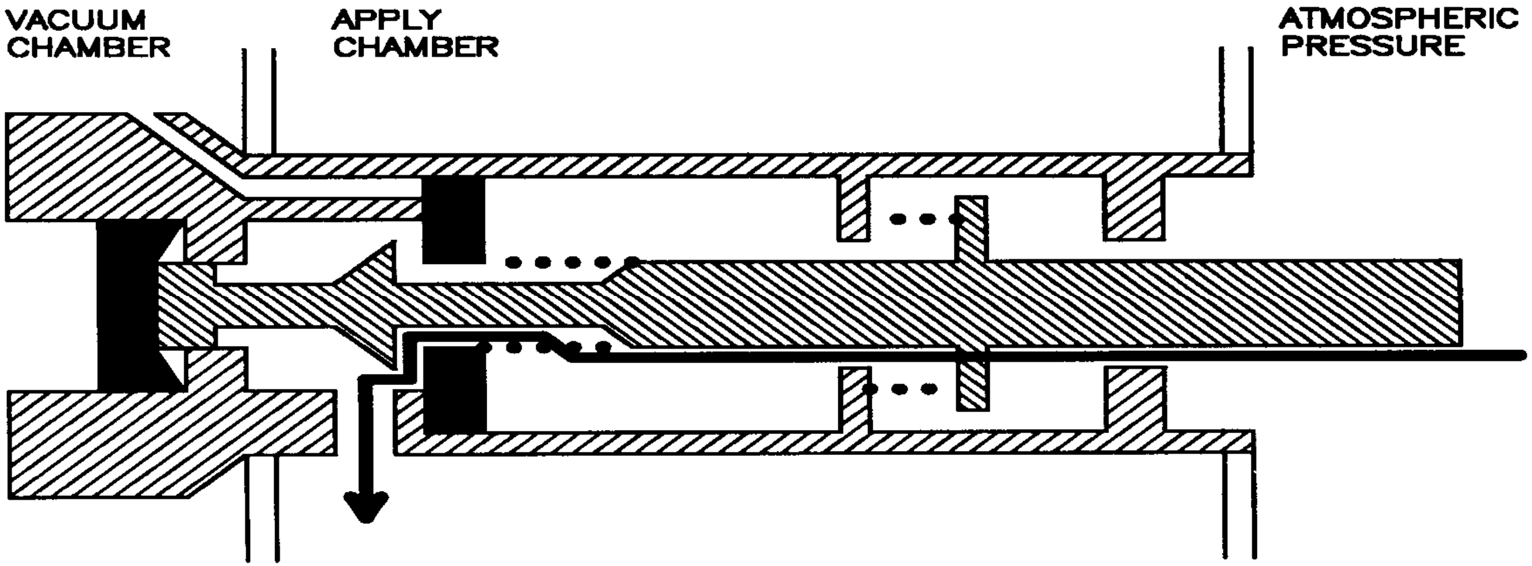
Master cylinder with leak

Vacuum Booster



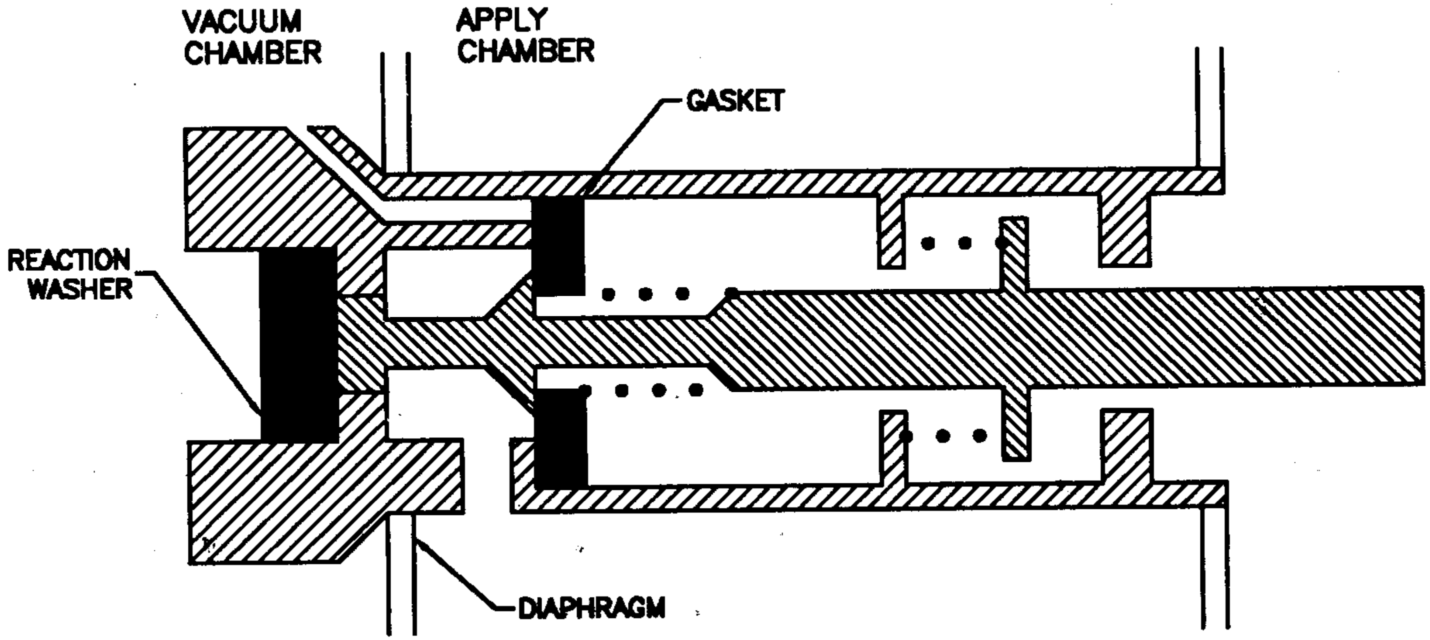
Vacuum Booster Diagram

Vacuum Booster Control Valve Model



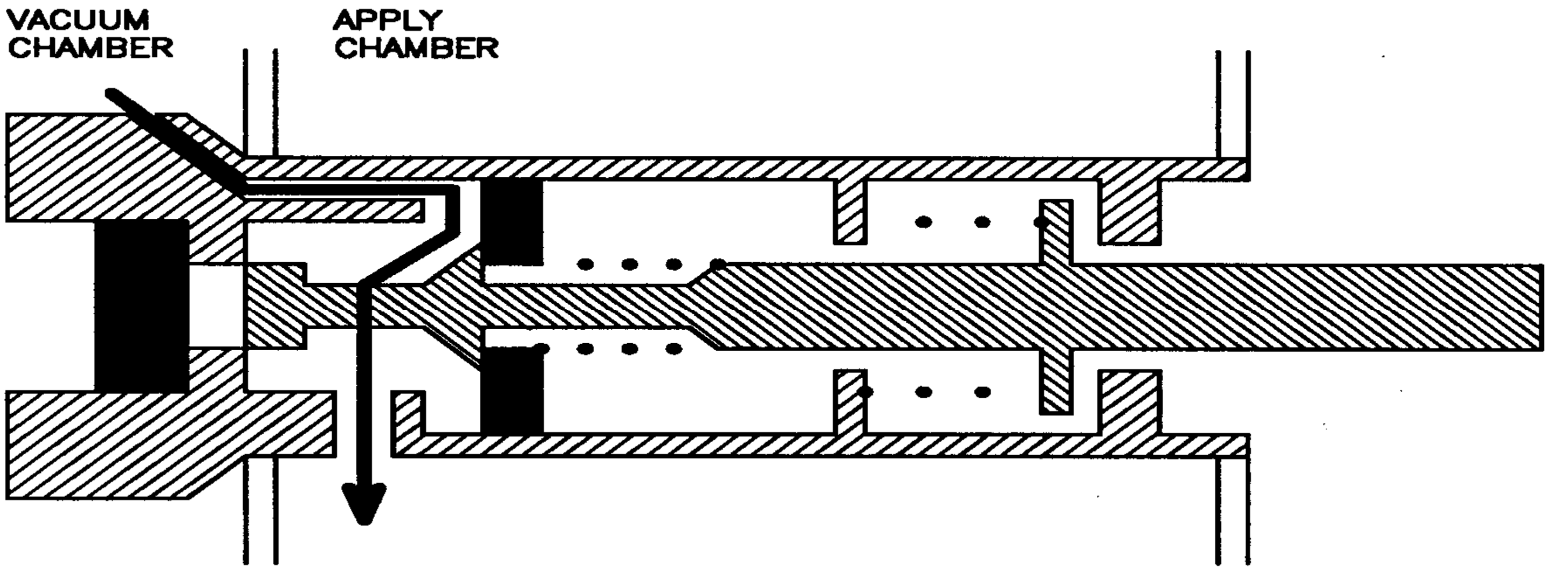
Control Valve – Apply stage

Vacuum Booster Control Valve Model



Control Valve – Hold stage

Vacuum Booster Control Valve Model



Control Valve- Release stage