

Thermal Systems



Thermal System

- $q = K\Delta\theta$

$\Delta\theta$: temperature difference [°C]

q : heat flow rate [kcal/sec]

K : coefficient [kcal/(sec·°C)]

specific heat : α [kcal/(kg·°C)]

Heat capacitance : $C = m \cdot \alpha$ [kcal/°C]



Heat Flow Rate

Coefficient K :

$$K = \frac{kA}{\Delta x} \quad (\text{conduction})$$
$$= HA \quad (\text{convection})$$

k = thermal conductivity, [kcal / ms°C]

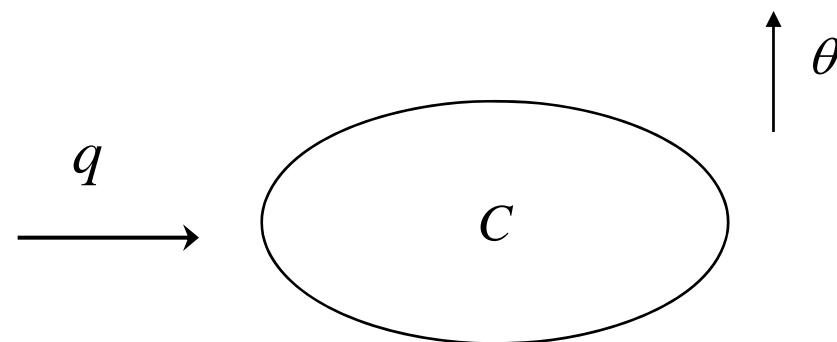
A = area normal to heat flow, [m^2]

Δx = thickness of conductor, [m]

H = convection coefficient, [kcal / $m^2 s^\circ C$]



Heat Balance Equation



$$q \cdot dt = C \cdot d\theta$$

$$\frac{d\theta}{dt} = \frac{q}{C}$$



Thermal Resistance / Capacitance

- Thermal resistance

$$R = \frac{\text{change in temperature difference } [{}^{\circ}\text{C}]}{\text{change in heat flow rate } [\text{kcal/sec}]}$$

$$q = \frac{\Delta\theta}{R}, \quad R = \frac{1}{K}$$

- Thermal capacitance

$$\begin{aligned} C &= \frac{q}{d\theta} = \frac{\text{change in heat stored } [\text{kcal}]}{\text{change in temperature } [{}^{\circ}\text{C}]} \\ &= m \cdot c \left(\text{mass } [\text{kg}] \cdot \text{specific heat } [\text{kcal}/(\text{kg} \cdot {}^{\circ}\text{C})] \right) \end{aligned}$$

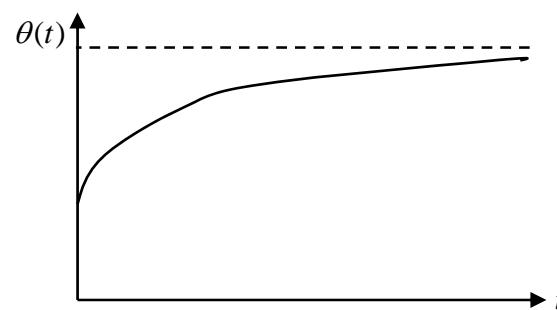
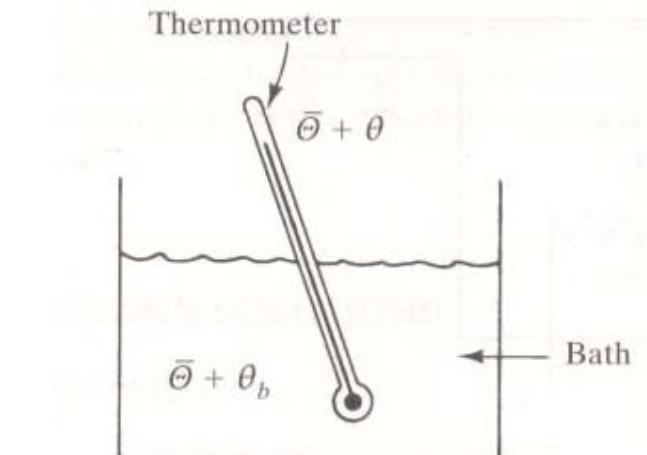
$$\frac{d\theta}{dt} = \frac{q}{C}$$



Thermal System : Thermometer System

$$q = K\Delta\theta, \quad R = \frac{1}{K}, \quad q = \frac{\Delta\theta}{R}$$

ambient temperature $\bar{\theta}$: constant
 bath temperature $\bar{\theta} + \theta_b$, θ_b : constant



$$qdt = Cd\theta$$

C : heat capacitance of the thermometer
 C_b : heat capacitance of the fluid
 R : thermal resistance

$$q = \frac{(\bar{\theta} + \theta_b) - (\bar{\theta} + \theta)}{R} = \frac{\theta_b - \theta}{R}$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{q}{C} = \frac{1}{RC}(\theta_b - \theta)$$

$$\therefore T.F = \frac{\theta(s)}{\theta_b(s)} = \frac{1}{RCs + 1}$$

$$\Rightarrow \theta_b(t) = \theta_b, \quad \theta(t) = \theta_b \left(1 - e^{-\frac{1}{RC}t} \right)$$



Thermal System : Thermometer System

When q_i applied,

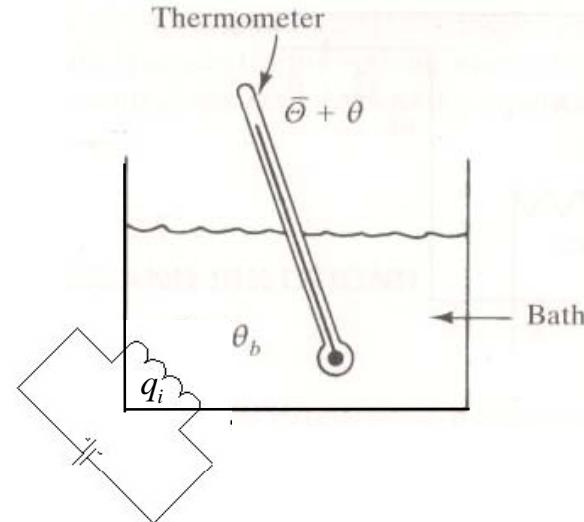
$$\left\{ \begin{array}{l} \frac{d\theta_b}{dt} = \frac{1}{C_b} (q_i - q) \\ q = \frac{1}{R} (\theta_b - \theta) \\ \frac{d\theta}{dt} = \frac{1}{C} q \end{array} \right.$$

$$\Rightarrow \frac{d\theta_b}{dt} = -\frac{1}{RC_b} \theta_b + \frac{1}{RC_b} \theta + \frac{1}{C_b} q_i, \quad \frac{d\theta}{dt} = -\frac{1}{RC} \theta + \frac{1}{RC} \theta_b$$

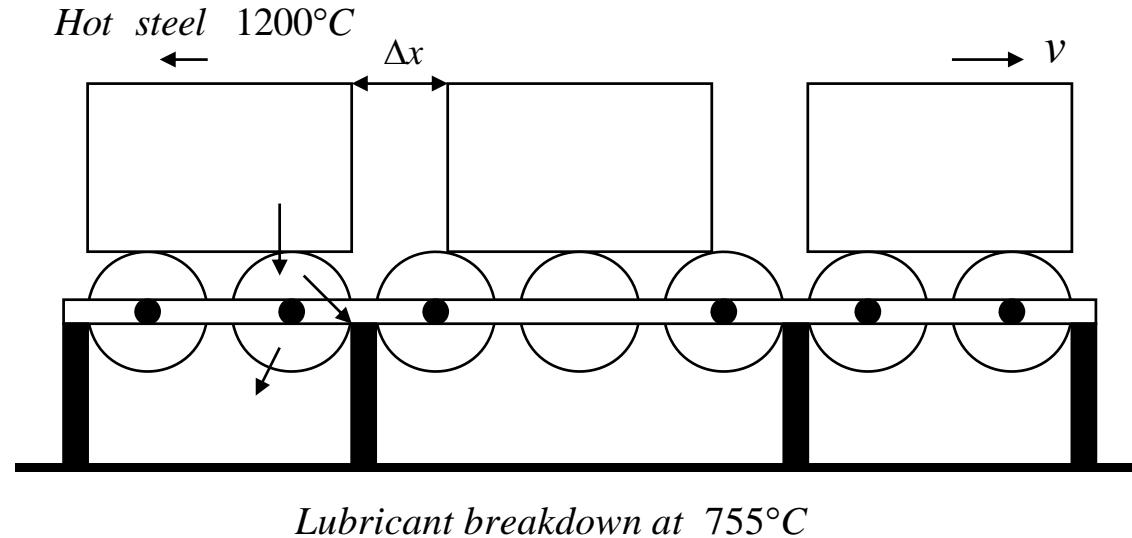
$$(RC_b s + 1)\Theta_b(s) = \Theta(s) + RQ_i(s), \quad (RCs + 1)\Theta(s) = \Theta_b(s)$$

$$\Rightarrow (R^2 C_b C s^2 + RCs + RC_b s)\Theta(s) = RQ_i(s)$$

$$\therefore \frac{\Theta(s)}{Q_i(s)} = \frac{1}{s(RC_b C s + C_b + C)} \approx \frac{1}{sC_b} \quad (C \ll C_b, \quad RC \text{ is small})$$



Thermal System : A Steel Processing Plant



- Large slabs of red-hot steel
- $T_{steel} = 1600°F$: almost constant
- $T_A = 100°F$ (ambient temperature)



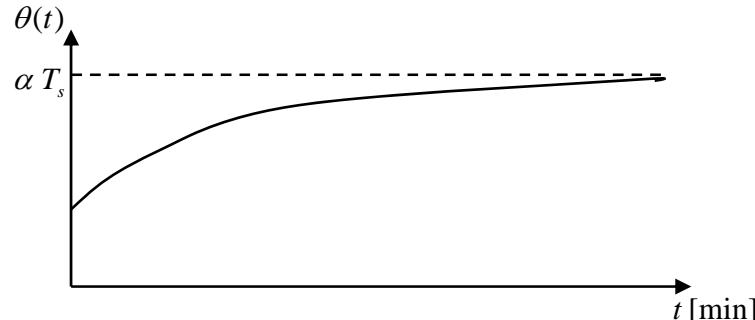
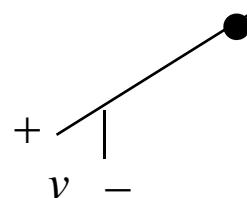
Thermal System : A Steel Processing Plant

T_A : ambient temperature

R_1 : thermal resistance between the slab and the rollers (conduction)

R_2 : thermal resistance between the rollers and the ambient air (convection)
and the bearing support (conduction)

Thermocouple : Produce voltage proportional to its temperature

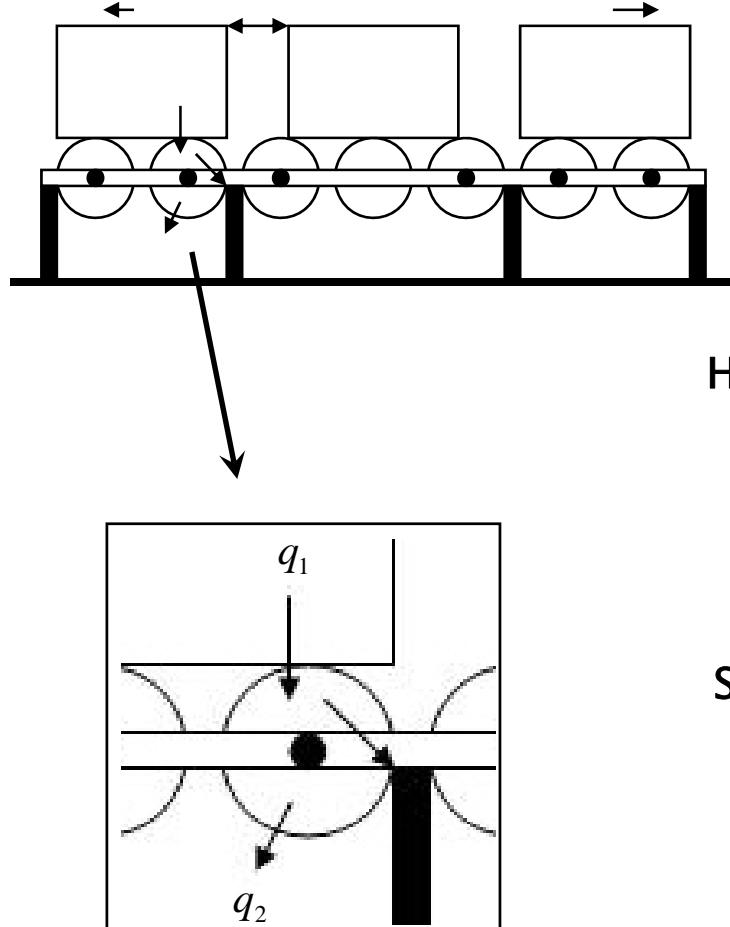


roller : mass = m , specific heat capacitance = C_p [$kcal / kg^\circ C$]

$$\text{heat capacitance} = C = m \cdot C_p$$



Thermal System : A Steel Processing Plant



$$q_1 = \frac{1}{R_1} (T_s - T_r)$$

$$q_2 = \frac{1}{R_2} (T_r - T_A)$$

Heat balance : $m \cdot C_p \cdot \frac{dT_r}{dt} = q_1 - q_2$

$$= -\left(\frac{1}{R_1} + \frac{1}{R_2} \right) T_r + \frac{1}{R_1} T_s + \frac{1}{R_2} T_A$$

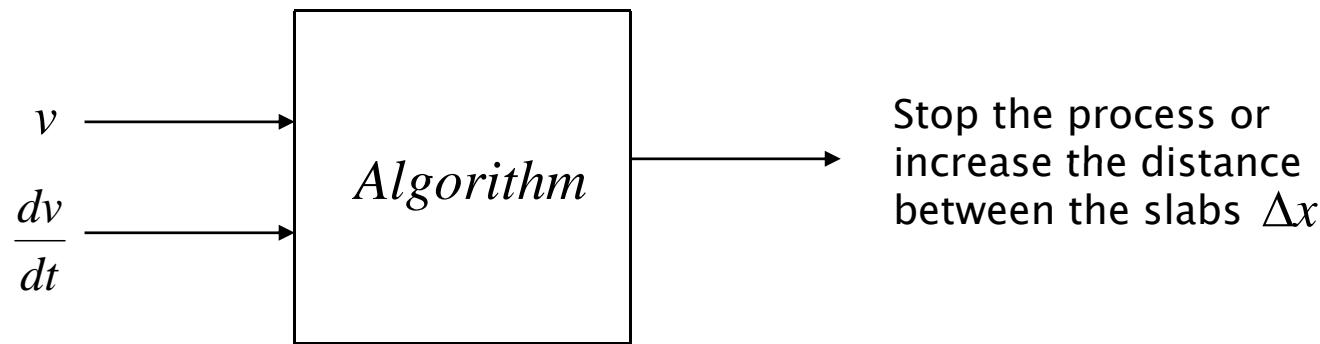
Sensor : $\frac{v(s)}{T_r(s)} = \frac{\alpha}{\tau s + 1}, \quad \tau \dot{v} = -v + \alpha T_r$
 $x = v + \beta \dot{v} \quad (\tau : \text{time constant})$

if, $x > x_{cr} \Rightarrow stop$

$x < x_{cr} \Rightarrow restart$



Thermal System : A Steel Processing Plant



$$x = v + \beta \dot{v}$$

if, $x > x_{cr}$ stop

