Polarization

Polarization (I)

$$\mathbf{E}(z,t) = \mathrm{Re}\left\{ \mathbf{A} \exp\left[j2\pi\nu\left(t - \frac{z}{c} \right) \right] \right\}$$

$$\mathrm{A} = A_x \hat{\mathbf{x}} + A_y \hat{\mathbf{y}}$$

$$A_x = a_x \exp(j\varphi_x), \qquad A_y = a_y \exp(j\varphi_y)$$

Polarization (II)

Different notation

$$\mathbf{E}(z,t) = \mathrm{Re}\left\{ \mathbf{A} \exp\left[-j2\pi\nu \left(t - \frac{z}{c} \right) \right] \right\}$$

$$A_x = a_x \exp(-j\varphi_x), \qquad A_y = a_y \exp(-j\varphi_y)$$

Polarization (III)

$$\mathbf{E}(z,t) = E_x\hat{\mathbf{x}} + E_y\hat{\mathbf{y}}$$

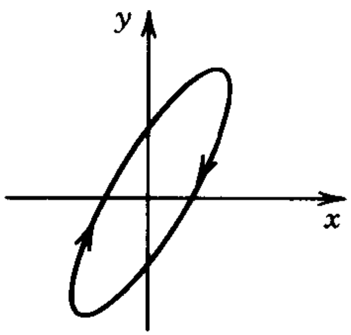
$$E_x = a_x \cos\left[2\pi\nu\left(t - \frac{z}{c}\right) + \varphi_x\right]$$

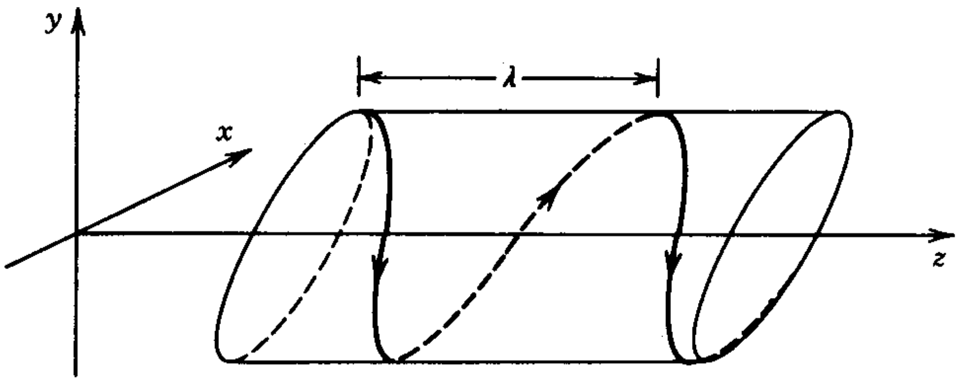
$$E_y = a_y \cos\left[2\pi\nu\left(t - \frac{z}{c}\right) + \varphi_y\right]$$

Polarization Ellipsoid

$$\frac{E_x^2}{a_x^2} + \frac{E_y^2}{a_y^2} - 2\cos\varphi \frac{E_x E_y}{a_x a_y} = \sin^2\varphi$$

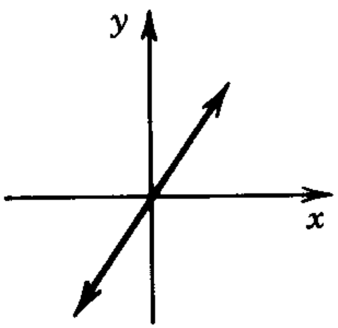
$$\varphi = \varphi_y - \varphi_x$$

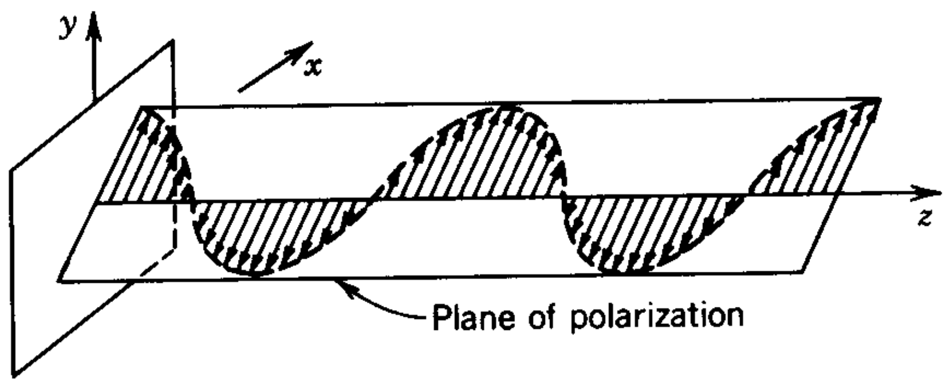




Linear Polarization

$$\varphi = 0 \quad \text{or} \quad \pi$$

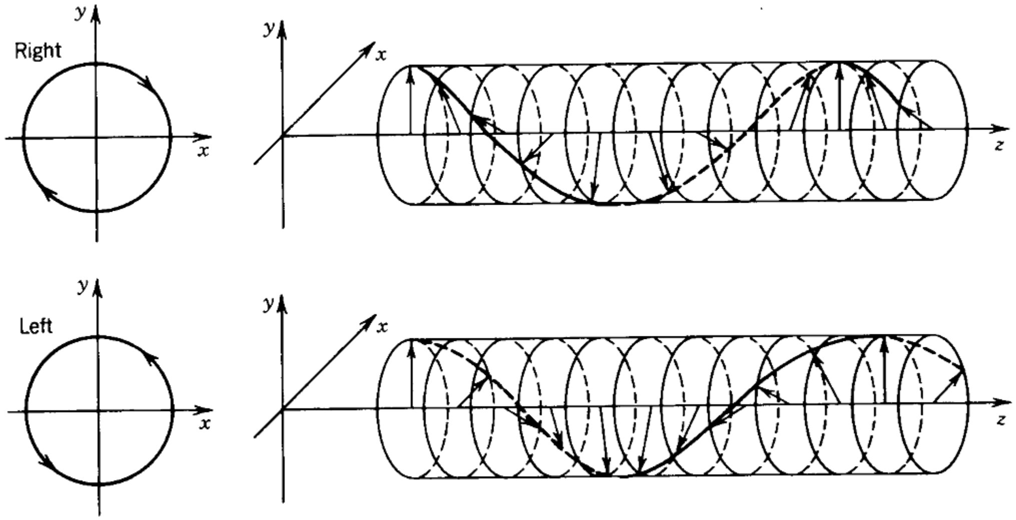




Circular Polarization

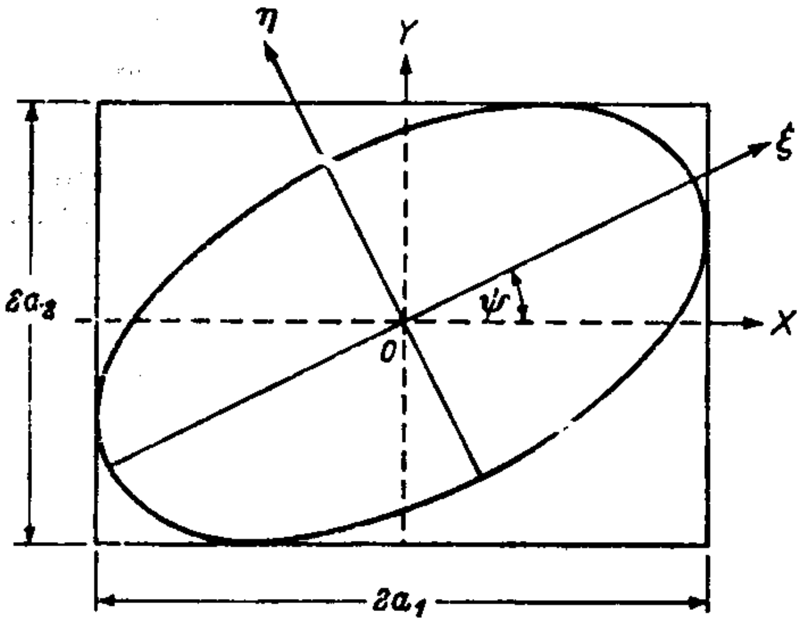
$$\text{Right Circularly Polarized}: \varphi = \frac{\pi}{2}$$

$$\text{Left Circularly Polarized}: \varphi = -\frac{\pi}{2}$$



Elliptical Polarization

Another notation $\quad a_x \to a_1, \; a_y \to a_2, \qquad \varphi = \varphi_y - \varphi_x \to \delta = \delta_2 - \delta_1$

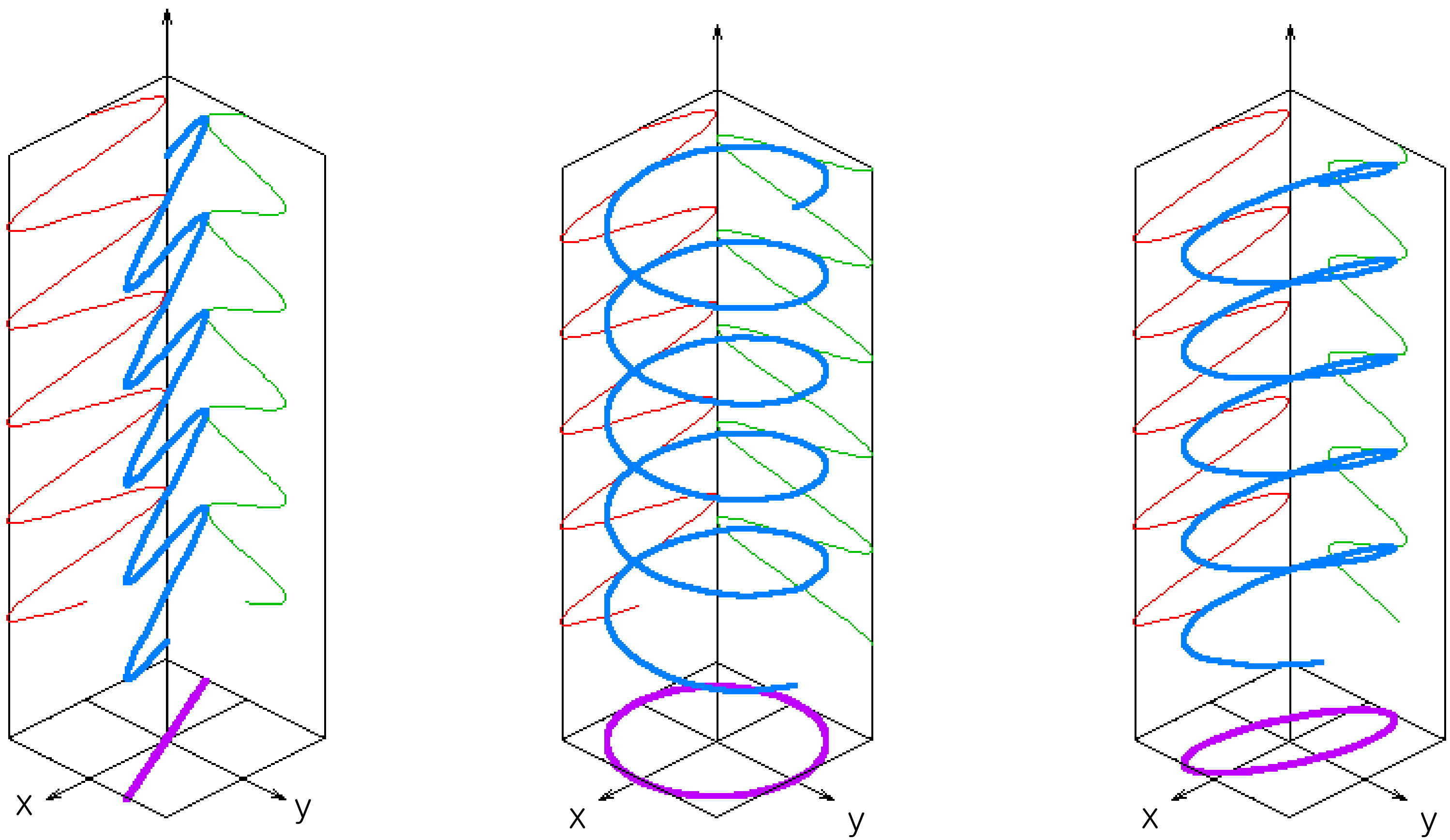


$$\tan\alpha = \frac{a_2}{a_1}, \qquad \tan\chi = \mp\frac{b}{a}$$

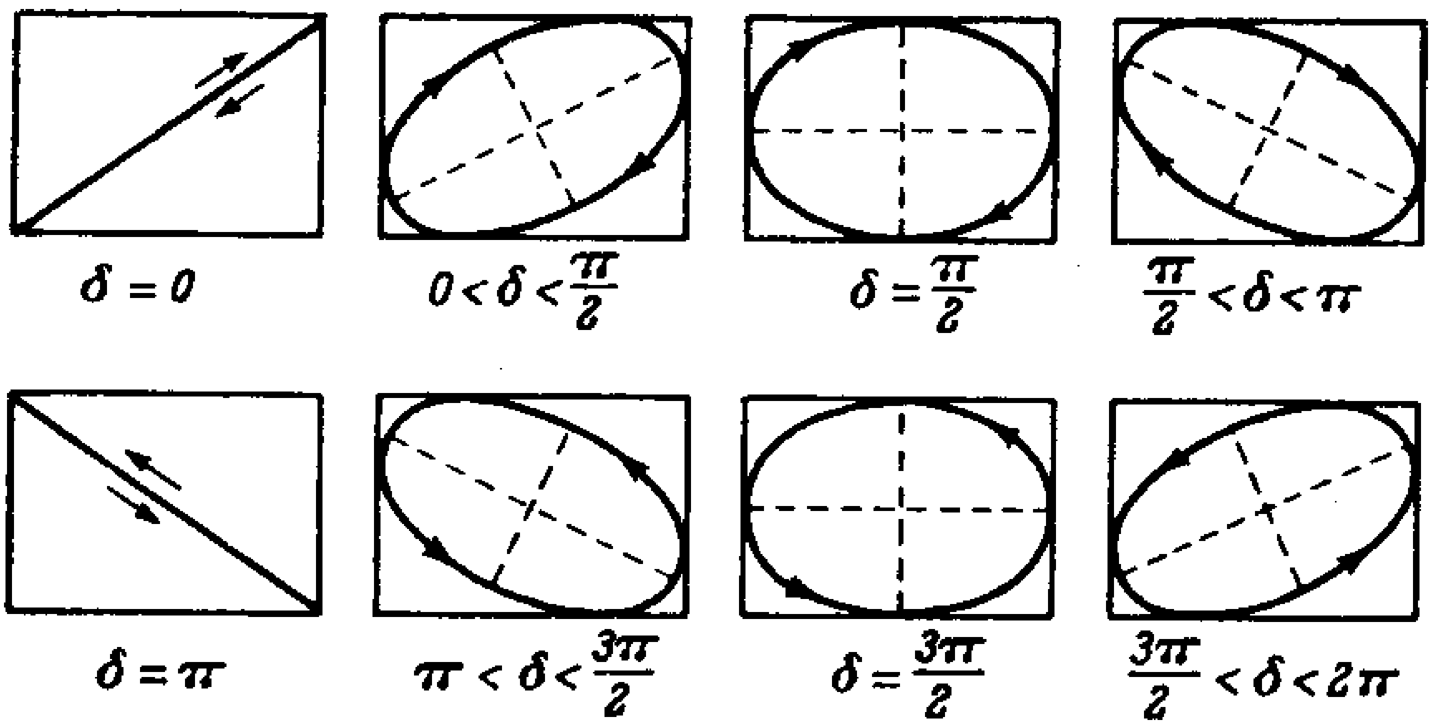
($2a$: long axis length,

$2b$: short axis length)

Comparison of Three Polarization Types

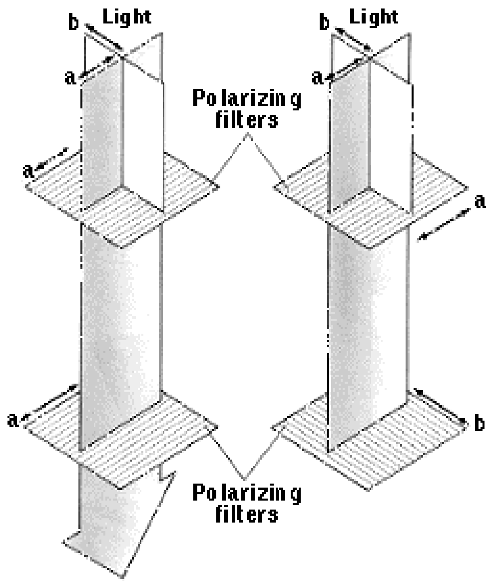


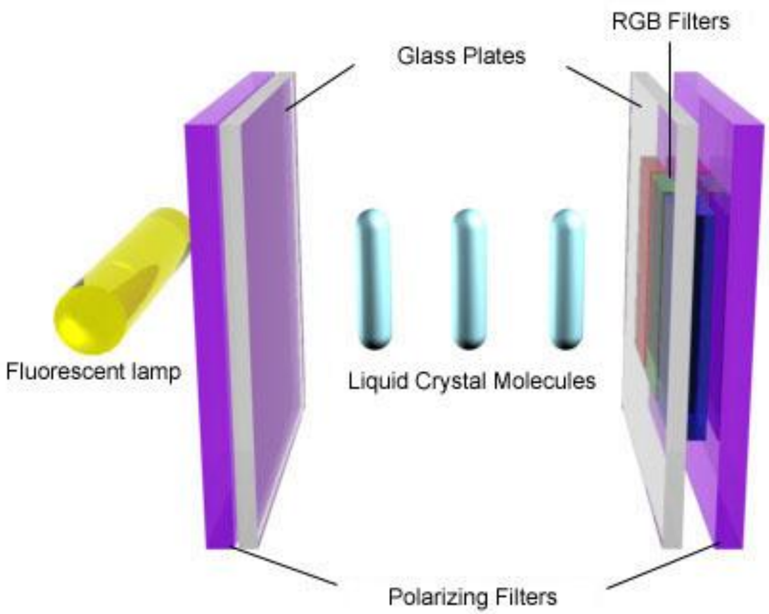
Types of Polarization



Liquid Crystal Display

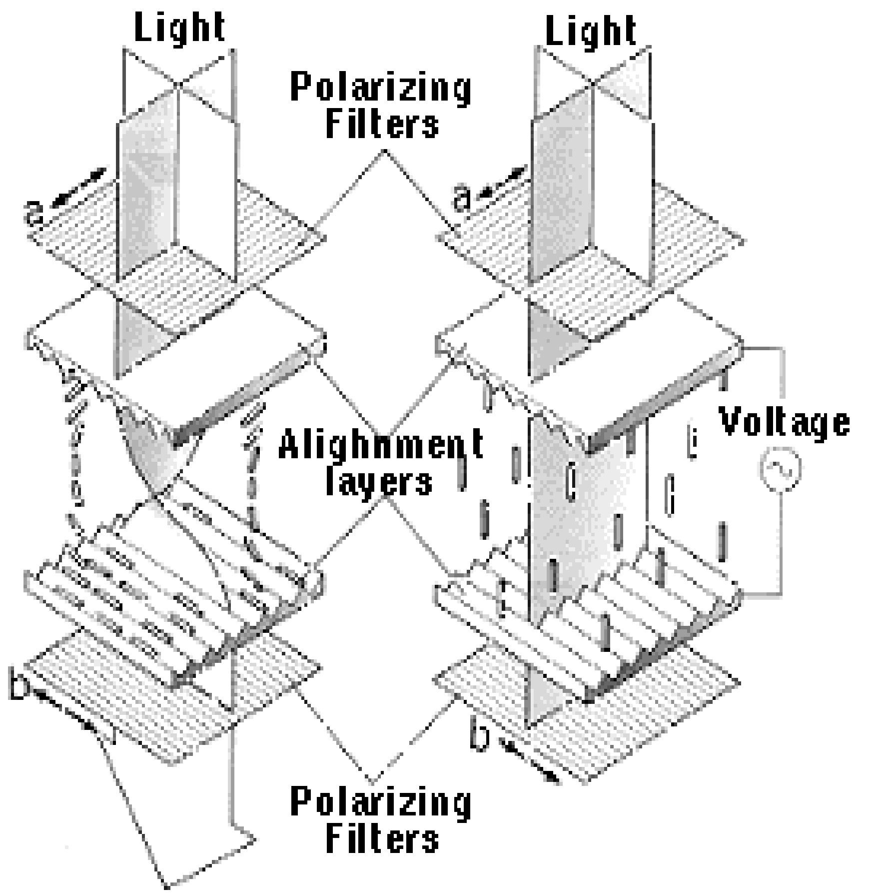
polarizer





LCD Cell

ON



OFF

Polarization Filter

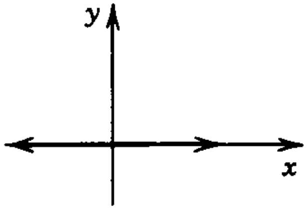
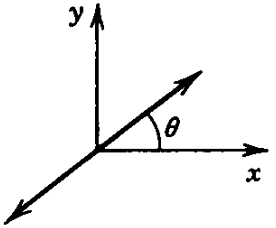
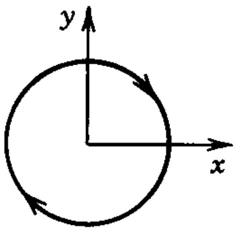
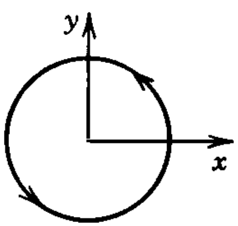
Jones Vector

R. Clark Jones (1916-2004)

$$A_x = a_x \exp(j\varphi_x), \quad A_y = a_y \exp(j\varphi_y)$$

$$\mathrm{J} = \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

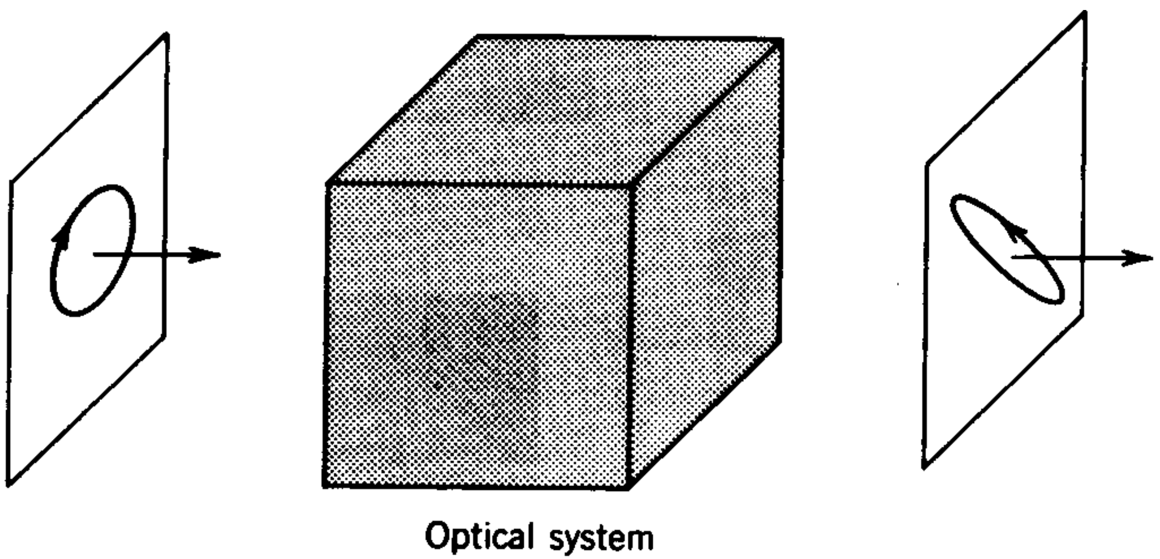
Jones Vectors

Linearly polarized wave, in x direction	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	
Linearly polarized wave, plane of polarization making angle θ with x axis	$\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$	
Right circularly polarized	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ j \end{bmatrix}$	
Left circularly polarized	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -j \end{bmatrix}$	

Jones Calculus

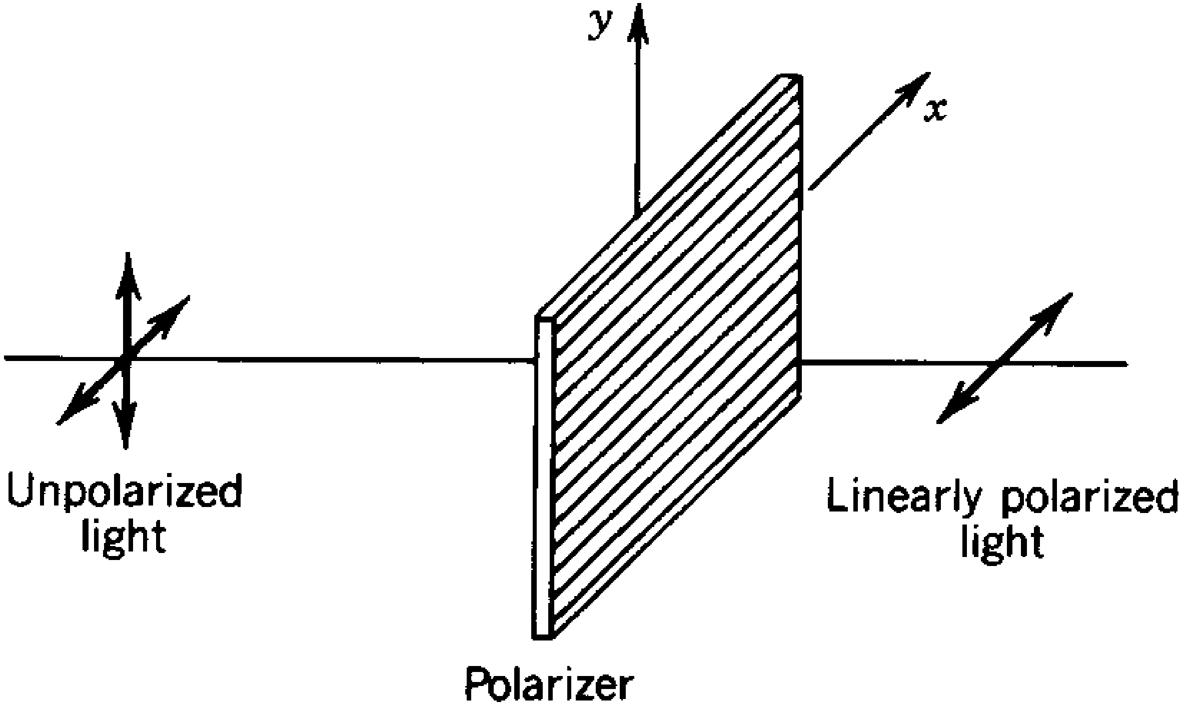
$$\begin{pmatrix} A_{2x} \\ A_{2y} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} A_{1x} \\ A_{1y} \end{pmatrix}$$

$$\mathbf{J}_2 = \mathbf{T}\mathbf{J}_1$$



Jones Matrix for Linear Polarizer

$$T = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

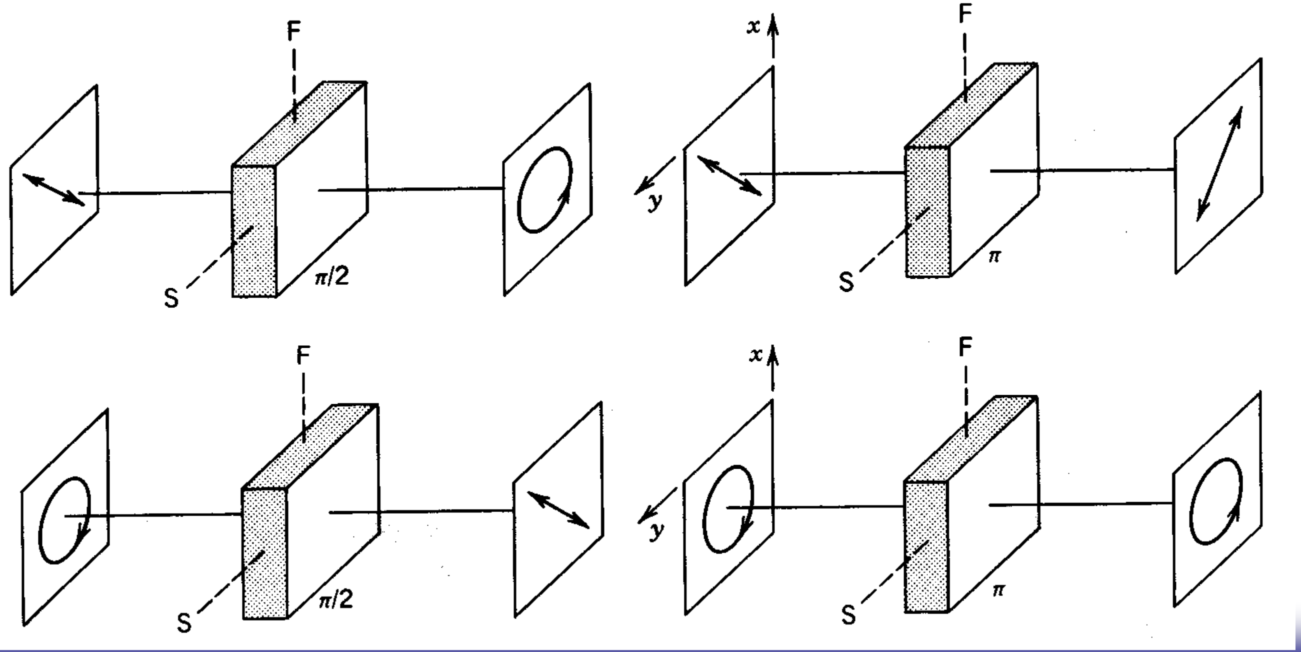


Jones Matrix for Wave Retarders

$$\mathbf{T} = \begin{pmatrix} 1 & 0 \\ 0 & \exp(-j\Gamma) \end{pmatrix}$$

$$\Gamma = \frac{\pi}{2} \quad : \quad \text{Quarter - wave retarder}$$

$$\Gamma = \pi \quad : \quad \text{Half - wave retarder}$$



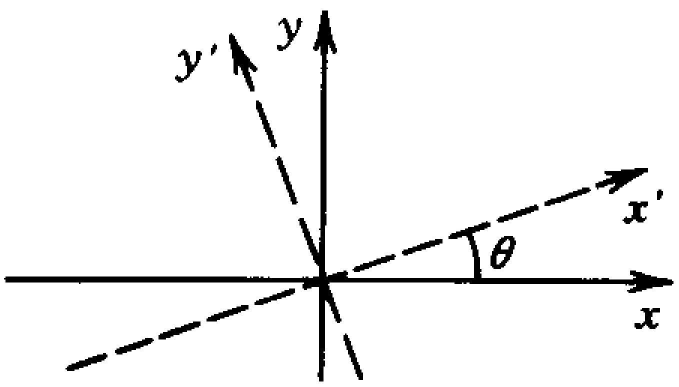
Jones Matrix for Polarization Rotators

$$\mathbf{T} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta_1 \\ \sin\theta_2 \end{pmatrix}$$

$$\theta_2 = \theta_1 + \theta$$

Coordinate Transformation



$$\mathbf{J}' = \mathbf{R}(\theta)\mathbf{J}$$

$$\mathbf{R}(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\mathbf{T}' = \mathbf{R}(\theta)\mathbf{T}\mathbf{R}(-\theta)$$

$$\mathbf{T} = \mathbf{R}(-\theta)\mathbf{T}'\mathbf{R}(\theta)$$

Stokes Parameters

$$s_0 = a_1^{\,2} + a_2^{\,2}$$

$$s_1 = a_1^{\,2} - a_2^{\,2}$$

$$s_2 = 2a_1a_2\cos\delta$$

$$s_3 = 2a_1a_2\sin\delta$$

$$s_0^{\,2} = s_1^{\,2} + s_2^{\,2} + s_3^{\,2}$$

Stokes Parameters (Polarized + Unpolarized) (I)

$$\begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix}$$

S_0 = Total power(polarized + unpolarized)

S_1 = Power through LH polarizer – power through LV polarizer

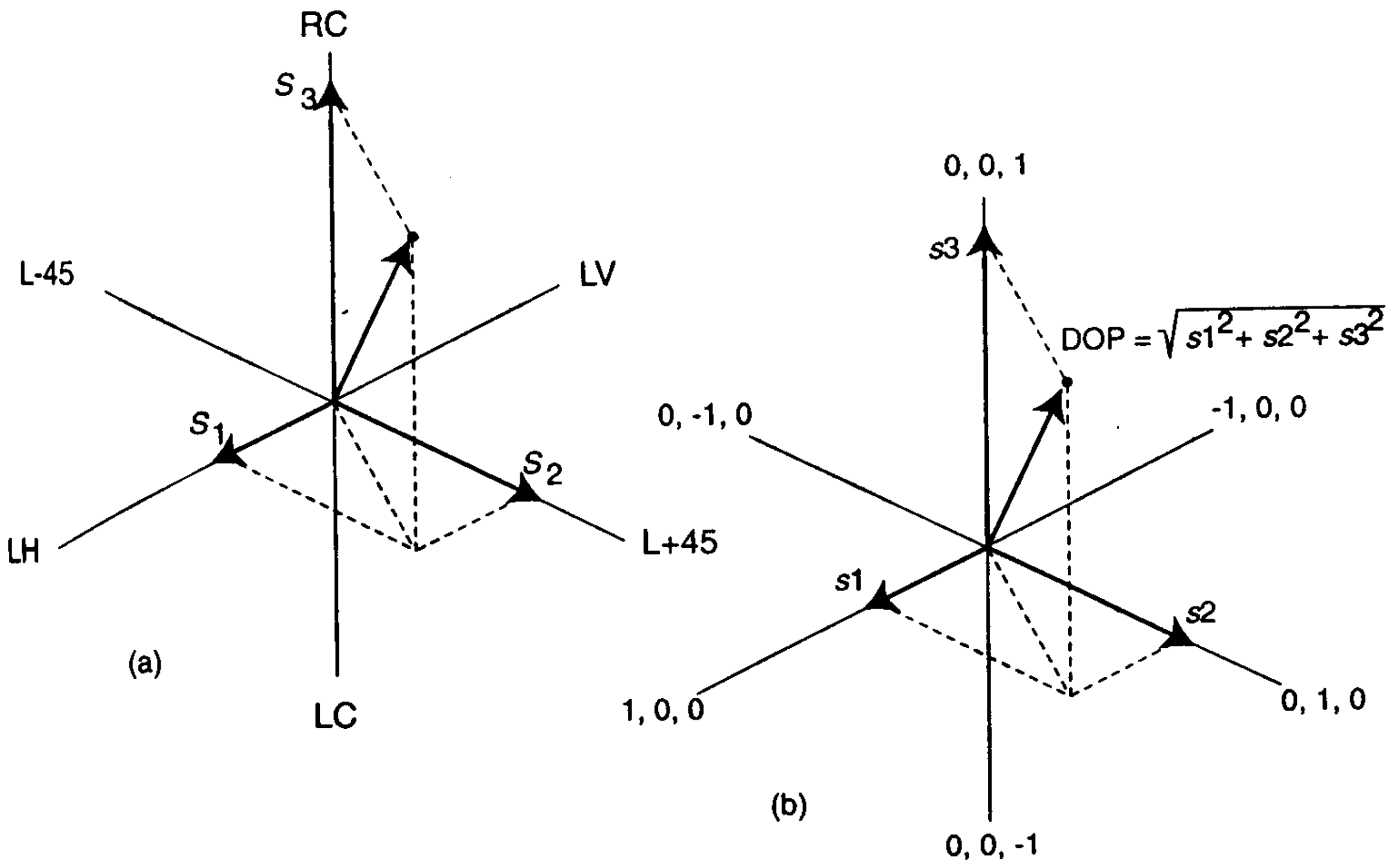
S_2 = Power through L+45 polarizer – power through L-45 polarizer

S_3 = Power through RC polarizer – power through LC polarizer

$$P_{\text{polarized}} = \sqrt{S_1^2 + S_2^2 + S_3^2}$$

$$s_1 = \frac{S_1}{S_0} \qquad s_2 = \frac{S_2}{S_0} \qquad s_3 = \frac{S_3}{S_0}$$

Stokes Parameters (Polarized + Unpolarized) (II)



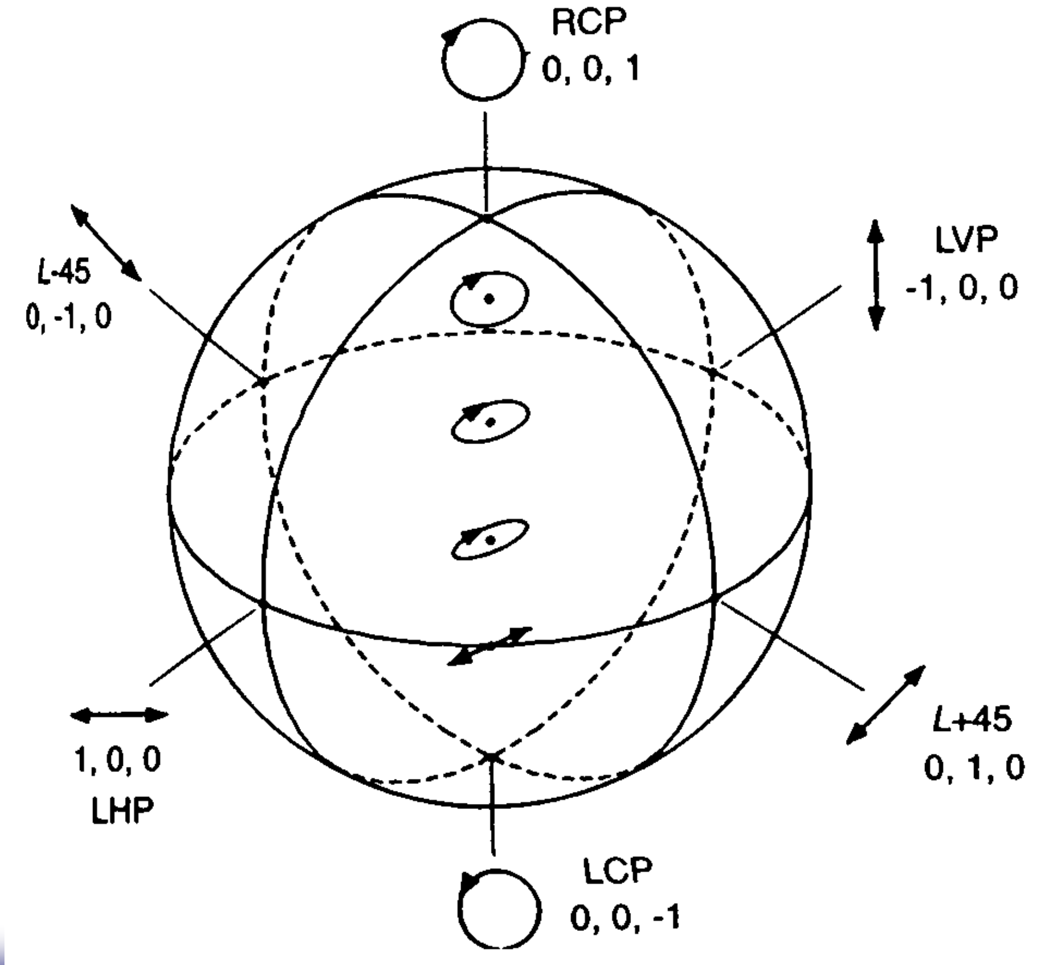
Degree of Polarization

$$\mathrm{DOP} = \frac{P_{\text{polarized}}}{P_{\text{polarized}} + P_{\text{unpolarized}}}$$

$$\mathrm{DOP} = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$$

$$\mathrm{DOP} = \sqrt{s_1{}^2 + s_2{}^2 + s_3{}^2}$$

Poincaré Sphere (I)



Poincaré Sphere (II)

$$s_1 = s_0 \cos 2\chi \cos 2\psi$$

$$s_2 = s_0 \cos 2\chi \sin 2\psi$$

$$s_3 = s_0 \sin 2\chi$$