EXPERIMENT
OF
ELEMENTARY FLUID MECHANICS

Free & Forced Vortices

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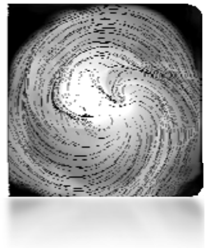
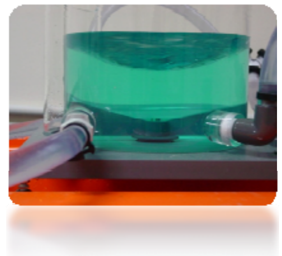
Objectives

Chapter 3. Free and Forced Vortices

A vortex can be seen in the spiraling motion of air or liquid around a center of rotation.
Circular current of water of conflicting tides form vortex shapes.
Turbulent flow makes many vortices.
- Atmospheric phenomenon of a whirlwind or a tornado or dust devil.

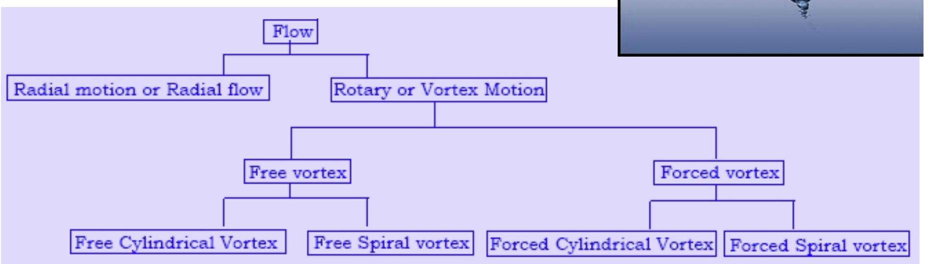
In this experiment,
through the generating of free and forced vortex,
we can compare the properties of vortices.

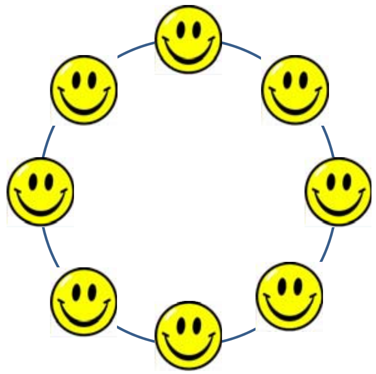
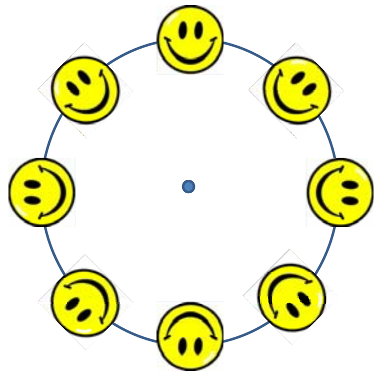
Background Theory

Two types of motion
- **Translation: vector specified**
- **Rotation: specified by angle, has center**

Vortex : spiraling motion of air or liquid around a center of rotation



Background Theory

-Translation
-Vector specified
- Irrotational flow

-Rotation
-Specified by angle
-Rotational Flow

Background Theory

Forced vortex motion

In a forced vortex, the fluid rotates as a solid body,
Motion is similar to a dish of fluid on a turntable rotating

Navier - Stokes Equation (r - component) $\frac{v_\theta^2}{r} = \frac{1}{\rho}\frac{\partial \rho}{r}$

Solid boundary rotation $v_\theta = \omega r \quad \Rightarrow \quad \rho\omega^2 r = \frac{\partial \rho}{\partial r}$ $\quad \omega$: angular velocity

Navier -Stokes Equation (z - component) $0 = -g - \frac{1}{\rho}\frac{\partial \rho}{\partial z} = -g - \frac{1}{\rho}\frac{\partial \rho}{\partial h}$

Hydrostatic condition $\frac{\partial \rho}{\partial h} = -\rho g = -\gamma$

Background Theory

Substitute into the differential

$$dp = \frac{\partial \rho}{\partial r}dr + \frac{\partial \rho}{\partial h}dh = \rho\omega^2 r dr - \gamma dh$$

Integrate to obtain

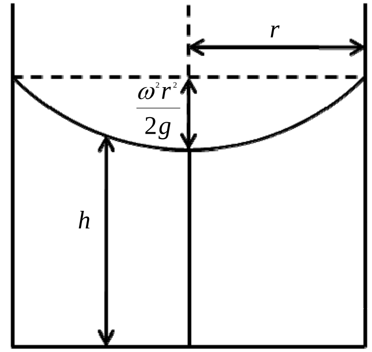
$$p = \rho\frac{\omega^2 r^2}{2} - \gamma h + C$$

Boundary conditions $r = 0, \quad h = h_0, \quad p = p_0$

Result

$$p - p_0 = -\gamma(h - h_0) + \rho\frac{\omega^2 r^2}{2}$$

Free surface location, $p = p_0 \quad \Rightarrow \quad h = h_0 + \frac{\omega^2 r^2}{2g}$



Background Theory

Free vortex motion

If the motion of the particles is purely translational,
the flow is irrotational and the vorticity is presented as $\frac{\partial v_y}{\partial x}\frac{\partial v_x}{\partial y} = \text{constant}$

Each particle moves in a circular path with speed varying inversely as the distance from the center. The tangential velocity is given by $q = \frac{k}{r}$

In the steady state flow condition, Bernoulli's equation is applied

$$\frac{p}{\rho g} + \frac{q^2}{2g} + z = \text{constant}$$

Free surface condition, $p = 0$, $h = h_0 - \frac{k^2}{r^2 2g}$, $k = qr = v_\theta r$

Flow velocity (tangential) $q = \sqrt{2gH}$ $\quad H$: Hydraulic head

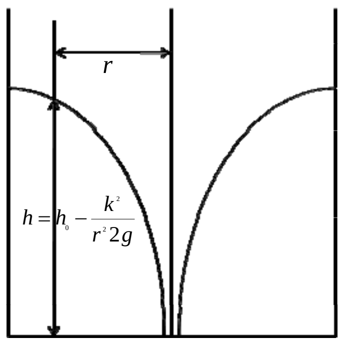
Background Theory

Free vortex motion

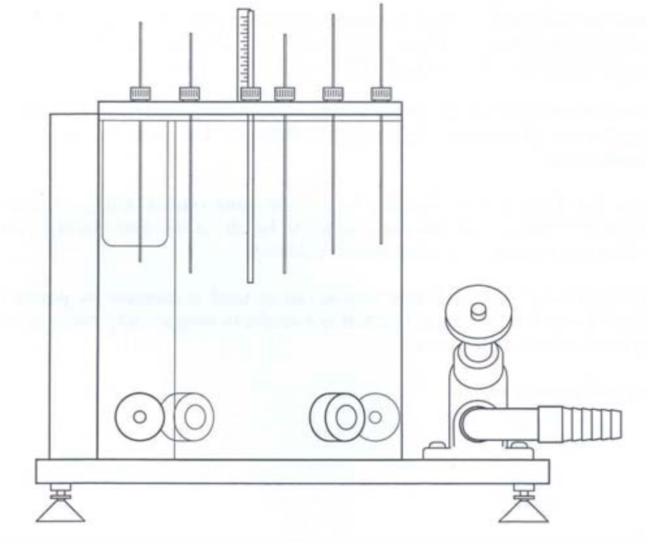
Free surface condition, $p = 0$, $h = h_0 - \frac{k^2}{r^2 2g}$, $k = qr = v_\theta r$

Flow velocity (tangential) $q = \sqrt{2gH}$ $\quad H$: Hydraulic head

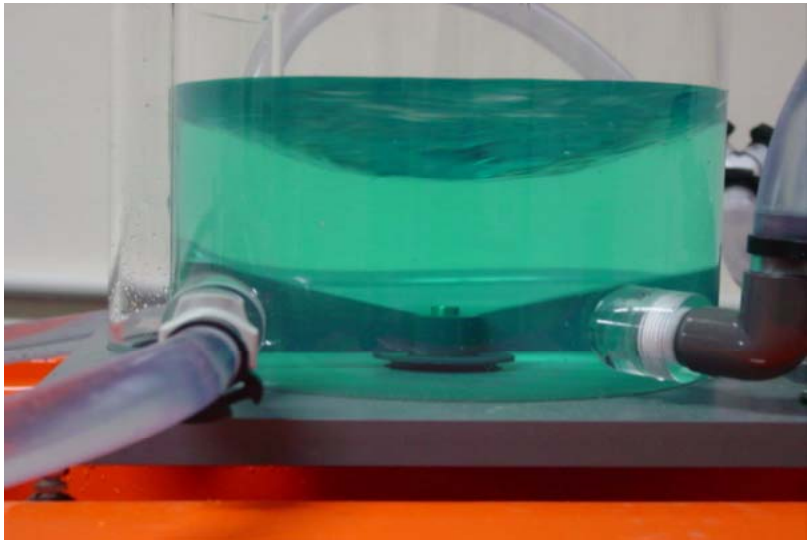
h_0: height when $r \to \infty$



Experimental Apparatus



Forced Vortex



Procedure – Forced Vortex

1. Set the vortex apparatus on the Hydraulics Bench and keep it level
2. Connect the Y-hose to 9 mm inlet of cylinder and outlet of Hydraulics Bench
3. Connect the hose to 12 mm outlet of cylinder and volumetric tank of Hydraulics Bench
4. Attach the rotating paddle to the bottom of cylinder.
5. Attach the depth gages on the cylinder.
6. Turn on the Hydraulics Bench and regulate the water inflow using control valve to keep the water elevation steady.
7. Calculate the angular velocity of flow. Using stopwatch and rotating paddle.
8. Record the water surface profile of Forced vortex using depth gauges.
9. Repeat 6 - 8 with changing angular velocity.

Experiment results

Forced Vortex

	Number of revolution	Angular Velocity	Radius (m)	Measured Needle Length (m)	Height from Datum (m)	Calculated Height (m)	Measured Height (m)
1	47	4.922	0.110	0.1070	0.1700	0.1700	
2							
3							
4							
5							
6							
7							
8							

Free Vortex

Procedure - Free Vortex

1. Set the vortex apparatus on the Hydraulics Bench and keep it level
2. Connect the Y-hose to 12mm inlet of cylinder and outlet of the Hydraulics Bench
3. Close 9mm outlet of cylinder, the water will be discharged only through the orifice of cylinder.
4. Attach the orifice to the bottom of cylinder.
5. Attach the depth gages on the cylinder.
6. Turn on the Hydraulics Bench and regulate the water inflow using control valve to keep the water elevation steady.
7. Record the water surface profile of Free vortex using depth gauges.
8. If the free vortex is not formed at the center of cylinder, it will be difficult to measure the water surface profile with depth gauges. (Use only one needle of depth gauges to measure the water surface profile.)
9. Measure the hydraulic head using three kinds of pitot tube.
10. Repeat 4-9 by changing the diameter of the orifice.

Experiment results

Free Vortex

	Pitot tube arm length (m)	Pitot tube water height (m)	$q=\sqrt{2gh}$ (m/sec)	h_0 (m)	Calculated Height (m)	Observed Height (m)
1	0.015	0.021	0.642	0.172		
2						
3						
4						
5						
6						
7						
8						

Problems

1. Plot the water surface profile, and compare calculated result with measured data.

2. Discuss the reason of difference between calculated result with measured data.

3. Discuss the examples of free and forced vortices.