EXPERIMENT
OF
ELEMENTARY FLUID MECHANICS

Bernoulli's Theorem Experiment

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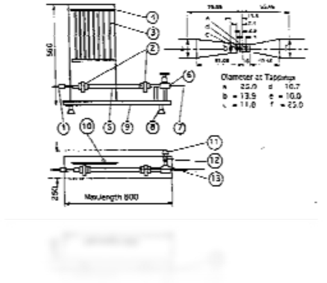
Objectives

Chapter 4. Bernoulli's Theorem Experiment

To investigate the validity of Bernoulli's Theorem as applied to the flow of water in a tapering circular duct.

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + Z_1 = \frac{V_2^2}{2g} + \frac{P_1}{\gamma} + Z_2 + h_L = H$$



Bernoulli Theorem

Daniel Bernoulli
(1700-1782)

Born in Netherland

Mathematician, physicist

Hydrodynamique (1738)

Conservation of Energy

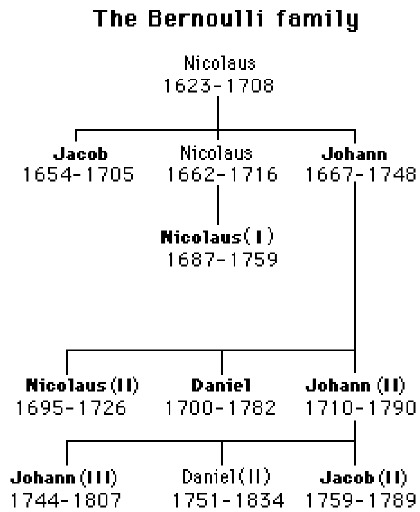
Exposition of a New Theory on the Measuremet of Risk (1738)

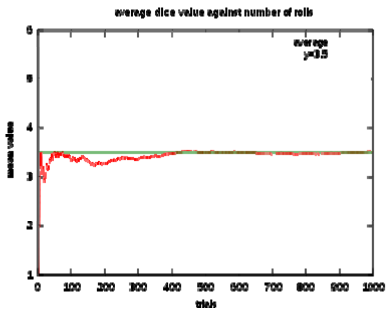
St. Petersburg Paradox

Beam theory

$$\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right) = q$$

Bernoulli Theorem





Bernoulli Theorem

Considering flow at two sections in a pipe
Bernoulli's equation

$$\frac{V_1^2}{2g}+\frac{P_1}{\gamma}+Z_1=\frac{V_2^2}{2g}+\frac{P_2}{\gamma}+Z_2=H$$

$\frac{V}{2g}$ = velocity head $\quad \frac{(\text{m/s})^2}{\text{m/s}}=\text{m}$

$\frac{P}{\gamma}$ = pressure head $\quad \frac{\text{kg}\cdot\text{m/s}^2}{\text{m}^2}\Big/\frac{\text{kg}\cdot\text{m/s}^2}{\text{m}^3} = \text{m}$

Z = potential(elevation) head

H = total head

V = velocity

g = gravitational acceleration

P = pressure

γ = specific weight

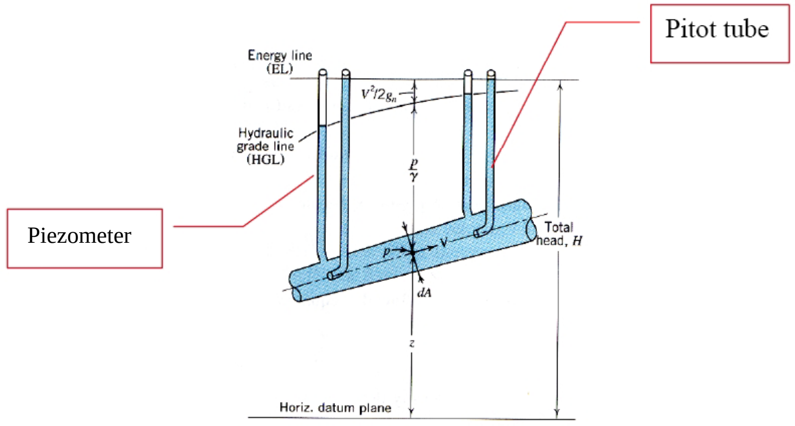
Z = height

h_f = head loss

H = total head

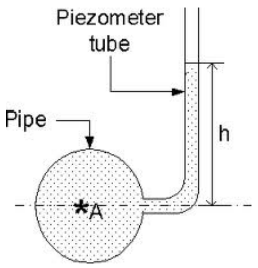
Bernoulli Theorem

$$\frac{V_1^2}{2g}+\frac{P_1}{\gamma}+Z_1=\frac{V_2^2}{2g}+\frac{P_2}{\gamma}+Z_2=H$$

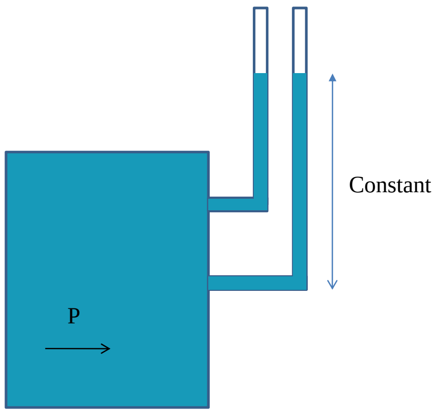


Piezometer

Small diameter observation well to measure the hydraulic head

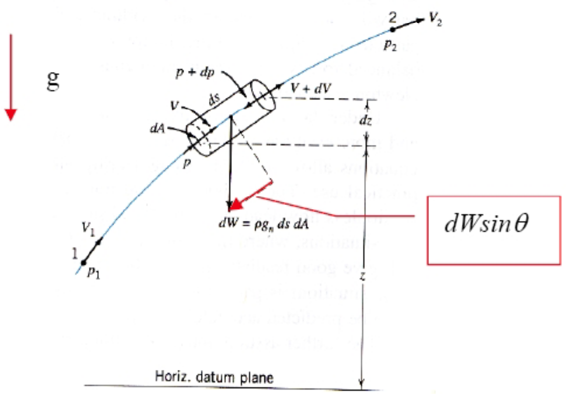


$$\frac{p}{\gamma}+z=\text{constant}$$



Derivation of Theorem

Apply Newton's 2nd law to the motion of fluid particles
Consider a streamline and select a small cylindrical fluid system



Derivation of Theorem

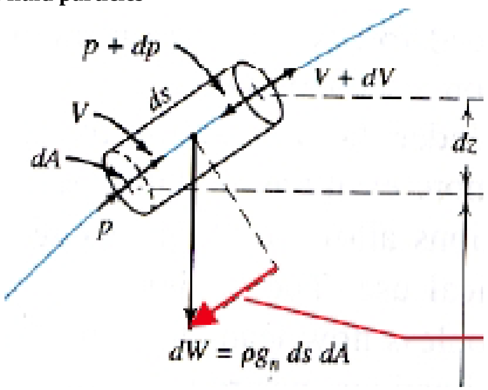
Apply Newton's 2nd law to the motion of fluid particles

$$\sum F = ma$$

$$dF = pdA - (p+dp)dA - dW\sin\theta$$

$$= -dp\cdot dA = -\rho g dA\cdot ds\cdot\frac{dz}{ds}$$

$$= -dp\cdot dA - \rho g\cdot dA\cdot dz$$



$dm = \rho ds\cdot dA$ **(density X volume)**

$$a = \frac{dV}{dt} = \frac{dV}{ds}\cdot\frac{ds}{dt} = V\frac{dV}{ds}$$

$$\therefore\quad -dpdA - \rho g\cdot dA\cdot dz = \rho\cdot ds\cdot dA\cdot V\frac{dV}{ds}$$

Derivation of Theorem

$$dpdA + \rho\cdot ds\cdot dA\cdot V\frac{dV}{ds} + \rho g\cdot dA\cdot dz = 0$$

Divide by ρdA

$$\frac{dp}{\rho} + VdV + gdz = 0 \qquad \gamma = \rho g$$

$$\frac{dp}{\gamma} + \frac{1}{g}VdV + dz = 0$$

$$\frac{dp}{\gamma} + d\left(\frac{V^2}{2g}\right) + dz = 0 \qquad d(V^2) = 2V\cdot dV$$

$$d\left(\frac{p}{\gamma} + \frac{V^2}{2g} + z\right) = 0$$

Euler's equation

Derivation of Theorem

Bernoulli's equation?

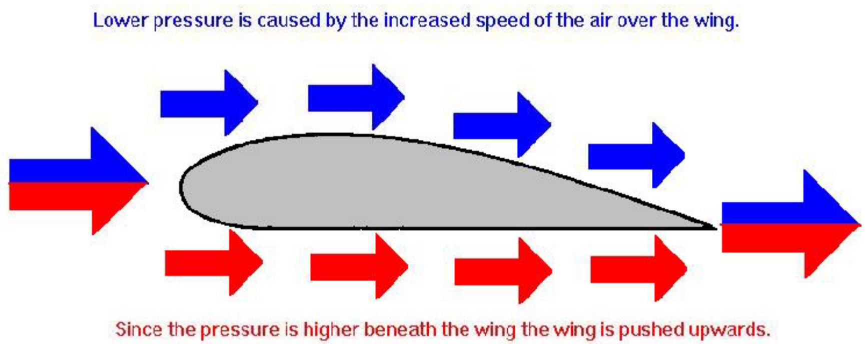
Integrate Euler's equation $d\left(\frac{p}{\gamma} + \frac{V^2}{2g} + z\right) = 0$

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{const}$$

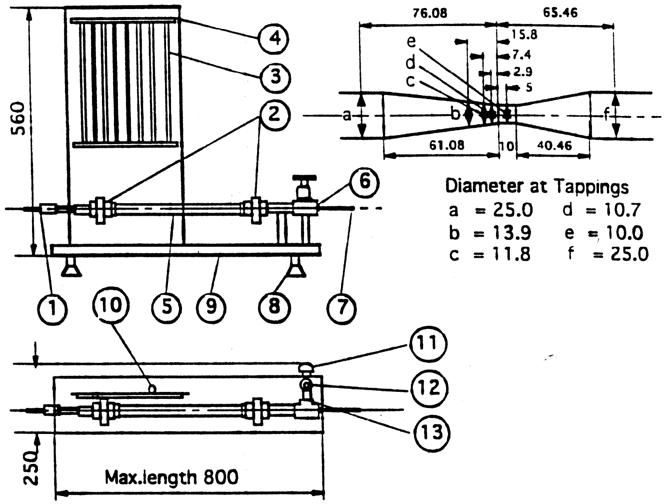
$$\boxed{\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + Z_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + Z_2 = H}$$

Application of Theorem

Air lift? **Not the main reason**



Experimental Apparatus



Procedure

1. Obtain the area of cross sections of the duct point connected to the manometer.
2. Calculate the flowrate with a stopwatch and the volumetric tank level.
3. Calculate mean velocity of each cross section with flowrate and area of cross sections.
4. Compute Reynolds number and velocity head using mean velocity.
5. Measure the pressure head by reading Manometer level.
6. The sum of velocity head(4) and pressure head(5) and potential head is the total head. (Potential head is zero, we assumed that the centerline of the duct is datum)
7. Measure the total head of each cross section using Pitot tube.
8. Compare the computed total head(6) with measured total head(7).
9. Repeat process (2-5) 5 times with each other flowrate.

Results

Point	1	2	3	4	5	6
Diameter (mm)	25	13.9	11.8	10.7	10	25
Area (mm^2)	490.874					
Speed (mm/s)						
Velocity Head (mm)						
Pressure Head (mm)						
Potential Head (mm)						
Calculated Total Head						
Measured Total Head						
Difference						

Results

1. Using Bernoulli Theorem, discuss relations of diameter of duct, mean velocity, pressure.

2. Compare the computed total head with measured total head, discuss why does the difference occurs.