Naval Architecture & Ocean Engineering

Computer Aided Ship Design

Part I. Optimization Method

Ch. 1 Overview of Optimal Design

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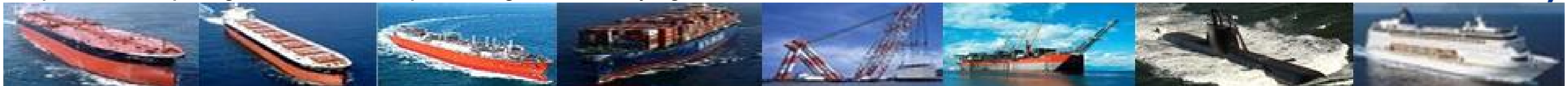

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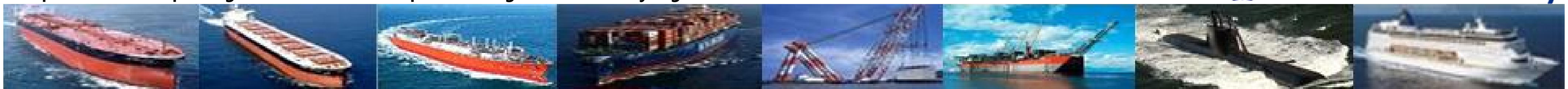

Indeterminate and Determinate Problems (1/2)

Variables: x_1, x_2, x_3

Equation: $x_1 + x_2 + x_3 = 3$

- ✓ Number of variables: 3
- ✓ Number of equations: 1

Because the number of variables is larger than that of equations, this problem forms an **indeterminate system**.

Solution for the indeterminate problem:

We assume two unknown variables

Number of variables(3) - Number of equations(1)

Example) assume that $x_1 = 1, x_2 = 0$

➡ $x_3 = 2$

Equation of straight line

$$y = a_0 + a_1 x$$

Where a_0, a_1 are given.

- ✓ Number of variables: 2 x, y
- ✓ Number of equations: 1

☞ We can get the value of y by assuming x.

Finding intersection point (x^*, y^*) of two straight lines

$$y = a_0 + a_1 x$$

$$y = b_0 + b_1 x$$

Where a_0, a_1, b_0, b_1 are given.

- ✓ Number of variables: 2 x, y
- ✓ Number of equations: 2

Because the number of variables is equals to that of equations, this problem forms an **determinate system**.

Indeterminate and Determinate Problems (2/2)

Determinate problem

Variables: x_1, x_2, x_3

Equations: $f_1(x_1, x_2, x_3)=0$

$f_2(x_1, x_2, x_3)=0$

$f_3(x_1, x_2, x_3)=0$

If f_1, f_2, and f_3 are linearly independent,

- ✓ Number of variables: 3
- ✓ Number of equations: 3

Since the number of equations is equal to that of variables, this problem can be solved.

What happens if $2 \times f_3 = f_2$?

Then f_2 and f_3 are linearly dependent.

Since the number of equations, which are linearly independent, is less than that of variables, this problem forms an indeterminate system.

Indeterminate problem

Variables: x_1, x_2, x_3

Equations: $f_1(x_1, x_2, x_3)=0$

$f_2(x_1, x_2, x_3)=0$

$f_3(x_1, x_2, x_3)=0$

If f_1 and f_2 are only linearly independent, then

- ✓ Number of variables: 3
- ✓ Number of equations: 2

Since the number of equations is less than that of variables, one equation should be added to solve this problem.

Added Equation	Solution
$f_4^1 = 0$	(x_1^1, x_2^1, x_3^1)
$f_4^2 = 0$	(x_1^2, x_2^2, x_3^2)
:	

We can obtain many sets of solutions by assuming different equations.
➡ Indeterminate problem

We need a certain criteria to determine the proper solution. By adding the criteria, this problem can be formulated as an optimization problem.

Example of a Design Problem

Esthetic* Design of a Dress

Find(Design variables)

- Size, material, color, etc.

Constraints

- There are some requirements, but it is difficult to formulate them.
- By using the sense of a designer, the requirements are satisfied.

Objective function(Criteria to determine the proper design variables)

- There are many design alternatives.
- Among them, we should select the best one. How?
- Criteria: Preference, cost, etc.
- It is difficult to formulate the objective function.

➡ **Indeterminate, optimization problem**

Mathematical Model for Determination of the Principal Dimensions(L, B, D, T, C_B) of a Ship(Summary)

- "Conceptual Ship Design Equation"

Find(Design variables) $L,\ B,\ D,\ C_B$

length breadth depth block coefficient

Given(Owner's requirements) $DWT,\ V_{H\_req},\ T_{\max}(=T),\ V$

deadweight, Required cargo hold capacity, maximum draft, ship speed

Physical constraint

→ Displacement - Weight equilibrium (Weight equation) – Equality constraint

$$L \cdot B \cdot T \cdot C_B \cdot \rho_{sw} \cdot C_\alpha = DWT_{given} + LWT(L, B, D, C_B)$$

$$= DWT_{given} + C_s \cdot L^{1.6}(B + D) + C_o \cdot L \cdot B$$

$$+ C_{power} \cdot (L \cdot B \cdot T \cdot C_B)^{2/3} \cdot V^3 \quad \cdots (2.3)$$

Economical constraints(Owner's requirements)

→ Required cargo hold capacity (Volume equation) - Equality constraint

$$V_{H\_req} = C_H \cdot L \cdot B \cdot D \cdots (3.1)$$

- DFOC(Daily Fuel Oil Consumption)
  : It is related with the resistance and propulsion.
- Delivery date
  : It is related with the shipbuilding process.

Regulatory constraint

→ Freeboard regulation(1966 ICLL) - Inequality constraint

$$D \geq T + C_{FB} \cdot D \cdots (4)$$

$$Min.\ Roll\ Period : e.g.$$

$$T_R \geq 12 \sec ....(6)$$

Stability regulation(MARPOL, SOLAS, ICLL)

$$GM \geq GM_{\text{Required}} \quad \cdots (5)$$

$$GZ \geq GZ_{\text{Required}}$$

Objective function(Criteria to determine the proper principal dimensions)

$$Building\ \ Cost = C_{PS} \cdot C_s \cdot L^{1.6}(B + D) + C_{PO} \cdot C_o \cdot L \cdot B + C_{PM} \cdot C_{power} \cdot (L \cdot B \cdot T \cdot C_B)^{2/3} \cdot V^3$$

4 variables(L, B, D, C_B), 2 equality constraints((2.3), (3.1)), 3 inequality constraints((4), (5), (6))

➡ Optimization problem

Determination of the Optimal Principal Dimensions of a Ship

Optimization problem

➡ Minimize/maximize an objective function with constraints on design variables

Find(Design variables)

x_1, x_2, x_3, x_4

Equality constraint

$h(x_1, x_2, x_3, x_4) = 0$

Inequality constraint

$g(x_1, x_2, x_3, x_4) \le 0$

Objective function

$f(x_1, x_2, x_3, x_4)$

Engineering Design

Find(Design variables)

$L(=x_1), B(=x_2), D(=x_3), C_B(=x_4)$

length, breadth, depth, block coefficient

Weight equation

$$L \cdot B \cdot T \cdot C_B \cdot \rho_{sw} \cdot C_\alpha = DWT_{given} + LWT(L, B, D, C_B)$$

$$x_1 \cdot x_2 \cdot x_4 \cdot C_1 = C_2 + h'(x_1, x_2, x_3, x_4)$$

$$x_1 \cdot x_2 \cdot x_4 \cdot C_1 - C_2 - h'(x_1, x_2, x_3, x_4) = h(x_1, x_2, x_3, x_4) = 0$$

Objective function(Criteria to determine the proper principal dimensions)

$$Building\ Cost = C_{PS} \cdot C_s \cdot L^{1.6}(B + D) + C_{PO} \cdot C_o \cdot L \cdot B + C_{PM} \cdot C_{ma} \cdot NMCR$$

$$f(x_1, x_2, x_3, x_4) = C_3 \cdot x_1^{1.6} \cdot (x_2 + x_3) + C_4 \cdot x_1 \cdot x_2 + C_5$$

- $T, C_\alpha, \rho_{sw}, DWT_{given}, C_{PS}, C_s, C_{PO}, C_o, C_{PM}, C_{ma}, NMCR$ *are* Given

Characteristics of the constraint

✓ Physical constraints are usually formulated as equality constraints.
(Example of ship design: Weight equation)

✓ Economical constraints, regulatory constraints, and constraints related with politics and culture are formulated as inequality constraints.
(Example of ship design: Required cargo hold capacity(Volume equation), Freeboard regulation(1966 ICLL))

Classification of Optimization Problems and Optimization Methods

<table>
<tr><th rowspan="2"></th><th colspan="2">Unconstrained optimization problem</th><th colspan="3">Constrained optimization problem</th></tr>
<tr><th>Linear</th><th>Nonlinear</th><th>Linear</th><th colspan="2">Nonlinear</th></tr>
<tr><td>Objective function (example)</td><td>Minimize $f(\mathbf{x})$
$f(\mathbf{x}) = x_1 + 2x_2$</td><td>Minimize $f(\mathbf{x})$
$f(\mathbf{x}) = x_1^2 + x_2^2 - 3x_1x_2$</td><td>Minimize $f(\mathbf{x})$
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<tr><td>Constraint (example)</td><td>None</td><td>None</td><td>$h(\mathbf{x}) = x_1 + 5x_2 = 0$
$g(\mathbf{x}) = -x_1 \le 0$</td><td>$h(\mathbf{x}) = x_1 + 5x_2 = 0$
$g(\mathbf{x}) = -x_1 \le 0$</td><td>$g_1(\mathbf{x}) = \frac{1}{6}x_1^2 + \frac{1}{6}x_2^2 - 1.0 \le 0$
$g_2(\mathbf{x}) = -x_1 \le 0$</td></tr>
<tr><td rowspan="3">Optimization methods for continuous value</td><td colspan="2" rowspan="3">① Direct search method
- Hooke & Jeeves method
- Nelder & Mead method

② Gradient method
- Steepest descent method
- Conjugate gradient method
- Newton method
- Davidon-Fletcher-Powell(DFP) method
- Broyden-Fletcher-Goldfarb-Shanno(BFGS) method</td><td>Linear programming (LP) method is usually used.</td><td colspan="2">Penalty Function Method: Converting the constrained optimization problem to the unconstrained optimization problem by using the penalty function, the problem can be solved using unconstrained optimization method.</td></tr>
<tr><td>Simplex Method
(Linear programming)</td><td rowspan="2">Quadratic programming(QP) method</td><td>SLP(Sequential Linear Programming)
First, linearize the nonlinear problem and then obtain the solution to this linear approximation problem using the linear programming method.
And ,then, repeat the linearization</td></tr>
<tr><td></td><td>Sequential Quadratic Programming(SQP) method
First, approximate a quadratic objective function and linear constraints, find the search direction and then obtain the solution to this quadratic programming problem in this direction. And ,then, repeat the approximation</td></tr>
<tr><td>Optimization methods for discrete value</td><td colspan="5">Integer programming: ① Cut algorithm ② Enumeration algorithm ③ Constructive algorithm</td></tr>
<tr><td>Heuristic optimization</td><td colspan="5">Genetic algorithm(GA), Ant algorithm, Simulated annealing, etc.</td></tr>
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