Ship Stability

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Ship Stability

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- ☑ Ch. 2 Review of Fluid Mechanics
- ☑ Ch. 3 Transverse Stability
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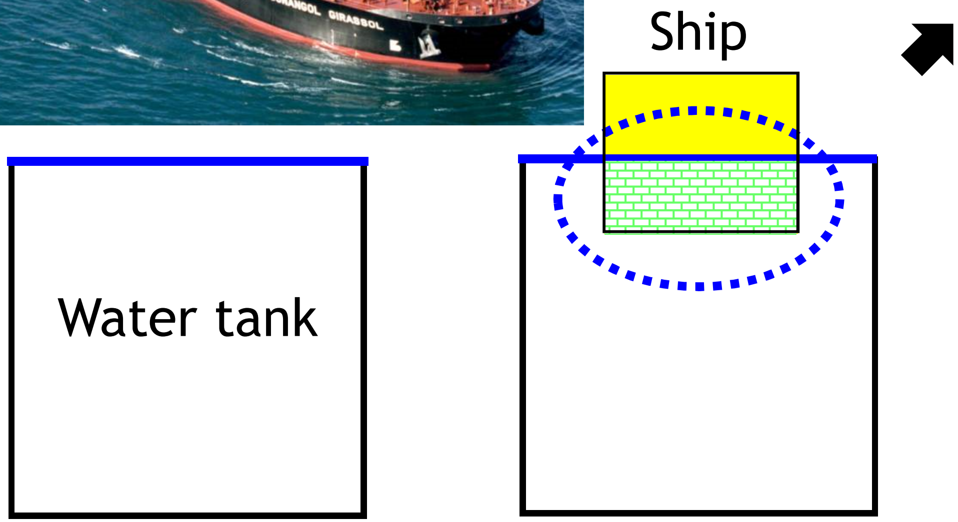
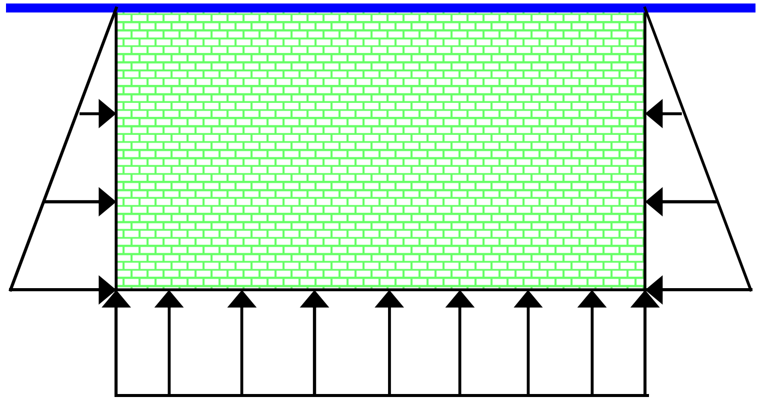
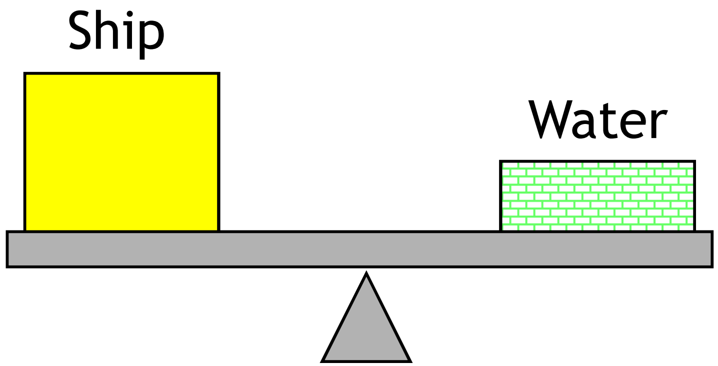
How does a ship float? (1/3)

☑ **The force that enables a ship to float** ➡ "Buoyant Force"

- **It is directed upward.**
- **It has a magnitude equal to the weight of the fluid which is displaced by the ship.**







How does a ship float? (2/3)

☑ Archimedes' Principle

- The magnitude of the buoyant force acting on a floating body in the fluid is equal to the weight of the fluid which is displaced by the floating body.
- The direction of the buoyant force is opposite to the gravitational force.

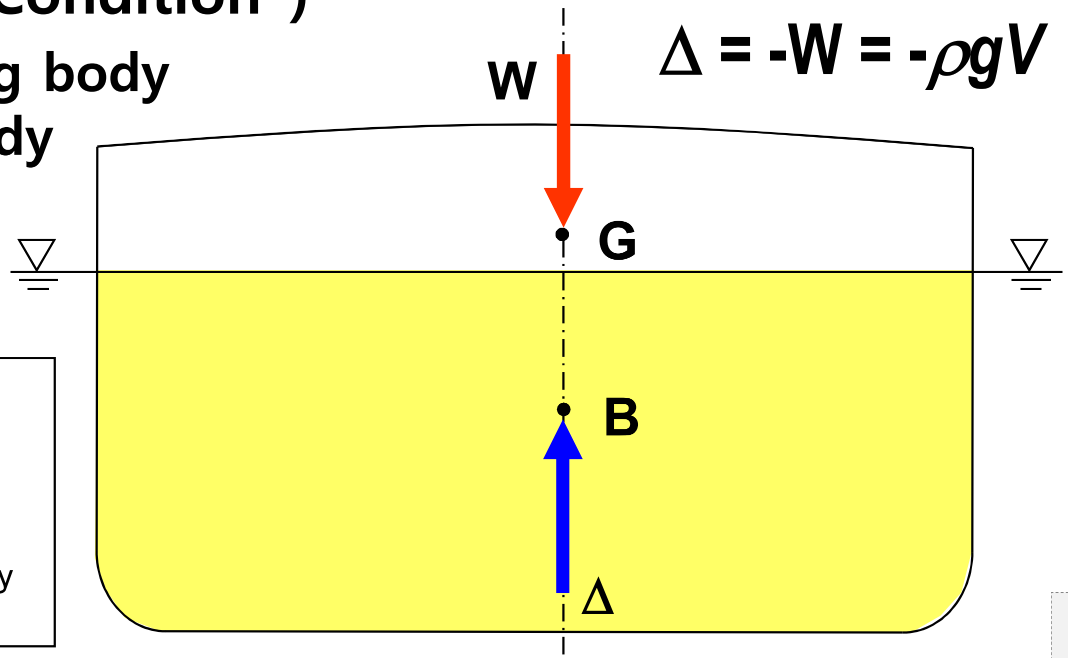
Buoyant force of a floating body
= the weight of the fluid which is displaced by the floating body ("Displacement")
➡ Archimedes' Principle

☑ Equilibrium State ("Floating Condition")

- Buoyant force of the floating body
= Weight of the floating body

$\therefore$ Displacement = Weight

$$\Delta = -W = -\rho g V$$



G: Center of gravity
B: Center of buoyancy
W: Weight, Δ: Displacement
ρ: Density of fluid
V: Submerged volume of the floating body
(Displacement volume, ∇)

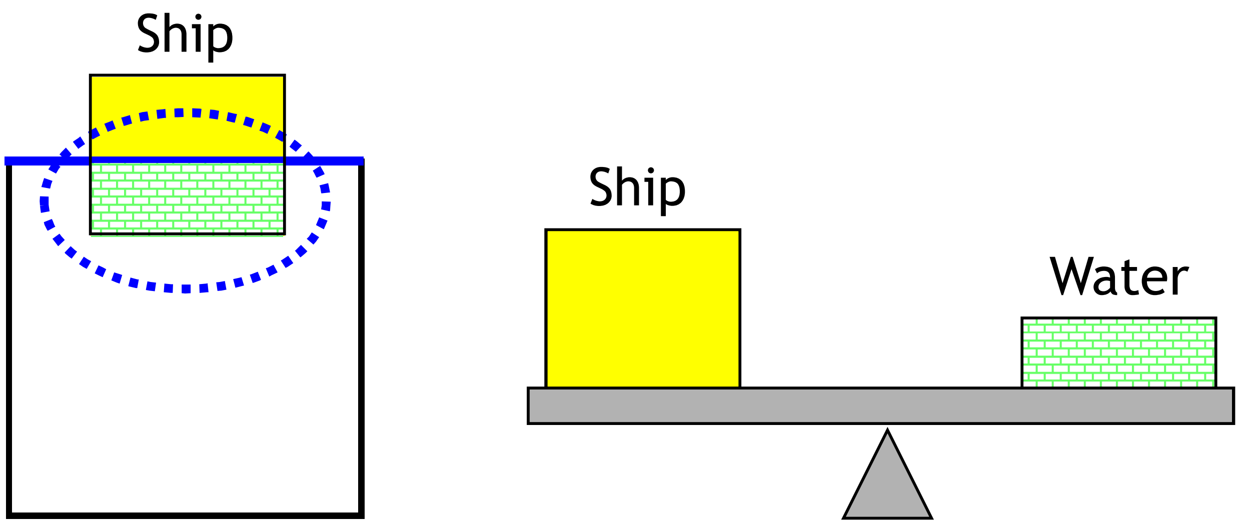
How does a ship float? (3/3)

☑ **Displacement(Δ) = Buoyant Force = Weight(W)**

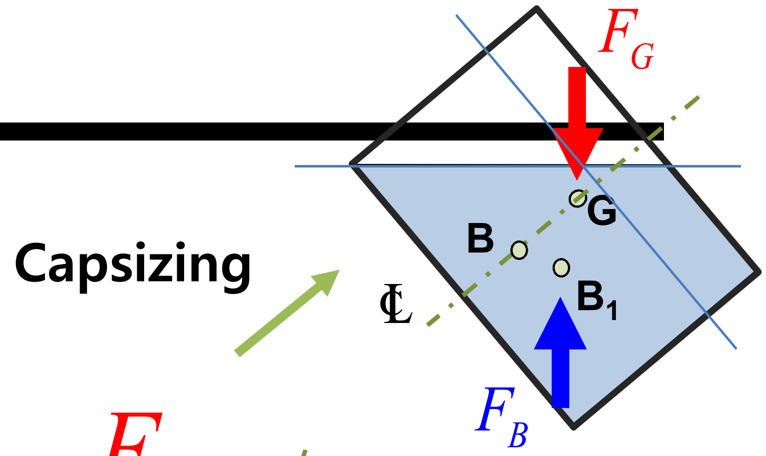
$$\boxed{\Delta = L \cdot B \cdot T \cdot C_B \cdot \rho} = \boxed{W = LWT + DWT}$$

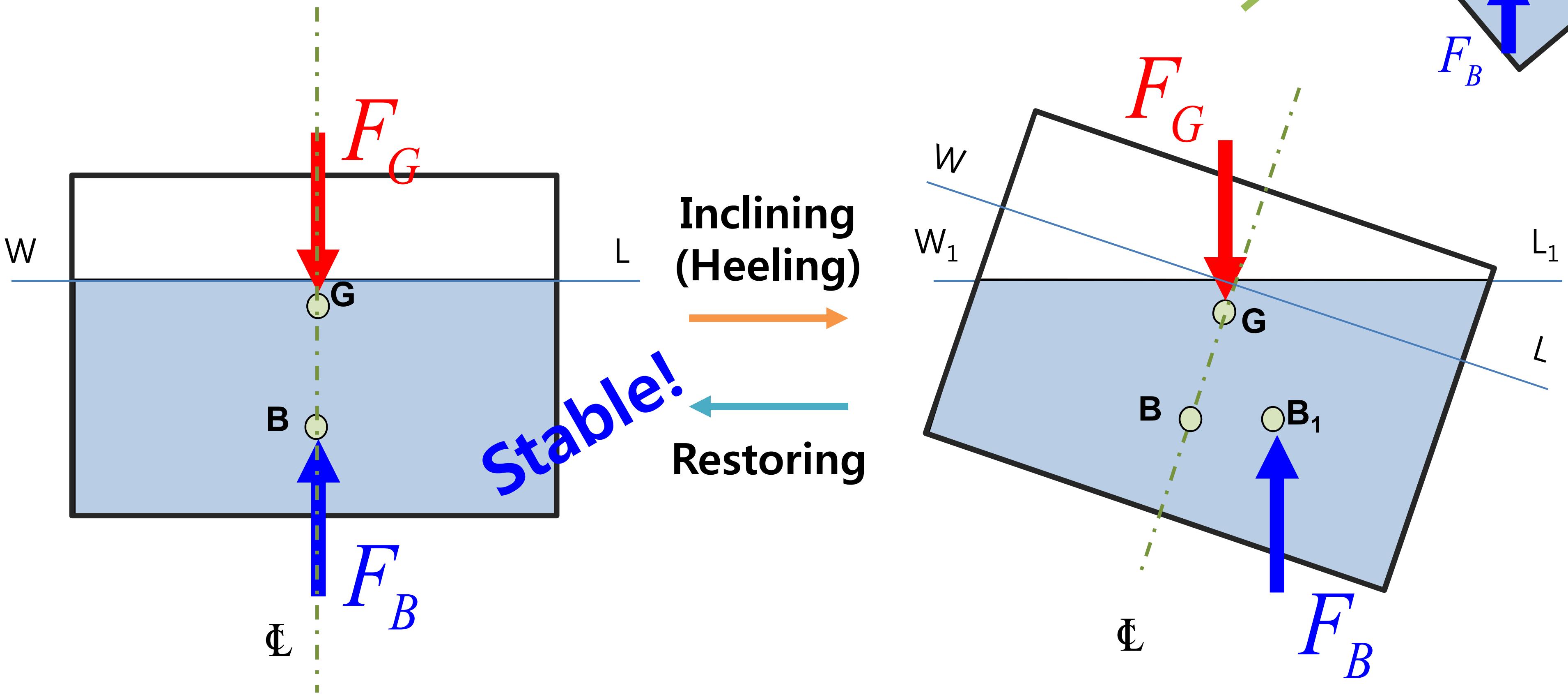
T: Draft
C_B: Block coefficient
ρ: Density of sea water
LWT: Lightweight
DWT: Deadweight

☑ **Weight = Ship weight (Lightweight) + Cargo weight(Deadweight)**



What is "Stability"?





Stability = Stable + Ability

Ch. 1 Introduction to Ship Stability

What is a "Hull form"?

☑ **Hull form**

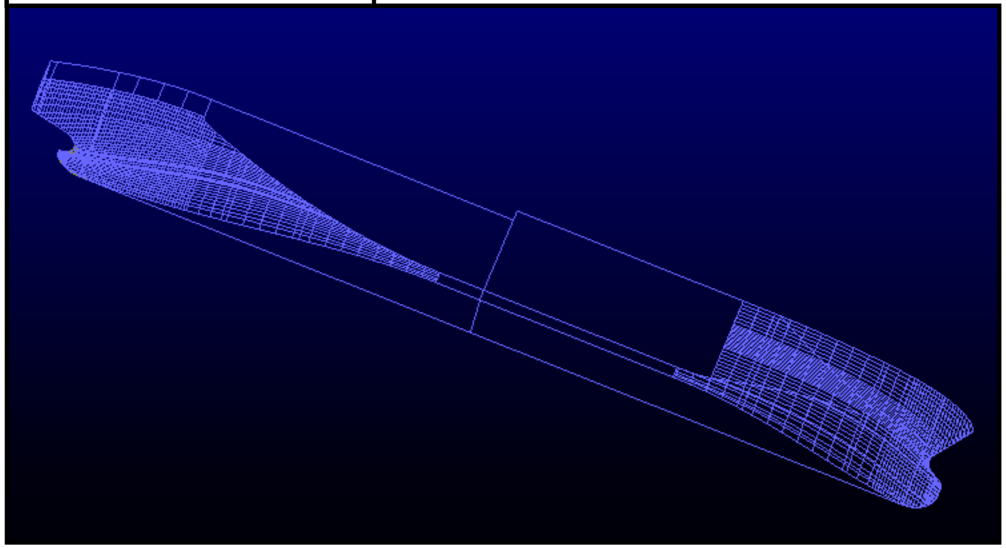
- **Outer shape of the hull** that is streamlined in order to satisfy requirements of a ship owner such as a deadweight, ship speed, and so on
- Like a skin of human

☑ **Hull form design**

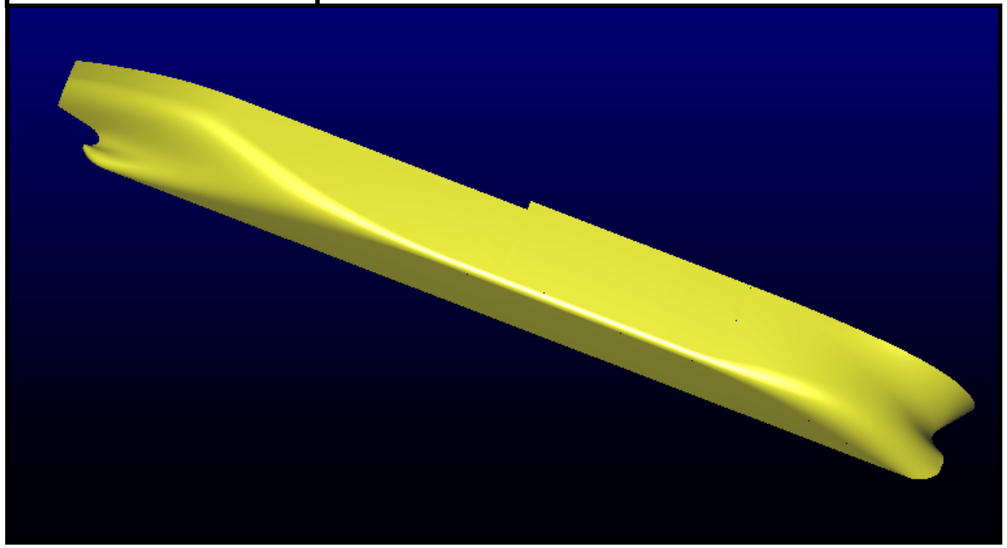
- Design task that designs the hull form

Hull form of the VLCC(Very Large Crude oil Carrier)

Wireframe model

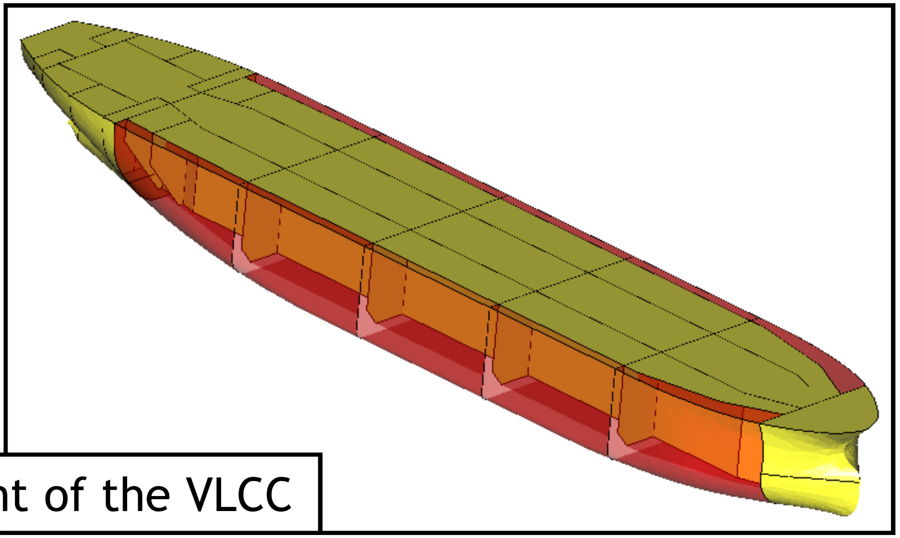


Surface model



What is a “Compartment”?

- ☑ Compartment
  - Space to load cargos in the ship
  - It is divided by a bulkhead which is a diaphragm or peritoneum of human.
- ☑ Compartment design (General arrangement design)
  - Compartment modeling + Ship calculation
- ☑ Compartment modeling
  - Design task that divides the interior parts of a hull form into a number of compartments
- ☑ Ship calculation (Naval architecture calculation)
  - Design task that evaluates whether the ship satisfies the required cargo capacity by a ship owner and, at the same time, the international regulations related to stability, such as MARPOL and SOLAS, or not



Compartment of the VLCC

What is a "Hull structure"?

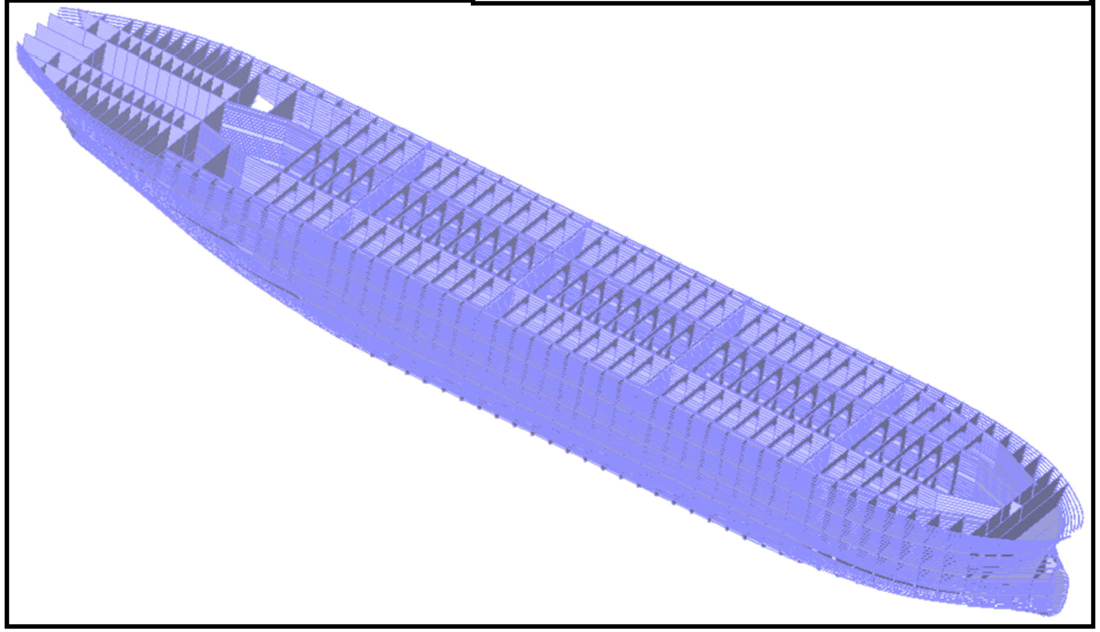
☑ **Hull structure**

- **Frame of a ship** comprising of a number of hull structural parts such as plates, stiffeners, brackets, and so on
- Like a skeleton of human

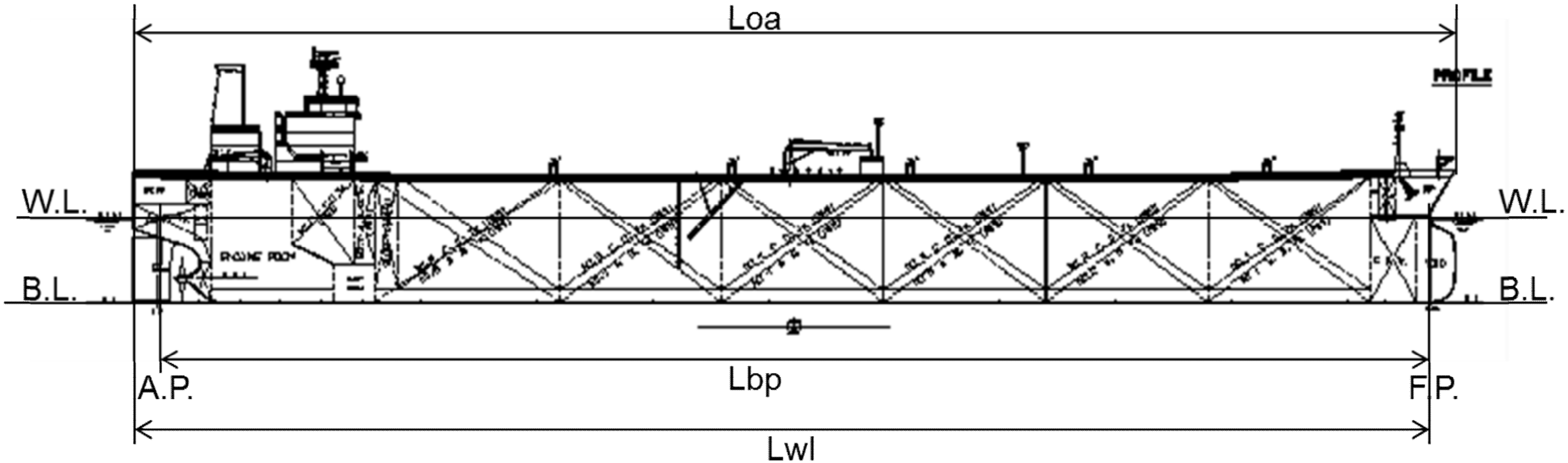
☑ **Hull structural design**

- Design task that determines the specifications of the hull structural parts such as the size, material, and so on

Hull structure of the VLCC



Principal Characteristics (1/2)



☑ LOA (Length Over All) [m] : Maximum Length of Ship

☑ LBP (Length Between Perpendiculars (A.P. ~ F.P.)) [m]

- A.P. : After perpendicular (normally, center line of the rudder stock)
- F.P. : Inter-section line between designed draft and fore side of the stem, which is perpendicular to the baseline

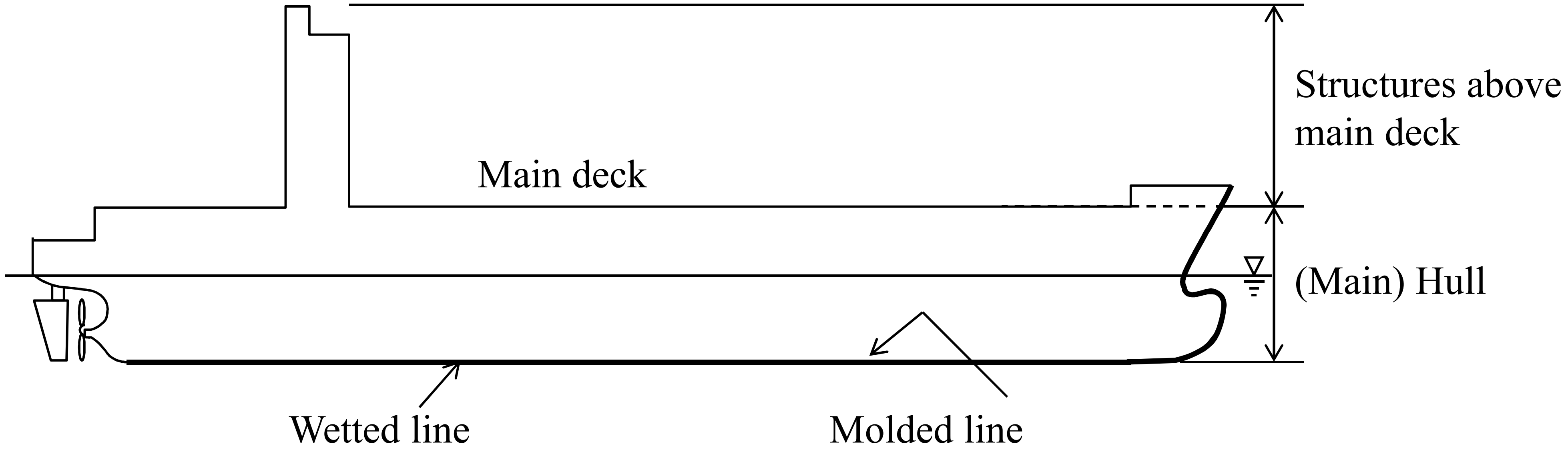
☑ Lf (Freeboard Length) [m] : Basis of freeboard assignment, damage stability calculation

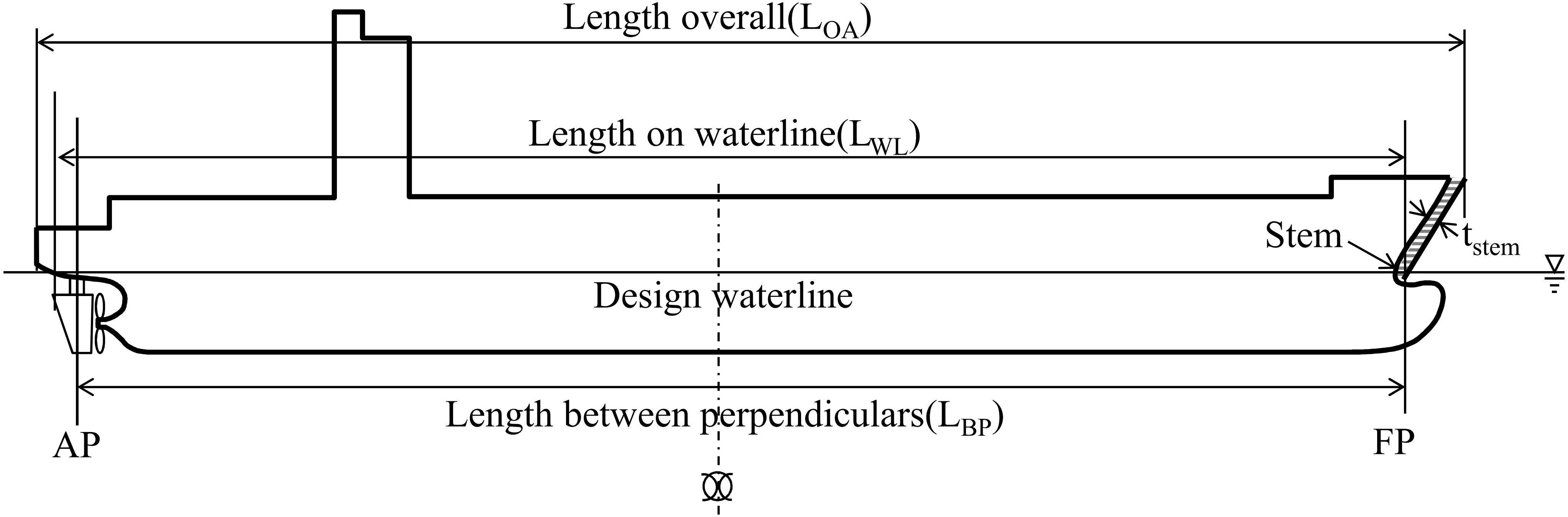
- 96% of Lwl at 0.85D or Lbp at 0.85D, whichever is greater

☑ Rule Length (Scantling Length) [m] : Basis of structural design and equipment selection

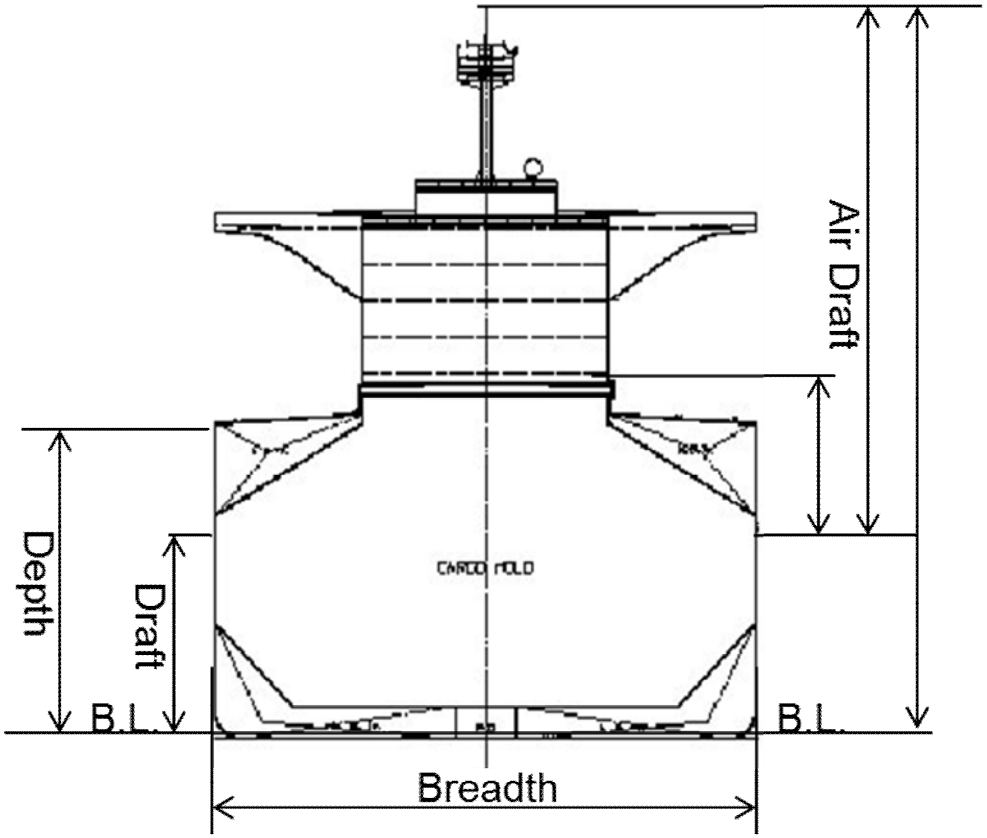
- Intermediate one among (0.96 Lwl at Ts, 0.97 Lwl at Ts, Lbp at Ts)

Definitions for the Length of a Ship





Principal Characteristics (2/2)



- **B (Breadth) [m] : Maximum breadth of the ship, measured amidships**
  - B,molded : excluding shell plate thickness
  - B,extreme : including shell plate thickness

- **D (Depth) [m] : Distance from the baseline to the deck side line**
  - D,molded : excluding keel plate thickness
  - D,extreme : including keel plate thickness

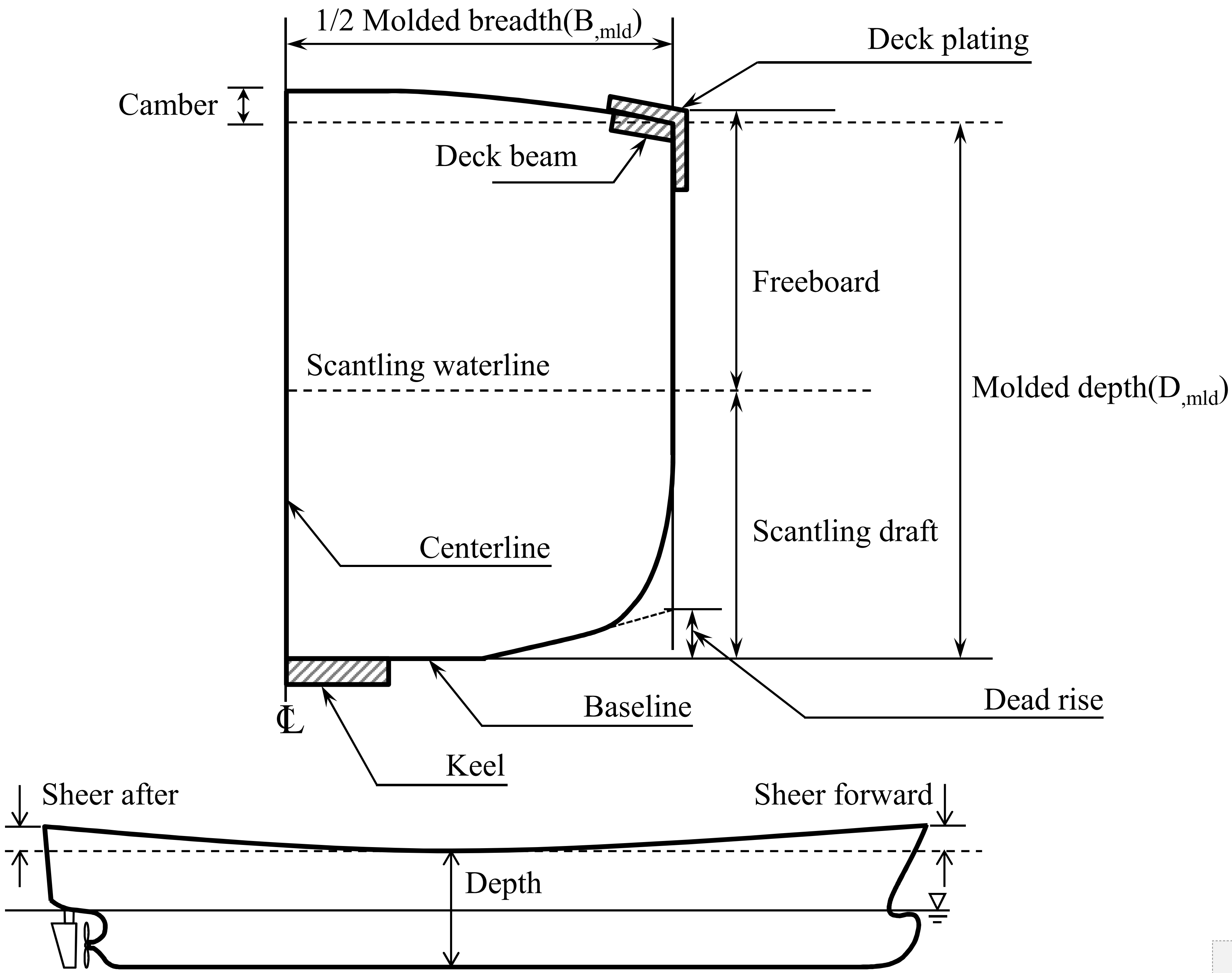
- **Td (Designed Draft) [m] : Main operating draft**
  - In general, basis of ship's deadweight and speed/power performance

- **Ts (Scantling Draft) [m] : Basis of structural design**

- **Air Draft**

  Distance (height above waterline only or including operating draft) restricted by the port facilities, navigating route, etc.
  - Air draft from baseline to the top of the mast
  - Air draft from waterline to the top of the mast
  - Air draft from waterline to the top of hatch cover
  - …

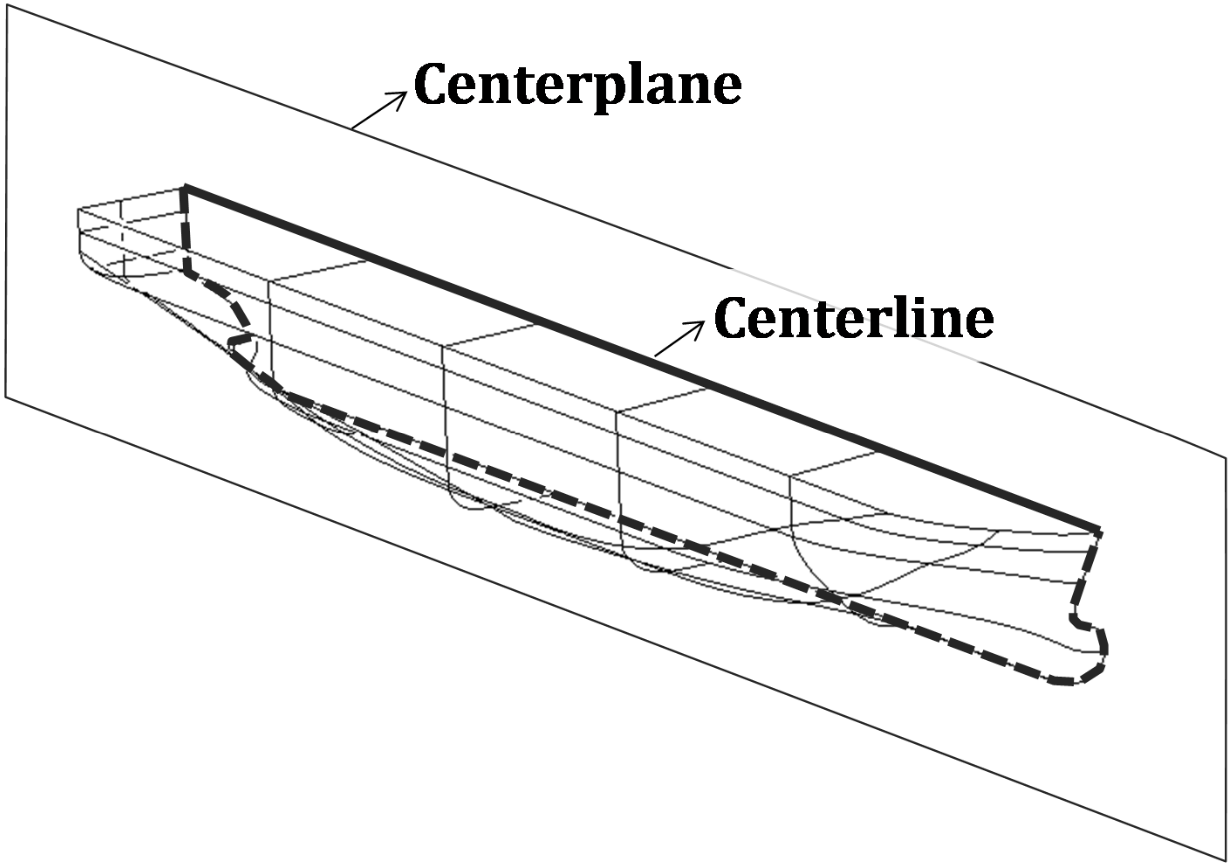
Definitions for the Breadth and Depth of a Ship



Static Equilibrium

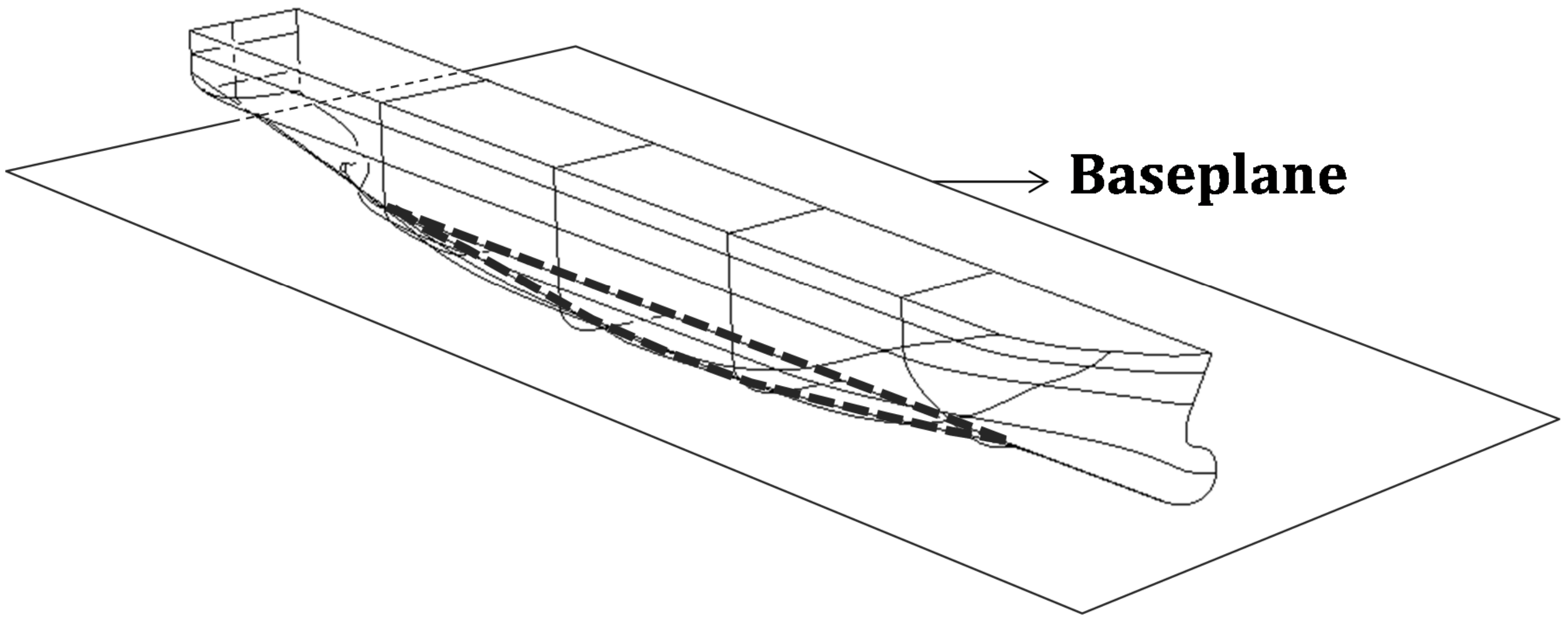
Center plane

Before defining the coordinate system of a ship, we first introduce three planes, which are all standing perpendicular to each other.



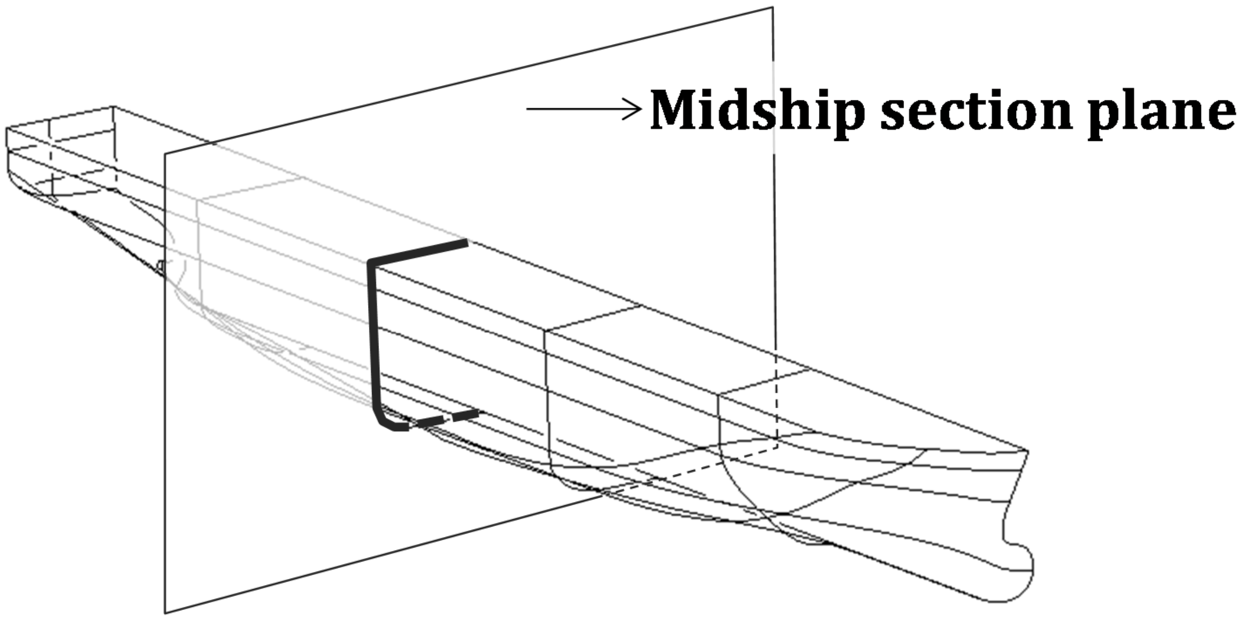
Generally, a ship is **symmetrical** about starboard and port.
The first plane is the vertical longitudinal plane of symmetry, or **center plane**.

Base plane



The second plane is the horizontal plane, containing the bottom of the ship, which is called **base plane**.

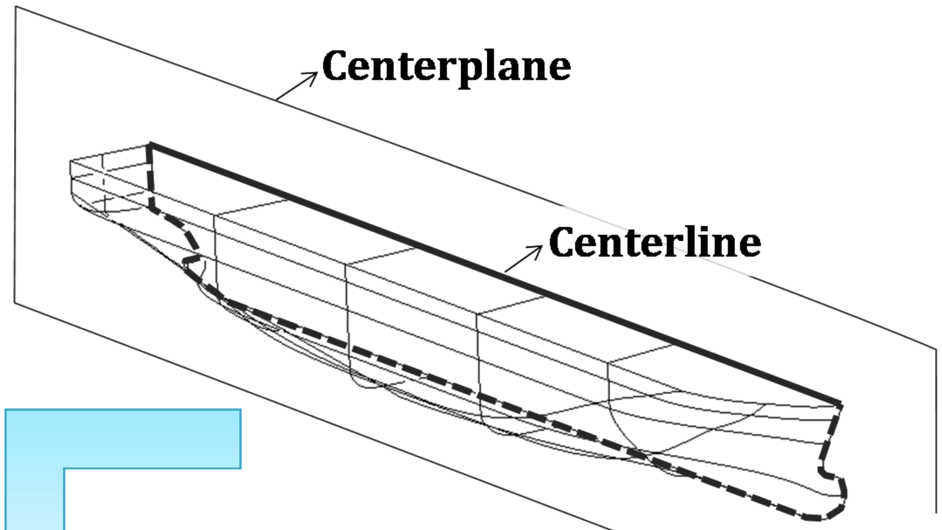
Midship section plane

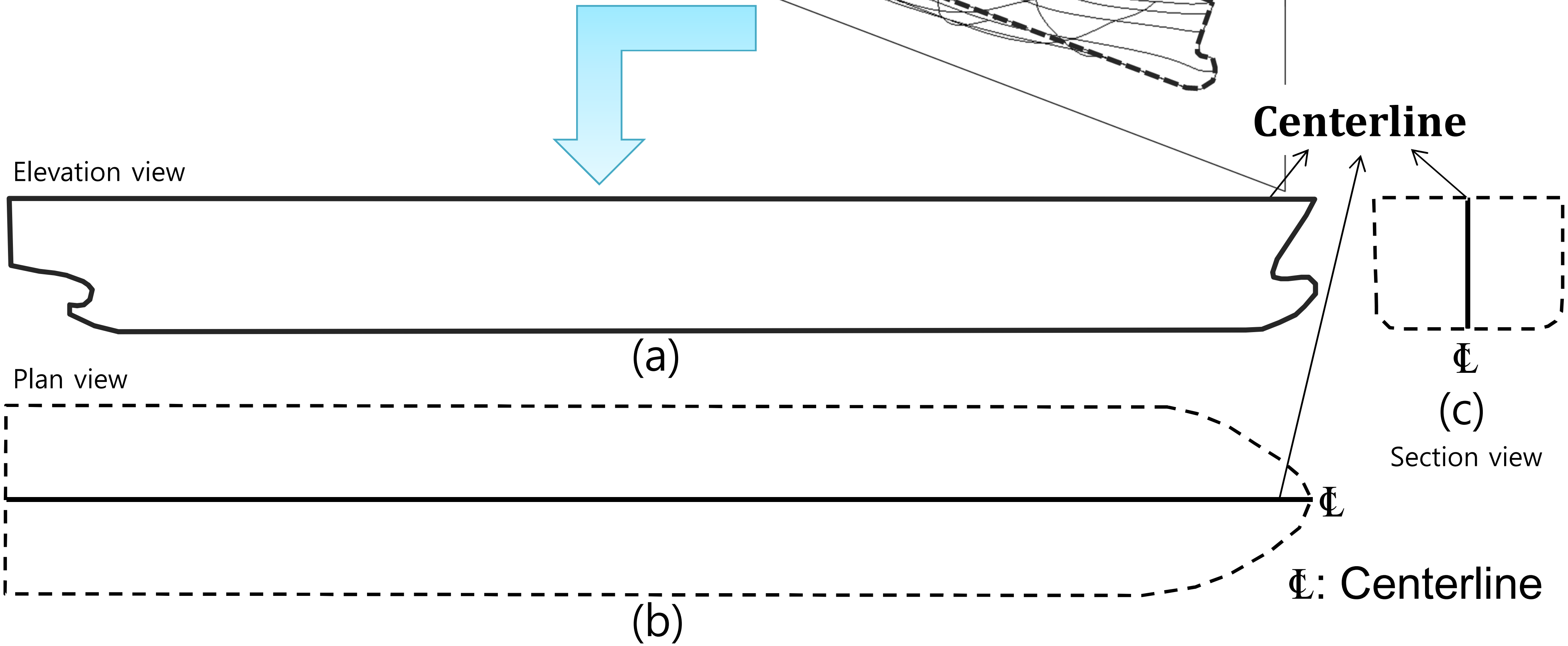


The third plane is the vertical transverse plane through the midship, which is called **midship section plane**.

Centerline in (a) Elevation view, (b) Plan view, and (c) Section view

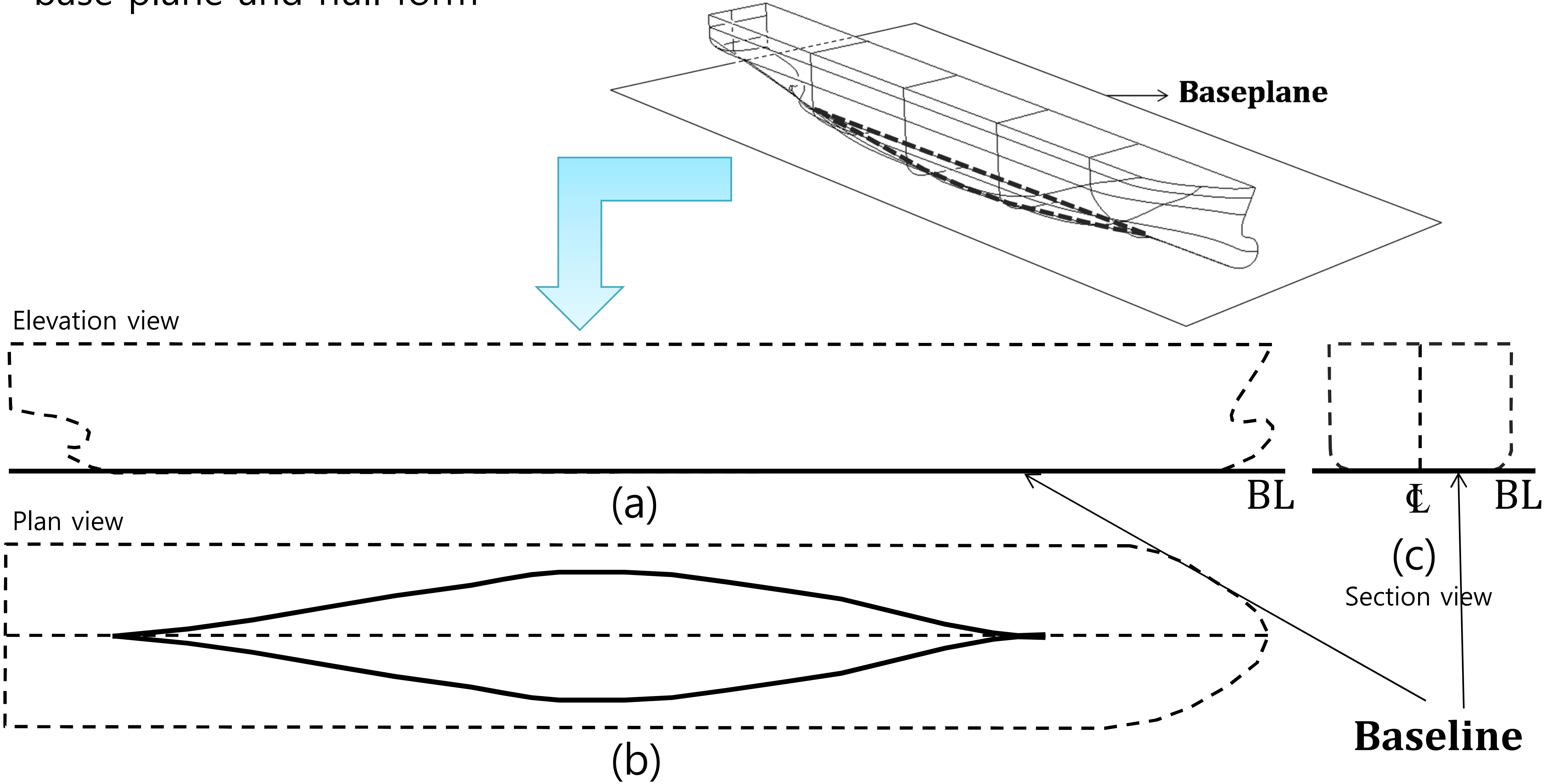
Centerline:
Intersection curve between center plane and hull form



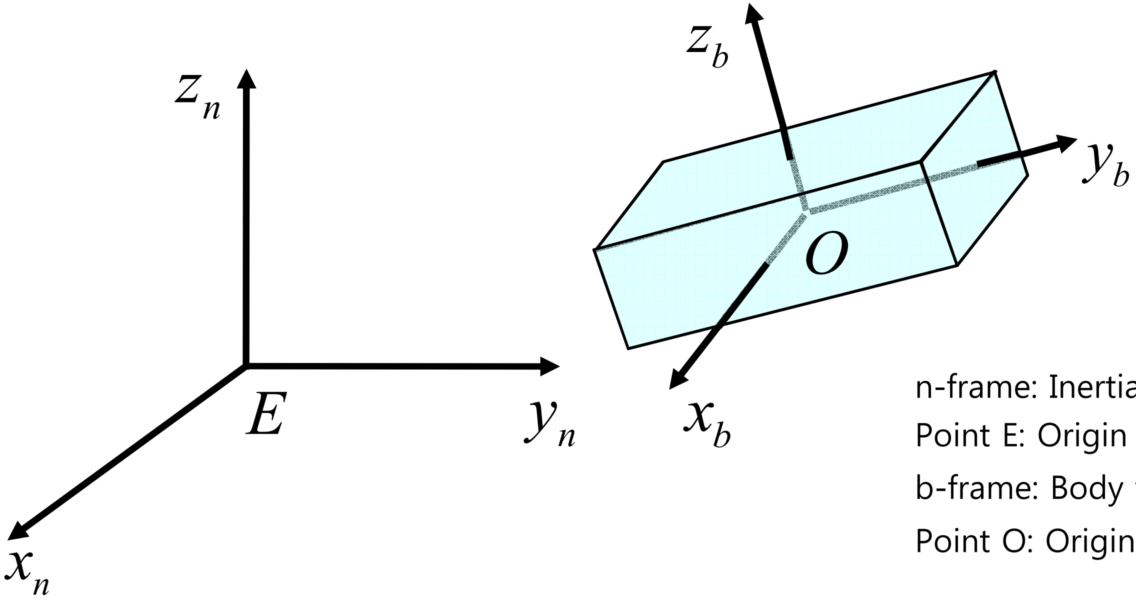


Baseline in
(a) Elevation view, (b) Plan view, and (c) Section view

Baseline:
Intersection curve between base plane and hull form



System of coordinates



n-frame: Inertial frame $x_n\, y_n\, z_n$ or $x\, y\, z$
Point E: Origin of the inertial frame(n-frame)
b-frame: Body fixed frame $x_b\, y_b\, z_b$ or $x'\, y'\, z'$
Point O: Origin of the body fixed frame(b-frame)

1) ***Body fixed coordinate system***
The right handed coordinate system with the axis called x_b(or x'), y_b(or y'), and z_b(or z') is **fixed to the object**. This coordinate system is called ***body fixed coordinate system*** or ***body fixed reference frame(b-frame)***.

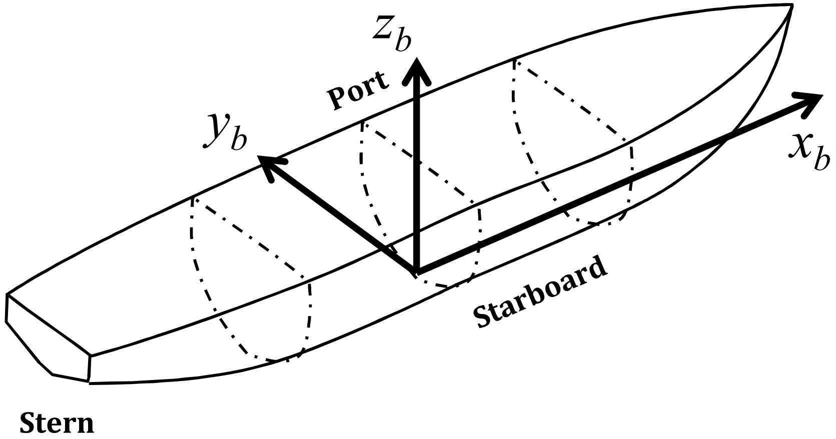
2) ***Space fixed coordinate system***
The right handed coordinate system with the axis called x_n(or x), y_n(or y) and z_n(or z) is **fixed to the space**. This coordinate system is called ***space fixed coordinate system*** or ***space fixed reference frame*** or ***inertial frame(n-frame)***.

In general, a change in the position and orientation of the object is described with respect to the inertial frame. Moreover Newton's 2nd law is only valid for the inertial frame.

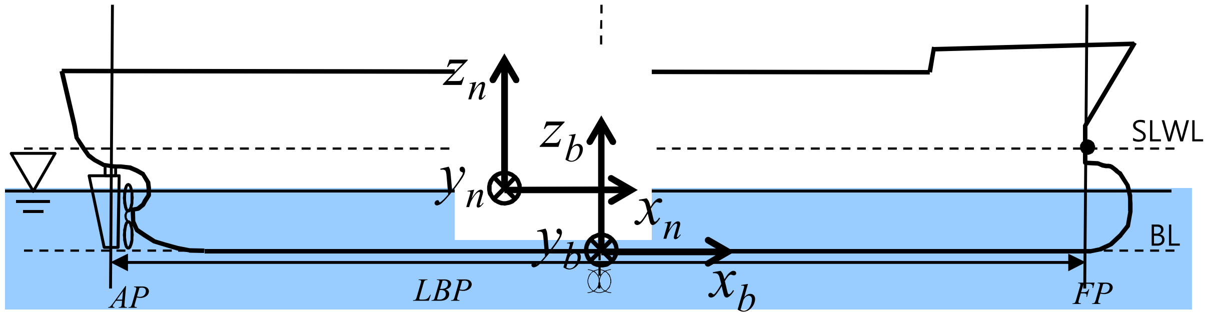
System of coordinates for a ship

Body fixed coordinate system(b-frame): Body fixed frame $x_b\ y_b\ z_b$ or $x'\ y'\ z'$

Space fixed coordinate system(n-frame): Inertial frame $x_n\ y_n\ z_n$ or $x\ y\ z$



(a)



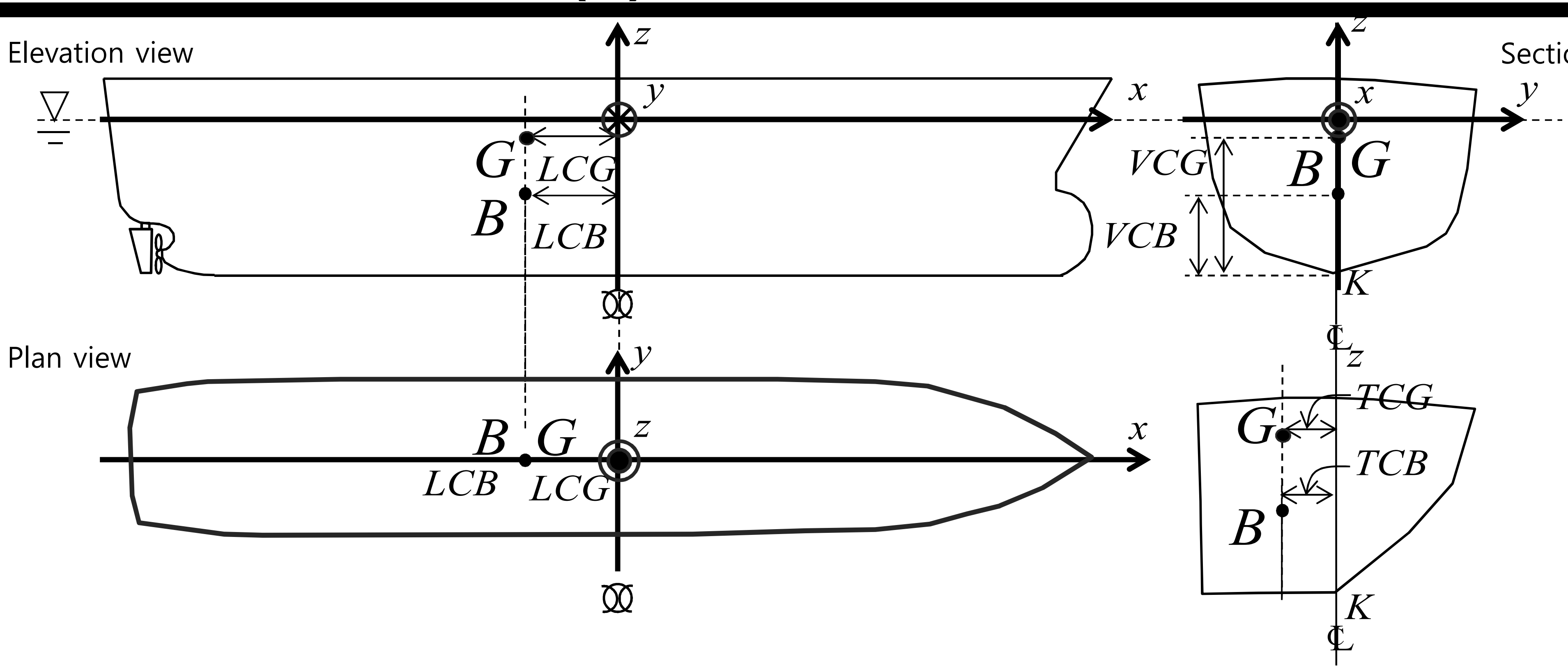
AP: aft perpendicular
FP: fore perpendicular
LBP: length between perpendiculars.
BL: baseline
SLWL: summer load waterline

⌘: midship

(b)

Center of buoyancy (B) and Center of mass (G)

K: keel
LCB: longitudinal center of buoyancy
VCB: vertical center of buoyancy
TCB: transverse center of buoyancy
LCG: longitudinal center of gravity
VCG: vertical center of gravity
TCG: transverse center of gravity



※ In the case that the shape of a ship is asymmetrical with respect to the centerline.

Center of buoyancy (B)

It is the point at which all the vertically upward forces of support (buoyant force) can be considered to act.

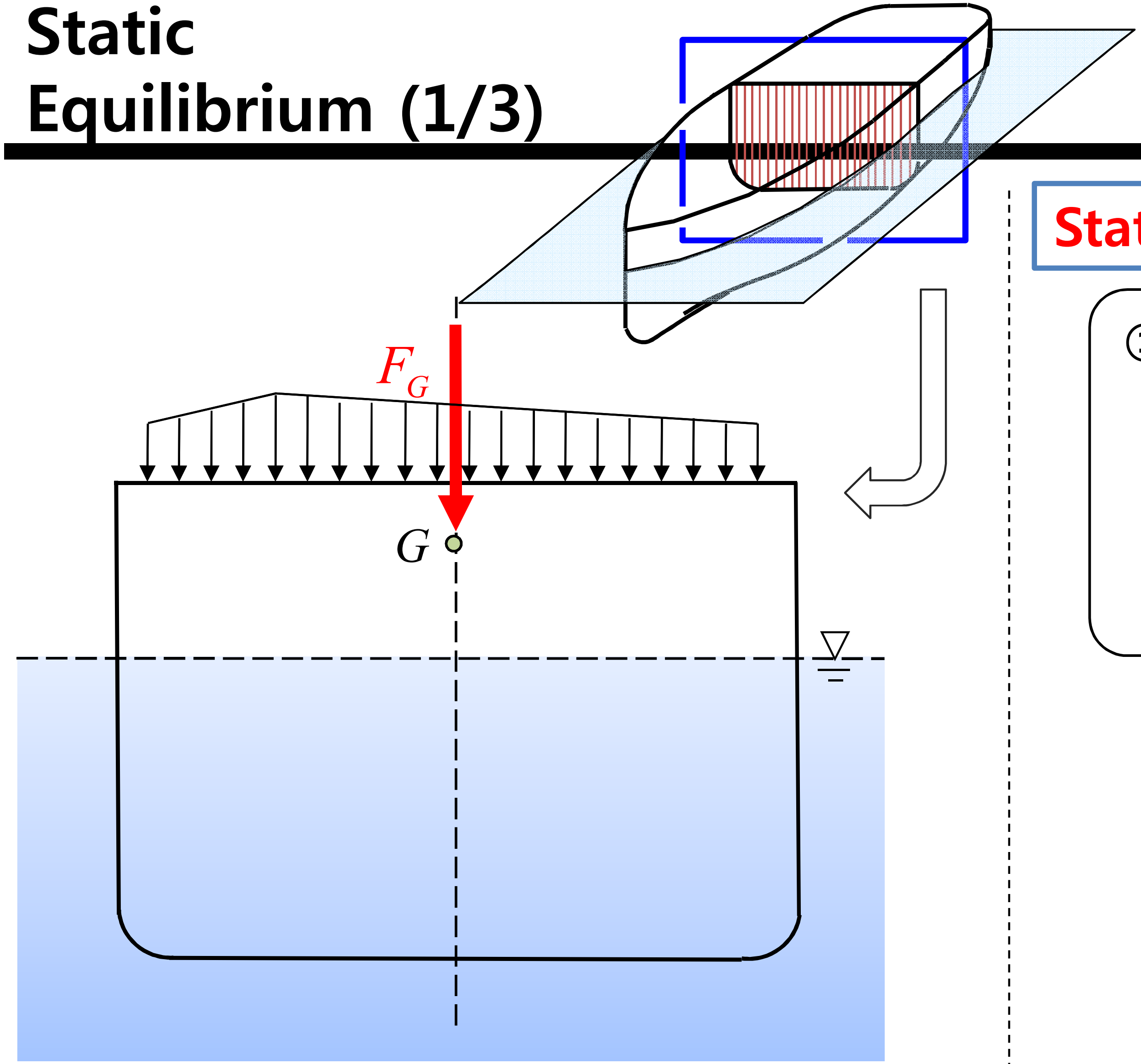
It is equal to the center of volume of the submerged volume of the ship. Also, It is equal to the first moment of the submerged volume of the ship about particular axis divided by the total buoyant force (displacement).

Center of mass or Center of gravity (G)

It is the point at which all the vertically downward forces of weight of the ship(gravitational force) can be considered to act.

It is equal to the first moment of the weight of the ship about particular axis divided by the total weight of the ship.

Static Equilibrium (1/3)



m: mass of ship
a: acceleration of ship

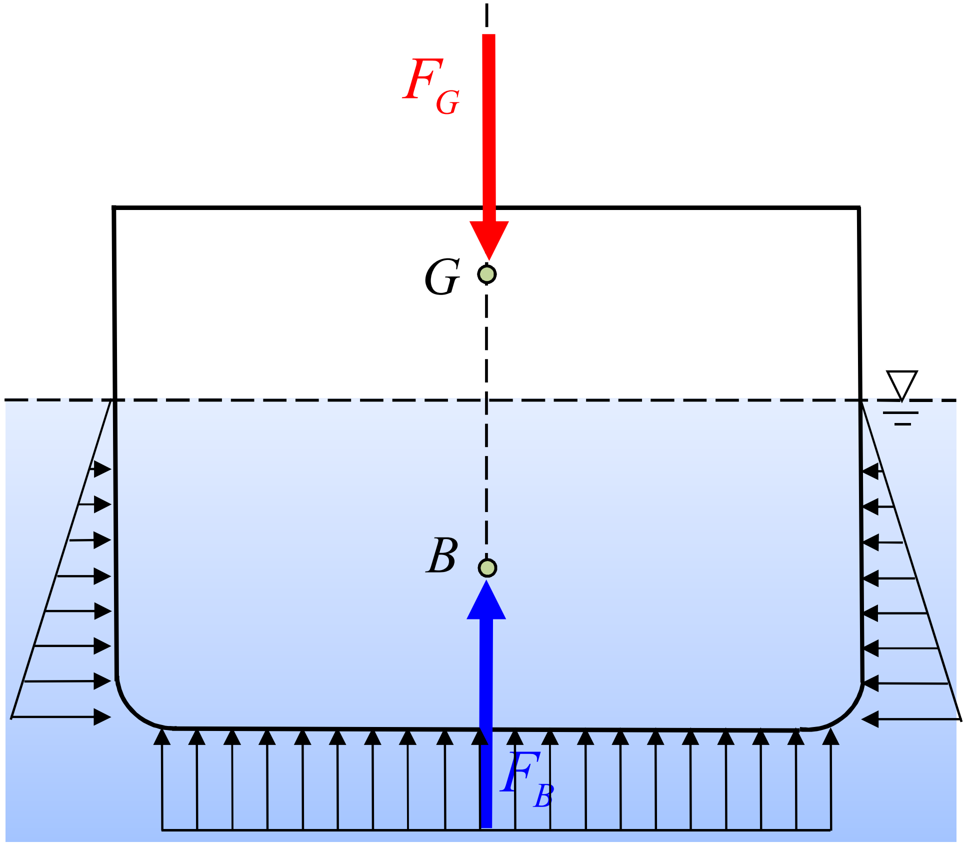
G: Center of mass
F_G : Gravitational force of ship

Static Equilibrium

① **Newton's 2nd law**

$$ma = \sum F$$
$$= -F_G$$

Static Equilibrium (2/3)



B: Center of buoyancy at upright position(center of volume of the submerged volume of the ship)

F_B : Buoyant force acting on ship

Static Equilibrium

① **Newton's 2nd law**

$$ma = \sum F$$

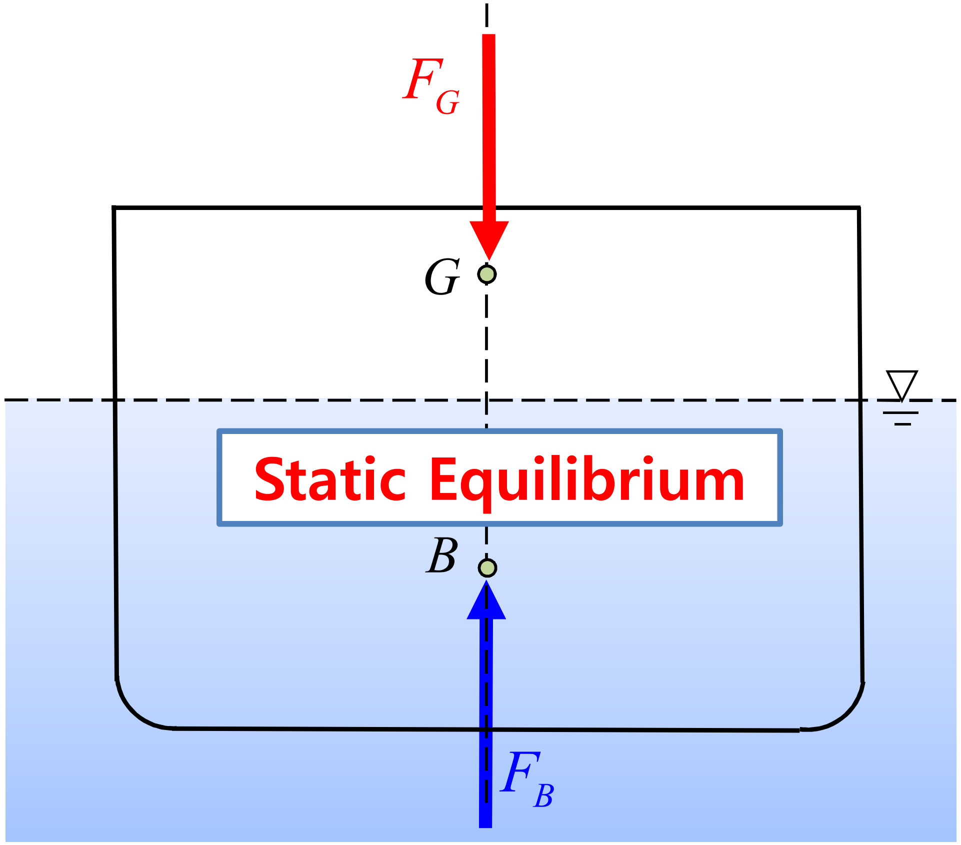
$$= -F_G + F_B$$

for the ship to be in static equilibrium

$$0 = \sum F \quad ,(\because a = 0)$$

$$\boxed{\therefore F_G = F_B}$$

Static Equilibrium (3/3)



τ : Moment
I : Mass moment of interia
ω : Angular velocity

Static Equilibrium

① **Newton's 2nd law**

$$ma = \sum F$$
$$= -F_G + F_B$$

for the ship to be in static equilibrium

$$0 = \sum F \quad ,(\because a = 0)$$

$$\boxed{\therefore F_G = F_B}$$

② **Euler equation**

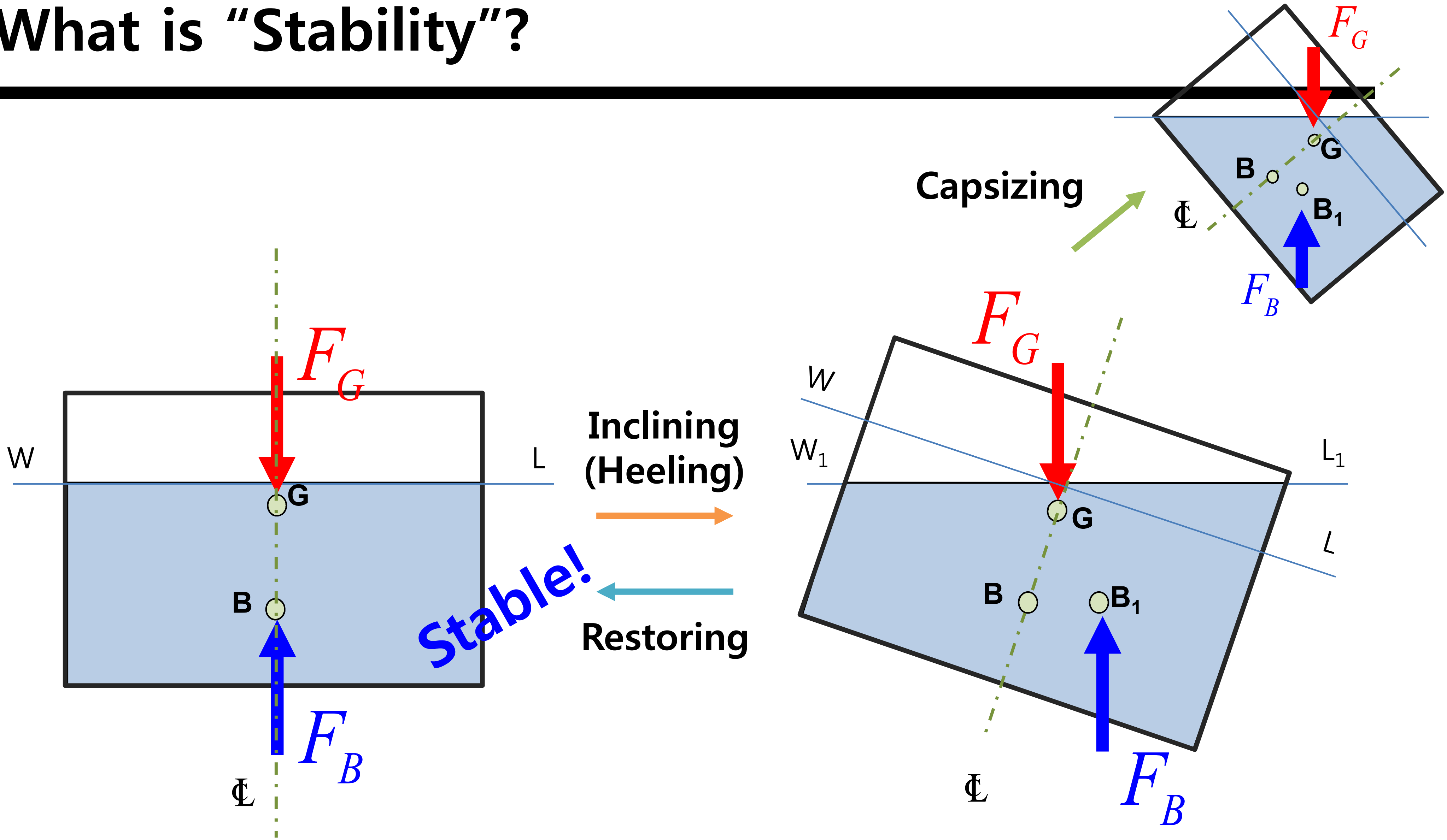
$$I\dot{\omega} = \sum \tau$$

for the ship to be in static equilibrium

$$0 = \sum \tau \quad ,(\because \dot{\omega} = 0)$$

When the buoyant force(F_B) lies on the **same line** of action as the gravitational force(F_G), total summation of the moment becomes 0.

What is "Stability"?

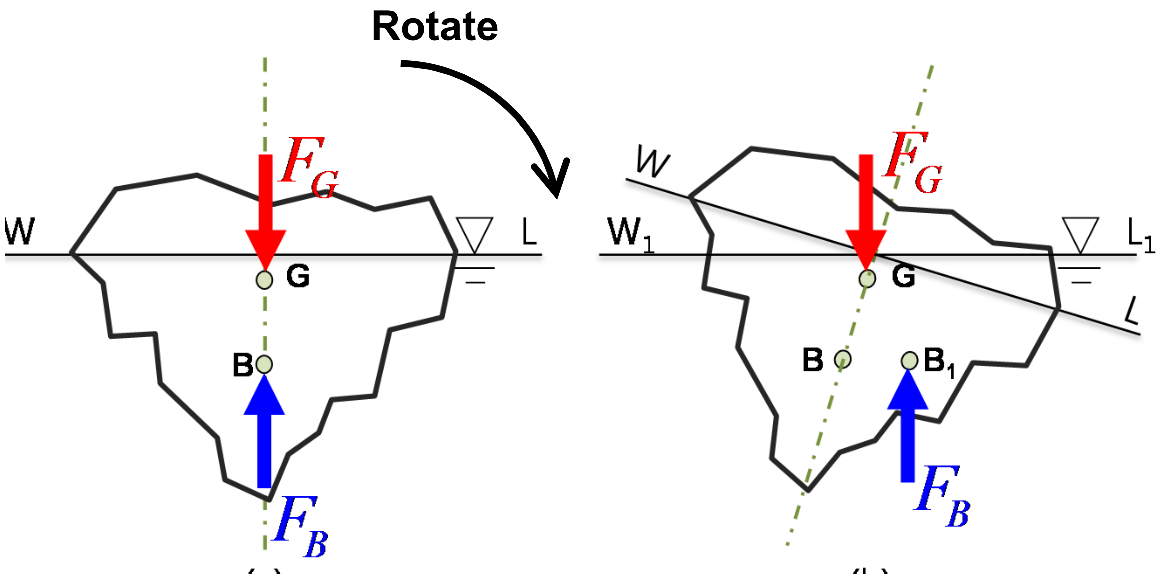


Stability = Stable + Ability

Stability of a floating object

- You have a torque on this object relative to any point that you choose. It does not matter where you pick a point.

- The torque will only be zero when the buoyant force and the gravitational force are on one line. Then the torque becomes zero.



(a) (b)

Static Equilibrium

① **Newton's 2nd law**

$$ma = \sum F$$
$$= -F_G + F_B$$

for the ship to be in static equilibrium

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Stability of a ship

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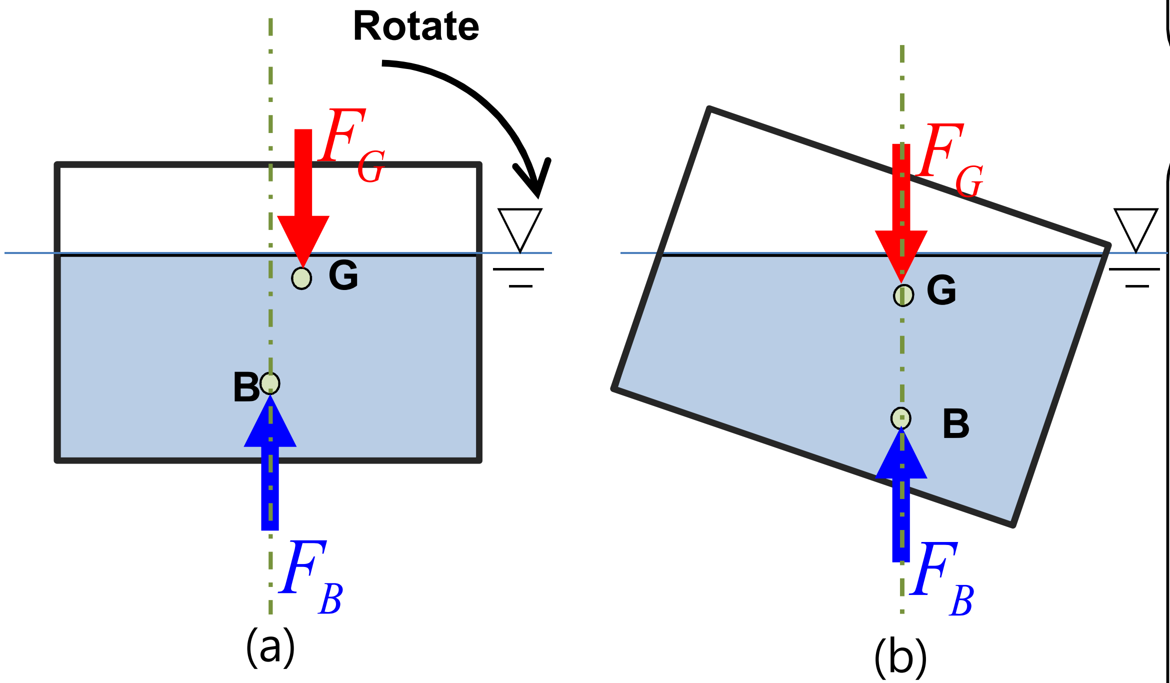

Static Equilibrium

Static Equilibrium

① **Newton's 2nd law**

$$ma = \sum F$$
$$= -F_G + F_B$$

for the ship to be in static equilibrium

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② **Euler equation**

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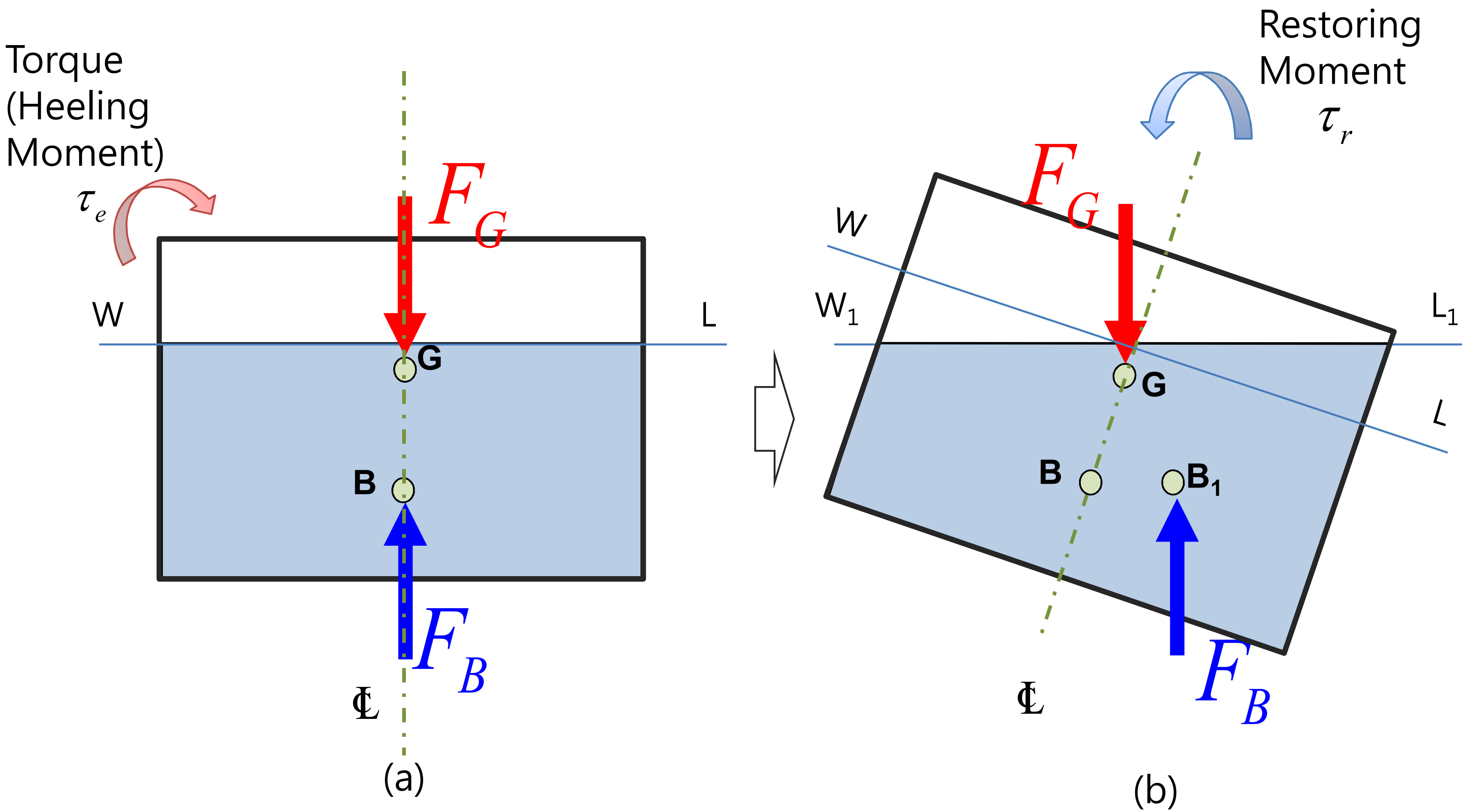
When the buoyant force(F_B) lies on the **same line** of action as the gravitational force(F_G), total summation of the moment becomes 0.

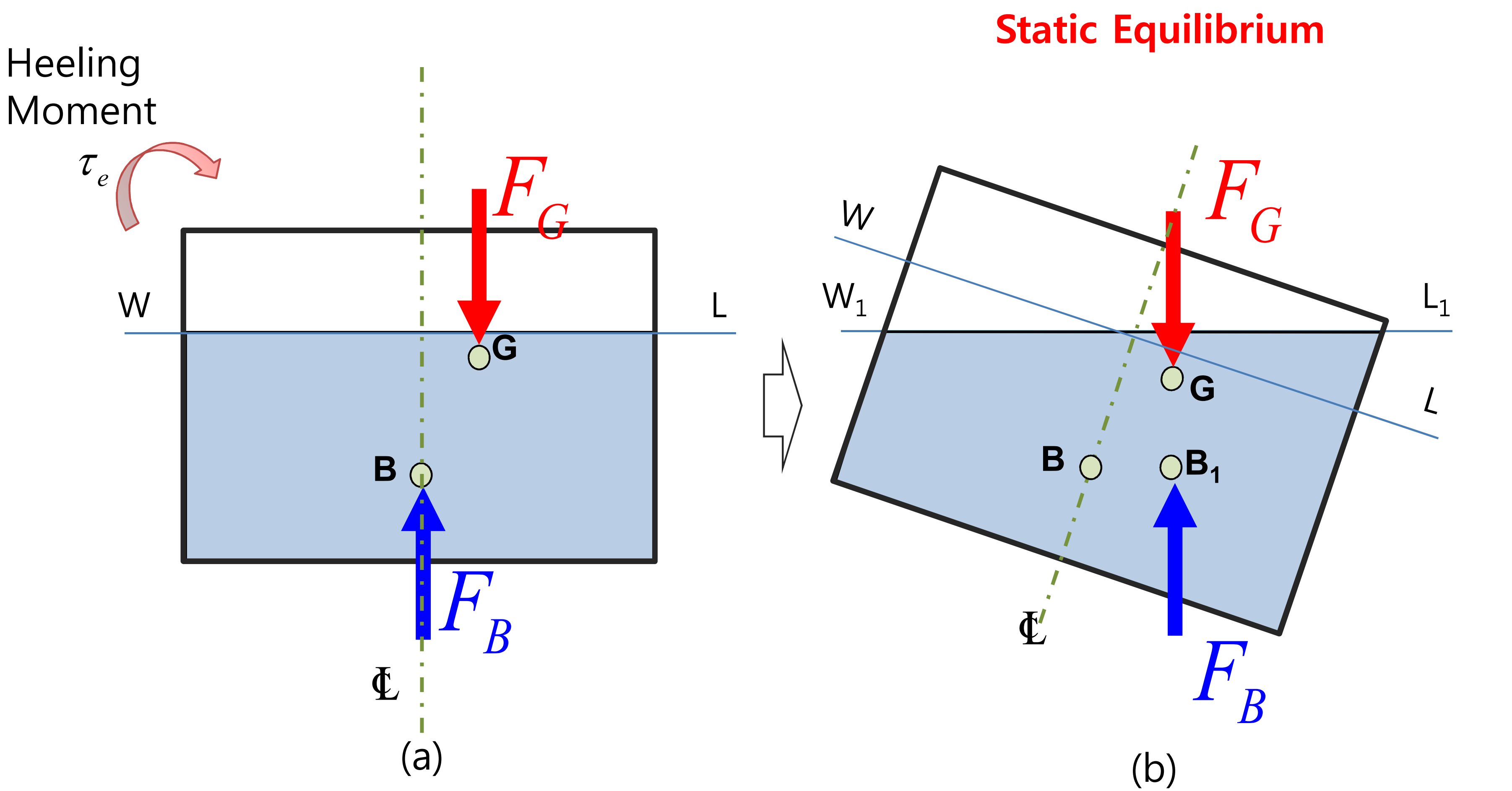
Interaction of weight and buoyancy of a floating body (1/2)



Euler equation: $I\dot{\omega} = \sum \tau \Rightarrow \dot{\omega} \neq 0$

Interaction of weight and buoyancy resulting in intermediate state

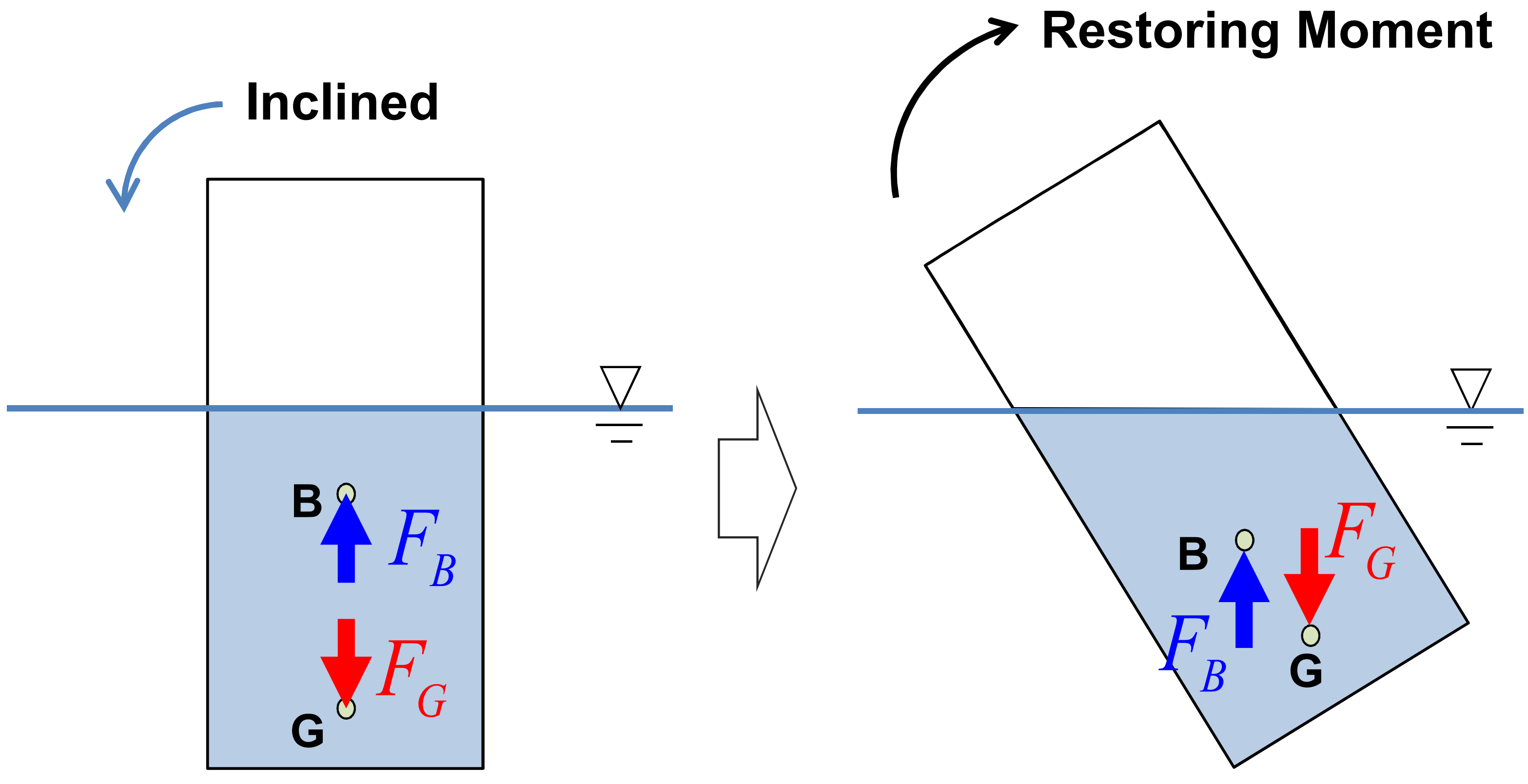
Interaction of weight and buoyancy of a floating body (2/2)



Euler equation: $I\dot{\omega} = \sum \tau \Rightarrow \dot{\omega} = 0$

Interaction of weight and buoyancy resulting in static equilibrium state

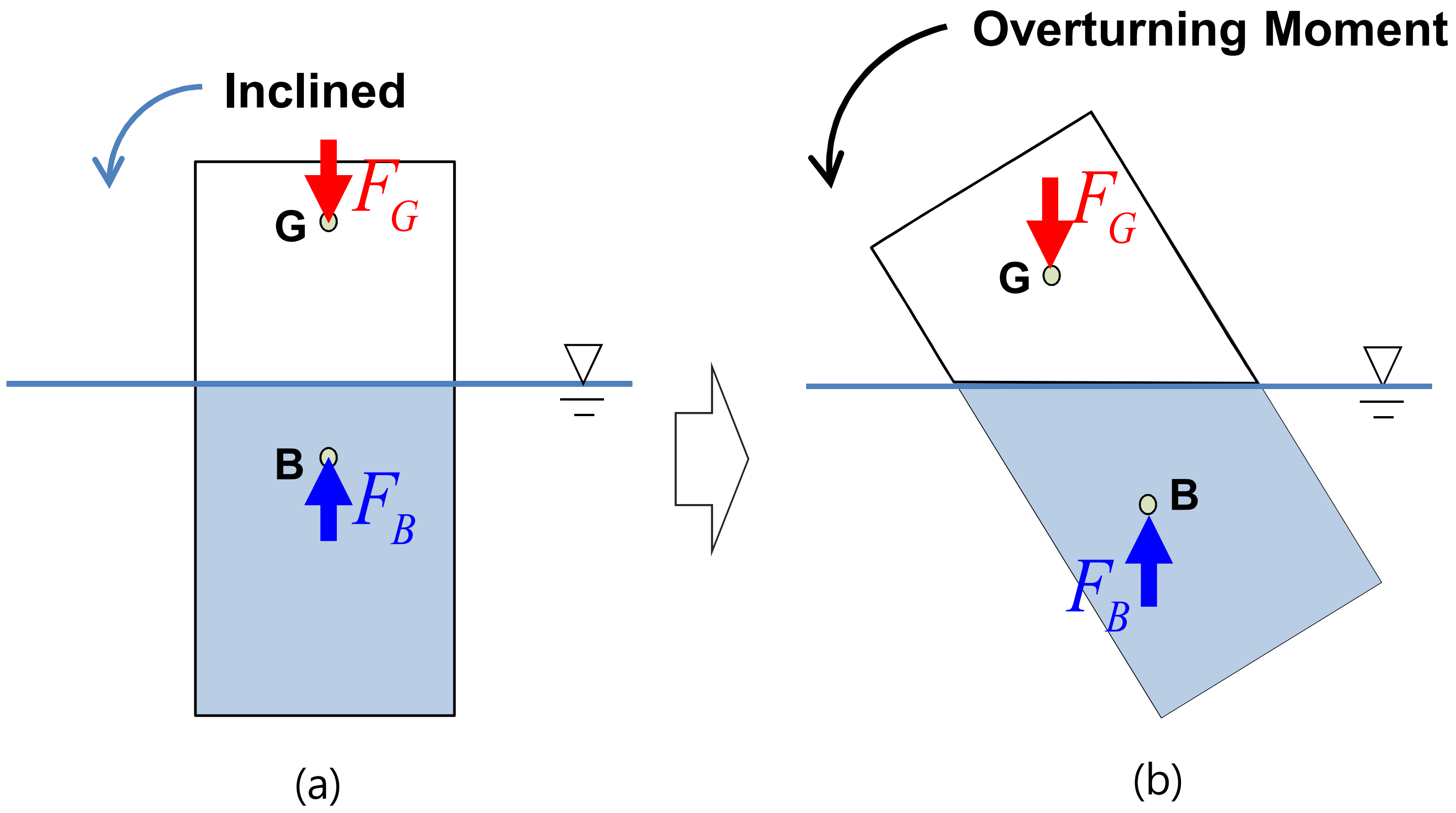
Stability of a floating body (1/2)



(a) (b)

Floating body in stable state

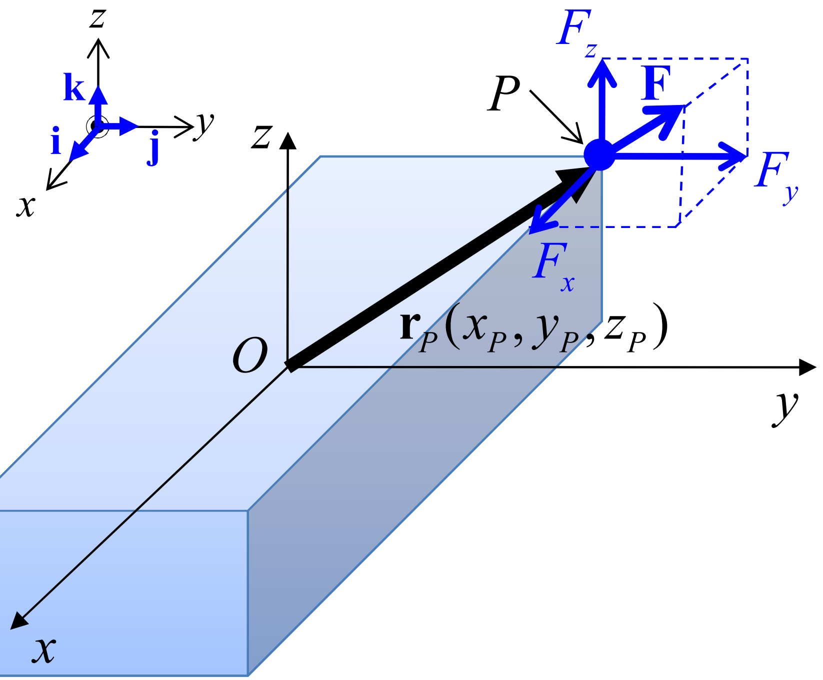
Stability of a floating body (2/2)



Floating body in unstable state

Transverse, longitudinal, and yaw moment

Question) **If the force F is applied on the point of rectangle object, what is the moment?**



$$\mathbf{M} = \mathbf{r}_P \times \mathbf{F}$$

Transverse moment | Longitudinal moment | Yaw moment

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_P & y_P & z_P \\ F_x & F_y & F_z \end{bmatrix} = \mathbf{i}(\underbrace{y_P \cdot F_z - z_P \cdot F_y}_{M_x}) + \mathbf{j}(\underbrace{-x_P \cdot F_z + z_P \cdot F_x}_{M_y}) + \mathbf{k}(\underbrace{x_P \cdot F_y - y_P \cdot F_x}_{M_z})$$

The x-component of the moment, i.e., the bracket term of unit vector **i,** indicates the **transverse moment,** which is the moment caused by the force F acting on the point P **about x axis**. Whereas the y-component, the term of unit vector **j,** indicates the **longitudinal moment about y axis**, and the z-component, the last term **k**, represents the **yaw moment about z axis**.

Equations for Static Equilibrium (1/3)

Suppose there is a floating ship. The force equilibrium states that the sum of total forces is zero.

$$\sum F = F_{G,z} + F_{B,z} = 0$$

, where
$F_{G.z}$ and $F_{B.z}$ are the z component of the gravitational force vector and the buoyant force vector, respectively, and all other components of the vectors are zero.

Also the moment equilibrium must be satisfied, this means, the resultant moment should be also zero.

$$\sum \boldsymbol{\tau} = \mathbf{M}_G + \mathbf{M}_B = \mathbf{0}$$

where $\mathbf{M}_G$ is the moment due to the gravitational force and $\mathbf{M}_B$ is the moment due to the buoyant force.

Equations for Static Equilibrium (2/3)

$$\sum \boldsymbol{\tau} = \mathbf{M}_G + \mathbf{M}_B = \mathbf{0}$$

where $\mathbf{M}_G$ is the moment due to the gravitational force and $\mathbf{M}_B$ is the moment due to the buoyant force.

From the calculation of a moment we know that $\mathbf{M}_G$ and $\mathbf{M}_B$ can be written as follows:

$$\mathbf{M}_G = \mathbf{r}_G \times \mathbf{F}_G$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_G & y_G & z_G \\ F_{G,x} & F_{G,y} & F_{G,z} \end{bmatrix}$$

$$= \mathbf{i}(y_G \cdot F_{G,z} - z_G \cdot F_{G,y}) + \mathbf{j}(-x_G \cdot F_{G,z} + z_G \cdot \cancel{F_{G,x}}) + \mathbf{k}(x_G \cdot \cancel{F_{G,y}} - y_G \cdot \cancel{F_{G,x}})$$

$$\mathbf{M}_B = \mathbf{r}_B \times \mathbf{F}_B$$

$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_B & y_B & z_B \\ F_{B,x} & F_{B,y} & F_{B,z} \end{bmatrix}$$

$$= \mathbf{i}(y_B \cdot F_{B,z} - z_B \cdot F_{B,y}) + \mathbf{j}(-x_B \cdot F_{B,z} + z_B \cdot \cancel{F_{B,x}}) + \mathbf{k}(x_B \cdot \cancel{F_{B,y}} - y_B \cdot \cancel{F_{B,x}})$$

$$\Rightarrow \quad \mathbf{M}_G = \mathbf{i}(y_G \cdot F_{G,z} - z_G \cdot \cancel{F_{G,y}}) + \mathbf{j}(-x_G \cdot F_{G,z}) \text{ and } \mathbf{M}_B = \mathbf{i}(y_B \cdot F_{B,z} - z_B \cdot \cancel{F_{B,y}}) + \mathbf{j}(-x_B \cdot F_{B,z})$$

$$\Rightarrow \quad \mathbf{M}_G = \mathbf{i}(y_G \cdot F_{G,z}) + \mathbf{j}(-x_G \cdot F_{G,z}) \text{ and } \mathbf{M}_B = \mathbf{i}(y_B \cdot F_{B,z}) + \mathbf{j}(-x_B \cdot F_{B,z})$$

Equations for Static Equilibrium (3/3)

$$\sum \boldsymbol{\tau} = \mathbf{M}_G + \mathbf{M}_B = \mathbf{0}$$

where $\mathbf{M}_G$ is the moment due to the gravitational force and $\mathbf{M}_B$ is the moment due to the buoyant force.

$$\mathbf{M}_G = \mathbf{i}(y_G \cdot F_{G,z}) + \mathbf{j}(-x_G \cdot F_{G,z}) \ \text{ and } \ \mathbf{M}_B = \mathbf{i}(y_B \cdot F_{B,z}) + \mathbf{j}(-x_B \cdot F_{B,z})$$

$$\Rightarrow \sum \boldsymbol{\tau} = \mathbf{M}_G + \mathbf{M}_B = \mathbf{i}(y_G \cdot F_{G,z} + y_B \cdot F_{B,z}) + \mathbf{j}(-x_G \cdot F_{G,z} - x_B \cdot F_{B,z}) = \mathbf{0}$$

$$y_G \cdot F_{G,z} + y_B \cdot F_{B,z} = 0 \quad \text{and} \quad -x_G \cdot F_{G,z} - x_B \cdot F_{B,z} = 0$$

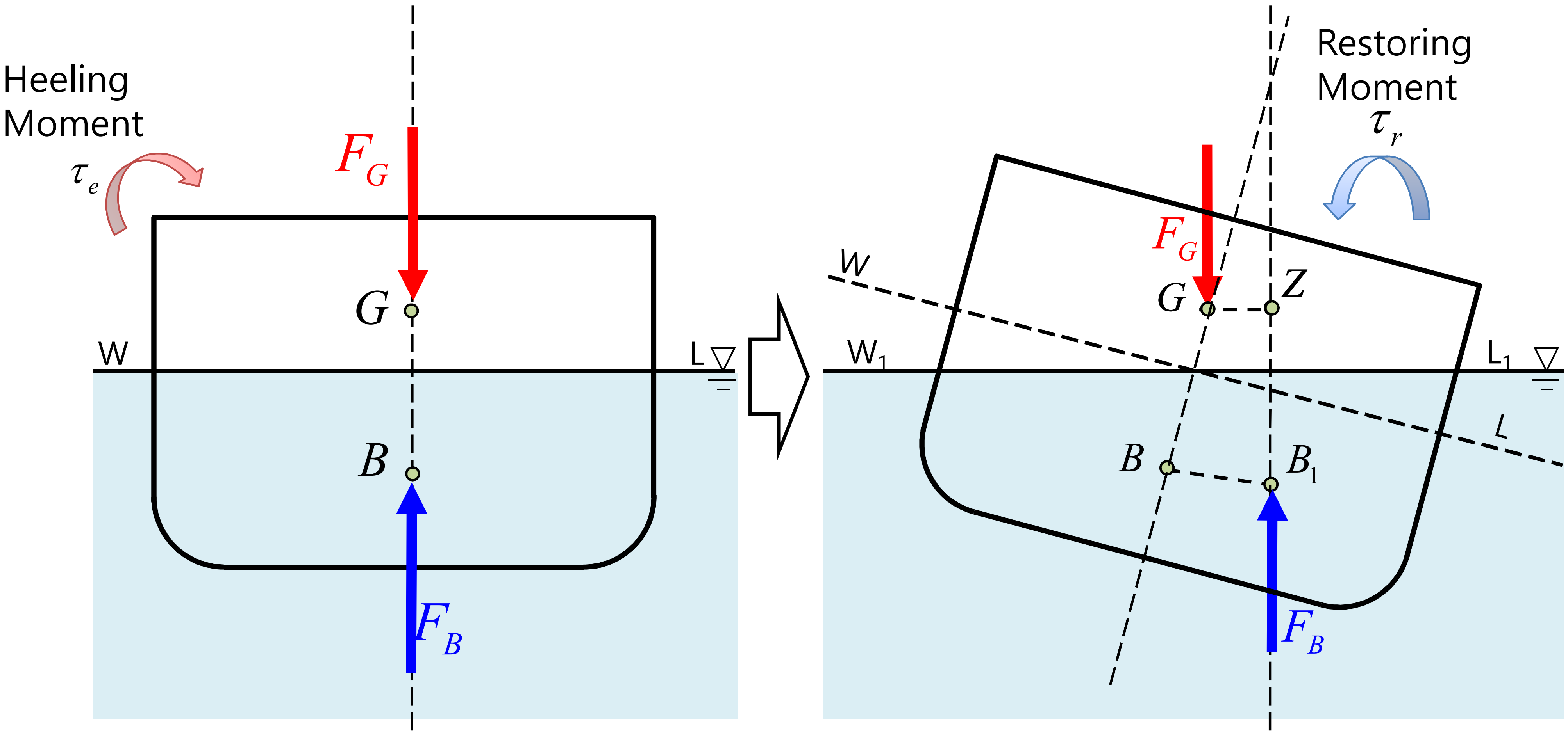
Substituting $F_{G,z} = -F_{B,z}$ (force equilibrium)

$$y_G - y_B = 0 \qquad\qquad x_G - x_B = 0$$

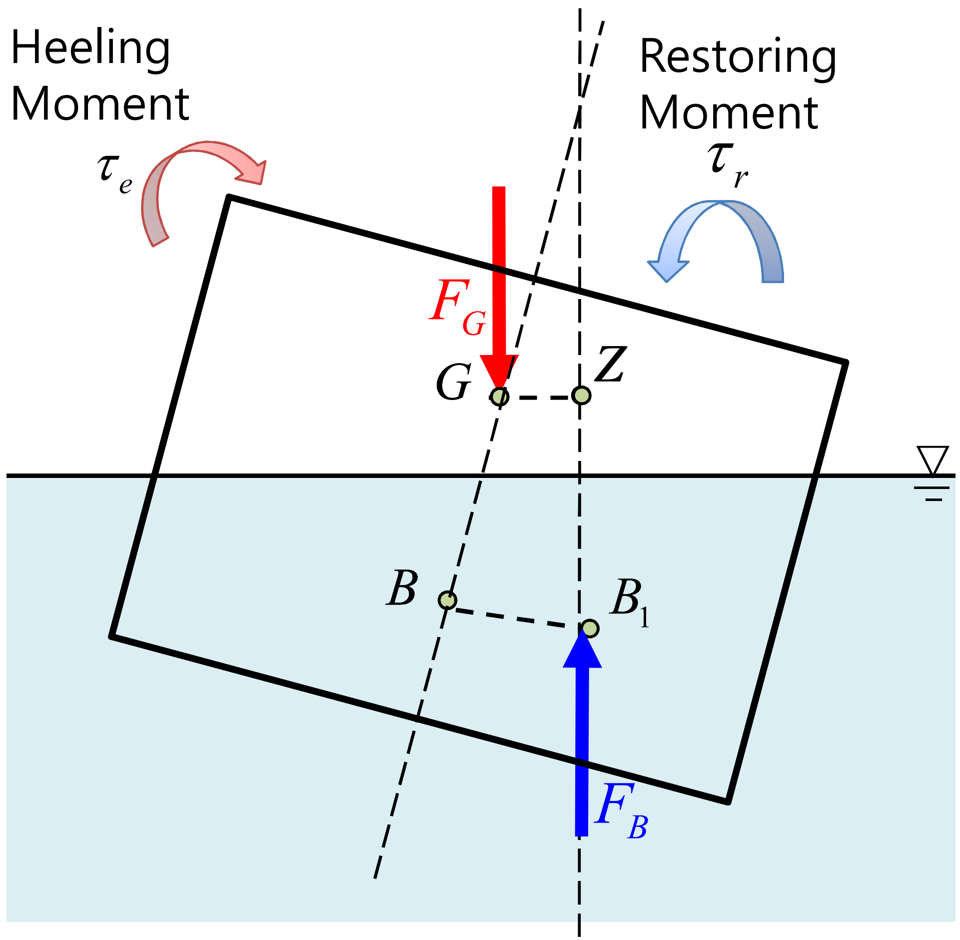
$$\therefore y_G = y_B \qquad\qquad \therefore x_G = x_B$$

Restoring Moment and Restoring Arm

Restoring moment acting on an inclined ship



Restoring Arm (GZ, Righting Arm)



- The value of the restoring moment is found by multiplying the buoyant force of the ship (displacement), F_B, by the perpendicular distance from G to the line of action of F_B.
- It is customary to label as Z the point of intersection of the line of action of F_B and the parallel line to the waterline through G to it.
- This distance GZ is known as the '**restoring arm**' or '**righting arm**'.

- **Transverse Restoring Moment**

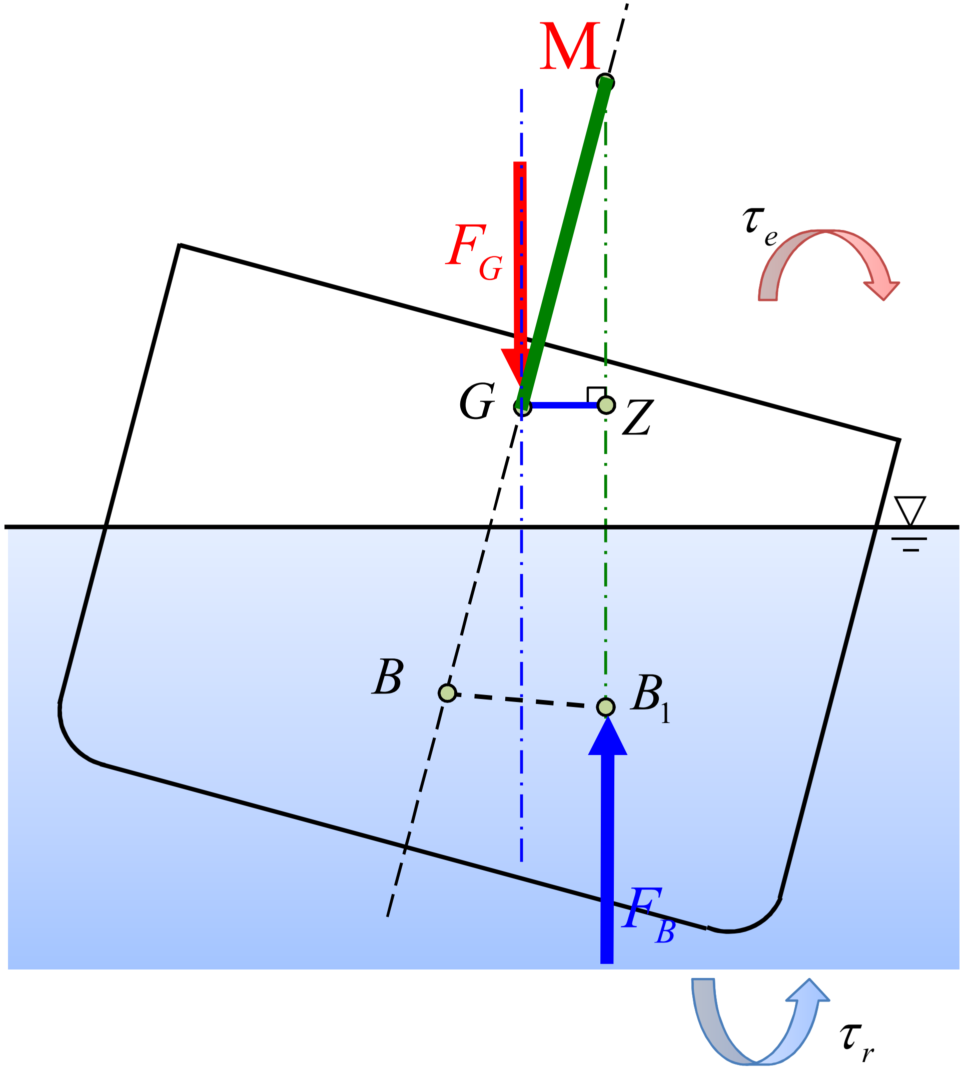
$$\tau_{restoring} = F_B \cdot GZ$$

G: Center of mass K: Keel
B: Center of buoyancy at upright position
B_1: Changed center of buoyancy
F_G: Weight of ship F_B: Buoyant force acting on ship

Metacenter (M)

• **Restoring Moment**

$$\tau_{restoring} = F_B \cdot GZ$$



Z: The intersection point of the line of buoyant force through B_1 with the transverse line through G

Definition of M (Metacenter)

• The intersection point of the vertical line through the center of buoyancy at **previous position** (*B*) with the vertical line through the center of buoyancy at **new position** (*B_1*) **after inclination**

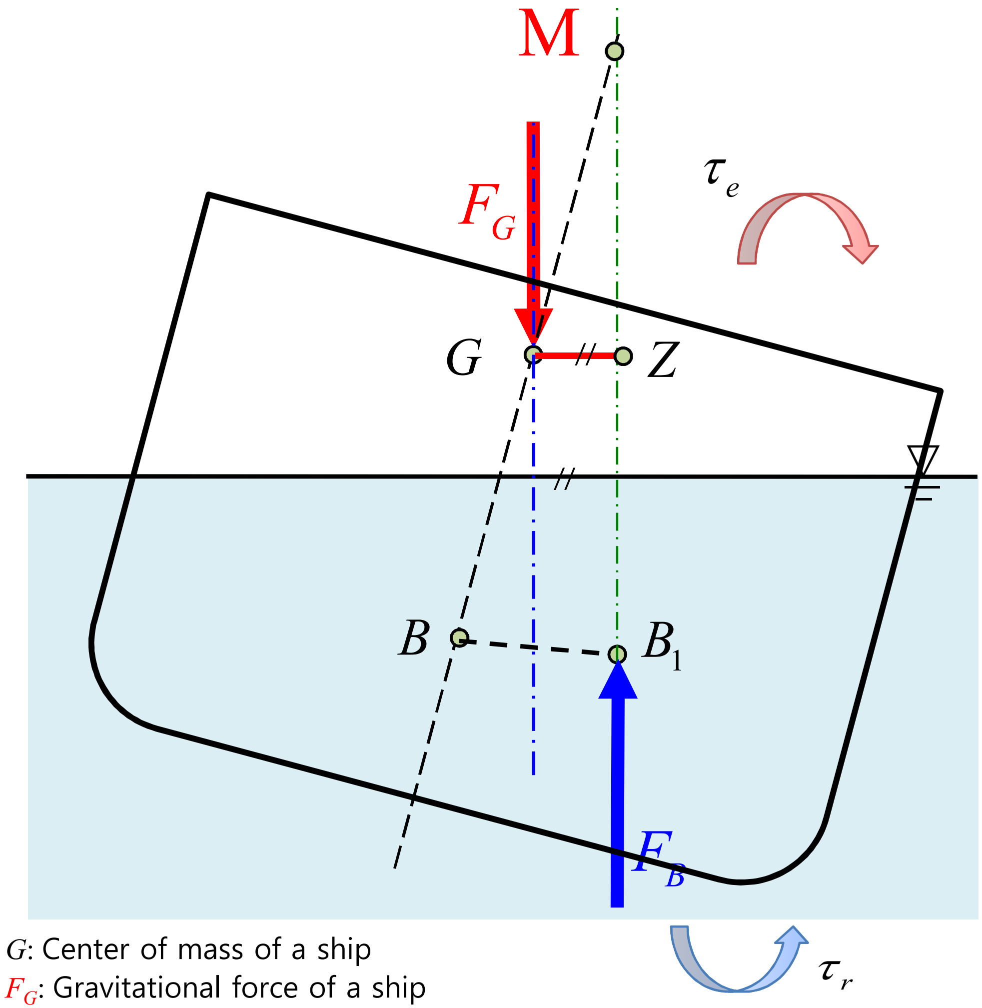
• The term **meta** was selected as a prefix for center because its Greek meaning implies **movement**. The **metacenter** therefore is a **moving center**.

• GM ➡ **Metacentric height**

• From the figure, GZ can be obtained with assumption that *M* does not change within a **small angle of inclination** (about 7° to 10°), as below.

$$GZ \approx GM \cdot \sin\phi$$

Restoring moment at large angle of inclination (1/3)



G: Center of mass of a ship
F_G: Gravitational force of a ship
B: Center of buoyancy in the previous state (before inclination)
F_B: Buoyant force acting on a ship
B_1: New position of center of buoyancy after the ship has been inclined
Z : The intersection point of a vertical line through the new position of the center of buoyancy(B_1) with the transversely parallel line to a waterline through the center of mass(G)

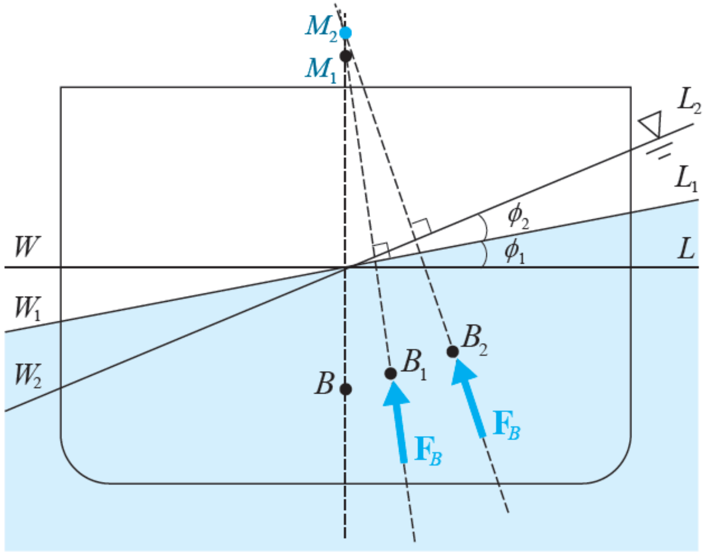
$$GZ \approx GM \cdot \sin\phi$$

For a small angle of inclination
(about 7° to 10°)

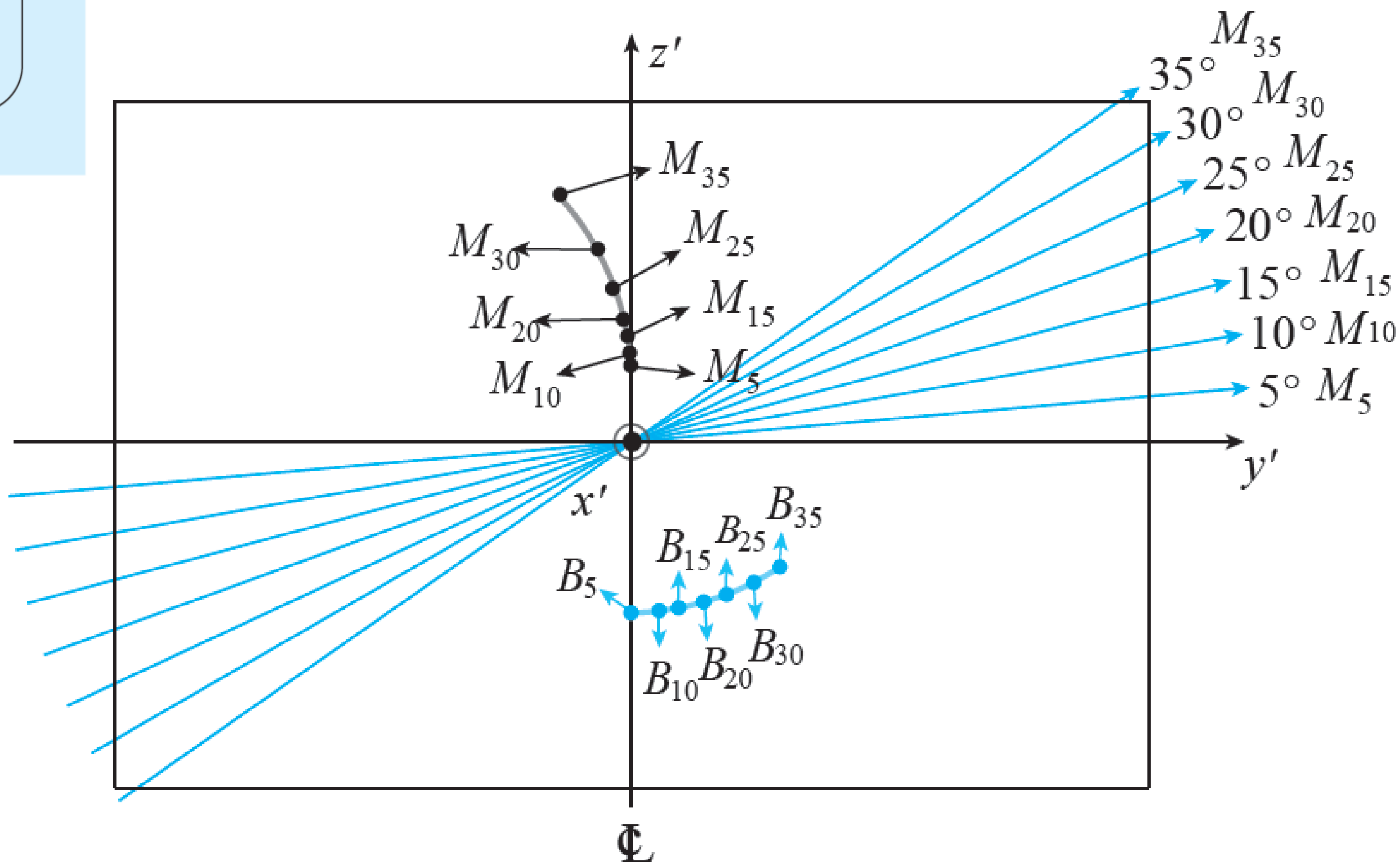
- The use of metacentric height(GM) as the restoring arm is not valid for a ship at a large angle of inclination.

To determine the restoring arm "GZ", it is necessary to know the positions of the center of mass(G) and the new position of the center of buoyancy(B_1).

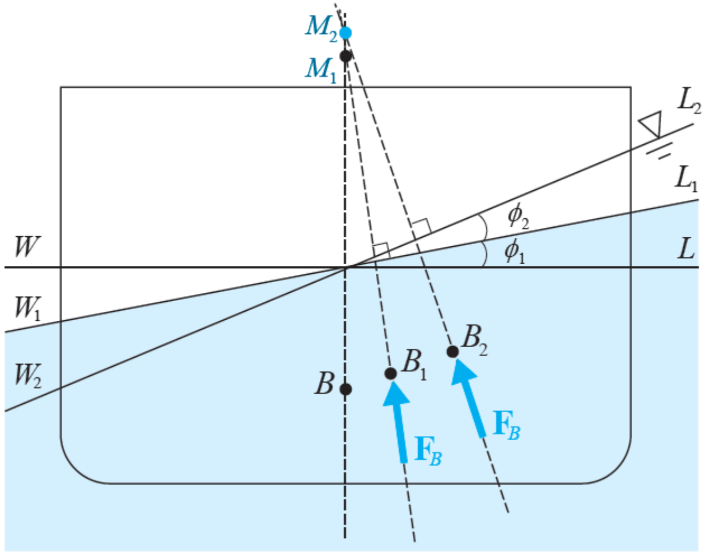
Restoring moment at large angle of inclination (2/3)



M: The intersection point of the vertical line through the center of buoyancy at **previous position** (B_{i-1}) with the vertical line through the center of buoyancy at **present position** (B_i) **after inclination**

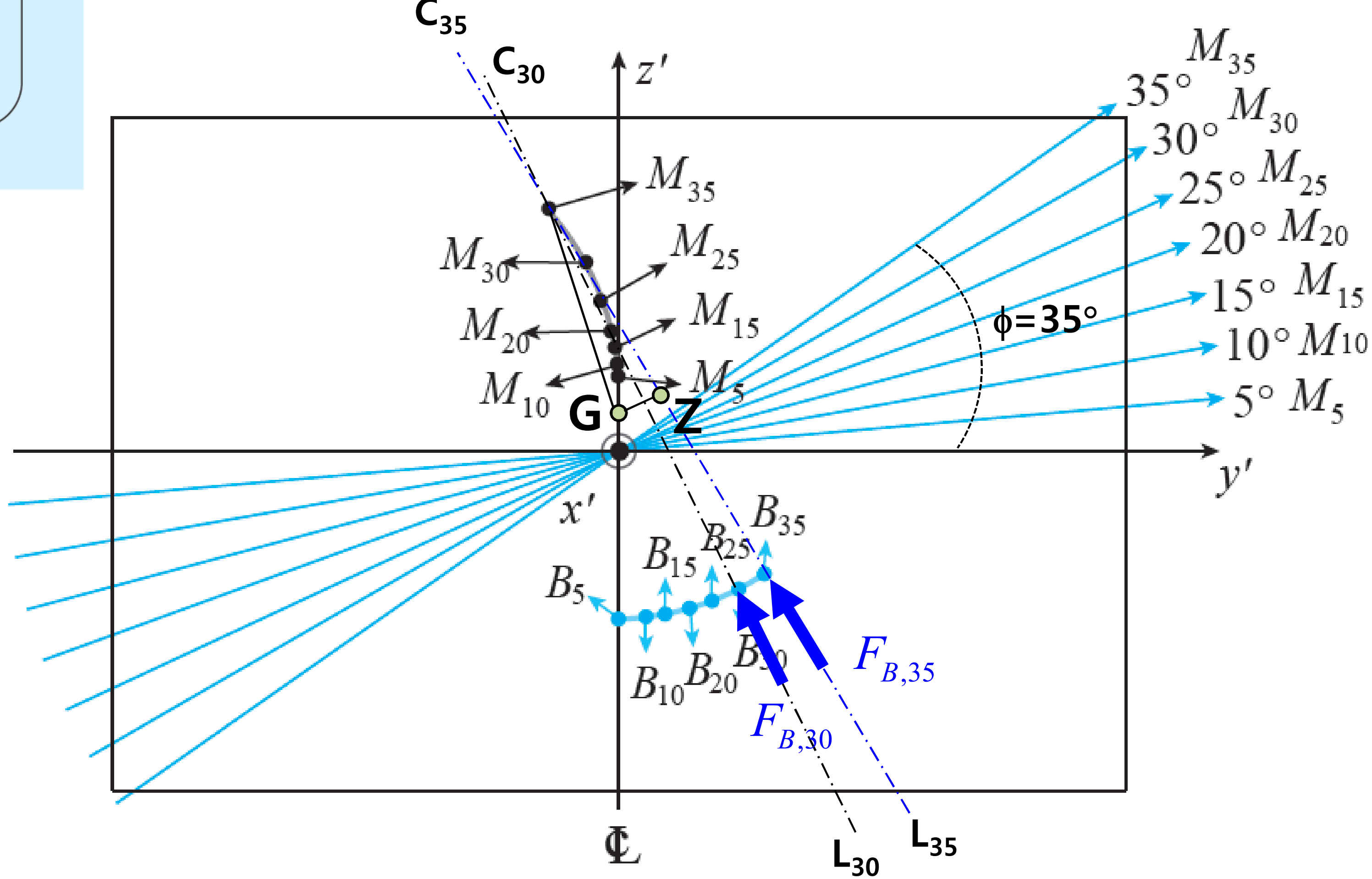


Restoring moment at large angle of inclination (3/3)



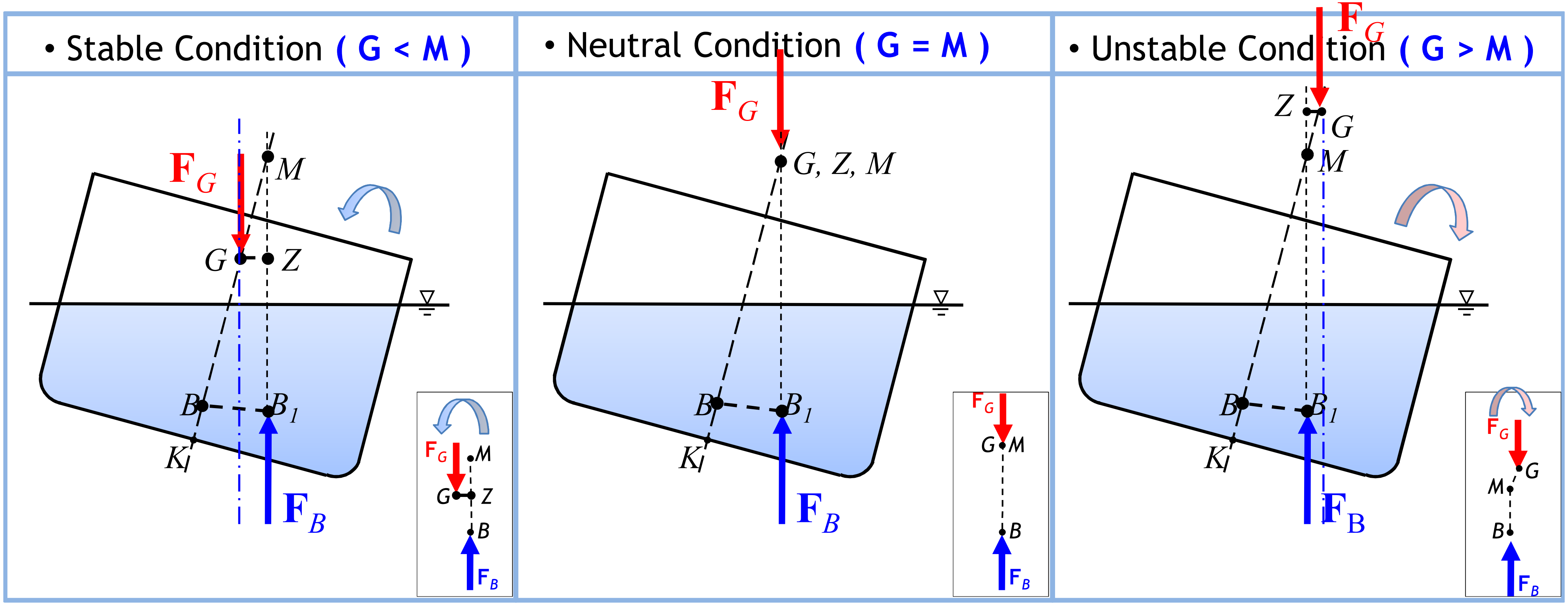
M: The intersection point of the vertical line through the center of buoyancy at **previous position** (B_{i-1}) with the vertical line through the center of buoyancy at **present position** (B_i) **after inclination**

$$GZ \neq GM_{35} \cdot \sin\phi_{35}$$



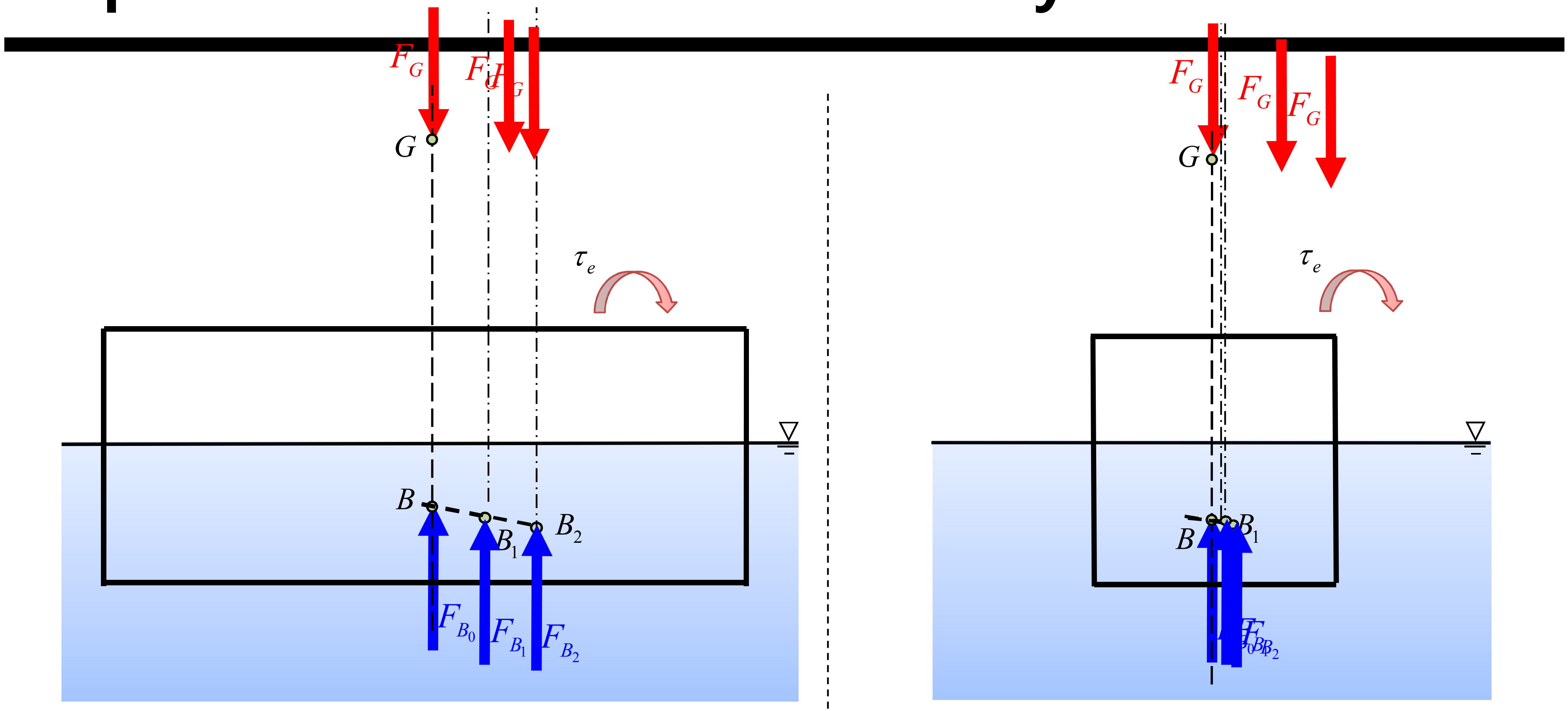
Stability of a ship according to relative position between "G", "B", and "M" at small angle of inclination

- **Righting(Restoring) Moment :** Moment to return the ship to the upright floating position
- **Stable / Neutral / Unstable Condition :** Relative height of G with respect to M is one measure of stability.



G: Center of mass　　*K* : Keel
B: Center of buoyancy at upright position　　B_1: Changed center of buoyancy
F_G : Weight of ship　　F_B : Buoyant force acting on ship
Z : The intersection of the line of buoyant force through B1 with the transverse line through G
M : The intersection of the line of buoyant force through B1 with the centerline of the ship

Importance of transverse stability



The ship is inclined further from it.

The ship is in static equilibrium state.

The ship is inclined further from it.

Because of the limit of the breadth, "B" can not move further. the ship will capsize.

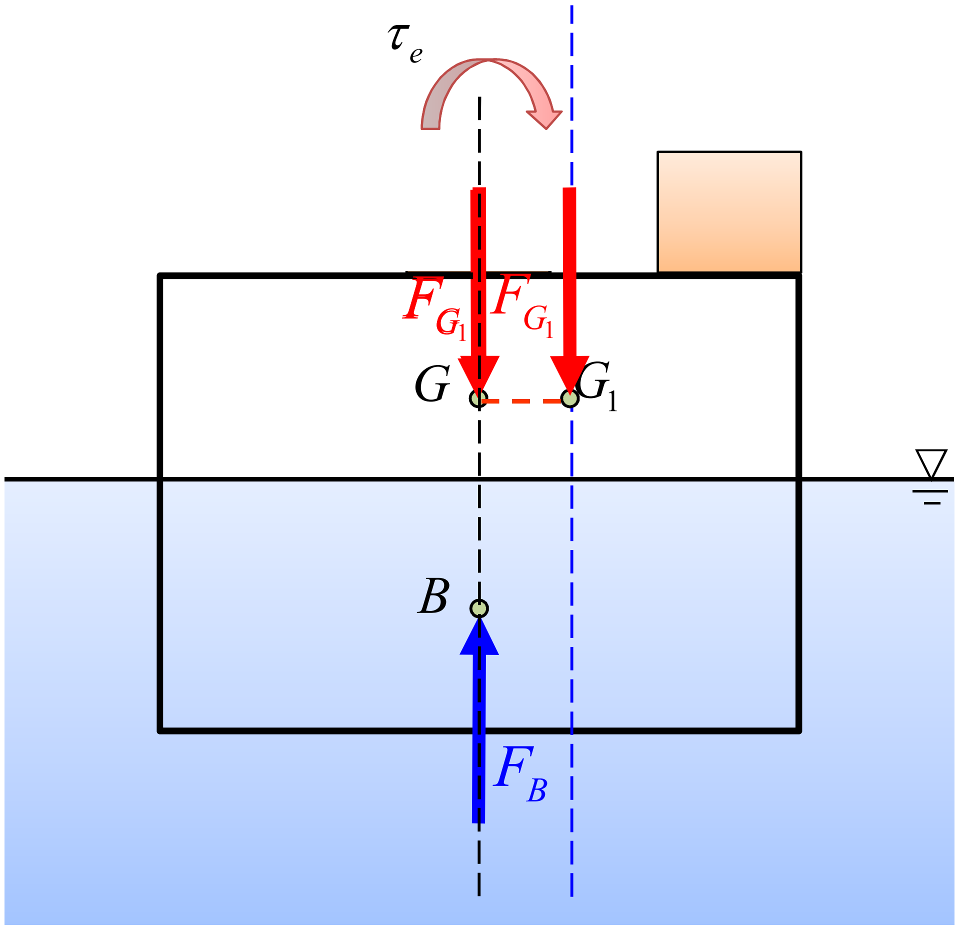
As the ship is inclined, the position of the center of buoyancy "B" is changed.

Also the **position of the center of mass "G" relative to inertial frame is changed.**

One of the most important factors of stability is the breadth.

So, we usually consider that transverse stability is more important than longitudinal stability.

Summary of static stability of a ship (1/3)

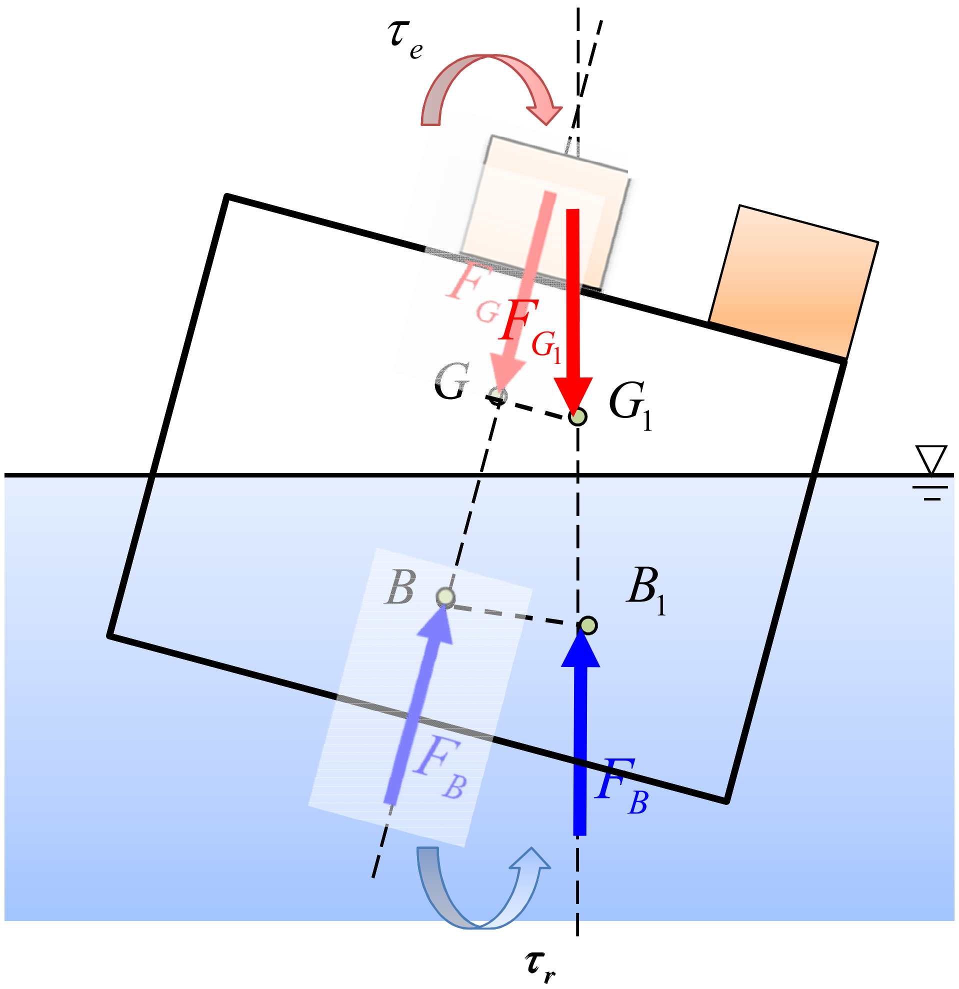


- When an object on the deck moves to the right side of a ship, the total center of mass of the ship moves to the point G_1, off the centerline.
- Because the buoyant force and the gravitational force are not on one line, the forces induces a moment to incline the ship.

  * **We have a moment on this object relative to any point that we choose. It does not matter where we pick a point.**

G: Center of mass of a ship
G_1: New position of center of mass after the object on the deck moves to the right side
F_G: Gravitational force of a ship
B: Center of buoyancy at initial position
F_B: Buoyant force acting on a ship
B_1: New position of center of buoyancy after the ship has been inclined
Z: The intersection of a line of buoyant force(F_B) through the new position of the center of buoyancy (B_1) with the transversely parallel line to the waterline through the center of mass of a ship(G)

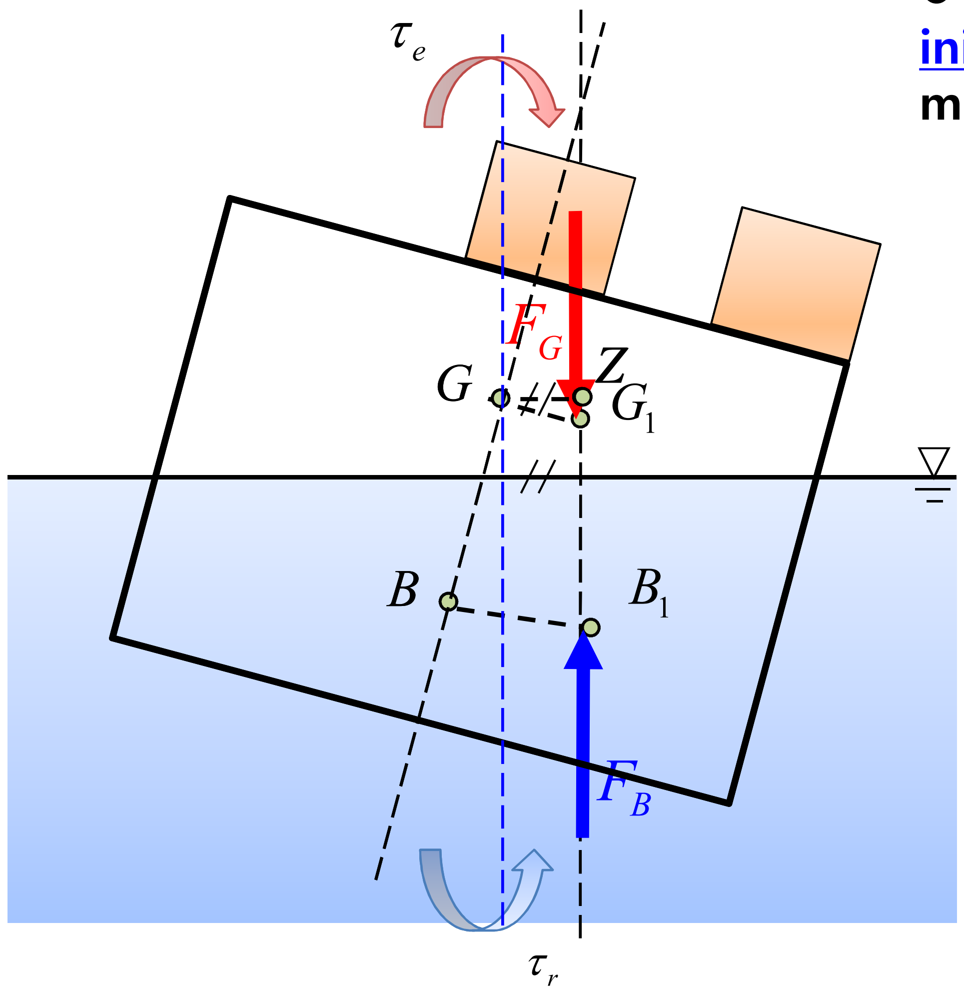
Summary of static stability of a ship (2/3)



G: Center of mass of a ship
G_1: New position of center of mass after the object on the deck moves to the right side
F_G: Gravitational force of a ship
B: Center of buoyancy at initial position
F_B: Buoyant force acting on a ship
B_1: New position of center of buoyancy after the ship has been inclined
Z: The intersection of a line of buoyant force(F_B) through the new position of the center of buoyancy (B_1) with the transversely parallel line to the waterline through the center of mass of a ship(G)

- **The total moment will only be zero when the buoyant force and the gravitational force are on one line. If the moment becomes zero, the ship is in static equilibrium state.**

Summary of static stability of a ship (3/3)



G: Center of mass of a ship
G_1: New position of center of mass after the object on the deck moves to the right side
F_G: Gravitational force of a ship
B: Center of buoyancy at initial position
F_B: Buoyant force acting on a ship
B_1: New position of center of buoyancy after the ship has been inclined
Z: The intersection of a line of buoyant force(F_B) through the new position of the center of buoyancy (B_1) with the transversely parallel line to the waterline through the center of mass of a ship(G)

- When the object on the deck returns to the initial position in the centerline, the center of mass of the ship returns to the initial point G.
  - Then, because the buoyant force and the gravitational force are not on one line, the forces induces a restoring moment to return the ship to the initial position.

    ※ Naval architects refer to the restoring moment as "righting moment".

  - The moment arm of the buoyant force and gravitational force about G is expressed by GZ, where Z is defined as the intersection point of the line of buoyant force(F_B) through the new position of the center of buoyancy(B_1) with the transversely parallel line to the waterline through the center of mass of the ship(G)

- Transverse Righting Moment

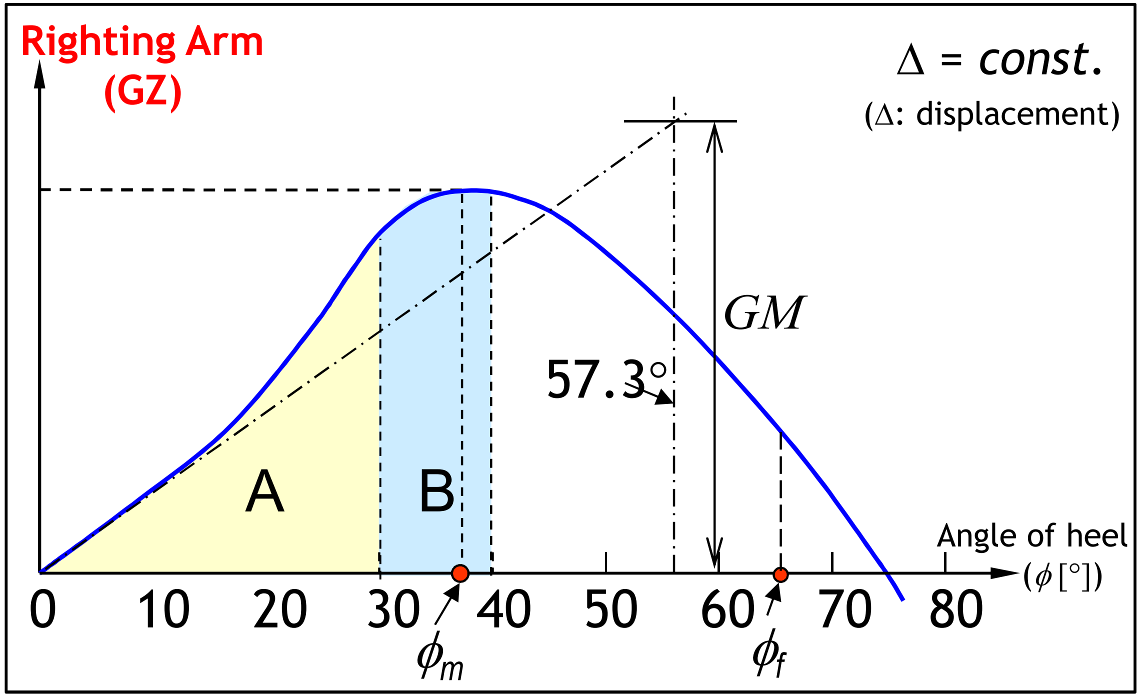
$$\tau_{righting} = F_B \cdot \underline{GZ}$$

- By the restoring moment, the ship returns to the initial position.

Evaluation of Stability
: Merchant Ship Stability Criteria – IMO Regulations for Intact Stability

(IMO Res.A-749(18) ch.3.1)

☑ **IMO recommendation on intact stability for passenger and cargo ships.**



Area A: Area under the righting arm curve between the heel angle of 0° and 30°

Area B: Area under the righting arm curve between the heel angle of 30° and min(40°, ϕ_f)

※ ϕ_f: Heel angle at which openings in the hull

ϕ_m: Heel angle of maximum righting arm

※ After receiving approval of calculation of IMO regulation from Owner and Classification Society, ship construction can proceed.

IMO Regulations for Intact Stability

(a) Area A ≥ 0.055 m-rad

(b) Area A + B ≥ 0.09 m-rad

(c) Area B ≥ 0.030 m-rad

The work and energy considerations (dynamic stability)

(d) GZ ≥ 0.20 m at an angle of heel equal to or greater than 30°

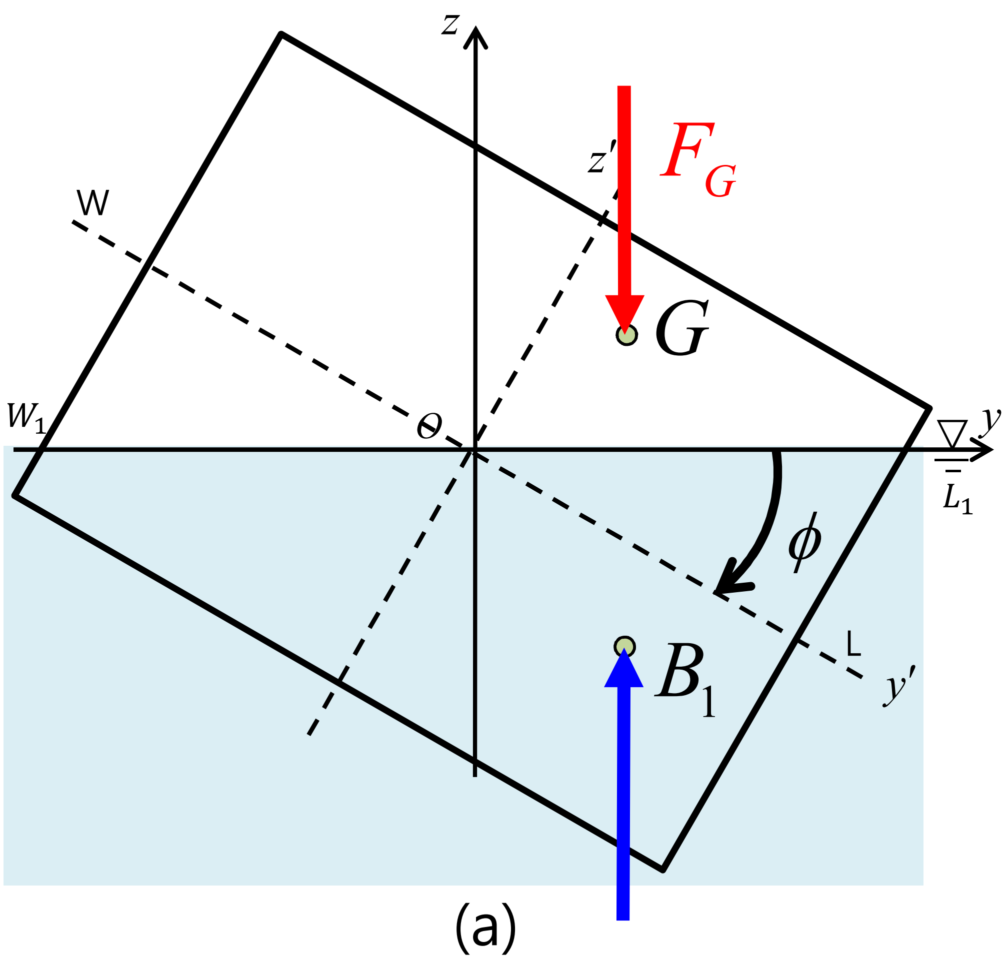
(e) GZ_{max} should occur at an angle of heel preferably exceeding 30° but not less than 25°.

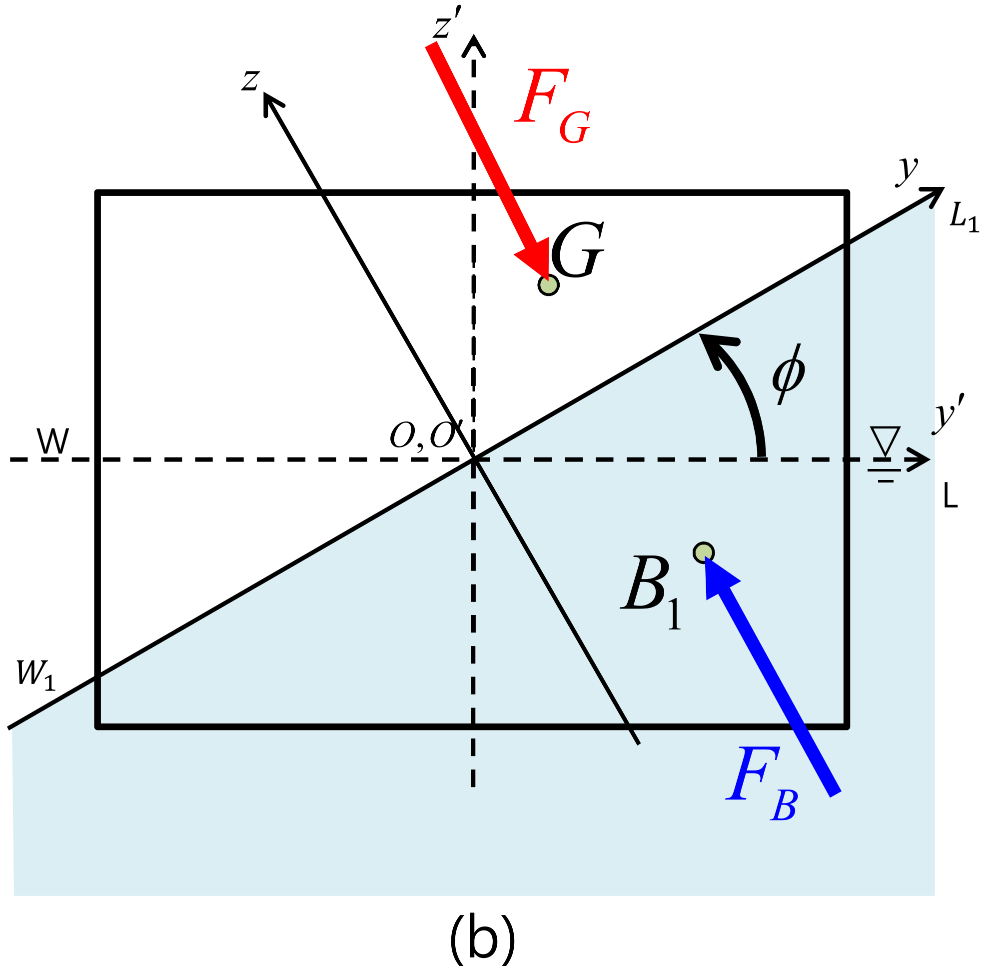
(f) The initial metacentric height GM_o should not be less than 0.15 m.

Static considerations

Rotational Transformation of a Position Vector to a Body in Fluid

Orientation of a ship with respect to the different reference frame





Space(Water plane) fixed reference frame **Body fixed reference frame**

Body fixed coordinate system(b-frame): Body fixed reference frame $x'y'z'$

Space fixed coordinate system(n-frame): Inertial reference frame $x\,y\,z$

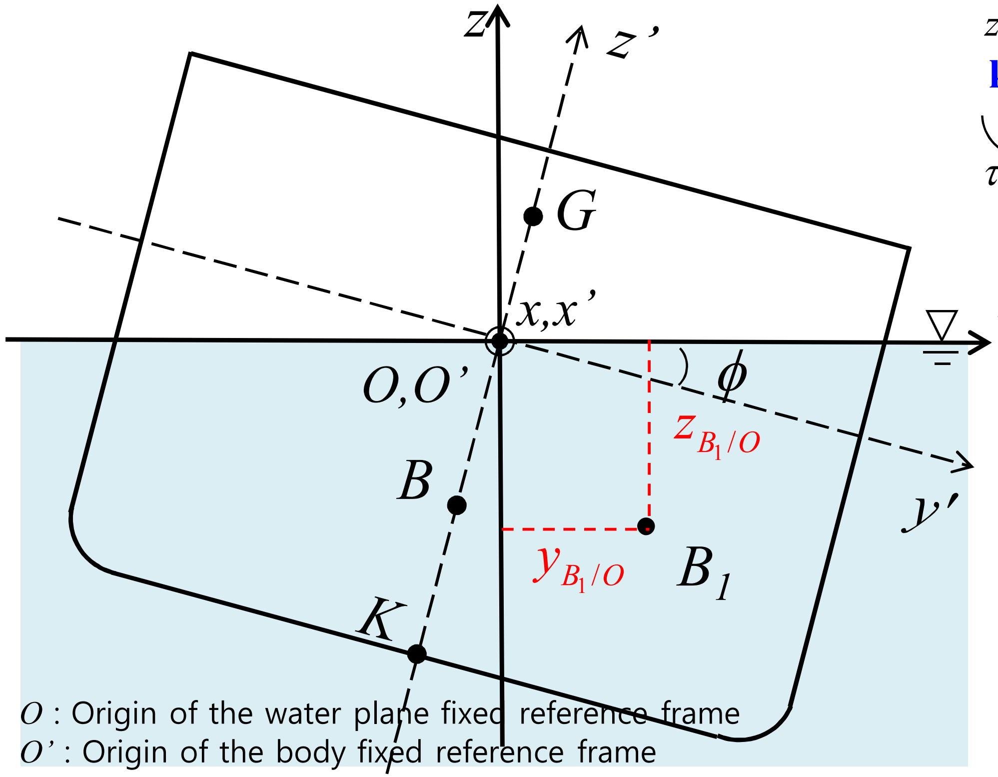
Reference)
- Water Plane Fixed Reference Frame vs. Body Fixed Reference Frame

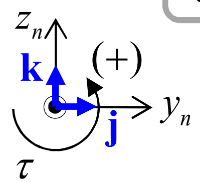
How can we calculate ship's center of buoyancy(B_1)?

We can calculate the center of buoyancy with respect to the water plane fixed reference frame (inertial reference frame).

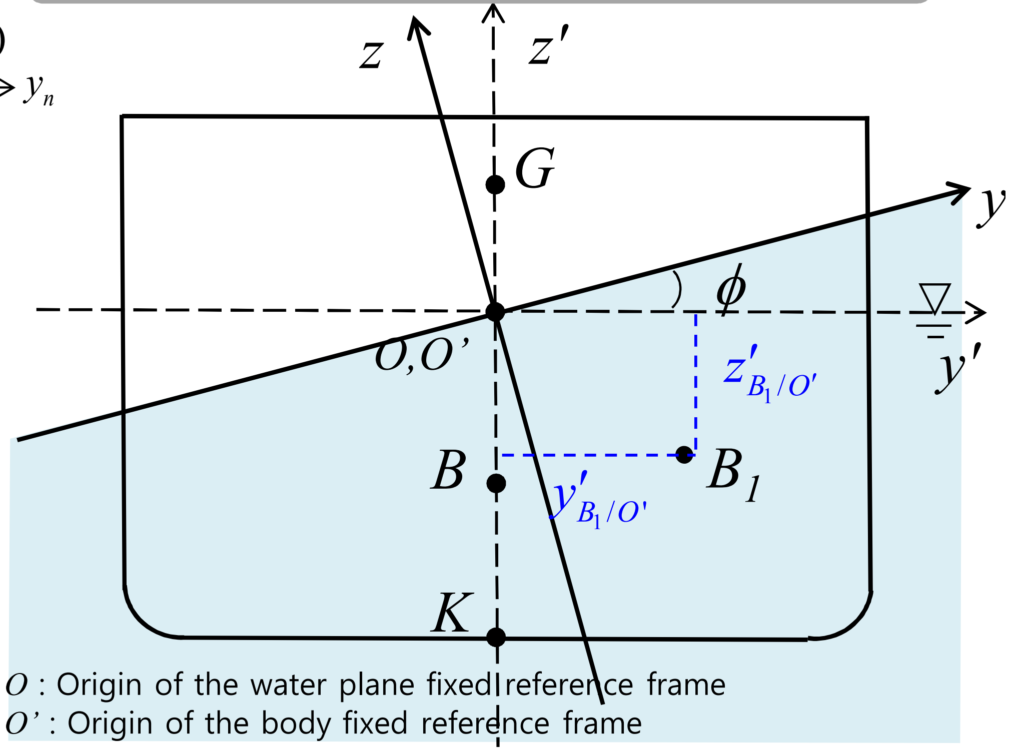
Alternatively, we can calculate the center of buoyancy with respect to the body fixed reference frame (non-inertial reference frame).

Method 1. Calculate center of buoyancy B_1 directly with respect to the **water plane reference fixed frame**.





Method 2. Calculate center of buoyancy B_1 with respect to the **body fixed reference frame**, then **transform** B_1 to the water plane fixed reference frame.



Water plane fixed reference frame

Body fixed reference frame

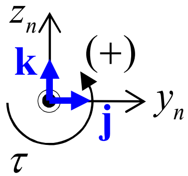
Reference)

Question : How to calculate center of the buoyancy(B_1) with respect t

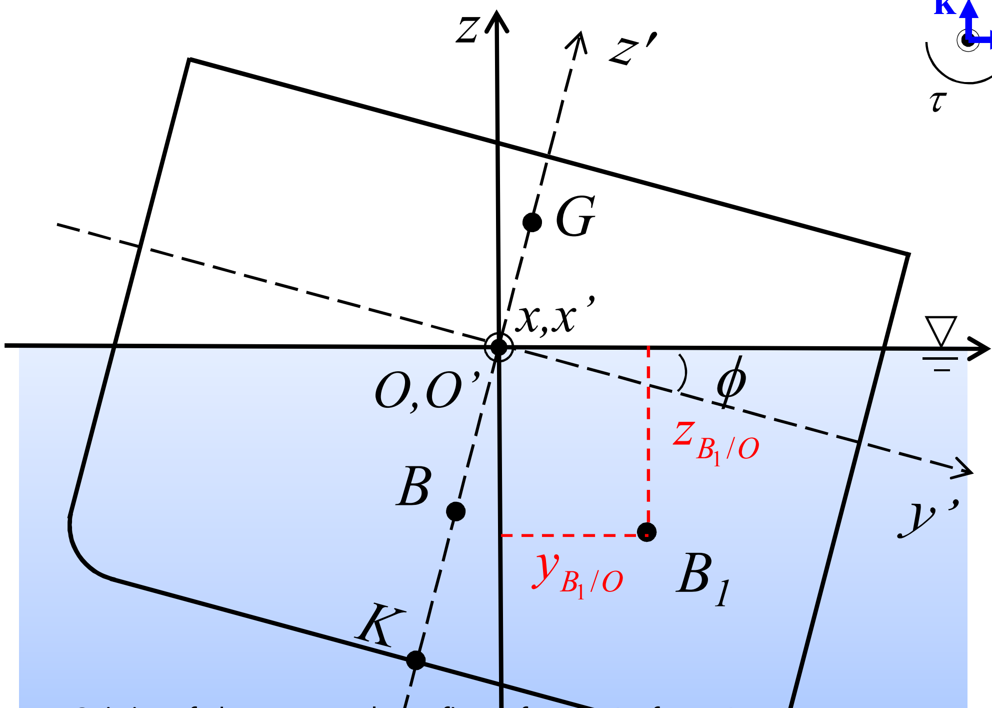
✓Comparison between Method 1 and Method 2 (1/2)

$$\begin{bmatrix} {}^{n}y_{P/O} \\ {}^{n}z_{P/O} \end{bmatrix} = \boxed{\begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}} \begin{bmatrix} {}^{b}y_{P/O} \\ {}^{b}z_{P/O} \end{bmatrix}$$

Method 1. Calculate center of buoyancy B_1 directly with respect to the water plane fixed reference frame.

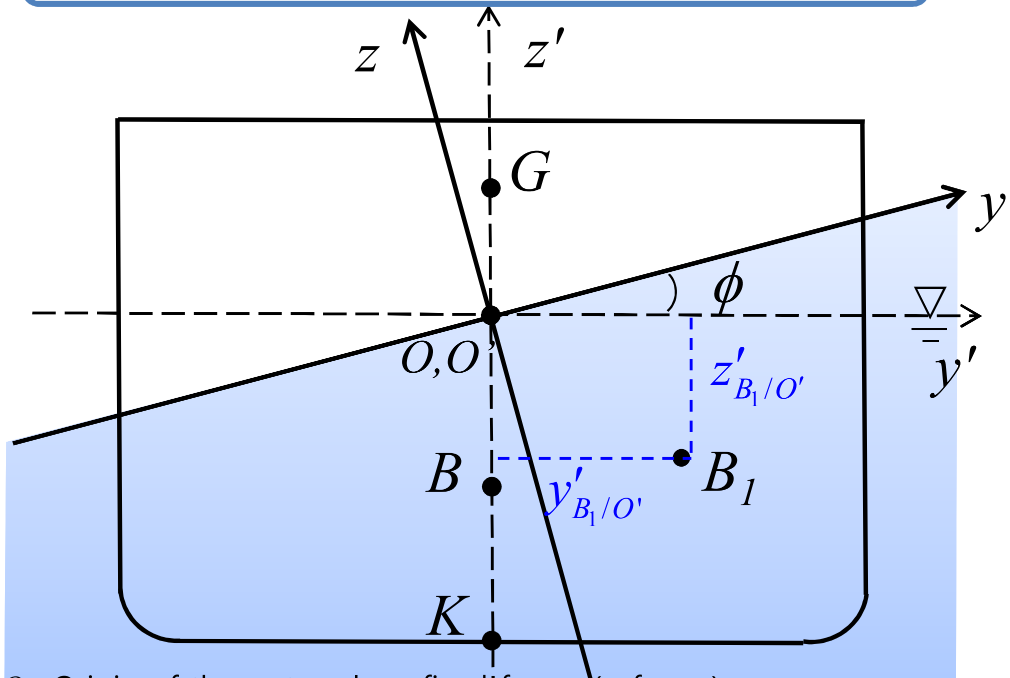


Method 2. Calculate center of buoyancy B_1 with respect to the body fixed reference frame, then transform B_1 to the water plane fixed reference frame.



O : Origin of the water plane fixed frame (n-frame)

O' : Origin of the body fixed reference frame (b-frame)



O : Origin of the water plane fixed frame (n-frame)

O' : Origin of the body fixed reference frame (b-frame)

✓ A, M_z, M_y with respect to the water plane fixed frame

$$dA = dydz \qquad A = \int dA$$

$$M_{A,z} = \int y dA \qquad M_{A,y} = \int z dA$$

✓ Center of buoyancy with respect to the water plane fixed frame

$$(y_{B_1/O}, z_{B_1/O}) = \left(\frac{M_{A,z}}{A}, \frac{M_{A,y}}{A} \right)$$

✓ A, $M_{A,z'}$, $M_{A,y'}$ with respect to the body fixed frame

$$dA' = dy'dz' \qquad M_{A,z'} = \int y' dA \qquad M_{A,y'} = \int z' dA$$

✓ Center of buoyancy with respect to the body fixed frame

$$(y'_{B_1/O'}, z'_{B_1/O'}) = \left(\frac{M_{A,z'}}{A}, \frac{M_{A,y'}}{A} \right)$$

✓ Rotational transformation

$$\begin{bmatrix} y_{B_1/O} \\ z_{B_1/O} \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} y'_{B_1/O'} \\ z'_{B_1/O'} \end{bmatrix}$$

$M_{A,z}$: The moment of sectional area under the water plane about z-axis

$M_{A,y}$: The moment of sectional area under the water plane about y-axis

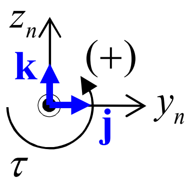
Reference)

Question : How to calculate center of the buoyancy(B_1) with respect t

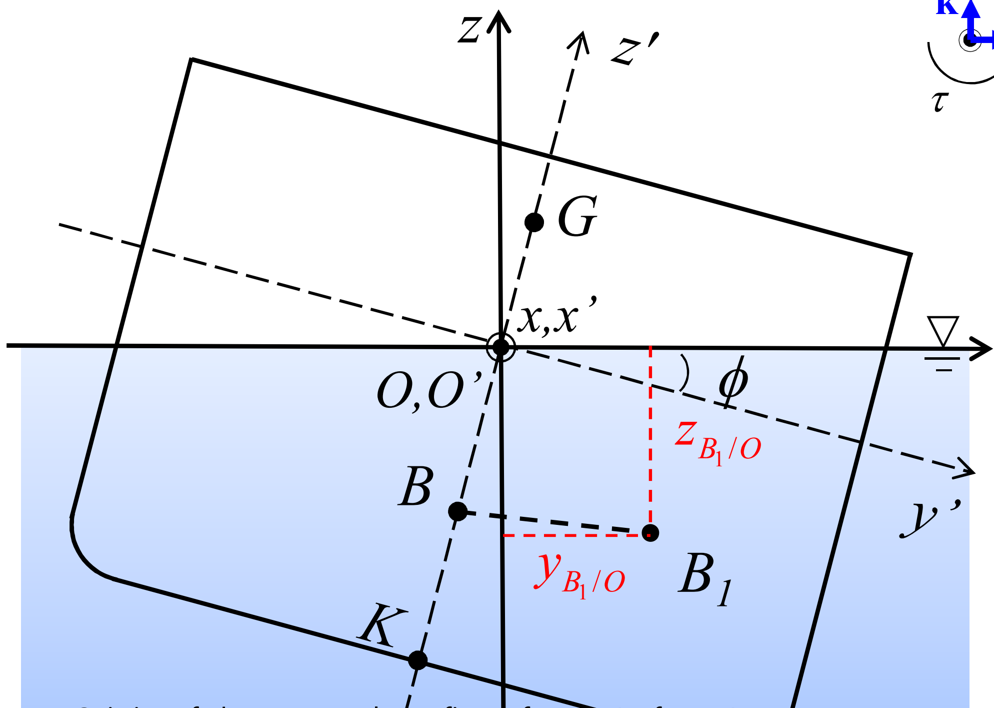
$$\begin{bmatrix} {}^{n}y_{P/O} \\ {}^{n}z_{P/O} \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} {}^{b}y_{P/O} \\ {}^{b}z_{P/O} \end{bmatrix}$$

✓Comparison between Method 1 and Method 2 (2/2)

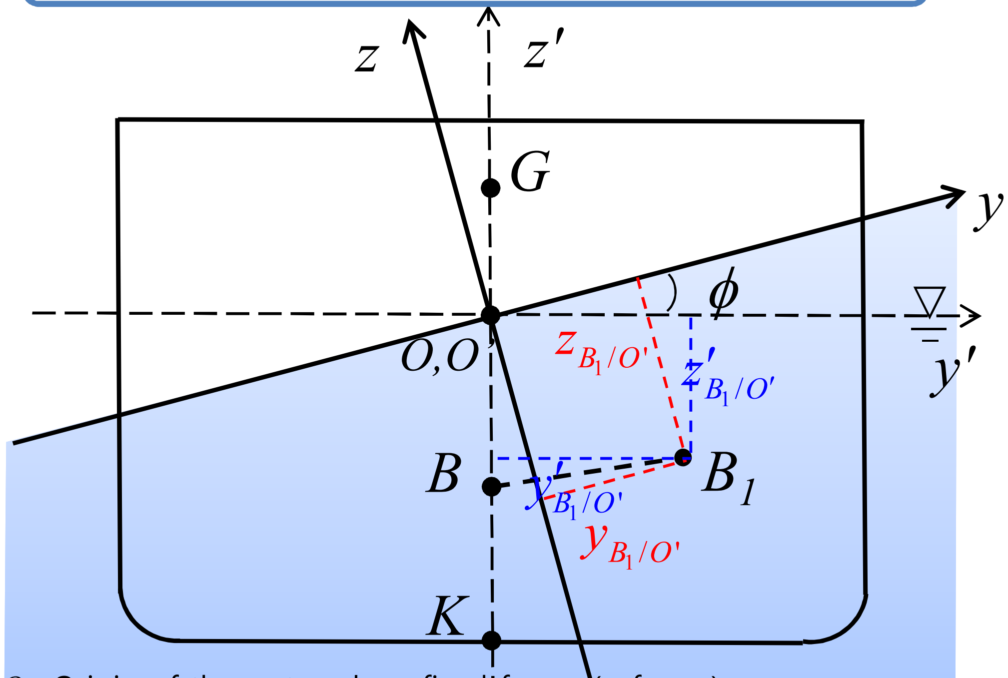
Method 1. Calculate center of buoyancy B_1 directly with respect to the water plane fixed reference frame.



Method 2. Calculate center of buoyancy B_1 with respect to the body fixed reference frame, then transform B_1 to the water plane fixed reference frame.



O : Origin of the water plane fixed frame (n-frame)
O' : Origin of the body fixed reference frame (b-frame)



O : Origin of the water plane fixed frame (n-frame)
O' : Origin of the body fixed reference frame (b-frame)

✓ A, M_z, M_y with respect to the water plane fixed frame

$$dA = dydz \qquad A = \int dA$$

$$M_{A,z} = \int y dA \qquad M_{A,y} = \int z dA$$

✓ Center of buoyancy with respect to the water plane fixed frame

$$(y_{B_1/O}, z_{B_1/O}) = \left(\frac{M_{A,z}}{A}, \frac{M_{A,y}}{A} \right)$$

Same

✓ $A, M_{A,z'}, M_{A,y'}$ with respect to the body fixed frame

$$dA' = dy'dz' \qquad M_{A,z'} = \int y'dA \qquad M_{A,y'} = \int z'dA$$

✓ Center of buoyancy with respect to the body fixed frame

$$(y'_{B_1/O'}, z'_{B_1/O'}) = \left(\frac{M_{A,z'}}{A}, \frac{M_{A,y'}}{A} \right)$$

✓ Rotational transformation **Convenient**

$$\begin{bmatrix} y_{B_1/O} \\ z_{B_1/O} \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} y'_{B_1/O'} \\ z'_{B_1/O'} \end{bmatrix}$$

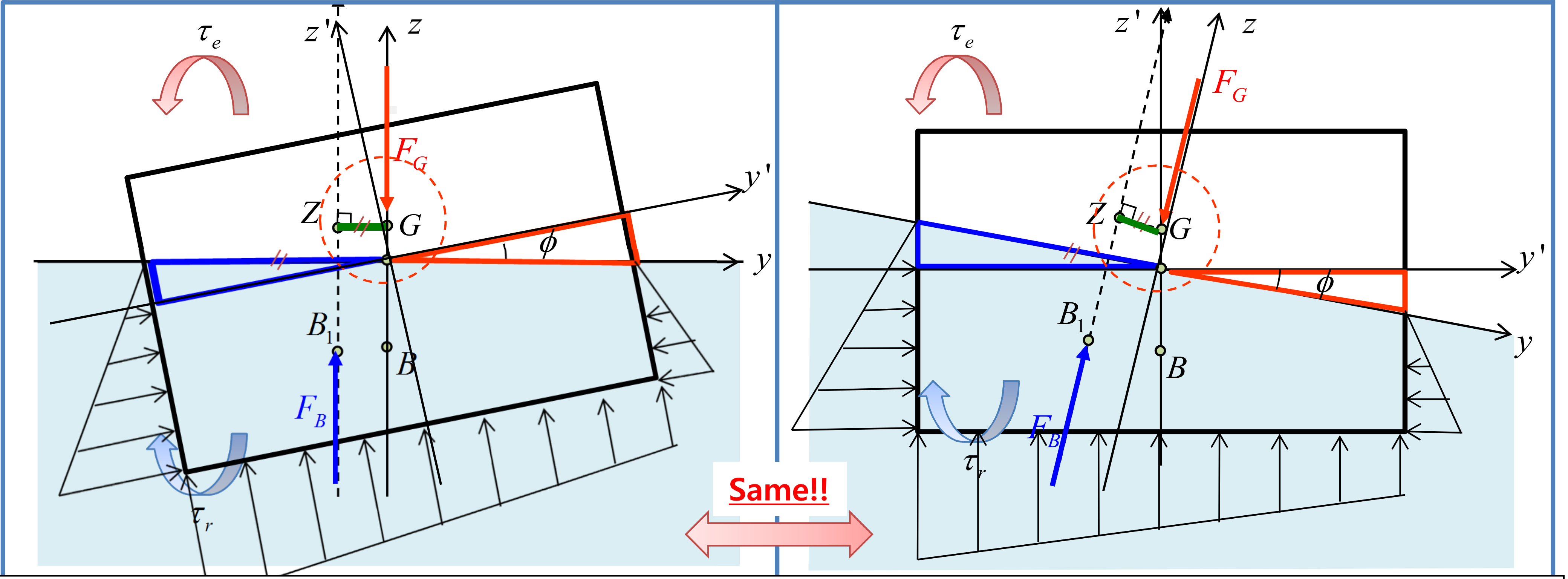
Reference)

Orientation of a ship with respect to the different reference frame

Inclination of a ship can be represented either with respect to the **water plane fixed frame("inertial reference frame")** or the **body fixed reference frame**.

Are these two phenomena with respect to the different reference frames the same?

Rotation of a ship with respect to the water plane fixed reference frame	Rotation of a ship with respect to the body fixed reference frame

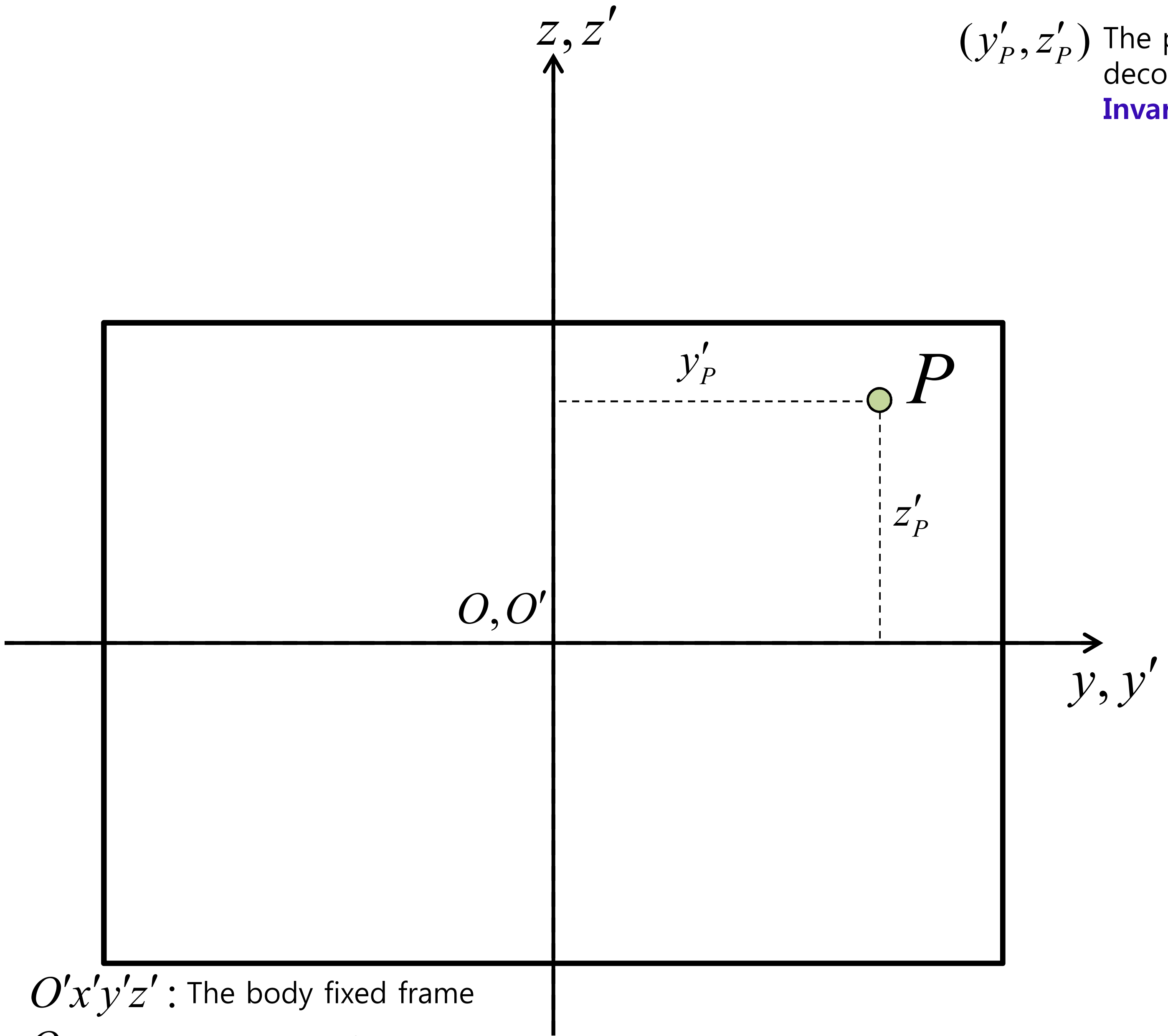


Submerged volume and emerged volume do not change with respect to the frame, that means volume is invariant with respect to the reference frame. Also is the pressure acting on the ship invariant with respect to the reference frame.

In addition, the magnitude of the moment arm "GZ" also does not change. However, the position vectors of the center of mass "G" and the center of buoyancy "B_1" are variant with respect to the water plane fixed reference frame.

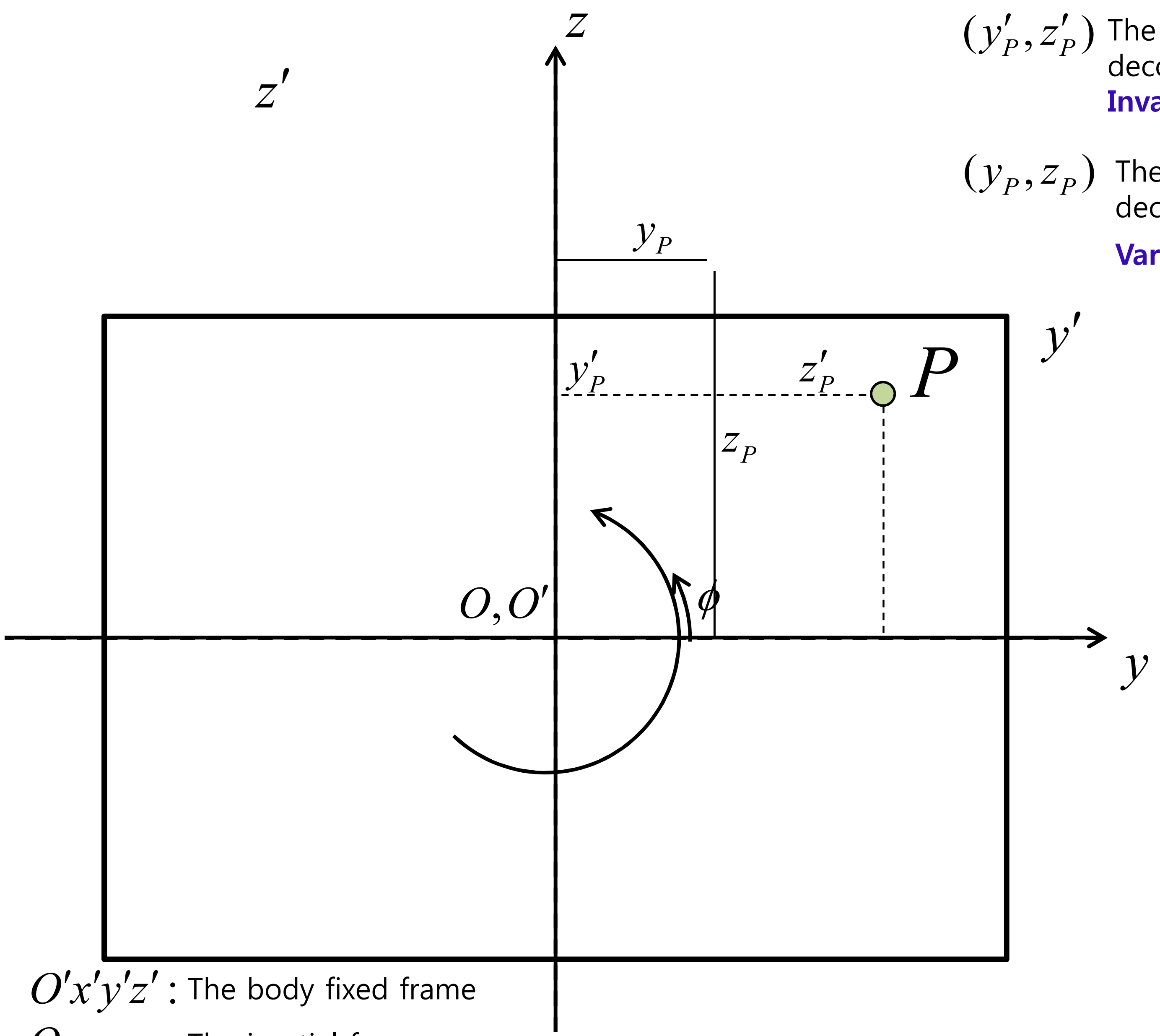
Representation of a Point "P" on the object with respect to the body fixed frame (decomposed in the body fixed frame)

(y'_P, z'_P) The position vector of the point P decomposed in the body fixed frame
Invariant with respect to the body fixed frame



$O'x'y'z'$: The body fixed frame
$Oxyz$: The inertial frame

Rotate the object with an angle of ϕ and then represent the point "P" on the object with respect to the inertial frame.



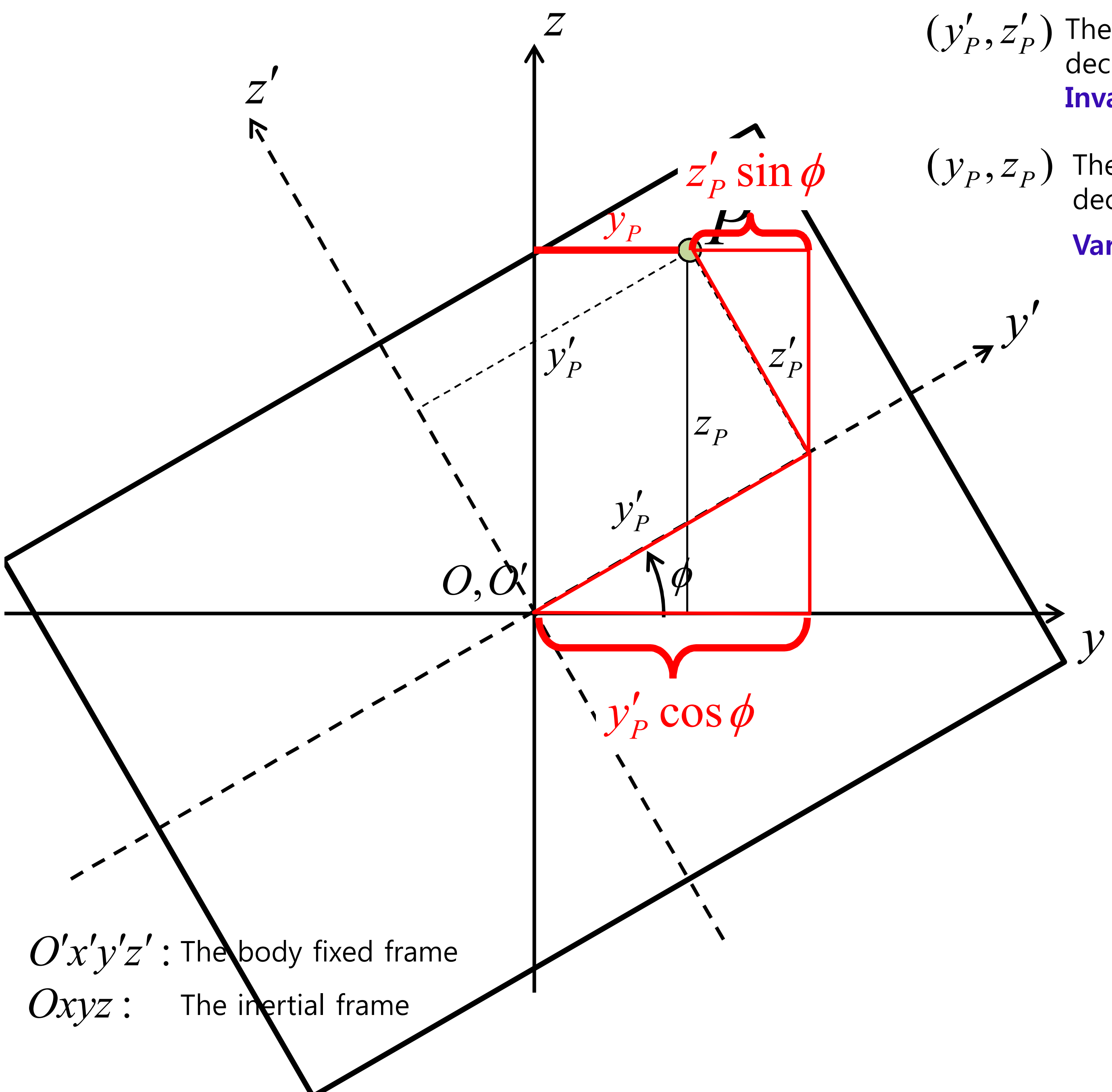
(y'_P, z'_P) The position vector of the point P decomposed in the body fixed frame
Invariant with respect to the body fixed frame

(y_P, z_P) The position vector of the point P decomposed in the initial frame
Variant with respect to the inertial frame

$O'x'y'z'$: The body fixed frame

$Oxyz$: The inertial frame

Coordinate Transformation of a Position Vector



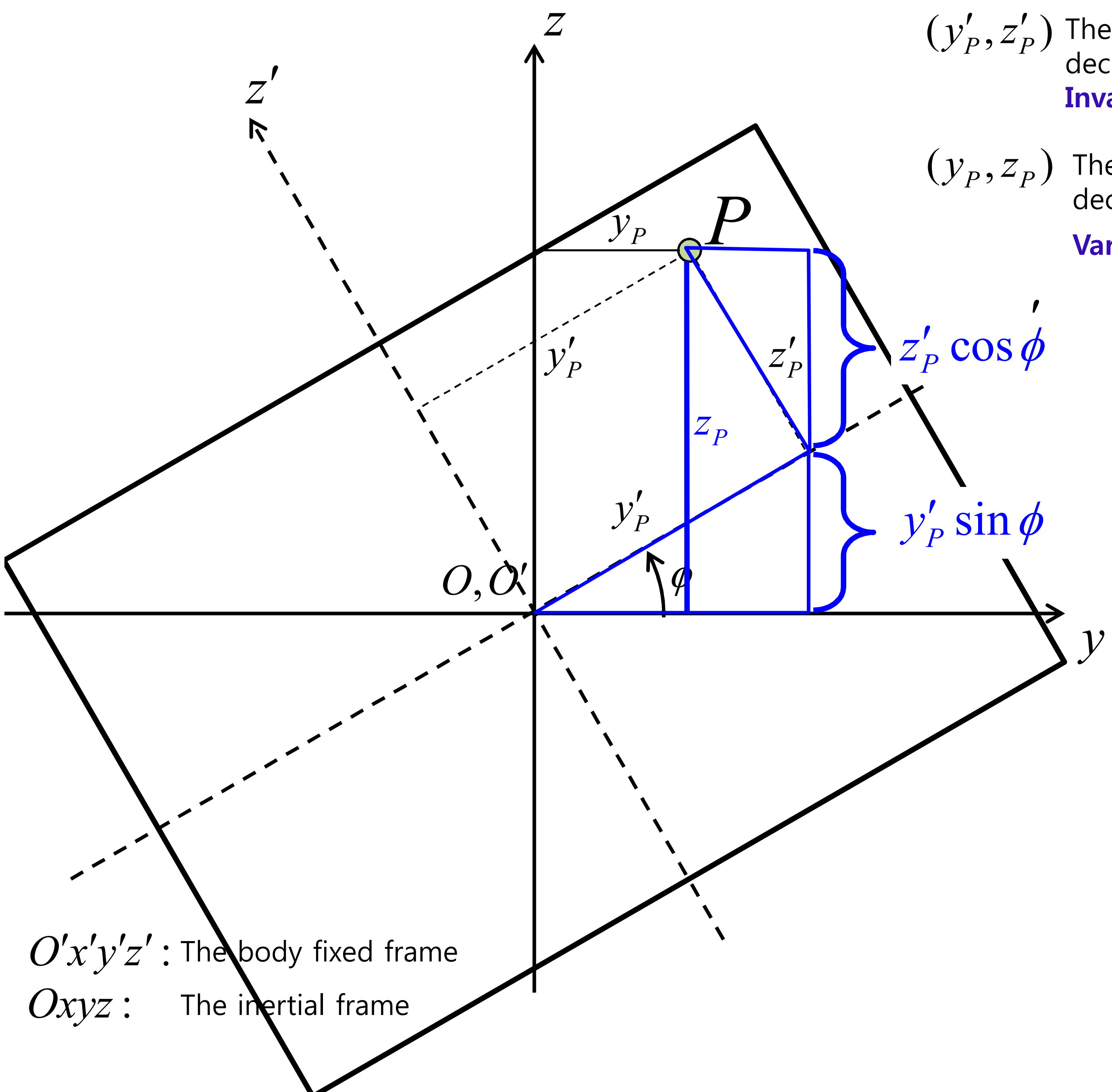
(y'_P, z'_P) The position vector of the point P decomposed in the body fixed frame
Invariant with respect to the body fixed frame

(y_P, z_P) The position vector of the point P decomposed in the initial frame
Variant with respect to the inertial frame

$$y_P = y'_P \cos\phi - z'_P \sin\phi$$

$O'x'y'z'$: The body fixed frame
$Oxyz$: The inertial frame

Coordinate Transformation of a Position Vector



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$$y_P = y'_P \cos\phi - z'_P \sin\phi$$

$$z_P = y'_P \sin\phi + z'_P \cos\phi$$

$O'x'y'z'$: The body fixed frame
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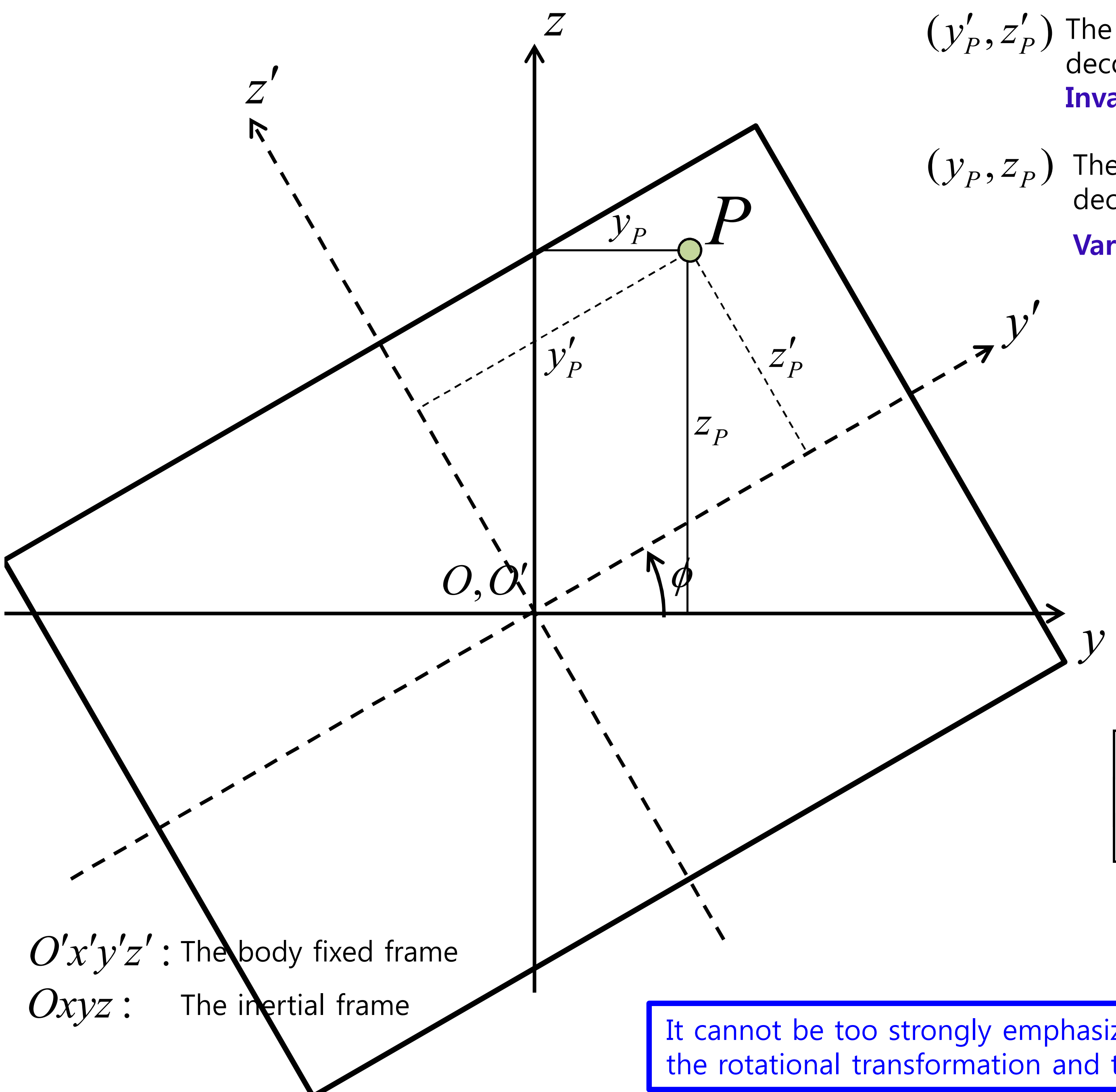
Coordinate Transformation of a Position Vector



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(y_P, z_P) The position vector of the point P decomposed in the initial frame
Variant with respect to the inertial frame

$$y_P = y'_P \cos\phi - z'_P \sin\phi$$

$$z_P = y'_P \sin\phi + z'_P \cos\phi$$

Matrix Form

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$

$$^n\mathbf{r}_P = {}^n\mathbf{R}_b \, {}^b\mathbf{r}_P$$

$O'x'y'z'$: The body fixed frame

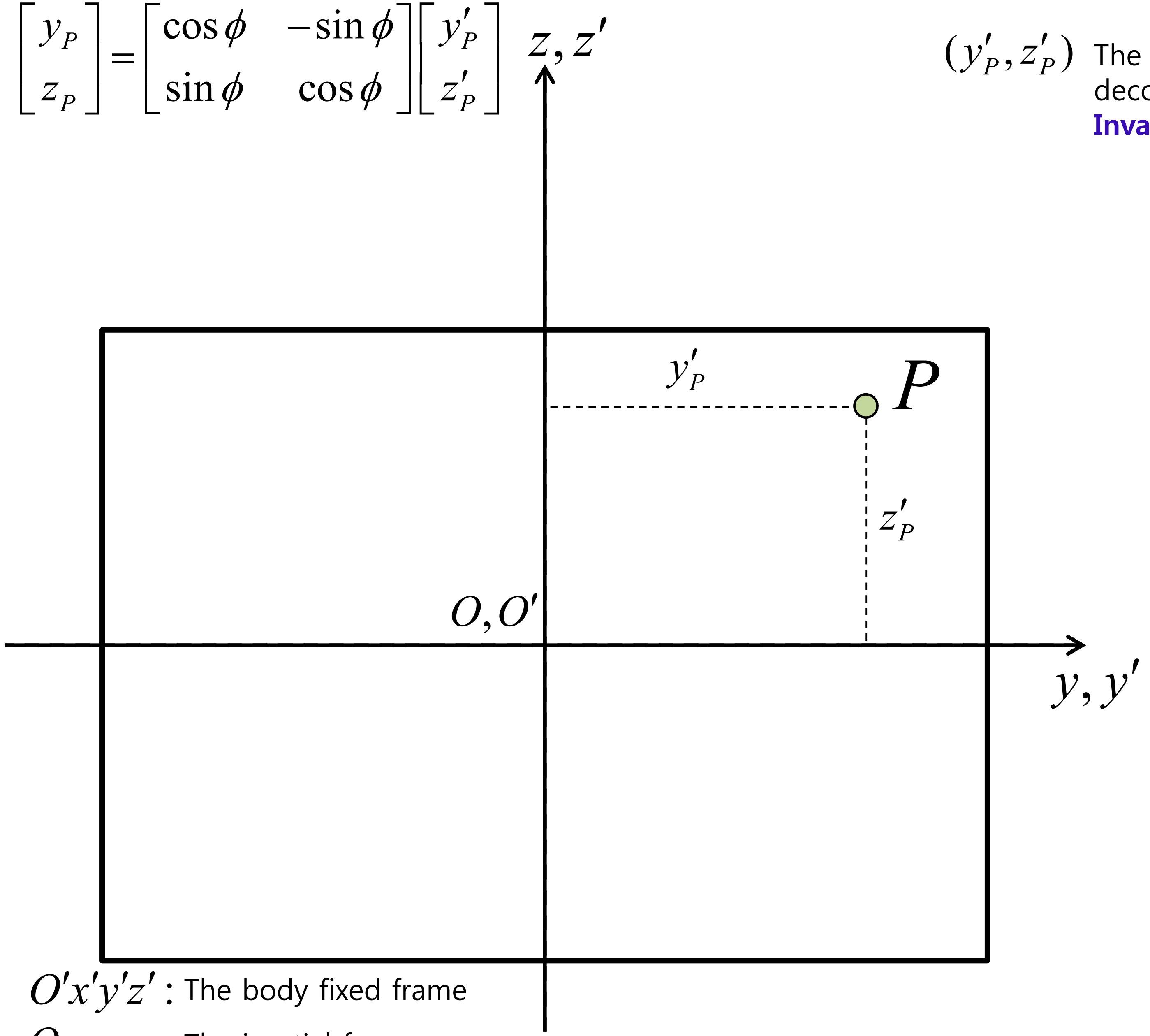
$Oxyz$: The inertial frame

It cannot be too strongly emphasized that
the rotational transformation and the coordinate transformation are important.

Representation of a Point "P" on the object with respect to the body fixed frame (decomposed in the body fixed frame)

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$

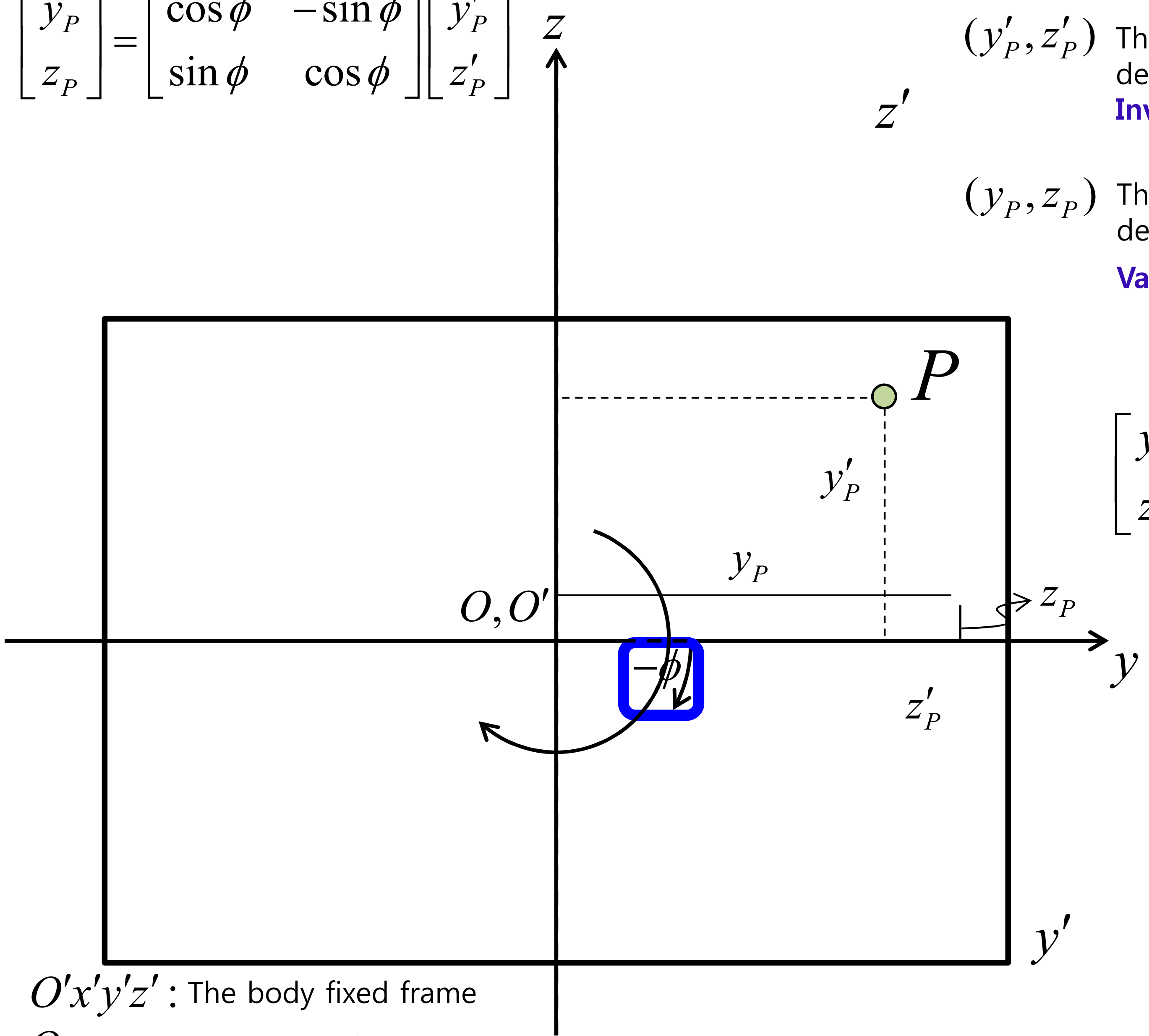
(y'_P, z'_P) The position vector of the point P decomposed in the body fixed frame
Invariant with respect to the body fixed frame



$O'x'y'z'$: The body fixed frame
$Oxyz$: The inertial frame

Coordinate Transformation of a Position Vector

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$



(y'_P, z'_P) The position vector of the point P decomposed in the body fixed frame

Invariant with respect to the body fixed frame

(y_P, z_P) The position vector of the point P decomposed in the initial frame

Variant with respect to the inertial frame

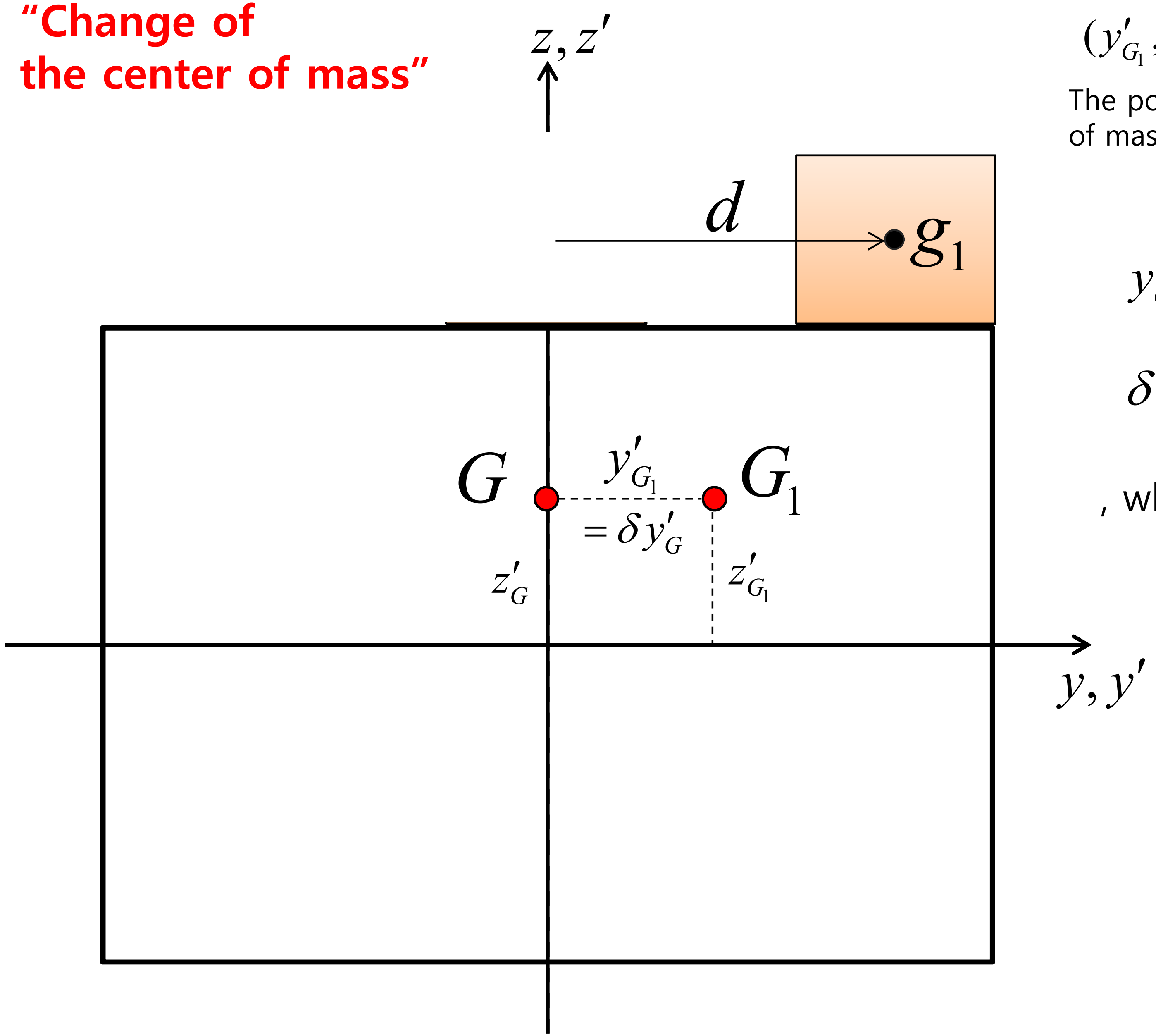
$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$

$O'x'y'z'$: The body fixed frame

$Oxyz$: The inertial frame

Change of the total center of mass caused by moving a load of weight "w" with distance "d" from "g" to "g_1"

"Change of the center of mass"



(y'_{G_1}, z'_{G_1})

The position vector of the changed total center of mass G_1 decomposed in the body fixed frame

$$y'_G = \delta y'_G$$

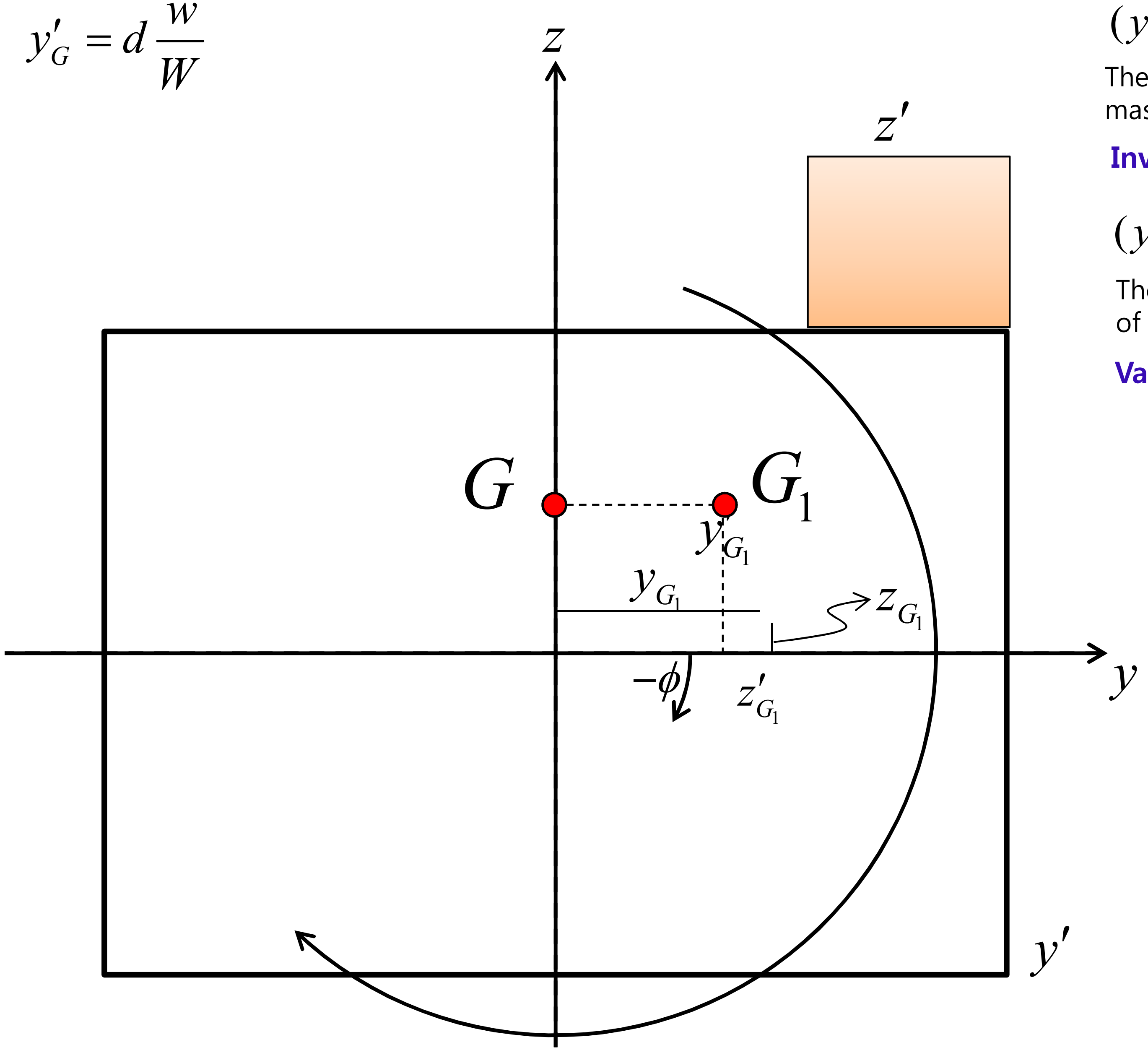
$$\delta y'_G = d \frac{w}{W}$$

, where w is the weight of the moving load

W is total weight of the object.

Rotate the object with an angle of "-ϕ" and then represent the total center of mass with respect to the inertial frame

$$y'_G = d\frac{w}{W}$$



(y'_{G_1}, z'_{G_1})

The position vector of the changed total center of mass G_1 decomposed in the body fixed frame

Invariant with respect to the body fixed frame

(y_{G_1}, z_{G_1})

The position vector of the changed total center of mass G_1 decomposed in the initial frame

Variant with respect to the inertial frame

$$\begin{bmatrix} y_{G_1} \\ z_{G_1} \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_{G_1} \\ z'_{G_1} \end{bmatrix}$$

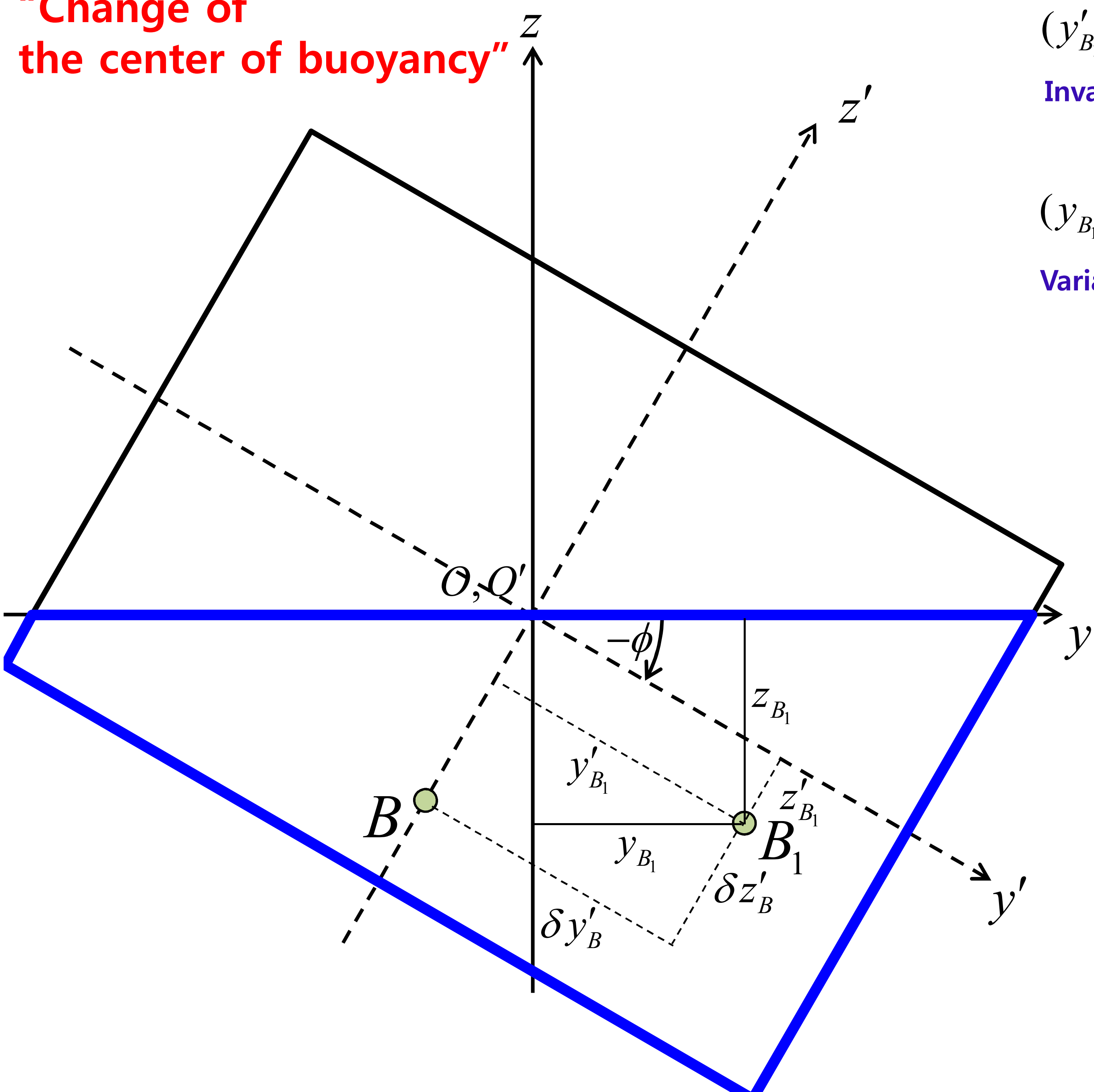
$$= \begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} y'_{G_1} \\ z'_{G_1} \end{bmatrix}$$

$$y_{G_1} = y'_{G_1}\cos\phi + z'_{G_1}\sin\phi$$

$$z_{G_1} = -y'_{G_1}\sin\phi + z'_{G_1}\cos\phi$$

Change of the center of buoyancy caused by changing the shape of immersed volume

"Change of the center of buoyancy"



(y'_{B_1}, z'_{B_1}) The position vector of the point B_1 decomposed in the body fixed frame

Invariant with respect to the body fixed frame

(y_{B_1}, z_{B_1}) The position vector of the point B_1 decomposed in the initial frame

Variant with respect to the inertial frame

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$

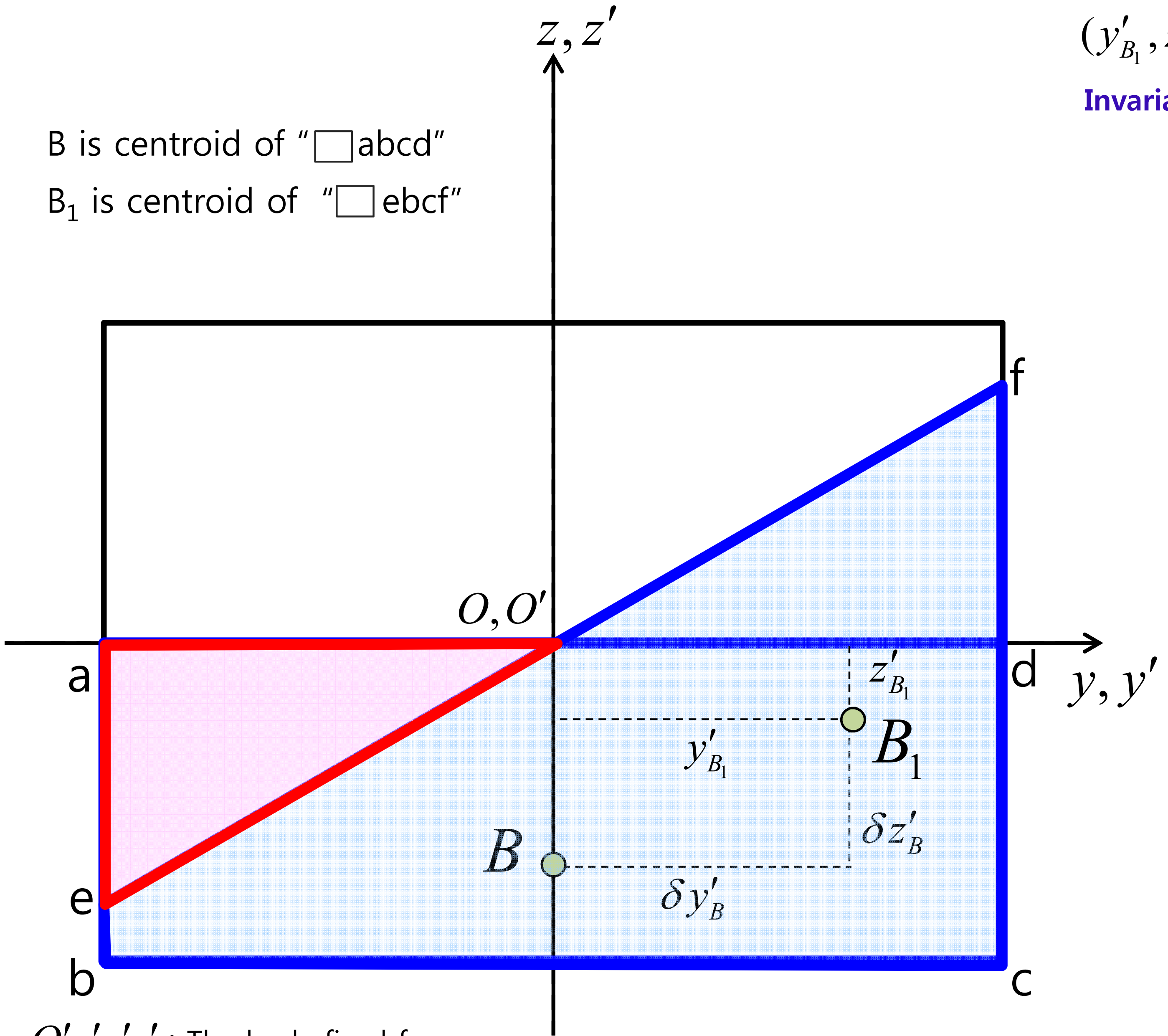
(1) Calculate the initial centroid "B" of the rectangle for z'<0 with respect to the body fixed frame.
(2) Then calculate new centroid "B_1" caused by moving a partial triangular area with respect to the body fixed frame.

(y'_{B_1}, z'_{B_1}) The position vector of the point B_1 decomposed in the body fixed frame

Invariant with respect to the body fixed frame

B is centroid of "□abcd"

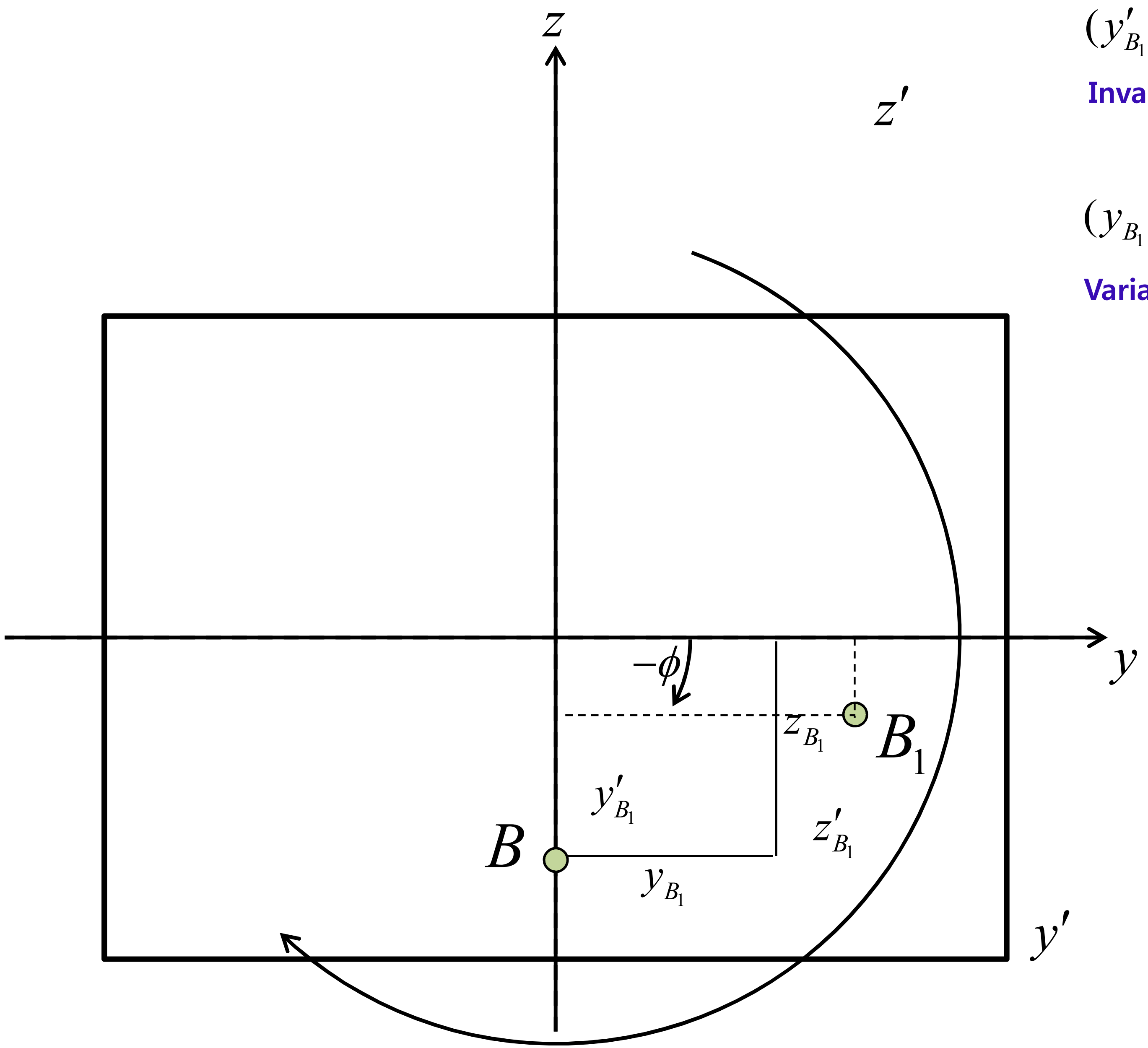
B_1 is centroid of "□ebcf"



$O'x'y'z'$: The body fixed frame

$Oxyz$: The inertial frame

(3) Rotate the new centroid "B_1" with an angle of "$-\phi$"(clockwise direction).
(4) Then calculate the position vector of the point "B_1" with respect to the inertial frame.



(y'_{B_1}, z'_{B_1}) The position vector of the point B_1 decomposed in the body fixed frame

Invariant with respect to the body fixed frame

(y_{B_1}, z_{B_1}) The position vector of the point B_1 decomposed in the initial frame

Variant with respect to the inertial frame

$$\begin{bmatrix} y_{B_1} \\ z_{B_1} \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_{B_1} \\ z'_{B_1} \end{bmatrix}$$

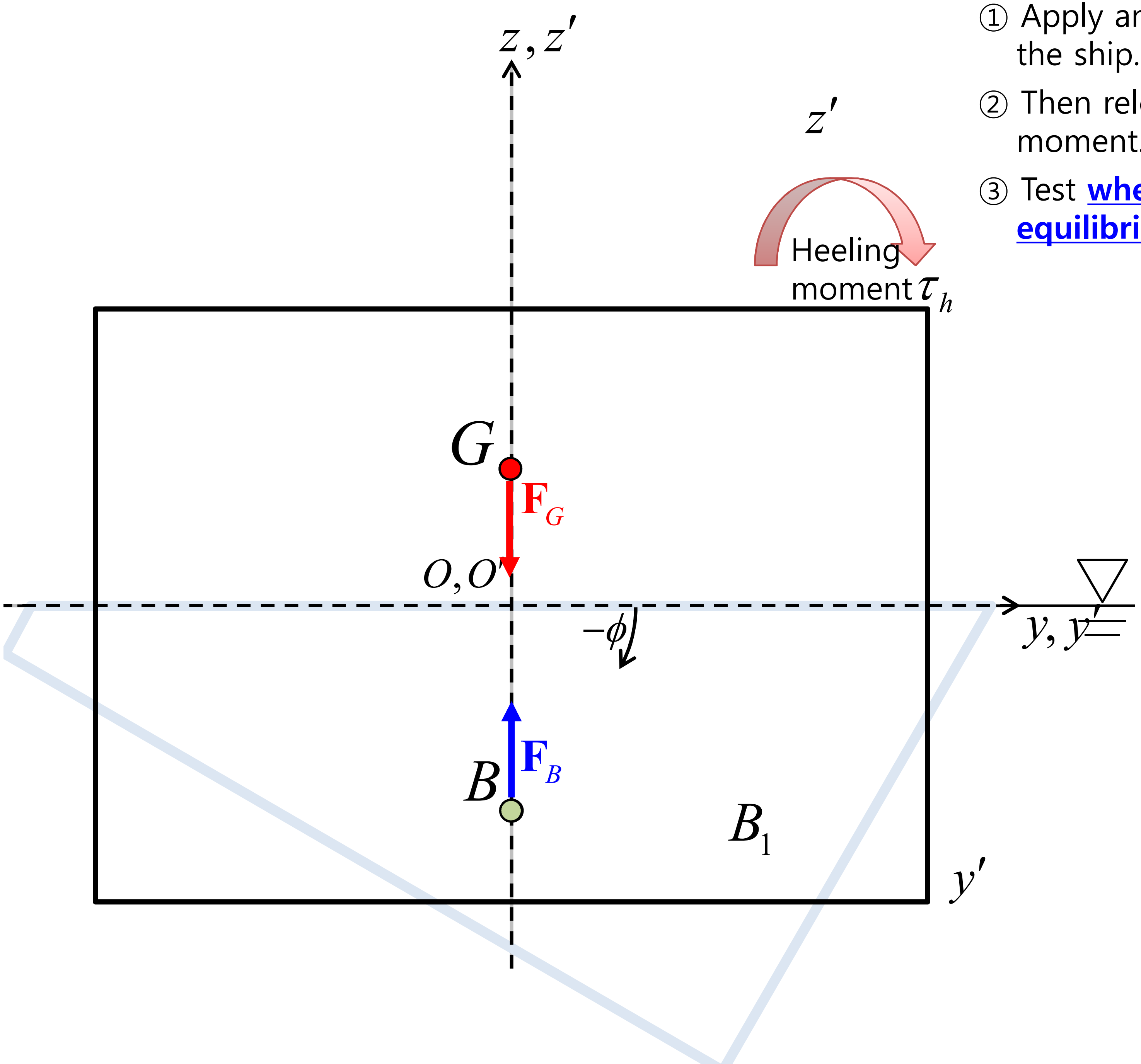
$$= \begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} y'_{B_1} \\ z'_{B_1} \end{bmatrix}$$

$$y_{B_1} = y'_{B_1} \cos\phi + z'_{B_1} \sin\phi$$

$$z_{B_1} = -y'_{B_1} \sin\phi + z'_{B_1} \cos\phi$$

Stability of a ship
- Stable Condition (1/3)

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$



① Apply an external heeling moment to the ship.

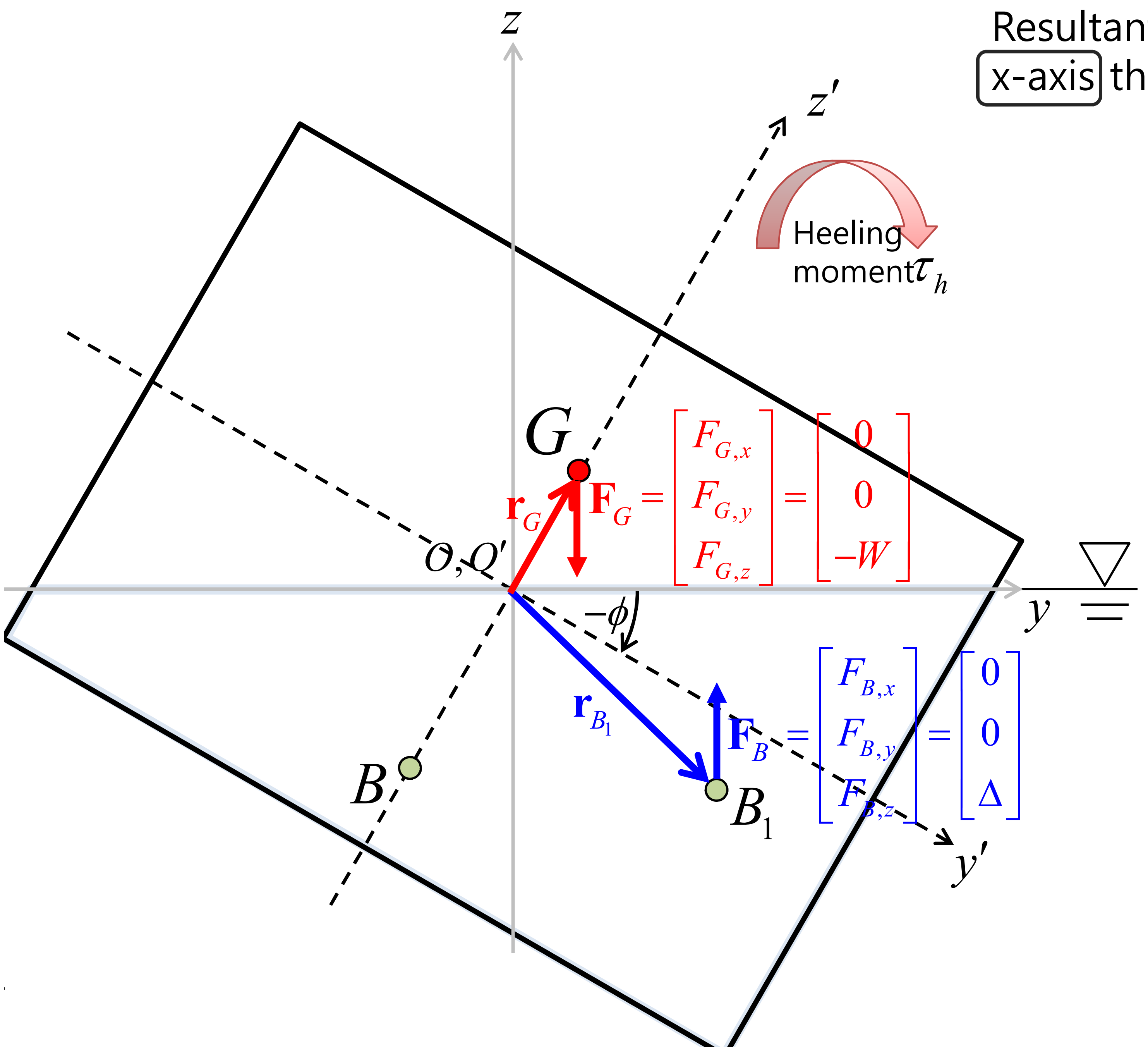
② Then release the external moment.

③ Test **whether it returns to its initial equilibrium position**.

Stability of a ship
- Stable Condition (2/3)

$$\mathbf{r}_G \times \mathbf{F}_G = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_G & y_G & z_G \\ F_{G,x} & F_{G,y} & F_{G,z} \end{vmatrix} = \begin{array}{l} \mathbf{i}(y_G \cdot F_{G,z} - z_G \cdot F_{G,y}) \\ +\mathbf{j}(-x_G \cdot F_{G,z} + z_G \cdot F_{G,x}) \\ +\mathbf{k}(x_G \cdot F_{G,y} - y_G \cdot F_{G,x}) \end{array}$$

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$



Resultant moment about $\boxed{\text{x-axis}}$ through point O ($\boldsymbol{\tau}^e$) :

$$\boldsymbol{\tau}^e = \mathbf{r}_G \times \mathbf{F}_G + \mathbf{r}_{B_1} \times \mathbf{F}_B$$

$$= \boxed{\mathbf{i}(y_G \cdot F_{G,z} - z_G \cdot F_{G,y})}$$
$$+\mathbf{j}(-x_G \cdot F_{G,z} + z_G \cdot F_{G,x})$$
$$+\mathbf{k}(x_G \cdot F_{G,y} - y_G \cdot F_{G,x})$$
$$\boxed{+\mathbf{i}(y_{B_1} \cdot F_{B,z} - z_{B_1} \cdot F_{B,y})}$$
$$+\mathbf{j}(-x_{B_1} \cdot F_{B,z} + z_{B_1} \cdot F_{B,x})$$
$$+\mathbf{k}(x_{B_1} \cdot F_{B,y} - y_{B_1} \cdot F_{B,x})$$

$$= \mathbf{i}(y_G \cdot F_{G,z} - z_G \cdot \cancel{F_{G,y}})$$
$$+\mathbf{i}(y_{B_1} \cdot F_{B,z} - z_{B_1} \cdot \cancel{F_{B,y}})$$

$$= \mathbf{i}(y_G \cdot (-W) + y_{B_1} \cdot \Delta)$$

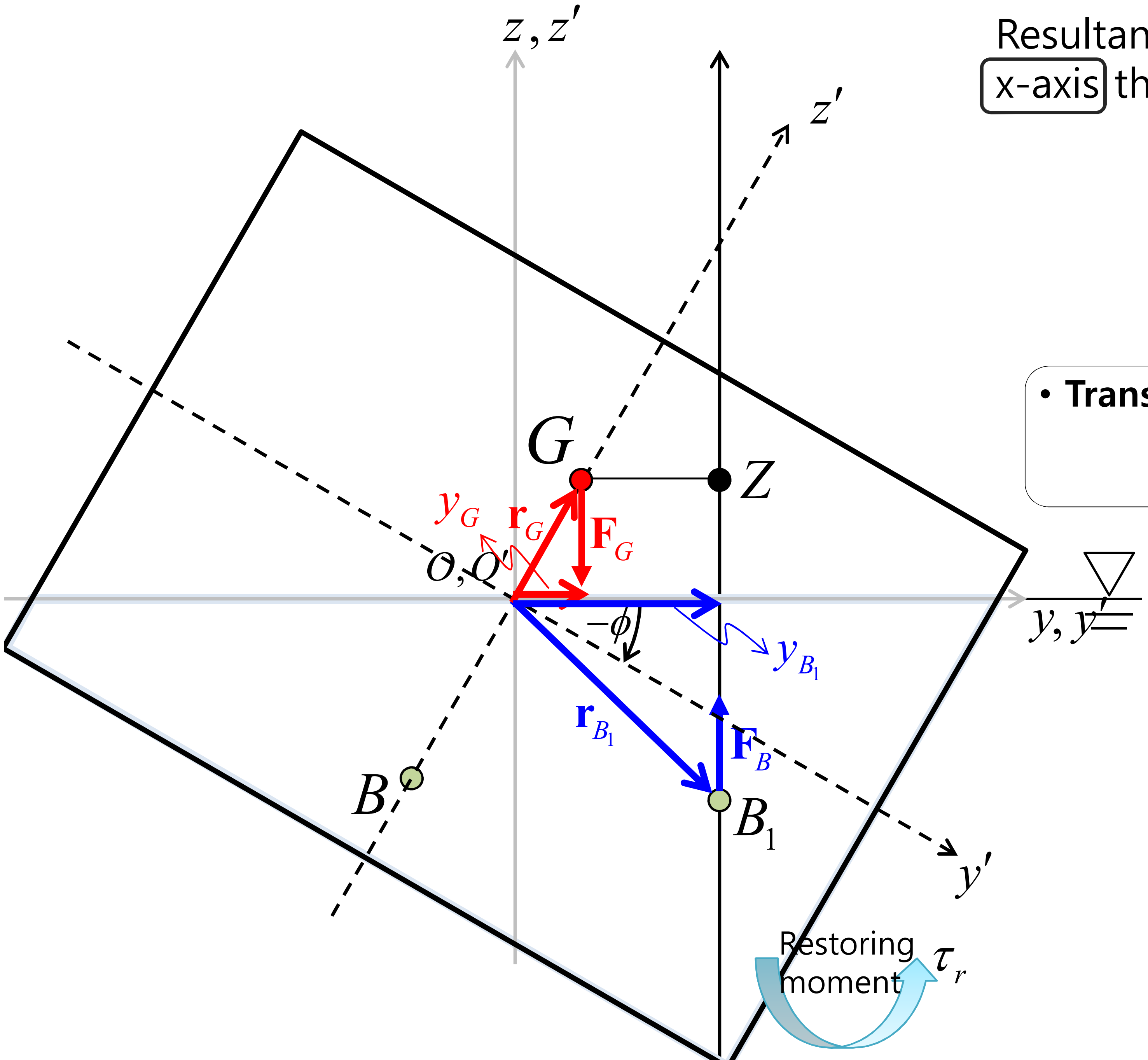
If $W = \Delta$

$$= \mathbf{i}(y_G \cdot (-\Delta) + y_{B_1} \cdot \Delta)$$

$$= \mathbf{i} \cdot \Delta(y_{B_1} - y_G)$$

Stability of a ship
- Stable Condition (3/3)

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$



Resultant moment about $\boxed{\text{x-axis}}$ through point O ($\boldsymbol{\tau}^e$) :

$$\boldsymbol{\tau}^e = \mathbf{r}_G \times \mathbf{F}_G + \mathbf{r}_{B_1} \times \mathbf{F}_B$$

$$= \mathbf{i} \cdot \Delta (y_{B_1} - y_G)$$

$$= \mathbf{i} \cdot \Delta \cdot GZ$$

- **Transverse Righting Moment**

$$\tau_r = \Delta \cdot GZ$$

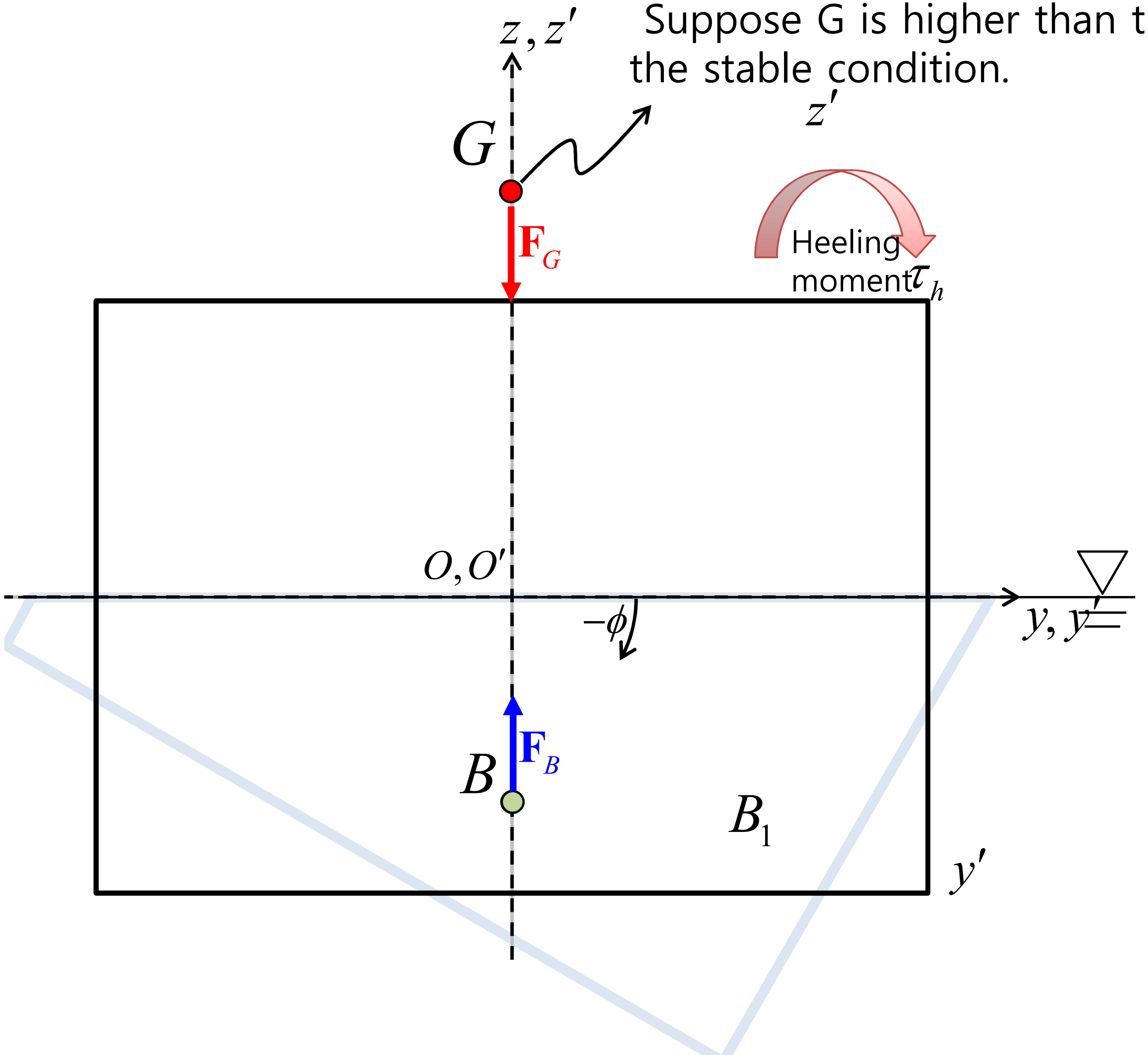
The moment arm induced by the buoyant force and gravitational force is expressed by GZ, where Z is the intersection point of the line of buoyant force(Δ) through the new position of the center of buoyancy(B_1) with a transversely parallel line to a waterline through the center of the ship's mass(G).

Stable!!

Stability of a ship
- Neutral Condition (1/3)

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$

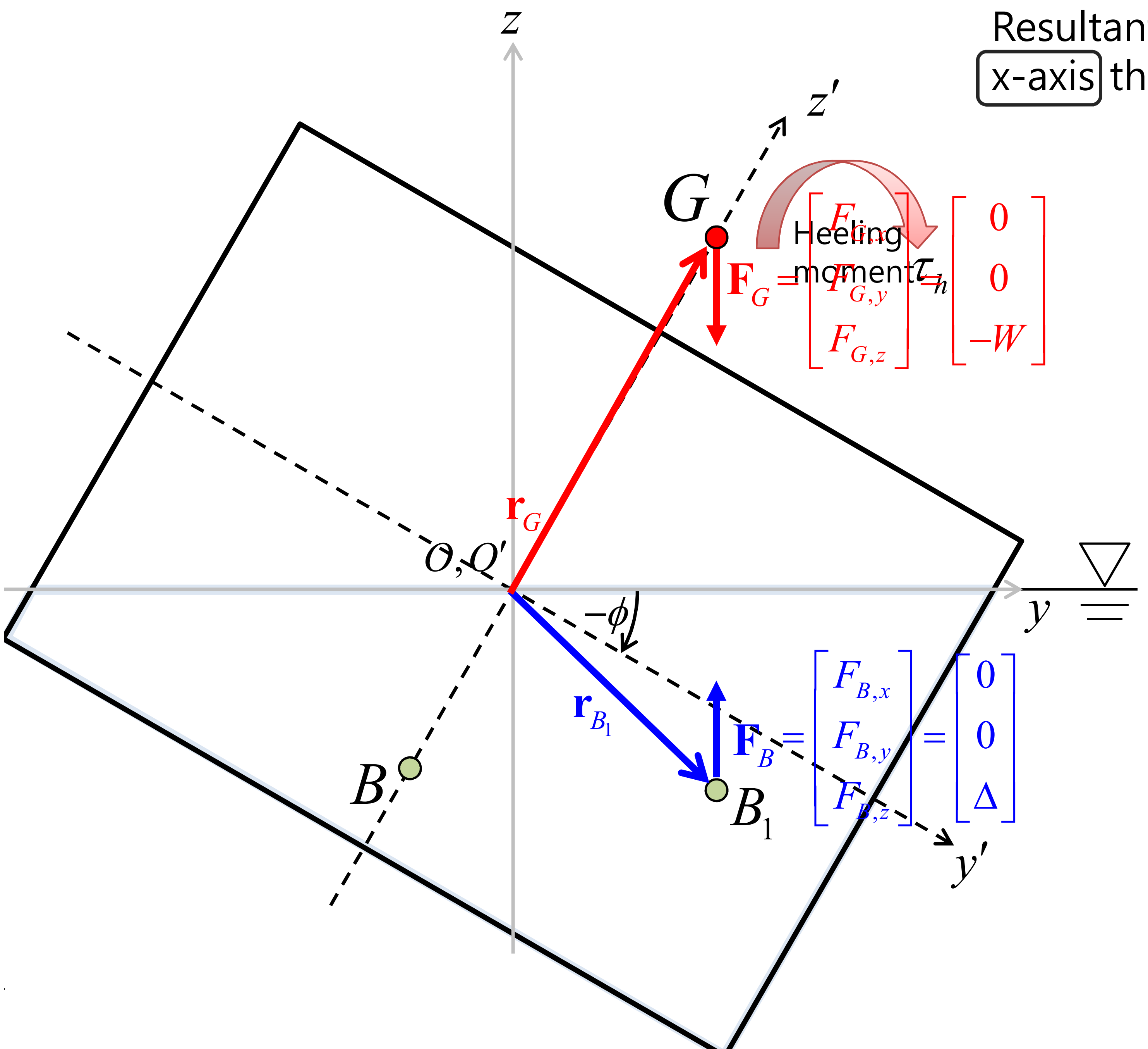
Suppose G is higher than that of the stable condition.



Stability of a ship - Neutral Condition (2/3)

$$\mathbf{r}_G \times \mathbf{F}_G = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_G & y_G & z_G \\ F_{G,x} & F_{G,y} & F_{G,z} \end{vmatrix} = \begin{matrix} \mathbf{i}(y_G \cdot F_{G,z} - z_G \cdot F_{G,y}) \\ +\mathbf{j}(-x_G \cdot F_{G,z} + z_G \cdot F_{G,x}) \\ +\mathbf{k}(x_G \cdot F_{G,y} - y_G \cdot F_{G,x}) \end{matrix}$$

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$



Resultant moment about $\boxed{\text{x-axis}}$ through point O ($\boldsymbol{\tau}^e$) :

$$\boldsymbol{\tau}^e = \mathbf{r}_G \times \mathbf{F}_G + \mathbf{r}_{B_1} \times \mathbf{F}_B$$

$$= \boxed{\mathbf{i}(y_G \cdot F_{G,z} - z_G \cdot F_{G,y})}$$
$$+\mathbf{j}(-x_G \cdot F_{G,z} + z_G \cdot F_{G,x})$$
$$+\mathbf{k}(x_G \cdot F_{G,y} - y_G \cdot F_{G,x})$$
$$\boxed{+\mathbf{i}(y_{B_1} \cdot F_{B,z} - z_{B_1} \cdot F_{B,y})}$$
$$+\mathbf{j}(-x_{B_1} \cdot F_{B,z} + z_{B_1} \cdot F_{B,x})$$
$$+\mathbf{k}(x_{B_1} \cdot F_{B,y} - y_{B_1} \cdot F_{B,x})$$

$$= \mathbf{i}(y_G \cdot F_{G,z} - z_G \cdot \cancel{F_{G,y}})$$
$$+\mathbf{i}(y_{B_1} \cdot F_{B,z} - z_{B_1} \cdot \cancel{F_{B,y}})$$

$$= \mathbf{i}(y_G \cdot (-W) + y_{B_1} \cdot \Delta)$$

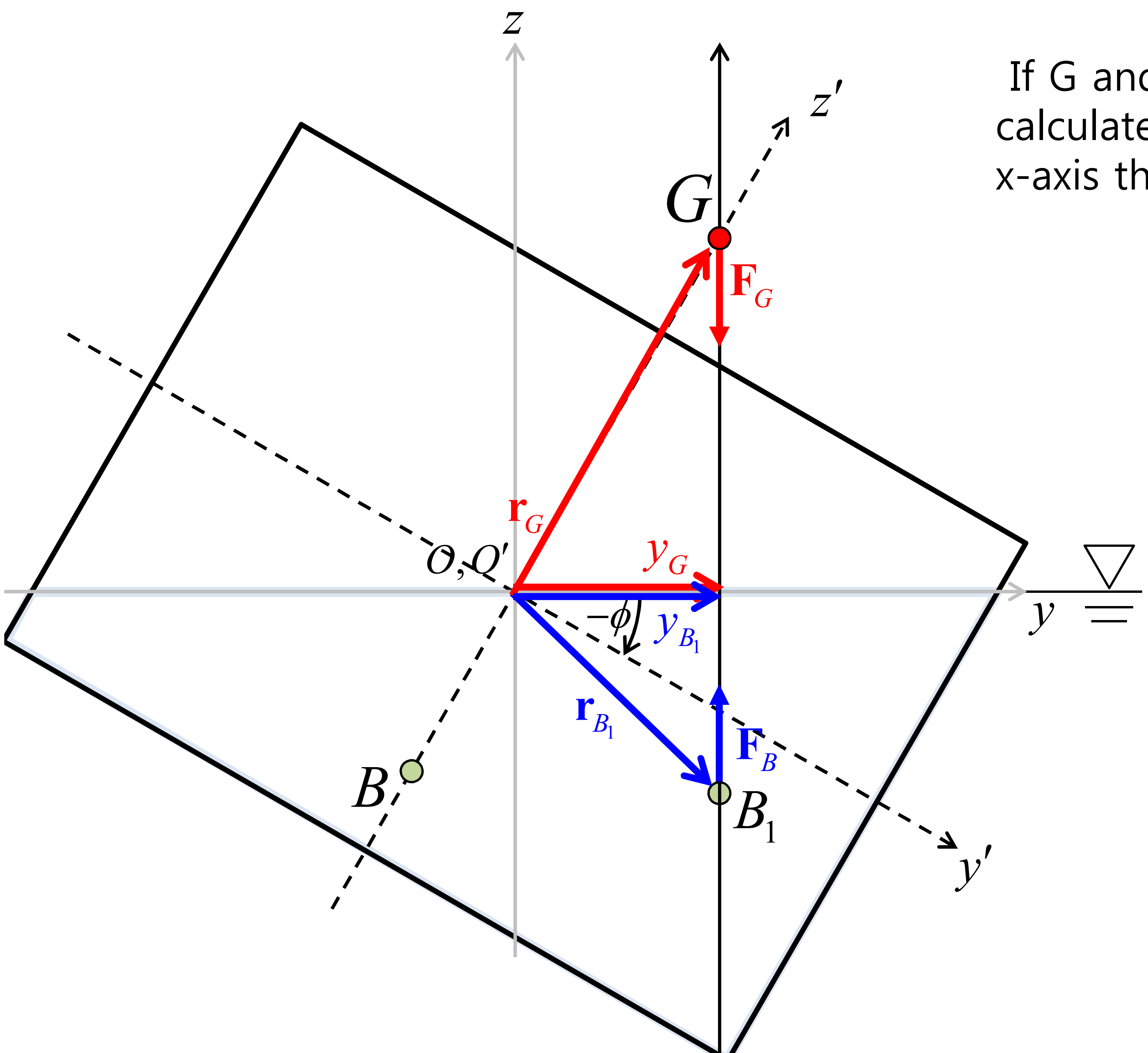
If $W = \Delta$

$$= \mathbf{i}(y_G \cdot (-\Delta) + y_{B_1} \cdot \Delta)$$

$$= \mathbf{i} \cdot \Delta(y_{B_1} - y_G)$$

Stability of a ship
- Neutral Condition (3/3)

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$



If G and B_1 are on one line, calculate resultant moment about x-axis through point O ($\boldsymbol{\tau}^e$) :

$$\boldsymbol{\tau}^e = \mathbf{r}_G \times \mathbf{F}_G + \mathbf{r}_{B_1} \times \mathbf{F}_B$$

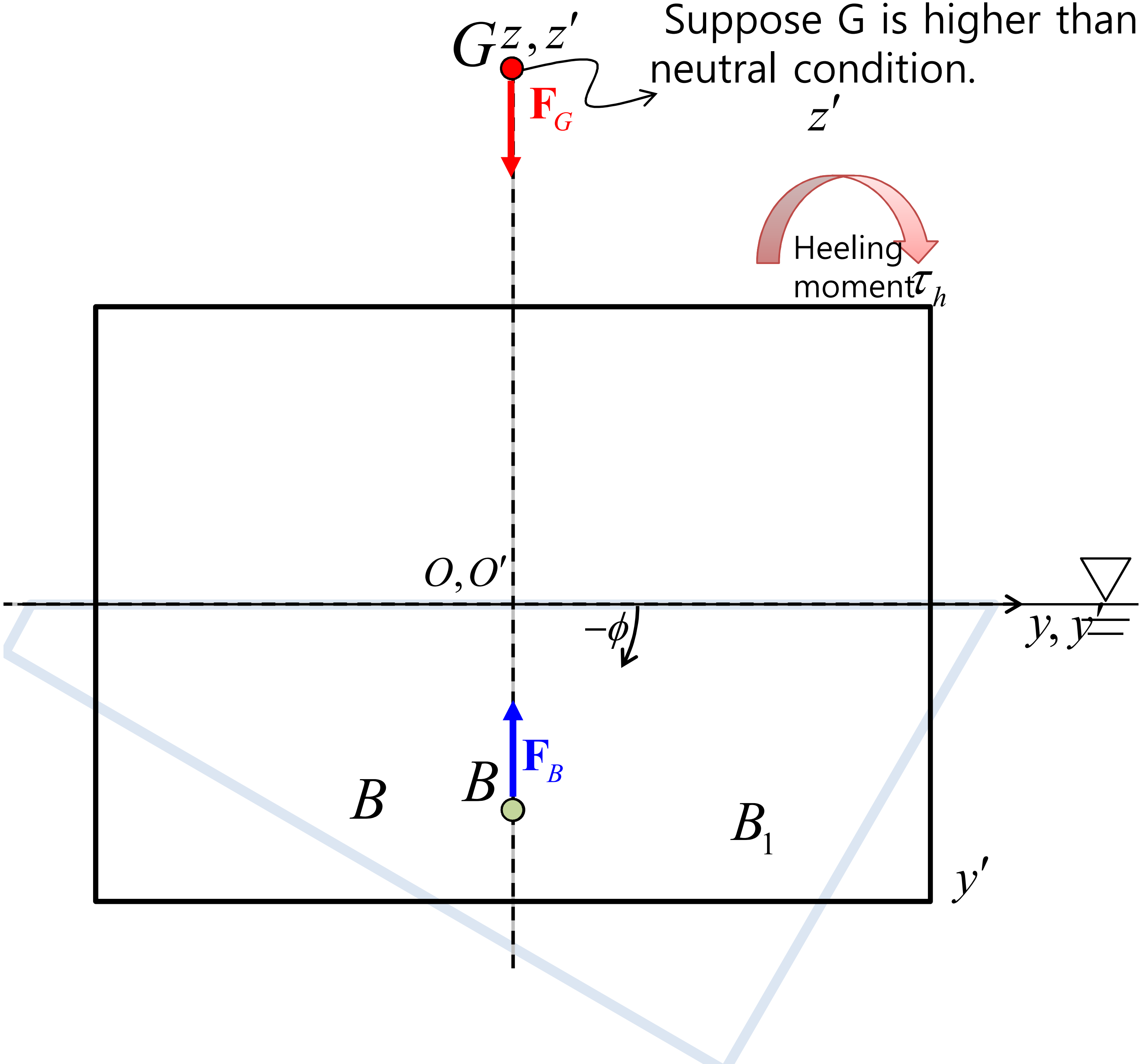
$$= \mathbf{i} \cdot \Delta(\underbrace{y_{B_1} - y_G}_{\Downarrow \; 0})$$

Neutral!!

Stability of a ship
- Unstable Condition (1/3)

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$

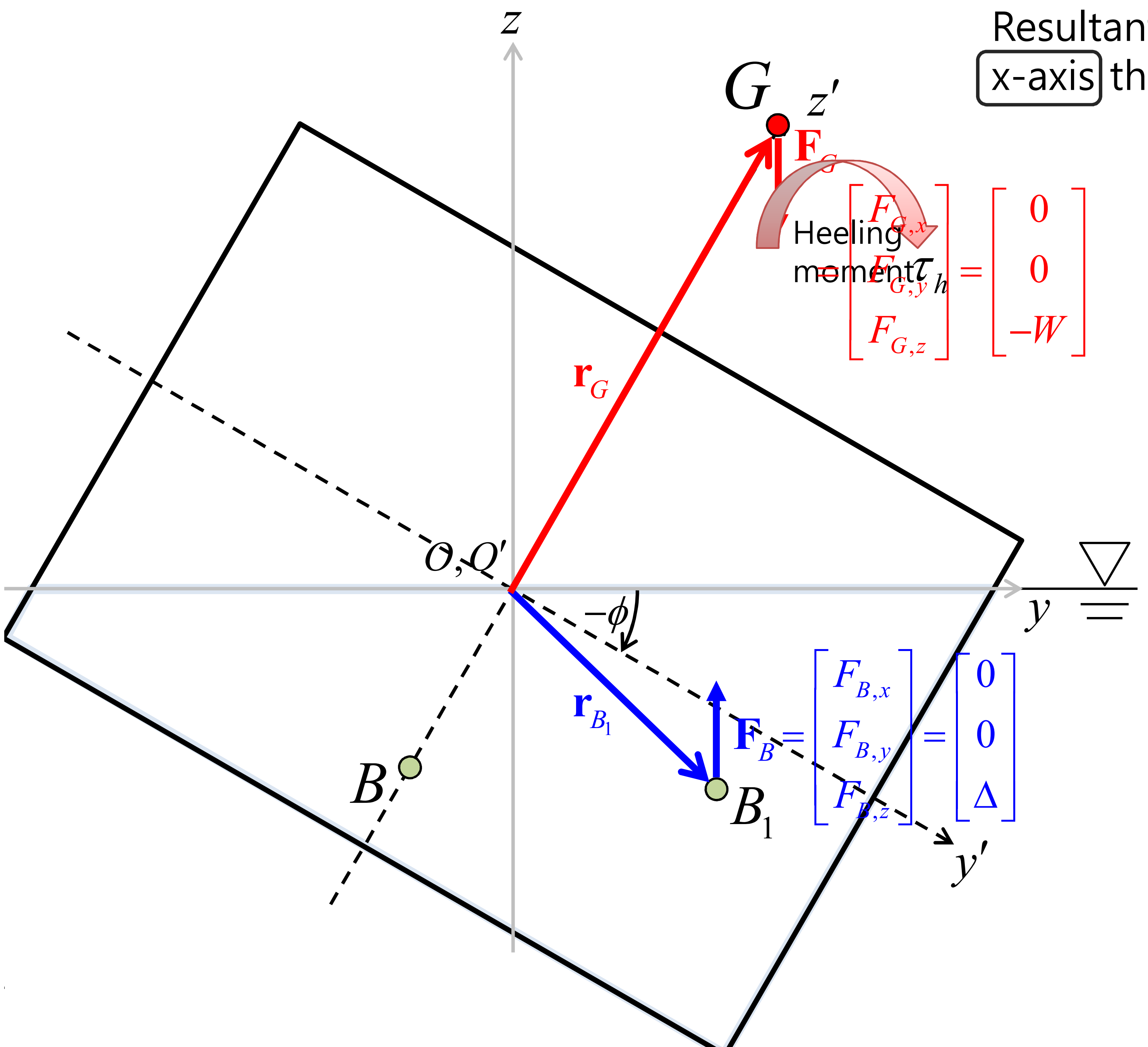
Suppose G is higher than that of the neutral condition.



Stability of a ship
- Unstable Condition (2/3)

$$\mathbf{r}_G \times \mathbf{F}_G = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_G & y_G & z_G \\ F_{G,x} & F_{G,y} & F_{G,z} \end{vmatrix} = \begin{array}{l} \mathbf{i}(y_G \cdot F_{G,z} - z_G \cdot F_{G,y}) \\ +\mathbf{j}(-x_G \cdot F_{G,z} + z_G \cdot F_{G,x}) \\ +\mathbf{k}(x_G \cdot F_{G,y} - y_G \cdot F_{G,x}) \end{array}$$

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$



Resultant moment about $\boxed{\text{x-axis}}$ through point O ($\boldsymbol{\tau}^e$) :

$$\boldsymbol{\tau}^e = \mathbf{r}_G \times \mathbf{F}_G + \mathbf{r}_{B_1} \times \mathbf{F}_B$$

$$= \boxed{\mathbf{i}(y_G \cdot F_{G,z} - z_G \cdot F_{G,y})}$$
$$+\mathbf{j}(-x_G \cdot F_{G,z} + z_G \cdot F_{G,x})$$
$$+\mathbf{k}(x_G \cdot F_{G,y} - y_G \cdot F_{G,x})$$
$$\boxed{+\mathbf{i}(y_{B_1} \cdot F_{B,z} - z_{B_1} \cdot F_{B,y})}$$
$$+\mathbf{j}(-x_{B_1} \cdot F_{B,z} + z_{B_1} \cdot F_{B,x})$$
$$+\mathbf{k}(x_{B_1} \cdot F_{B,y} - y_{B_1} \cdot F_{B,x})$$

$$= \mathbf{i}(y_G \cdot F_{G,z} - z_G \cdot \cancel{F_{G,y}})$$
$$+\mathbf{i}(y_{B_1} \cdot F_{B,z} - z_{B_1} \cdot \cancel{F_{B,y}})$$

$$= \mathbf{i}(y_G \cdot (-W) + y_{B_1} \cdot \Delta)$$

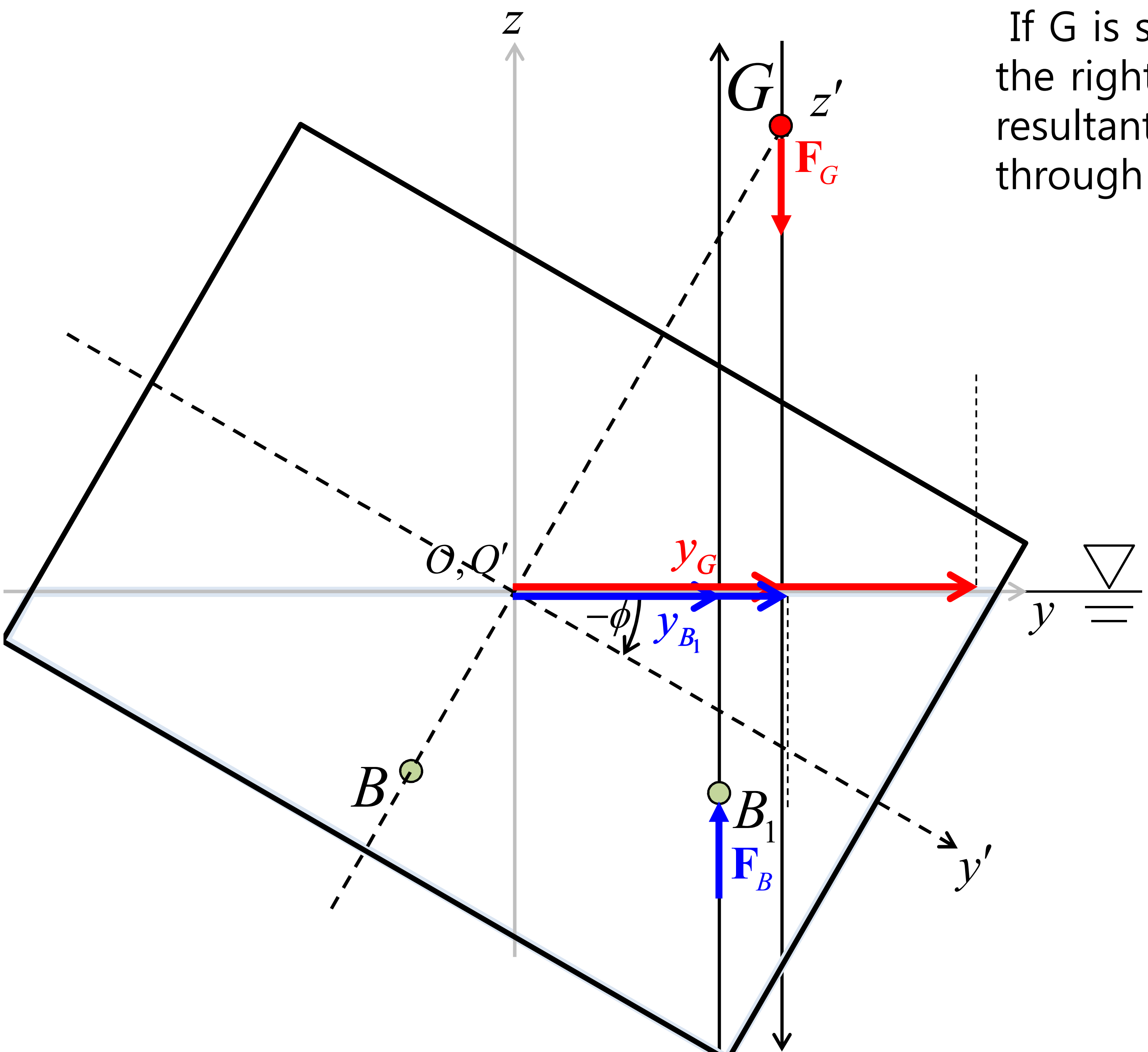
If $W = \Delta$

$$= \mathbf{i}(y_G \cdot (-\Delta) + y_{B_1} \cdot \Delta)$$

$$= \mathbf{i} \cdot \Delta(y_{B_1} - y_G)$$

Stability of a ship
- Unstable Condition (3/3)

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$



If G is so high that G locates on the right side of B_1, calculate resultant moment about x-axis through point O ($\boldsymbol{\tau}^e$) :

$$\boldsymbol{\tau}^e = \mathbf{r}_G \times \mathbf{F}_G + \mathbf{r}_{B_1} \times \mathbf{F}_B$$

$$= \mathbf{i} \cdot \Delta(\underline{y_{B_1} - y_G})$$

$$y_{B_1} - y_G < 0$$

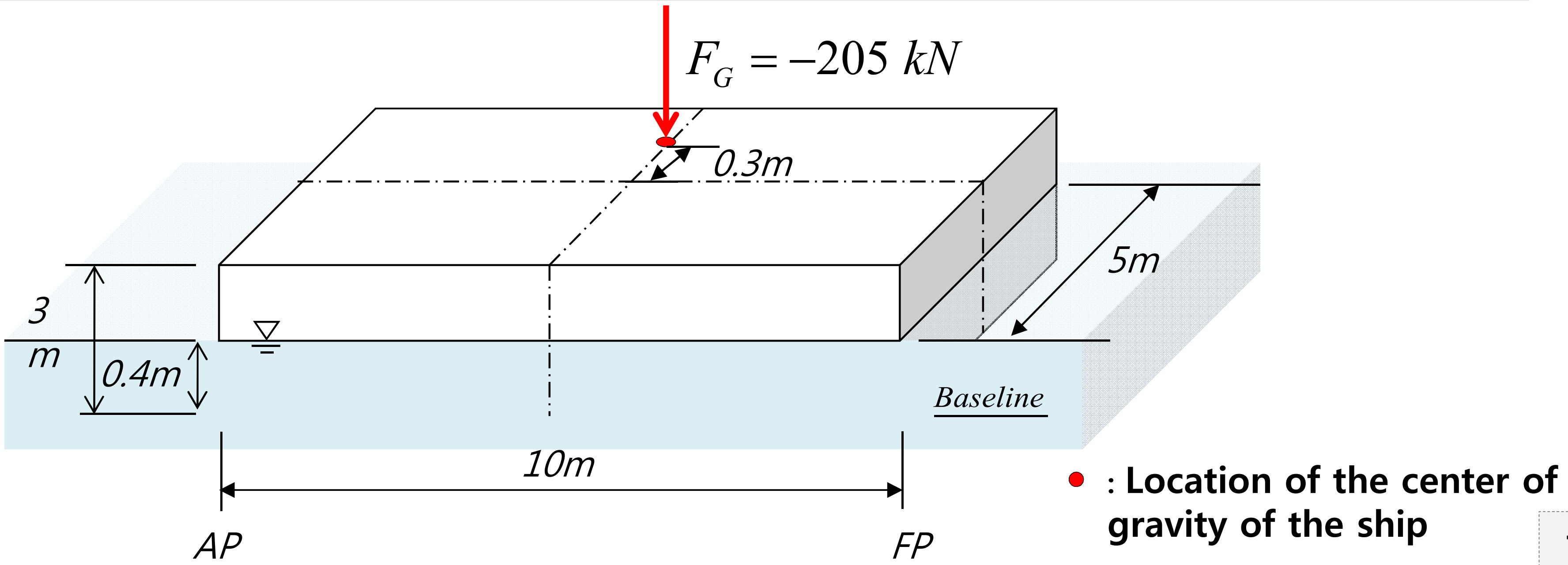
Unstable!!

Question 1) The center of mass is moved to 0.3 [m] in the direction of the starboard side.

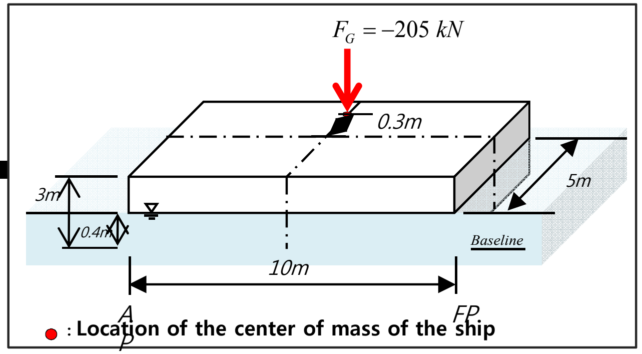
A box-shaped ship of 10 meter length, 5 meter breadth and 3 meter height weights 205 [kN].
The center of mass is moved 0.3 [m] to the left side of the center of the deck.
When the ship is in static equilibrium state, determine the angle of heel(ϕ) of the ship.

Given : Length(L):10m, Breadth(B):5m, Depth(D):3m, Weight(W): 205kN,
Location of the Center of Gravity: 0.3m to the left side of the center of the deck
Find : Angle of Heel(φ)

Assumption)
(1) Gravitational acceleration = 10 [m/s^2], Density of sea water = 1.025 [ton/m^3](Mg/m^3)
(2) When the ship will be in the static equilibrium finally, the deck will not be immersed and the bottom will not emerge.



Solution)
(1) Static Equilibrium



When the ship is floating in sea water, the requirement for ship to be in static equilibrium state is derived from Newton's 2nd law and Euler equation as follows.

(1-1) Newton's 2nd Law: Force Equilibrium

The resultant force should be zero to be in static equilibrium.

$$\sum {}^{n}F = {}^{n}F_{G,z} + {}^{n}F_{B,z} = 0$$

, where
${}^{n}F_{G.z}$: z_n-coordinate of the gravitational force
${}^{n}F_{B.z}$: z_n-coordinate of the buoyant force

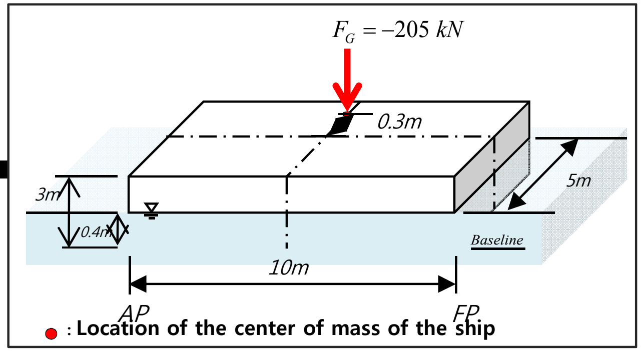
(1-2) Euler Equation: Moment Equilibrium

The resultant moment should be zero to be in static equilibrium.

$$\sum {}^{n}\boldsymbol{\tau} = {}^{n}\mathbf{M}_G + {}^{n}\mathbf{M}_B = \mathbf{0}$$

, where
${}^{n}\mathbf{M}_G$: the moment due to the gravitational force
${}^{n}\mathbf{M}_B$: the moment due to the buoyant force.

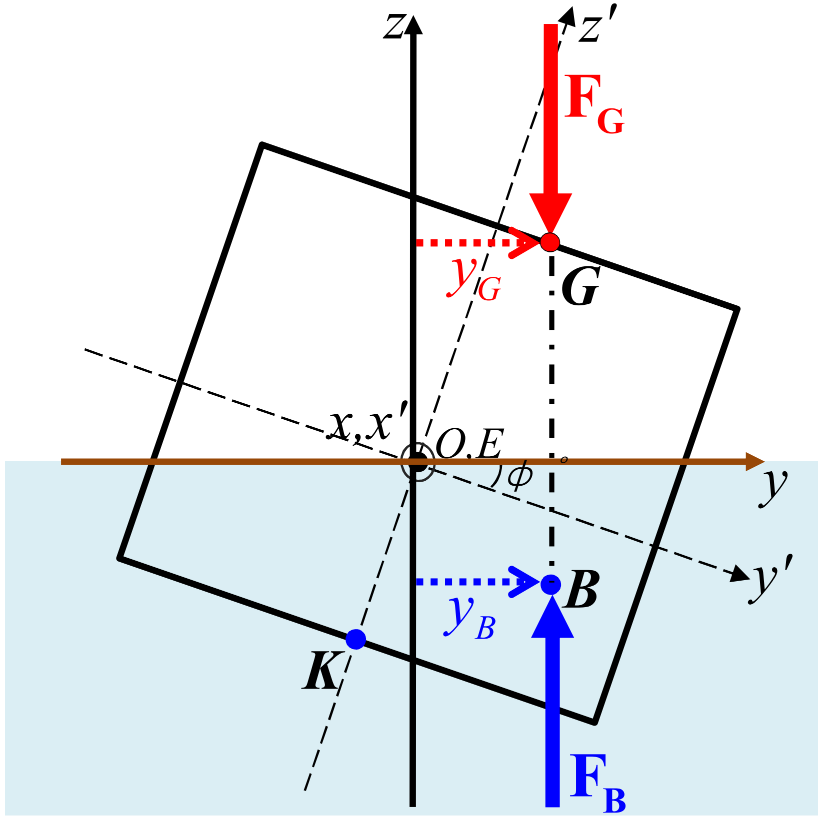
Solution)
(1) Static Equilibrium



The first step is to satisfy the Newton-Euler equation which requires that the sum of total forces and moments acting on the ship is zero.

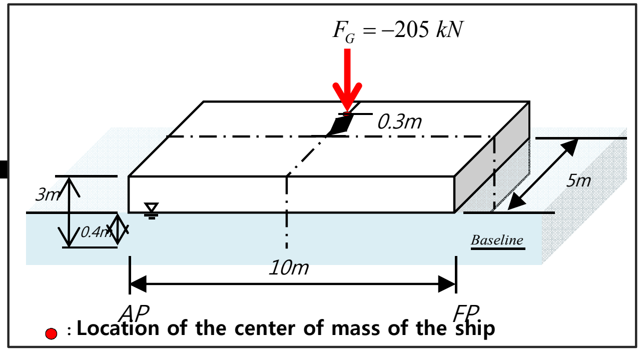
As described earlier, in order to satisfy a stable equilibrium, **the buoyant force and gravitational force should act on the same vertical line**, therefore, the moment arm of the buoyant force and gravitational force must be same.

$$\boxed{y_G = y_B}$$

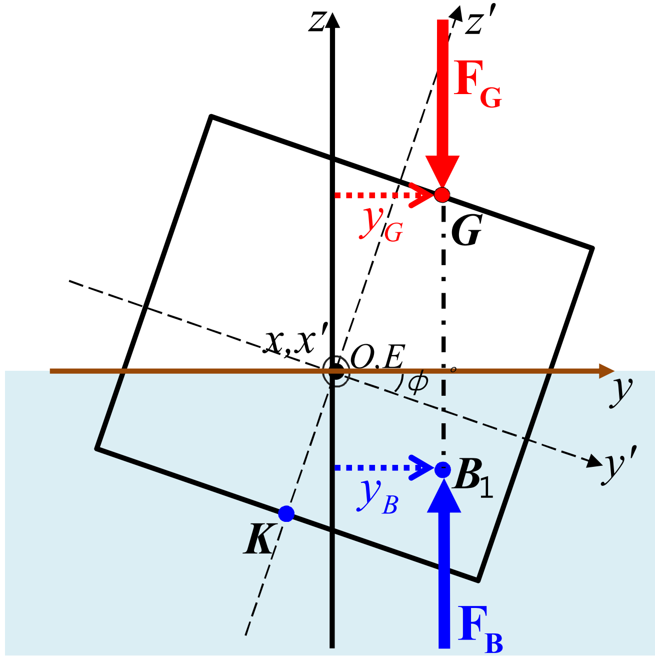


Solution)

(1) Static Equilibrium



$$\boxed{y_G = y_B}$$



$$\begin{bmatrix} y_G \\ z_G \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} y'_G \\ z'_G \end{bmatrix} \qquad \begin{bmatrix} y_B \\ z_B \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} y'_B \\ z'_B \end{bmatrix}$$

By representing y_G and y_B with y'_G, z'_G, y'_B, and z'_B, we can get

$$y'_G \cdot \cos\phi + z'_G \cdot \sin\phi = y'_B \cdot \cos\phi + z'_B \cdot \sin\phi$$

In this equation, we suppose that y'_G and z'_G are already given, and y'_B and z'_B can be geometrically calculated.

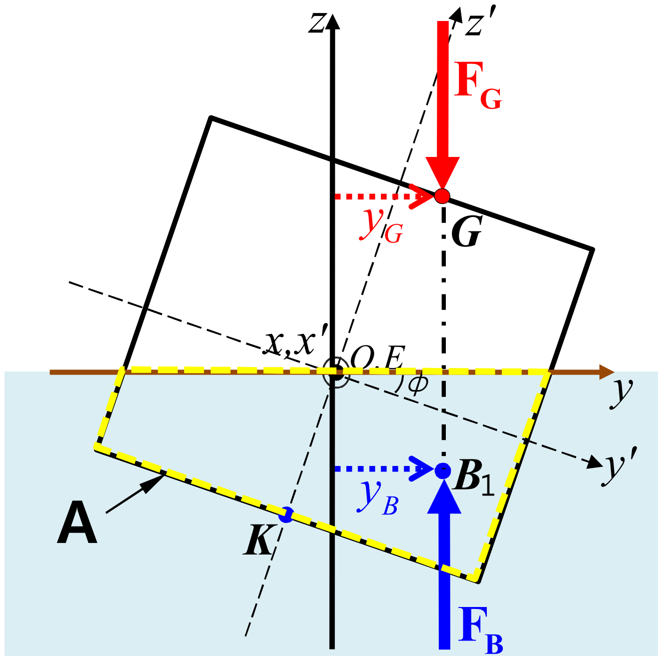
Body fixed coordinate system(b-frame): Body fixed frame $x'\,y'\,z'$

Space fixed coordinate system(n-frame): Inertial frame $x\,y\,z$

Solution)

$y_G = y_B$

(2-1) Changed center of buoyancy, B_1, with respect to the body fixed frame



The centroid of A with respect to the body fixed frame:

$$\left(y'_{C\_A}, z'_{C\_A}\right) = \left(\frac{M_{A,z'}}{A_A}, \frac{M_{A,y'}}{A_A}\right)$$

, where
A_A : the area of A
$M_{A,z'}$: 1st moment of area of A about z′ axis
$M_{A,y'}$: 1st moment of area of A about y′ axis.

To obtain the centroid of A, the followings are required.
- **The area of A**
- **1st moment of area of A about z′ axis**
- **1st moment of area of A about y′ axis**

Solution)

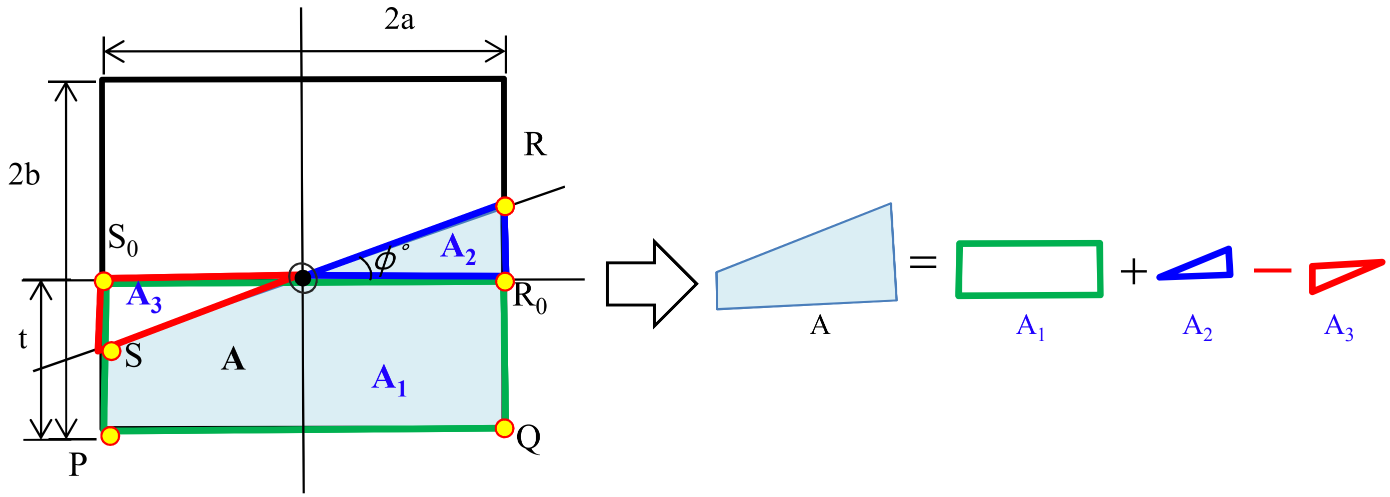
(2-2) Center of buoyancy and center of gravity with respect to the body fixed frame

1) Center of buoyancy, B_1, with respect to the body fixed frame

To calculate the centroid of A using the geometrical relations, we use **the areas, A_1, A_2, and A_3.**

The centroid of A with respect to the body fixed frame:

$$\left(y'_{C\_A}, z'_{C\_A}\right) = \left(\frac{M_{A,z'}}{A_A}, \frac{M_{A,y'}}{A_A}\right)$$



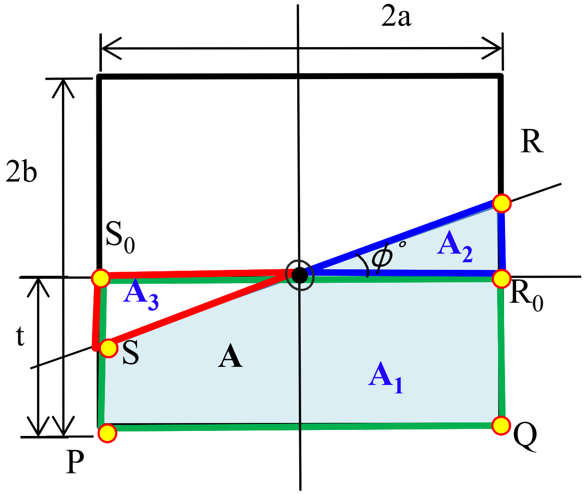
To describe the values of A_1, A_2, and A_3 using the geometrical parameters (a, t, and ϕ), **y′ and z′ coordinate of the points P, Q, R, R_0, S, S_0 with respect to the body fixed frame is used**, which are given as follows.

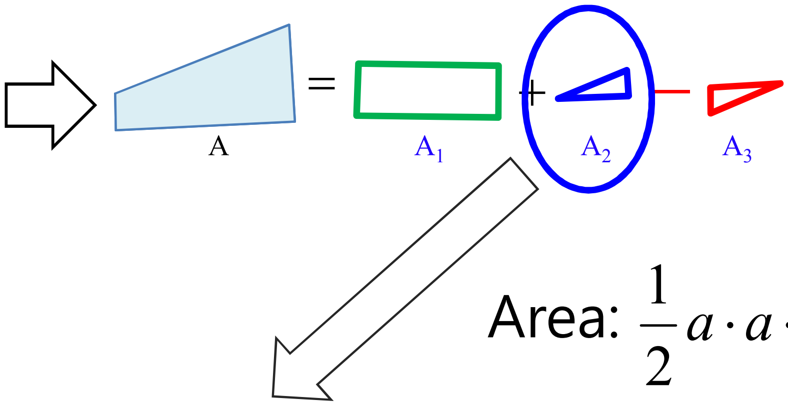
$$P(y'_P, z'_P) = (-a, \ -t), \ Q(y'_Q, z'_Q) = (a, -t)$$

$$R(y'_R, z'_R) = (a, a\cdot\tan\phi), \ R_0(y'_{R_0}, z'_{R_0}) = (a, 0)$$

$$S(y'_S, z'_S) = (-a, -a\cdot\tan\phi), \ S_0(y'_{S_0}, z'_{S_0}) = (-a, 0)$$

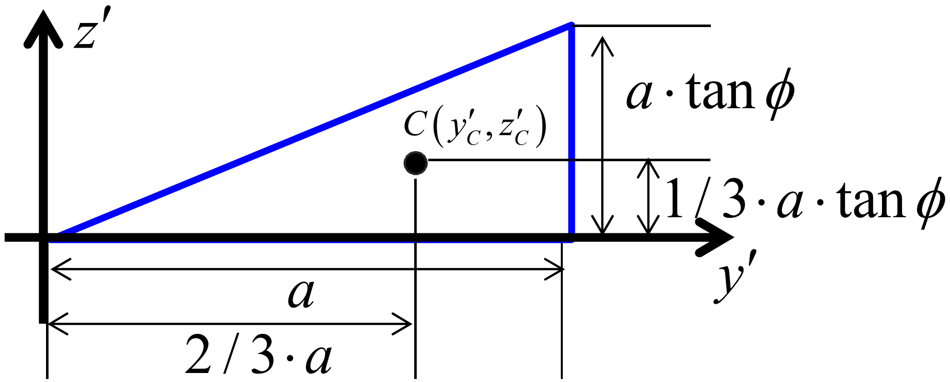
Calculation of area, centroid, and moment of area





Area: $\frac{1}{2} a \cdot a \cdot \tan\phi$

A_2



Centroid: $\left(y'_C, z'_C\right) = \frac{2}{3}a, \frac{1}{3}a\tan\phi$

Moment of area about z′ axis:

$$Area \times y'_C = \frac{1}{2} a \cdot a \cdot \tan\phi \times \frac{2}{3} a = \frac{1}{3} a^3 \tan\phi$$

Moment of area about y′ axis:

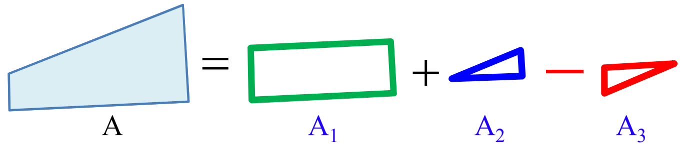
$$Area \times z'_C = \frac{1}{2} a \cdot a \cdot \tan\phi \times \frac{1}{3} a \cdot \tan\phi = \frac{1}{6} a^3 \tan^2\phi$$

Solution)

$y_G = y_B$

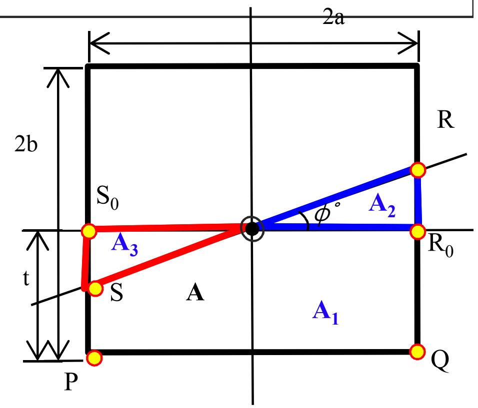
(2-3) Center of buoyancy and center of gravity with respect to the body fixed frame

1) Center of buoyancy, B_1, with respect to the body fixed frame



The centroid of A with respect to the body fixed frame:

$$\left(y'_{C\_A}, z'_{C\_A}\right) = \left(\frac{M_{A,z'}}{A_A}, \frac{M_{A,y'}}{A_A}\right)$$



The table blow summarizes **the results of the area, centroid with respect to the body fixed frame and 1st moment of area with respect to the body fixed frame of A_1, A_2, A_3, and A.**

	Area (A_A)	Centroid (y'_C, z'_C)	Moment of area about z'-axis $(y'_C \cdot A)$	Moment of area about y'-axis $(z'_C \cdot A)$
A_1	$2a \cdot t$	$\left(0, -\frac{t}{2}\right)$	0	$-a \cdot t^2$
A_2	$\frac{1}{2} \cdot a \cdot a \cdot \tan\phi$	$\left(\frac{2a}{3}, \frac{a \cdot \tan\phi}{3}\right)$	$\frac{a^3 \cdot \tan\phi}{3}$	$\frac{a^3 \cdot (\tan\phi)^2}{6}$
A_3	$\frac{1}{2} \cdot a \cdot a \cdot \tan\phi$	$\left(-\frac{2a}{3}, -\frac{a \cdot \tan\phi}{3}\right)$	$-\frac{a^3 \cdot \tan\phi}{3}$	$-\frac{a^3 \cdot (\tan\phi)^2}{6}$
A (=A_1+A_2-A_3)	$2a \cdot t$	-	$\frac{2a^3 \cdot \tan\phi}{3}$	$-a \cdot t^2 + \frac{a^3 \cdot (\tan\phi)^2}{3}$

The center of buoyancy, B_1, with respect to the body fixed frame is

$$\left(y'_B, z'_B\right) = \left(\frac{\boxed{M_{A,z'}}}{\boxed{A_A}}, \frac{\boxed{M_{A,y'}}}{\boxed{A_A}}\right) = \left(\frac{a^2 \cdot \tan\phi}{3t}, -\frac{t}{2} + \frac{a^2 \cdot (\tan\phi)^2}{6t}\right)$$

Solution)

$y_G = y_B$

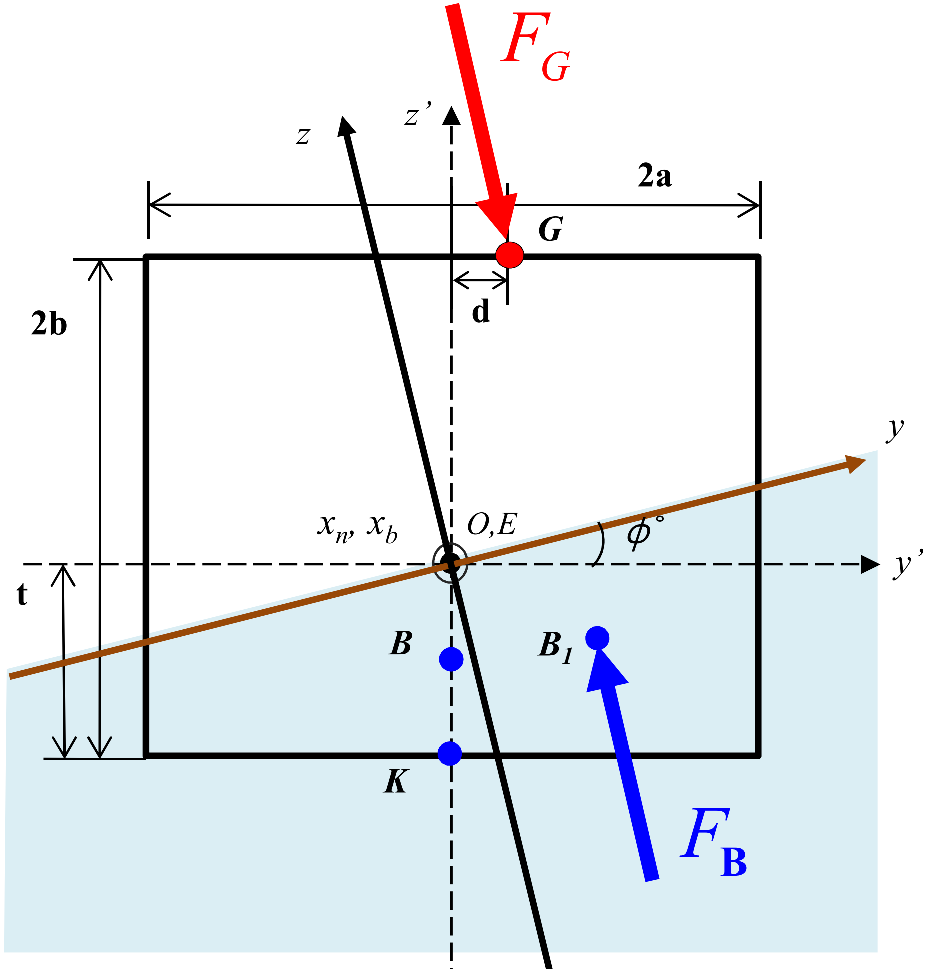
(2-3) Center of buoyancy and center of gravity with respect to the body fixed frame

2) Center of gravity, G, with respect to the body fixed frame

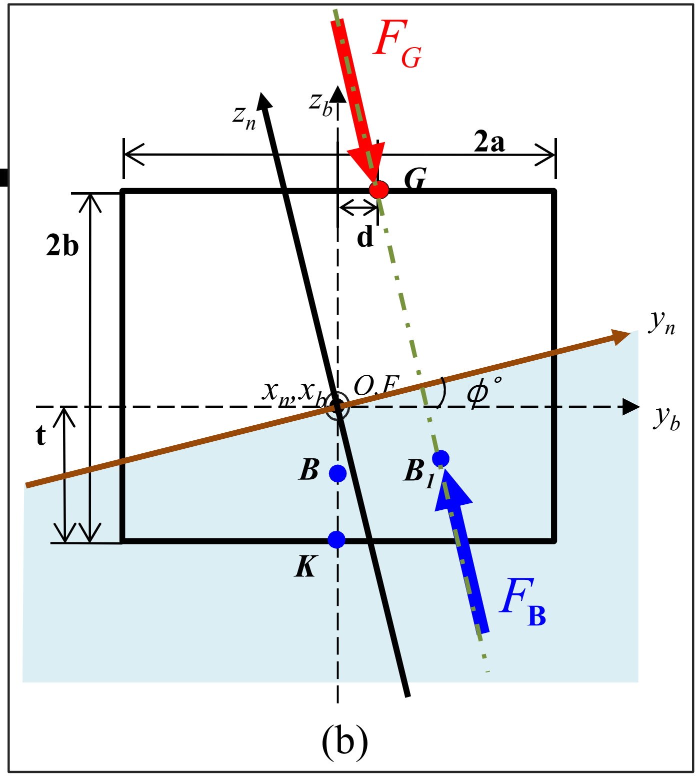
$$(y'_B, z'_B) = \left(\frac{a^2 \cdot \tan\phi}{3t}, -\frac{t}{2} + \frac{a^2 \cdot (\tan\phi)^2}{6t} \right)$$

The center of gravity, G, with respect to the body fixed frame is given by geometrical relations as shown in the figure, which is

$$\boxed{(y'_G, z'_G) = (d, 2b-t)}$$



Solution)
(3) Comparison between the figure describing the ship inclined and the figure describing the water plane inclined



(b)

Let us calculate the center of buoyancy, B_1, and the center of gravity, G, using the Fig. (b).

- The center of buoyancy, B_1, and the center of gravity, G, **with respect to the body fixed frame**

$$\boxed{\left(y'_B, z'_B\right) = \left(\frac{a^2 \cdot \tan\phi}{3t}, -\frac{t}{2} + \frac{a^2 \cdot (\tan\phi)^2}{6t}\right)}$$

$$\boxed{\left(y'_G, z'_G\right) = \left(d, 2b - t\right)}$$

Next, we use the condition that the moment arm of the buoyant force and gravitational force must be same and substitute the coordinates of the center of gravity and buoyancy with respect to the body fixed frame into the following equation.

$$y'_G \cdot \cos\phi + z'_G \cdot \sin\phi = y'_B \cdot \cos\phi + z'_B \cdot \sin\phi$$

Solution)
(3) Comparison between the figure describing the ship inclined and the figure describing the water plane inclined

$$y'_G \cdot \cos\phi + z'_G \cdot \sin\phi = y'_B \cdot \cos\phi + z'_B \cdot \sin\phi$$

$$\left(y'_B, z'_B\right) = \left(\frac{a^2 \cdot \tan\phi}{3t}, -\frac{t}{2} + \frac{a^2 \cdot (\tan\phi)^2}{6t}\right)$$

$$\left(y'_G, z'_G\right) = \left(d, 2b - t\right)$$

$$d \cdot \cos\phi + (2b - t) \cdot \sin\phi = \frac{\left\{-3t^2 + 2a^2 + a^2 \cdot (\tan\phi)^2\right\} \cdot \sin\phi}{6t}$$

Substituting a=2.5m, b=1.5m, t=0.4m, d=0.3m into this equation and rearranging

$$2.6 \cdot \sin\phi + 0.3 \cdot \cos\phi = \sin\phi \left(\frac{15.025}{3} + \frac{15.625}{6}(\tan\phi)^2\right)$$

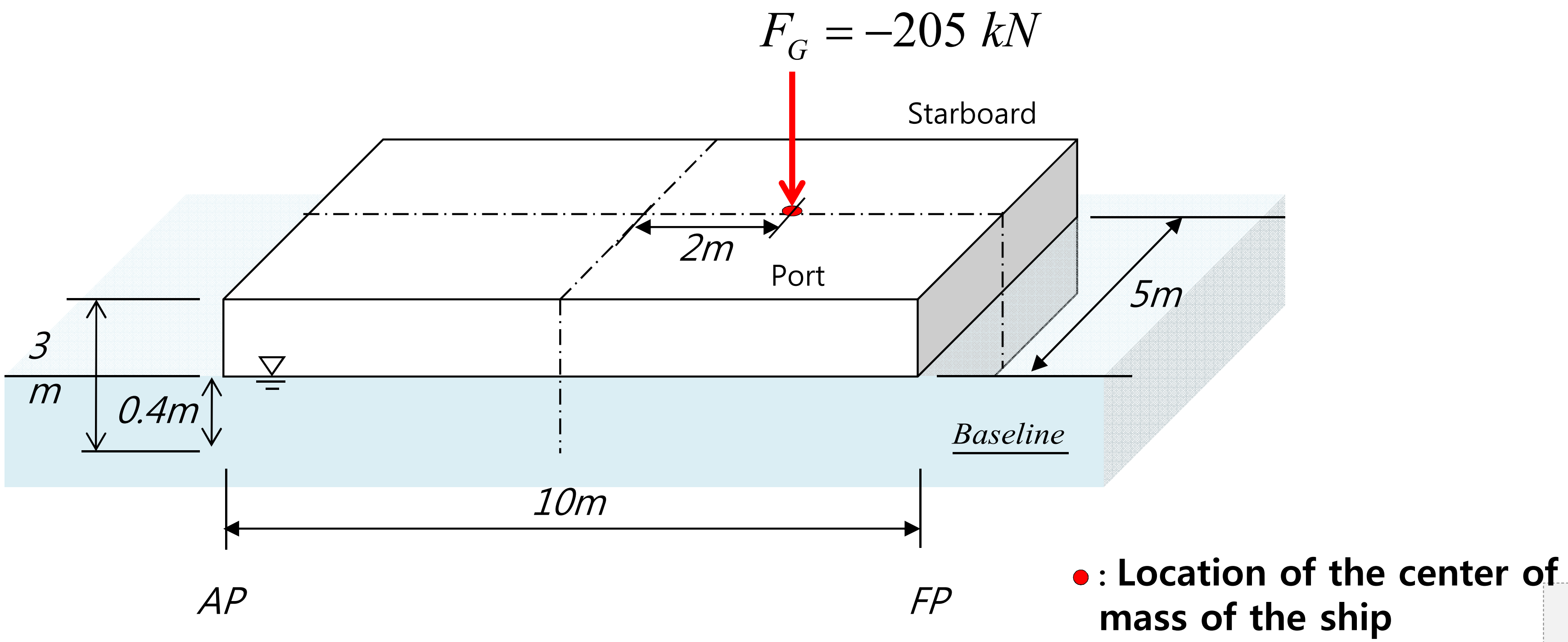
$$\tan\phi = 0.123 \text{ [rad]} \quad \Rightarrow \quad \boxed{\therefore \phi = 7.047 \text{ [deg]}}$$

A box-shaped ship of 10 meter length, 5 meter breadth and 3 meter height weights 205 [kN].
The center of mass is moved to 2 [m] in the direction of the forward perpendicular. When the ship is in static equilibrium state, determine the equilibrium position and orientation of the ship.

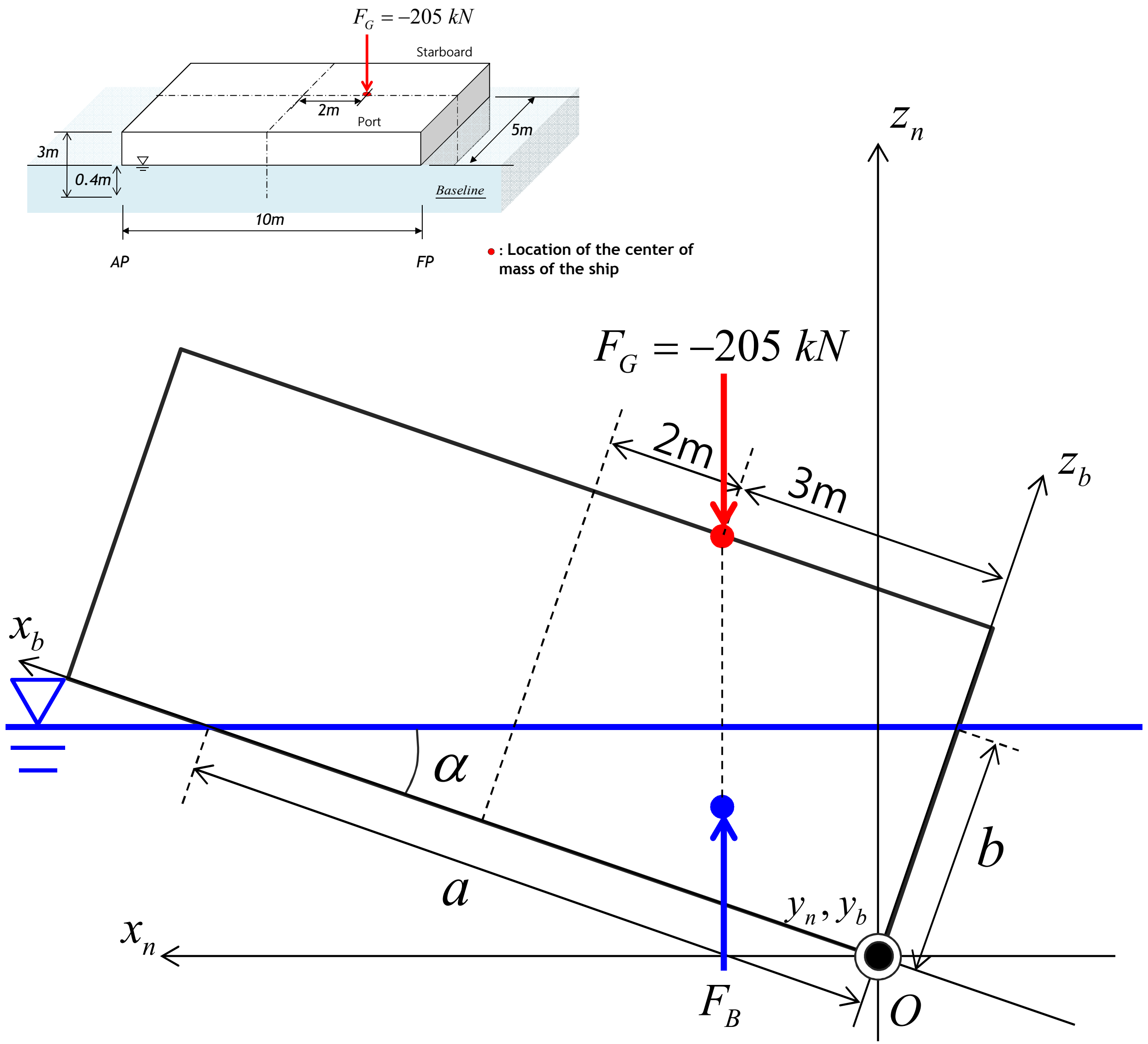
Assumption)

(1) Gravitational acceleration = 10 [m/s^2], Density of sea water = 1.025 [ton/m^3](Mg/m^3)

(2) When the ship will be in the static equilibrium finally, the deck will not be immersed and the bottom will emerge.



Question 2) The center of mass is moved to 2 [m] in the direction of the forward perpendicular.
Solution)



Force Equilibrium

$$\sum F = F_G + F_B = 0$$

$$F_G = -250$$

$$F_B = -\rho \cdot g \cdot V$$

$$= 1.025 \cdot 10 \cdot \left(\frac{1}{2} \cdot a \cdot b \cdot 5 \right)$$

$$= 25.625 \cdot a \cdot b$$

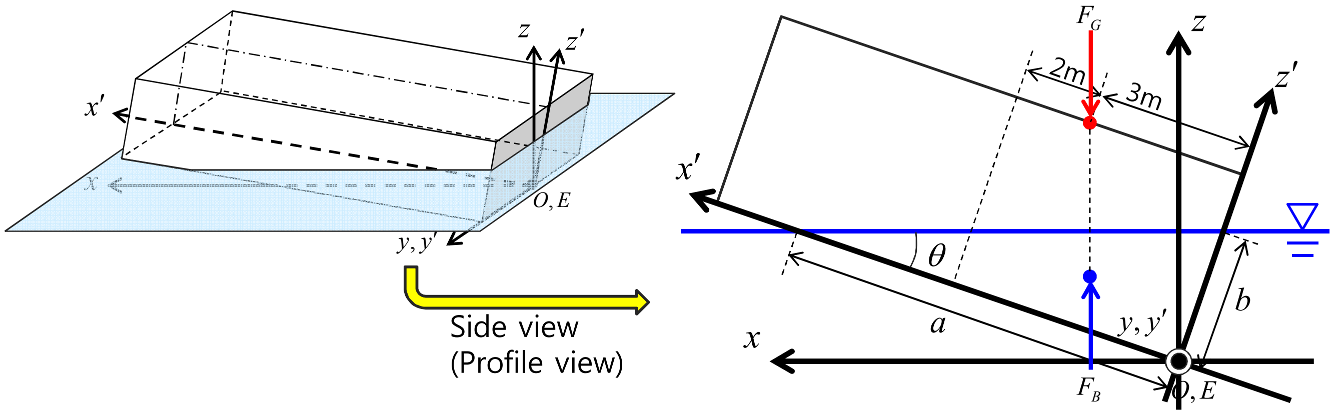
$$\sum F = F_G + F_B$$

$$= -250 + 25.625 \cdot a \cdot b$$

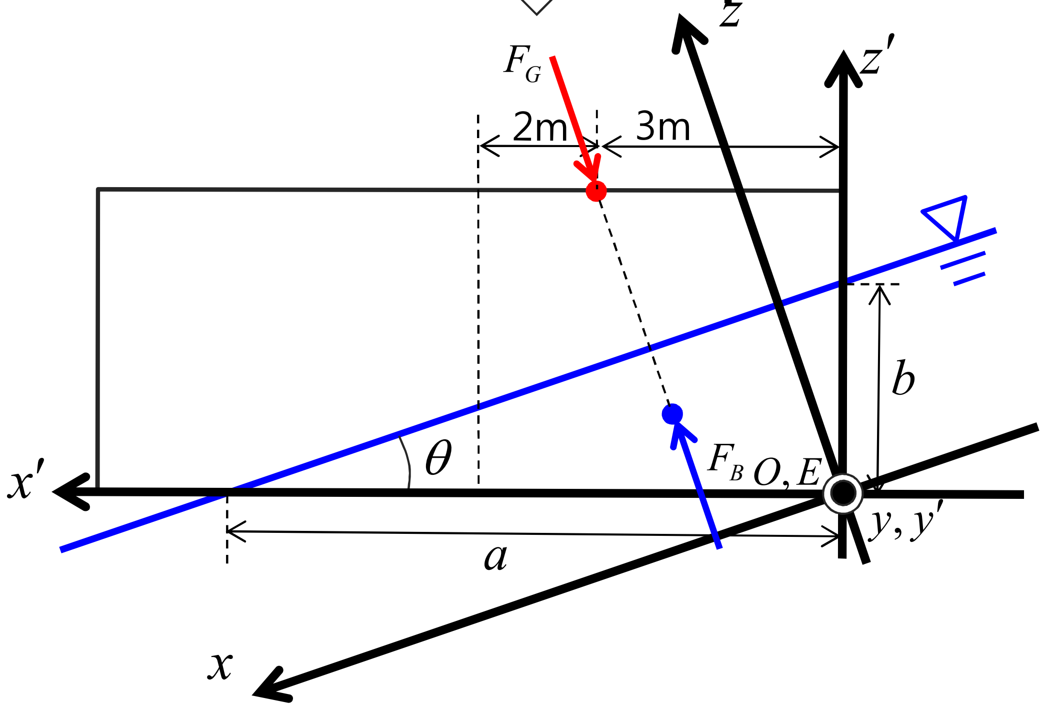
$$= 0$$

$$\therefore a \cdot b = 8$$

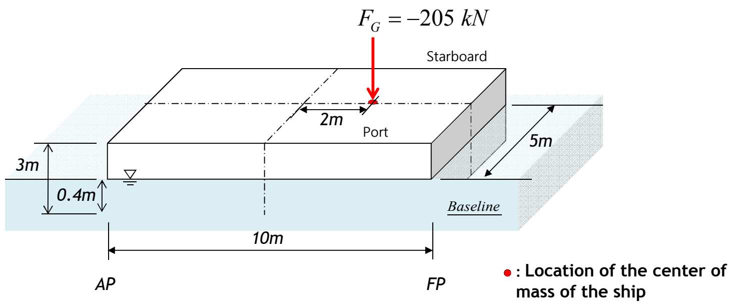
Question 2) <u>**The center of mass is moved to 2 [m] in the direction of the forward perpendicular.**</u>
Solution)

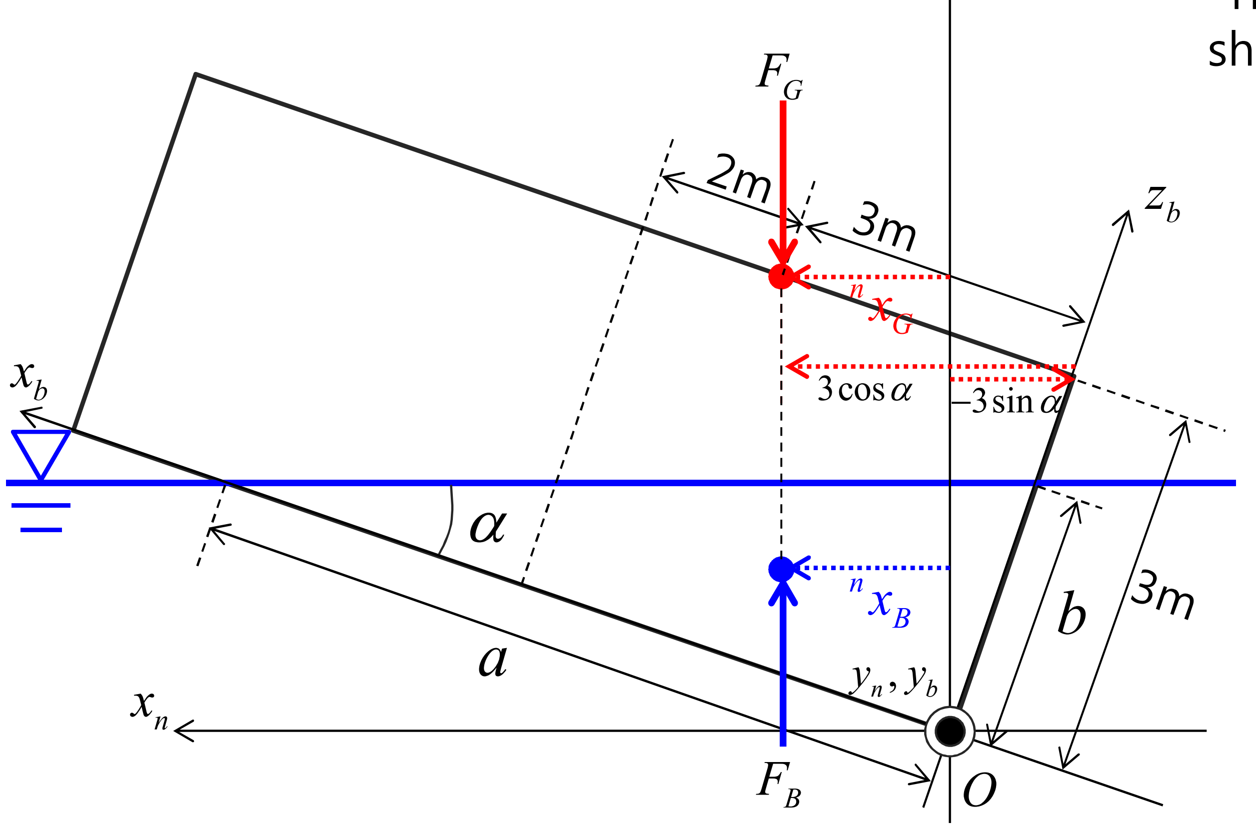


Instead of rotating the ship, we can consider the waterline rotated with an angle of θ while keeping the ship constant.



Question 2) **The center of mass is moved to 2 [m] in the direction of the forward perpendicular.**
Solution)





Moment Equilibrium

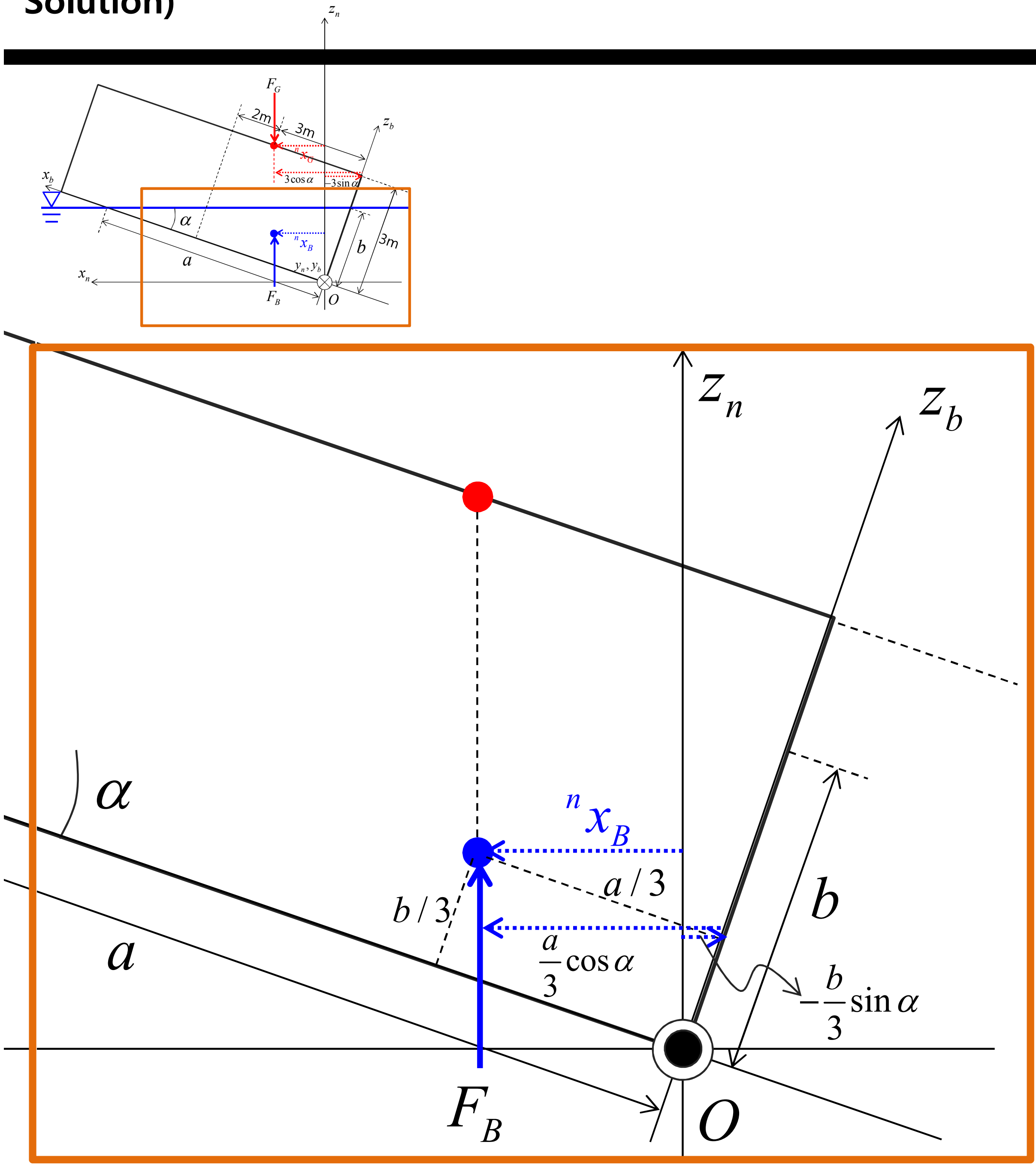
$$\sum M = M_G + M_B = 0$$

The centers of buoyancy B and gravity G should be in the same vertical line.

$$x_G = x_B$$

$$x_G = 3\cos\alpha - 3\sin\alpha$$

Question 2) <u>**The center of mass is moved to 2 [m] in the direction of the forward perpendicular.**</u>
Solution)



Moment Equilibrium

$$\sum M = M_G + M_B = 0$$

The centers of buoyancy B and gravity G should be in the same vertical line.

$$^n x_G = {}^n x_B$$

$$^n x_G = 3\cos\alpha - 3\sin\alpha$$

$$^n x_B = \frac{a}{3}\cos\alpha - \frac{b}{3}\sin\alpha$$

$$3\cos\alpha - 3\sin\alpha = \frac{a}{3}\cos\alpha - \frac{b}{3}\sin\alpha$$

Question 2) **The center of mass is moved to 2 [m] in the direction of the forward perpendicular.**
Solution)

$$3\cos\alpha - 3\sin\alpha = \frac{a}{3}\cos\alpha - \frac{b}{3}\sin\alpha$$

dividing the both side of equation by $\cos\alpha$

$$3 - 3\tan\alpha = \frac{a}{3}\alpha - \frac{b}{3}\tan\alpha$$

$$\tan\alpha = \frac{b}{a}$$

$$3 - 3\frac{b}{a} = \frac{a}{3} - \frac{b}{3}\cdot\frac{b}{a}$$

multiplying 3a to the both side of equation

$$9a - 9b = a^2 - b^2$$

$$9(a-b) = (a+b)(a-b)$$

From the force equilibrium

$$a \cdot b = 8$$

From the moment equilibrium

$$9(a-b) = (a+b)(a-b)$$

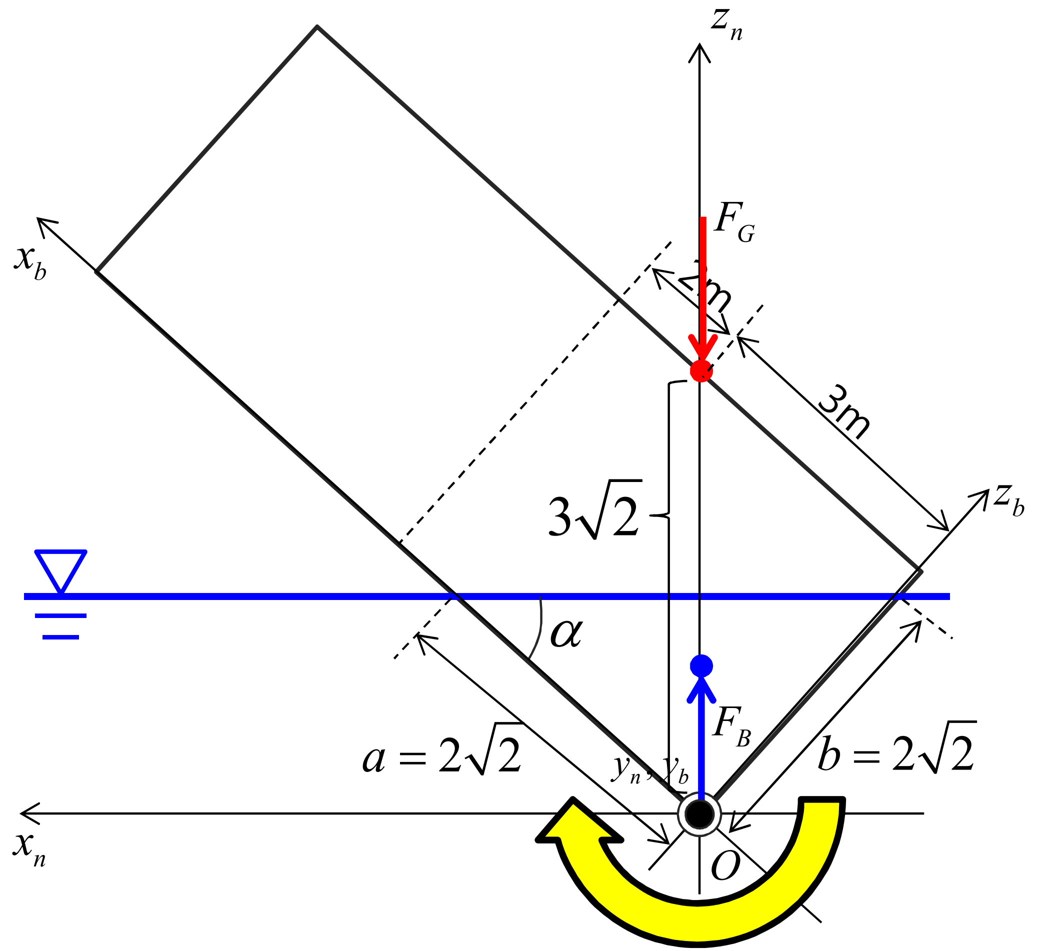
$if\ a = b \qquad a = b = 2\sqrt{2}$ **Unstable**

$if\ a \neq b \qquad a = 8$ **Stable**

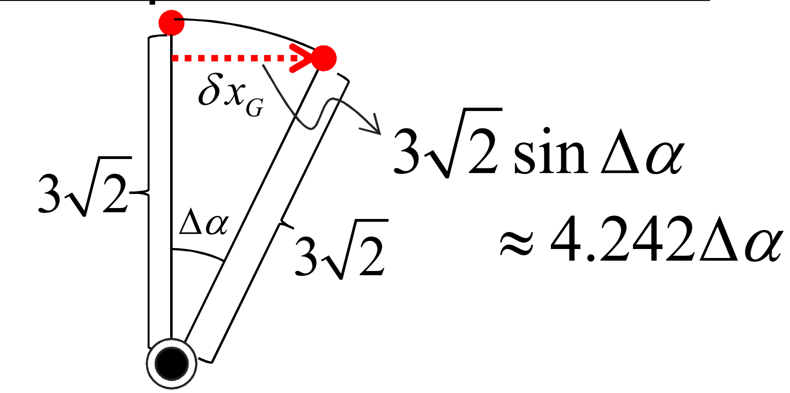
$b = 1$

Question 2) The center of mass is moved to 2 [m] in the direction of the forward perpendicular.
Solution)

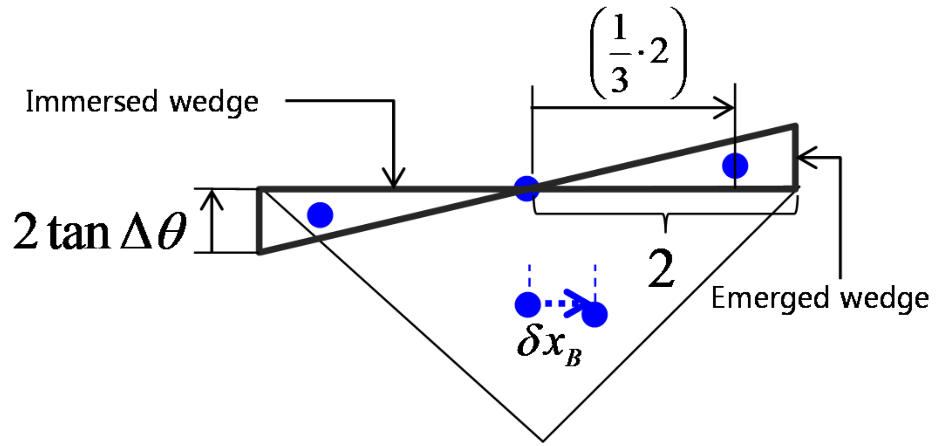
Why is the ship unstable, when $a = b = 2\sqrt{2}$?



Horizontal displacement of center of mass



Horizontal displacement of center of buoyancy



$$\frac{\delta x_B}{\delta x_b} = \frac{\nabla_{wedge}}{\nabla_{total}} \Rightarrow \delta x_B = \frac{\nabla_{wedge}}{\nabla_{total}} \delta x_b$$

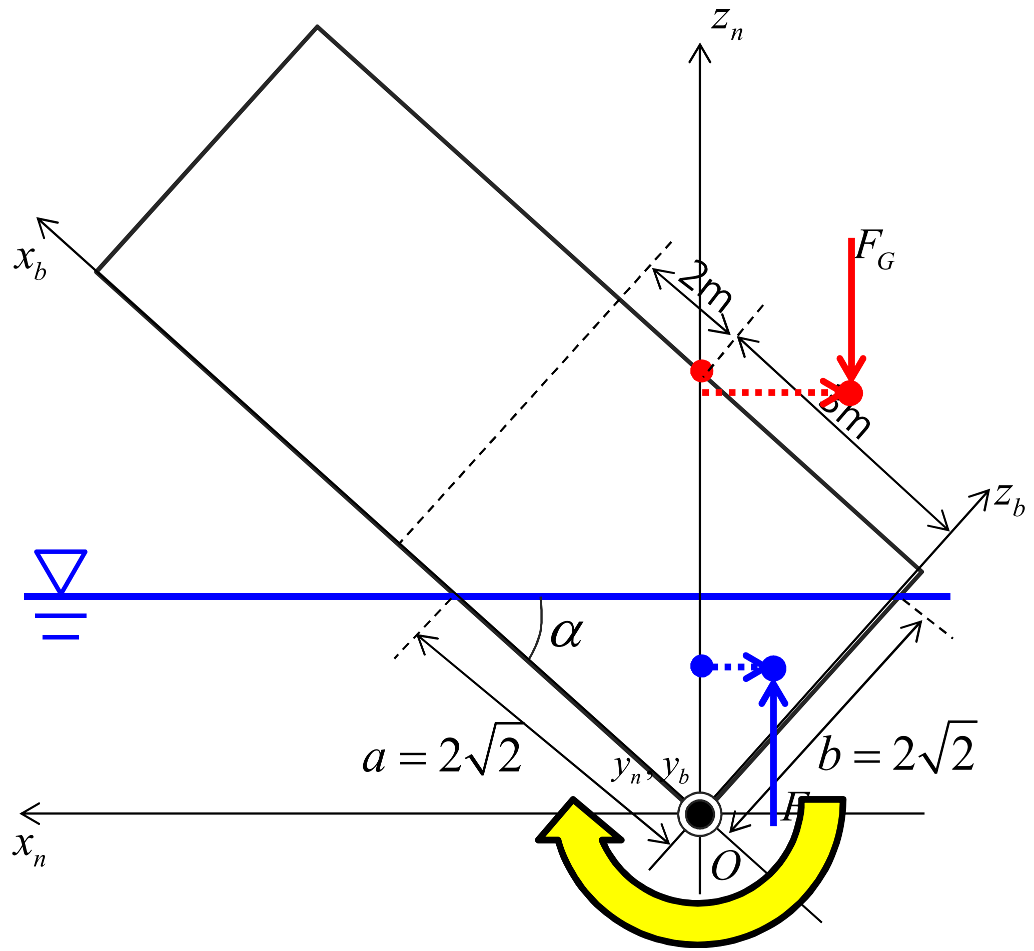
$$\nabla_{wedge} = 2\cdot 2\cdot \frac{1}{2}\cdot \tan(\Delta\theta) = 2\tan(\Delta\theta)$$

$$\nabla_{total} = \frac{2\sqrt{2}\cdot 2\sqrt{2}}{2} = 4,\ \delta x_b = 2\left(\frac{2}{3}\right) = \frac{4}{3}$$

$$\Rightarrow \delta x_B = \frac{2\tan(\Delta\theta)}{4}\cdot\frac{4}{3} \Rightarrow \delta x_B \approx 0.\overline{66}\Delta\theta$$

Question 2) **The center of mass is moved to 2 [m] in the direction of the forward perpendicular.**
Solution)

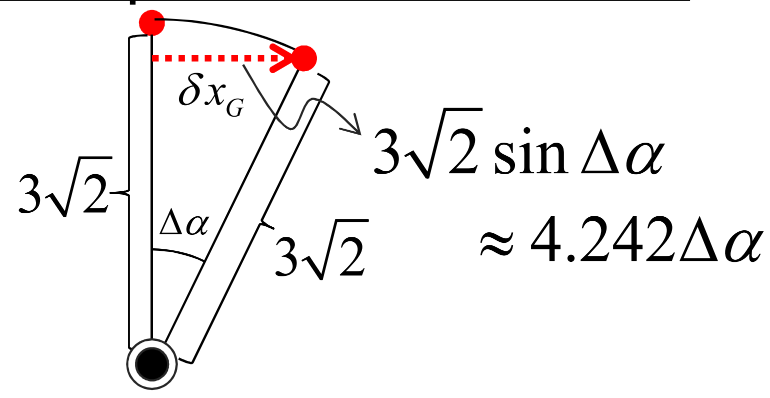
Why is the ship unstable, when $a = b = 2\sqrt{2}$?



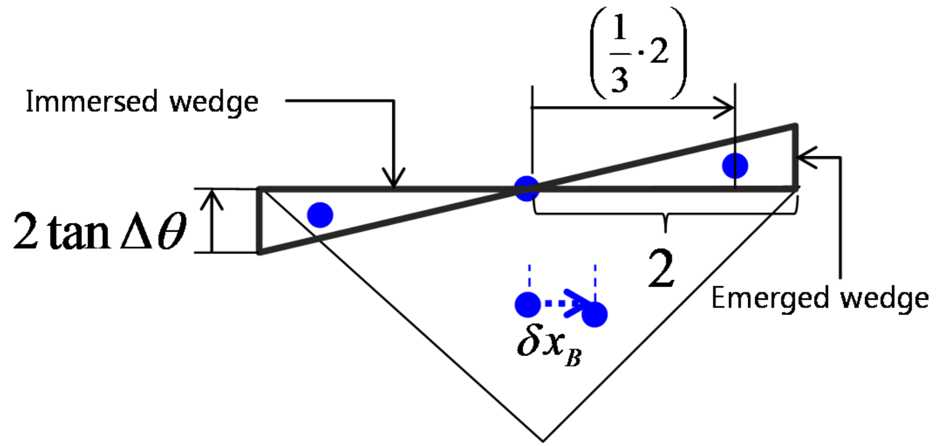
$\Delta\alpha \ll 1$

Unstable

Horizontal displacement of center of mass



Horizontal displacement of center of buoyancy



$$\frac{\delta x_B}{\delta x_b} = \frac{\nabla_{wedge}}{\nabla_{total}} \Rightarrow \delta x_B = \frac{\nabla_{wedge}}{\nabla_{total}}\delta x_b$$

$$\nabla_{wedge} = 2\cdot 2\cdot\frac{1}{2}\cdot\tan(\Delta\theta) = 2\tan(\Delta\theta)$$

$$\nabla_{total} = \frac{2\sqrt{2}\cdot 2\sqrt{2}}{2} = 4,\ \delta x_b = 2\left(\frac{2}{3}\right) = \frac{4}{3}$$

$$\Rightarrow \delta x_B = \frac{2\tan(\Delta\theta)}{4}\cdot\frac{4}{3} \Rightarrow \delta x_B \approx 0.\overline{66}\Delta\theta$$

More Examples for Ship Stability

Example) Heel Angle caused by Movement of Passengers in Ferry (1/2)

- Given : KB, KG, I_T, Heeling moment M_h
- Find : An angle of heel ϕ
- GZ of wall sided ship

$$GZ = \left(GM + \frac{1}{2} BM \tan^2 \phi \right) \sin \phi$$

Question) Emergency circumstance happens in Ferry with displacement (mass) 102.5 ton. Heeling moment of 8 ton·m occurs due to passengers moving to the right of the ship. What will be an angle of heel?
Assume that wall sided ship with KB=0.6m, KG=2.4m, I_T=200m^4.

Solution) If it is in static equilibrium at an angle of heel ϕ

Righting moment in wall sided ship(M_r) = Heeling moment (M_h)

$$\Delta \left(GM + \frac{1}{2} BM \tan^2 \phi \right) \sin \phi = 8\, ton \cdot m$$

① Calculation of BM

$$\Delta = 102.5 \text{ ton} \quad \rightarrow \quad \nabla = \Delta / 1.025 = 100 \text{ m}^3$$

$$BM = \frac{I_T}{\nabla} = \frac{200}{100} = 2m$$

② Calculation of GM

$$GM = KB + BM - KG$$
$$= 0.6 + 2 - 2.4 = 0.2\ m$$

$$\left(0.2 + \tan^2 \phi \right) \sin \phi = \frac{8}{102.5}$$

Non linear equation about ϕ ?

예제5.2

Example) Heel Angle caused by Movement of Passengers in Ferry (2/2)

- Given : KB, KG, I_T, Heeling moment M_h
- Find : An angle of heel ϕ
- GZ of wall sided ship

$$GZ = \left(GM + \frac{1}{2} BM \tan^2\phi\right)\sin\phi$$

Question) Emergency circumstance happens in Ferry with displacement (mass) 102.5 ton. Heeling moment of 8 ton·m occurs due to passengers moving to the right of the ship. What will be an angle of heel?
Assume that wall sided ship with KB=0.6m, KG=2.4m, I_T=200m^4.

Solution) If it is in static equilibrium at an angle of heel ϕ

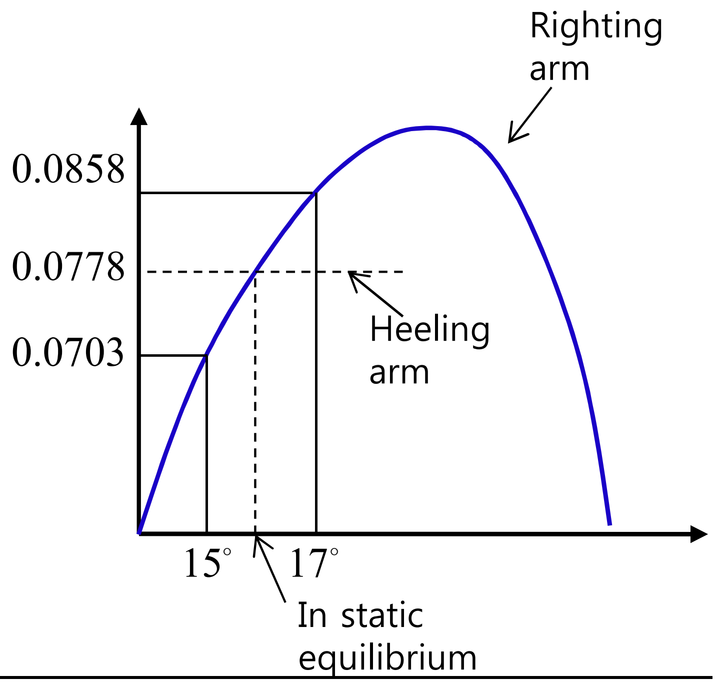
Righting moment in wall sided ship(M_r)	=	Heeling moment (M_h)
$\Delta\left(GM + \frac{1}{2} BM \tan^2\phi\right)\sin\phi$	=	$8\,ton \cdot m$

$$\left(0.2 + \tan^2\phi\right)\sin\phi = 0.078$$

Because of nonlinear equation, solve it by numerical method.

Result of calculation is about ϕ=16.0˚.

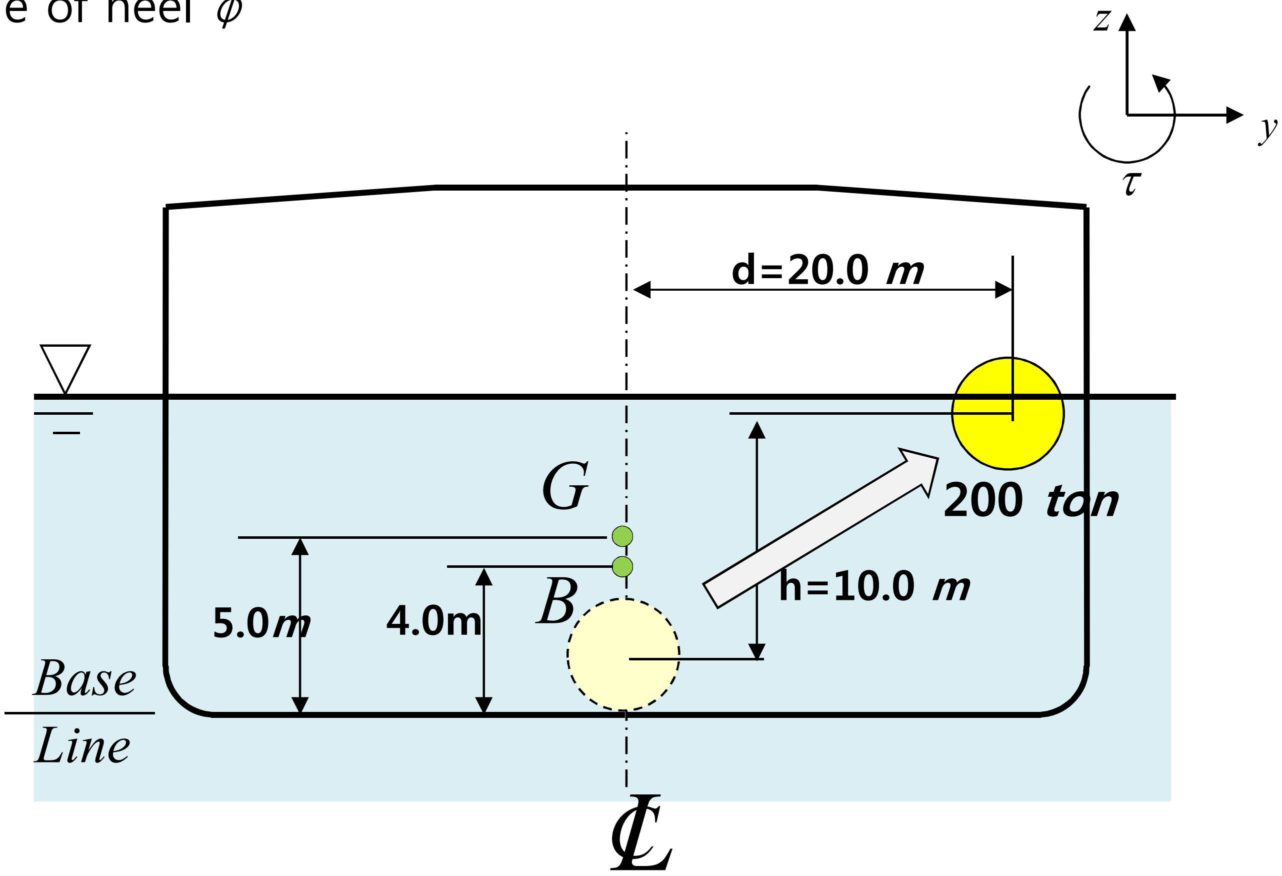
ϕ	LHS (Righting arm)	RHS (Heeling arm)
15˚	0.0703	0.0780
16˚	0.0778	0.0780
17˚	0.0858	0.0780



Example) Heel Angle caused by Movement of Cargo

Question) A cargo carrier of 10,000 ton displacement is floating. KB=4.0m, BM=2.5m, KG=5.0m. Cargo in hold of cargo carrier is shifted in vertical direction through a 10 meter, and shifted in transverse direction through a 20 meters. Find an angle of heel.

- Given : displacement (Δ), KB, BM, KG, weight of cargo(w) and moving distance
- Find : angle of heel ϕ

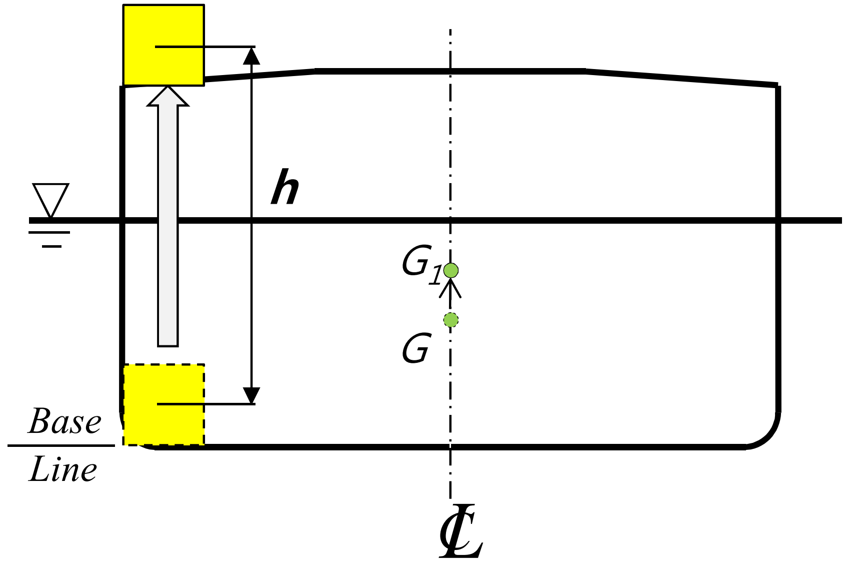


예제5.3

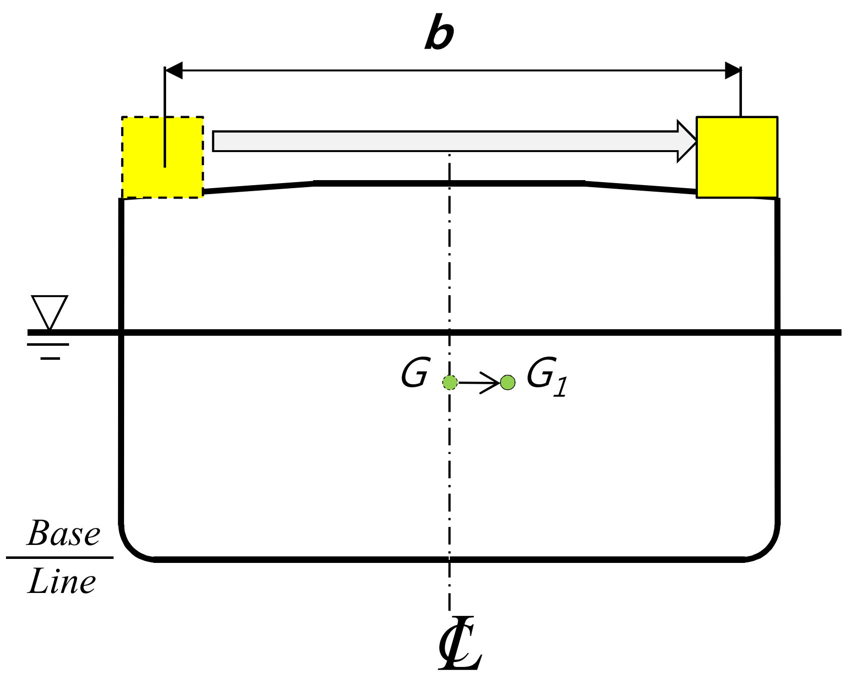
Example) Change of Center caused by Movement of Cargo

Question) As below cases partial weight w of the ship is shifted. What is the shift distance of center of mass of the ship?

Case 1) Vertical shift of the partial weight



Case 2) Horizontal shift of the partial weight



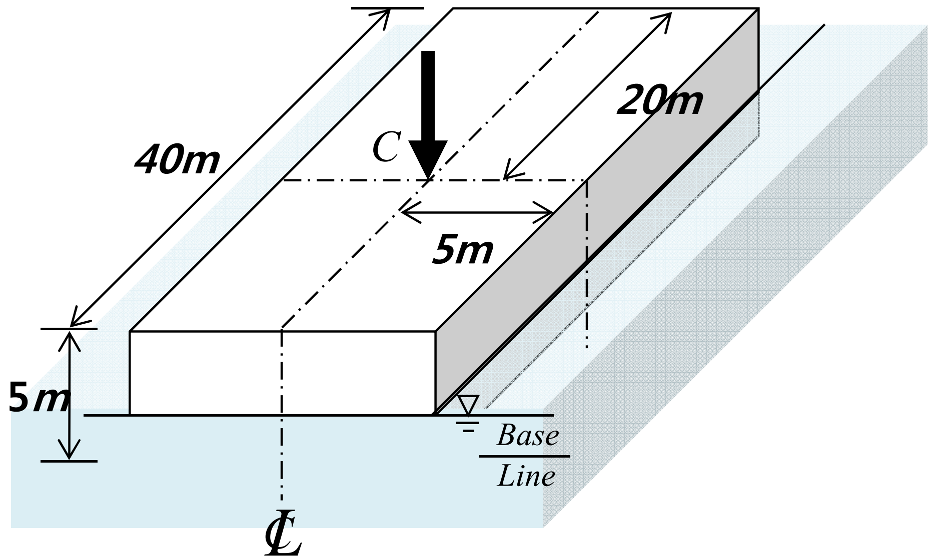
예제5.5

Example) Calculation of Deadweight of Barge

Question)

A barge is 40m length, 10m breadth, 5m depth, and is floating at 1 m draft. The vertical center of mass of the ship is located in 2 m from the baseline.

A cargo is supposed to be loaded in center of the deck. Find the maximum loadable weight that keeps the stability of ship.



Problem to calculated position of the ship when external force are applied.

Example) Calculation of Position of Ship when Cargo is moved by Crane

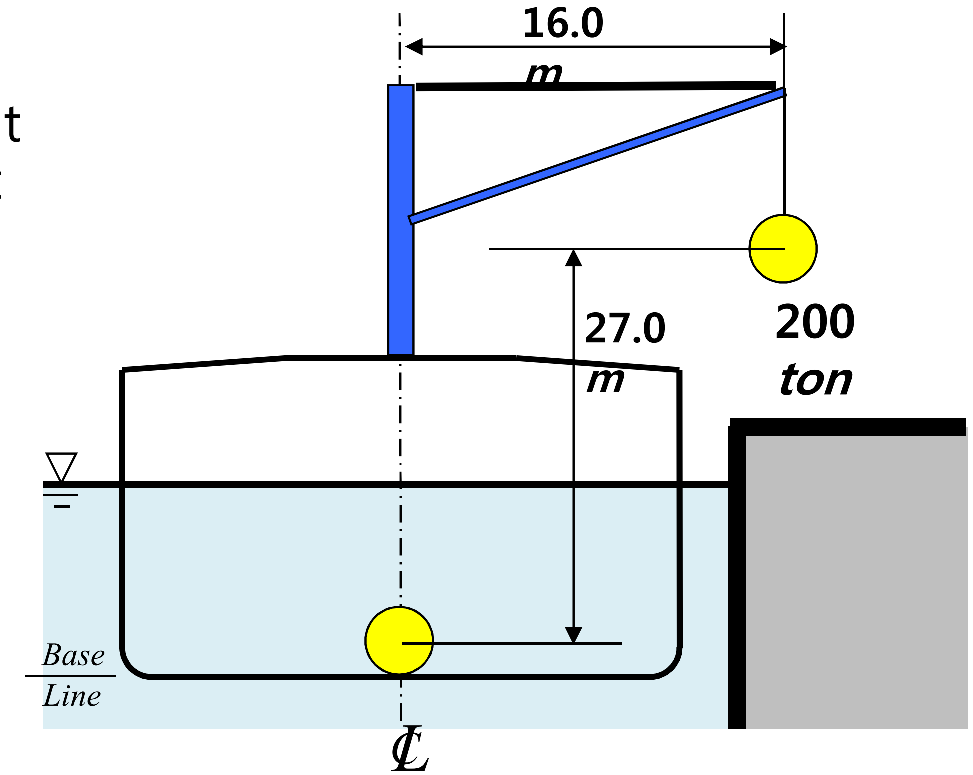
Question)

A Cargo carrier of 18,000 ton displacement is afloat and has GM = 1.5m. And we want to transfer the cargo of 200 ton weight from bottom of the ship to land.

A lifting height of cargo is 27.0 m from the original position.

After lifting the cargo, turn the cargo to the right through a distance of 16.0 m from the centerline.

What will be the angle of heel of the ship?



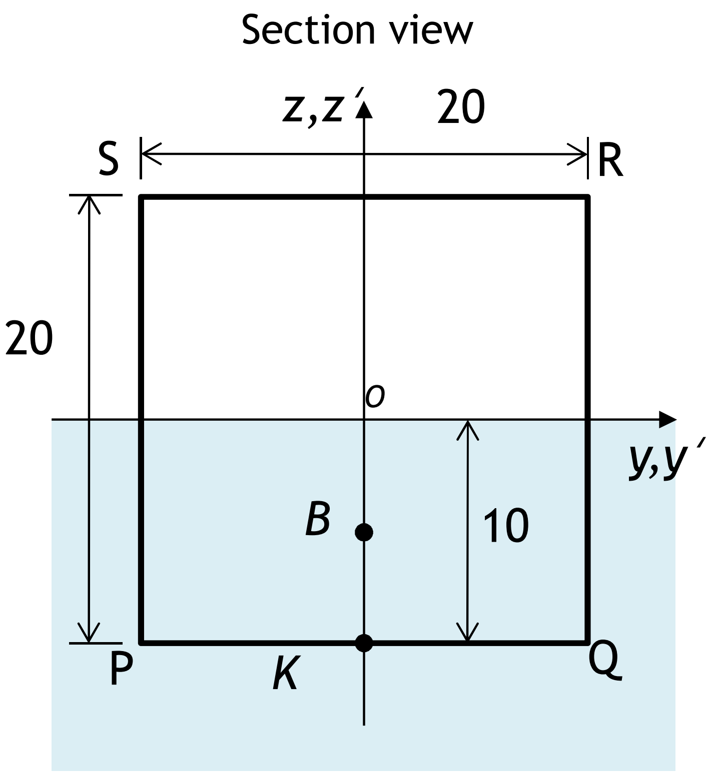
Problem to calculated position of the ship when external force are applied.

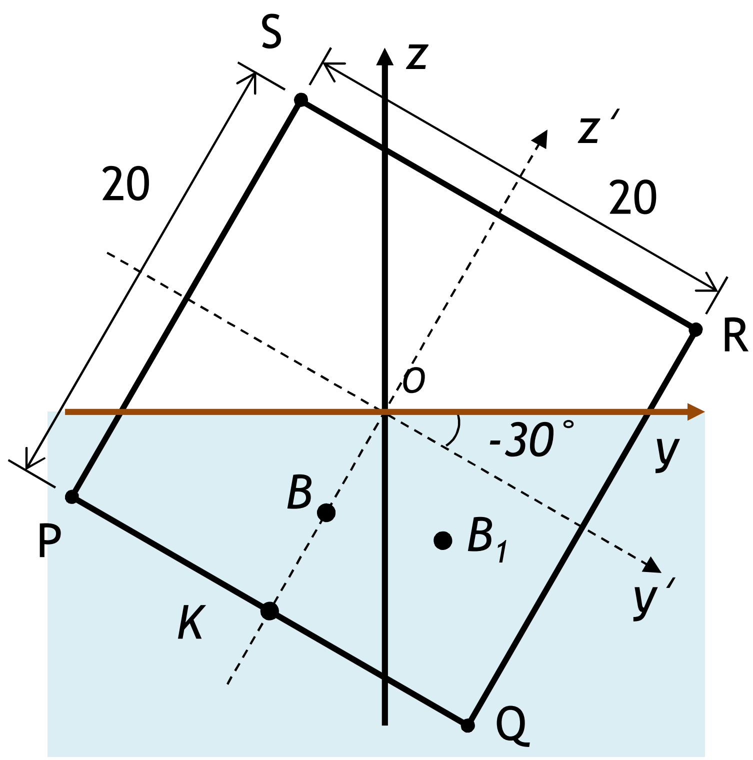
Example) Calculation of Center of Buoyancy of Ship with Constant Section

Example) A ship is inclined about x-axis through origin O with an angle of -30°. Calculate center of buoyancy with respect to the water plane fixed frame.

- Given: Breadth(B) 20m, Depth(D) 20m, Draft(T) 10m, Angle of Heel(ϕ) -30˚
- Find: Center of buoyancy(y_B, z_B)

G: Center of mass　　K:Keel
B: Center of buoyancy　B_1 : Changed center of buoyancy

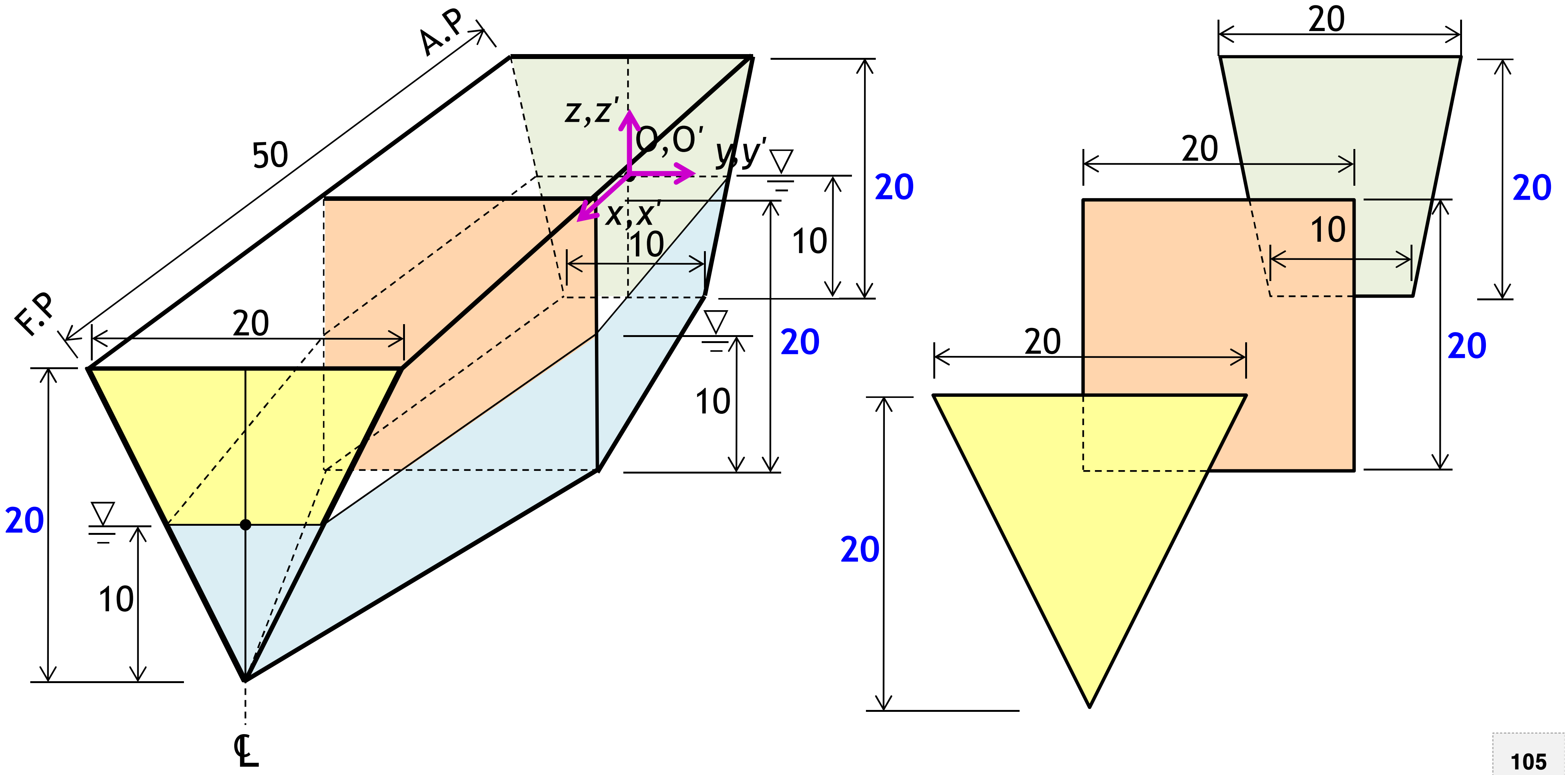




Example) Calculation of Center of Buoyancy of Ship with Various Station Shapes

A ship with three varied section shape is given. When this ship is inclined about x axis with an angle of -30°, calculate y and z coordinates of the center of buoyancy (with respect to the water plane fixed frame).

- Given: Length(L) 50m, Breadth(B) 20m, Depth(D) 20m, Draft(T) 10m, Angle of Heel(ϕ) -30˚
- Find: Center of buoyancy($y_{\nabla,c}$, $z_{\nabla,c}$) after heeling



Reference Slides

1) Gere, Mechanics of Materials, 6th ,Ch.12.3, 2006

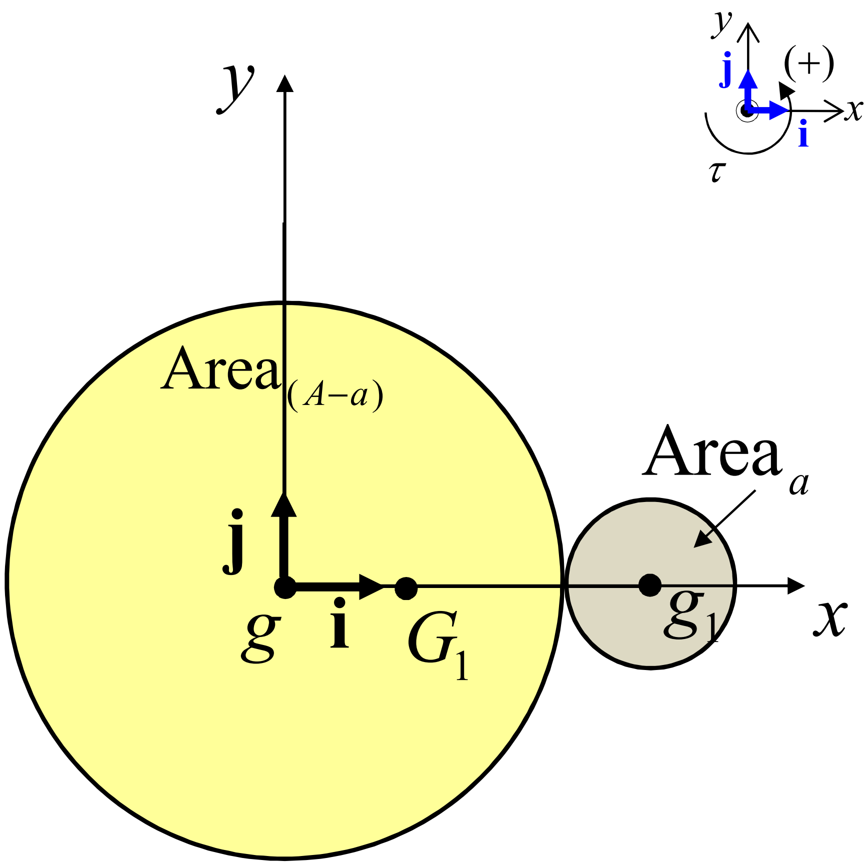
Movement of Centroid Caused by Movement of Area (1/3)

First Moment of Composite Area(Q_x)[1)]

$$Q_x = \sum_{n=1}^{n} A_i \cdot \bar{x}_i$$

$$A \cdot \bar{x} = \sum_{n=1}^{n} A_i \cdot \bar{x}_i$$

Q_x : 1st Moment
A_i : Each Area
A : Total Area
$\bar{x}$: Coordinate of Centroid



<1st moment of area>

Let us consider 1st moment of area about z axis through origin g.

$$gG_1 \cdot \text{Area}_A = \cancel{gg} \cdot \text{Area}_{(A-a)} + gg_1 \cdot \text{Area}_a$$

$$,(gg = 0)$$

$$gG_1 \cdot \text{Area}_A = gg_1 \cdot \text{Area}_a$$

$$\frac{gG_1}{gg_1} = \frac{\text{Area}_a}{\text{Area}_A} \quad \cdots ①$$

G_1 : Centroid of total area, Area_A : Total area

g : Centroid of the large circle, Area_{A-a} : Area of the large circle

g_1 : Centroid of the small circle, Area_a : Area of the small circle

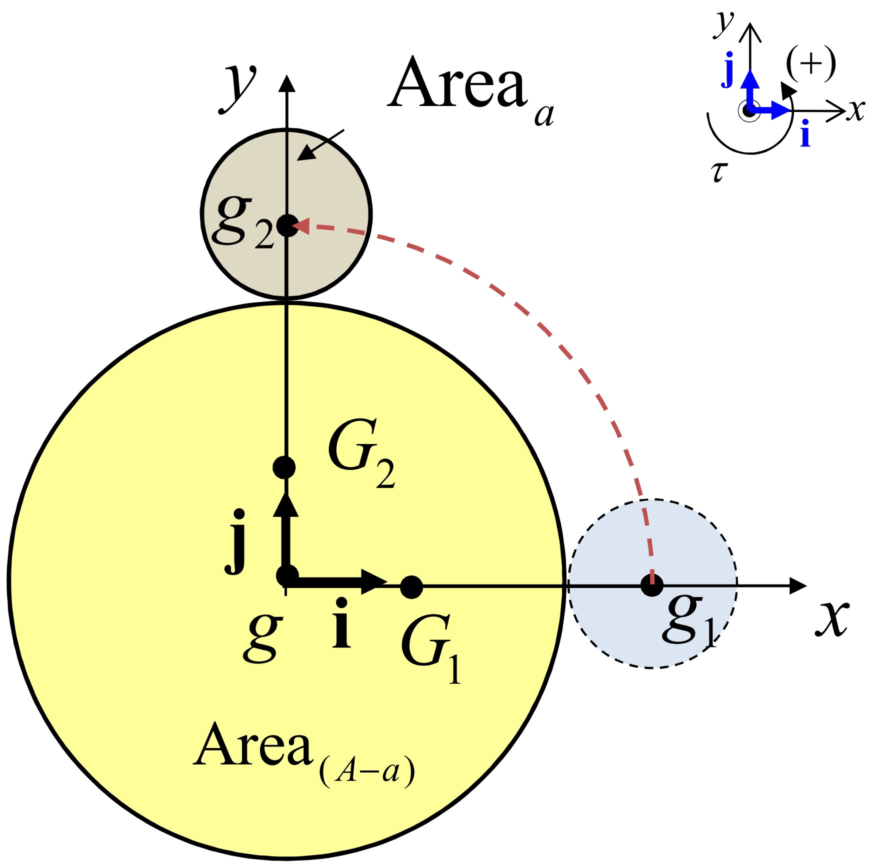
Movement of Centroid Caused by Movement of Area (2/3)

First Moment of Composite Area(Q_x)[1)]

$$Q_x = \sum_{n=1}^{n} A_i \cdot \overline{x}_i$$

$$A \cdot \overline{x} = \sum_{n=1}^{n} A_i \cdot \overline{x}_i$$

Q_x : 1st Moment
A_i : Each Area
A : Total Area
$\overline{x}$: Coordinate of Centroid



When the center of the small circle moves from g1 to g2, the total moment of area about z axis through origin g is

$$gG_2 \cdot \text{Area}_A = \cancel{gg} \cdot \text{Area}_{(A-a)} + gg_2 \cdot \text{Area}_a$$

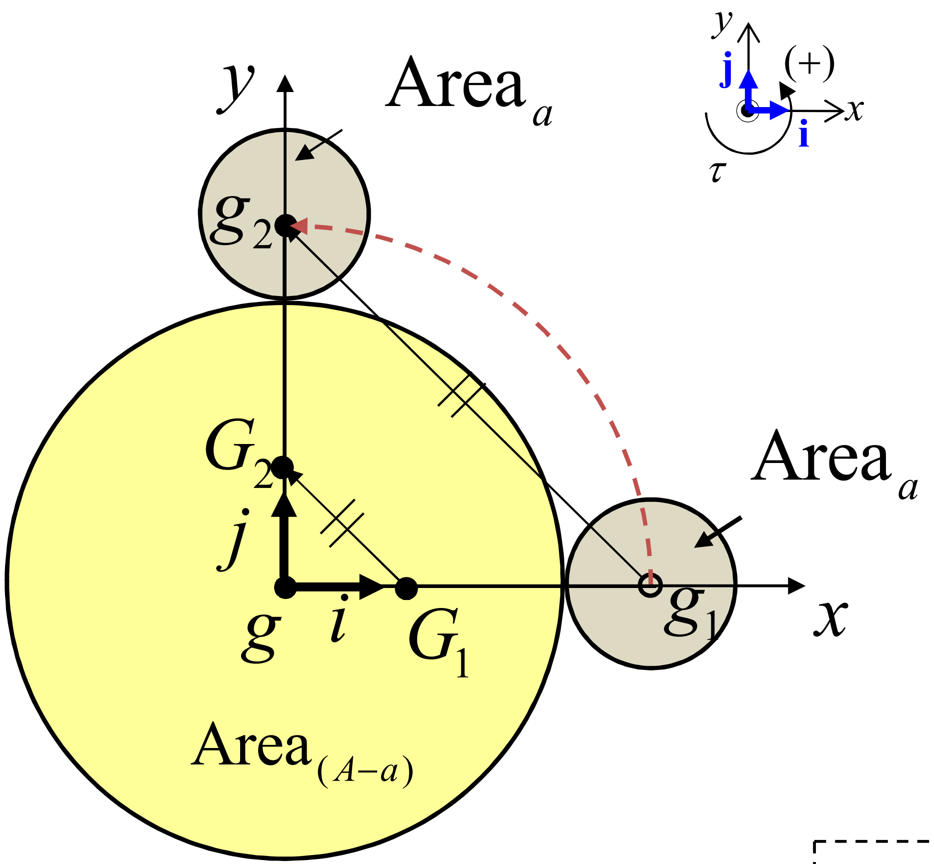
$$,(gg = 0)$$

$$gG_2 \cdot \text{Area}_A = gg_2 \cdot \text{Area}_a$$

$$\frac{gG_2}{gg_2} = \frac{\text{Area}_a}{\text{Area}_A} \quad \cdots ②$$

G_1 : Centroid of total area, Area_A : Total area
g : Centroid of the large circle, Area_{A-a} : Area of the large circle
g_1 : Centroid of the small circle, Area_a : Area of the small circle

Reference) Movement of Centroid Caused by Movement of Area (3/3)



Using the ratio of similitude

$$\frac{gG_1}{gg_1} = \frac{\text{Area}_a}{\text{Area}_A} \cdots ①$$

$$\frac{gG_2}{gg_2} = \frac{\text{Area}_a}{\text{Area}_A} \cdots ②$$

G_1 : Centroid of total area, Area_A : Total area

g : Centroid of the large circle, $\text{Area}_{A\text{-}a}$: Area of the large circle

g_1 : Centroid of the small circle, Area_a : Area of the small circle

$$\angle G_1 g G_2 = \angle g_1 g g_2 \cdots ③$$

From ①, ②, ③,

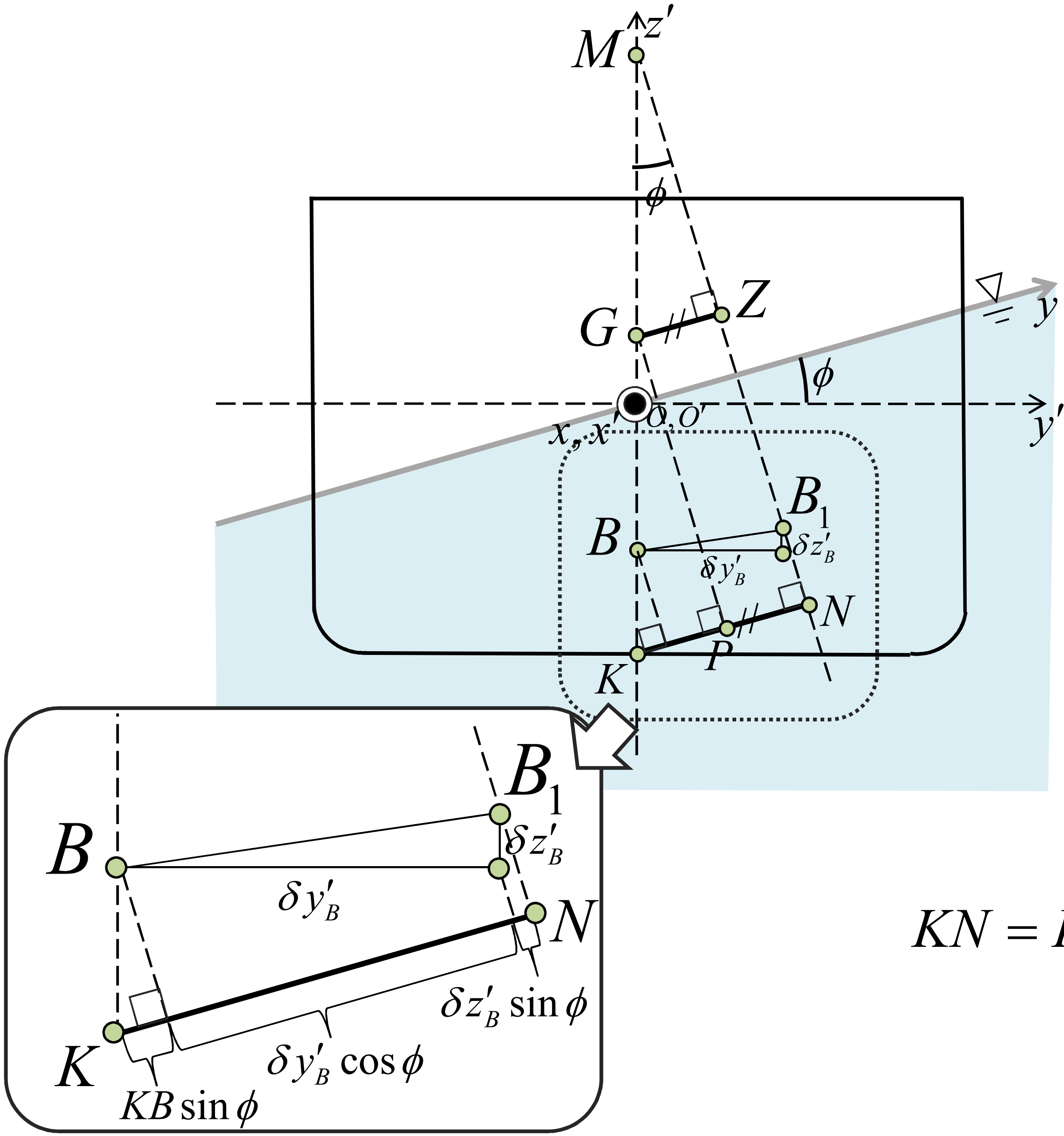
Triangle $\triangle G_1 g G_2$ and $\triangle g_1 g g_2$ are similar.
(by SAS(Side-Angle-Side) similarity theorem)

$$G_1G_2 // g_1g_2$$

$$\frac{G_1G_2}{g_1g_2} = \frac{\text{Area}_a}{\text{Area}_A} \Rightarrow G_1G_2 = \frac{\text{Area}_a}{\text{Area}_A} \times g_1g_2$$

The line G_1G_2 is parallel to the line g_1g_2.
Thus, the centroid of total area G_2 moves parallel to g_1g_2.

Calculation of GZ, when the ship is inclined with angle of ϕ without change of center of gravity



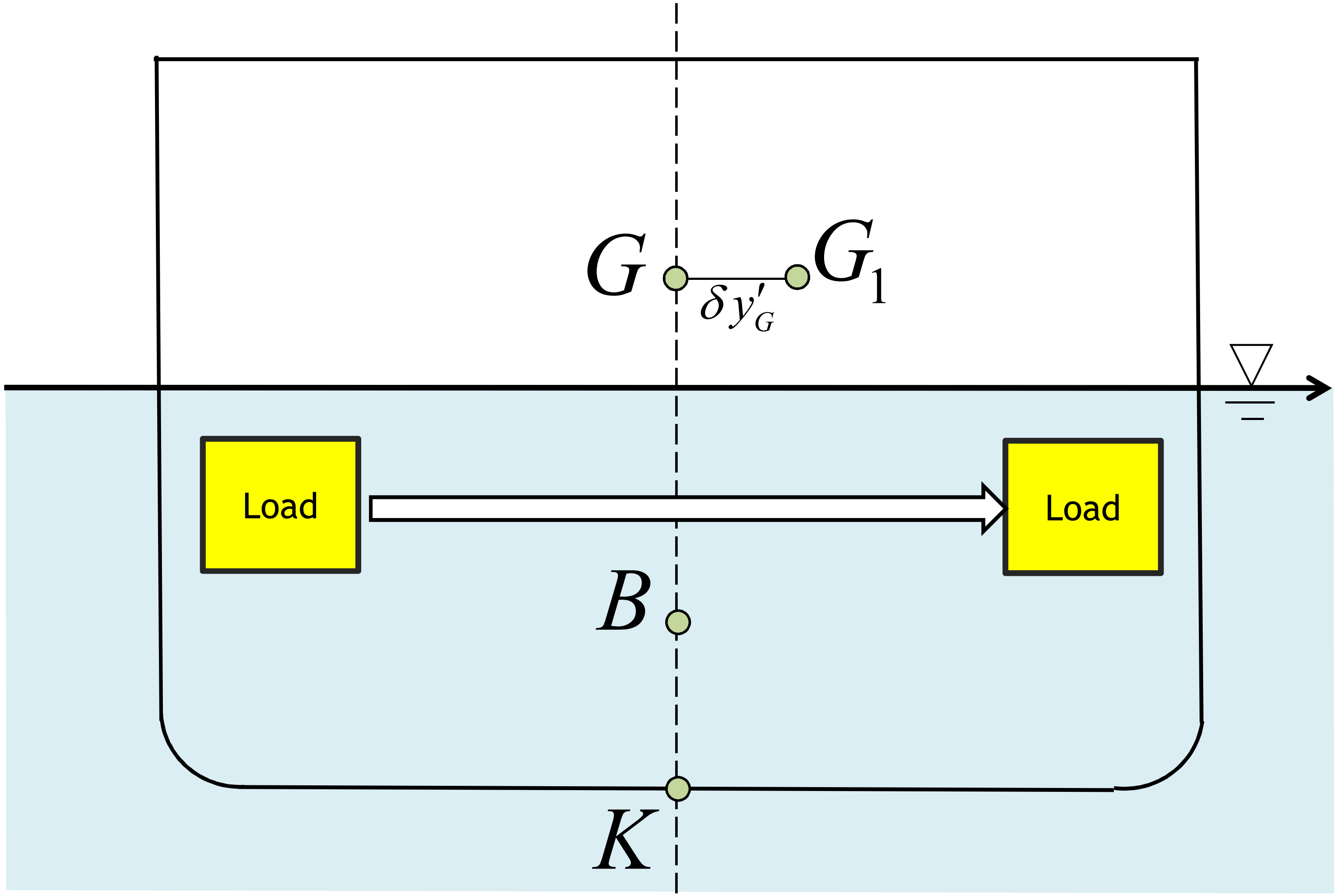
$$KN = KG\sin\phi + GZ$$

$$GZ = KN - KG\sin\phi$$

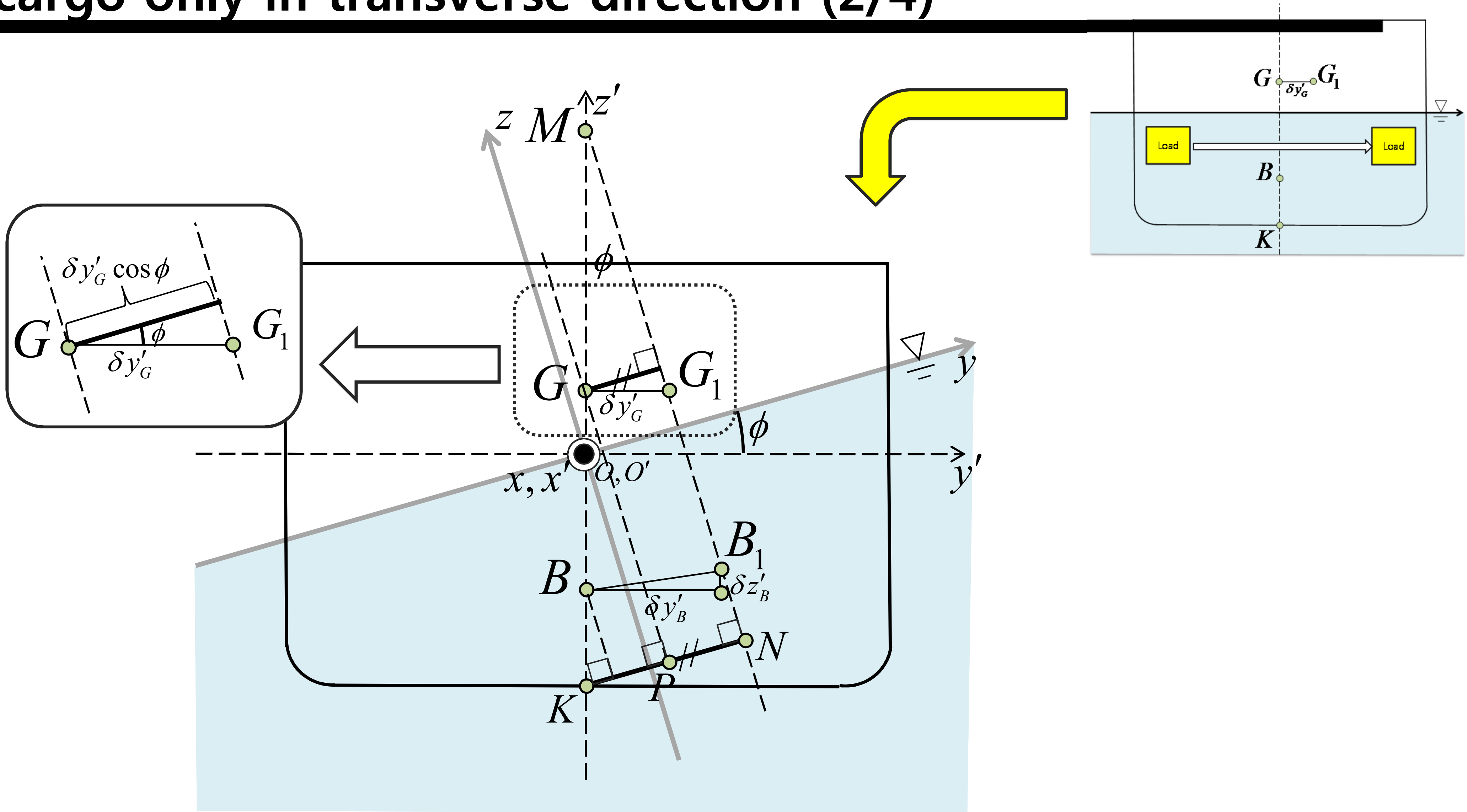
In this equation, KG can be measured by inclining test, and KN can be represented with the displacement of center of buoyancy with respect to the body fixed frame. If we define the horizontal and vertical displacement of the center of buoyancy as $\delta y'_B$ and $\delta z'_B$, respectively, then KN is given as

$$KN = KB\sin\phi + \delta y'_B \cos\phi + \delta z'_B \sin\phi$$

Determination of heeling angle for the case of moving a cargo only in transverse direction (1/4)

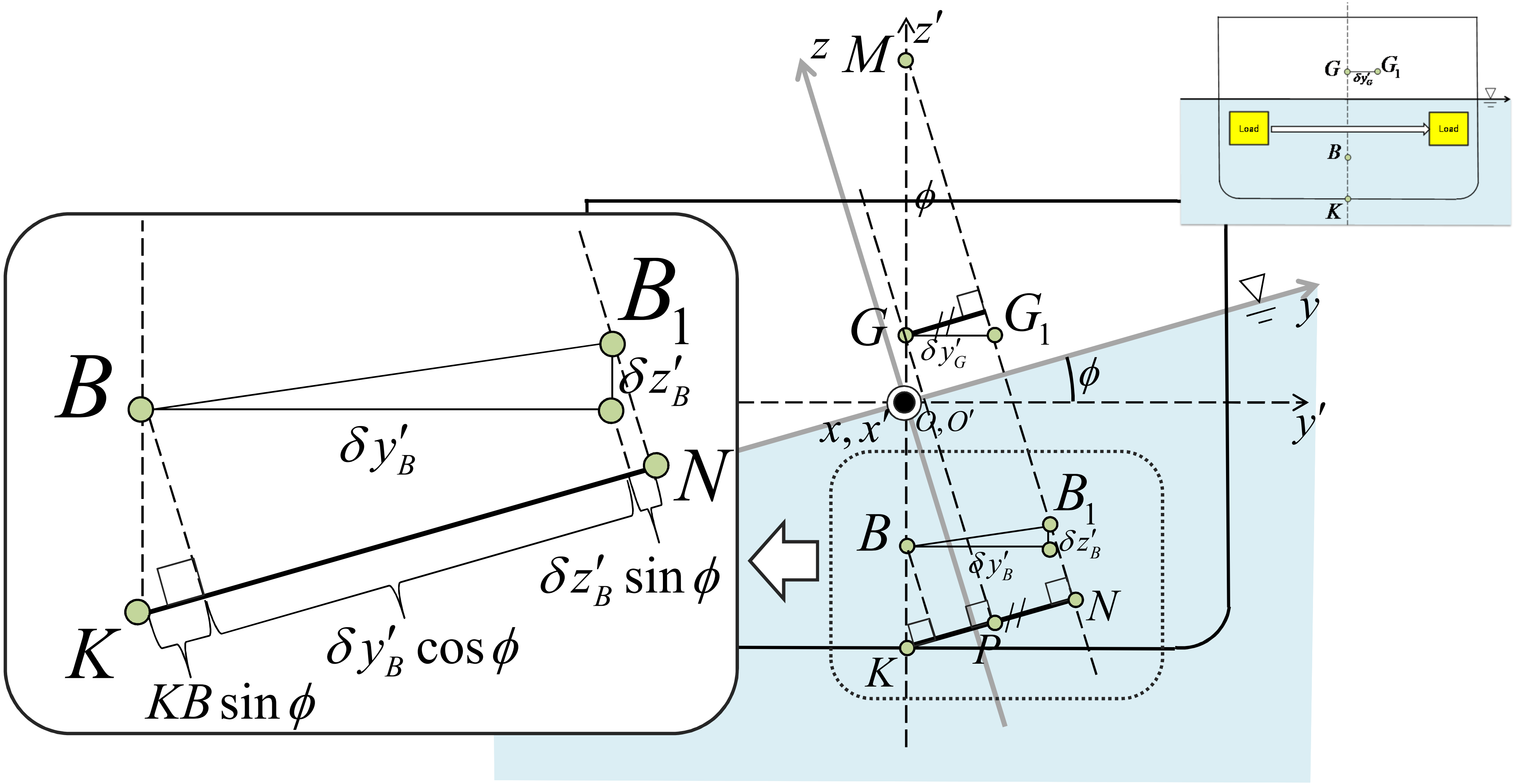


Determination of heeling angle for the case of moving a cargo only in transverse direction (2/4)



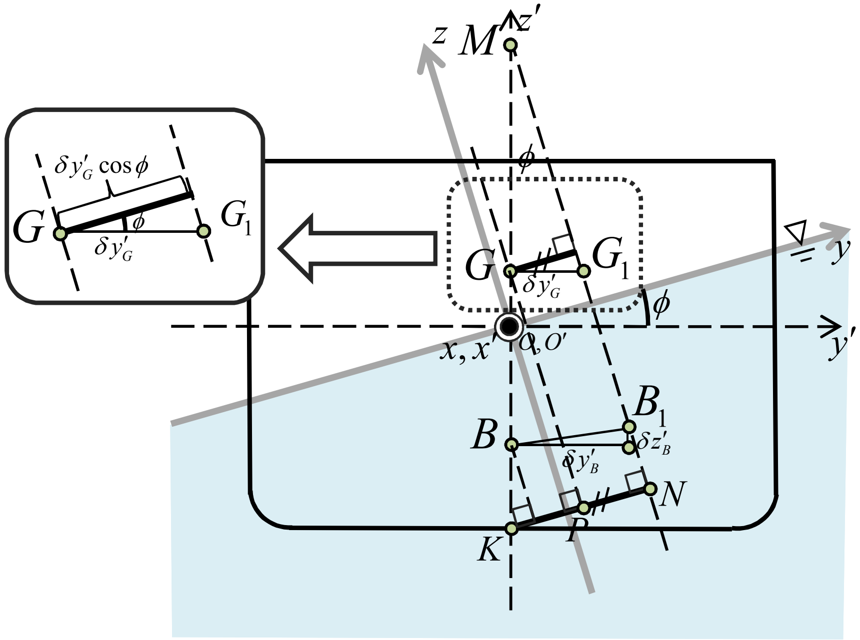
$$
\begin{aligned}
M_G &= -W \cdot \left(KP \qquad\quad + PN \right) \\
&= -W \cdot \left(KG\cos\phi + \delta y'_G \cos\phi \right)
\end{aligned}
$$

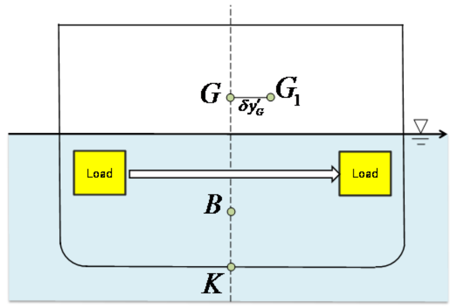
Determination of heeling angle for the case of moving a cargo only in transverse direction (3/4)



$$M_B = \Delta \cdot \left(KB\cos\phi + \delta y'_B \cos\phi + \delta z'_B \sin\phi \right)$$

Determination of heeling angle for the case of moving a cargo only in transverse direction (4/4)





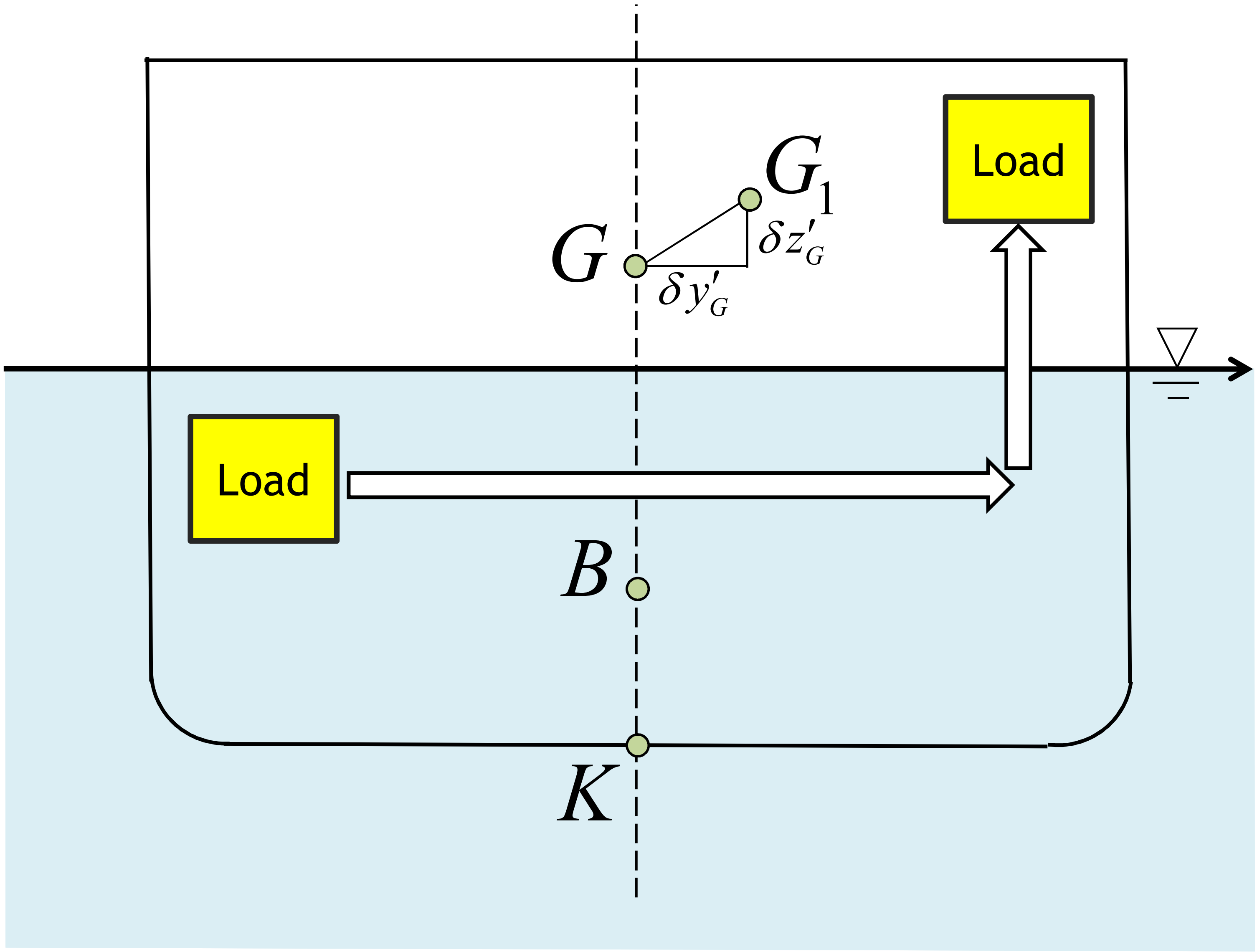
$$M_G + M_B = 0$$

$$-W \cdot \left(KG\cos\phi + \delta y'_G \cos\phi\right) + \Delta \cdot \left(KB\cos\phi + \delta y'_B \cos\phi + \delta z'_B \sin\phi\right) = 0$$

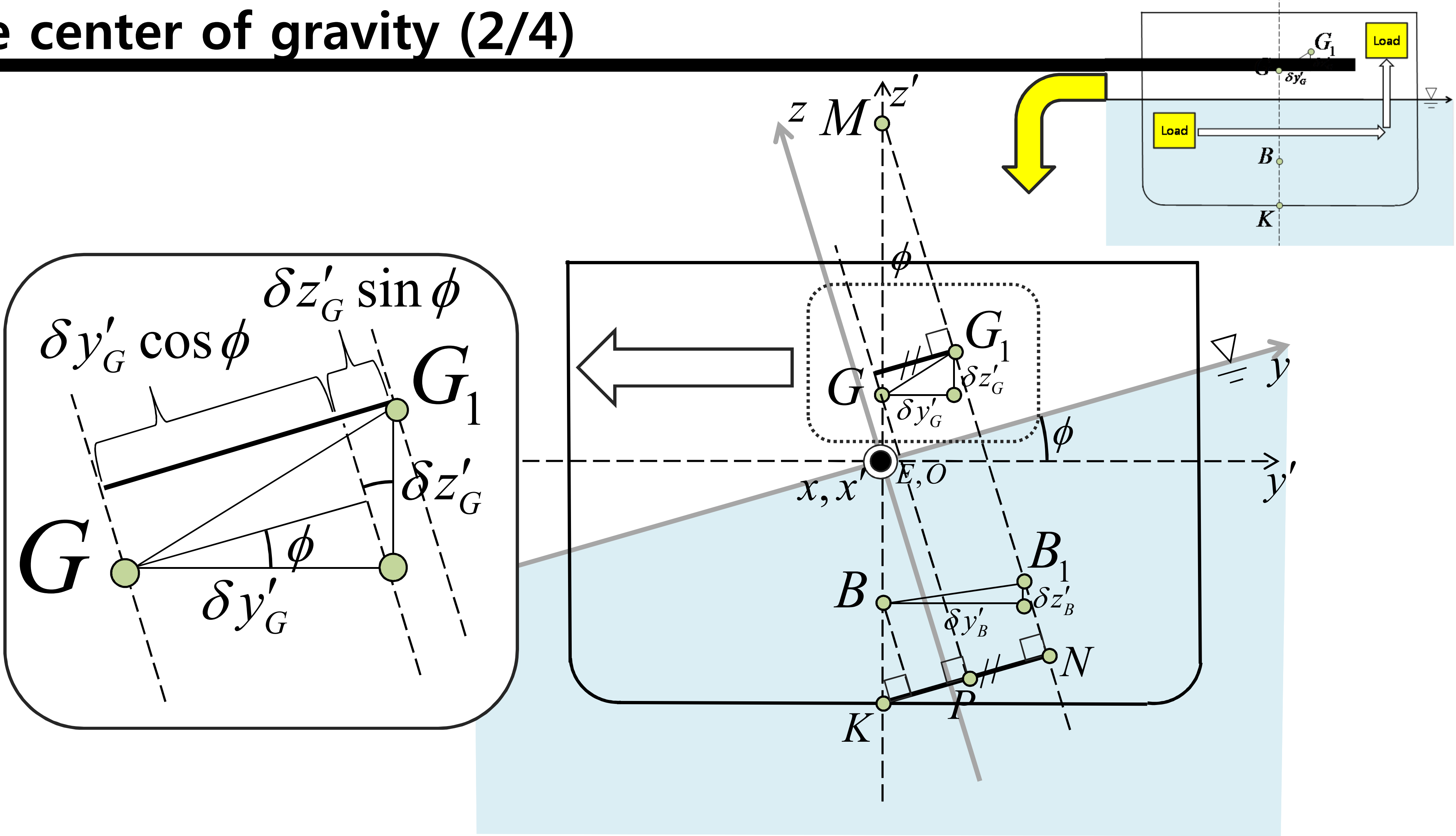
$$-\left(KG\cos\phi + \delta y'_G \cos\phi\right) + \left(KB\cos\phi + \delta y'_B \cos\phi + \delta z'_B \sin\phi\right) = 0 \qquad \because W = \Delta$$

In this equation, KG and KB are given. $\delta y'_G, \delta y'_B$ and $\delta z'_B$ are functions of ϕ.
Thus we can solve the equation and determine ϕ.

Determination of the heeling angle due to the movement of the center of gravity (1/4)

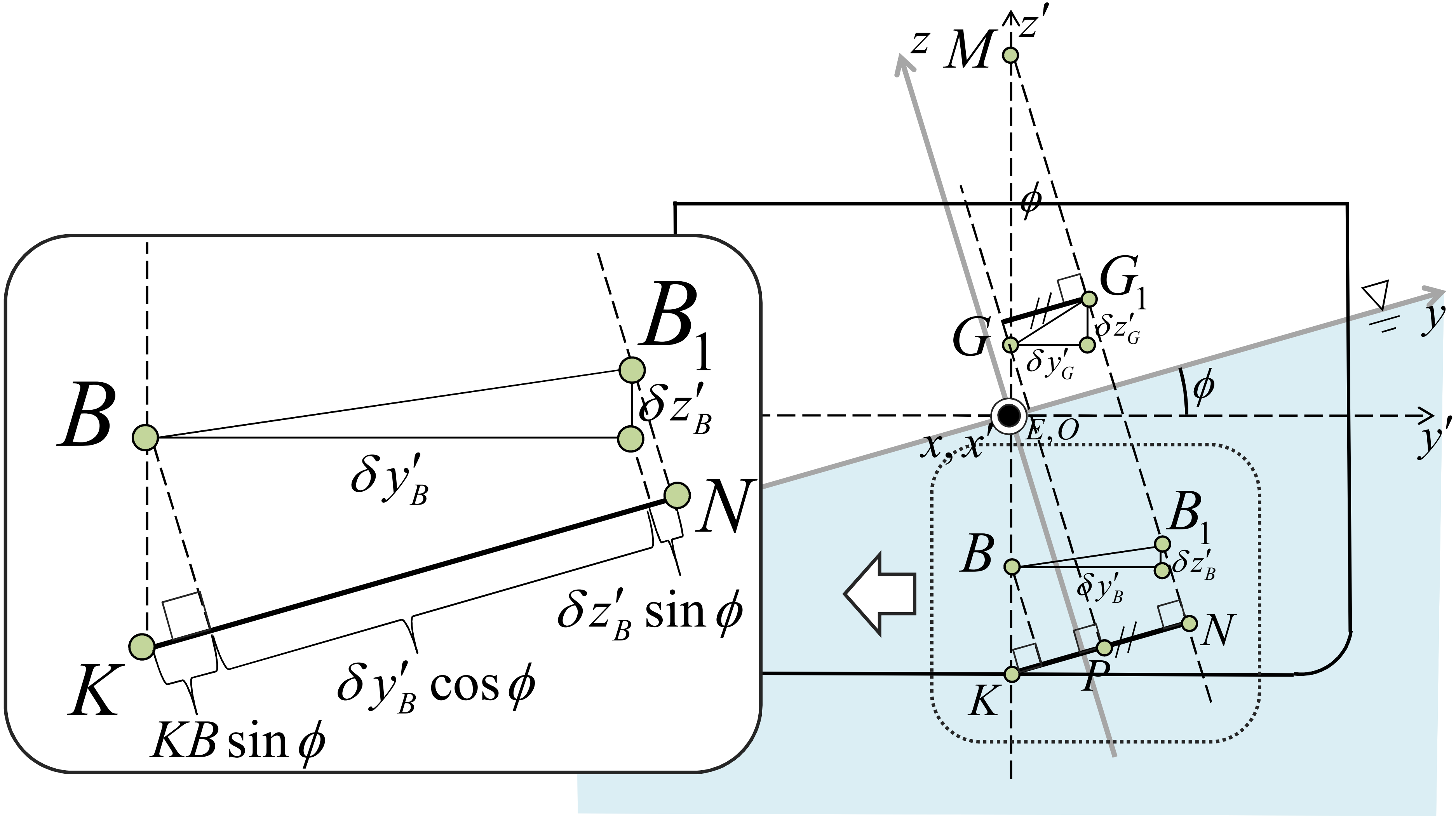


Determination of the heeling angle due to the movement of the center of gravity (2/4)



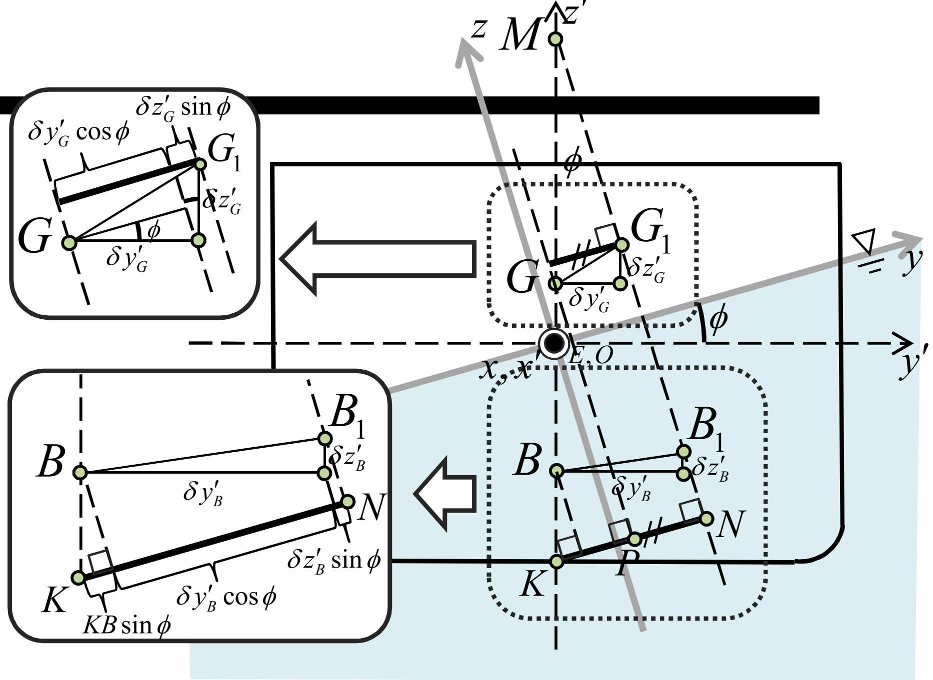
$$M_G = -W \cdot \left(KP + PN \right)$$
$$= -W \cdot \left(KG\cos\phi + \delta y'_G \cos\phi + \delta z'_G \sin\phi \right)$$

Determination of the heeling angle due to the movement of the center of gravity (3/4)



$$M_B = \Delta \cdot \left(KB\cos\phi + \delta y'_B \cos\phi + \delta z'_B \sin\phi \right)$$

Determination of the heeling angle due to the movement of the center of gravity (4/4)



$$M_G + M_B = 0$$

$$-W \cdot \left(KG\cos\phi + \delta y'_G \cos\phi + \delta z'_G \sin\phi\right) + \Delta \cdot \left(KB\cos\phi + \delta y'_B \cos\phi + \delta z'_B \sin\phi\right) = 0$$

$$-\left(KG\cos\phi + \delta y'_G \cos\phi + \delta z'_G \sin\phi\right) + \left(KB\cos\phi + \delta y'_B \cos\phi + \delta z'_B \sin\phi\right) = 0 \quad \because W = \Delta$$

In this equation, KG and KB are given. $\delta y'_G, \delta z'_G, \delta y'_B$ and $\delta z'_B$ are functions of ϕ.
Thus we can solve the equation and determine ϕ.