Ship Stability

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Ship Stability

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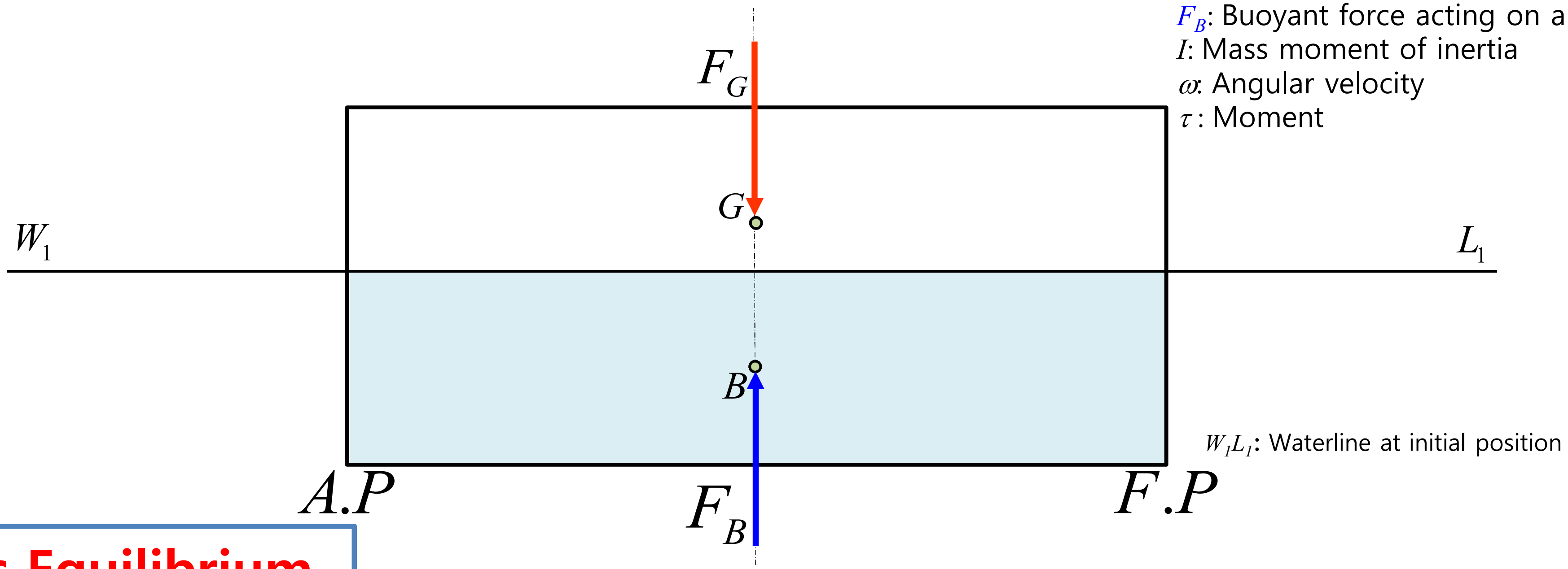
Static Equilibrium

Static Equilibrium

m: Mass of a ship
a: Acceleration of a ship
G: Center of mass of a ship
F_G: Gravitational force of a ship
B: Center of buoyancy at initial position
F_B: Buoyant force acting on a ship
I: Mass moment of inertia
ω: Angular velocity
τ : Moment



Static Equilibrium

① Newton's 2nd law

$$ma = \sum F$$
$$= -F_G + F_B$$

for the ship to be in static equilibrium

$$0 = \sum F \quad ,(\because a = 0)$$

$$\therefore F_G = F_B$$

② Euler equation

$$I\dot{\omega} = \sum \tau$$

for the ship to be in static equilibrium

$$0 = \sum \tau \quad ,(\because \dot{\omega} = 0)$$

When the buoyant force(F_B) and the gravitational force(F_G) are on one line,

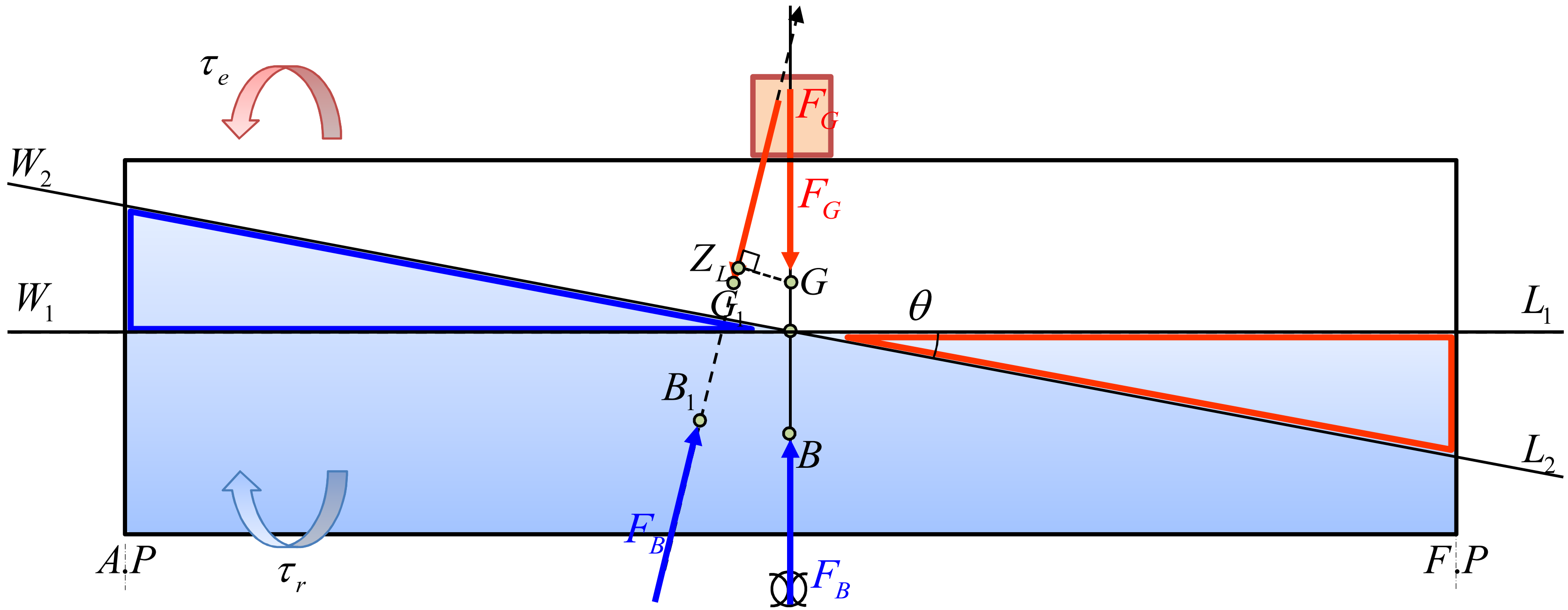
the total moment about the transverse axis **through any point** becomes 0.

It does not matter where the axis of rotation is selected if the two forces are on one line.

Longitudinal Stability

Longitudinal Stability
- Stable Equilibrium

B_1: Changed position of the center of buoyancy after the ship has been trimmed
W_1L_1: Waterline at initial position
W_2L_2: Waterline after trim
$A.P$: after perpendicular, $F.P$: forward perpendicular



① Produce an external trim moment by moving a weight in the longitudinal direction (τ_e).

② Then, release the external moment by moving the weight to its original position.

③ Test **whether it returns to its initial equilibrium position**.

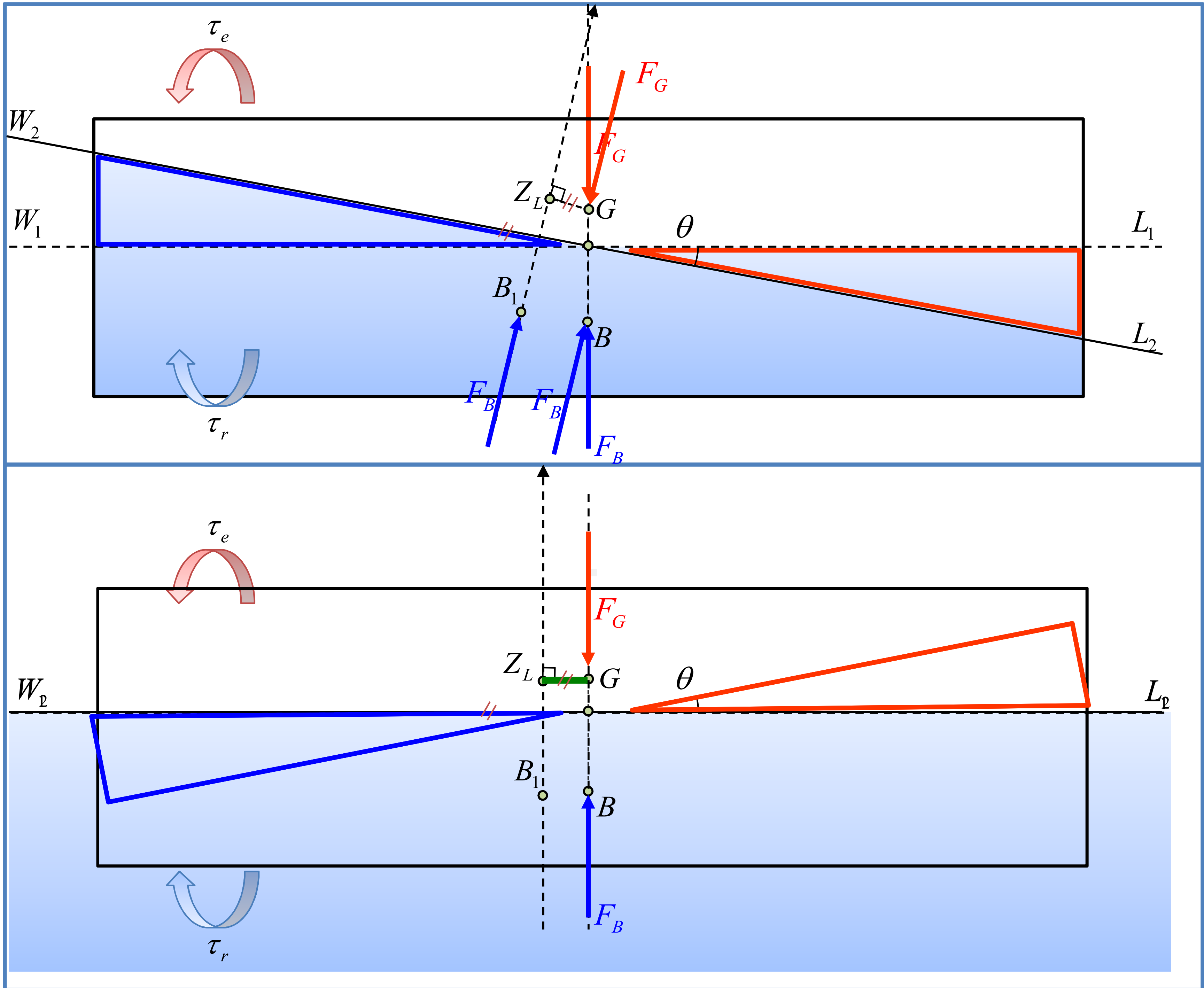
Z_L: Intersection point of the vertical line to the waterline W_2L_2 through the changed position of the center of buoyancy (B_1) with the horizontal line parallel to the waterline W_2L_2 through the center of mass of the ship(G)

Return to its initial equilibrium position

Stable

Longitudinal righting moment(τ_r) ⇨ **GZ_L ➡ Longitudinal righting arm**

Reference Frames

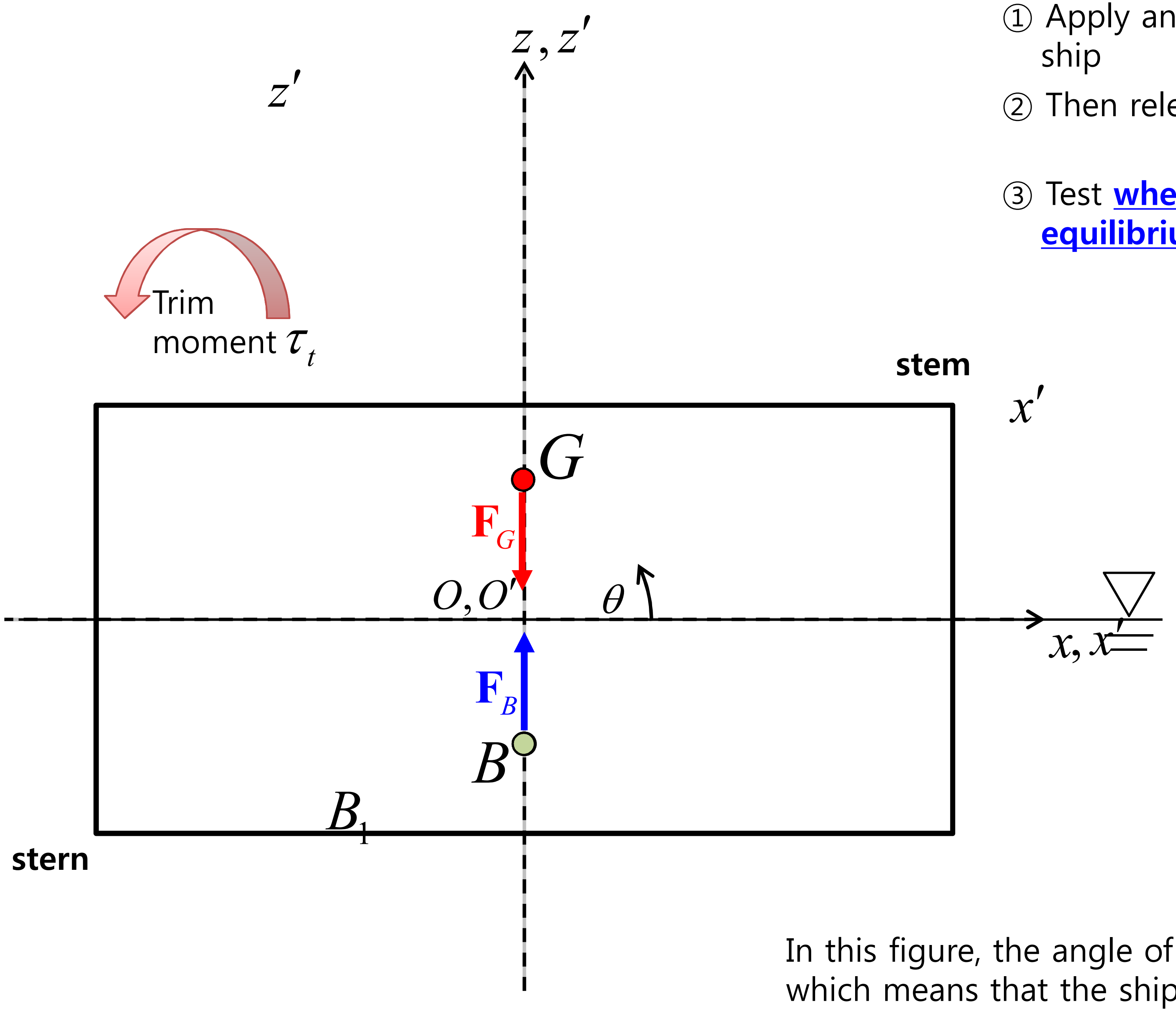


Rotation of the water plane fixed frame while the body(ship) fixed frame is fixed

Same in view of the Mechanics!!

Rotation of the body(ship) fixed frame with respect to the water plane fixed frame

Longitudinal stability of a ship
- Stable Condition (1/3)



① Apply an external trim moment to the ship

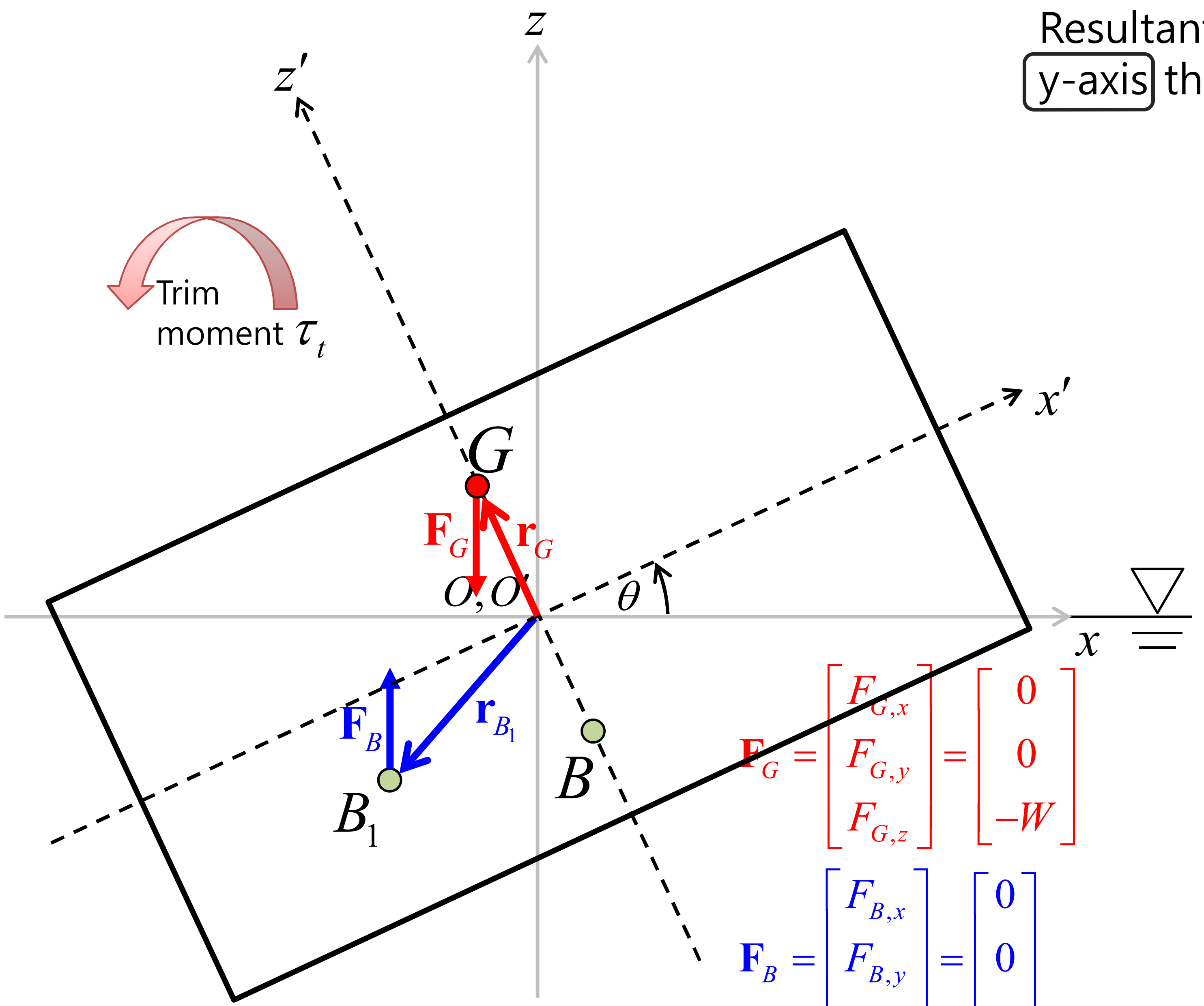
② Then release the external moment

③ Test **whether it returns to its initial equilibrium position**.

In this figure, the angle of inclination, θ, is negative, which means that the ship is trimmed by the stern.

Longitudinal stability of a ship - Stable Condition (2/3)

$$\mathbf{r}_G \times \mathbf{F}_G = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_G & y_G & z_G \\ F_{G,x} & F_{G,y} & F_{G,z} \end{vmatrix} = \begin{matrix} \mathbf{i}(y_G \cdot F_{G,z} - z_G \cdot F_{G,y}) \\ +\mathbf{j}(-x_G \cdot F_{G,z} + z_G \cdot F_{G,x}) \\ +\mathbf{k}(x_G \cdot F_{G,y} - y_G \cdot F_{G,x}) \end{matrix}$$



$$\mathbf{F}_G = \begin{bmatrix} F_{G,x} \\ F_{G,y} \\ F_{G,z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -W \end{bmatrix}$$

$$\mathbf{F}_B = \begin{bmatrix} F_{B,x} \\ F_{B,y} \\ F_{B,z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \Delta \end{bmatrix}$$

Resultant moment about y-axis through point O ($\boldsymbol{\tau}^e$) :

$$\boldsymbol{\tau}^e = \mathbf{r}_G \times \mathbf{F}_G + \mathbf{r}_{B_1} \times \mathbf{F}_B$$

$$\begin{aligned} = & \ \mathbf{i}(y_G \cdot F_{G,z} - z_G \cdot F_{G,y}) \\ & + \mathbf{j}(-x_G \cdot F_{G,z} + z_G \cdot F_{G,x}) \\ & + \mathbf{k}(x_G \cdot F_{G,y} - y_G \cdot F_{G,x}) \\ & + \mathbf{i}(y_{B_1} \cdot F_{B,z} - z_{B_1} \cdot F_{B,y}) \\ & + \mathbf{j}(-x_{B_1} \cdot F_{B,z} + z_{B_1} \cdot F_{B,x}) \\ & + \mathbf{k}(x_{B_1} \cdot F_{B,y} - y_{B_1} \cdot F_{B,x}) \end{aligned}$$

$$\begin{aligned} = & \ \mathbf{j}(-x_G \cdot F_{G,z} + z_G \cdot F_{G,x}) \\ & + \mathbf{j}(-x_{B_1} \cdot F_{B,z} + z_{B_1} \cdot F_{B,x}) \end{aligned}$$

$$= \mathbf{j}(-x_G \cdot (-W) - x_{B_1} \cdot \Delta)$$

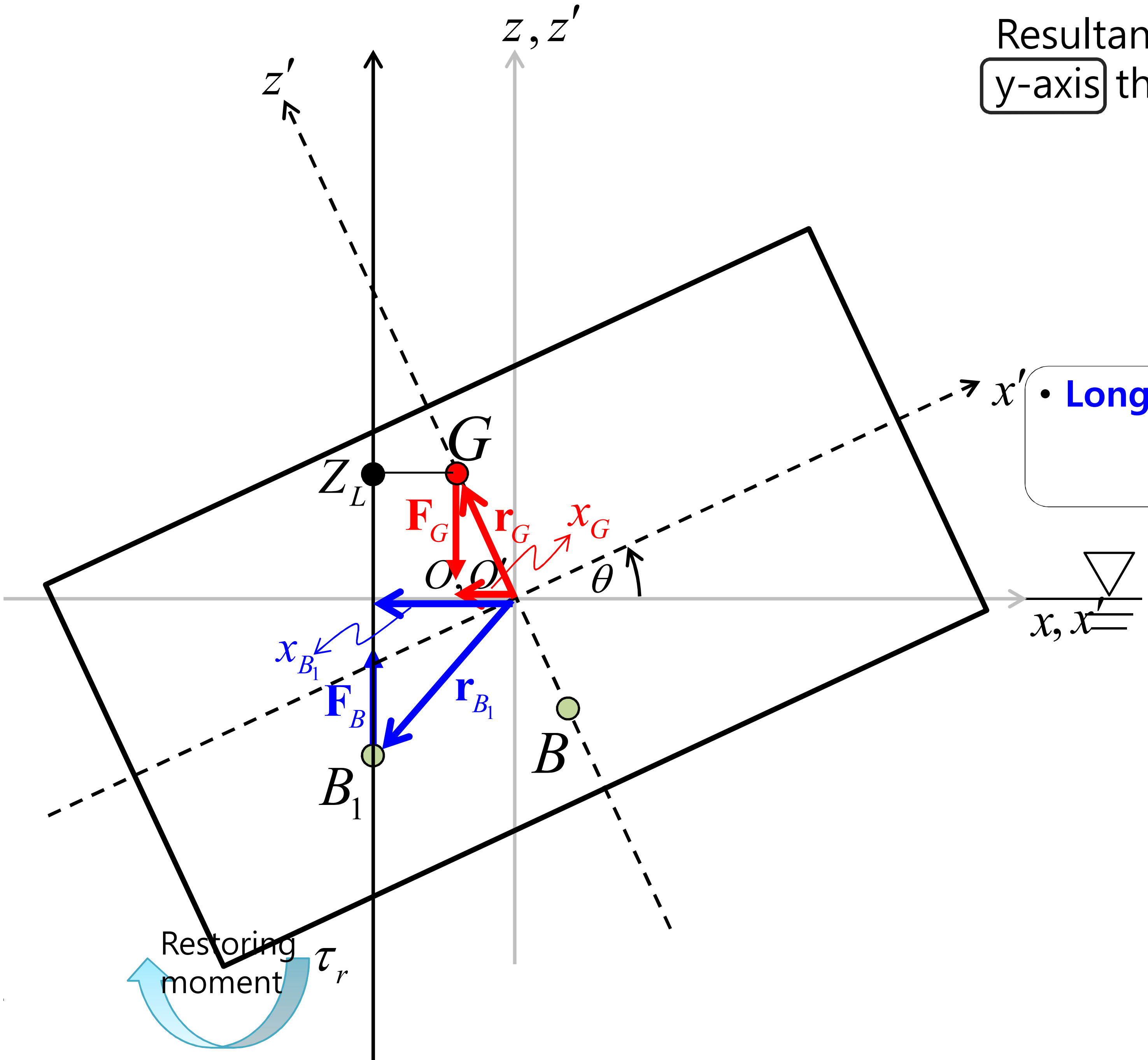
If $W = \Delta$

$$= \mathbf{j}(-x_G \cdot (-\Delta) - x_{B_1} \cdot \Delta)$$

$$= \mathbf{j} \cdot \Delta(x_G - x_{B_1})$$

In this figure, the angle of inclination, θ, is negative, which means that the ship is trimmed by the stern.

Longitudinal stability of a ship
- Stable Condition (3/3)



Resultant moment about y-axis through point O ($\boldsymbol{\tau}^e$) :

$$\boldsymbol{\tau}^e = \mathbf{r}_G \times \mathbf{F}_G + \mathbf{r}_{B_1} \times \mathbf{F}_B$$

$$= \mathbf{j} \cdot \Delta (x_G - x_{B_1})$$

$$= \mathbf{j} \cdot \Delta \cdot GZ_L$$

- **Longitudinal Righting Moment**

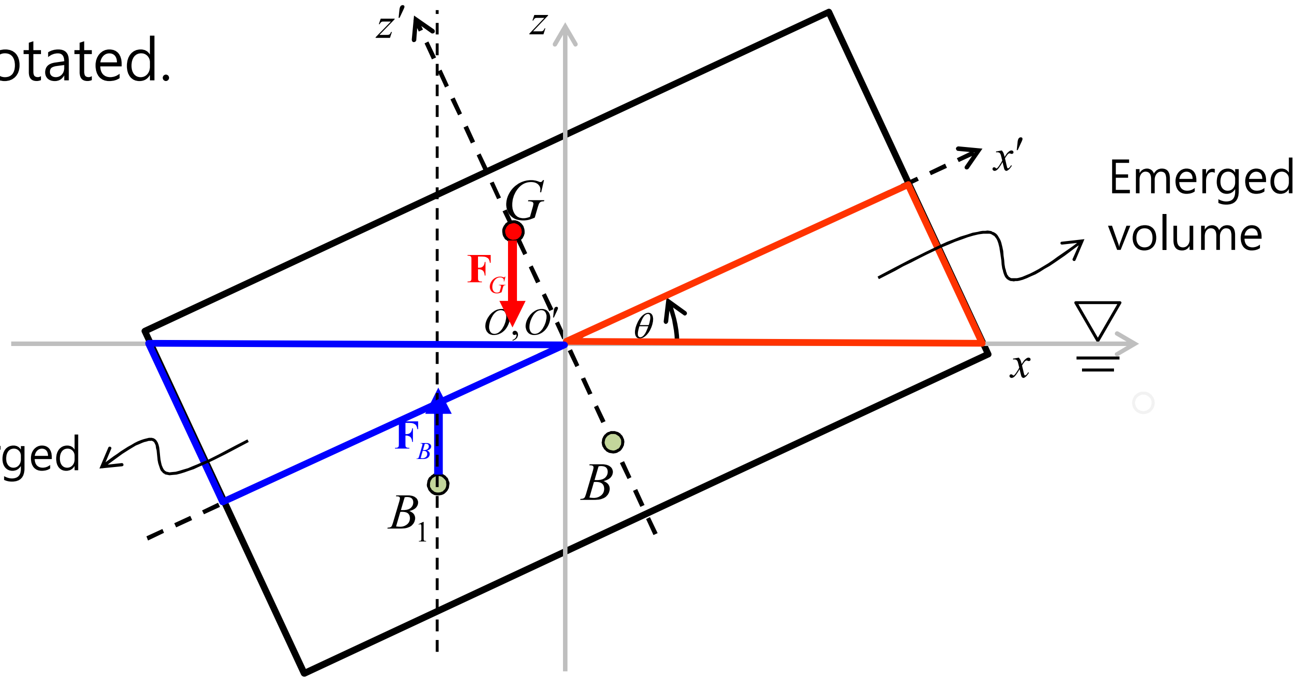
$$\tau_r = \Delta \cdot GZ_L$$

The moment arm induced by the buoyant force and gravitational force is expressed by GZ_L, where Z_L is the intersection point of the line of buoyant force(Δ) through the new position of the center of buoyancy(B_1) with a transversely parallel line to a waterline through the center of the ship's mass(G).

Stable !!

Position and Orientation of a Ship with Respect to the Water Plane Fixed and Body(Ship) Fixed Frame

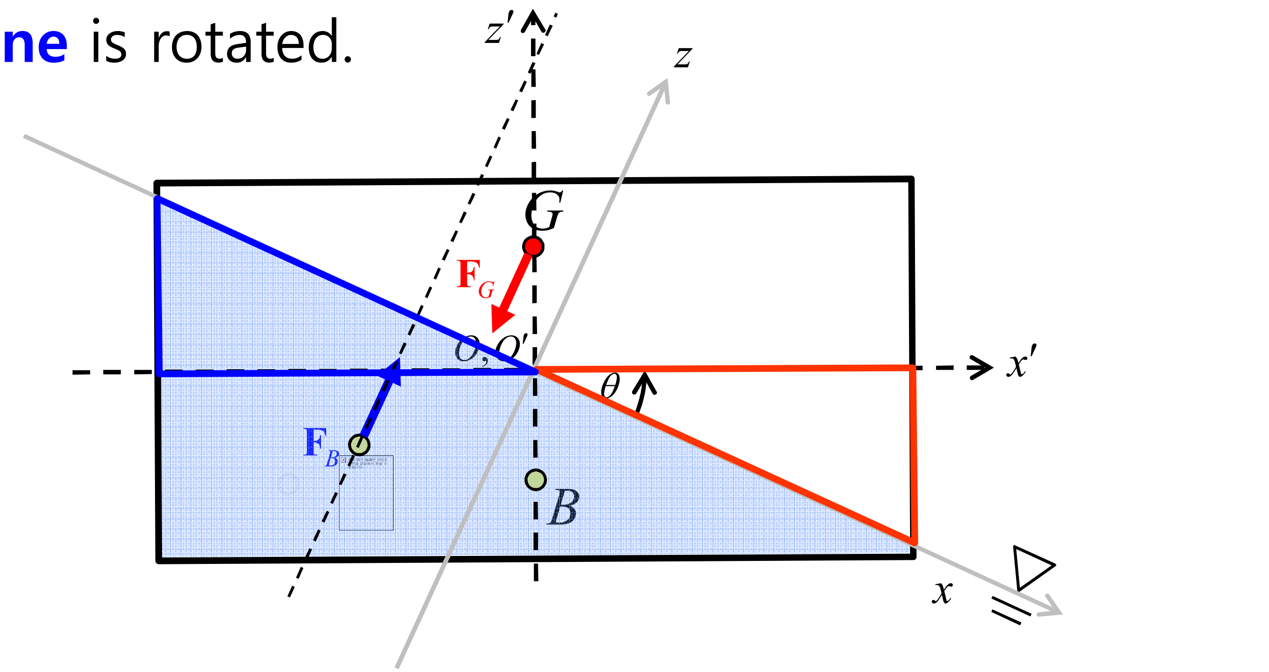
The ship is rotated.



Rotation of the body(ship) fixed frame with respect to the water plane fixed frame

Same in view of the Mechanics!!

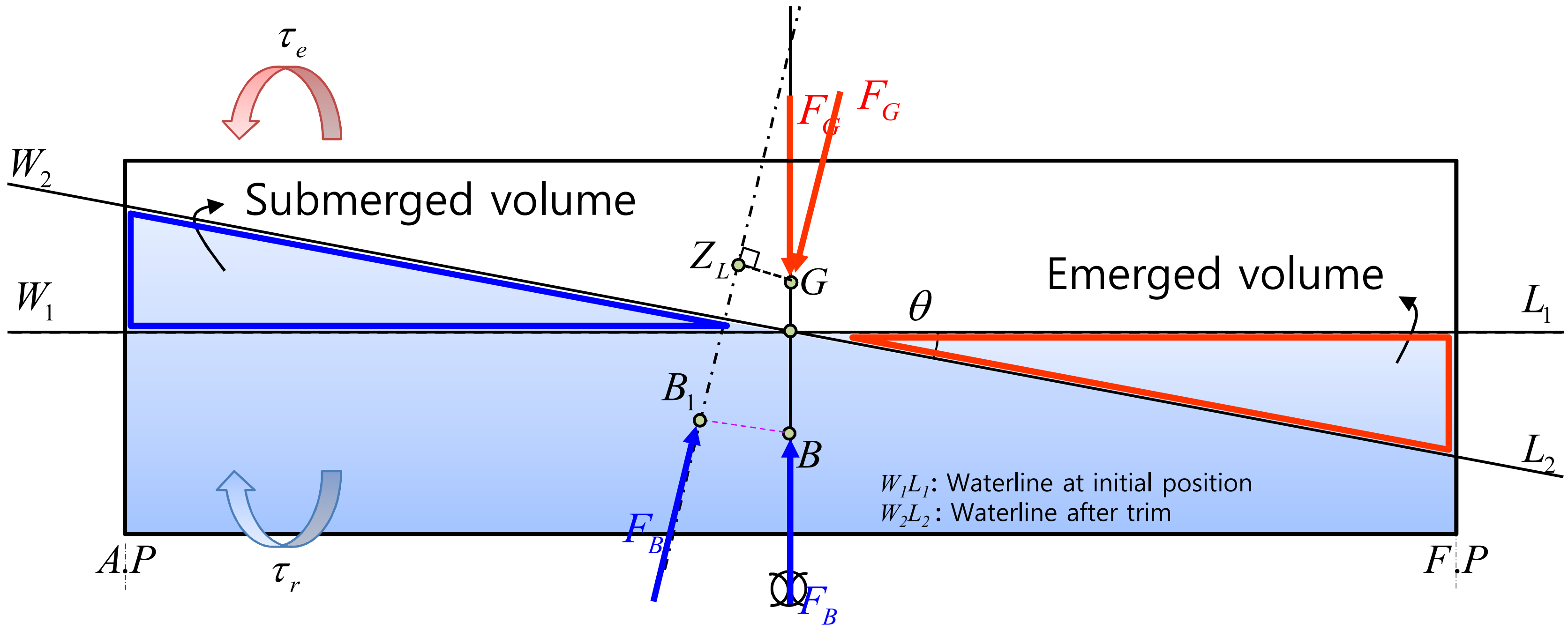
The water plane is rotated.



Rotation of the water plane fixed frame while the body(ship) fixed frame is fixed

Longitudinal Stability in Case of Small Angle of Trim

Assumptions for Small Angle of Trim

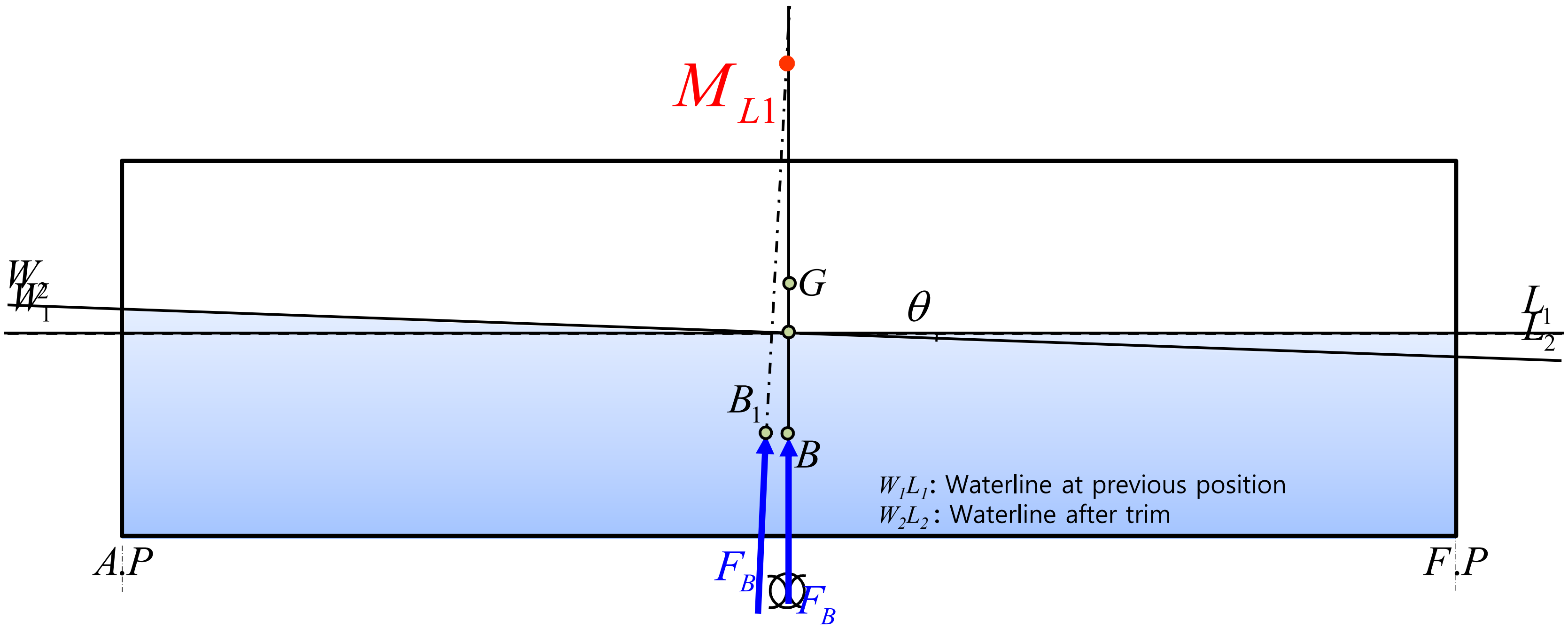


Assumptions

① **Small angle of inclination** (3°~5° for trim)

② **The submerged volume** and **the emerged volume** are to be the **same**.

Longitudinal Metacenter (M_L) (1/3)



Longitudinal Metacenter (M_L) ※ Metacenter "M" is valid for small angle of inclination.

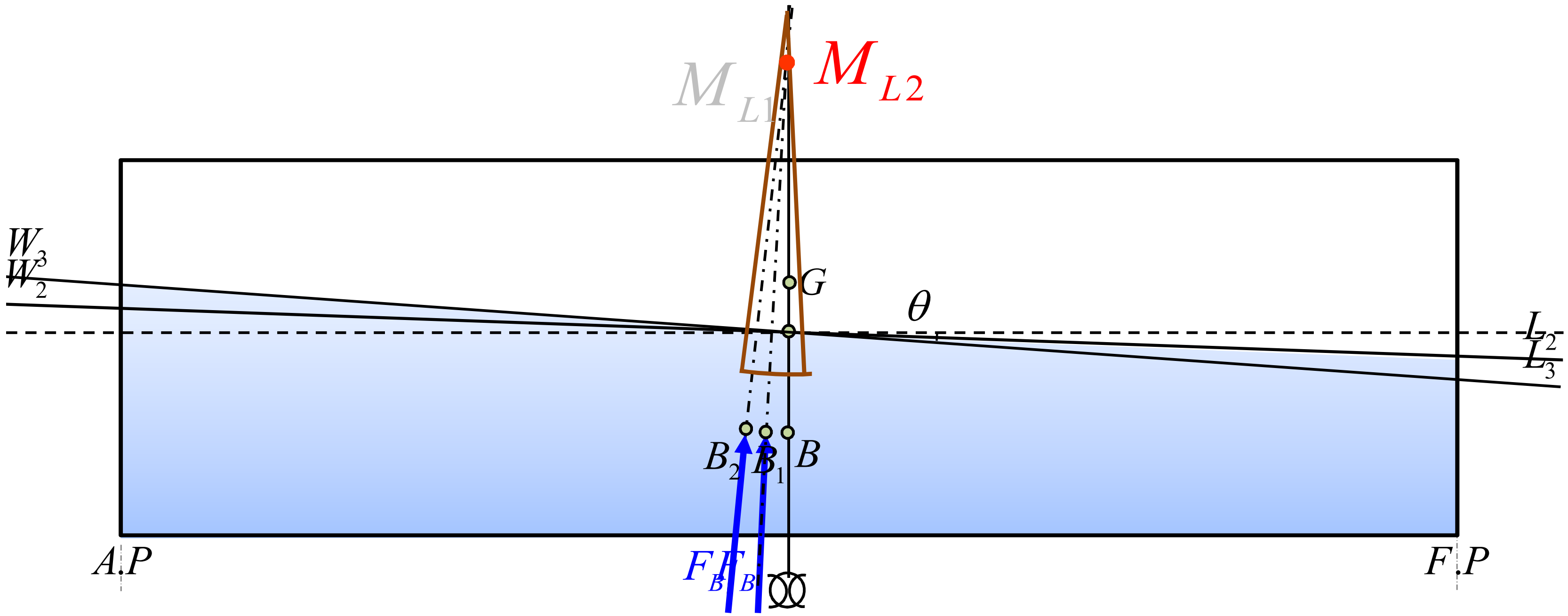
The **intersection point** of

a vertical line through the center of buoyancy **at a previous position** (***B***)

with a vertical line through the center of buoyancy **at the present position** (B_1)

after the ship has been trimmed.

Longitudinal Metacenter (M_L) (2/3)



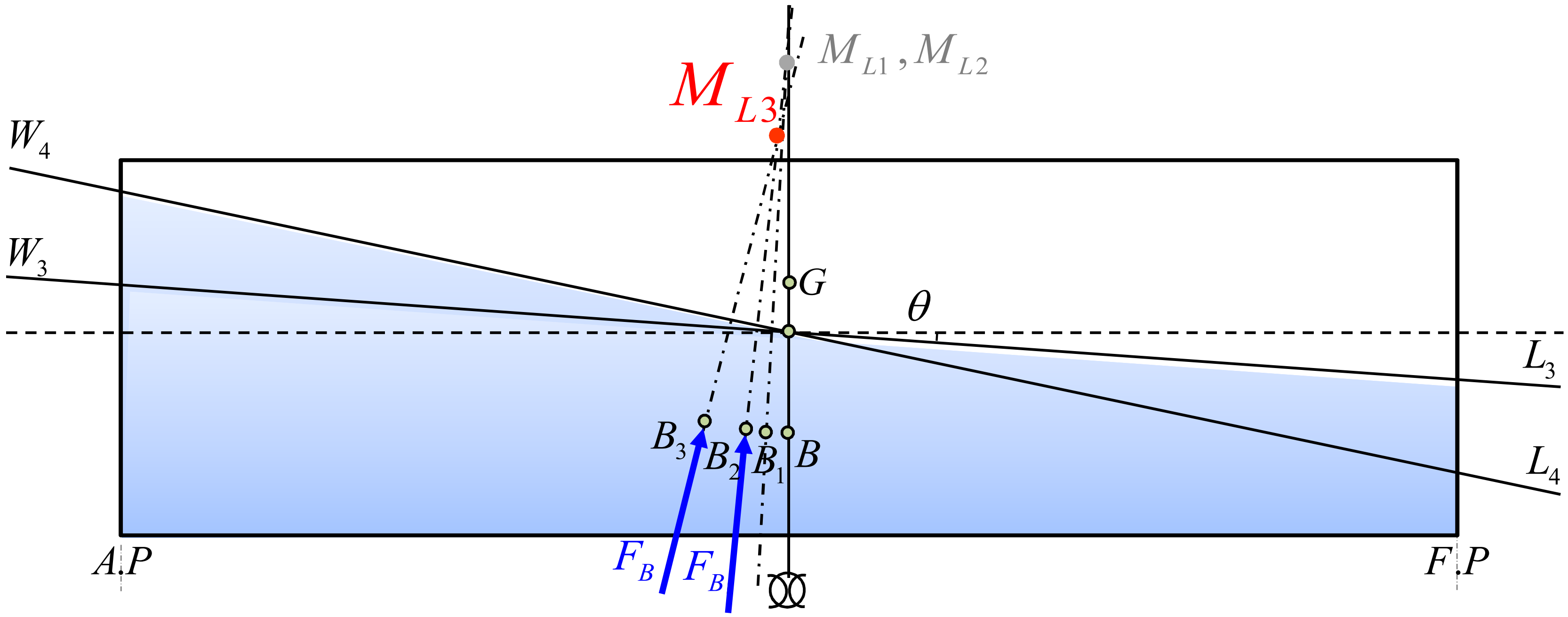
M_L **remains at the same position** for small angle of trim, up to **about 2~5 degrees**.

As the ship is inclined with a small trim angle, **B** moves on the arc of circle whose center is at M_L.

The BM_L is **longitudinal metacentric radius**.

The GM_L is **longitudinal metacentric height**.

Longitudinal Metacenter (M_L) (3/3)



M_L **does not remain in the same position** for large trim angles **over 5 degrees**.

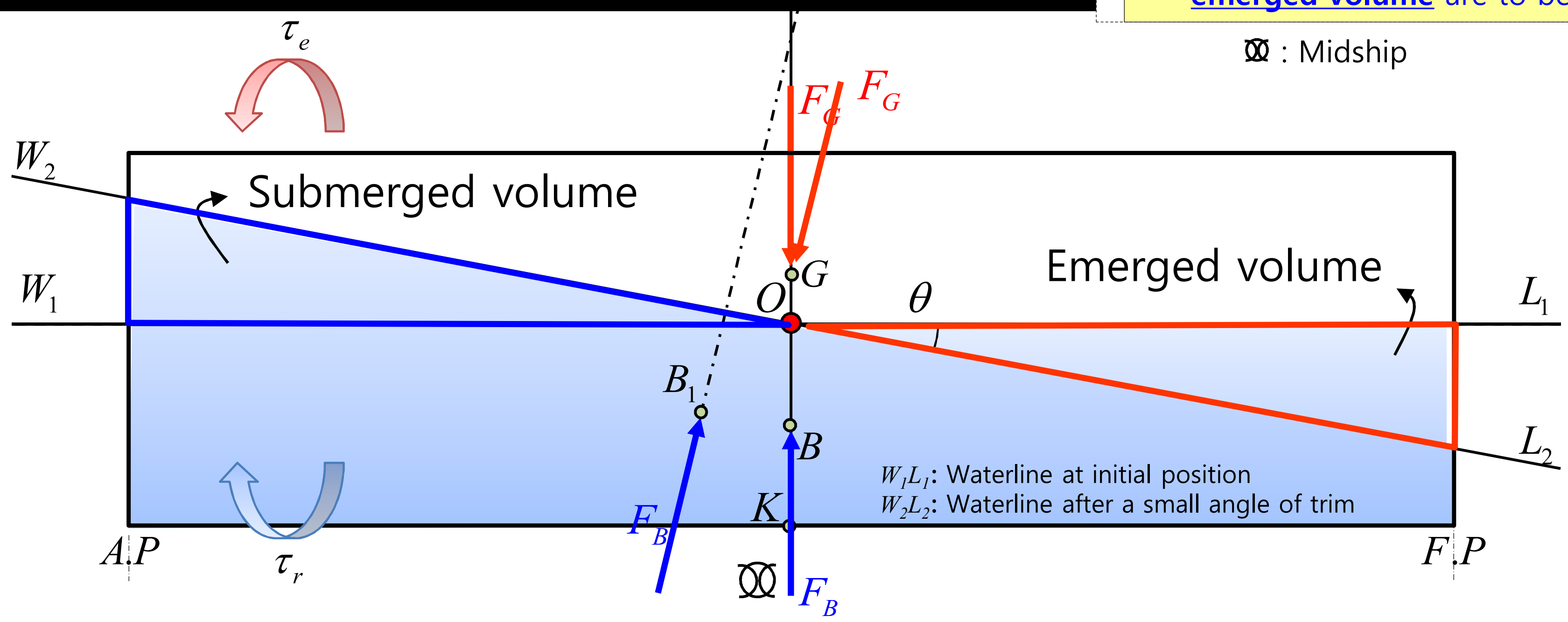
Thus, the longitudinal metacenter, M_L, is only **valid for a small trim angle**.

Longitudinal Stability for a Box-Shaped Ship

Assumption

① **A small trim angle** (3°~5°)

② The **submerged volume** and the **emerged volume** are to be the **same**.



About which point a box-shaped ship rotates, while the submerged volume and the emerged volume are to be the same?

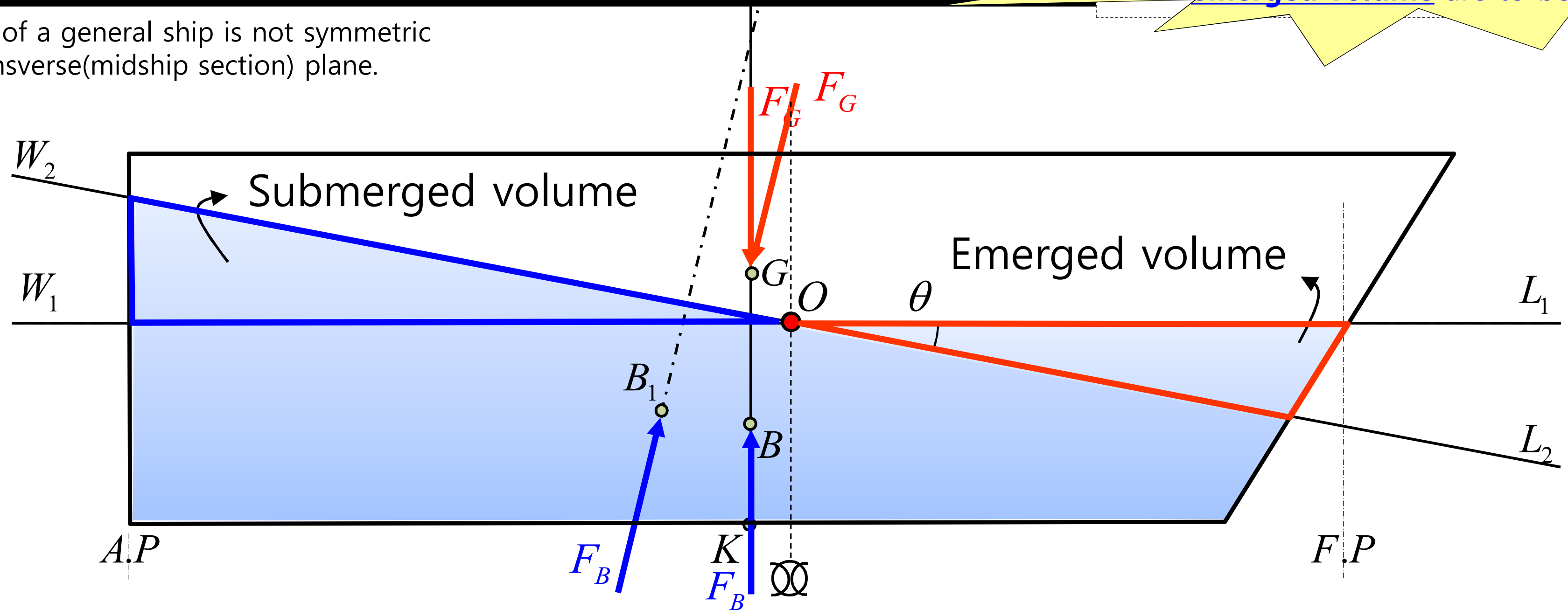
① Apply an external trim moment(τ_e) which results in the ship to incline with a trim angle θ.

② For the **submerged volume** and the **emerged volume** are to be the **same**, the ship rotates **about the transverse axis through the point O**.

Longitudinal Stability for a General Ship

Assumption
① **A small angle of inclination** (3°~5° for trim)
② The **submerged volume** and the **emerged volume** are to be the **same**.

The hull form of a general ship is not symmetric about the transverse(midship section) plane.

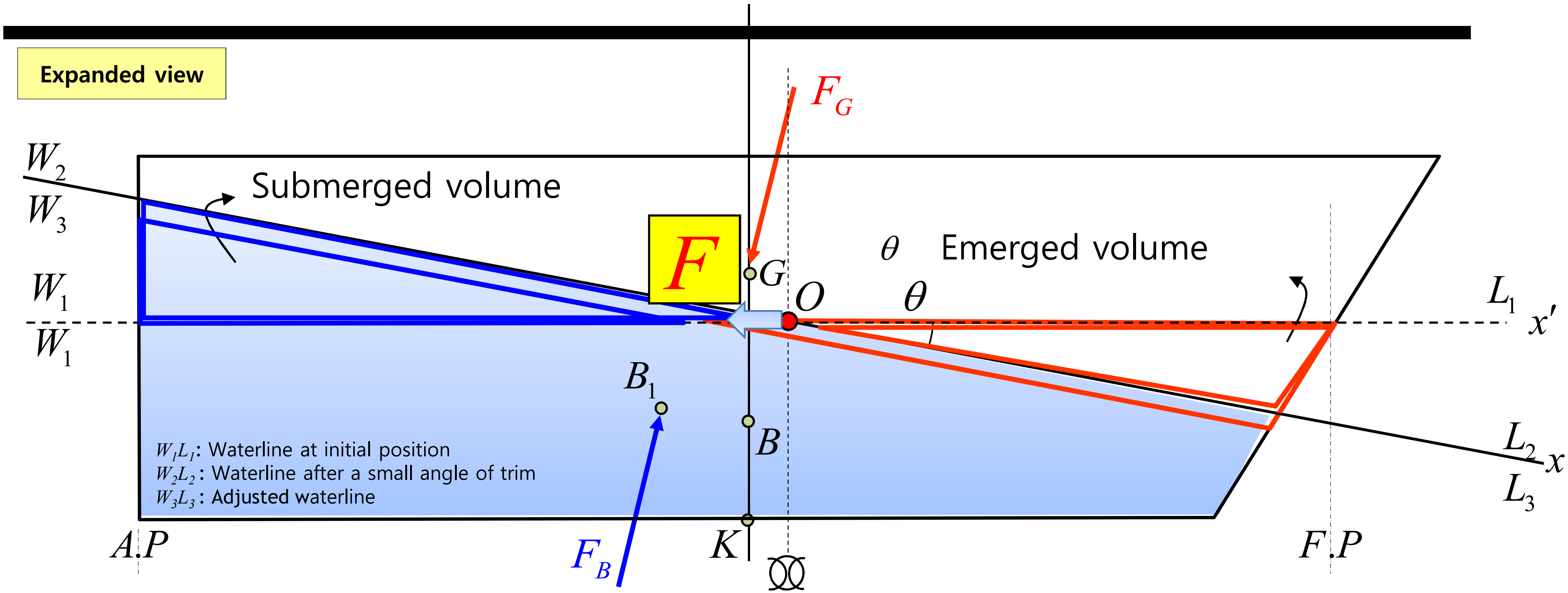


What will happen if the hull form of a ship is **not symmetric** about the transverse(midship section) plane through point O?

The submerged volume and the emerged volume are **not same**!

So, **the draft** must **be adjusted** to maintain same displacement.

Rotation Point (F)

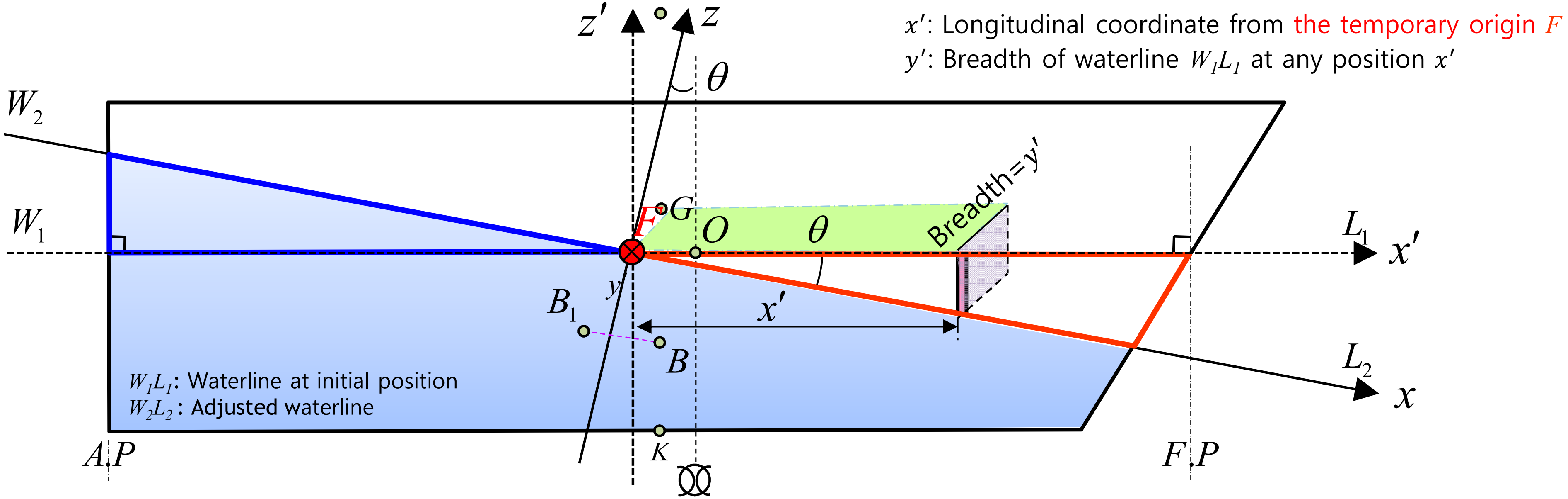


So, **the draft** must **be adjusted** to maintain same displacement.

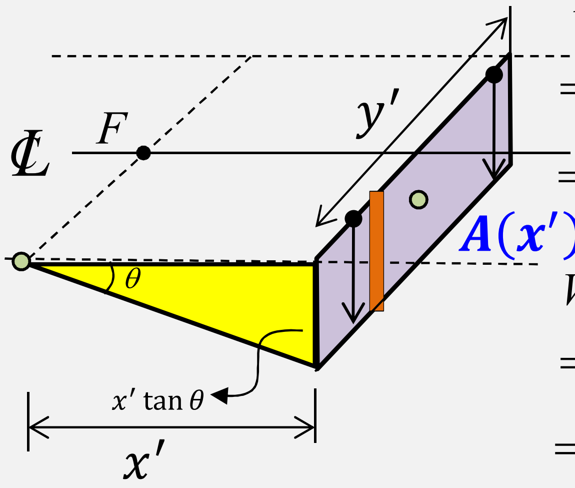
The intersection of the initial waterline(W_1L_1) with the adjusted waterline(W_3L_3) is a point ***F***, on which the submerged volume and the emerged volume are supposed to be the same.

What we want to find out is the point ***F***. How can we find the point ***F*** ?

Longitudinal Center of Floatation (LCF) (1/3)



From now on, the adjusted waterline is indicated as W_2L_2.



$A(x')$

$= (y') \cdot (x' \cdot \tan\theta)$

$= x' \cdot y' \cdot \tan\theta$

$V(x')$

$= \int A(x')dx'$

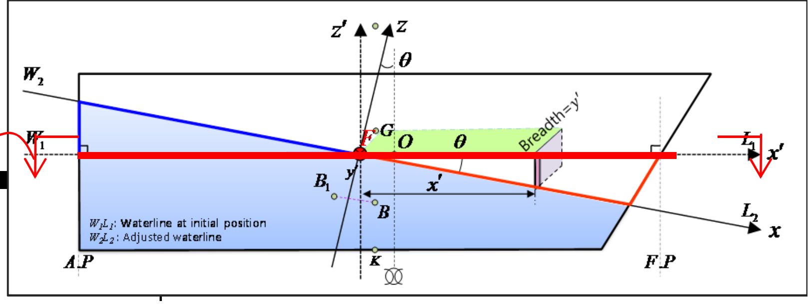
$= \int x' \cdot y' \cdot \tan\theta dx'$

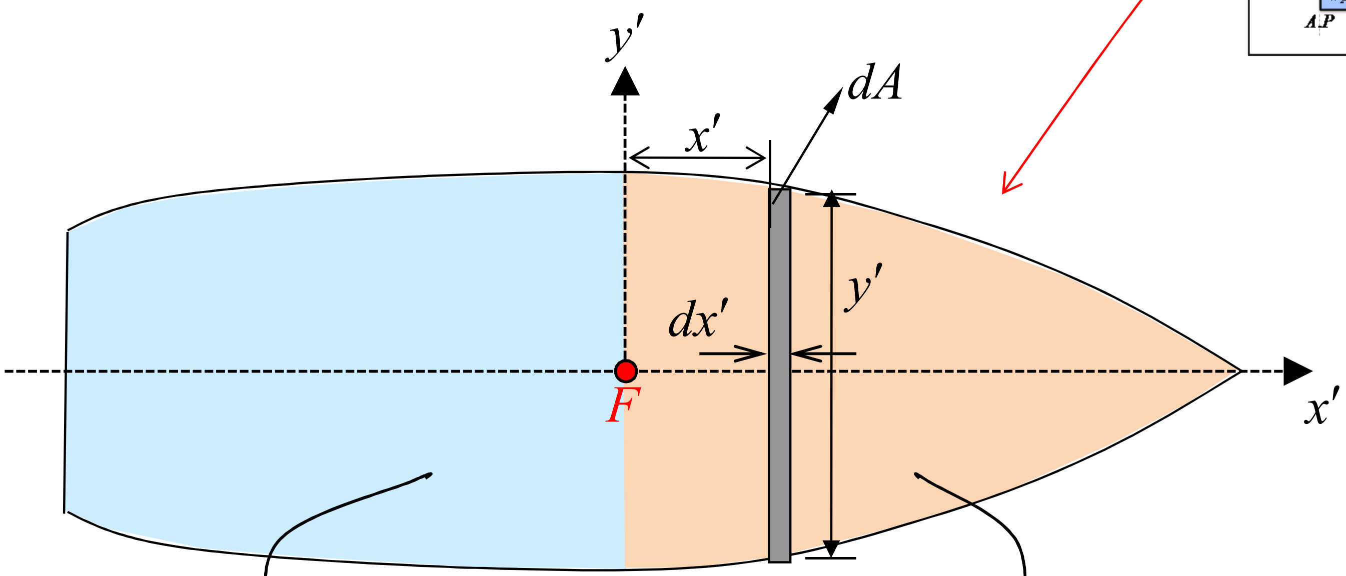
Submerged volume(v_a) = Emerged volume(v_f)

$$\int_{A.P}^{F} y' \cdot (x' \cdot \tan\theta)dx' = \int_{F}^{F.P} y' \cdot (x' \cdot \tan\theta)dx'$$

$$\int_{A.P}^{F} x' \cdot y' \, dx' = \int_{F}^{F.P} x' \cdot y' \, dx'$$

What does this equation mean?





<Plan view of waterplane>

Longitudinal moment of the **after** water plane area from F about the transverse axis(y') through the point F

$$\int_{A.P}^{F} x'\,dA = \int_{A.P}^{F} x'\cdot y'\,dx'$$

Longitudinal moment of the **forward** water plane area from F about the transverse axis(y') through the point F

$$\int_{F}^{F.P} x'\,dA = \int_{F}^{F.P} x'\cdot y'\,dx$$

$v_a = v_f \Rightarrow$

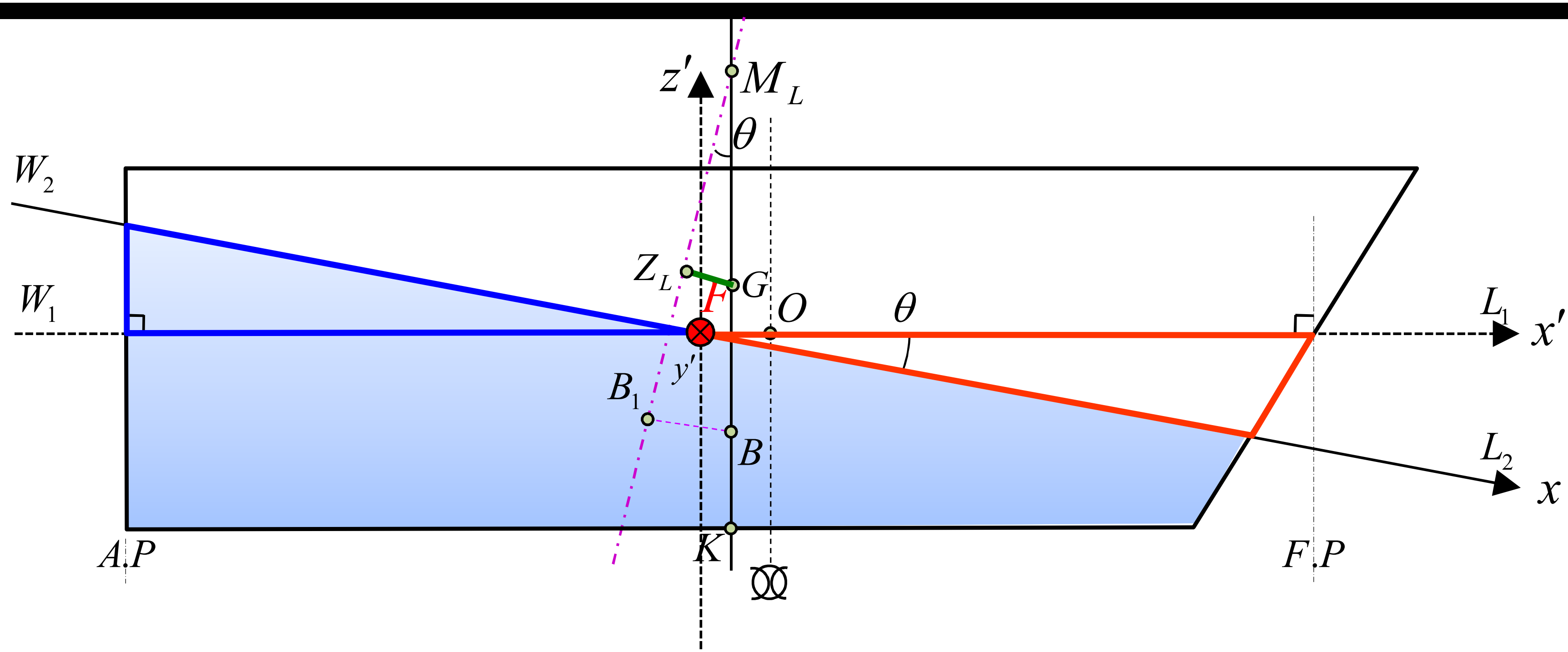
$$\int_{A.P}^{F} y'\cdot(x'\cdot\tan\theta)dx' = \int_{F}^{F.P} y'\cdot(x'\cdot\tan\theta)dx'$$

$$\int_{A.P}^{F} x'\cdot y'\,dx' = \int_{F}^{F.P} x'\cdot y'\,dx'$$

Since these moments are **equal and opposite**, the longitudinal moment of the entire water plane area about the transverse axis through point F is **zero**.

That means the point F lies on the transverse axis through the **centroid of the water plane**, called longitudinal center of floatation(LCF).

Longitudinal Center of Floatation (LCF) (3/3)

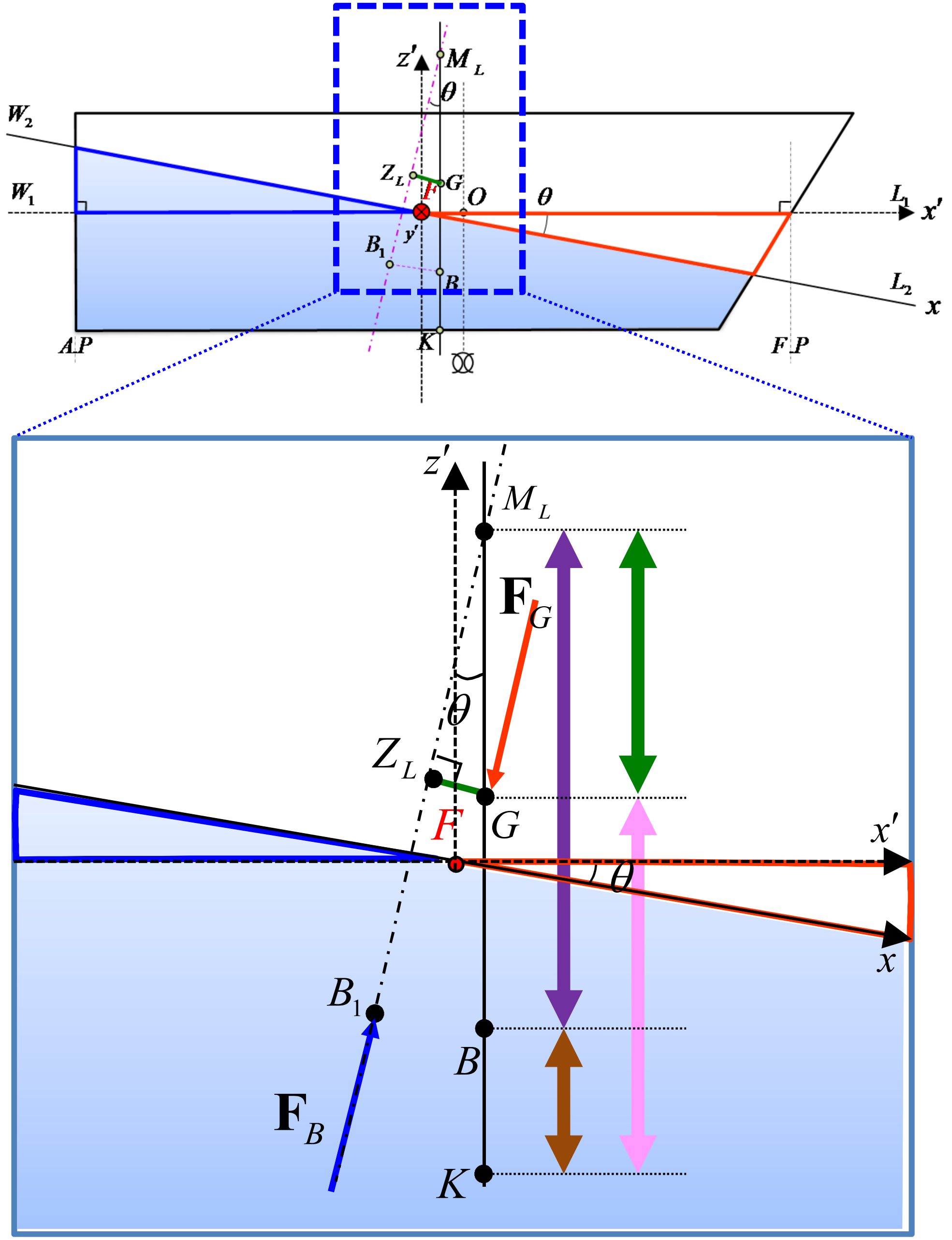


Therefore, for the ship to incline under the condition that the submerged volume and emerged volume are to be the same,
the ship rotates about the transverse axis through the longitudinal center of floatation (LCF).

F ⇨ Longitudinal Center of Floatation (LCF)

Longitudinal Righting Moment Arm

Longitudinal Righting Moment Arm (GZ_L)



Longitudinal Righting Moment

$$= GZ_L \cdot \mathbf{F}_B$$

From geometrical configuration

$$GZ_L \cong GM_L \cdot \sin\theta$$

with assumption that M_L remains at the same position within a small angle of trim (about 2°~5°)

$$GM_L = KB + BM_L - KG$$

KB ≒ 51~52% draft

How can you get the value of the BM_L?

Vertical center of mass of the ship

GM_L: Longitudinal metacentric height

KB: Vertical center of buoyancy at initial position

BM_L: Longitudinal metacentric radius

KG: Vertical center of mass of the ship

K: Keel M_L: Longitudinal Metacenter

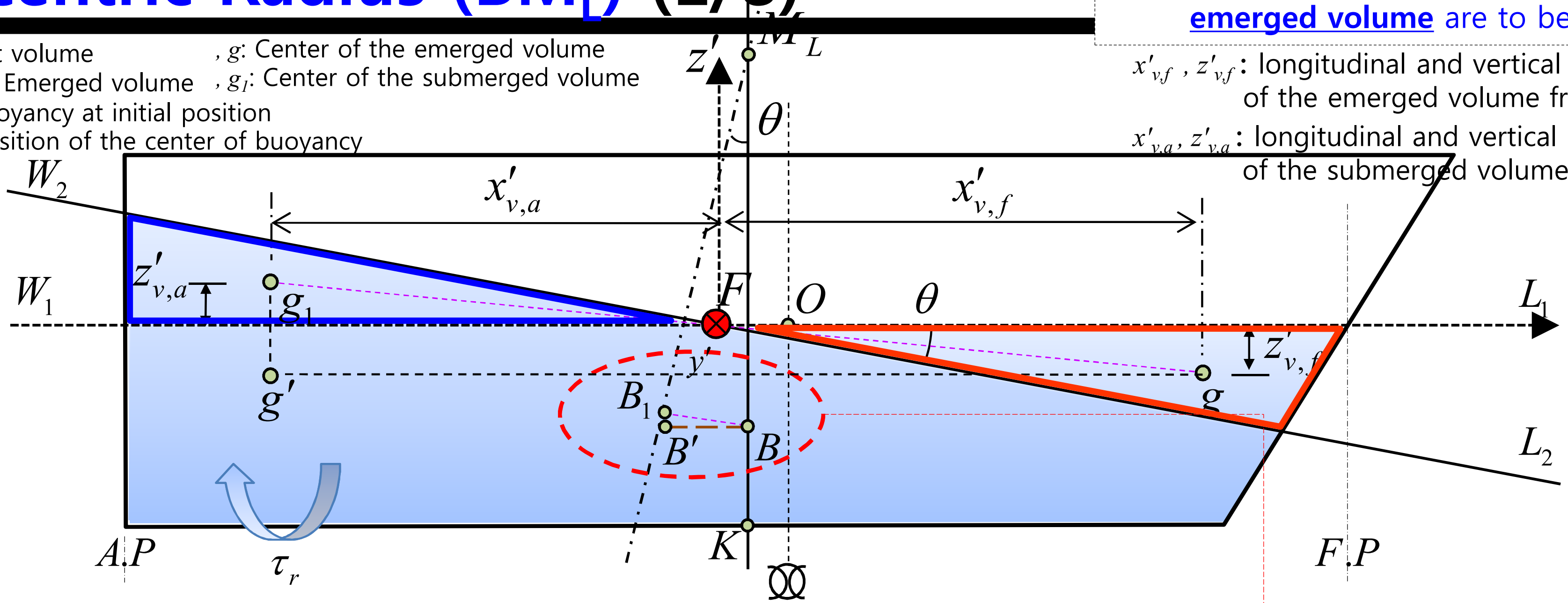
Derivation of Longitudinal Metacentric Radius (BM_L)

Derivation of Longitudinal Metacentric Radius (BM_L) (1/8)

Assumption
① **A small trim angle** (3°~5°)
② The **submerged volume** and the **emerged volume** are to be the **same**.

∇: Displacement volume
v: Submerged / Emerged volume
B: Center of buoyancy at initial position
B_1: Changed position of the center of buoyancy
, g: Center of the emerged volume
, g_1: Center of the submerged volume

$x'_{v,f}$, $z'_{v,f}$: longitudinal and vertical distance of the emerged volume from F
$x'_{v,a}$, $z'_{v,a}$: longitudinal and vertical distance of the submerged volume from F



Let's derive longitudinal metacentric radius BM_L when a ship is trimmed.

The center of buoyancy at initial position(B) moves parallel to gg_1.

The distance BB_1 equals to $\frac{v}{\nabla}\cdot gg_1$

$$BB_1 = \frac{v}{\nabla}\cdot gg_1$$

Because the triangle $gg'g_1$ is similar with $BB'B_1$

$$\frac{BB_1}{gg_1} = \frac{BB'}{gg'} = \frac{\delta x'_B}{(x'_{v,a}+x'_{v,f})}$$

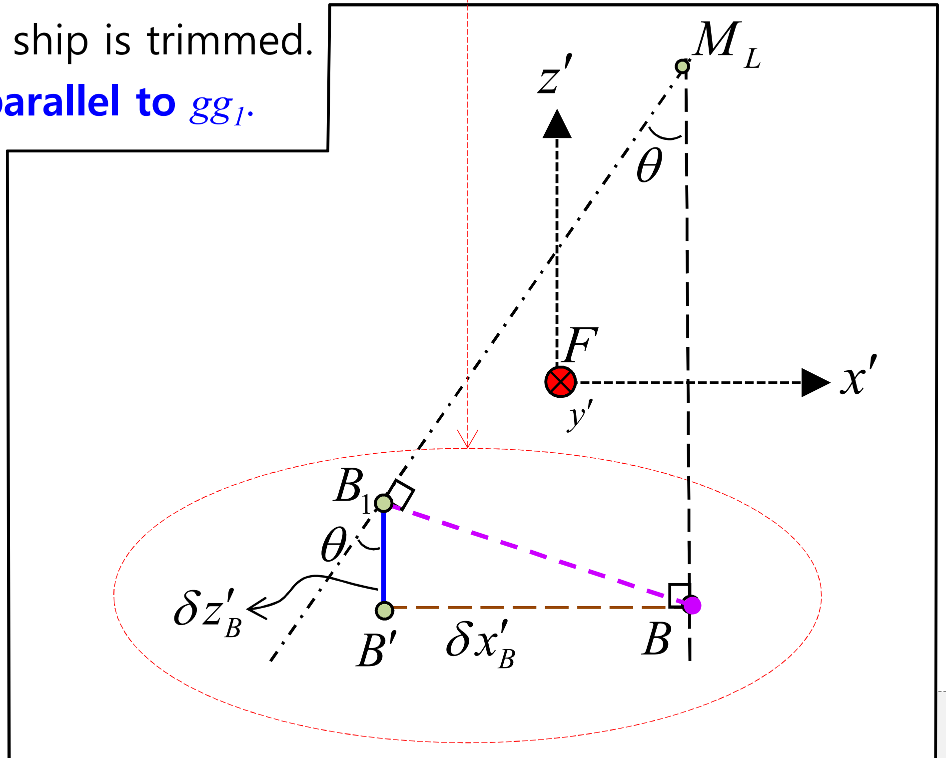
$$\frac{BB_1}{gg_1} = \frac{B'B_1}{g'g_1} = \frac{\delta z'_B}{(z'_{v,a}+z'_{v,f})}$$

$$\delta x'_B = \frac{v}{\nabla}\cdot(x'_{v,a}+x'_{v,f}) \quad ...(1)$$

: **Longitudinal translation** of B to B_1

$$\delta z'_B = \frac{v}{\nabla}\cdot(z'_{v,a}+z'_{v,f}) \quad ...(2)$$

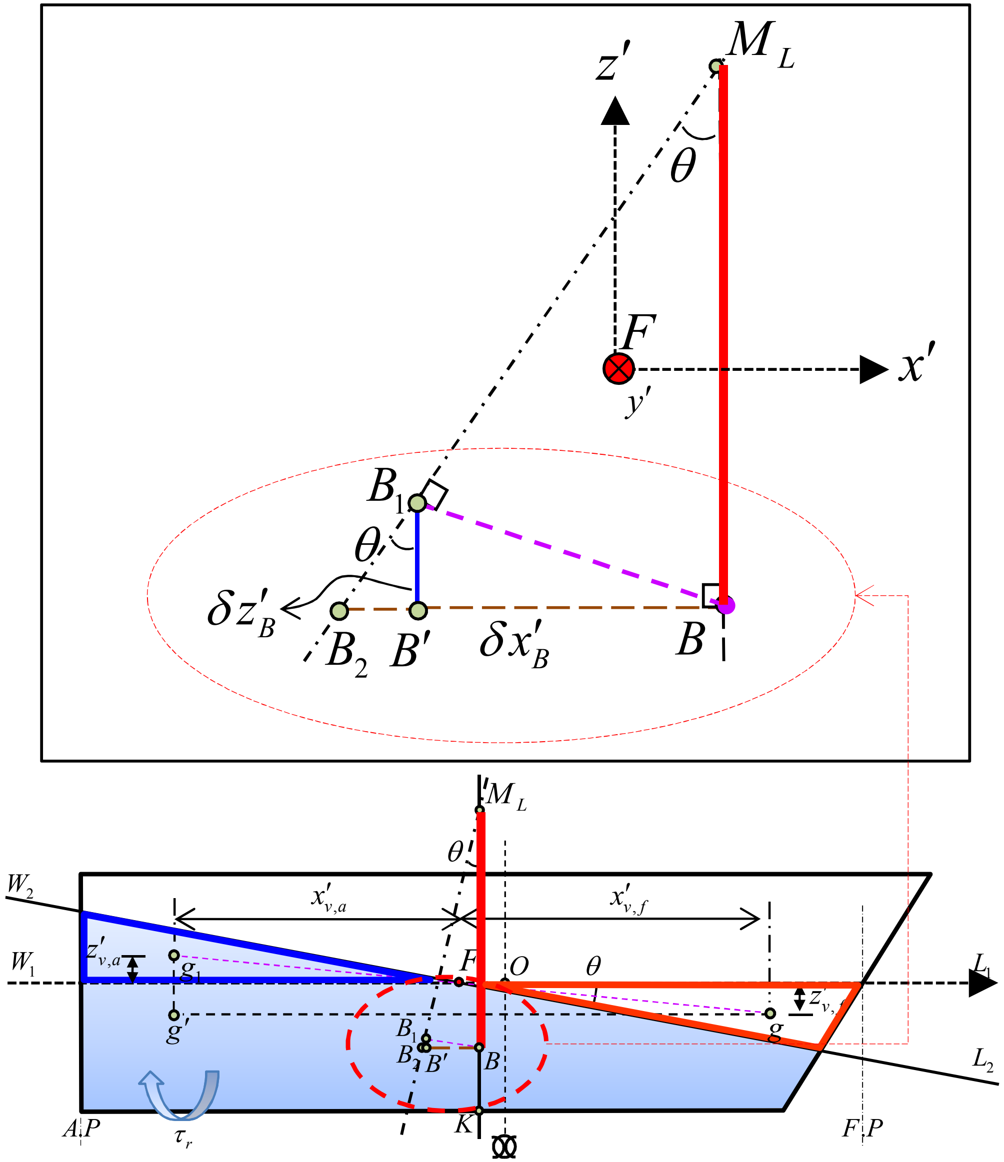
: **Vertical translation** of B to B_1



Derivation of Longitudinal Metacentric Radius (BM_L) (2/8)

$$\delta x'_B = \frac{v}{\nabla}\cdot(x'_{v,a} + x'_{v,f}) \quad ...(1)$$

$$\delta z'_B = \frac{v}{\nabla}\cdot(z'_{v,a} + z'_{v,f}) \quad ...(2)$$



$$BM_L \cdot \tan\theta = BB_2$$

$$BM_L = \frac{BB_2}{\tan\theta}$$

$$BB_2 = BB' + B'B_2$$

$$= \frac{1}{\tan\theta}\left(BB' + B'B_2\right)$$

$$= \frac{1}{\tan\theta}\left(\delta x'_B + \delta z'_B \tan\theta\right)$$

Substituting (1), (2) into $\delta x'_B$ and $\delta z'_B$

$$= \frac{1}{\tan\theta}\left(\frac{v}{\nabla}\cdot(x'_{v,a} + x'_{v,f}) + \frac{v}{\nabla}\cdot(z'_{v,a} + z'_{v,f})\tan\theta\right)$$

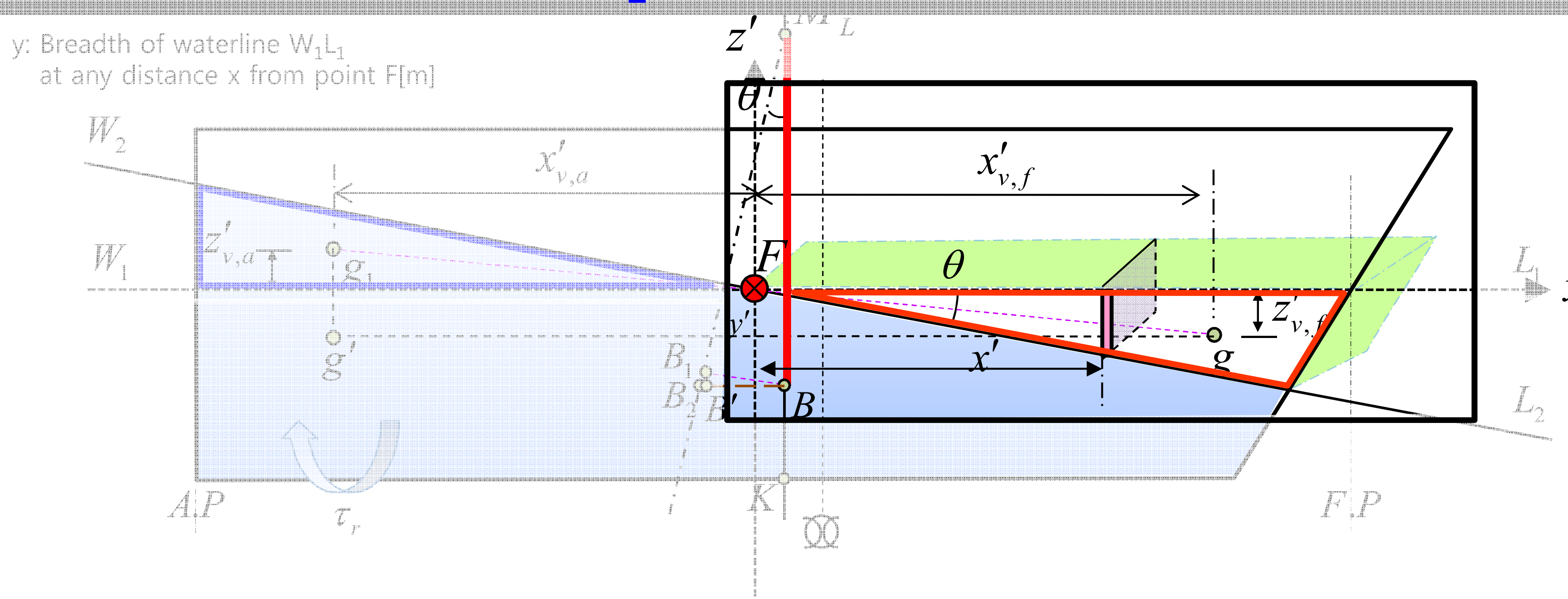
$$BM_L = \frac{1}{\nabla\cdot\tan\theta}\left(v\cdot x'_{v,a} + v\cdot x'_{v,f} + (v\cdot z'_{v,a} + v\cdot z'_{v,f})\tan\theta\right)$$

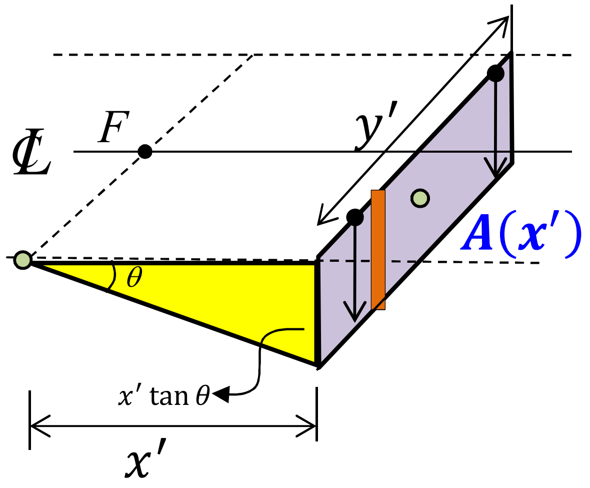
Find!

Derivation of Longitudinal Metacentric Radius (BM_L) (3/8)

$$BM_L = \frac{1}{\nabla \cdot \tan\theta}\left(v \cdot x'_{v,f} + v \cdot x'_{v,a} + \left(v \cdot z'_{v,f} + v \cdot z'_{v,a}\right)\tan\theta\right)$$

(A) (B) (C) (D)





$A(x')$

$= (y') \cdot (x' \cdot \tan\theta)$

$= x' \cdot y' \cdot \tan\theta$

(A) $v \cdot x'_{v,f}$: **Moment** of the emerged volume **about y'-z' plane**

$$= \int_F^{F.P} A(x') \cdot x' \, dx'$$

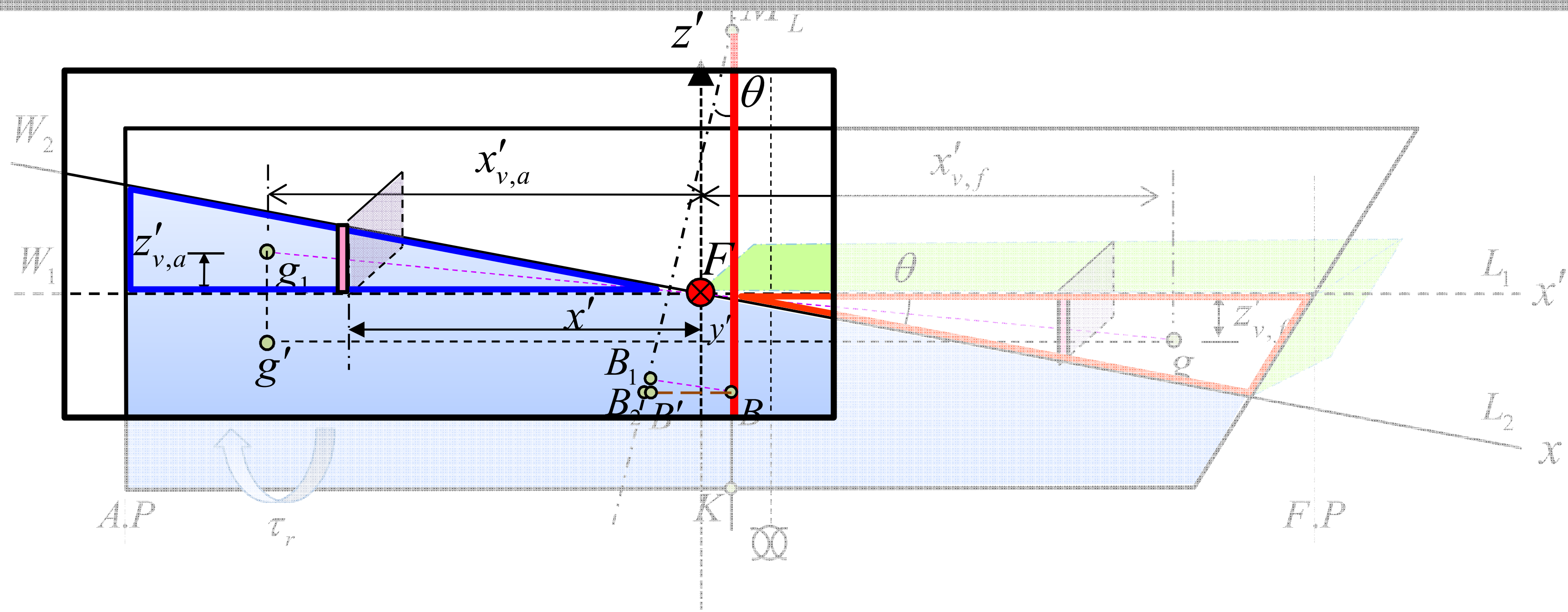
$$= \int_F^{F.P} x' \cdot y' \cdot \tan\theta \cdot x' \, dx'$$

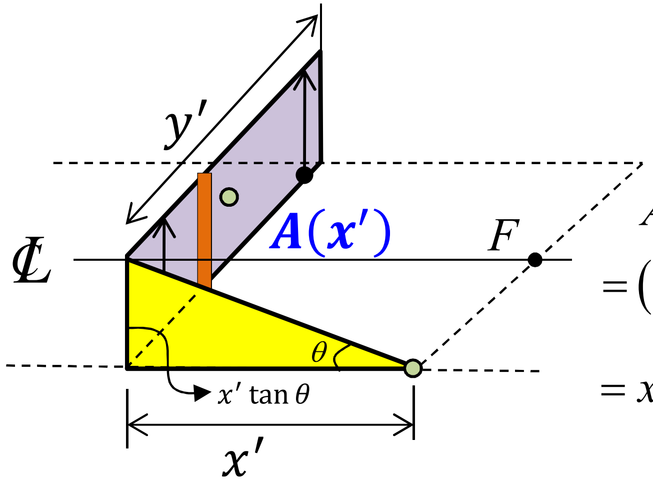
$$= \tan\theta \int_F^{F.P} (x')^2 \cdot y' \, dx'$$

Derivation of Longitudinal Metacentric Radius (BM_L) (4/8)

$$BM_L = \frac{1}{\nabla \cdot \tan\theta}\left(v \cdot x'_{v,f} + v \cdot x'_{v,a} + (v \cdot z'_{v,f} + v \cdot z'_{v,a})\tan\theta\right)$$

(A) (B) (C) (D)





$A(x')$

$= (y') \cdot (x' \cdot \tan\theta)$

$= x' \cdot y' \cdot \tan\theta$

(B) $v \cdot x'_{v,a}$: **Moment** of the submerged volume **about y'-z' plane**

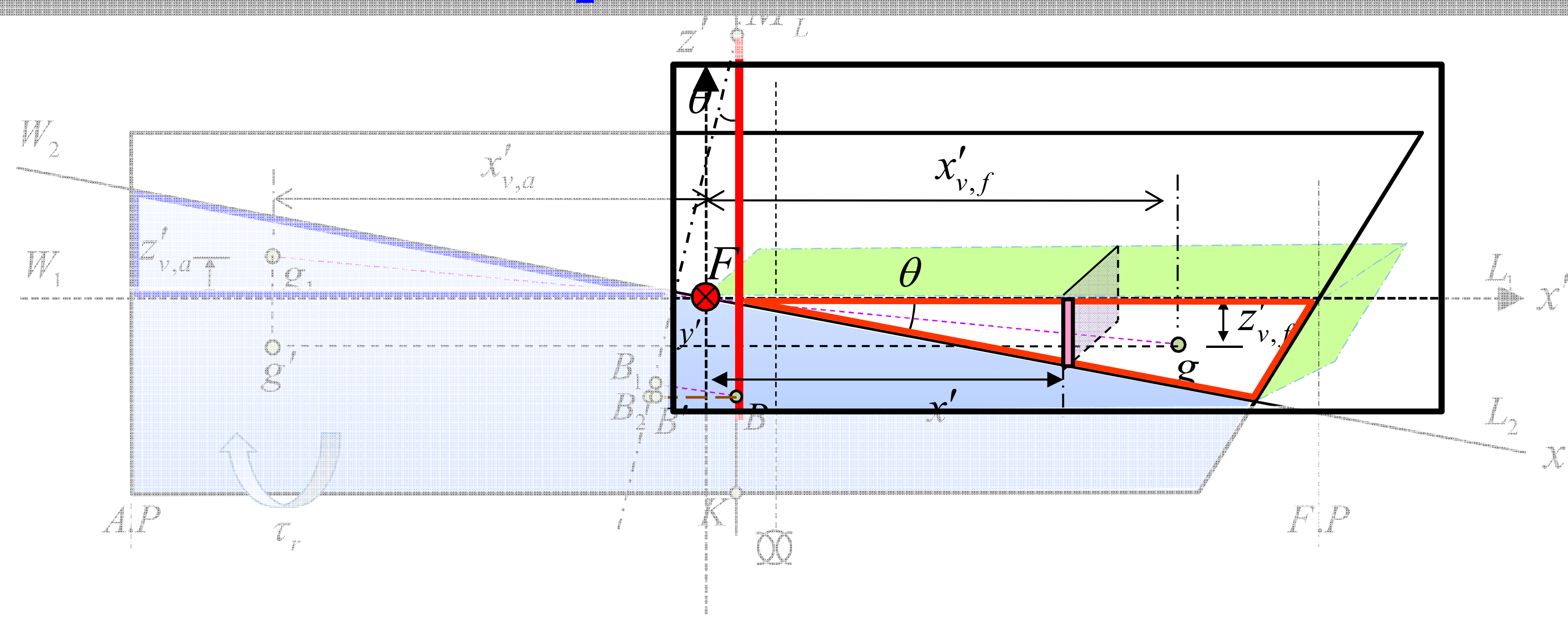
$$= \int_{A.P}^{F} A(x') \cdot x' \, dx'$$

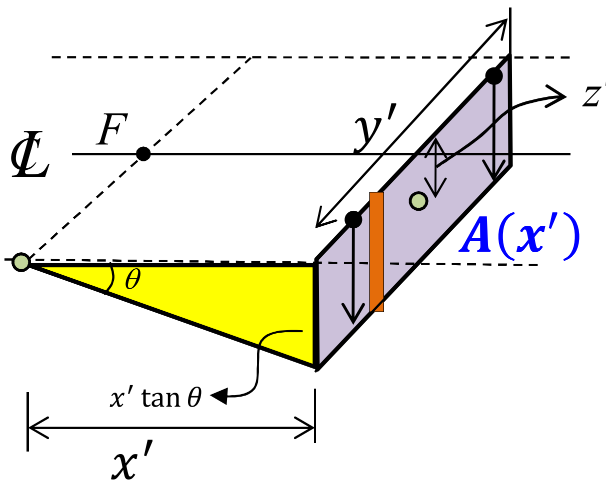
$$= \int_{A.P}^{F} (x' \cdot y' \cdot \tan\theta \cdot x') \, dx'$$

$$= \tan\theta \int_{A.P}^{F} (x')^2 \cdot y' dx'$$

Derivation of Longitudinal Metacentric Radius (BM_L) (5/8)

$$BM_L = \frac{1}{\nabla \cdot \tan\theta}\left(\underbrace{v \cdot x'_{v,f}}_{(A)} + \underbrace{v \cdot x'_{v,a}}_{(B)} + \left(\underbrace{v \cdot z'_{v,f}}_{(C)} + \underbrace{v \cdot z'_{v,a}}_{(D)}\right)\tan\theta\right)$$





$$z' = \frac{1}{2}x' \cdot \tan\theta$$

$$A(x') = (y') \cdot (x' \cdot \tan\theta) = x' \cdot y' \cdot \tan\theta$$

(C) $v \cdot z'_{v,f}$: **Moment** of the emerged volume **about x′-y′ plane**

$$= \int_F^{F.P} A(x') \cdot z' \, dx$$

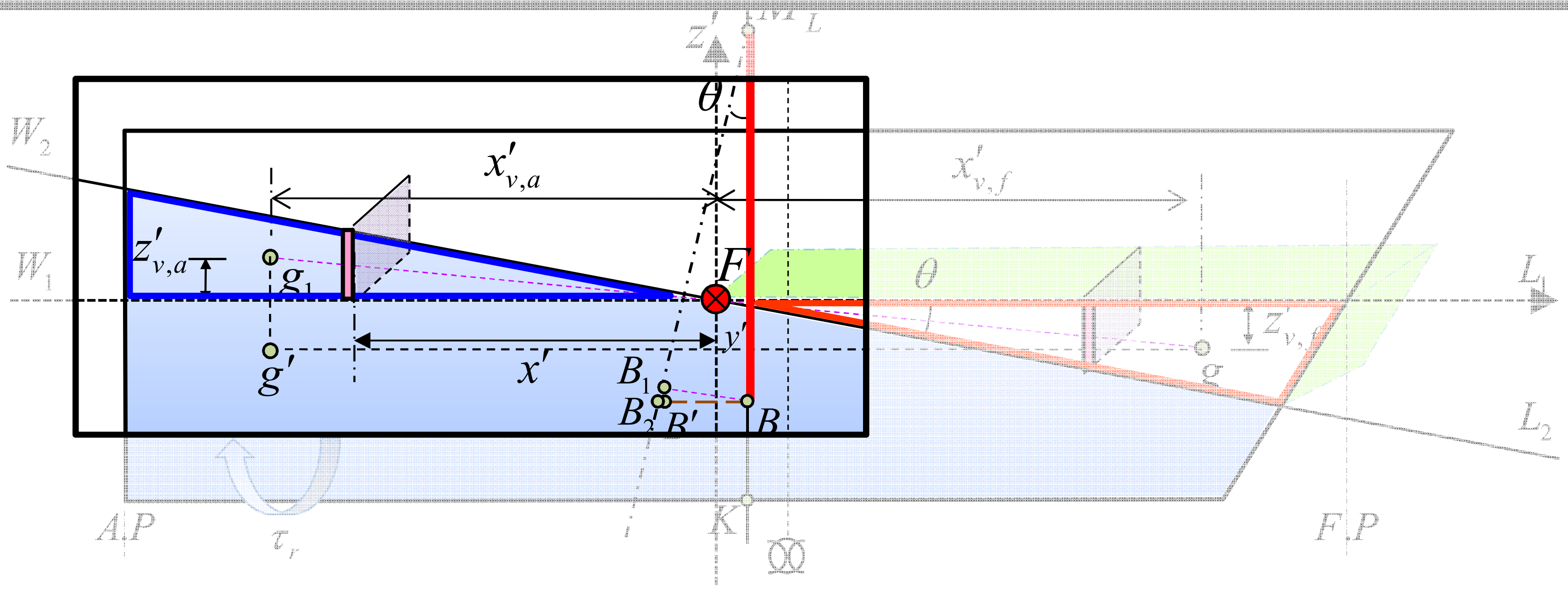
$$= \int_F^{F.P} (x' \cdot y' \cdot \tan\theta)\left(\frac{x'}{2}\tan\theta\right) dx'$$

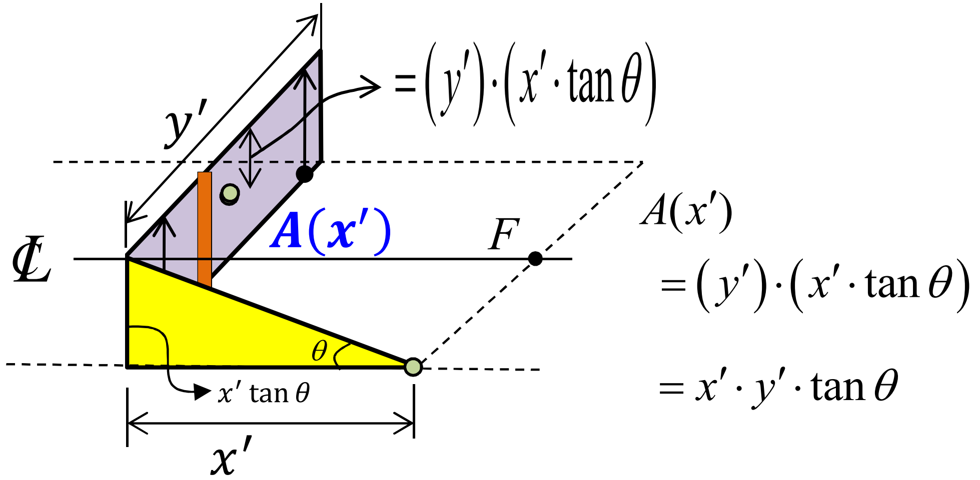
$$= \frac{\tan^2\theta}{2}\int_F^{F.P} (x')^2 \cdot y' \, dx'$$

Derivation of Longitudinal Metacentric Radius (BM_L) (6/8)

$$BM_L = \frac{1}{\nabla \cdot \tan\theta}\left(v \cdot x'_{v,f} + v \cdot x'_{v,a} + \left(v \cdot z'_{v,f} + v \cdot z'_{v,a}\right)\tan\theta\right)$$

(A) (B) (C) (D)





$A(x') = (y')\cdot(x'\cdot\tan\theta) = x'\cdot y'\cdot\tan\theta$

(D) $v \cdot z'_{v,a}$: **Moment** of the submerged volume **about x′-y′ plane**

$$= \int_{A.P}^{F} A(x')\cdot z'\,dx'$$

$$= \int_{A.P}^{F} (x'\cdot y'\cdot\tan\theta)\left(\frac{x'}{2}\tan\theta\right)dx'$$

$$= \frac{\tan^2\theta}{2}\int_{A.P}^{F} (x')^2\cdot y'\,dx'$$

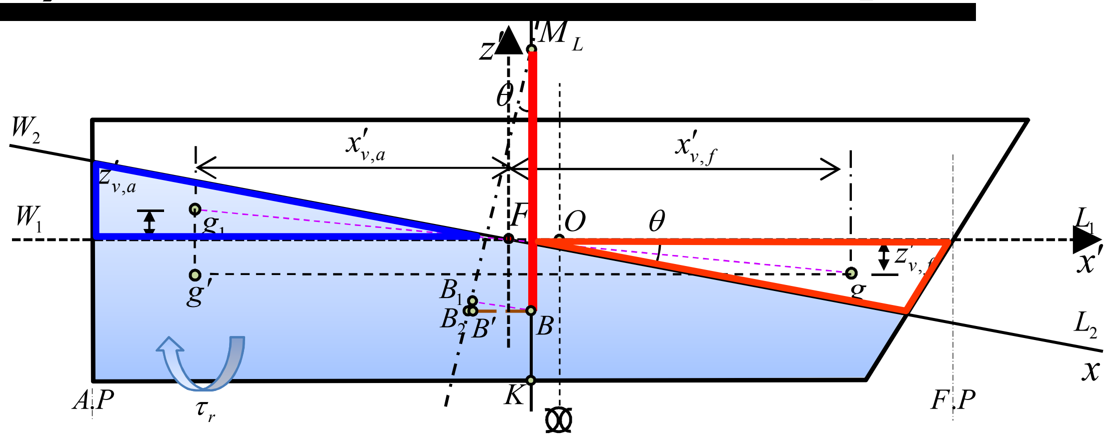
Derivation of Longitudinal Metacentric Radius (BM_L) (7/8)

(A) $v \cdot x'_{v,f} = \tan\theta \int_F^{FP} (x')^2 \cdot y' dx'$

(B) $v \cdot x'_{v,a} = \tan\theta \int_{AP}^{F} (x')^2 \cdot y' dx'$

(C) $v \cdot z'_{v,f} = \dfrac{\tan^2\theta}{2} \int_F^{FP} (x')^2 \cdot y' dx'$

(D) $v \cdot z'_{v,a} = \dfrac{\tan^2\theta}{2} \int_{AP}^{F} (x')^2 \cdot y' dx'$



$$BM_L = \frac{1}{\nabla \cdot \tan\theta}\left(v \cdot x'_{v,f} + v \cdot x'_{v,a} + (v \cdot z'_{v,f} + v \cdot z'_{v,a})\tan\theta\right)$$

(A) (B) (C) (D)

By substituting (A), (B), (C), and (D) into the BM_L equation

$$= \frac{1}{\nabla \cdot \tan\theta}\left(\tan\theta \cdot \int_F^{F.P} (x')^2 \cdot y' dx' + \tan\theta \cdot \int_{A.P}^{F} (x')^2 \cdot y' dx' + \left(\frac{\tan^2\theta}{2}\int_F^{F.P} (x')^2 \cdot y' dx' + \frac{\tan^2\theta}{2}\int_{A.P}^{F} (x')^2 \cdot y' dx'\right)\tan\theta\right)$$

$$= \frac{1}{\nabla \cdot \tan\theta}\left(\tan\theta\left(\int_F^{F.P} (x')^2 \cdot y' dx' + \int_{A.P}^{F} (x')^2 \cdot y' dx'\right) + \frac{\tan^3\theta}{2}\left(\int_F^{F.P} (x')^2 \cdot y' dx' + \int_{A.P}^{F} (x')^2 \cdot y' dx'\right)\right)$$

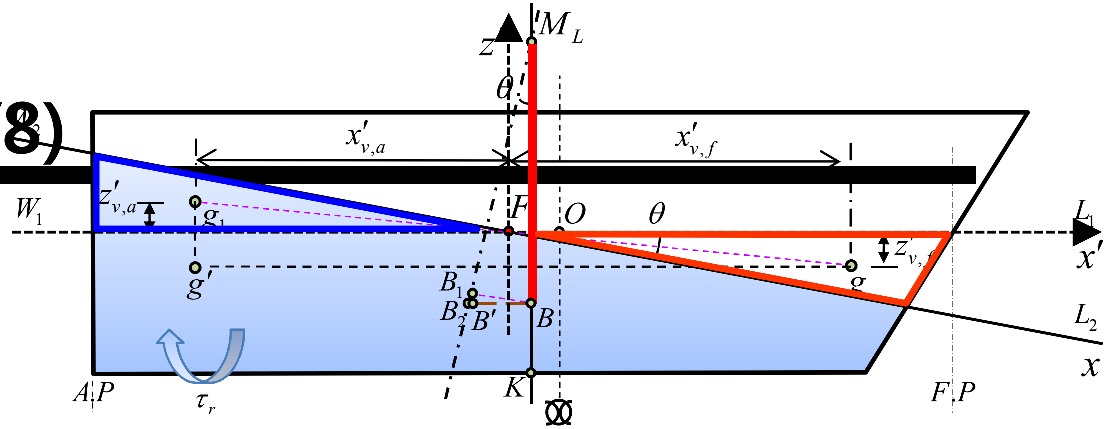
$$,\left(I_L = \int_F^{F.P} (x')^2 \cdot y' dx' + \int_{A.P}^{F} (x')^2 \cdot y' dx'\right)$$

I_L : Moment of inertia of the entire water plane about transverse axis through its centroid F

$$= \frac{1}{\nabla \cdot \tan\theta}\left(\tan\theta \cdot I_L + \frac{1}{2}\tan^3\theta \cdot I_L\right)$$

$$BM_L = \frac{I_L}{\nabla}\left(1 + \frac{1}{2}\tan^2\theta\right)$$

Derivation of Longitudinal Metacentric Radius (BM_L) (8/8)



$$BM_L = \frac{I_L}{\nabla}\left(1 + \frac{1}{2}\tan^2\theta\right)$$

If θ is small, $\tan^2\theta \approx \theta^2 = 0$

$$BM_L = \frac{I_L}{\nabla}$$

↓

which is generally known as BM_L.

The BM_L does not consider the change of center of buoyancy **in vertical direction**.

In order to distinguish between them, the two will be indicated as follows:

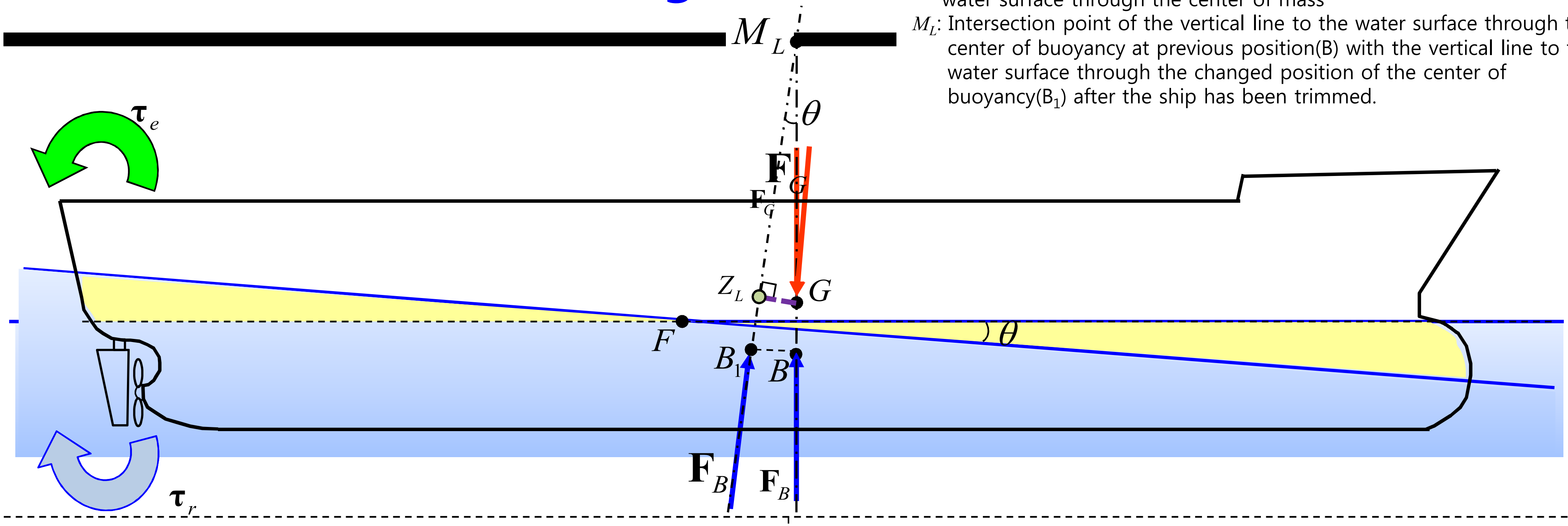
$$BM_{L0} = \frac{I_L}{\nabla}\left(1 + \frac{1}{2}\tan^2\theta\right)$$ (**Considering** the change of center of buoyancy in vertical direction)

$$BM_L = \frac{I_L}{\nabla}$$ (**Without considering** the change of center of buoyancy in vertical direction)

Moment to Trim One Degree and Moment to Trim One Centimeter (MTC)

Moment to Trim One Degree

LCF: Longitudinal center of floatation
Z_L: Intersection point of the vertical line to the water surface through the changed center of buoyancy with the horizontal line parallel to the water surface through the center of mass
M_L: Intersection point of the vertical line to the water surface through the center of buoyancy at previous position(B) with the vertical line to the water surface through the changed position of the center of buoyancy(B_1) after the ship has been trimmed.



Definition

It is **the moment of external forces** required to produce **one degree trim**.

Assumption

Small trim where the metacenter is not changed (about 2~5°)

When the ship is trimmed, the static equilibrium states:

Moment to trim = Righting moment

$$= F_B \cdot GZ_L$$

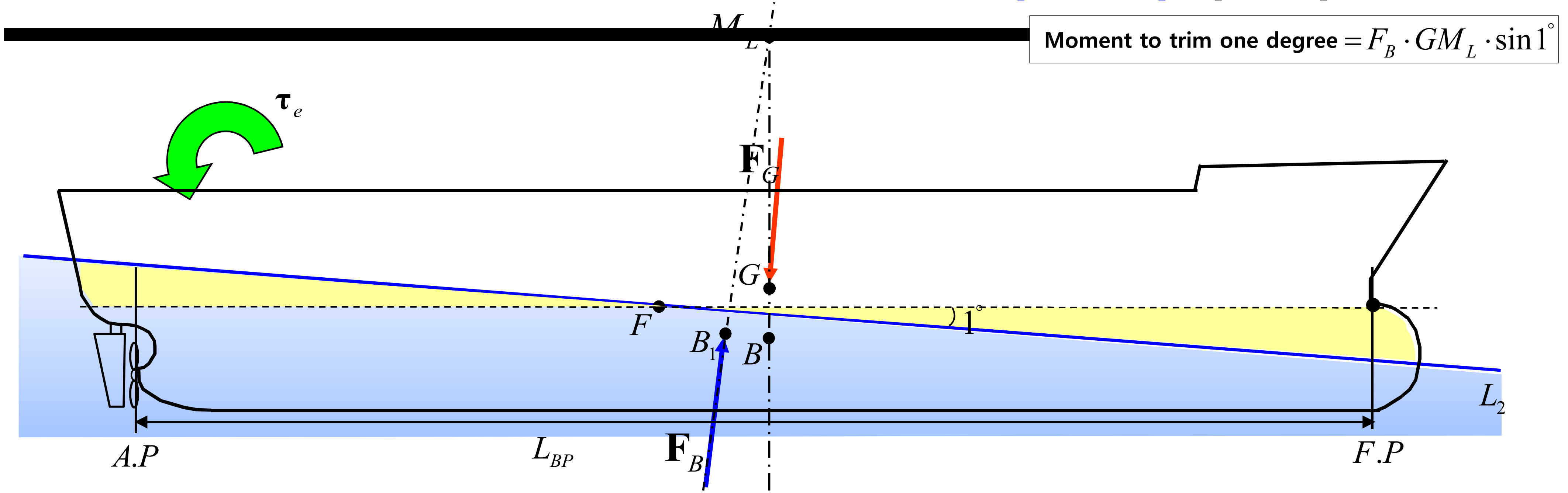
$$\text{(at small angle } \theta) \quad \simeq F_B \cdot GM_L \cdot \sin\theta$$

Substituting $\theta = 1^\circ$

$$\text{Moment to trim one degree} = F_B \cdot GM_L \cdot \sin 1^\circ$$

Moment to Trim One Centimeter (MTC) (1/2)

Moment to trim one degree $= F_B \cdot GM_L \cdot \sin 1^\circ$

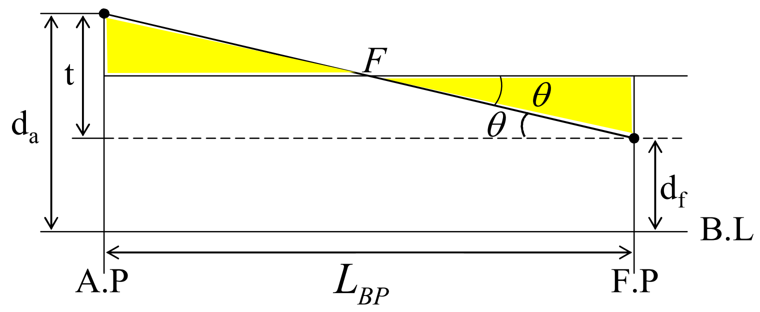


Often, we are more interested in the **changes in draft** produced by a trim moment than the changes in trim angle.

$$MTC = F_B \cdot GM_L \cdot \frac{1}{L_{BP} \cdot 100}$$

MTC: Moment to trim 1 cm

L_{BP}: Length between perpendiculars [m]



Trim(t): $d_a - d_f$,

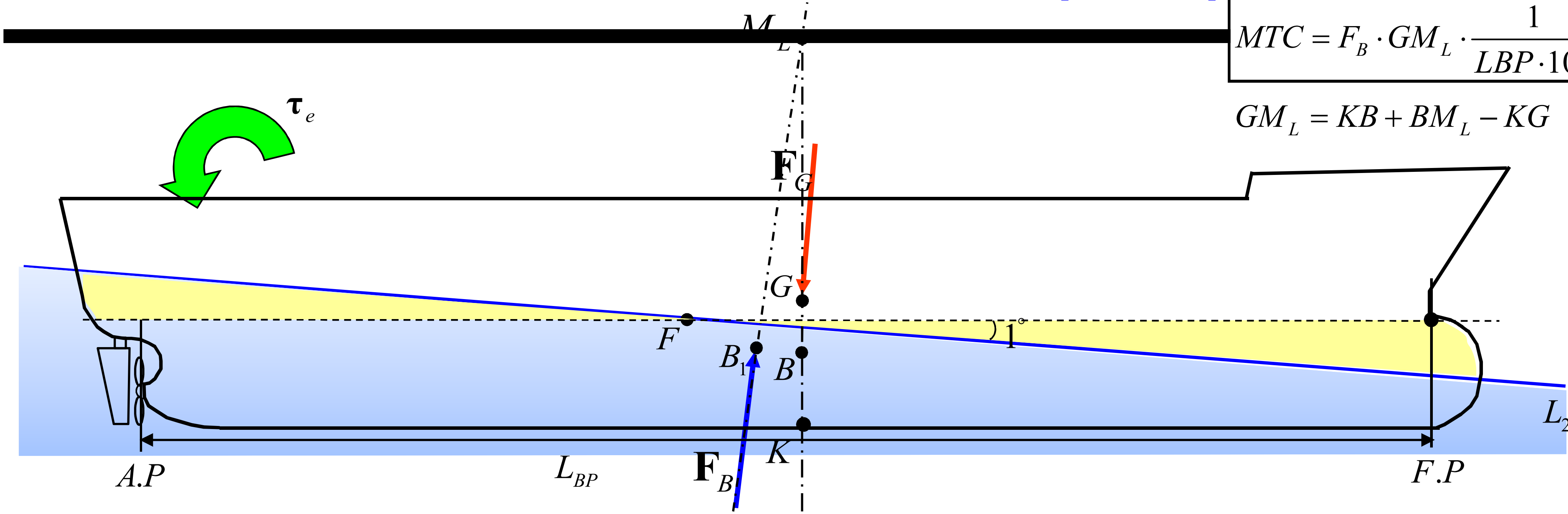
d_a: Draft after

d_f: Draft forward

$$\sin\theta \approx \tan\theta = \frac{t}{L_{BP}}$$

(*If* θ *is small*)

Moment to Trim One Centimeter (MTC) (2/2)



$$MTC = F_B \cdot GM_L \cdot \frac{1}{LBP \cdot 100}$$

$$GM_L = KB + BM_L - KG$$

As practical matter, KB and KG are usually so small compared to GM_L that **BM_L can be substituted for GM_L**.

$$MTC = F_B \cdot GM_L \cdot \frac{1}{L_{BP} \cdot 100} \approx F_B \cdot BM_L \cdot \frac{1}{L_{BP} \cdot 100}$$

Substituting $F_B = \rho \cdot \nabla,\ BM_L = \frac{I_L}{\nabla}$

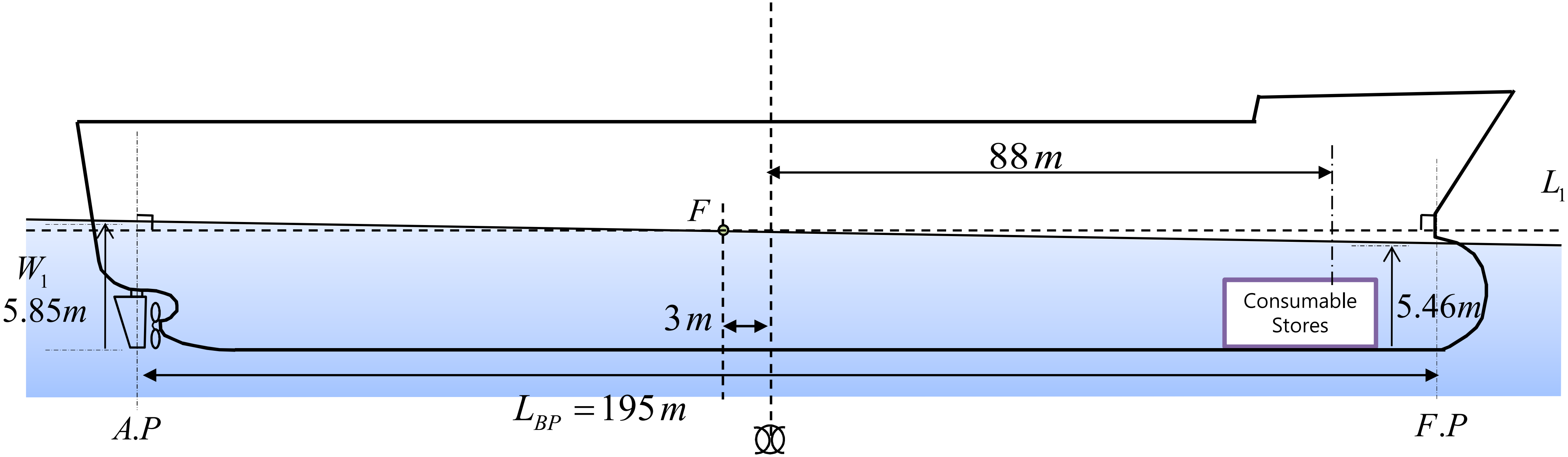
I_L: Moment of inertia of the water plane area about y' axis

$$MTC = \rho \cdot \nabla \cdot \frac{I_L}{\nabla} \cdot \frac{1}{L_{BP} \cdot 100}$$

$$= \frac{\rho \cdot I_L}{L_{BP} \cdot 100}$$

Example

Example) Calculation of Draft Change Due to Fuel Consumption (1/4)



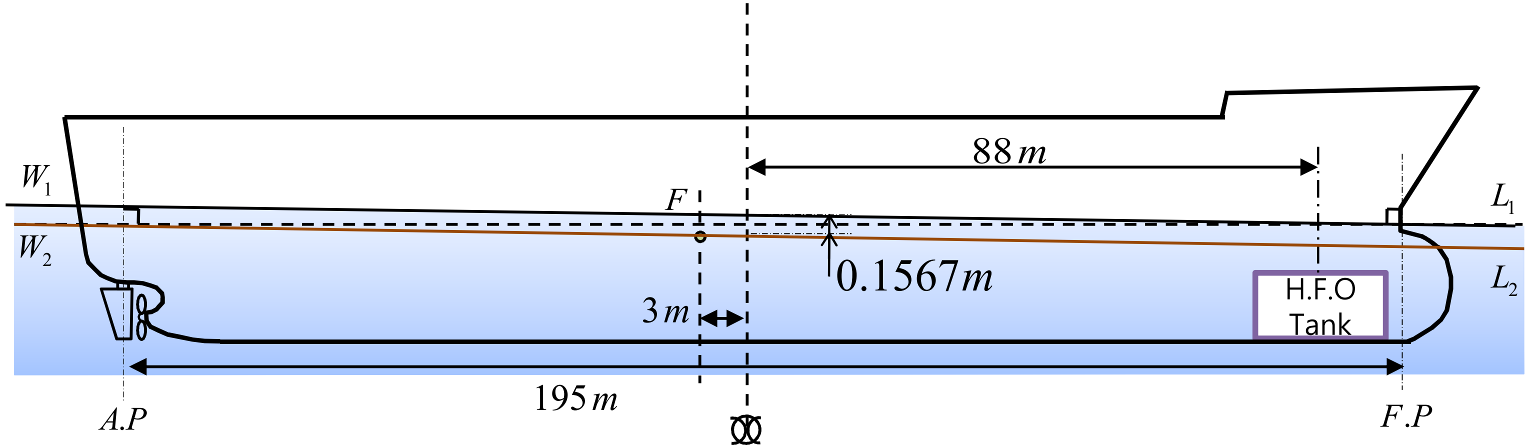
During a voyage, a cargo ship uses up 320 tones of consumable stores (H.F.O: Heavy Fuel Oil), located 88 m forward of the midships.

Before the voyage, the forward draft marks at forward perpendicular recorded 5.46 m, and the after marks at the after perpendicular, recorded 5.85 m.

At the mean draft between forward and after perpendicular, the hydrostatic data show the ship to have LCF after of midship = 3 m, Breadth = 10.47 m, moment of inertia of the water plane area about transverse axis through point F = 6,469,478 m^4, Cwp = 0.8.

Calculate the draft mark the readings at the end of the voyage, assuming that there is no change in water density(ρ=1.0 ton/m^3).

Example) Calculation of Draft Change Due to Fuel Consumption (2/4)



① Calculation of parallel rise (draft change)

$$A_{WP} = C_{WP} \cdot L \cdot B = 0.8 \cdot 195 \cdot 10.47 = 1,633.3[m^2]$$

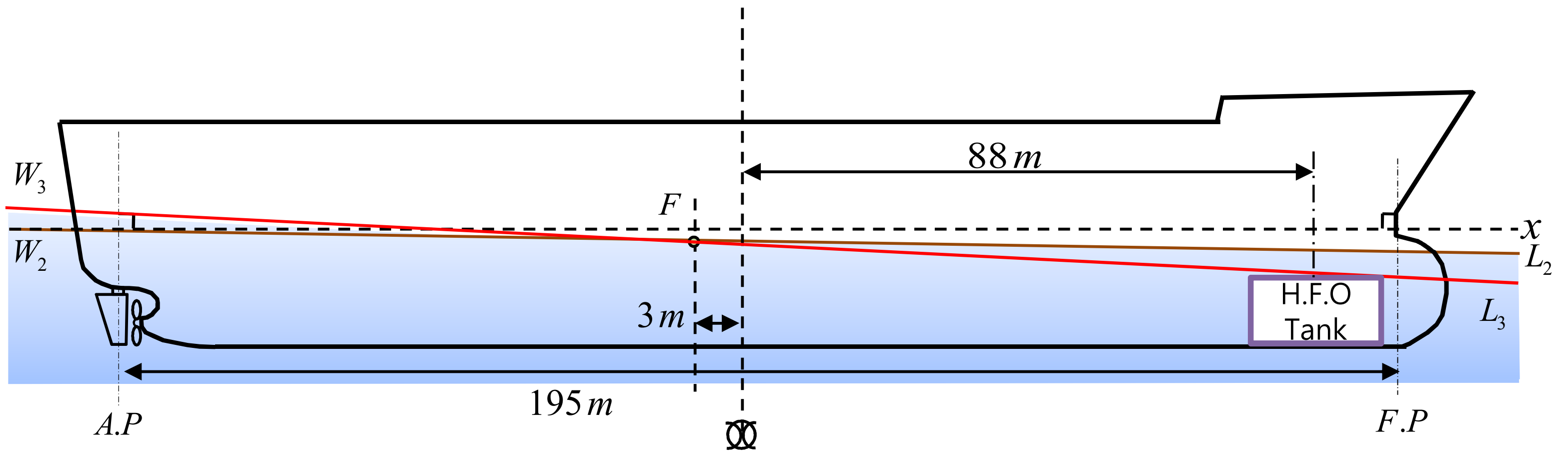
- Tones per 1 cm immersion (TPC)

$$:TPC = \rho \cdot A_{WP} \cdot \frac{1}{100} = 1[ton/m^3] \cdot 1,633.3[m^2] \cdot \frac{1}{100[cm/m]} = 20.4165[ton/cm]$$

- Parallel rise

$$:\delta d = \frac{weight}{TPC} = \frac{320[ton]}{20.4165[ton/cm]} = 15.6736[cm] = 0.1567[m]$$

Example) Calculation of Draft Change Due to Fuel Consumption (3/4)



② Calculation of trim

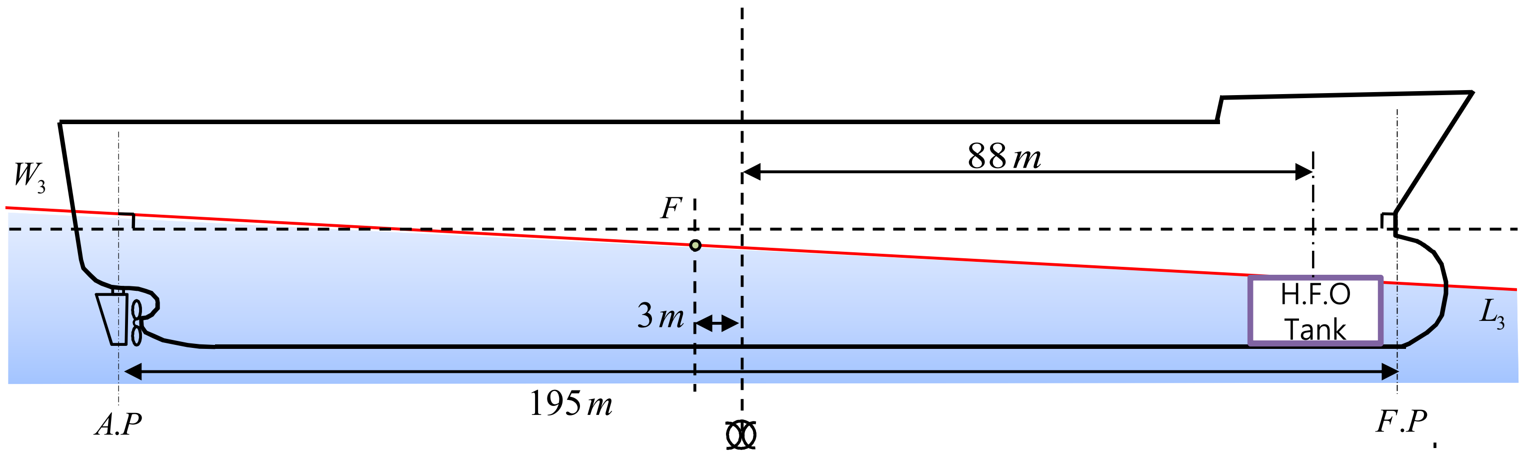
- Trim moment $: \tau_{trim} = 320[ton]\cdot 88[m] = 28{,}160[ton\cdot m]$
- Moment to trim 1 cm (MTC)

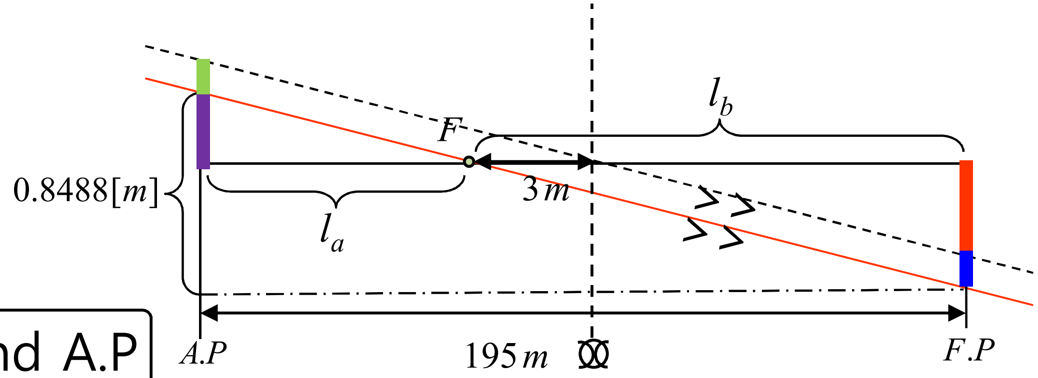
$$: MTC = \frac{\rho \cdot I_L}{100\cdot L_{BP}} = \frac{1[ton/m^3]}{100[cm/m]\cdot 195[m]}\cdot 6{,}469{,}478[m^4] = 331.7949[ton\cdot m/cm]$$

- Trim

$$: Trim = \frac{\tau_{trim}}{MTC} = \frac{28{,}160[ton\cdot m]}{331.7949[ton\cdot m/cm]} = 84.8785[cm] = 0.8488[m]$$

Example) Calculation of Draft Change Due to Fuel Consumption (4/4)





(195 : 0.8488 = l_a : ?)
(195 : 0.8488 = l_b : ?)

③ Calculation of changed draft at F.P and A.P

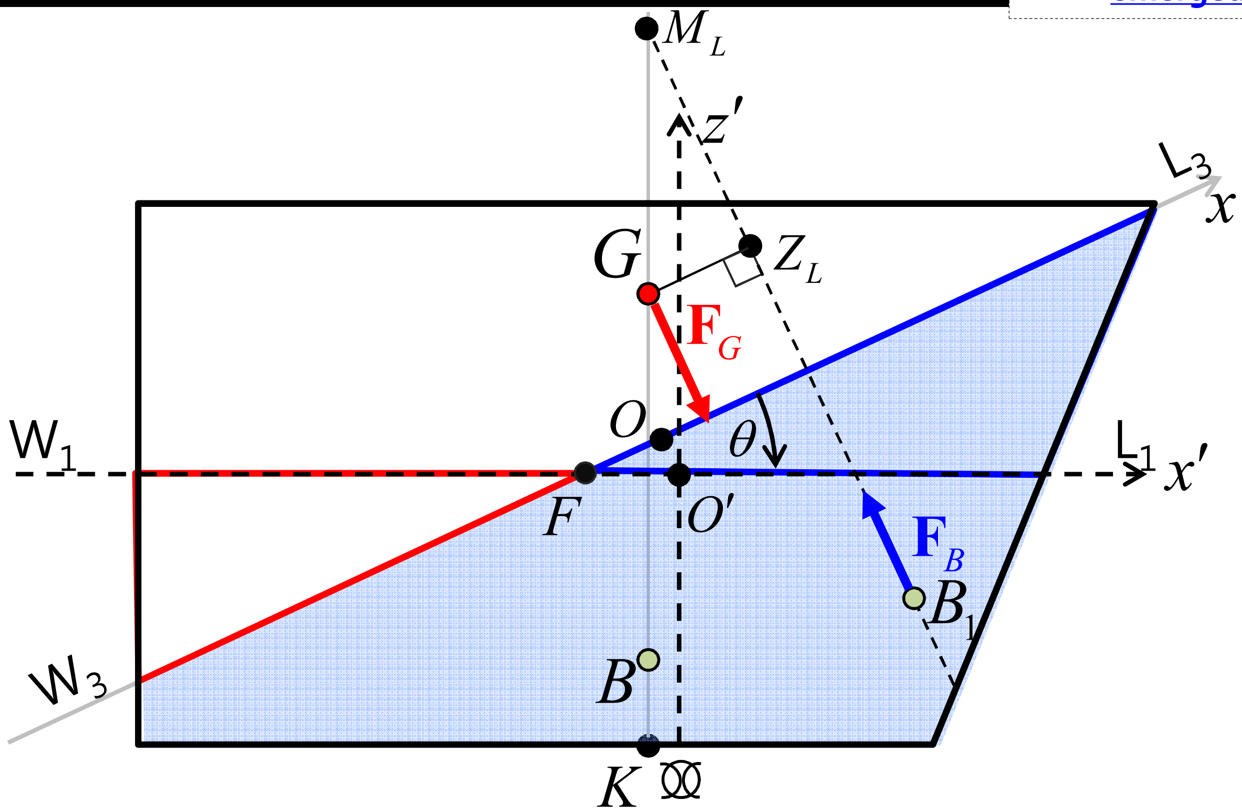
- Draft change at F.P due to trim $= -\dfrac{195/2+3}{195} \times 0.8488 = -0.4375[m]$
- Draft change at A.P due to trim $= \dfrac{195/2-3}{195} \times 0.8488 = 0.4113[m]$
- Changed Draft at F.P : draft – parallel rise - draft change due to trim
  $= 5.46[m] - 0.1567[m] - 0.4375[m] \quad = 4.8658[m]$
- Changed Draft at A.P : draft – parallel rise + draft change due to trim
  $= 5.85[m] - 0.1567[m] + 0.4113[m] \quad = 6.1046[m]$

Reference Slides

Derivation of Longitudinal Metacentric Radius (BM_L) by Using the Origin of the Body Fixed Frame

Derivation of BM_L (1/12)

Assumption
① **A small trim angle** (3°~5°)
② The **submerged volume** and the **emerged volume** are to be the **same**.



Let us derive longitudinal metacentric radius "BM_L".

Assumption

1. A main deck is not submerged.

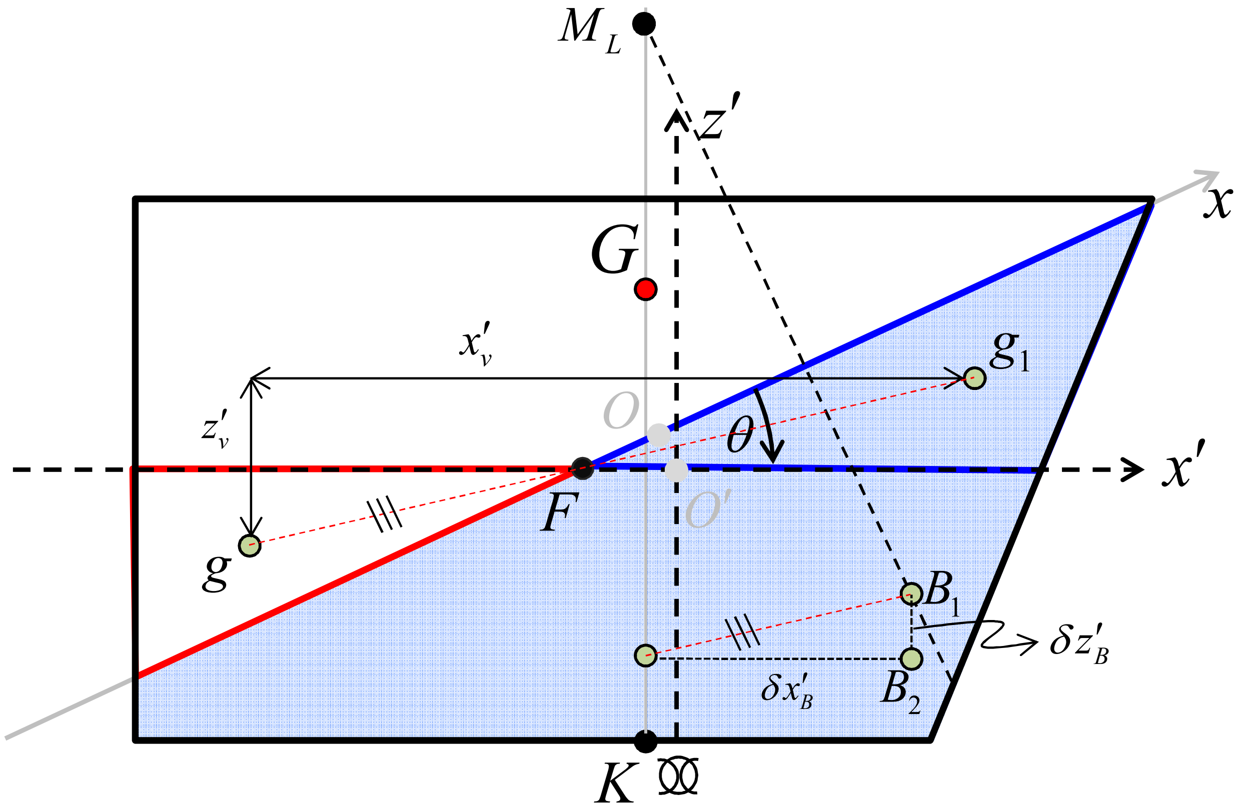
2. Small angle of inclination (3°~5° for trim)

※ The ship is not symmetrical with respect to midship section. Thus to keep the same displaced volume, the axis of inclination does not stand still. In small angle of inclination, the axis of inclination passes through the point "F" (longitudinal center of floatation).

Derivation of BM_L (2/12)

Assumption
① **A small trim angle** (3°~5°)
② The **submerged volume** and the **emerged volume** are to be the **same**.

∇: Displacement volume
v: Submerged / Emerged volume
B: The center of buoyancy before inclination
B_1: The center of buoyancy after inclination
g: The center of the emerged volume
g_1: The center of the submerged volume



Translation of the center of buoyancy caused by the movement of the small volume v

$$\begin{cases} \delta x'_B = (x'_v \cdot v) / \nabla \quad ...(1) \\ \delta z_B = (z'_v \cdot v) / \nabla \quad ...(2) \end{cases}$$

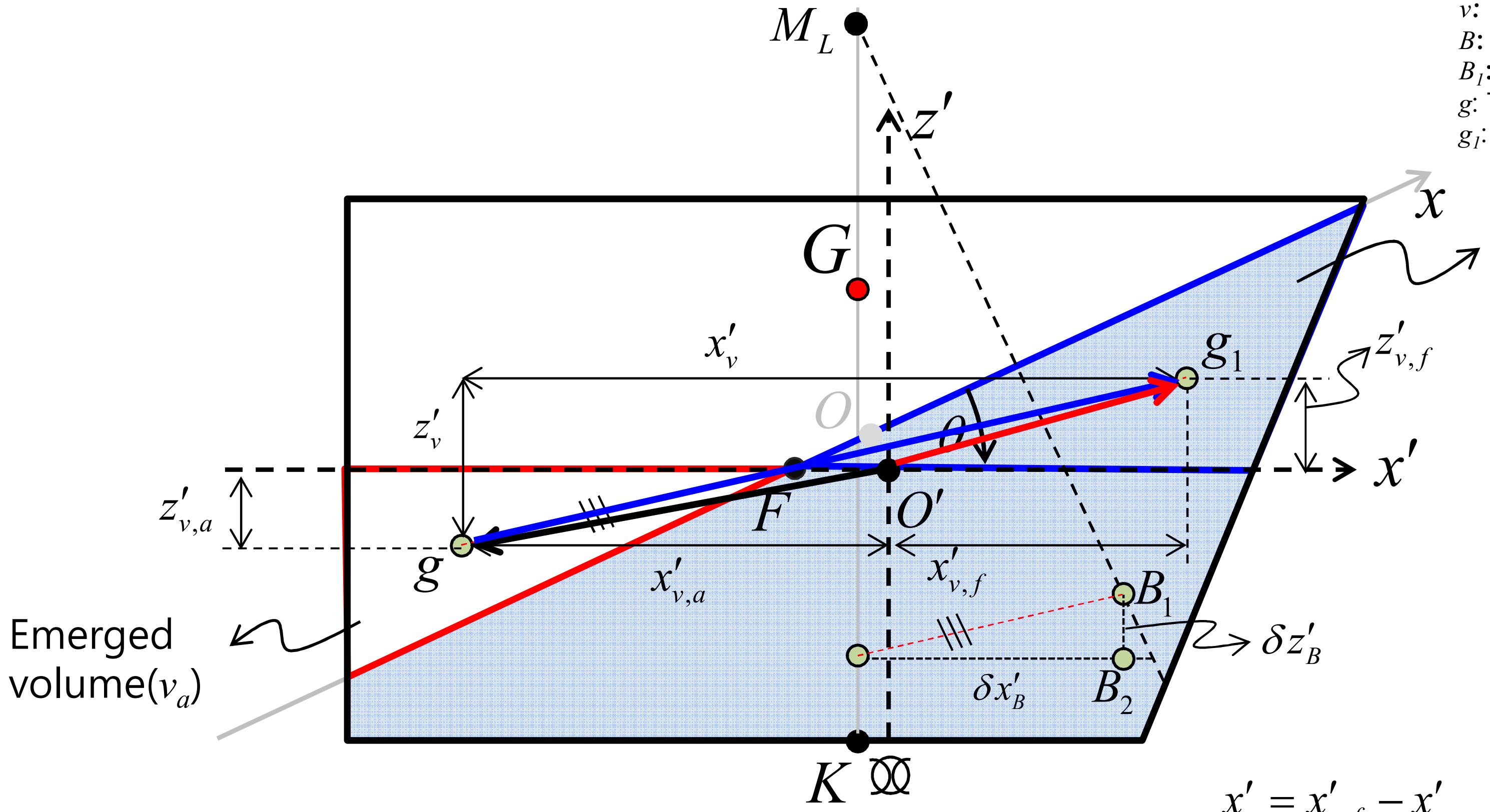
, where v is the each volume of the submerged and emerged volume.

∇ is total volume of the ship.

Derivation of BM_L (3/12)

Assumption
① **A small trim angle** (3°~5°)
② The **submerged volume** and the **emerged volume** are to be the **same**.

∇: Displacement volume
v: Submerged / Emerged volume
B: The center of buoyancy before inclination
B_1: The center of buoyancy after inclination
g: The center of the emerged volume
g_1: The center of the submerged volume



$$\overrightarrow{O'g} + \overrightarrow{gg_1} = \overrightarrow{O'g_1}$$

$$\overrightarrow{gg_1} = \overrightarrow{O'g_1} - \overrightarrow{O'g}$$

$$(x'_g, z'_g) = (x'_{v,f}, z'_{v,f}) - (x'_{v,a}, z'_{v,a})$$

$$x'_v = x'_{v,f} - x'_{v,a} \quad \text{(3)}$$

$$z'_v = z'_{v,f} - z'_{v,a} \quad \text{(4)}$$

Translation of the center of buoyancy caused by the movement of the small volume v

$$\begin{cases} \delta x'_B = (x'_v \cdot v)/\nabla \quad ...(1) \\ \delta z_B = (z'_v \cdot v)/\nabla \quad ...(2) \end{cases}$$

, where v is the each volume of the submerged and emerged volume.

∇ is total volume of the ship.

Substituting Eq. (3), (4) into the Eq. (1), (2), respectively.

$$\begin{cases} \delta x'_B = (x'_{v,f} - x'_{v,a}) \cdot v/\nabla \\ \delta z'_B = (z'_{v,f} - z'_{v,a}) \cdot v/\nabla \end{cases}$$

⇩ ⟵ $v = v_f, -v = v_a$

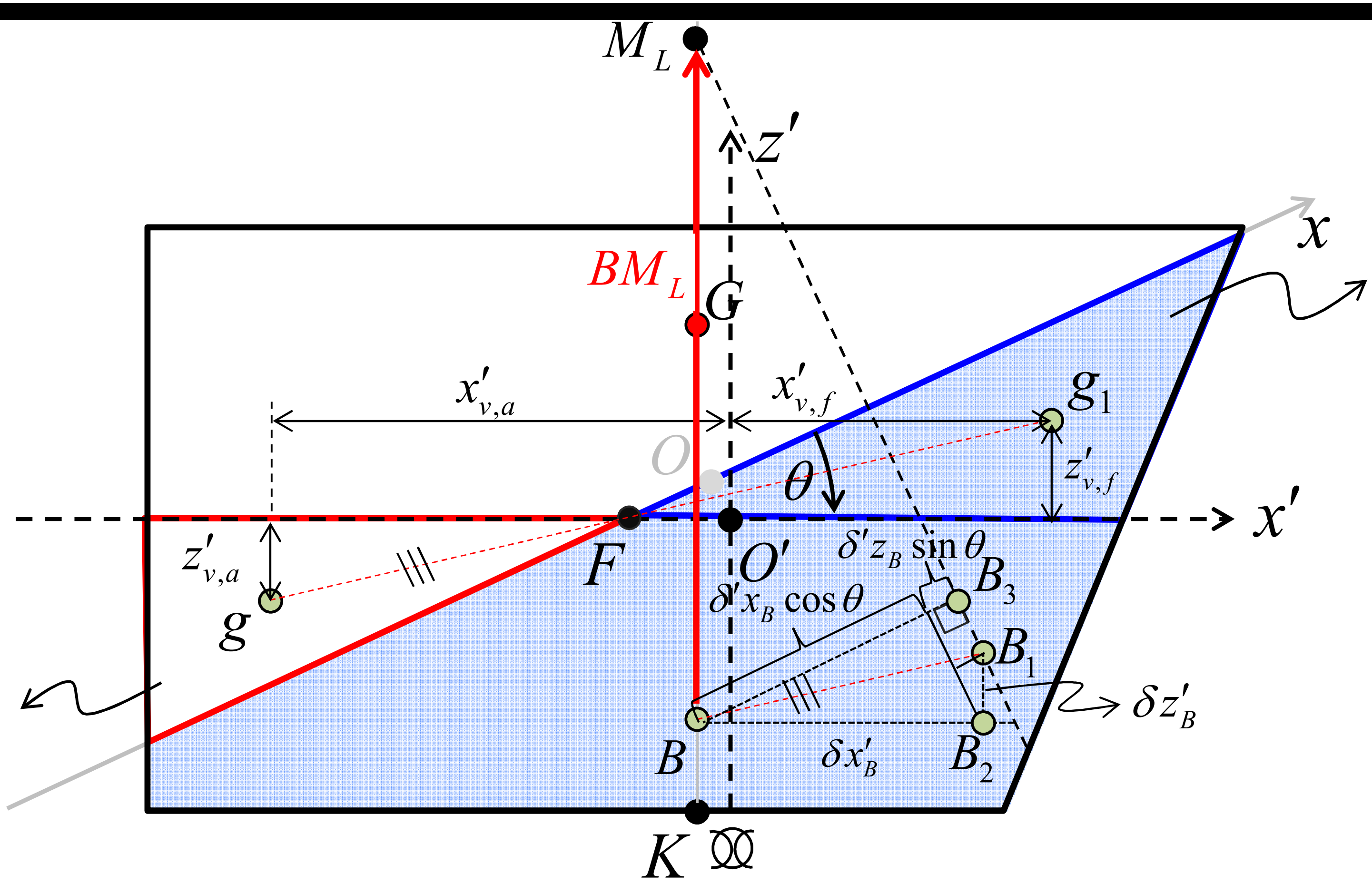
$$\begin{cases} \delta x'_B = (x'_{v,f} \cdot v_f + x'_{v,a} \cdot v_a)/\nabla \\ \delta z'_B = (z'_{v,f} \cdot v_f + z'_{v,a} \cdot v_a)/\nabla \end{cases}$$

Derivation of BM_L (4/12)

$$\begin{cases} \delta x'_B = (x'_{v,a} \cdot v_a + x'_{v,f} \cdot v_f) / \nabla \ldots (1) \\ \delta z'_B = (z'_{v,a} \cdot v_a + z'_{v,f} \cdot v_f) / \nabla \ \ldots (2) \end{cases}$$

(1): Moment about the y_t axis through the point F due to the force in z' direction

(2): Moment about the y_t axis through the point F due to the force in x' direction



$$BM_L \sin\theta = BB_3$$

$$BM_L = \frac{BB_3}{\sin\theta}$$

$$= \frac{1}{\sin\theta}(\delta' x_B \cos\theta + \delta' z_B \sin\theta)$$

$$= \frac{\cos\theta}{\sin\theta}\left(\delta' x_B + \delta' z_B \frac{\sin\theta}{\cos\theta}\right)$$

$$= \frac{1}{\tan\theta}(\delta' x_B + \delta' z_B \tan\theta)$$

$$BM_L = \frac{1}{\tan\theta}(\delta x'_B + \delta z'_B \tan\theta)$$

Substituting (1), (2) into $\delta x'_B, \delta z'_B$

$$= \frac{1}{\tan\theta}\left(\frac{1}{\nabla}\cdot(x'_{v,a} \cdot v_a + x'_{v,f} \cdot v_f) + \frac{1}{\nabla}\cdot(z'_{v,a} \cdot v_a + z'_{v,f} \cdot v_f)\tan\theta\right)$$

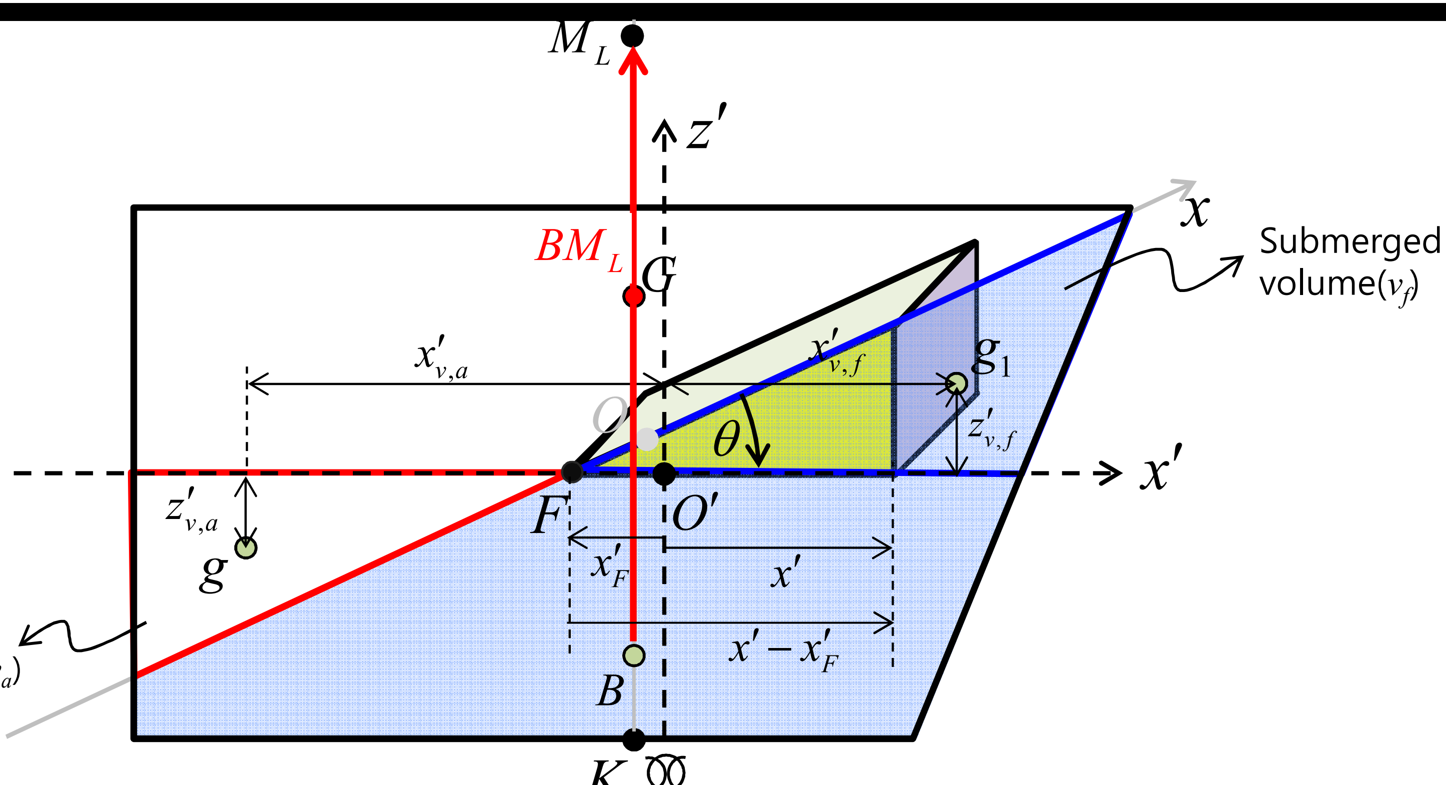
$$BM_L = \frac{1}{\nabla \cdot \tan\theta}\left(x'_{v,a} \cdot v_a + x'_{v,f} \cdot v_f + (z'_{v,a} \cdot v_a + z'_{v,f} \cdot v_f)\tan\theta\right)$$

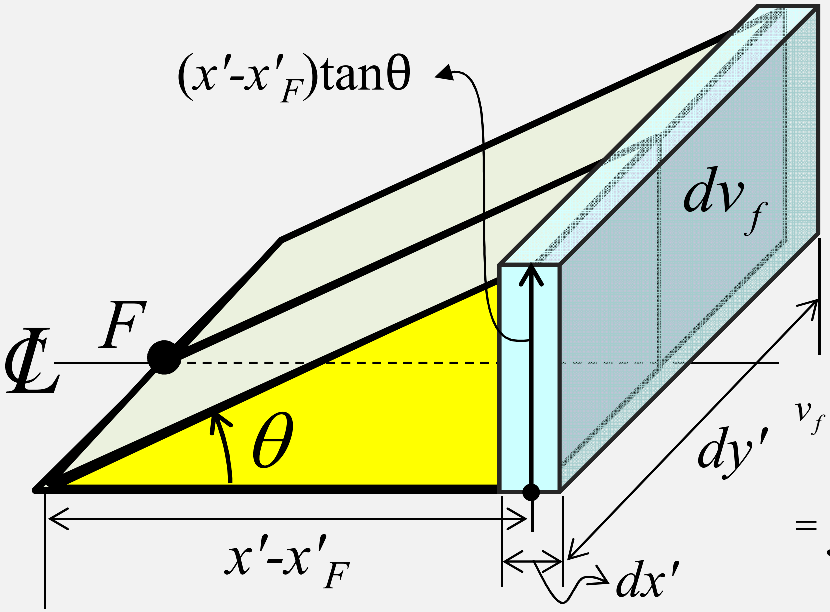
Find!

Derivation of BM_L (5/12)

$$BM_L = \frac{1}{\nabla \cdot \tan\theta}\left(x'_{v,f}\cdot v_f + x'_{v,a}\cdot v_a + (z'_{v,f}\cdot v_f + z'_{v,a}\cdot v_a)\tan\theta\right)$$

(A) (B) (C) (D)





$$dv_f = (x'-x'_F)\cdot\tan\theta\cdot dy'\cdot dx'$$

$$v_f = \int_F^{fore}\int_{star}^{port} dv_f$$

$$= \int_F^{fore}\int_{star}^{port}(x'-x'_F)\cdot\tan\theta\cdot dy'\cdot dx'$$

(A) $x'_{v,f}\cdot v_f$: **Moment about transverse axis through point O** of the submerged volume

$$= \int_F^{fore}\int_{star}^{port} x'\cdot dv_f$$

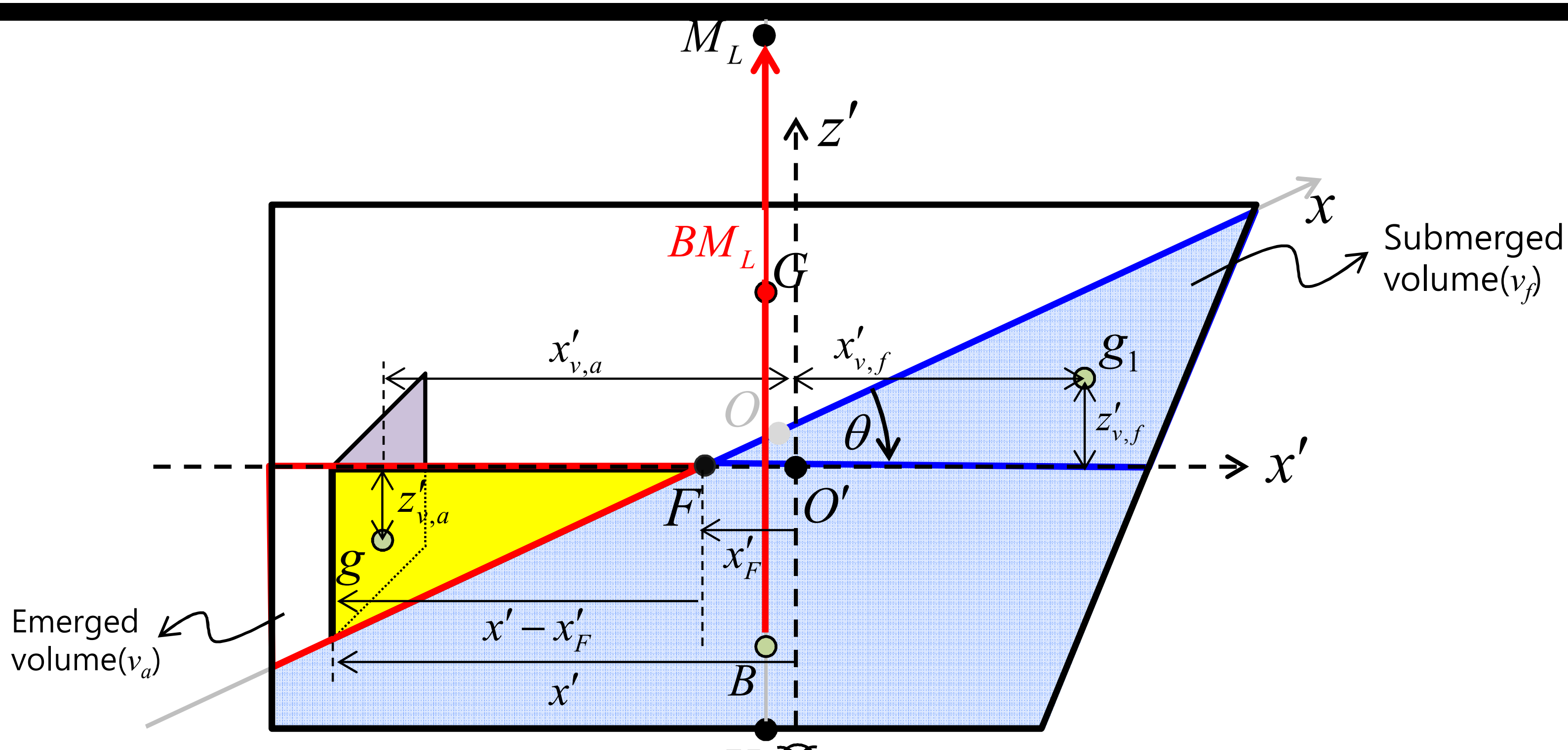
$$= \int_F^{fore}\int_{star}^{port} x'\cdot(x'-x'_F)\cdot\tan\theta\cdot dy'\cdot dx'$$

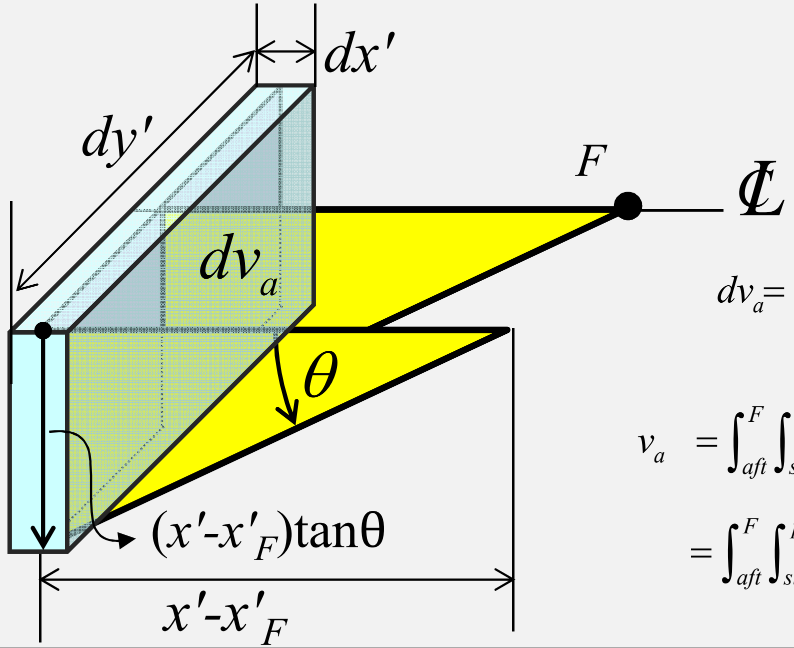
$$= \tan\theta\cdot\int_F^{fore}\int_{star}^{port} x'\cdot(x'-x'_F)\cdot dy'\cdot dx'$$

Derivation of BM_L (6/12)

$$BM_L = \frac{1}{\nabla \cdot \tan\theta}\left(x'_{v,f} \cdot v_f + x'_{v,a} \cdot v_a + (z'_{v,f} \cdot v_f + z'_{v,a} \cdot v_a)\tan\theta \right)$$

(A) (B) (C) (D)





$$dv_a = (x' - x'_F) \cdot \tan\theta \cdot dy' \cdot dx'$$

$$v_a = \int_{aft}^{F}\int_{star}^{port} dv_a$$

$$= \int_{aft}^{F}\int_{star}^{port} (x' - x'_F) \cdot \tan\theta \cdot dy' \cdot dx'$$

(B) $x'_{v,a} \cdot v_a$: **Moment about transverse axis through point O** of the emerged volume

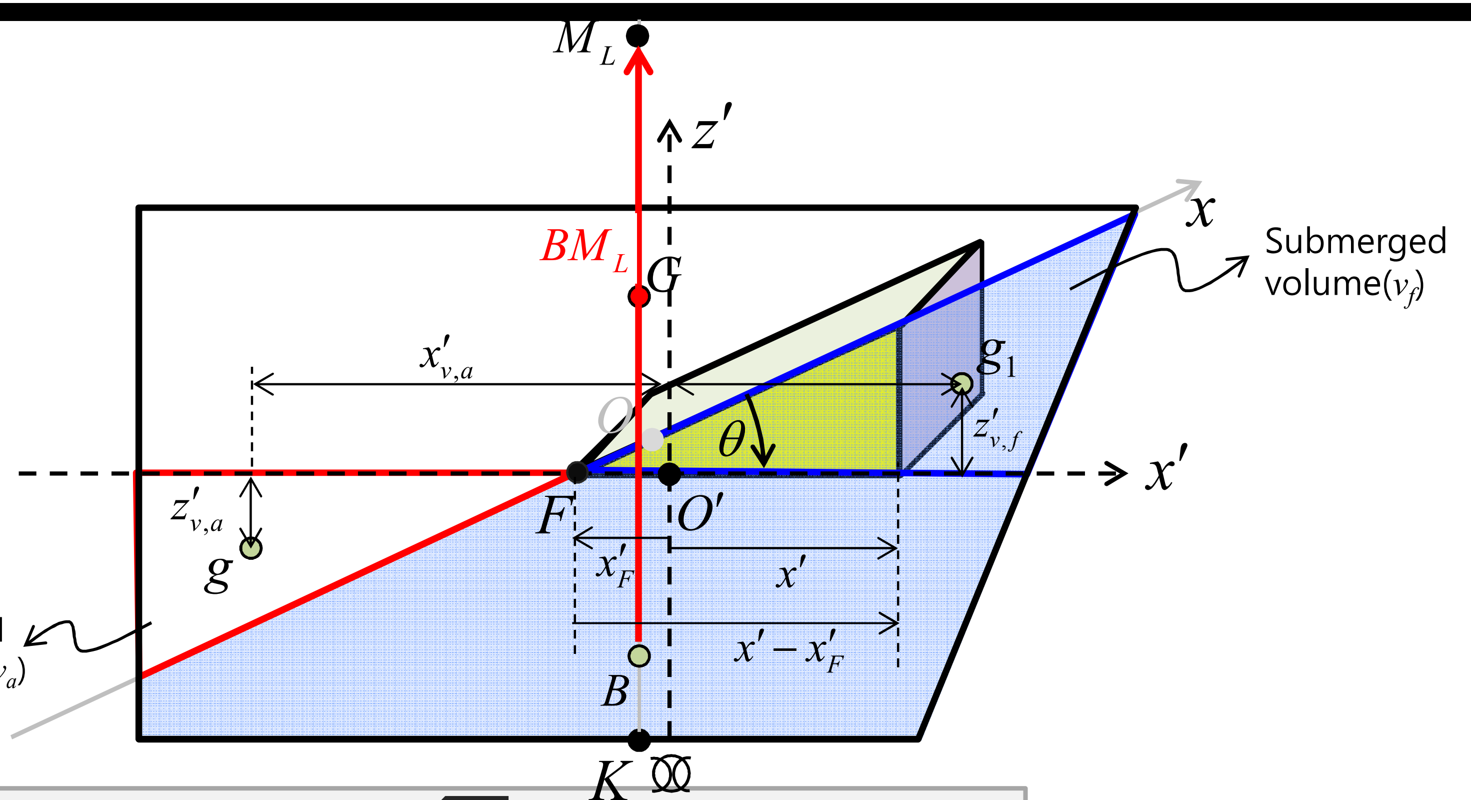
$$= \int_{aft}^{F}\int_{star}^{port} x' \cdot dv_a$$

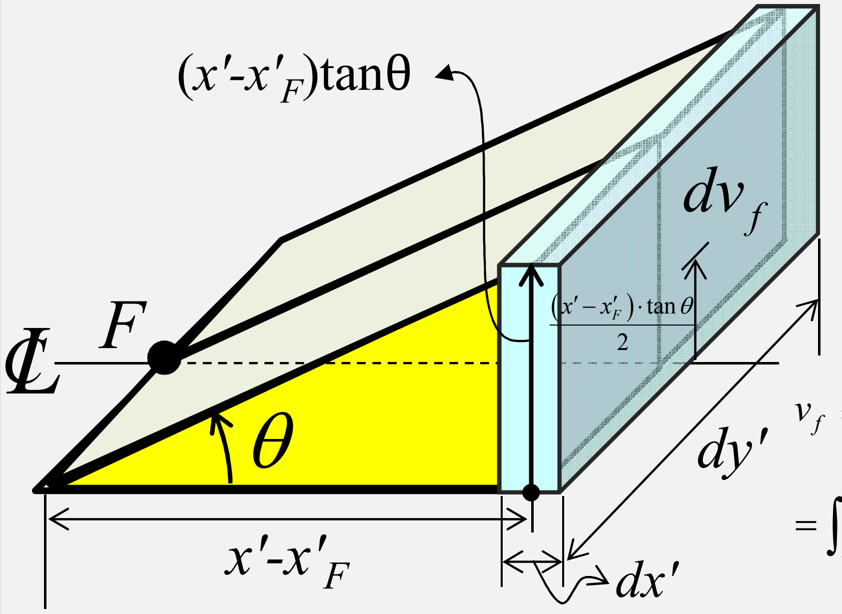
$$= \int_{aft}^{F}\int_{star}^{port} x' \cdot (x' - x'_F) \cdot \tan\theta \cdot dy' \cdot dx'$$

$$= \tan\theta \cdot \int_{aft}^{F}\int_{star}^{port} x' \cdot (x' - x'_F) \cdot dy' \cdot dx'$$

Derivation of BM$_L$ (7/12)

$$BM_L = \frac{1}{\nabla \cdot \tan\theta}\left(\underbrace{x'_{v,f}\cdot v_f}_{(A)} + \underbrace{x'_{v,a}\cdot v_a}_{(B)} + (\underbrace{z'_{v,f}\cdot v_f}_{(C)} + \underbrace{z'_{v,a}\cdot v_a}_{(D)})\tan\theta\right)$$





$$dv_f = (x' - x'_F)\cdot\tan\theta\cdot dy'\cdot dx'$$

$$v_f = \int_F^{fore}\int_{star}^{port} dv_f$$

$$= \int_F^{fore}\int_{star}^{port}(x' - x'_F)\cdot\tan\theta\cdot dy'\cdot dx'$$

(C) $z'_{v,f}\cdot v_f$: **Moment about transverse axis through point O** of the submerged volume

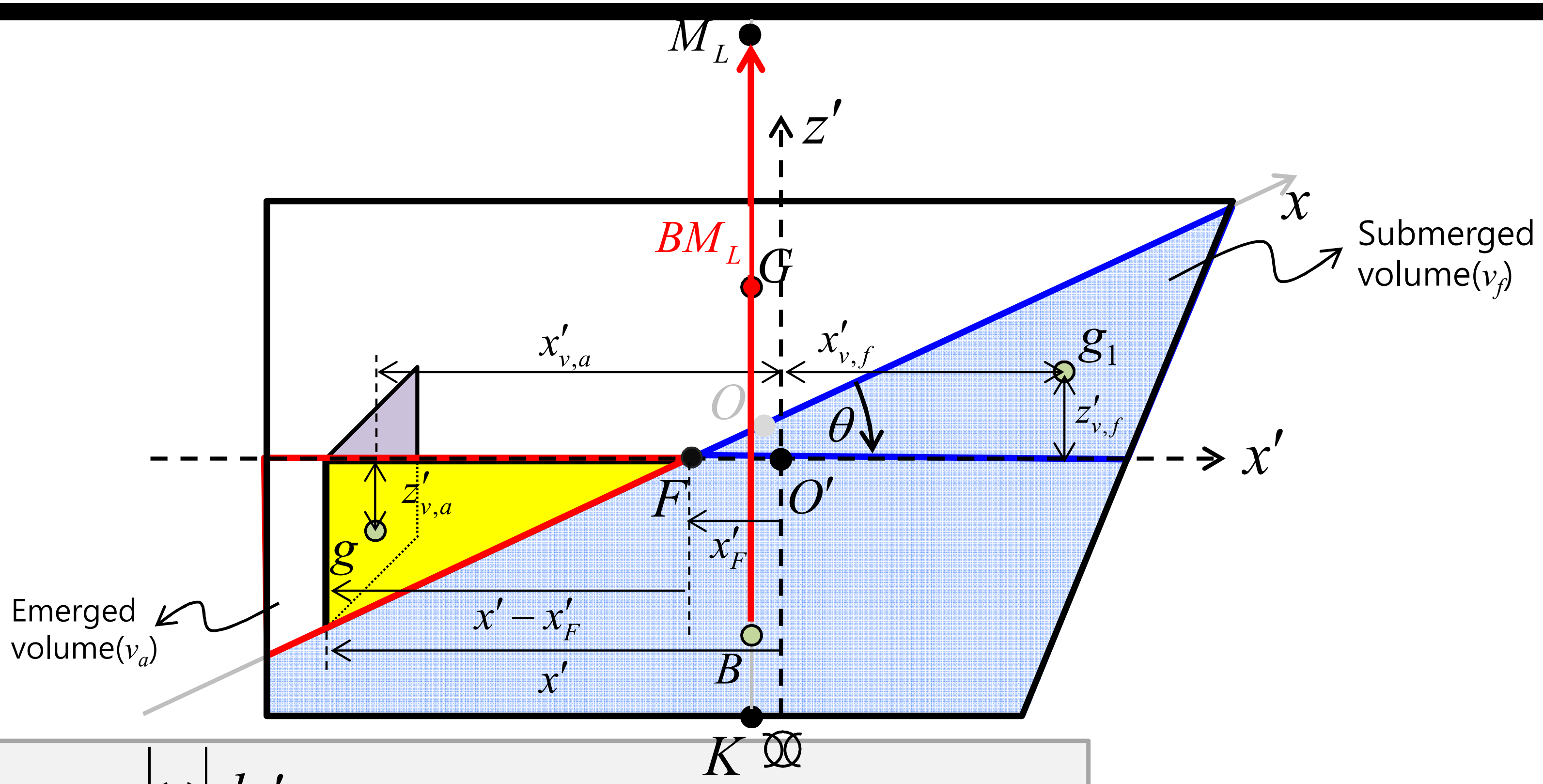
$$= \int_F^{fore}\int_{star}^{port}\frac{(x'-x'_F)\cdot\tan\theta}{2}\cdot dv_f$$

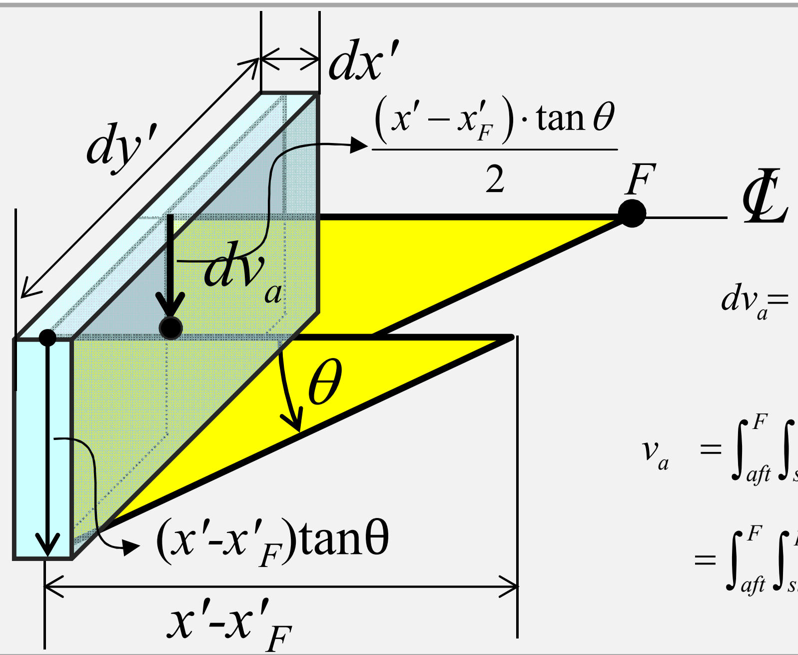
$$= \int_F^{fore}\int_{star}^{port}\frac{(x'-x'_F)\cdot\tan\theta}{2}\cdot(x'-x'_F)\cdot\tan\theta\cdot dy'\cdot dx'$$

$$= \frac{\tan^2\theta}{2}\int_F^{fore}\int_{star}^{port}(x'-x'_F)^2\cdot dy'\cdot dx'$$

Derivation of BM_L (8/12)

$$BM_L = \frac{1}{\nabla \cdot \tan\theta}\left(\underset{(A)}{x'_{v,f} \cdot v_f} + \underset{(B)}{x'_{v,a} \cdot v_a} + \left(\underset{(C)}{z'_{v,f} \cdot v_f} + \underset{(D)}{z'_{v,a} \cdot v_a}\right)\tan\theta\right)$$





$$dv_a = (x' - x'_F)\cdot\tan\theta\cdot dy'\cdot dx'$$

$$v_a = \int_{aft}^{F}\int_{star}^{port} dv_a$$

$$= \int_{aft}^{F}\int_{star}^{port} (x' - x'_F)\cdot\tan\theta\cdot dy'\cdot dx'$$

(D) $z'_{v,a} \cdot v_a$: **Moment about transverse axis through point O** of the emerged volume

$$= \int_{aft}^{F}\int_{star}^{port} \frac{(x' - x'_F)\cdot\tan\theta}{2}\cdot dv_a$$

$$= \int_{aft}^{F}\int_{star}^{port} \frac{(x' - x'_F)\cdot\tan\theta}{2}\cdot(x' - x'_F)\cdot\tan\theta\cdot dy'\cdot dx'$$

$$= \frac{\tan^2\theta}{2}\int_{aft}^{F}\int_{star}^{port} (x' - x'_F)^2\cdot dy'\cdot dx'$$

Derivation of BM_L (9/12)

(A) $x'_{v,f}\cdot v_f=\tan\theta\cdot\int_F^{fore}\int_{star}^{port}x'\cdot(x'-x'_F)\cdot dy'\cdot dx'$

(B) $x'_{v,a}\cdot v_a=\tan\theta\cdot\int_{aft}^{F}\int_{star}^{port}x'\cdot(x'-x'_F)\cdot dy'\cdot dx'$

(C) $z'_{v,f}\cdot v_f=\dfrac{\tan^2\theta}{2}\int_F^{fore}\int_{star}^{port}(x'-x'_F)^2\cdot dy'\cdot dx'$

(D) $z'_{v,a}\cdot v_a=\dfrac{\tan^2\theta}{2}\int_{aft}^{F}\int_{star}^{port}(x'-x'_F)^2\cdot dy'\cdot dx'$

∇: Displacement volume

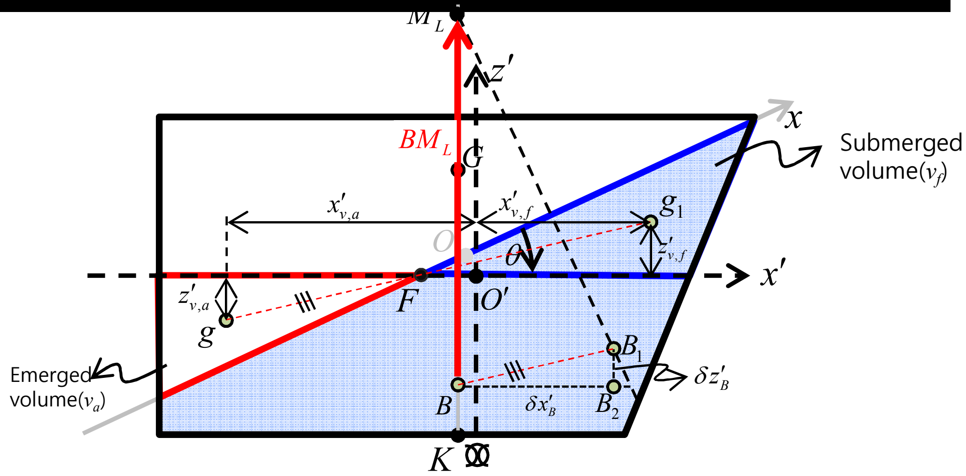
v: Submerged / Emerged volume

B: Center of buoyancy before inclination

B_1: Center of buoyancy after inclination

, g: Center of the emerged volume

, g_1: Center of the submerged volume



$$BM_L=\frac{1}{\nabla\cdot\tan\theta}\left(x'_{v,f}\cdot v_f+x'_{v,a}\cdot v_a+(z'_{v,f}\cdot v_f+z'_{v,a}\cdot v_a)\tan\theta\right)$$

(A) (B) (C) (D)

By substituting (A), (B), (C), (D) into above equation

$$=\frac{1}{\nabla\cdot\tan\theta}\left(\tan\theta\cdot\int_F^{fore}\int_{star}^{port}x'\cdot(x'-x'_F)\cdot dy'\cdot dx'+\tan\theta\cdot\int_{aft}^{F}\int_{star}^{port}x'\cdot(x'-x'_F)\cdot dy'\cdot dx'+\frac{\tan^2\theta}{2}\int_F^{fore}\int_{star}^{port}(x'-x'_F)^2\cdot dy'\cdot dx'+\frac{\tan^2\theta}{2}\int_{aft}^{F}\int_{star}^{port}(x'-x'_F)^2\cdot dy'\cdot dx'\right)$$

$$=\frac{1}{\nabla\cdot\tan\theta}\left(\tan\theta\cdot\int_{aft}^{fore}\int_{star}^{port}x'\cdot(x'-x'_F)\cdot dy'\cdot dx'+\frac{\tan^2\theta}{2}\int_{aft}^{fore}\int_{star}^{port}(x'-x'_F)^2\cdot dy'\cdot dx'\right)$$

$$=\frac{1}{\nabla\cdot\tan\theta}\left(\tan\theta\cdot\int_{aft}^{fore}\int_{star}^{port}(x'^2-x'\cdot x'_F)\cdot dy'\cdot dx'+\frac{\tan^2\theta}{2}\int_{aft}^{fore}\int_{star}^{port}(x'-x'_F)^2\cdot dy'\cdot dx'\right)$$

$$=\frac{1}{\nabla}\left(\int_{aft}^{fore}\int_{star}^{port}(x'^2-x'\cdot x'_F)\cdot dy'\cdot dx'+\frac{\tan\theta}{2}\int_{aft}^{fore}\int_{star}^{port}(x'-x'_F)^2\cdot dy'\cdot dx'\right)$$

$$=\frac{1}{\nabla}\left(\int_{aft}^{fore}\int_{star}^{port}x'^2\cdot dy'\cdot dx'-\int_{aft}^{fore}\int_{star}^{port}x'_F\cdot x'\cdot y'\cdot dx'+\frac{\tan\theta}{2}\left(\int_{aft}^{fore}\int_{star}^{port}x'^2\cdot dy'\cdot dx'-2\int_{aft}^{fore}\int_{star}^{port}x'^2\cdot dy'\cdot dx'+\int_{aft}^{fore}\int_{star}^{port}x_F'^2\cdot dy'\cdot dx'\right)\right)$$

Derivation of BM_L (10/12)

(A) $x'_{v,f} \cdot v_f = \tan\theta \int_F^{F.P} (x' - x'_F) x' \cdot y' \cdot dx'$

(B) $x'_{v,a} \cdot v_a = \tan\theta \int_{A.P}^F (x' - x'_F) \cdot x' \cdot y' \cdot d'x$

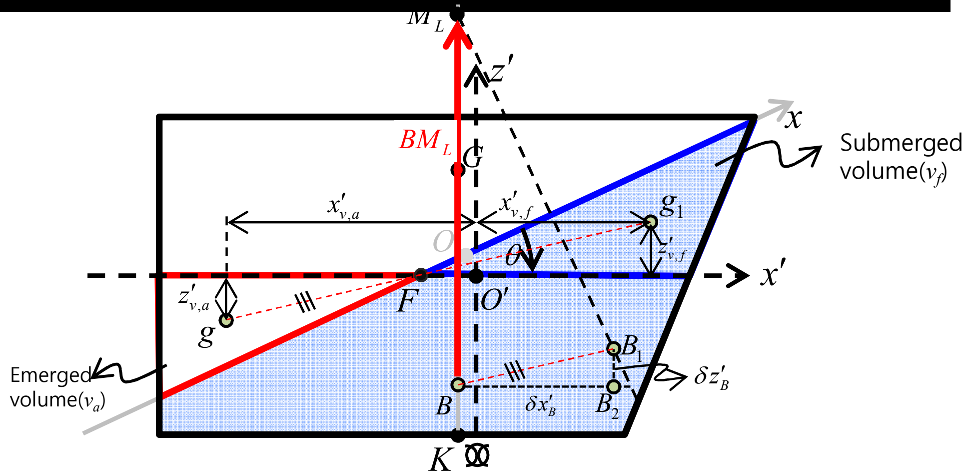
(C) $z'_{v,f} \cdot v_f = \frac{\tan^2\theta}{2} \int_F^{F.P} (x' - x'_F)^2 \cdot y' \cdot dx'$

(D) $z'_{v,a} \cdot v_a = \frac{\tan^2\theta}{2} \int_{A.P}^F (x' - x'_F)^2 \cdot y' \cdot dx'$

∇: Displacement volume, g: Center of the emerged volume
v: Submerged / Emerged volume, g_1: Center of the submerged volume
B: Center of buoyancy before inclination
B_1: Center of buoyancy after inclination

$$BM_L = \frac{1}{\nabla \cdot \tan\theta}\left(x'_{v,f} \cdot v_f + x'_{v,a} \cdot v_a + (z'_{v,f} \cdot v_f + z'_{v,a} \cdot v_a) \tan\theta \right)$$

(A) (B) (C) (D)



By substituting (A), (B), (C), and (D) into above equation

$$= \frac{1}{\nabla}\left(\int_{aft}^{fore}\int_{star}^{port} x'^2 \cdot dy' \cdot dx' - \int_{aft}^{fore}\int_{star}^{port} x'_F \cdot x' \cdot y' \cdot dx' + \frac{\tan\theta}{2}\left(\int_{aft}^{fore}\int_{star}^{port} x'^2 \cdot dy' \cdot dx' - 2\int_{aft}^{fore}\int_{star}^{port} x'_F \cdot x' \cdot dy' \cdot dx' + \int_{aft}^{fore}\int_{star}^{port} x'^2_F \cdot dy' \cdot dx' \right)\right)$$

$\int_{aft}^{fore}\int_{star}^{port} dy' \cdot dx' = A_{WP}$: The water plane area

$\int_{aft}^{fore}\int_{star}^{port} x' \cdot dy' \cdot dx' = M_{y'}$: The moment of the water plane area about the y′-axis through point O′

$\int_{aft}^{fore}\int_{star}^{port} x'^2_F \cdot dy' \cdot dx' = I_{L,y'}$: The moment of inertia of the water plane area about the y′-axis through point O′

$$= \frac{1}{\nabla}\left(I_{L,y'} - x'_F M_{y'} + \frac{\tan\theta}{2}\left(I_{L,O} - 2x'_F M_{y'} + x'^2_F A_{WP} \right)\right)$$

Derivation of BM_L (11/12)

(A) $x'_{v,f} \cdot v_f = \tan\theta \int_F^{F.P} (x' - x'_F) x' \cdot y' \cdot dx'$

(B) $x'_{v,a} \cdot v_a = \tan\theta \int_{A.P}^{F} (x' - x'_F) \cdot x' \cdot y' \cdot d'x$

(C) $z'_{v,f} \cdot v_f = \dfrac{\tan^2\theta}{2} \int_F^{F.P} (x' - x'_F)^2 \cdot y' \cdot dx'$

(D) $z'_{v,a} \cdot v_a = \dfrac{\tan^2\theta}{2} \int_{A.P}^{F} (x' - x'_F)^2 \cdot y' \cdot dx'$

∇: Displacement volume

v: Submerged / Emerged volume

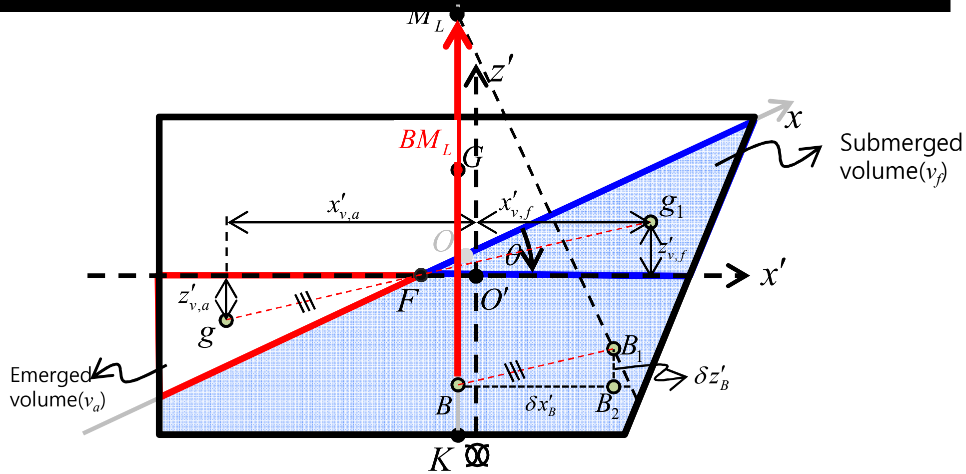
B: Center of buoyancy before inclination

B_1: Center of buoyancy after inclination

, g: Center of the emerged volume

, g_1: Center of the submerged volume

$$BM_L = \frac{1}{\nabla \cdot \tan\theta} \left(\underbrace{x'_{v,f} \cdot v_f}_{(A)} + \underbrace{x'_{v,a} \cdot v_a}_{(B)} + (\underbrace{z'_{v,f} \cdot v_f}_{(C)} + \underbrace{z'_{v,a} \cdot v_a}_{(D)}) \tan\theta \right)$$



By substituting (A), (B), (C), and (D) into above equation

$$= \frac{1}{\nabla} \left(I_{L,y'} - x'_F M_{y'} + \frac{\tan\theta}{2} \left(I_{L,O} - 2x'_F M_{y'} + x'^2_F A_{WP} \right) \right)$$

$$x'_F A_{WP} = M_{y'}$$

$$= \frac{1}{\nabla} \left(I_{L,y'} - x'_F M_{y'} + \frac{\tan\theta}{2} \left(I_{L,O} - 2x'_F M_{y'} + x'_F M_{y'} \right) \right)$$

$$= \frac{1}{\nabla} \left(I_{L,y'} - x'_F M_{y'} + \frac{\tan\theta}{2} \left(I_{L,O} - x'_F M_{y'} \right) \right)$$

$$= \frac{1}{\nabla} \left(I_{L,^t y} + \frac{\tan\theta}{2} I_{L,^t y} \right) \Rightarrow \boxed{BM_L = \frac{I_{L,^t y}}{\nabla} \left(1 + \frac{1}{2} \tan^2\theta \right)}$$

According to the parallel axis theorem

$$I_{L,y'} = I_{L,^t y} + x'^2_F A_{WP}$$

$$= I_{L,^t y} + x'_F \cdot x'_F \cdot A_{WP} \Rightarrow I_{L,^t y} = I_{L,y'} - x'_F \cdot M_{y'}$$

$$= I_{L,^t y} + x'_F \cdot M_{y'}$$

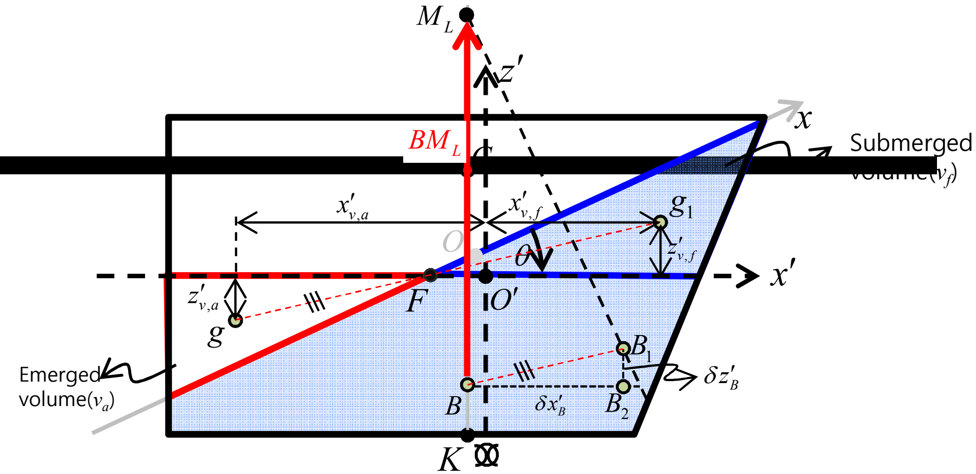
$I_{L,y'}$: The moment of inertia of the water plane area about y′-axis through the point O′

$I_{L,^t y}$: The moment of inertia of the water plane area about y_t-axis through the center of floatation F

A_{WP}: The water plane area

$M_{y'}$: The moment of the water plane area about y′-axis through point O′

Derivation of BM_L (12/12)



∇: Displacement volume , g: Center of the emerged volume
v: Submerged / Emerged volume , g_1: Center of the submerged volume
B: Center of buoyancy before inclination
B_1: Center of buoyancy after inclination

$$BM_L = \frac{I_{L,'y}}{\nabla}\left(1+\frac{1}{2}\tan^2\theta\right)$$

If θ is small, $\tan^2\theta \approx \theta^2 = 0$

$$BM_L = \frac{I_{L,'y}}{\nabla}$$

which is generally known as BM_L.

That BM_L does not consider change of center of buoyancy **in vertical direction**.

In order to distinguish between them, those will be indicated as follows :

$$BM_L = \frac{I_{L,'y}}{\nabla}(1+\frac{1}{2}\tan^2\theta)$$ (**Considering** change of center of buoyancy in vertical direction)

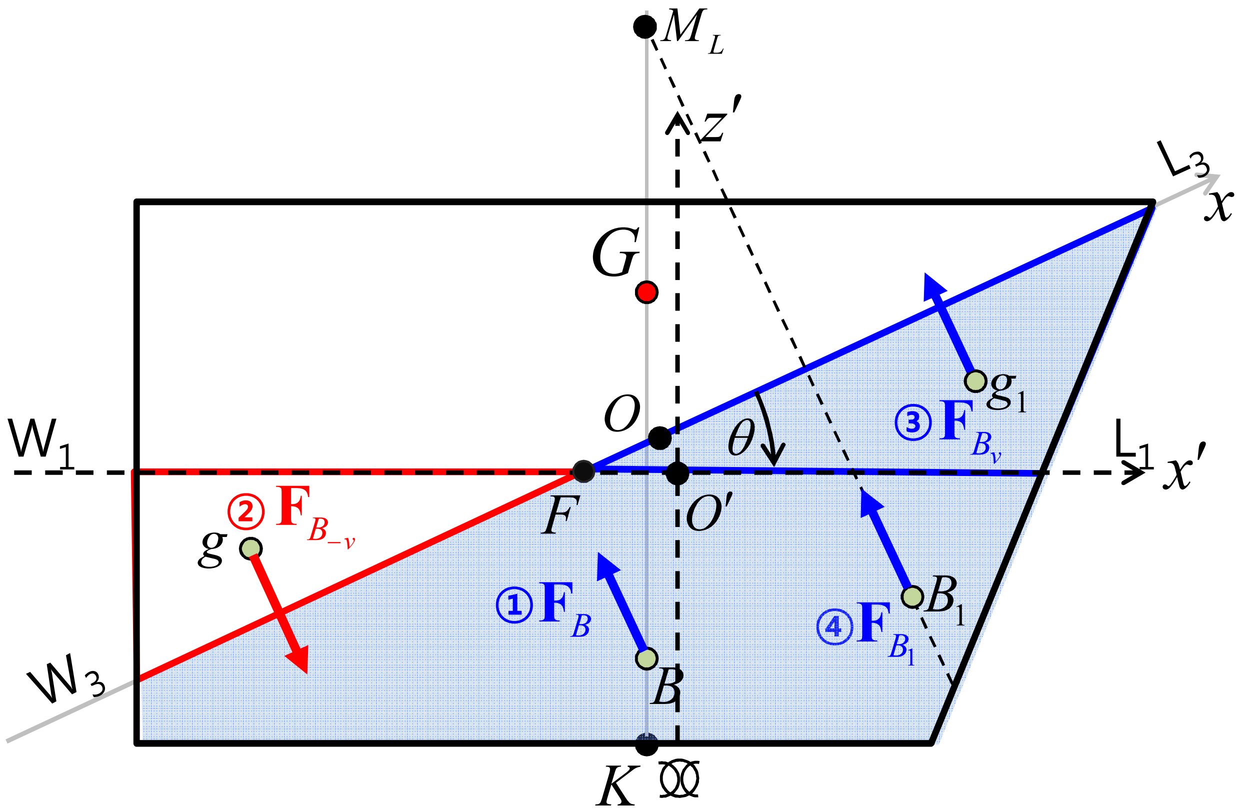
$$BM_L = \frac{I_{L,'y}}{\nabla}$$ (**Without considering** change of center of buoyancy in vertical direction)

Another Approach to Derive the Following Formula

$$\begin{cases} \delta\, {}^t x_B = ({}^t x_{v,f} \cdot v_f + {}^t x_{v,a} \cdot v_a) / \nabla \\ \delta\, {}^t z_B = ({}^t z_{v,f} \cdot v_f + {}^t z_{v,a} \cdot v_a) / \nabla \end{cases}$$

Derivation of BM_L (1/4)

Assumption
① **A small trim angle** (3°~5°)
② The **submerged volume** and the **emerged volume** are to be the **same**.



∇: Displacement volume
v: Submerged / Emerged volume
B: The center of buoyancy before inclination
B_1: The center of buoyancy after inclination
g: The center of the emerged volume
g_1: The center of the submerged volume

Another approach to derive the following Equations

$$\begin{cases} \delta^t x_B = ({}^t x_{v,f} \cdot v_f + {}^t x_{v,a} \cdot v_a) / \nabla \\ \delta^t z_B = ({}^t z_{v,f} \cdot v_f + {}^t z_{v,a} \cdot v_a) / \nabla \end{cases}$$

The change in moment about the y_t-axis due to the buoyant force caused by a small inclination, θ, consists of two different components:

1. The change in moment due to the movement of the previous center of buoyancy B by rotation of the ship : M①

2. The change in the displaced volume

1) The change in moment due to the emerged volume: M②

2) The change in moment due to the (additional) submerged volume: M③

The resultant moment: M④ = M①+M②+M③

Derivation of BM_L (2/4)

Assumption

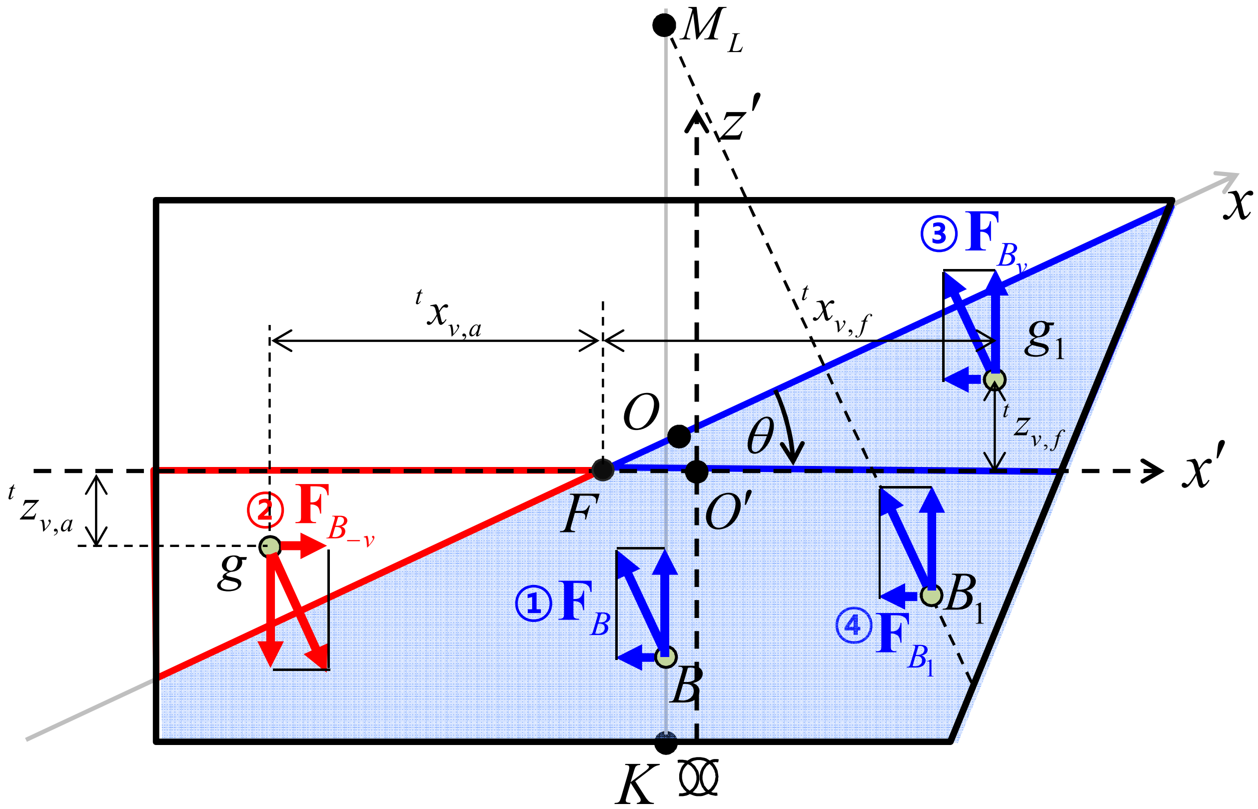
① **A small trim angle** (3°~5°)

② The **submerged volume** and the **emerged volume** are to be the **same**.

For the convenience of calculation, the forces are decomposed in the body fixed frame.

Body fixed frame

Moment: M①+M②+M③=M④



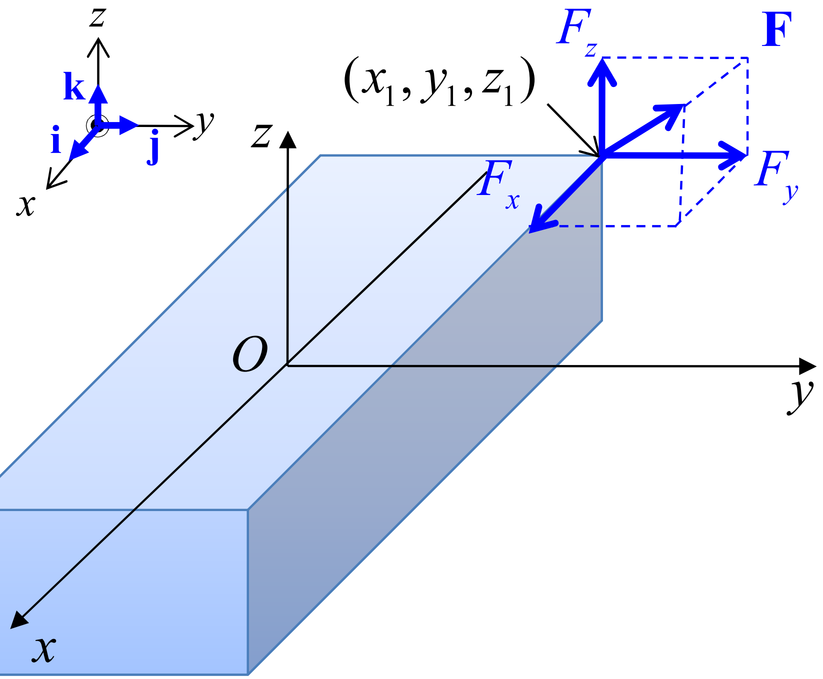
∇: Displacement volume
v: Changed displacement volume (wedge)
BB_1: Distance of changed center of buoyancy
gg_1: Distance of changed center of wedge
B: The center of buoyancy before inclination
B_1: The center of buoyancy after inclination
M_L: The intersection of the line of buoyant force through B_1 with the line of buoyant force B

For convenience, we define a new temporary body fixed frame F-x_t-y_t-z_t, whose origin is the point F.

[Reference] Moment about x axis

Question)

Force **F** is applied on the point of rectangle object, what is the moment about x axis?



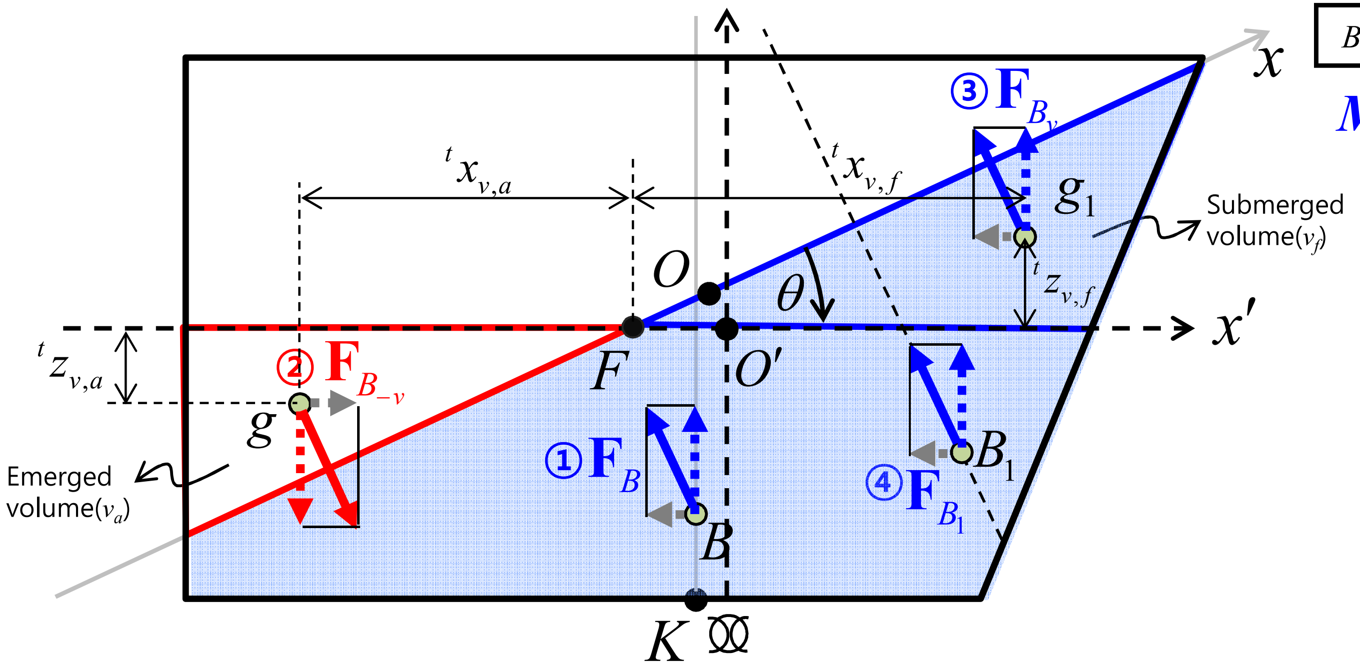
$$M = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{bmatrix} = \underbrace{\mathbf{i}(r_y \cdot F_z - r_z \cdot F_y)}_{M_x} + \underbrace{\mathbf{j}(-r_x \cdot F_z + r_z \cdot F_x)}_{M_y} + \underbrace{\mathbf{k}(r_x \cdot F_y - r_y \cdot F_x)}_{M_z}$$

Transverse moment

Derivation of BM_L (3/4)

$$M=\begin{bmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ r_x & r_y & r_z\\ F_x & F_y & F_z\end{bmatrix}=\mathbf{i}(r_y\cdot F_z-r_z\cdot F_y)+\mathbf{j}(-r_x\cdot F_z+r_z\cdot F_x)+\mathbf{k}(r_x\cdot F_y-r_y\cdot F_x)$$

M_{F,y_t}



Body fixed frame

Moment: M①+M②+M③=M④

∇: Displacement volume
v: **Changed displacement volume** (wedge)
BB_1: Distance of changed center of buoyancy
gg_1: Distance of changed center of wedge
B: The center of buoyancy before inclination
B_1: The center of buoyancy after inclination
M_L: The intersection of the line of buoyant force through B_1 with the line of buoyant force B

For convenience, we define a new temporary body fixed frame F-x_t-y_t-z_t, whose origin is the point F.

Moment about the y_t-axis through the point F

1. Moment about the y_t **axis** due to the force in z_t direction

$$-{}^t x_{B_1}\cdot(F_{B_1,z})=-{}^t x_B\cdot(F_{B,z})-{}^t x_{v,a}\cdot(F_{B\_v,z})-{}^t x_{v,f}(F_{B_v,z})$$

M④ ↓ M① M② M③

$$-{}^t x_{B_1}\cdot(\rho g\nabla\cdot\cos\theta)=-{}^t x_B\cdot(\rho g\nabla\cdot\cos\theta)-{}^t x_{v,a}\cdot(\rho g v_a\cdot\cos\theta)-{}^t x_{v,f}(\rho g v_f\cdot\cos\theta)$$

↓

$$-({}^t x_{B_1}-{}^t x_B)\cdot(\rho g\nabla\cdot\cos\theta)=(-{}^t x_{v,a}\cdot v_a-{}^t x_{v,f}\cdot v_f)\cdot(\rho g\cdot\cos\theta)$$

$=\delta x_B$

↓

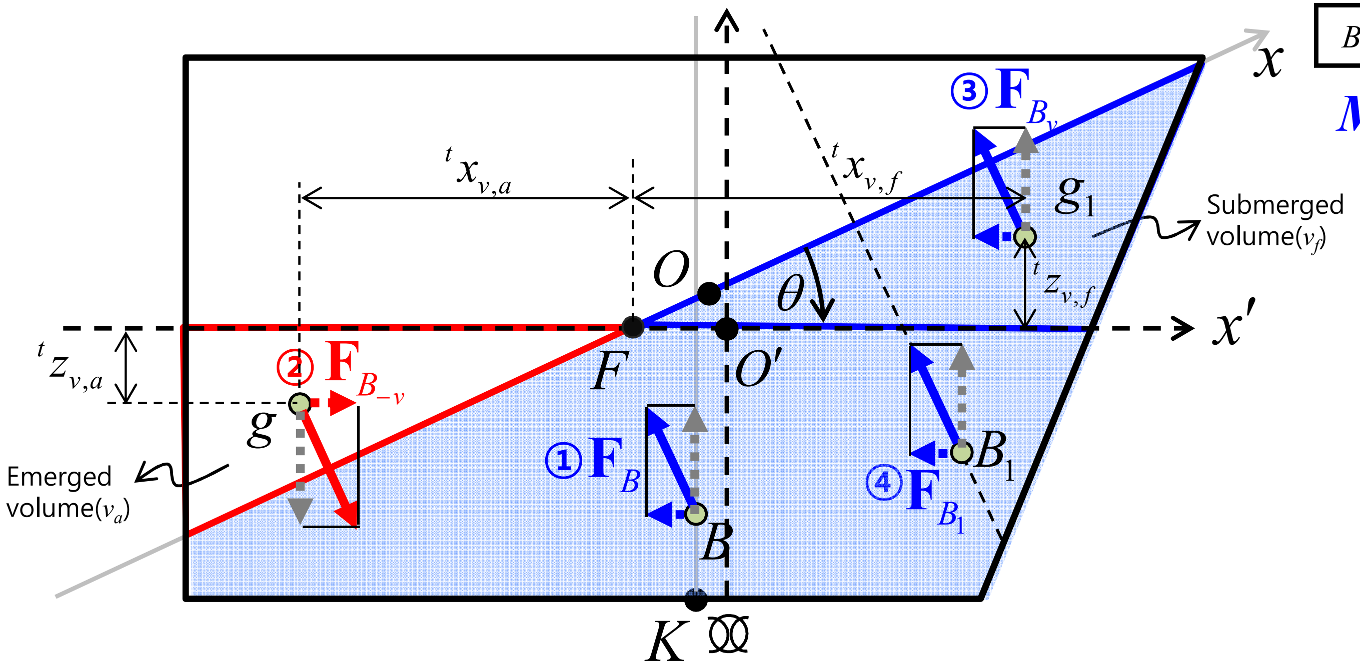
$$-(\delta\,{}^t x_B)\cdot\nabla=(-{}^t x_{v,a}\cdot v_a-{}^t x_{v,f}\cdot v_f)$$

↓

$$(\delta\,{}^t x_B)\cdot\nabla=({}^t x_{v,a}\cdot v_a+{}^t x_{v,f}\cdot v_f)$$

Derivation of BM_L (4/4)

$$M=\begin{bmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ r_x & r_y & r_z\\ F_x & F_y & F_z\end{bmatrix}=\mathbf{i}(r_y\cdot F_z-r_z\cdot F_y)+\underbrace{\mathbf{j}(-r_x\cdot F_z+r_z\cdot F_x)}_{M_{F,y_t}}+\mathbf{k}(r_x\cdot F_y-r_y\cdot F_x)$$



Moment: $M①+M②+M③=M④$

∇: Displacement volume
v: **Changed displacement volume** (wedge)
BB_1: Distance of changed center of buoyancy
gg_1: Distance of changed center of wedge
B: The center of buoyancy before inclination
B_1: The center of buoyancy after inclination
M_L: The intersection of the line of buoyant force through B_1 with the line of buoyant force B

For convenience, we define a new temporary body fixed frame F-x_t-y_t-z_t, whose origin is the point F

Moment about the y_t-axis through the point F

2. Moment about the y_t axis due to the force in x_t direction

$$\underbrace{{}^t z_{B_1}\cdot(F_{B_1,x})}_{M④}={}^t z_B\cdot\underbrace{(F_{B,x})}_{M①}+{}^t z_{v,a}\cdot\underbrace{(F_{B\_v,x})}_{M②}+{}^t z_{v,f}\underbrace{(F_{B_v,x})}_{M③}$$

$$\downarrow$$

$${}^t z_{B_1}\cdot(\rho g\nabla\cdot\cos\theta)={}^t z_B\cdot(\rho g\nabla\cdot\sin\theta)+{}^t z_{v,a}\cdot(\rho g v_a\cdot\sin\theta)+{}^t z_{v,f}(\rho g v_f\cdot\sin\theta)$$

$$\downarrow$$

$$\underbrace{({}^t z_{B_1}-{}^t z_B)}_{=\delta^t z_B}\cdot(\rho g\nabla\cdot\sin\theta)=({}^t z_{v,a}\cdot v_a+{}^t z_{v,f}\cdot v_f)\cdot(\rho g\cdot\sin\theta)$$

$$\downarrow$$

$$\delta^t z_B\cdot\nabla=({}^t z_{v,a}\cdot v_a+{}^t z_{v,f}\cdot v_f)$$