Chapter 20
Sequencing Method for the Mixed-Model Assembly Line to Realize Smoothed Production

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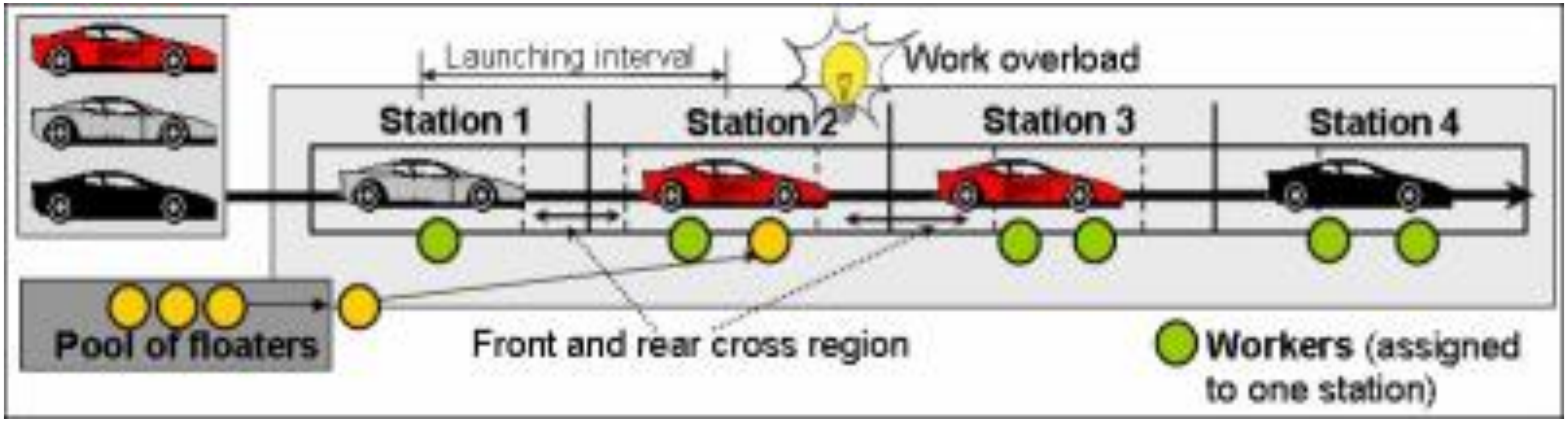
CONTENTS

20.1 GOALS OF CONTROLLING THE ASSEMBLY LINE

- A mixed-model includes several processes.
- It can operate different car models.
- It involves many types of parts.



20.1 GOALS OF CONTROLLING THE ASSEMBLY LINE

- Designing a mixed-model assembly line includes six steps.

- Determine the cycle time.
- Determine the minimum number of processes.
- Prepare a diagram of integrated precedence relationship among elemental jobs.
- Line balancing
- Determine the sequence schedule for introducing various products to the line.
- Determine the length of the operations range of each process.

20.1 GOALS OF CONTROLLING THE ASSEMBLY LINE

- There are two goals of controlling the mixed-model assembly line.

  - Leveling the total assembly time on each process within the line

  - Keeping a constant speed in consuming each part on the line

- Different goals require different sequences of introducing models to the line.

20.1 GOALS OF CONTROLLING THE ASSEMBLY LINE

- Indexes:

i	type of products (models)
l	process

- Parameters:

α	number of types of products
P	number of processes
Q_i	planned production quantity of product i per day
T_{il}	unit operation time of product i on process l
C	cycle time of the mixed-model assembly line

20.1 GOALS OF CONTROLLING THE ASSEMBLY LINE

- Goal one: work load streamlining

- A mixed-model assembly line has predetermined cycle time.

- Some products may have longer operation time than the cycle time.

20.1 GOALS OF CONTROLLING THE ASSEMBLY LINE

- In terms of averages, the total operation time for all models, each weighted by its production ratio, should satisfy

$$\max_{l=1,\dots,P}\left\{\frac{\sum_{i=1}^{\alpha} Q_i T_{il}}{\sum_{i=1}^{\alpha} Q_i}\right\} \le C$$

$$C = \text{cycle time} = \frac{\text{total operation time per day}}{\sum_{i=1}^{\alpha} Q_i}$$

20.1 GOALS OF CONTROLLING THE ASSEMBLY LINE

- Suppose a line that produces three models with two processes.

α=3, P=2

Q_1=4 units, Q_2=3 units, Q_3=2 units

$T_{1,1}$=2 hours/unit, $T_{2,1}$=5 hours/unit, $T_{3,1}$=2 hours/unit

$T_{1,2}$=3 hours/unit, $T_{2,2}$=1 hour/unit, $T_{3,2}$=4 hours/unit

20.1 GOALS OF CONTROLLING THE ASSEMBLY LINE

- For Process 1, the average operation time is

$$\frac{Q_1T_{1,1}+Q_2T_{2,1}+Q_3T_{3,1}}{Q_1+Q_2+Q_3}=\frac{4\times 2+3\times 5+2\times 2}{4+3+2}=3 \text{ hours/unit}$$

- For Process 2, the average operation time is

$$\frac{Q_1T_{1,2}+Q_2T_{2,2}+Q_3T_{3,2}}{Q_1+Q_2+Q_3}=\frac{4\times 3+3\times 1+2\times 4}{4+3+2}=\frac{23}{9} \text{ hours/unit}$$

$$\max\left\{3,\frac{23}{9}\right\}=3 \le C$$

20.1 GOALS OF CONTROLLING THE ASSEMBLY LINE

- If products with relatively longer operation time are successively introduced into the line, there would be a delay in completing the product.

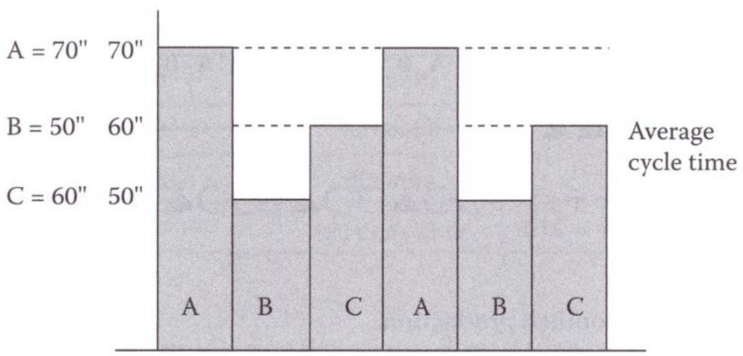


FIGURE 5.5
Sequence schedule that enables assembly within the average cycle time.

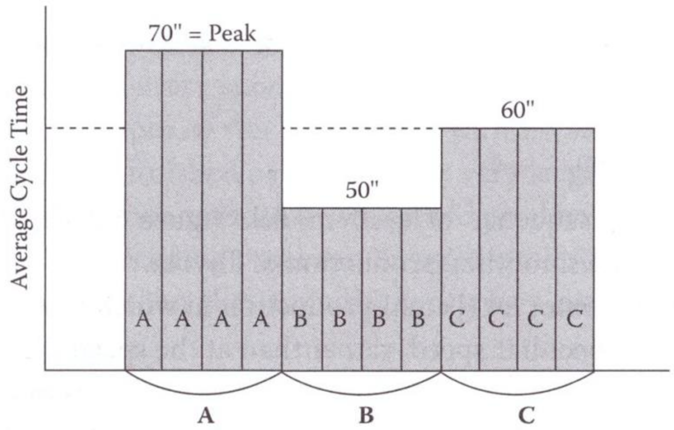


FIGURE 5.6
Sequence schedule that causes line stoppage.

20.1 GOALS OF CONTROLLING THE ASSEMBLY LINE

Goal two: sequencing model for parts usage streamlining

- In the pull system, the variation in production quantities or conveyance times at preceding processes should be minimized.

- Therefore, the quantity used per hour for each part in the mixed-model line should be kept as constant as possible.

20.1 GOALS OF CONTROLLING THE ASSEMBLY LINE

- Parameters:

Q	total production quantity of all products = $\sum_{i=1}^{\alpha} Q_i$
β	total number of types of parts
N_j	necessary quantity of part j for producing Q
$X_{j,k}$	necessary quantity of part j for producing the products of determined sequence from first to kth
N_j/Q	average necessary quantity of part j for one product
kN_j/Q	average necessary quantity of part j for k products

20.1 GOALS OF CONTROLLING THE ASSEMBLY LINE

- To keep the consumption speed of part j constant, the value of $X_{j,k}$ should be as close as possible to the value of kN_j/Q, k=1,2,…,Q.

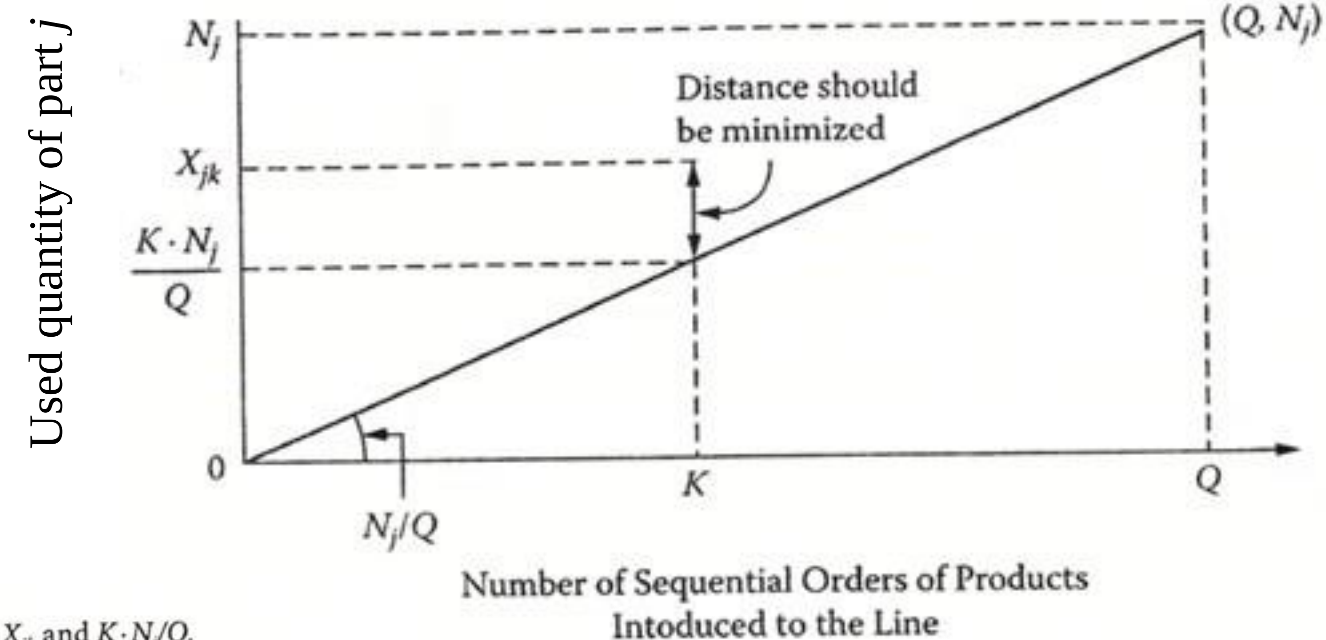


FIGURE 20.1
Relationship between X_{jk} and $K \cdot N_j/Q$.

20.1 GOALS OF CONTROLLING THE ASSEMBLY LINE

❖ It can be further defined that

Independent of the sequence

$$\text{A point } G_k = \left(kN_1 / Q, kN_2 / Q, \ldots, kN_\beta / Q \right)$$

$$\text{A point } P_k = \left(X_{1,k}, X_{2,k}, \ldots, X_{\beta,k} \right)$$

Dependent on the sequence

20.1 GOALS OF CONTROLLING THE ASSEMBLY LINE

- If a sequence schedule assures the constant speed of consuming each part, point P_k must be as close as possible to point G_k.

- Let the distance be D_k, we have

$$\min D_k = \left\| G_k - P_k \right\| = \sqrt{\sum_{j=1}^{\beta} \left(\frac{kN_j}{Q} - X_{j,k} \right)^2}$$

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

b_{ij} necessary quantity of part j for product i

$i=1,2,\ldots,\alpha$; $j=1,2,\ldots,\beta$

- Algorithm

  Step 1: set $k=1$, $X_{j,0}=0$, and $S_0=\{1,2,\ldots,\alpha\}$

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- Algorithm

Step 2: Set product i^* as the kth product in the sequence schedule that minimizes D_k.

$$D_{ki^*} = \min_{i \in S_{k-1}} \left\{ D_{ki} \right\}$$

$$D_{ki} = \sqrt{\sum_{j=1}^{\beta} \left(\frac{kN_j}{Q} - X_{j,k} \right)^2} = \sqrt{\sum_{j=1}^{\beta} \left(\frac{kN_j}{Q} - X_{j,k-1} - b_{ij} \right)^2}$$

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- Algorithm

Step 3: If all of product i^* have been included in the sequence, set $S_k = S_{k\text{-}1} - \{i^*\}$;
otherwise, set $S_k = S_{k\text{-}1}$.

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- Algorithm

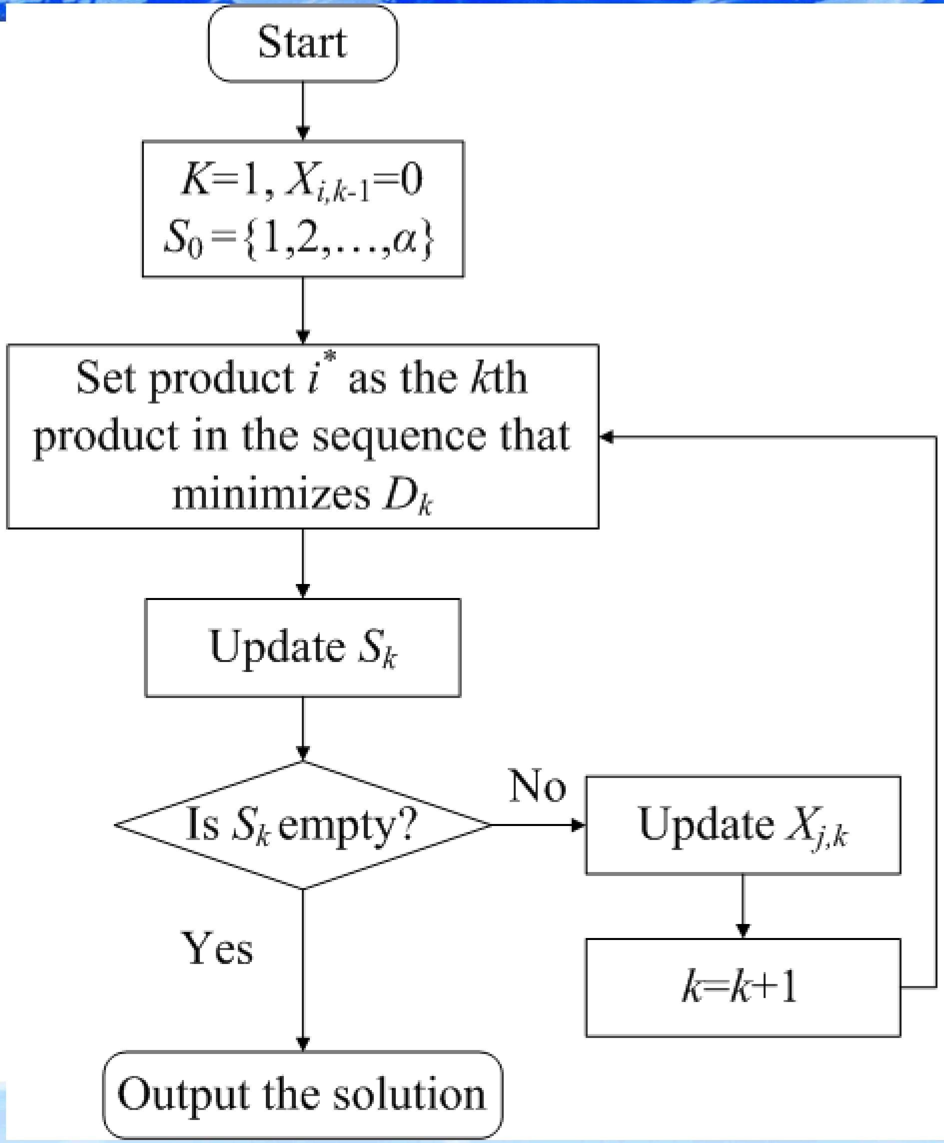
Step 4: If $S_k = \emptyset$, stop.

If $S_k \neq \emptyset$, then set $X_{i,k} = X_{i,k\text{-}1} + b_{i^*j}$, j=1,2,…, β.

set k = k+1;

go to Step 2.

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE



20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

Denote:

b_{ij} = Necessary quantity of the part a_i (j = 1,..., β) for producing one unit of the product A_i (i = 1,..., α).

Other notations are already defined.

Then,

Step 1 Set $K = 1$, $X_{j,k-1} = 0$, (j = 1,..., β), S_{k-1} = (1, 2, ..., α).

Step 2 Set as K th order in the sequence schedule the product A_i^* which minimizes the distance D_k. The minimum distance will be found by the following formula:

$$D_{ki^*} = \min\{D_{ki}\},\ i \in S_{k-1}\ \text{i}$$

should be removed

$$\text{where } D_{ki} = \sqrt{\sum_{j=1}^{\beta}\left(\frac{K \cdot N_j}{Q} - X_{j,k-1} - b_{ij}\right)^2}\ .$$

should be "all"

Step 3 If $\boxed{\text{some}}$ units of _ product A_{i^*} were ordered and included in the sequence schedule, then

Set $S_k = S_{k-1} - \{i^*\}$.

If some units of a product A_{i^*} are still remaining as being not ordered, then set $S_k = S_{k-1}$.

Step 4 If $S_k = \emptyset$ (empty set), the algorithm will end.

If $S_k \neq 0$, then compute $X_{jk} = X_{j,k-1} + b_{i^*j}$ (j = 1, ..., β) and go back to Step 2 by setting $K = K + 1$.

FIGURE 20.2
Goal-chasing method I.

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- An example

TABLE 20.1

Production Quantities Q_i and Parts Condition b_{ij}.

	Product A_t		
	A_1	A_2	A_3
Planned Production Quantity Q_j	2	3	5

	Parts a_j			
Products A_i	a_1	a_2	a_3	a_4
A_1	1	0	1	1
A_2	1	1	0	1
A_3	0	1	1	0

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- The total production quantity

$$\sum_{i=1}^{3} Q_i = 2+3+5=10$$

$$[N_j] = [Q_i]\,[b_{ij}]$$

$$= \left[2,3,5\right]\begin{bmatrix}1011\\ \\ 1101\\ \\ 0110\end{bmatrix} = \left[5,8,7,5\right]$$

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

$$\left[N_j / Q \right] = \left[\frac{5}{10}, \frac{8}{10}, \frac{7}{10}, \frac{5}{10} \right] \quad j = 1, 2, 3, 4$$

- Next, apply the values of $[N_j / Q]$ and $[b_{ij}]$ to the formula in Step 2.

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- When $k = 1$, D_{ki} can be calculated as:

$$D_{1,1} = \sqrt{\left(\frac{1\times 5}{10}-0-1\right)^2+\left(\frac{1\times 8}{10}-0-0\right)^2+\left(\frac{1\times 7}{10}-0-1\right)^2+\left(\frac{1\times 5}{10}-0-1\right)^2}$$

$$=1.11$$

$$D_{1,2} = 1.01 \qquad D_{1,3} = 0.79$$

$$D_{1,i^*} = \min\{1.11, 1.01, 0.79\} = 0.79$$

$$i^* = 3$$

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- Therefore, the first product introduced into the line is Product 3.

$$X_{j,k} = X_{j,k-1} + b_{3j}$$

$$X_{1,1} = 0 + 0 = 0 \qquad X_{2,1} = 0 + 1 = 1$$

$$X_{3,1} = 0 + 1 = 1 \qquad X_{4,1} = 0 + 0 = 0$$

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- When $k = 2$, D_{ki} can be calculated as:

$$D_{2,1} = \sqrt{\left(\frac{2\times 5}{10} - 0 - 1\right)^2 + \left(\frac{2\times 8}{10} - 1 - 0\right)^2 + \left(\frac{2\times 7}{10} - 1 - 1\right)^2 + \left(\frac{2\times 5}{10} - 0 - 1\right)^2}$$
$$= 0.85$$

$$D_{2,2} = 0.57 \qquad D_{2,3} = 1.59$$

$$D_{2,i^*} = \min\{0.85, 0.57, 1.59\} = 0.57$$

$$i^* = 2$$

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- Therefore, the second product introduced into the line is Product 2.

$$X_{j,k} = X_{j,k-1} + b_{2j}$$

$$X_{1,2} = 0 + 1 = 1 \qquad X_{2,2} = 1 + 1 = 2$$

$$X_{3,2} = 1 + 0 = 1 \qquad X_{4,2} = 0 + 1 = 1$$

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- By repeating the above steps, we can obtain the solution.

K	D_{k1}	D_{k2}	D_{k3}	Sequence Schedule	X_{1k}	X_{2k}	X_{3k}	X_{4k}
1	1.11	1.01	0.79	A_3	0	1	1	0
2	0.85	0.57*	1.59	$A_3 A_2$	1	2	1	1
3	0.82*	1.44	0.93	$A_3 A_2 A_1$	2	2	2	2
4	1.87	1.64	0.28*	$A_3 A_2 A_1 A_3$	2	3	3	2
5	1.32	0.87*	0.87	$A_3 A_2 A_1 A_3 A_2$	3	4	3	3
6	1.64	1.87	0.28*	$A_3 A_2 A_1 A_3 A_2 A_3$	3	5	4	3
7	0.93	1.21	0.82*	$A_3 A_2 A_1 A_3 A_2 A_3 A_3$	3	6	5	3
8	0.57*	0.85	1.59	$A_3 A_2 A_1 A_3 A_2 A_3 A_3 A_1$	4	6	6	4
9	1.56	0.77*	1.01	$A_3 A_2 A_1 A_3 A_2 A_3 A_3 A_1 A_2$	5	7	6	5
10	—	—	0*	$A_3 A_2 A_1 A_3 A_2 A_3 A_3 A_1 A_2 A_3$	5	8	7	5

FIGURE 20.3
Sequence schedule. (*Note:* * indicates smallest distance D_{kj}.)

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- Suppose that $[[kN_j/Q]]$ denotes the integer which is the closest to kN_j/Q.

- If $X_{j,k} = [[kN_j/Q]]$, the optimality is achieved in part j at the kth position of the sequence.

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- The values of kN_j/Q and $X_{j,k}$ for part j are depicted as follows.

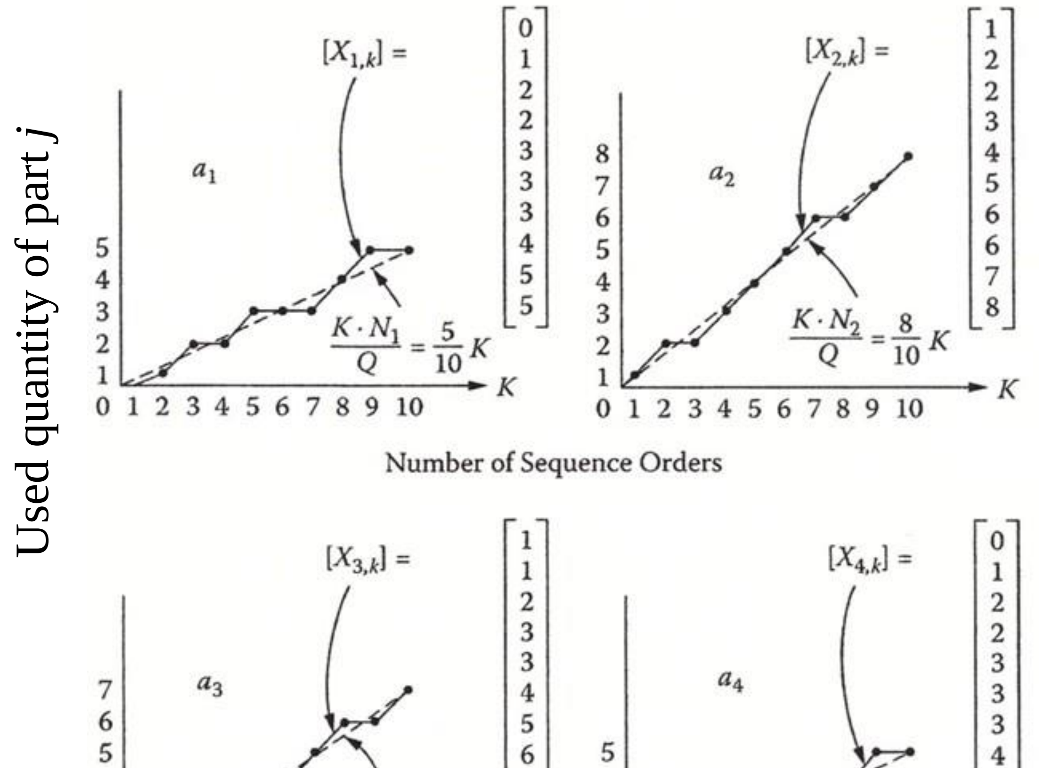


FIGURE 20.4
How X_{jk} approached $K \cdot N_j/Q$.

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- To further evaluate this algorithm, we calculate the mean and standard deviation of the following values.

$$\frac{kN_j}{Q} - X_{j,k} \qquad k = 1, 2, \ldots, Q$$

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- When the number of varieties in parts and/or the number of varieties in models are increased, both the mean and the standard deviation are increased.

- When the production quantity is increased, both the mean and the standard deviation are decreased.

The more the tendency to produce multi-varieties in each small quantity is promoted, the less likely smoothing of production will be attained.

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- Consider the case with a large total production quantity (e.g., Q=1000).

- The sequence determined by the algorithm can be divided into 16 equal ranges, each of which corresponds to one hour.

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- The quantity of each part in each range are computed.
- The standard deviation of these quantities are computed.

20.2 GOAL-CHASING METHOD: A NUMBERICAL EXAMPLE

- The values of σ are fairly small. The coefficient of variation is also small. $\left(\sigma / \bar{x}\right)$

Kind of front axies \ Range	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	$\bar{x}$	σ
a_1	9	7	7	9	8	7	8	8	8	8	7	8	9	7	7	8	7.8	0.73
a_2	6	5	7	6	5	6	7	5	7	6	5	7	6	6	5	6	5.9	0.75
a_3	5	6	5	5	6	6	4	6	4	6	6	5	4	6	5	6	5.3	0.77
a_4	3	3	3	2	3	3	3	3	3	2	3	3	3	2	3	3	2.8	0.33
a_5	2	2	2	2	3	2	2	2	2	3	2	1	3	2	2	2	2.1	0.48
a_6	1	1	1	1	1	2	1	1	2	0	2	1	1	1	1	1	1.1	0.48

FIGURE 20.5
Distribution of each kind of front axle used.

20.3 SIMULTANEOUS ACHIEVEMENT OF TWO SIMPLIFYING GOALS

- It is important to avoid successive processing of the products that have larger load of assembly time.

- Products have different loads at different processes.

20.3 SIMULTANEOUS ACHIEVEMENT OF TWO SIMPLIFYING GOALS

TOYOTA

The car models with longer assembly time	→	Larger loads at every process

- Avoid introducing successively the same product requiring longer assembly time to the line.

20.3 SIMULTANEOUS ACHIEVEMENT OF TWO SIMPLIFYING GOALS

- All models are classified according to large (a_l), medium (a_m), or small (a_s) total assembly time.

- Each model of a_j must be introduced to the line to keep the speed constant, j=l, m, s.

20.3 SIMULTANEOUS ACHIEVEMENT OF TWO SIMPLIFYING GOALS

- In practice, Toyota "weights" important subassemblies and provides constraints, such as facility capacities .

- The categories (a_l, a_m, a_s) are also given some weight to solve the conflict between the line balancing goal and the part smoothing goal.