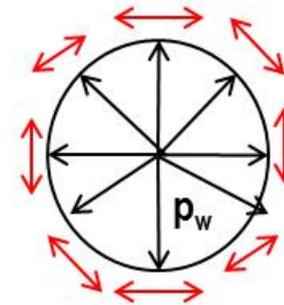
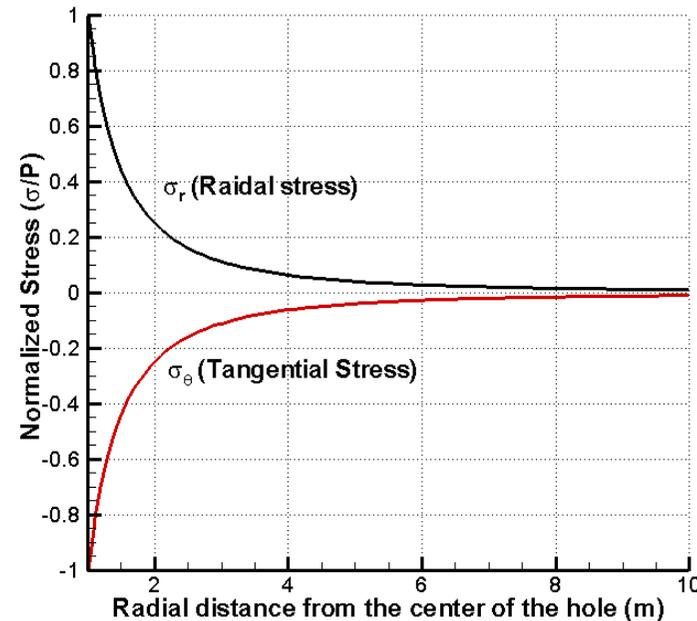


Topics in Energy and Environmental Geomechanics – Enhanced Geothermal Systems (EGS) Lecture 2. Fundamentals of Geomechanics (2)



Ki-Bok Min, PhD, Associate Professor
Department of Energy Resources Engineering
Seoul National University



Introduction

Contents of the course



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- Week 1 (2 Sept): Introduction to the course/Climate change & Emerging Subsurface Eng Application
- **Week 2 (9 Sept): Fundamentals of Geomechanics**
- Week 3 (16 Sept): Borehole Stability
- Week 4 (23 Sept): Borehole Stability
- Week 5 (30 Sept): No lecture (business trip)
- Week 6 (7 Oct) : Hydraulic Stimulation (focus on Hydraulic fracturing)
- Week 7 (14 Oct): Hydraulic Stimulation
- Week 8 (21 Oct): Induced seismicity
- Week 9 (28 Oct): Induced seismicity

Introduction

Contents of the course



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- Week 10 (4 Nov): Drilling Engineering (invited lecture)
- Week 11 (11 Nov): Well logging (invited lecture)
- Week 12 (18 Nov): EGS Case studies
- Week 13 (25 Nov): EGS Case studies
- Week 14 (2 Dec): Student Conference
- Week 15 (9 Dec): Final Exam (closed book or take-home exam)

Borehole stability problem



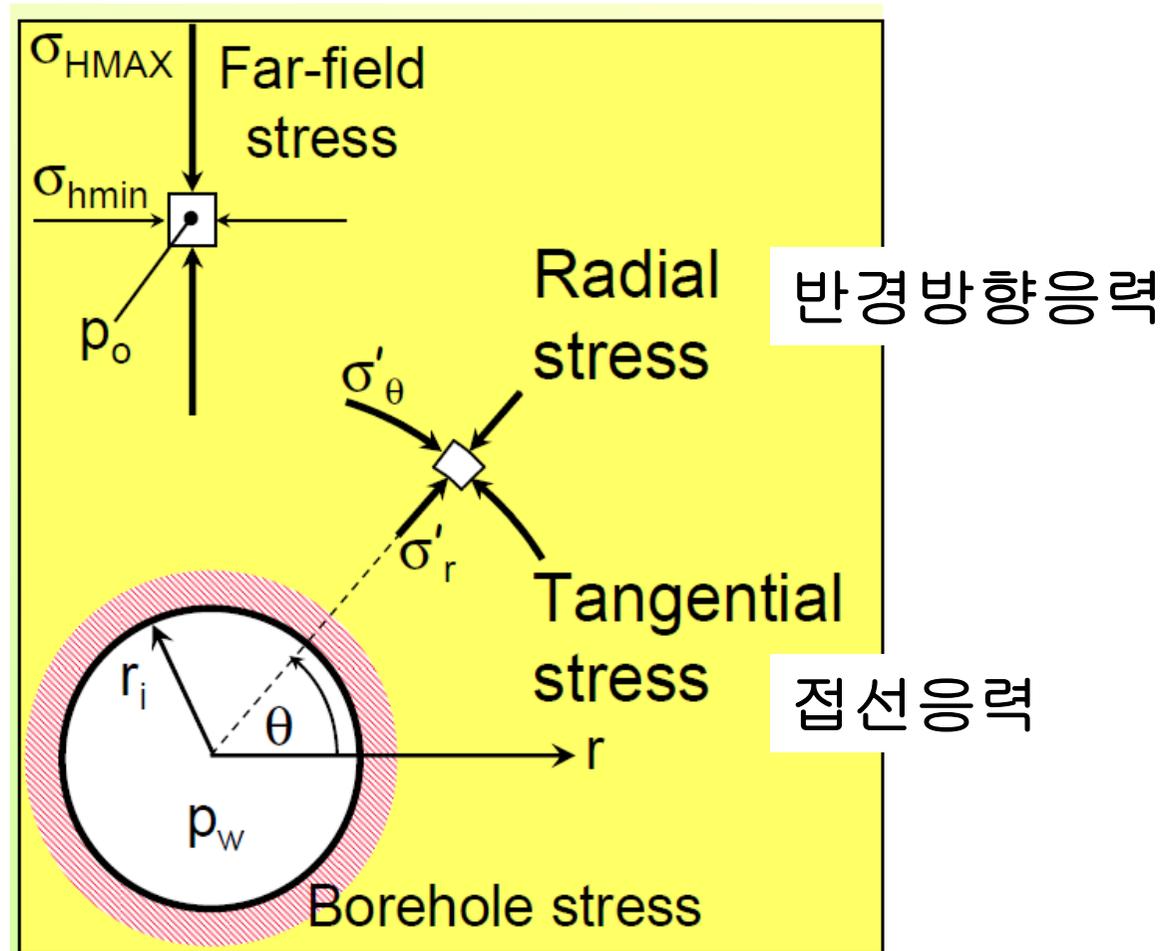
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- Importance?
 - Economy, Safety, Environment
 - Optimal trajectory and mud weight determination
- Factors
 - In situ stress
 - Injection pressure or mud weight
 - Reservoir rock mechanics properties
 - Anisotropy
 - Thermal effect
 - ...

Borehole stability problem



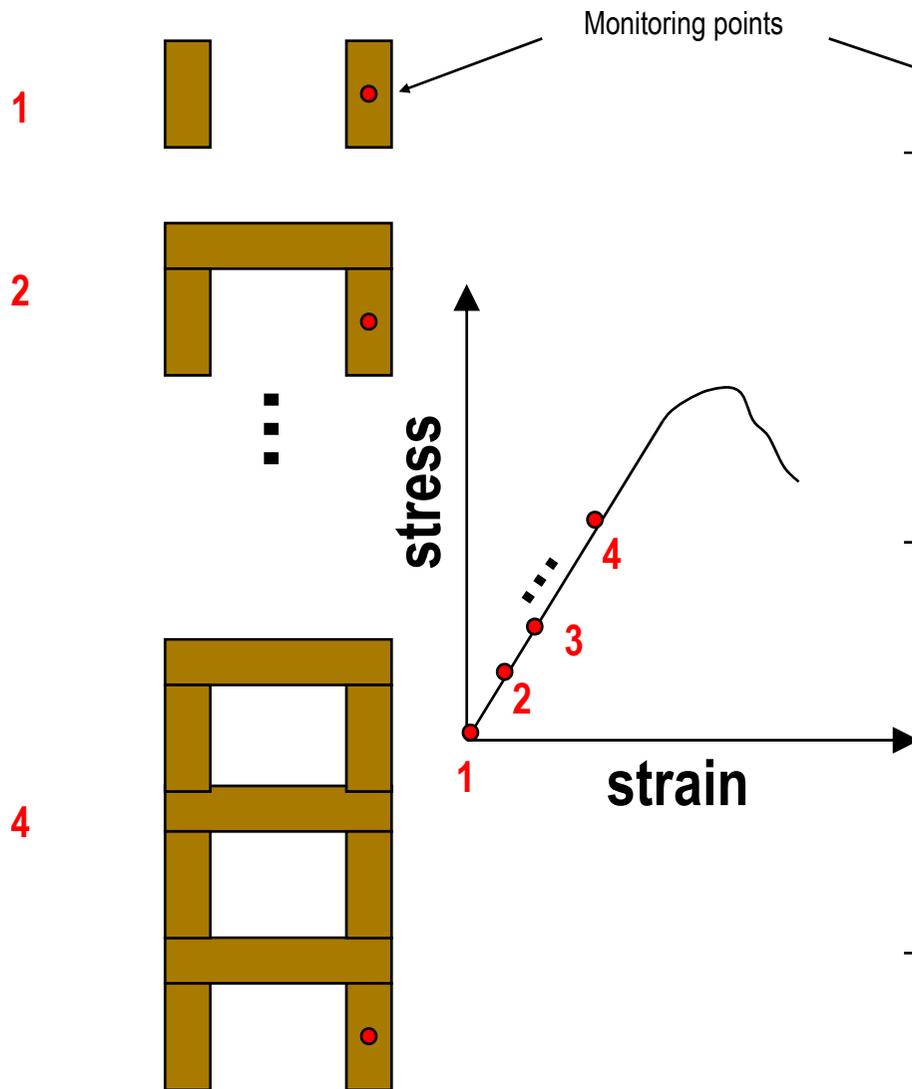
- Definitions – cylindrical coordinate



Nature of Underground Geomechanics

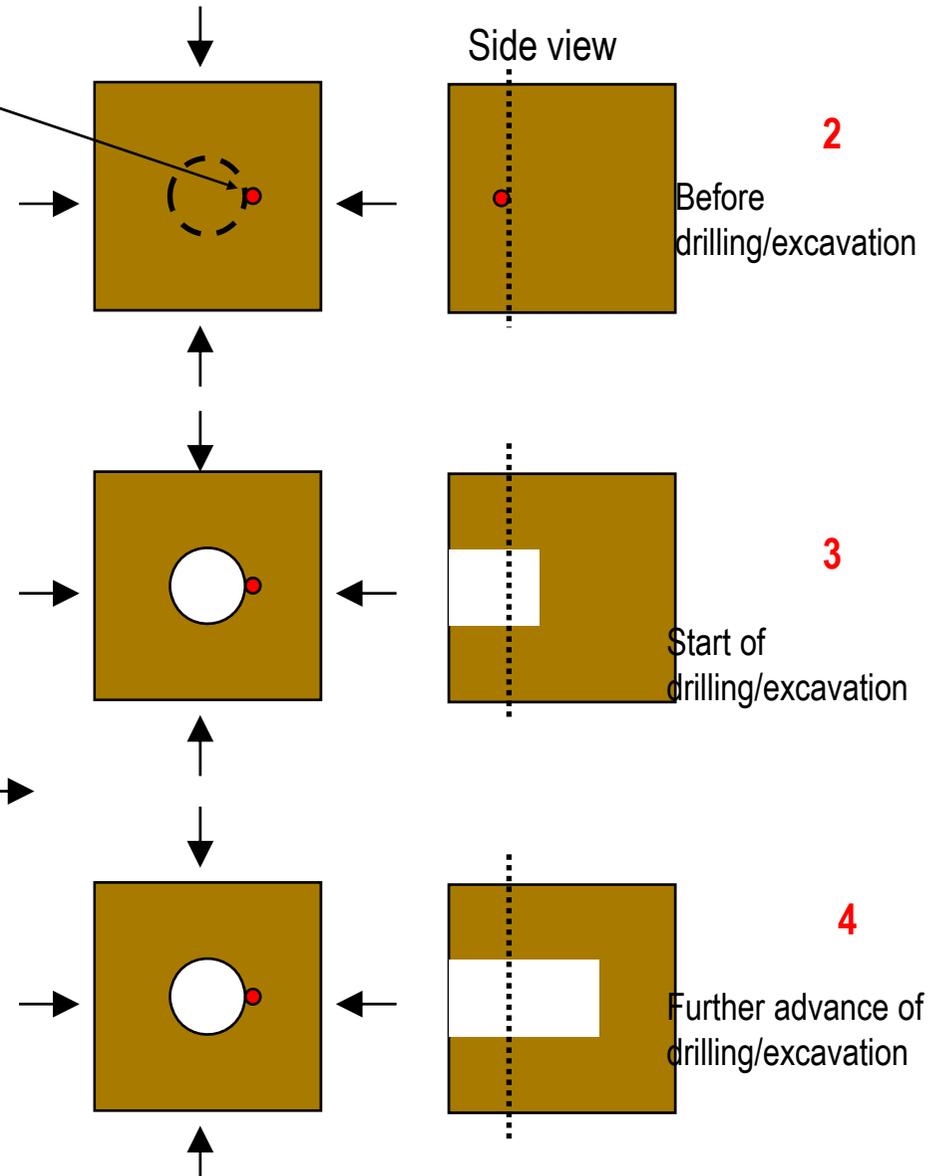
Civil structural problems:

Mechanics of **"Addition"**



Underground Geomechanics problems:

Mechanics of **"Removal"**



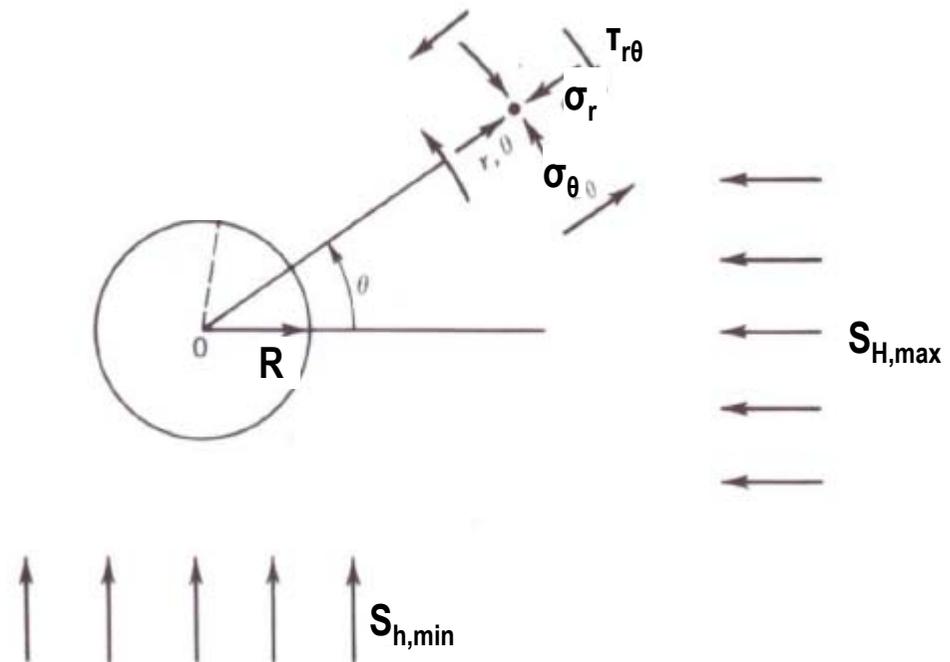
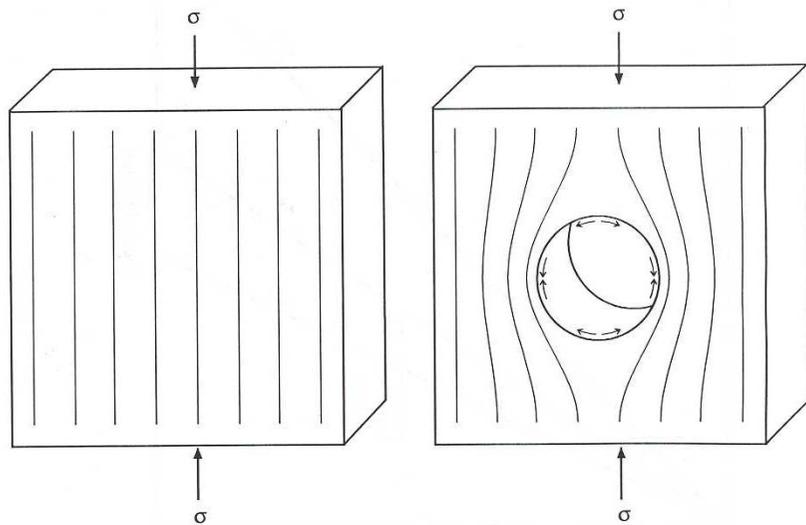
Borehole stability problem

Kirsch solution (1)



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- Stress distribution around circular borehole (Kirsch, 1898)
- Homogeneous, isotropic rock under plane strain condition



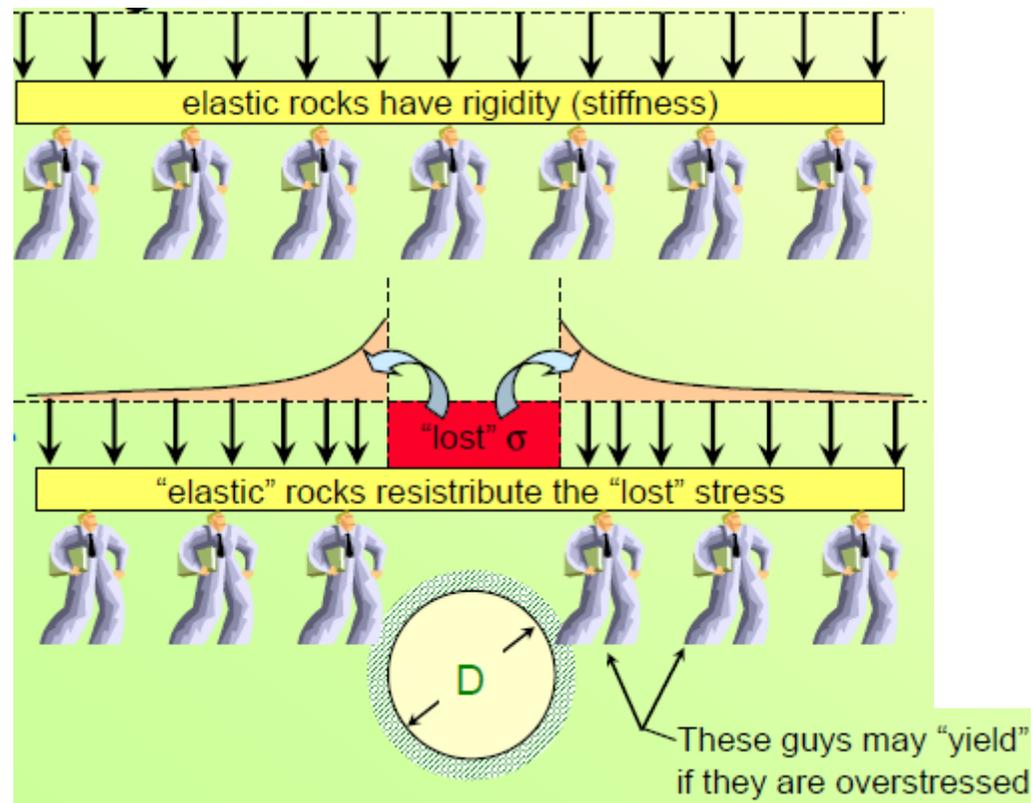
Borehole stability problem

Kirsch solution (2)



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- Stress redistribution(응력 재분배)



Borehole stability problem

Kirsch solution (3)



$$\sigma_r = \frac{S_{H \max} + S_{h \min}}{2} \left(1 - \frac{R^2}{r^2} \right) + \frac{S_{H \max} - S_{h \min}}{2} \left(1 - \frac{4R^2}{r^2} + \frac{3R^4}{r^4} \right) \cos 2\theta$$

$$\sigma_\theta = \frac{S_{H \max} + S_{h \min}}{2} \left(1 + \frac{R^2}{r^2} \right) - \frac{S_{H \max} - S_{h \min}}{2} \left(1 + \frac{3R^4}{r^4} \right) \cos 2\theta$$

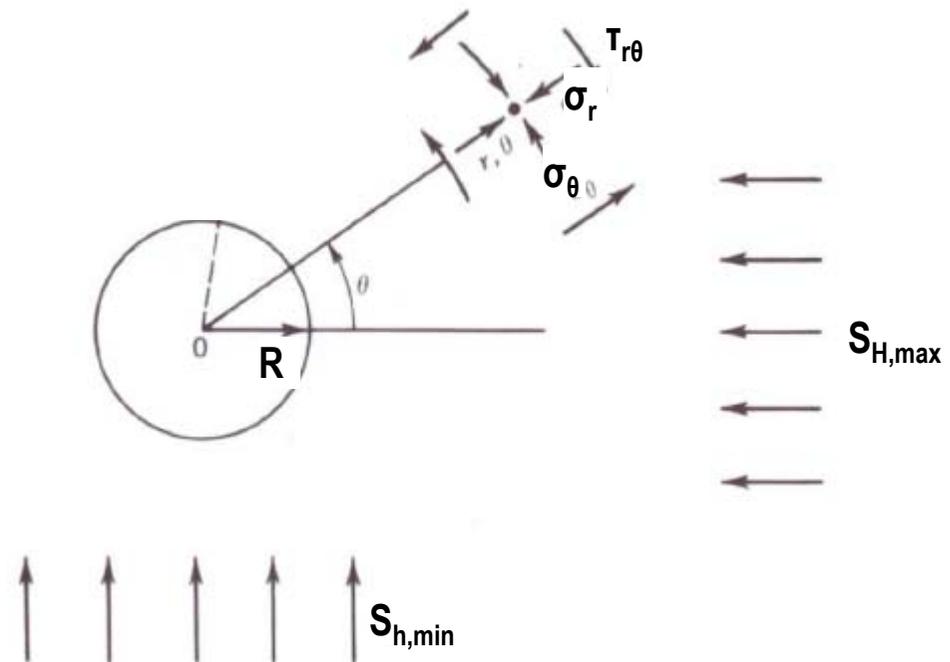
$$\tau_{r\theta} = \frac{S_{H \max} - S_{h \min}}{2} \left(1 + \frac{2R^2}{r^2} - \frac{3R^4}{r^4} \right) \sin 2\theta$$

R: 보어홀의 반경

r: 보어홀 중심에서
반경방향의 거리

θ : $S_{H \max}$ 으로부터 반시계
방향으로 측정

$S_{H \max}$, $S_{h \min}$: 최대 및 최소
수평응력

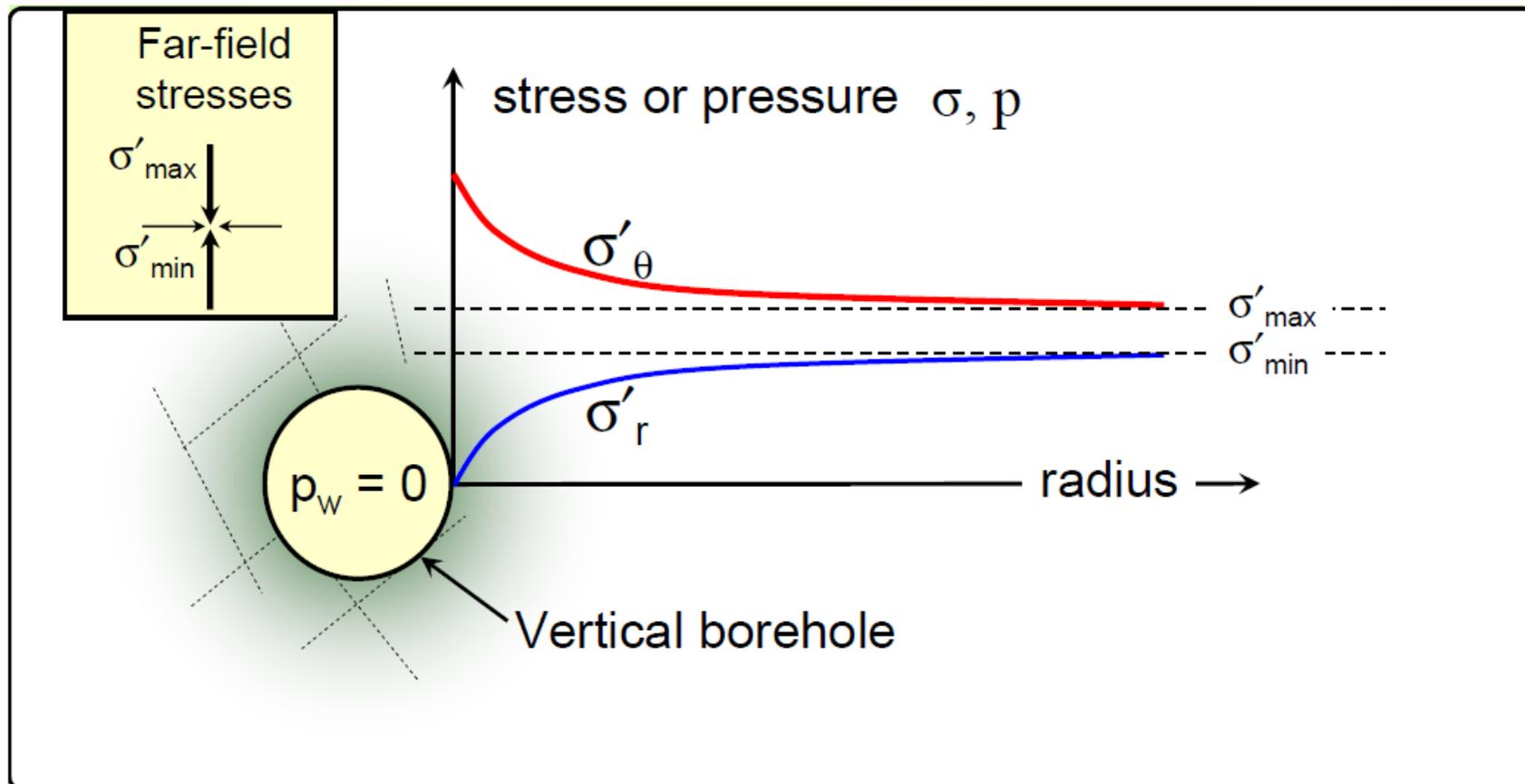


Borehole stability problem

Kirsch solution (4)



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Borehole stability problem

Kirsch solution (5)



- In the borehole surface ($R=r$)

$$\sigma_r = 0$$

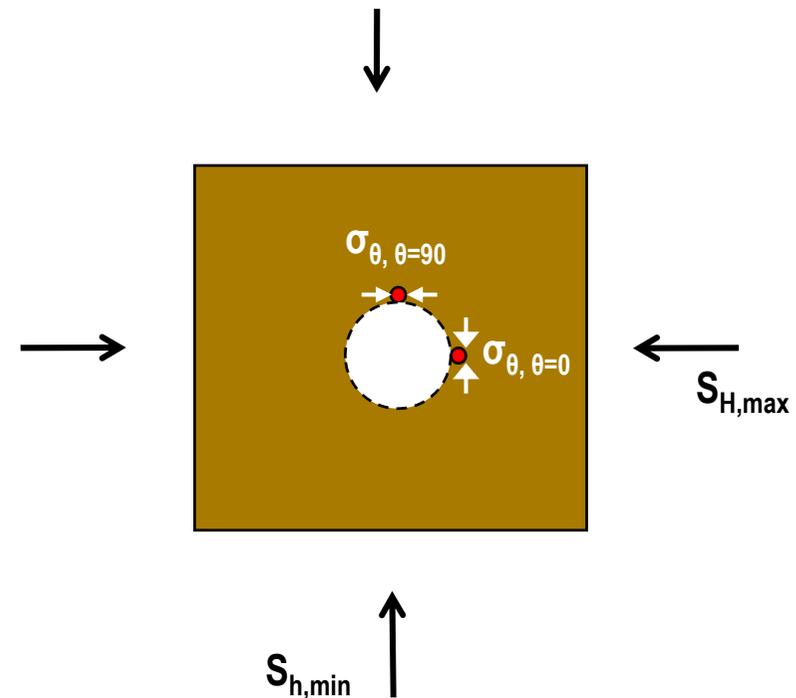
$$\sigma_\theta = S_{H \max} + S_{h \min} - 2(S_{H \max} - S_{h \min}) \cos 2\theta$$

$$\tau_{r\theta} = 0$$

$$\sigma_\theta = \frac{S_{H \max} + S_{h \min}}{2} \left(1 + \frac{R^2}{r^2} \right) - \frac{S_{H \max} - S_{h \min}}{2} \left(1 + \frac{3R^4}{r^4} \right) \cos 2\theta$$

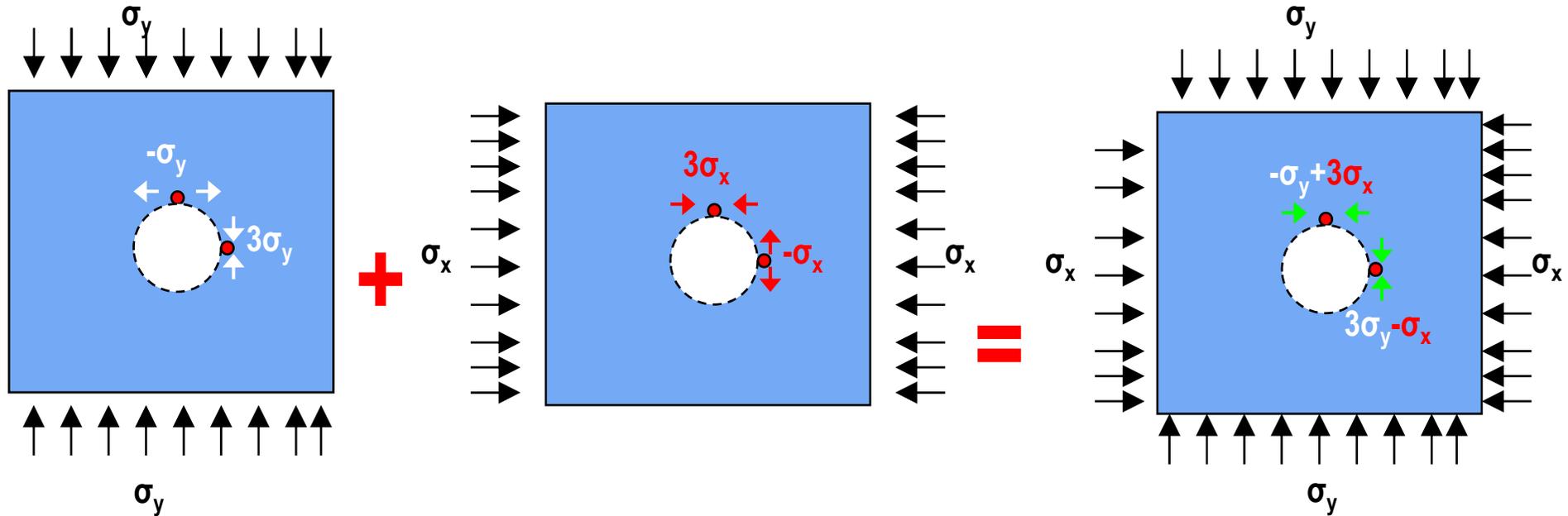
$$\theta = 0, \quad \sigma_\theta = -S_{H \max} + 3S_{h \min}$$

$$\theta = 90, \quad \sigma_\theta = 3S_{H \max} - S_{h \min}$$



Borehole stability problem

Kirsch solution (6)



Under uniaxial stress condition
 Maximum Stress concentration 3
 Minimum Stress concentration -1

Biaxial Stress
 condition:

By superposition

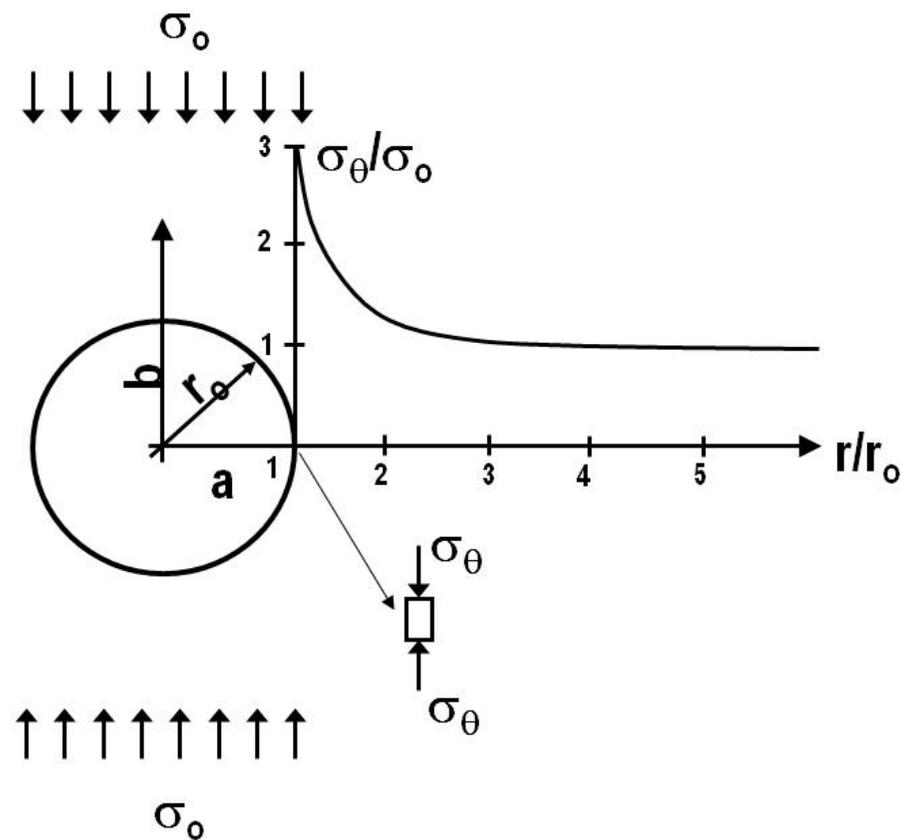
Borehole stability problem

Kirsch solution (7)



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- The area of stress disturbance is within 2~3 times of borehole radius

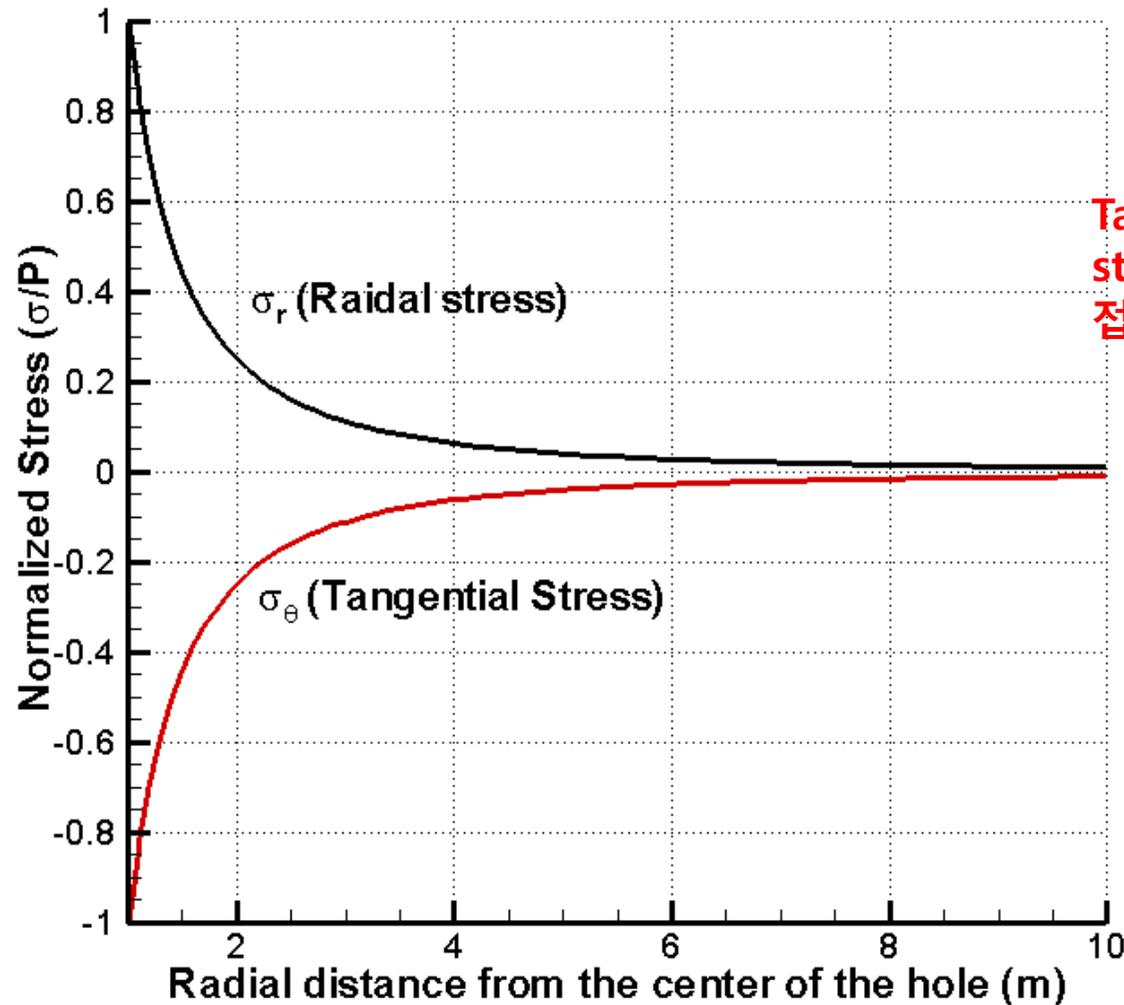


Borehole stability problem

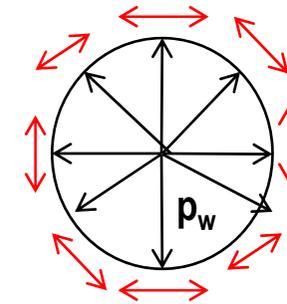
Internal pressure (1)



- Increase of internal mud/hydraulic pressure



Tangential stress
접선응력



$$\sigma_r = P_w \frac{R^2}{r^2}$$

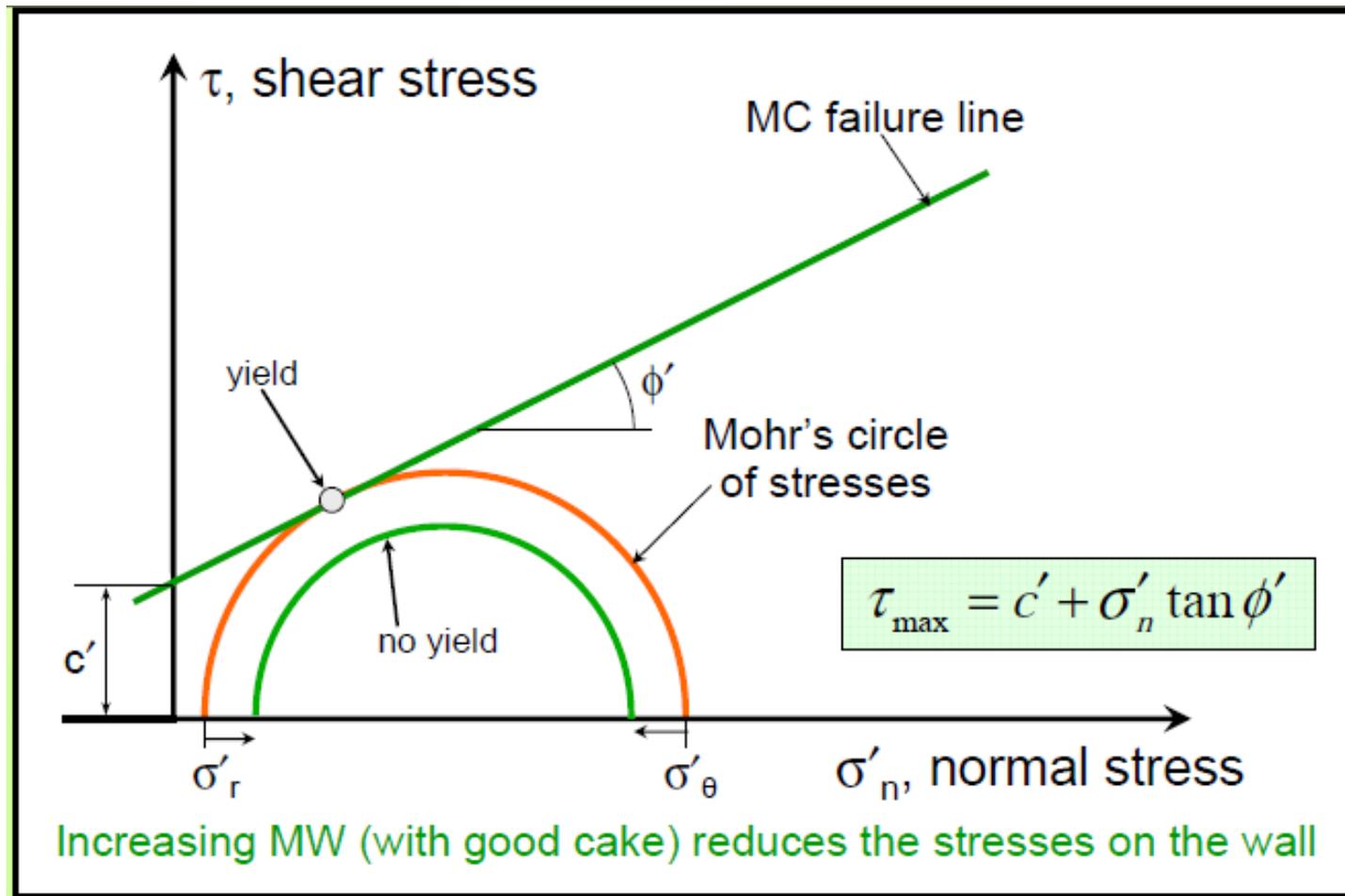
$$\sigma_\theta = -P_w \frac{R^2}{r^2}$$

Borehole stability problem

Internal pressure (2)



- 이수밀도의 증가 (or 주입압력증가) 는 공벽의 응력을 감소시킴.

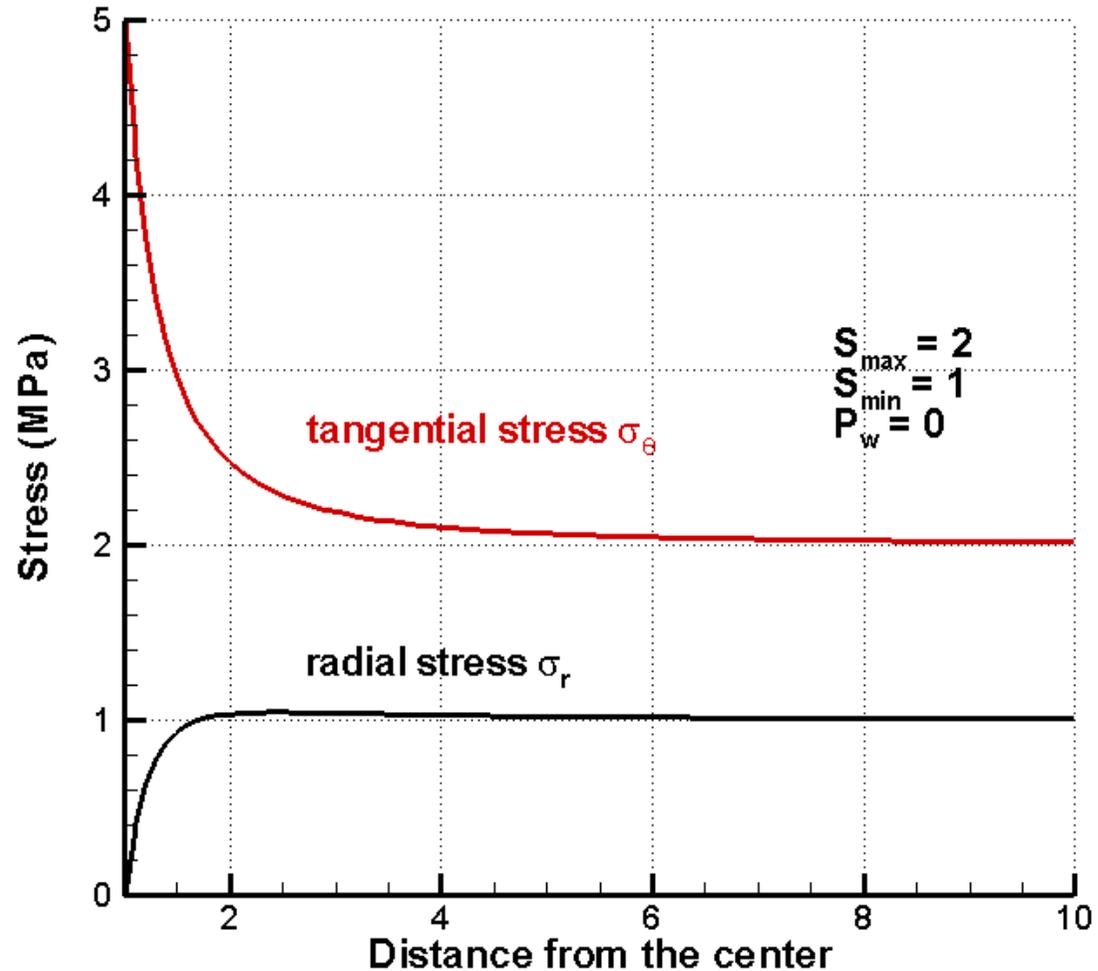
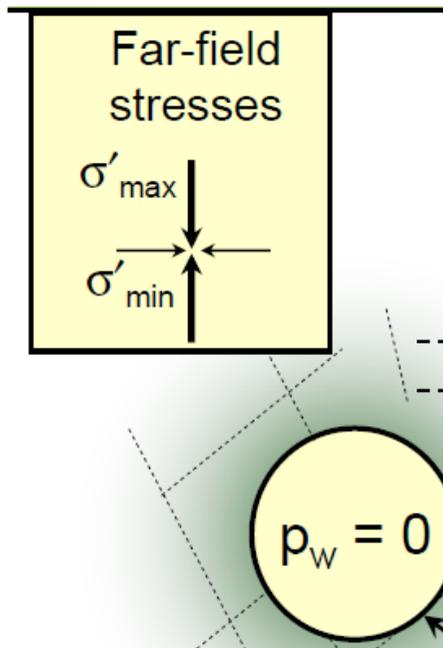


Borehole stability problem

Example (1)



- Anisotropic Boundary Stress

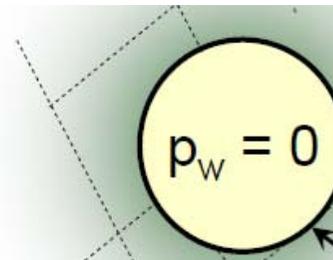
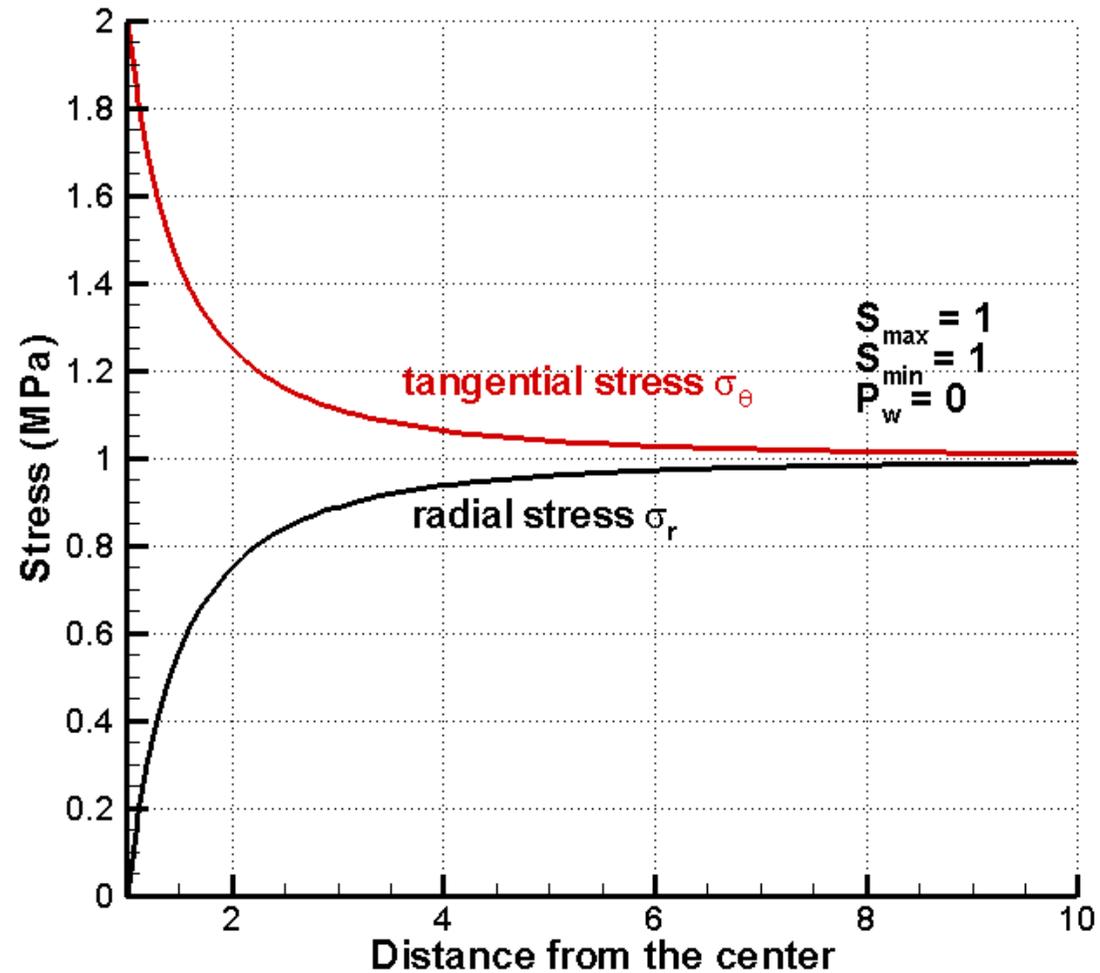


Borehole stability problem

Example (2)



- Isotropic Boundary Stress

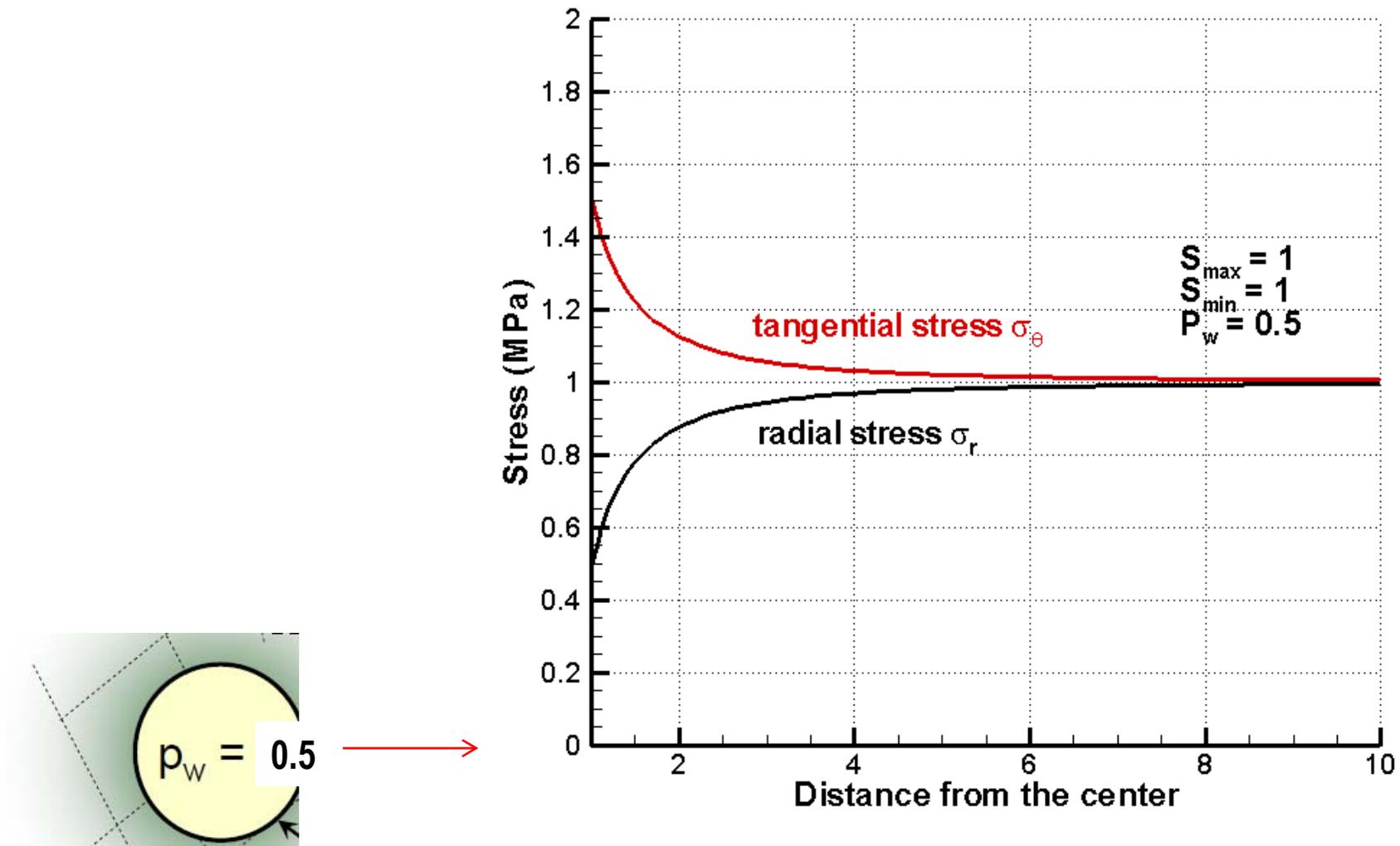


Borehole stability problem

Example (3)



- Isotropic boundary stress + Injection pressure (0.5)

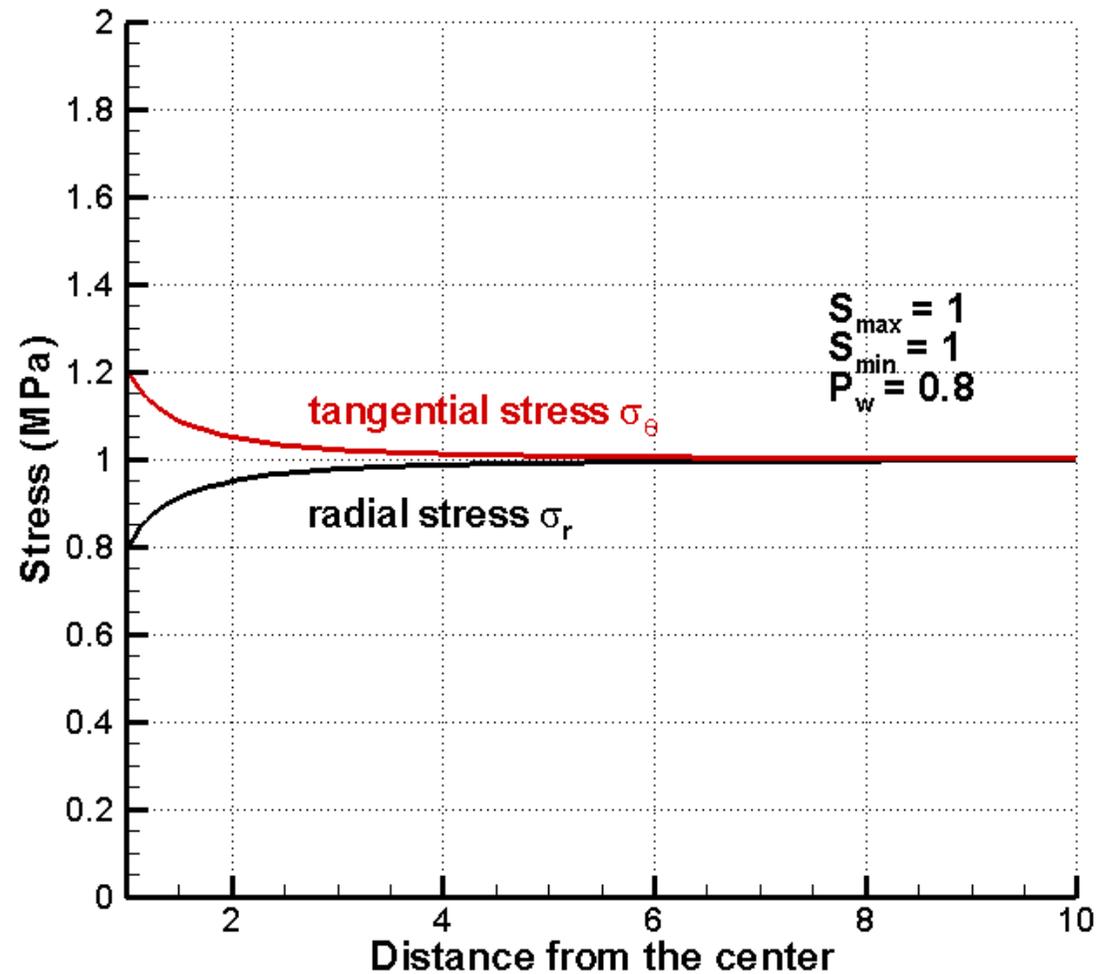
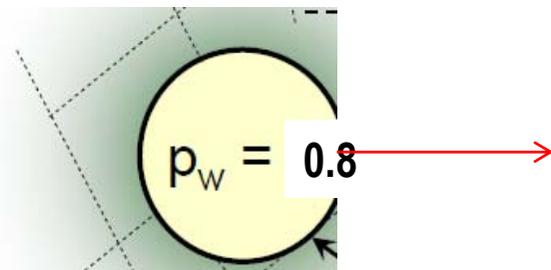


Borehole stability problem

Example (4)



- Isotropic boundary stress + Injection pressure (0.8)



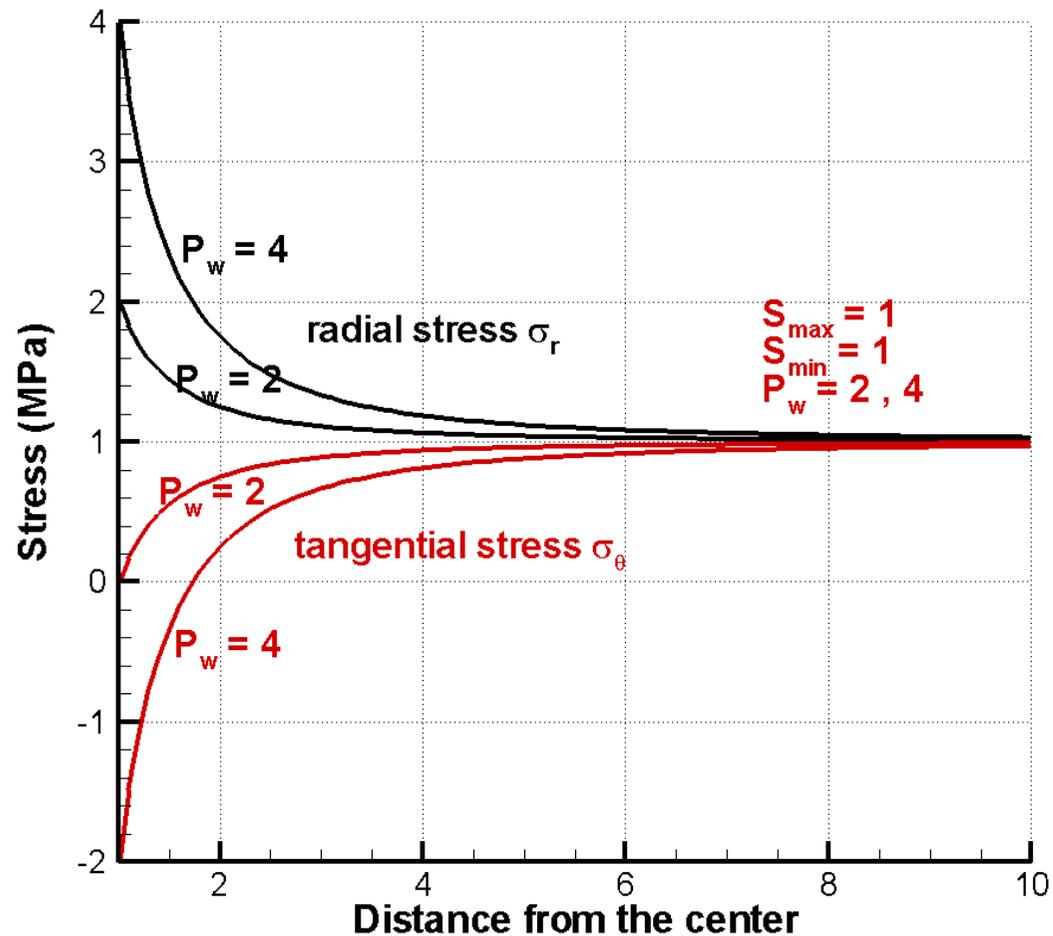
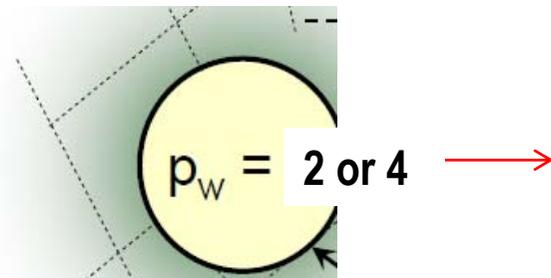
Borehole stability problem

Example (5)



- Isotropic boundary stress + Injection pressure (2-4)

- Hydraulic Fracturing can occur when internal pressure is big enough



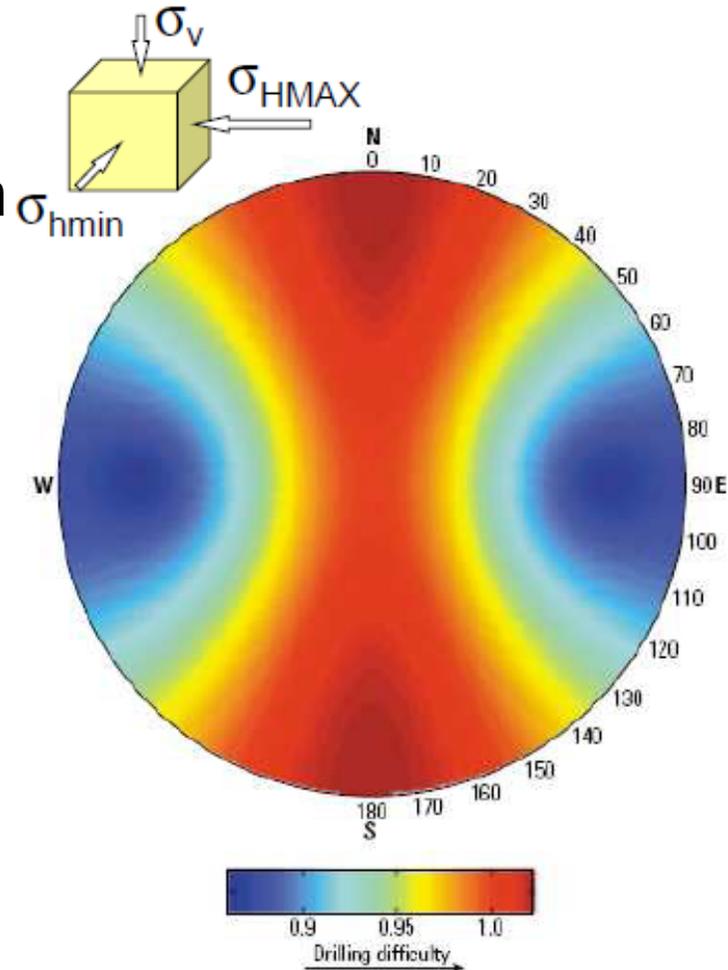
Borehole stability problem

Borehole Trajectory



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- Drilling trajectory (최적 시추 궤적 결정)
 - Determine the trajectory that can induce the least stress concentration



Borehole stability problem

Thermal stress(열팽창 및 열응력)



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- Linear thermal expansion coefficient (unit: /K)

$$\frac{\Delta l}{l} = \alpha(T - T_0)$$

- Thermal stress ← thermal expansion + mechanical restraint

- Thermal stress in 1D

- Thermal stress when a rock is completely (in all directions) restrained

$$\sigma_T = 3\alpha K(T - T_0) = \frac{E}{1 - 2\nu} \alpha(T - T_0)$$

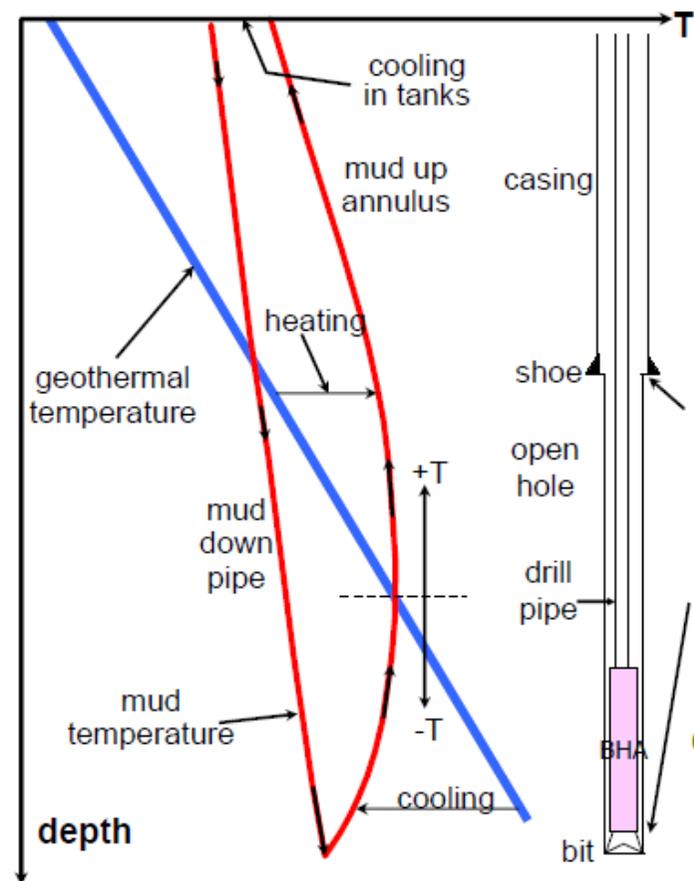
Borehole stability problem

Thermal stress(열팽창 및 열응력)



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- Downhole : Cooling \rightarrow stabilize
- Uphole: Heating \rightarrow can induce failure



Stress around circular borehole

Generalized Kirsch's solution



- Boundary stress + internal pressure + temperature;
 - 응력경계 (Principal in situ stress boundary)
 - 내부 주입압력 (Internal pore pressure (mud/water pressure))
 - 온도변화 (Temperature change)

$$\sigma_r = \frac{S_{H \max} + S_{h \min}}{2} \left(1 - \frac{R^2}{r^2} \right) + \frac{S_{H \max} - S_{h \min}}{2} \left(1 - \frac{4R^2}{r^2} + \frac{3R^4}{r^4} \right) \cos 2\theta + P_w \frac{R^2}{r^2}$$
$$\sigma_\theta = \frac{S_{H \max} + S_{h \min}}{2} \left(1 + \frac{R^2}{r^2} \right) - \frac{S_{H \max} - S_{h \min}}{2} \left(1 + \frac{3R^4}{r^4} \right) \cos 2\theta - P_w \frac{R^2}{r^2} + \frac{E}{1-\nu} \alpha (T_w - T_0)$$
$$\tau_{r\theta} = \frac{S_{H \max} - S_{h \min}}{2} \left(1 + \frac{2R^2}{r^2} - \frac{3R^4}{r^4} \right) \sin 2\theta$$

Stress around circular borehole

Generalized Kirsch's solution



- At the borehole wall ($r = R$), maximum and minimum hoop stresses are;

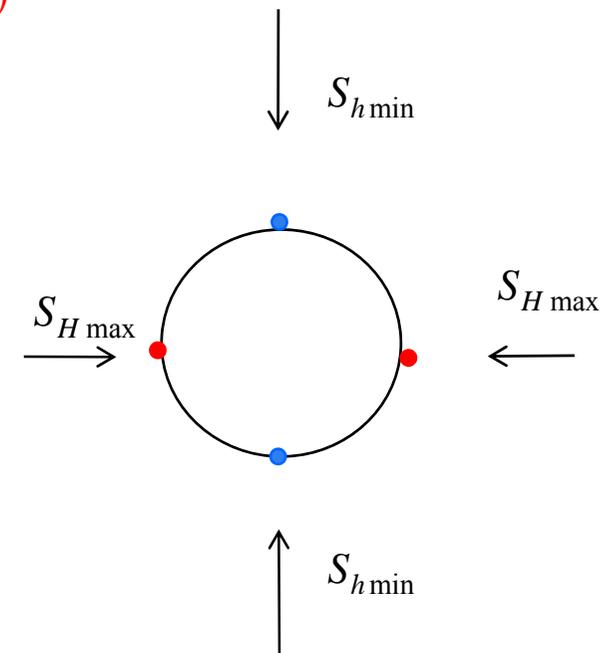
- $\sigma_{\theta, \min} = 3S_{h \min} - S_{H \max} - P_w + \frac{E}{1-\nu} \alpha (T_w - T_0)$

- $\sigma_{\theta, \max} = 3S_{h \max} - S_{h \min} - P_w + \frac{E}{1-\nu} \alpha (T_w - T_0)$

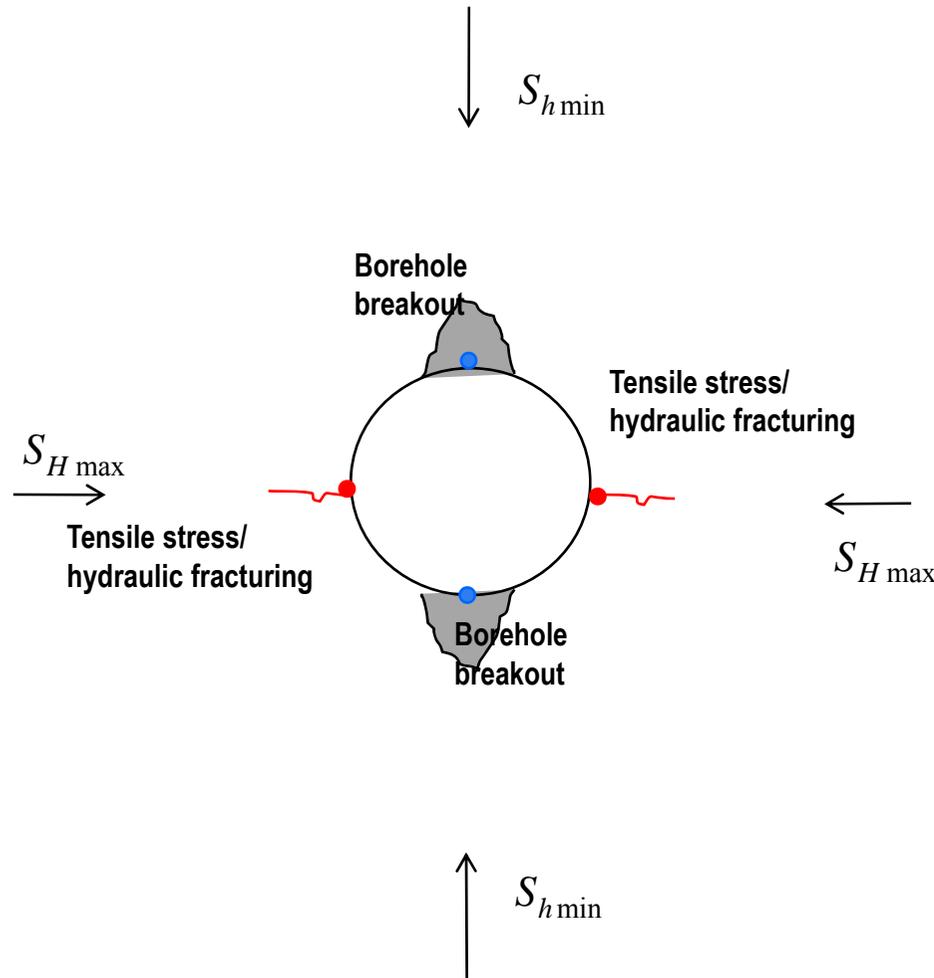
- Without considering temperature change,

$$\sigma_{\theta, \min} = 3S_{h \min} - S_{H \max} - P_w$$

$$\sigma_{\theta, \max} = 3S_{h \max} - S_{h \min} - P_w$$



Stress around circular borehole borehole breakout vs. hydraulic fracturing



– Required internal pressure to induce tensile stress (neglect T effect);

- $P_w > 3S_{h\min} - S_{H\max}$

– To induce hydraulic fracturing

- $P_w > 3S_{h\min} - S_{H\max} + T_0$

– Required uniaxial compressive strength not to have borehole breakout

- $\sigma_c > 3S_{h\max} - S_{h\min} - P_w$

Borehole breakout

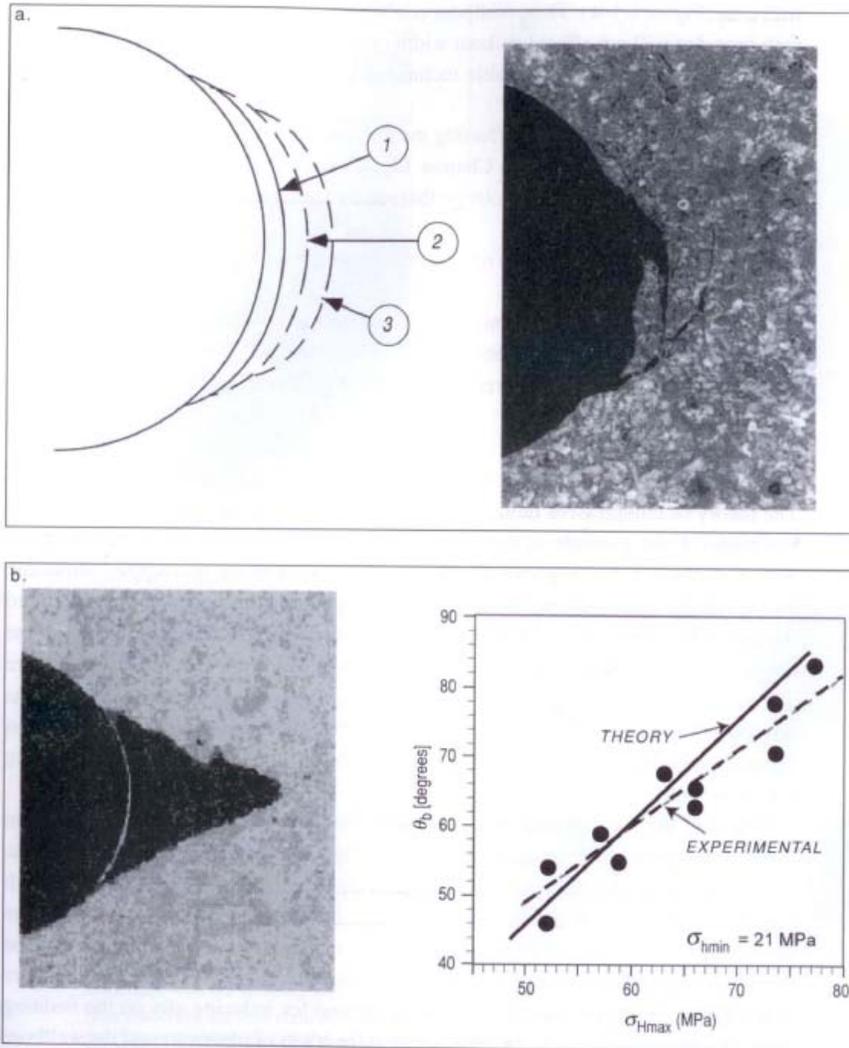
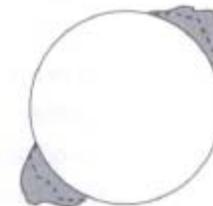


Figure 6.15. After the formation of wellbore breakouts, they are expected to increase in depth, but not width. This is as shown theoretically in (a) after Zoback, Moos *et al.* (1985) and confirmed by laboratory studies (Haimson and Herrick 1989). It can be seen photographically that breakouts in laboratory experiments deepen but do not widen after formation. As shown in (b), measured breakout widths compare very well with those predicted by the simple theory presented in Zoback, Moos *et al.* (1985) which form the basic for the breakout shapes illustrated in Figures 6.2 and 6.3.

- wellbore enlargements caused by stress-induced failure of a well occurring 180 degree apart
- Induced by compressive failure
- Occur in the direction of minimum horizontal stress



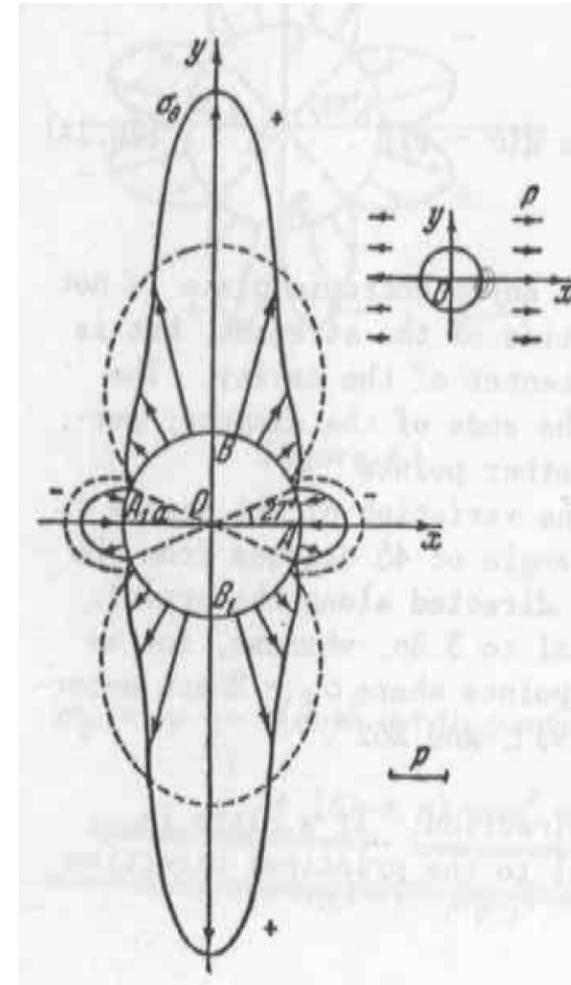
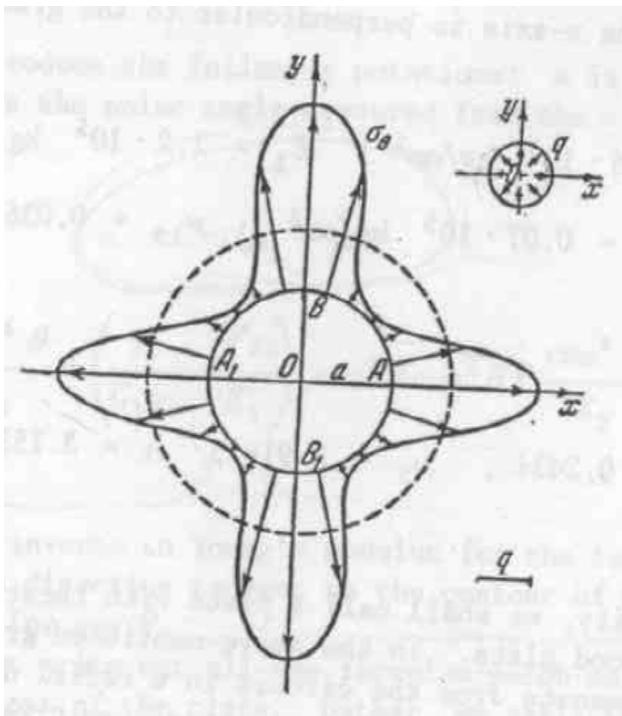
Borehole stability problem

Anisotropy



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- Some formulae considering anisotropy (internal pressure, uniaxial stress)



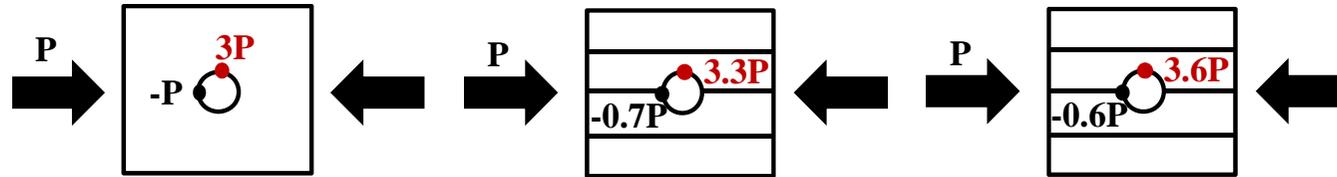
Lekhnitskii, 1963

Borehole stability problem

Anisotropy



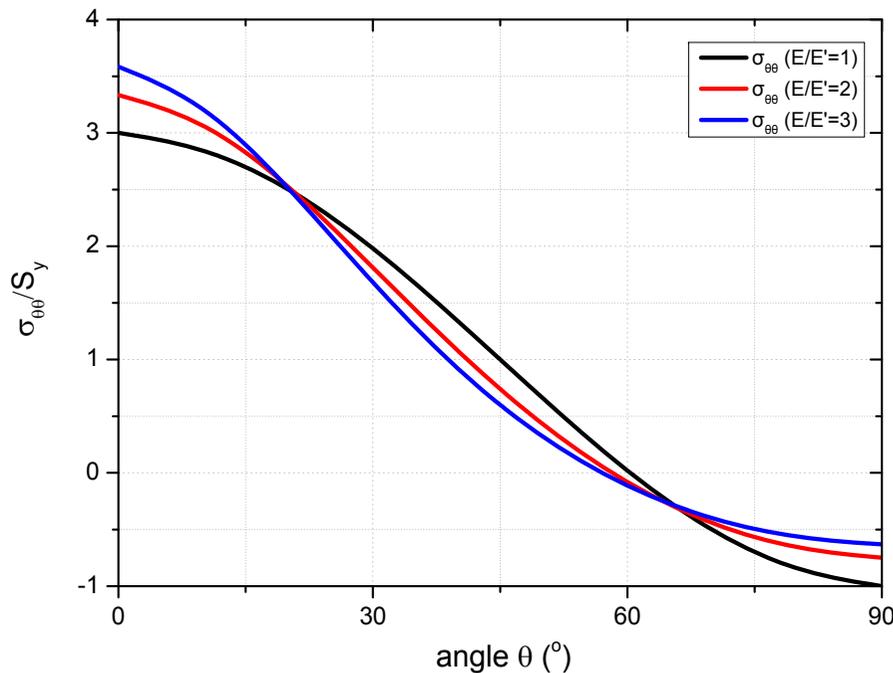
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Isotropic rock
($E/E' = 1$)

Transversely
isotropic rock
($E/E' = 2$)

Transversely
isotropic rock
($E/E' = 3$)



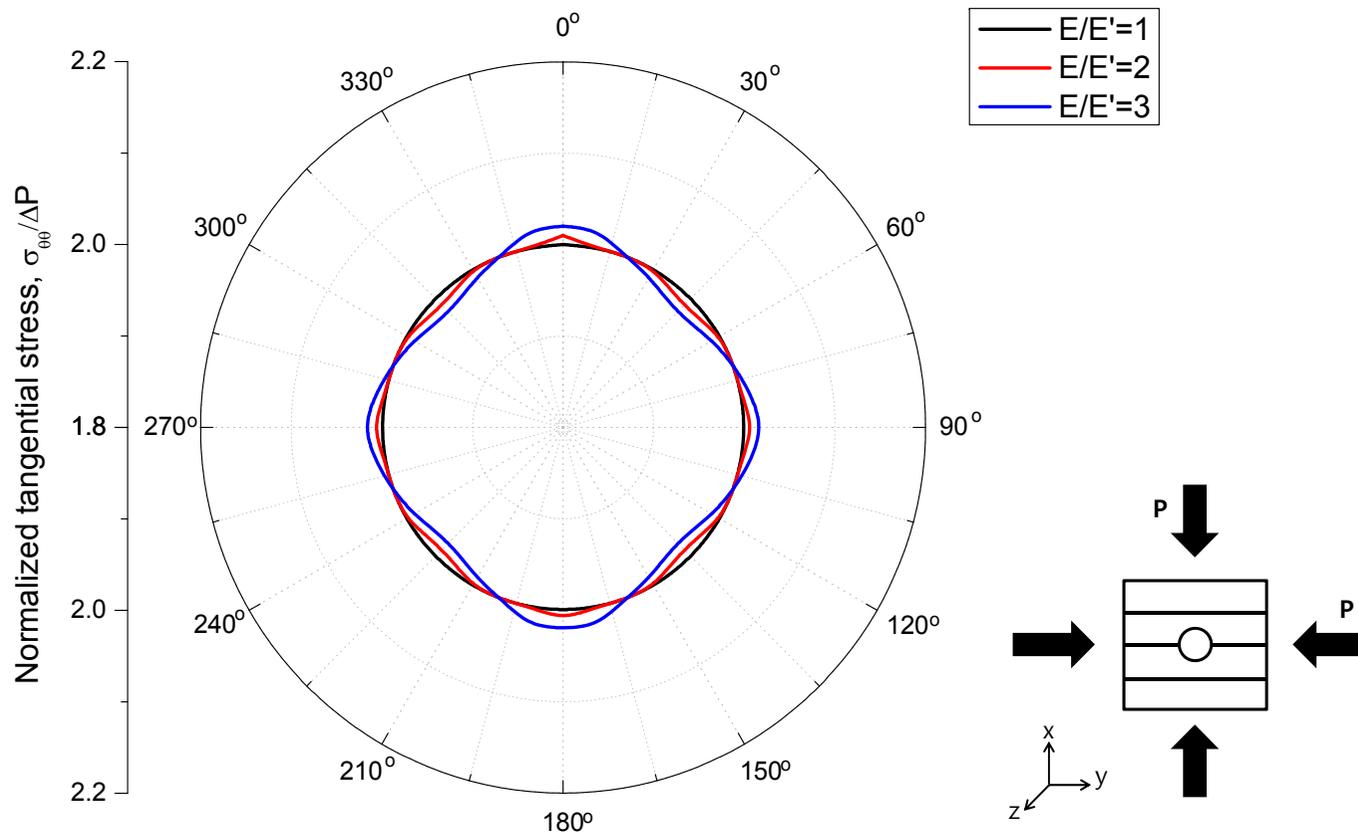
- 탄성계수의 이방성에 따라 응력집중의 정도가 달라짐.

Borehole stability problem

Anisotropy



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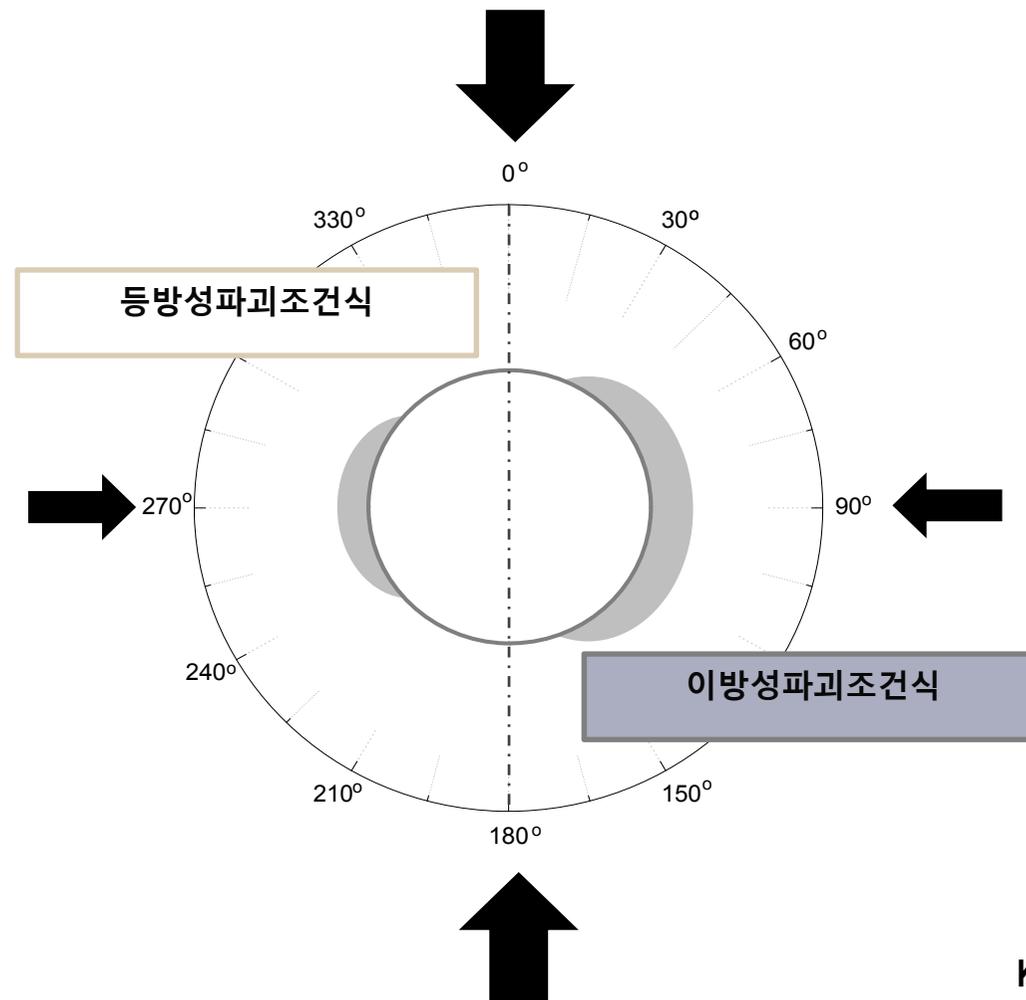


Borehole stability problem

Anisotropy



- 어떤 파괴조건식을 적용하느냐에 따라 파괴범위가 다르게 나타남



Borehole stability problem

Anisotropy – North Sea Example (1)

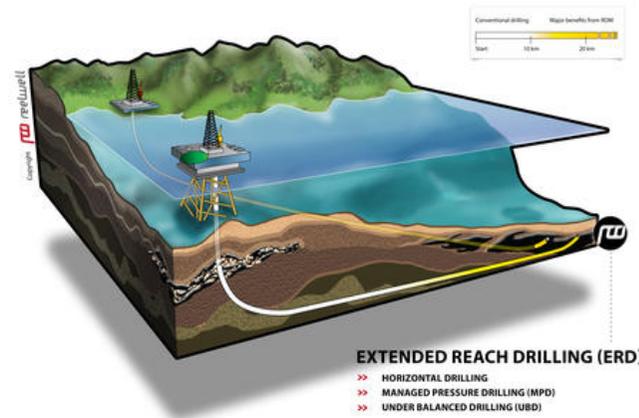


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Oseberg in North Sea (Norway)



- **Extended Reach Drilling (ERD)** has been employed for increasing oil recovery.
- Total Depth = **9,327 m**
- Since 1979, total depth for wells has increased steadily.



Okland & Cook, SPE, 1998

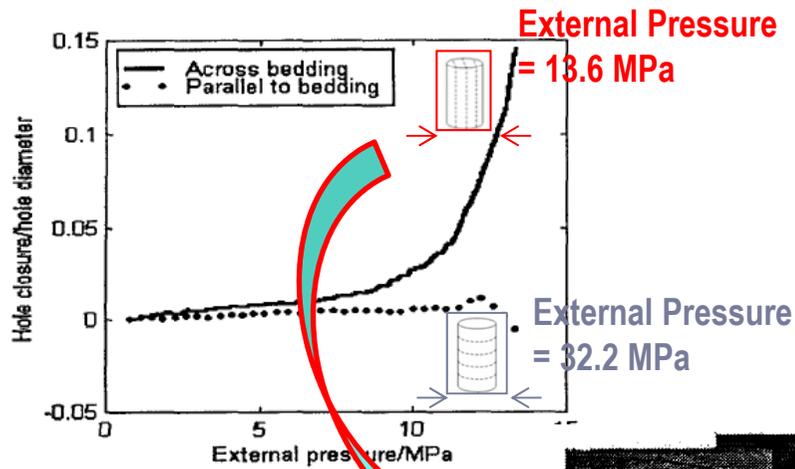
Borehole stability problem

Anisotropy – North Sea Example (2)

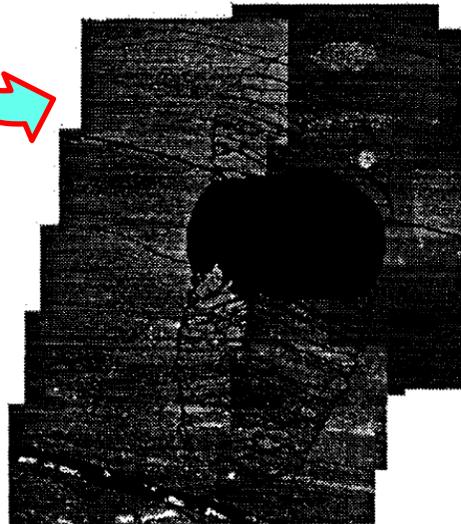


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a) Draupne-similar Shale

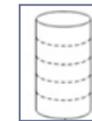
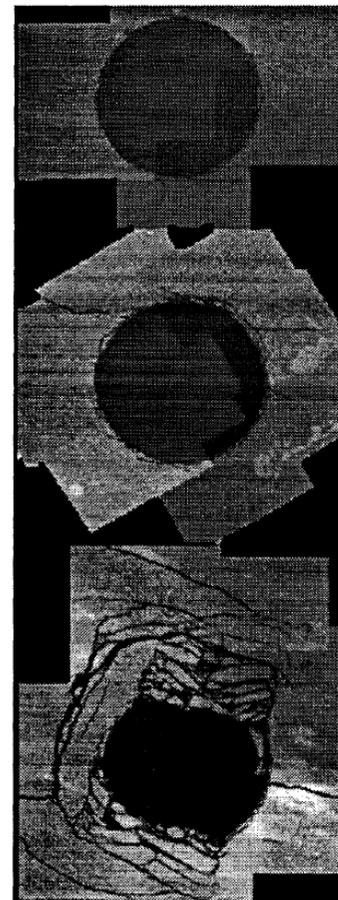


Severely Damaged!!



b) J₂ (Outcrop shale)

Loaded and unloaded at approximately 20 MPa

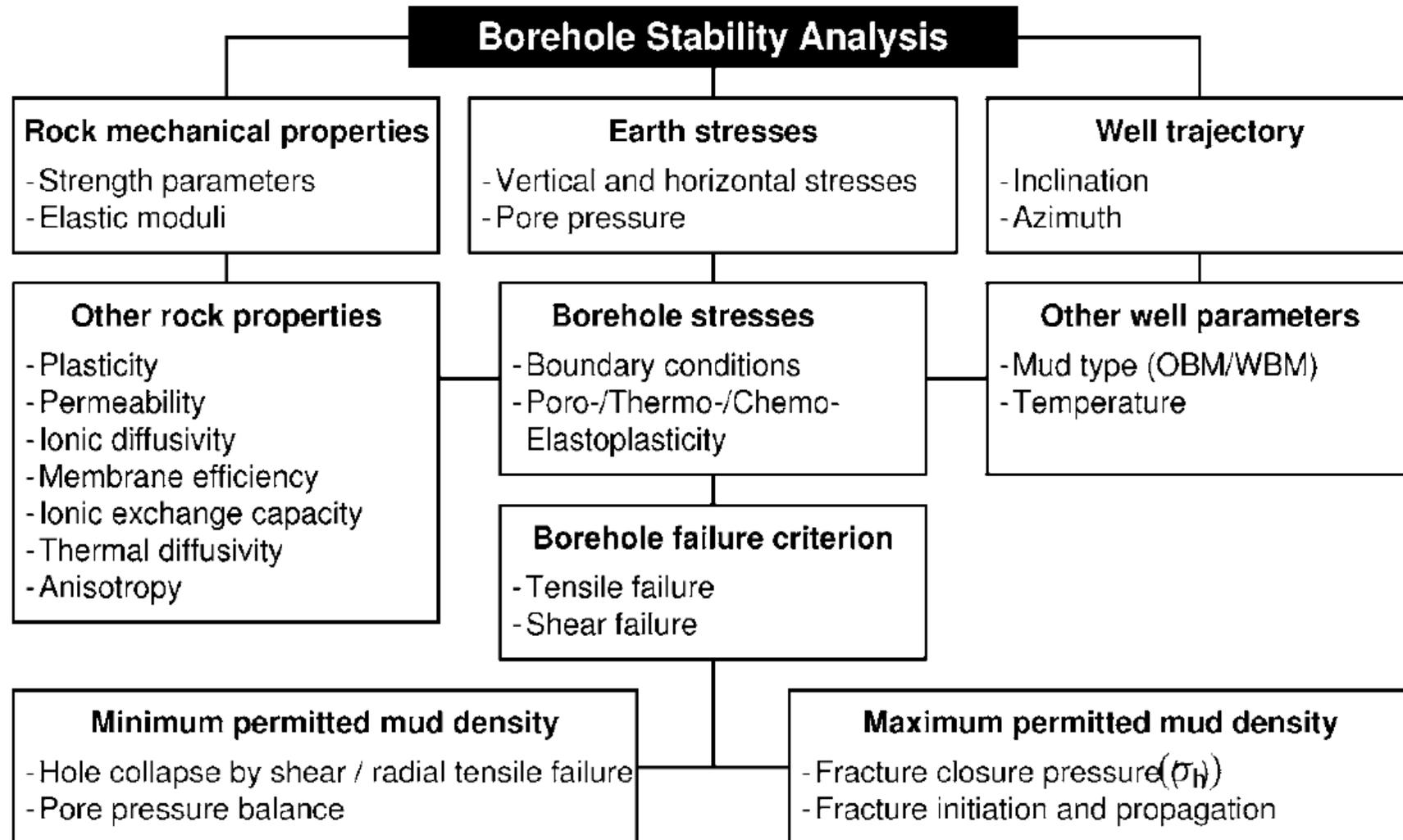


Undamaged



Borehole stability problem

Work Flow chart

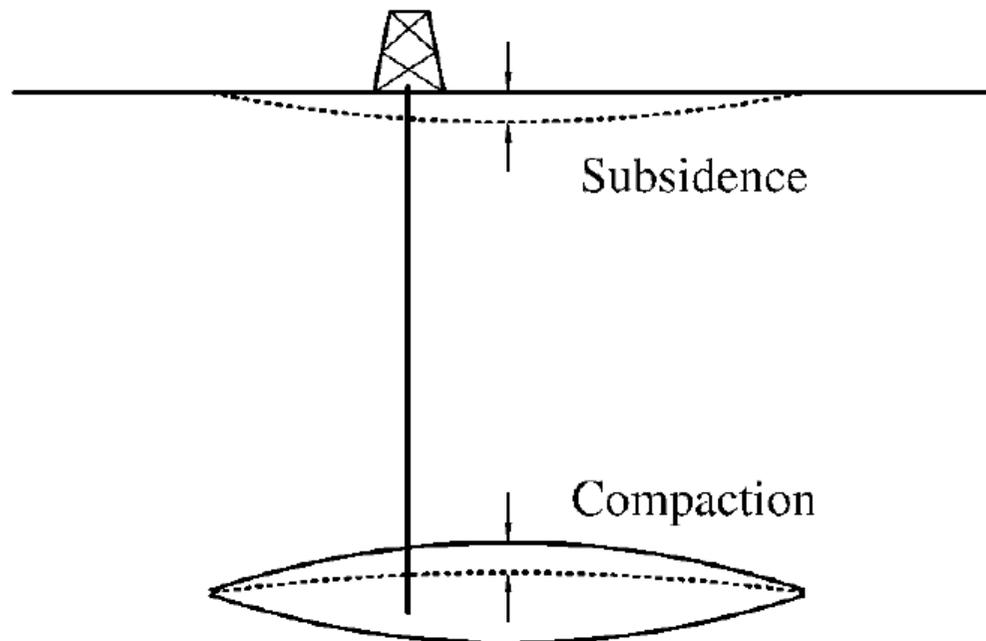


Reservoir subsidence (저류층 침하) Mechanism



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- Subsidence induced by the decrease of reservoir formation pressure
- $P \downarrow \rightarrow$ effective stress $\uparrow \rightarrow$ compression



Reservoir subsidence (저류층 침하) Mechanism



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- More serious when;
 - Bigger reservoir formation pressure decrease
 - Thick reservoir
 - Compressible reservoir
 - E.g.) Wilmington field in California: 9 m subsidence (Fjaer, 2008)

Reservoir subsidence (저류층 침하) Poroelastic stress change



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- Change of horizontal stress due to pore pressure change

$$\Delta S_h = \alpha \frac{(1-2\nu)}{(1-\nu)} \Delta p_p = k \Delta p_p$$

S_h : 수평방향응력 (horizontal stress)

ν : 암석의 포아송비 (Poisson's ratio)

k : 응력경로계수 (stress path factor, SPF)

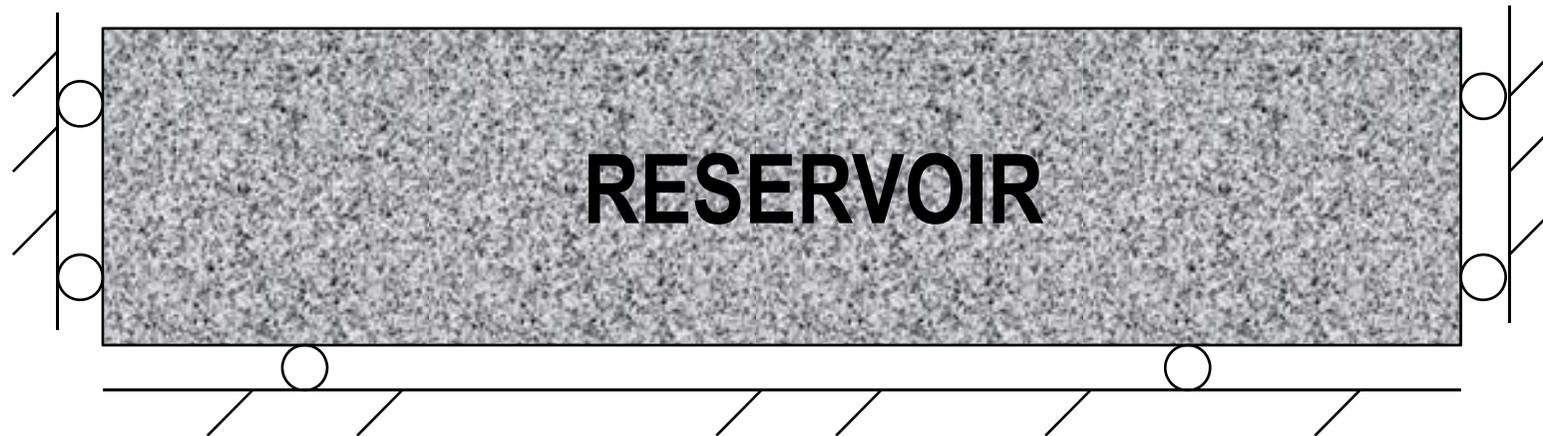
Reservoir subsidence (저류층 침하) Uniaxial compaction model



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- Compaction coefficient or uniaxial compressibility, C_m ;

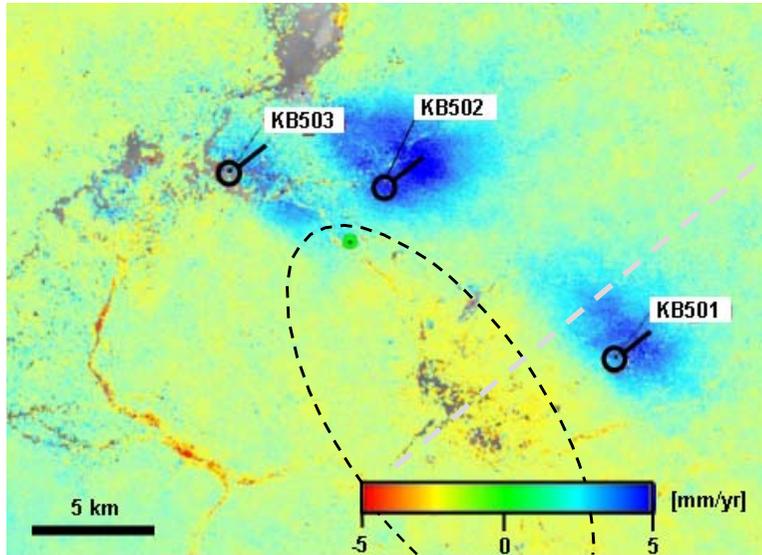
$$\frac{\Delta h}{h} = -C_m \alpha \Delta p = -\frac{1}{E} \frac{(1+\nu)(1-2\nu)}{1-\nu} \alpha \Delta p$$



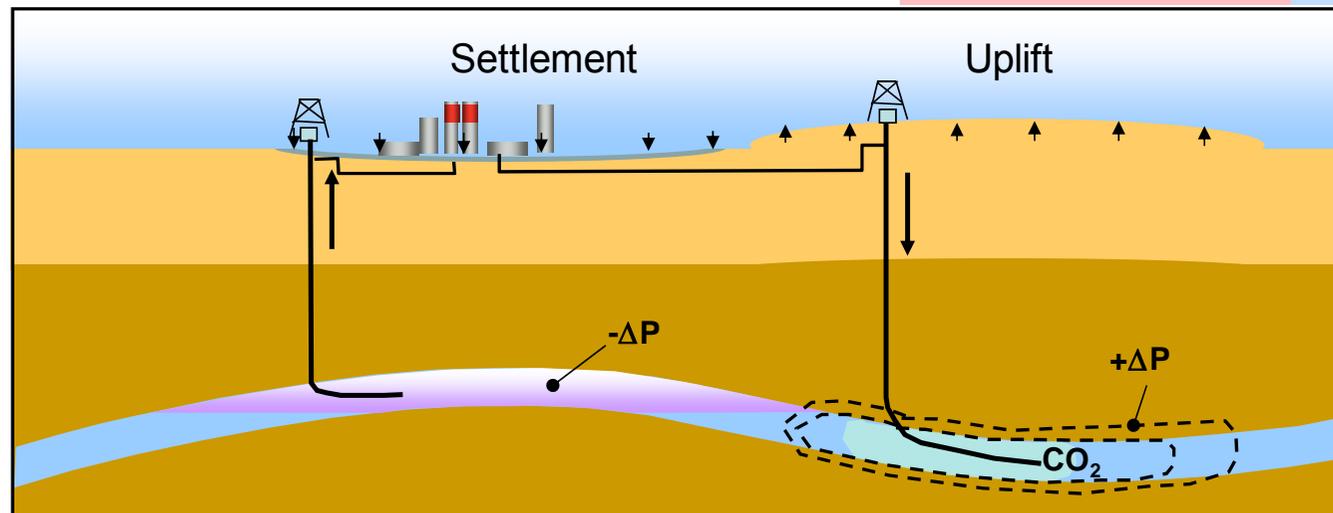
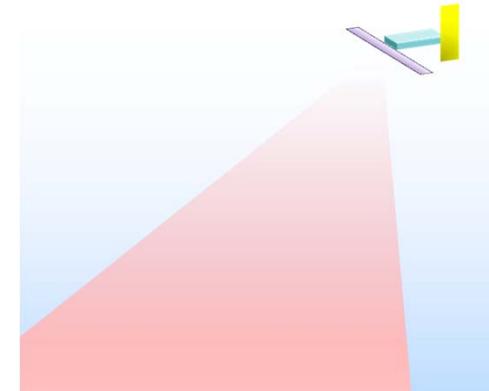
Reservoir subsidence (저류층 침하) Example(2): In Salah (Gas/CO₂ storage)



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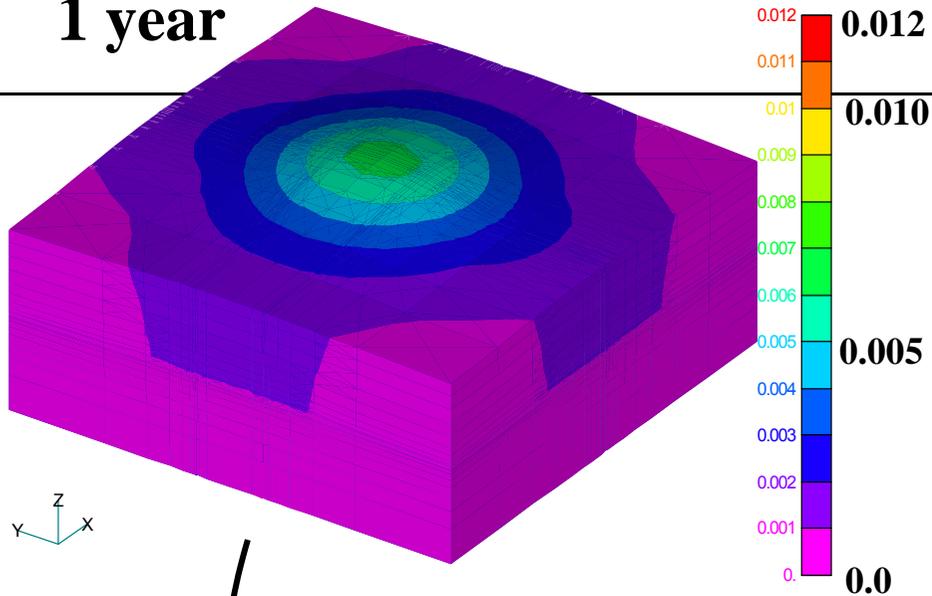
- Annually, 5 mm (above injection holes)
- Heaving and Subsidence



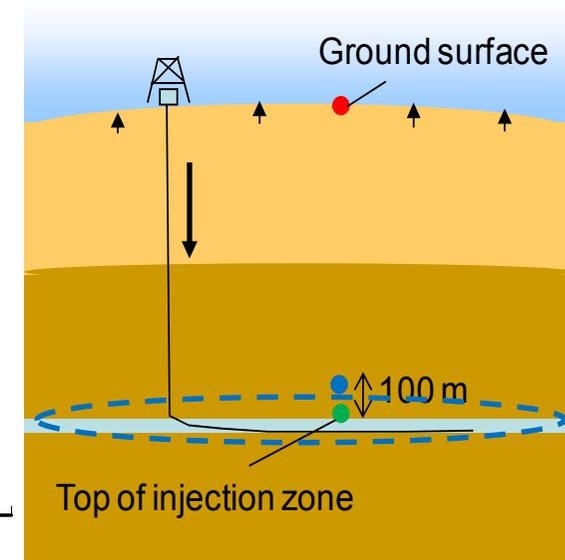
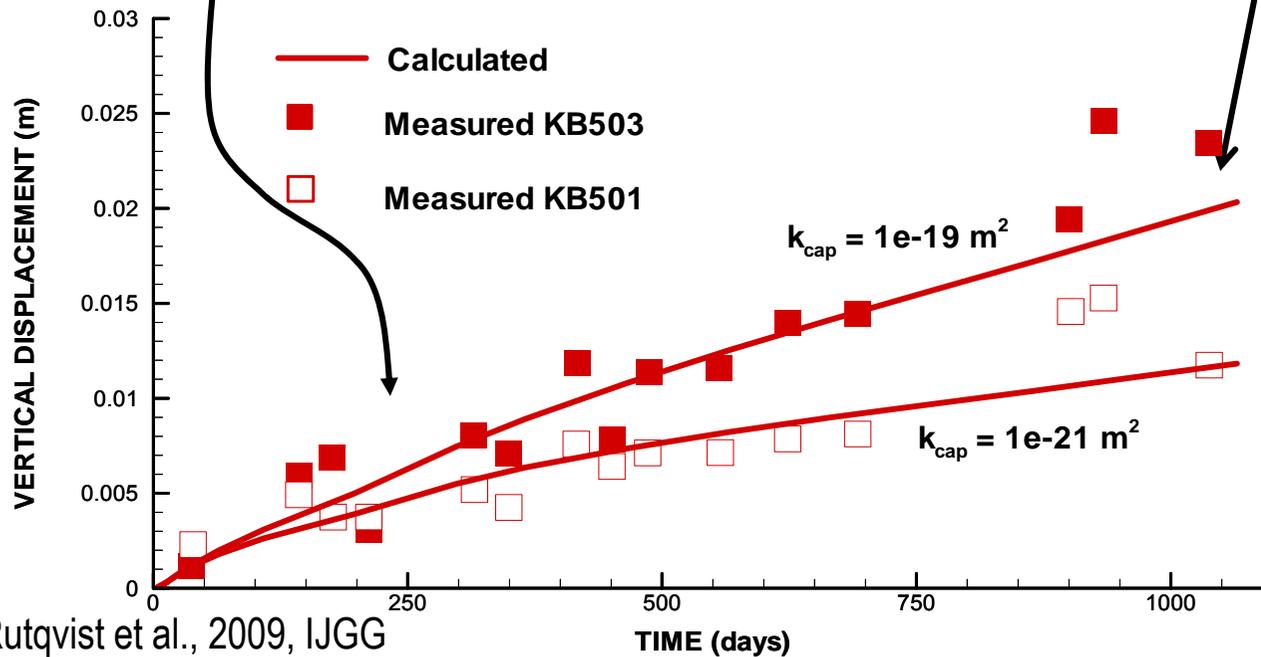
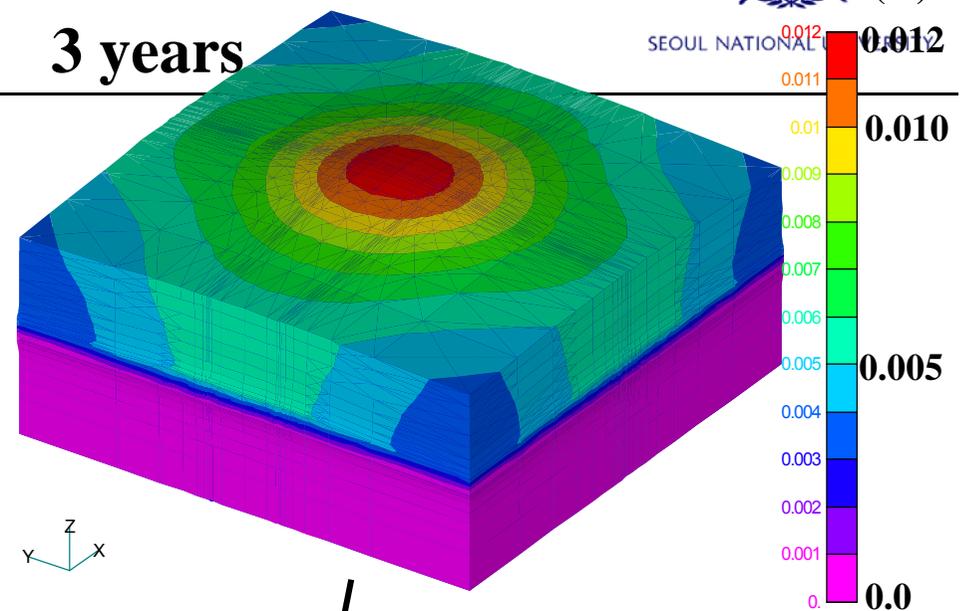
Prediction of Reservoir Heaving



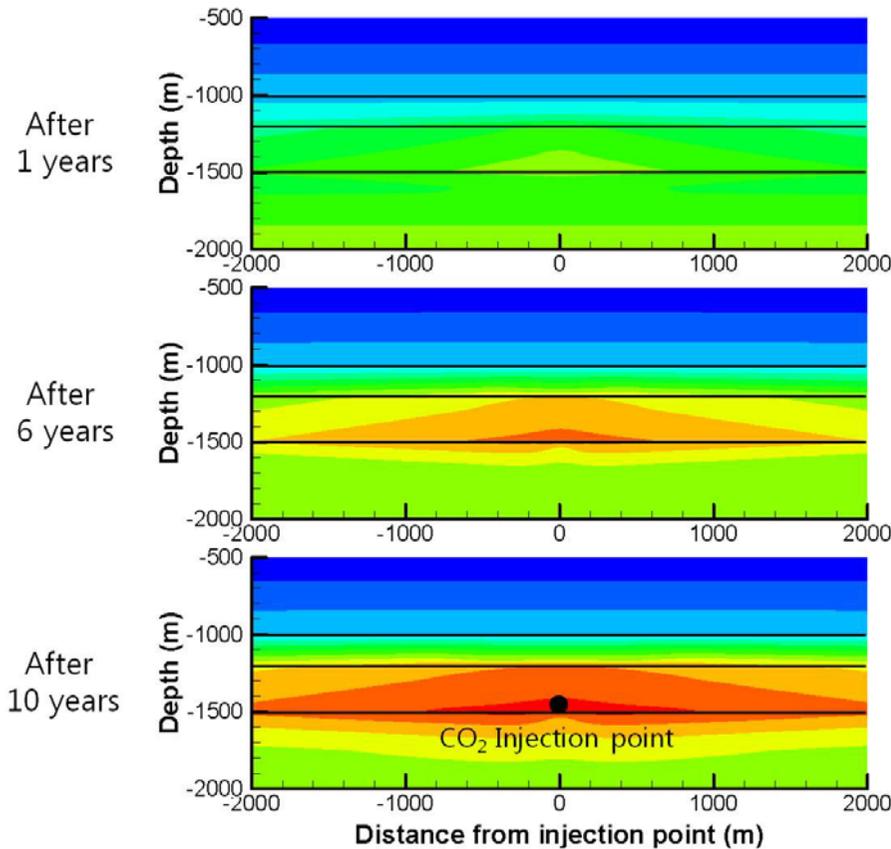
1 year



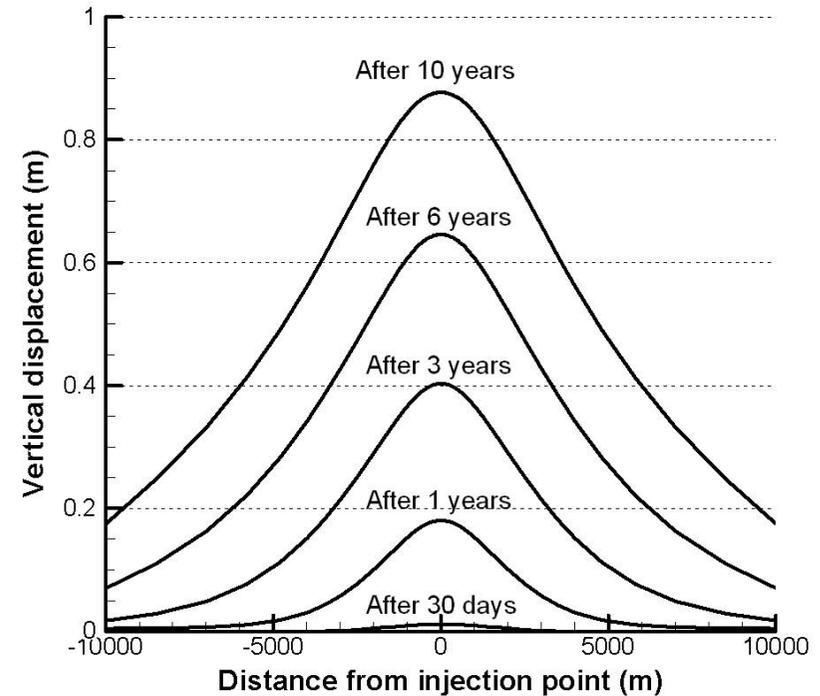
3 years



Reservoir subsidence (저류층 침하) Example(3): Heaving in CO₂ storage reservoir



Pressure change with time
near the injection point (Units: MPa)



Vertical displacement profile

- After 10 years
 - the pore pressure : about 12 MPa
 - the vertical displacement : 0.87 m

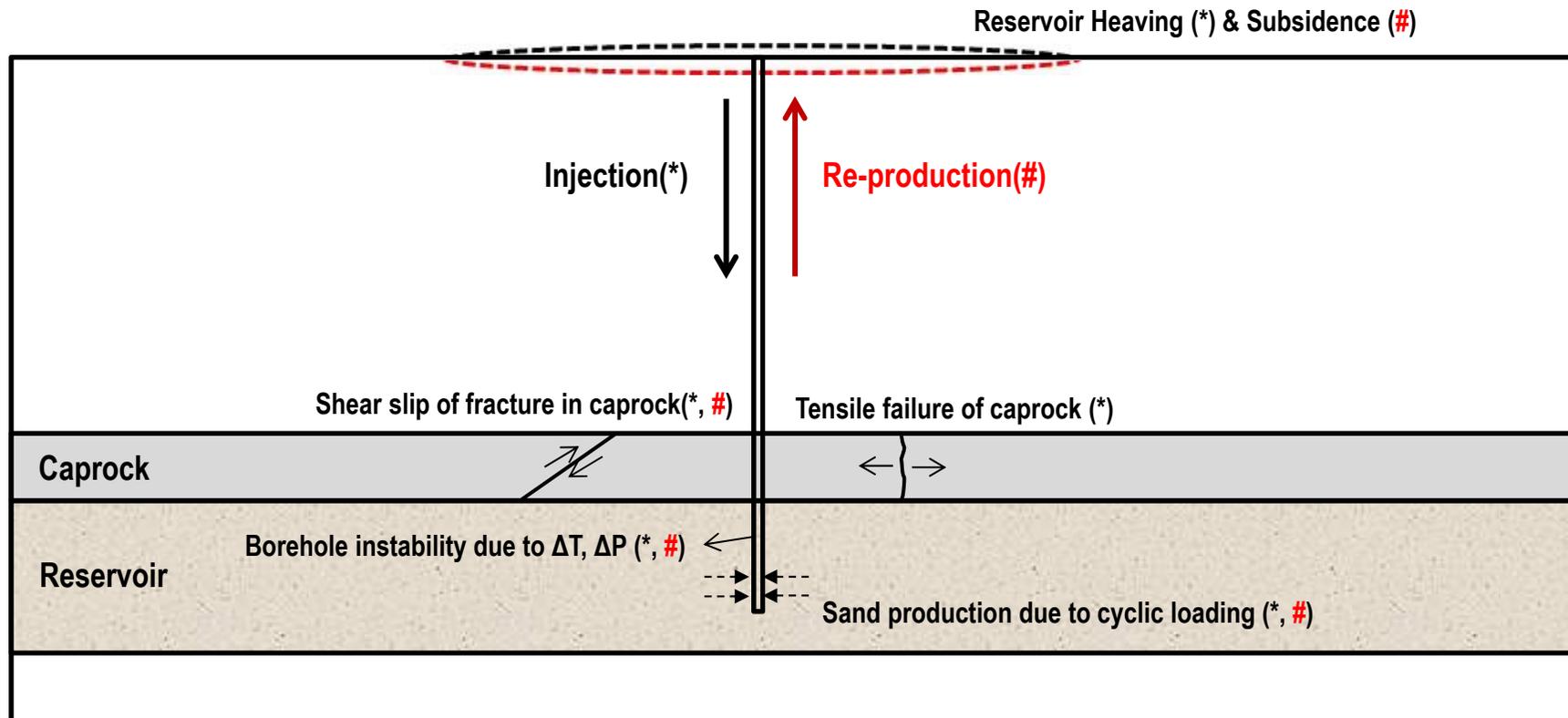
Example of Reservoir Geomechanics Application

Natural Gas storage in depleted Reservoir



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- Natural Gas need to be stored due to unbalanced consumption during summer and winter.



Summary

Contents – Applications



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- Fundamentals of Geomechanics
 - Stress
 - Strain
 - Stress-strain relationship
 - Uniaxial, tensile & triaxial strength
 - In situ rock stress
- Stress around a cavities
 - Circular hole
 - ↗ Anisotropic case
 - ↗ Elastic-plastic rock
 - Penny-shaped cracks

Things to remember



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- Stress (Equilibrium, Principal stress, Transformation)
- Hooke's Law (compare with Darcy's Law)
- Stress concentration around circular hole
(3 & -1)
- Concept of Effective Stress