Chapter 5. Coastal Water Level Fluctuation

5.0 Introduction

• Long period waves (few minutes to several days) compared to wind waves (~10 s)
  tides: sun, moon
  tsunamis: underwater earthquake or land sliding
  basin oscillation (seiches): resonance in closed or semi-closed basin
  storm surge: low pressure, onshore wind

• Measurement of long-period waves
  tide well (or stilling well): fixed structure for long-term measurement

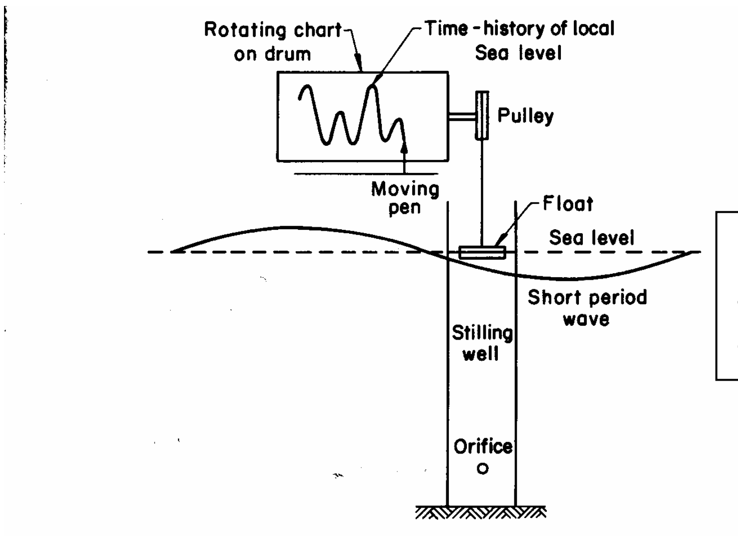
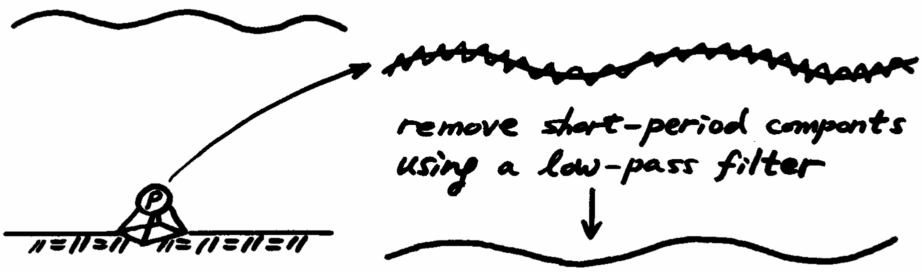


Figure 5.1. Float-stilling well water level gage.

Short-period components are filtered out through the orifice.

  pressure gauge: for short-term measurement



5.1 Long Wave Equations (Shallow Water Equations)

In shallow water, u, v ~ constant over depth
$w << u, v$

Vertical integration of $\left\{ \begin{matrix} \text{Continuity equation} \\ \text{Equations of motion} \end{matrix} \right\}$

$\rightarrow$ horizontal 2D (x, y) equations for continuity and motion

Include: (1) Coriolis force: effect of rotation of earth (important for large water mass)
(2) bottom friction: important for long waves
(3) surface wind shear stress: important for storm surge

Introduce depth-integrated volume flux:

$$q_x = \int_{-d}^{\eta} u dz; \quad q_y = \int_{-d}^{\eta} v dz$$

Depth-averaged velocities are

$$U = \frac{1}{d+\eta}\int_{-d}^{\eta} u dz; \quad V = \frac{1}{d+\eta}\int_{-d}^{\eta} v dz$$

Continuity equation:

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial \eta}{\partial t} = 0$$

Equations of motion:

$$x: \quad -g(d+\eta)\frac{\partial \eta}{\partial x} + fq_y + \frac{1}{\rho}(\tau_{sx} - \tau_{bx}) = \frac{\partial}{\partial x}\left(\frac{q_x^2}{d+\eta}\right) + \frac{\partial}{\partial y}\left(\frac{q_x q_y}{d+\eta}\right) + \frac{\partial q_x}{\partial t}$$

$$y: \quad -g(d+\eta)\frac{\partial \eta}{\partial y} - fq_x + \frac{1}{\rho}(\tau_{sy} - \tau_{by}) = \frac{\partial}{\partial x}\left(\frac{q_x q_y}{d+\eta}\right) + \frac{\partial}{\partial y}\left(\frac{q_y^2}{d+\eta}\right) + \frac{\partial q_y}{\partial t}$$

where τ_s = surface shear stress

τ_b = bottom shear stress

$f = 2\omega \sin\phi$ = Coriolis parameter

ω = earth's rotation speed = $2\pi / 24\text{hrs} = 7.27 \times 10^{-5}$ rad/s

ϕ = latitude (positive in northern hemisphere, negative in southern hemisphere)

In terms of U, V, η

$$\frac{\partial U(d+\eta)}{\partial x} + \frac{\partial V(d+\eta)}{\partial y} + \frac{\partial \eta}{\partial t} = 0$$

$$-g\frac{\partial \eta}{\partial x} + fV + \frac{1}{\rho(d+\eta)}\left(\tau_{sx} - \tau_{bx}\right) = U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + \frac{\partial U}{\partial t}$$

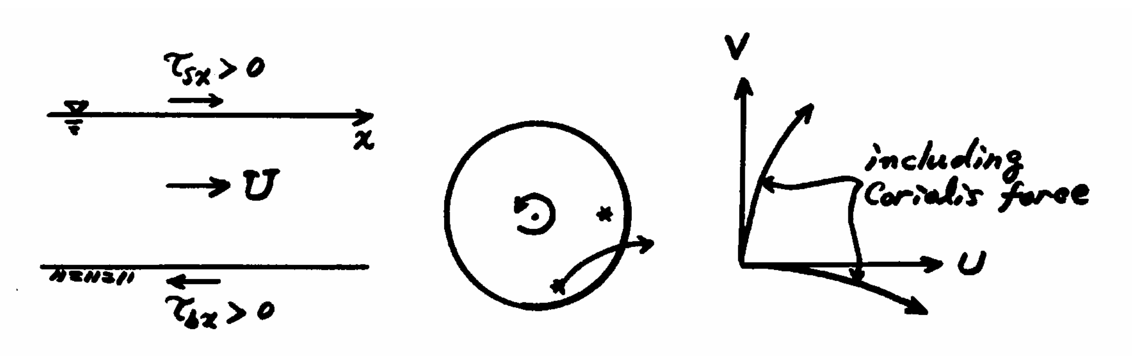
$$-g\frac{\partial \eta}{\partial y} - fU + \frac{1}{\rho(d+\eta)}\left(\tau_{sy} - \tau_{by}\right) = U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} + \frac{\partial V}{\partial t}$$

Linearized equations are

$$\frac{\partial (Ud)}{\partial x} + \frac{\partial (Vd)}{\partial y} + \frac{\partial \eta}{\partial t} = 0$$

$$-g\frac{\partial \eta}{\partial x} + fV + \frac{1}{\rho(d+\eta)}\left(\tau_{sx} - \tau_{bx}\right) = \frac{\partial U}{\partial t}$$

$$-g\frac{\partial \eta}{\partial y} - fU + \frac{1}{\rho(d+\eta)}\left(\tau_{sy} - \tau_{by}\right) = \frac{\partial V}{\partial t}$$



5.2 Astronomical Tide Generation and Characteristics

Tide predictions are important for

(1) hydrographic survey
(2) coastal flooding (high tide + storm surge)
(3) coastal structures (crest elevation, wave force, breaking, etc.)
(4) stability of inlets (tidal currents)
⋮

<u>Tide generation mechanism</u>

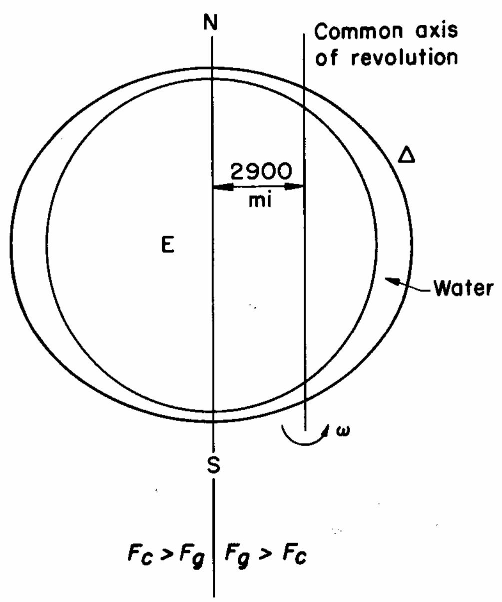
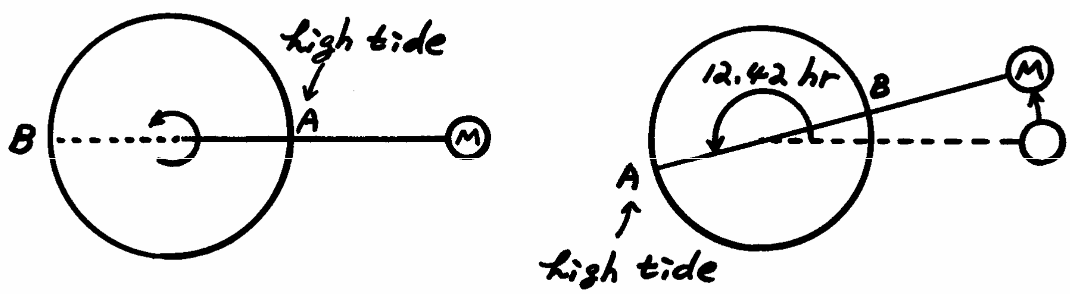


Figure 5.3. Tide generation—idealized earth/moon system.



Effect of sun

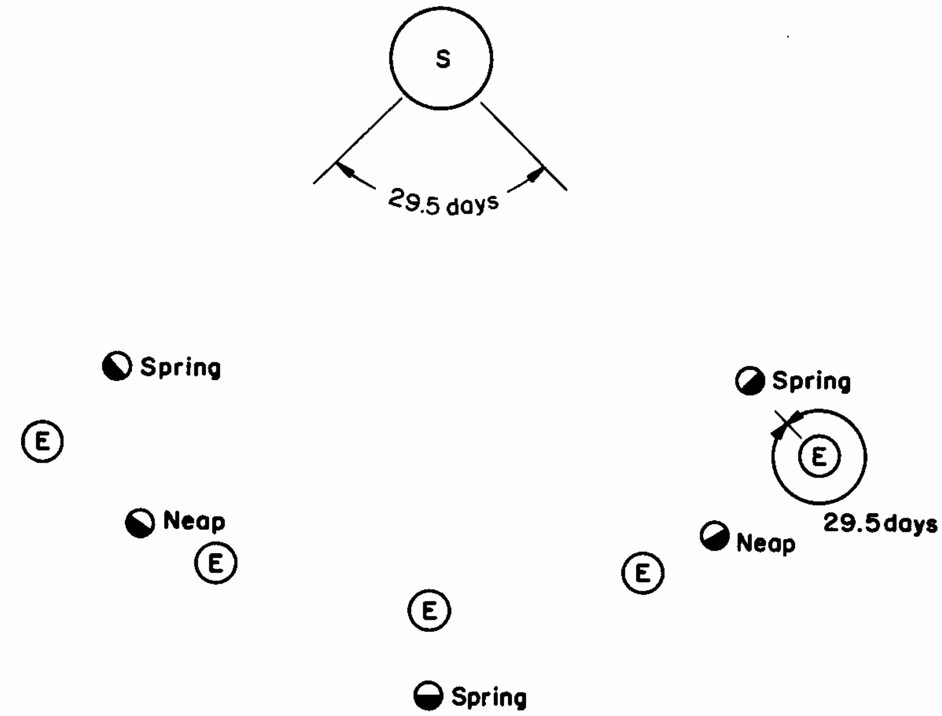
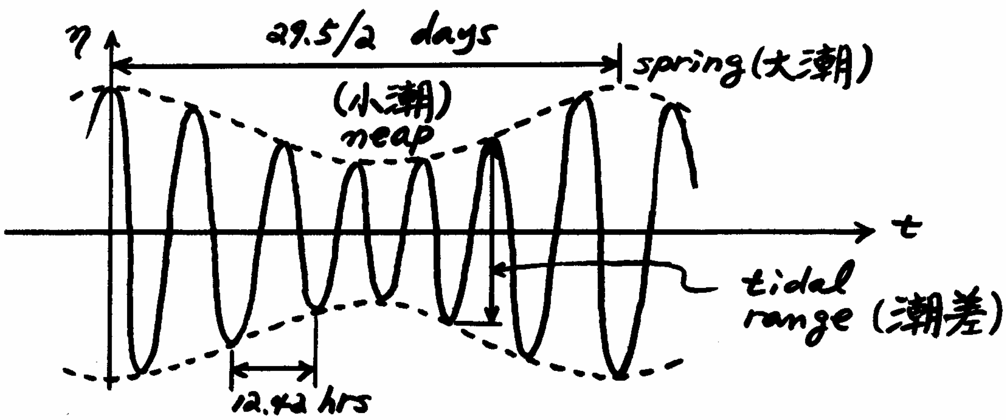
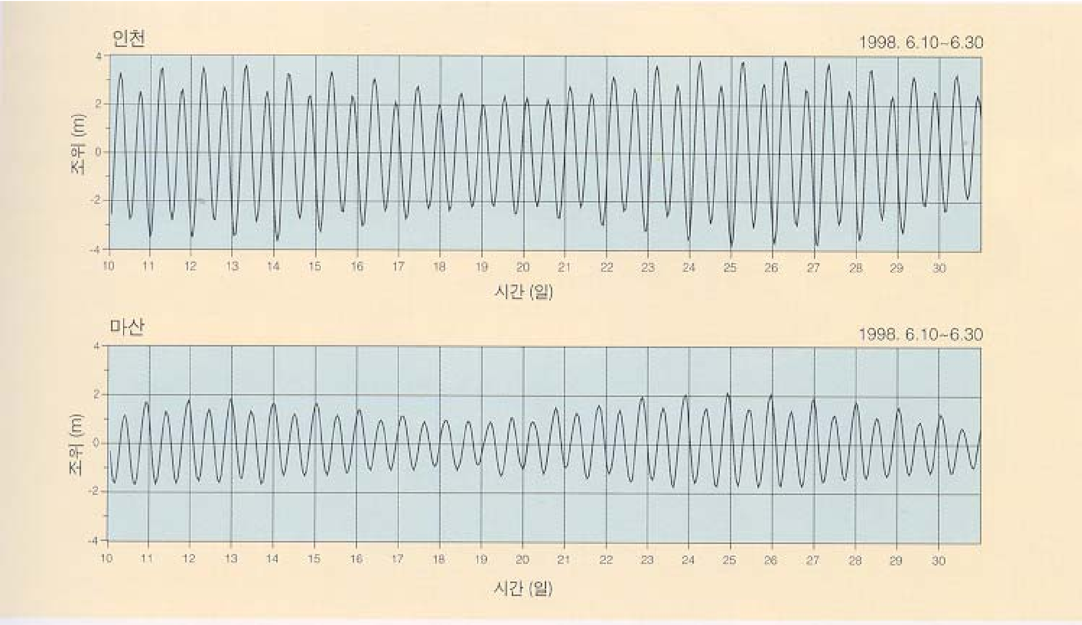


Figure 5.4. Earth, sun, and moon during a lunar month.





인천과 마산에서 관측된 조석 기록

Fourier analysis of water level record

↓

amplitude, period, phase of over 390 components

↓

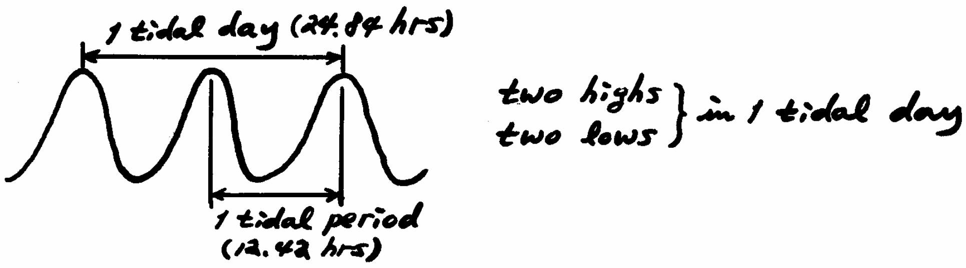
eight major components (Table 5.1) or four major components (M_2, S_2, K_1, O_1)

↓

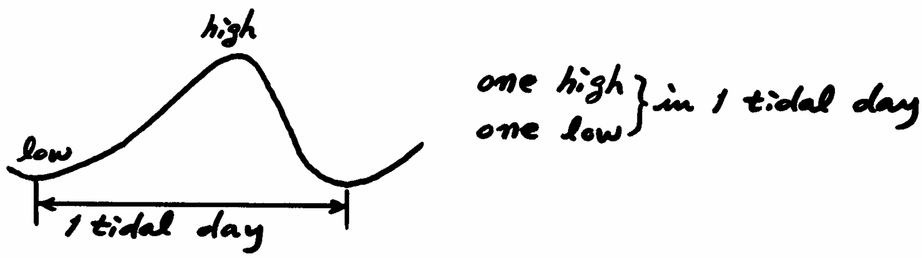
used for tide prediction

Types of tide

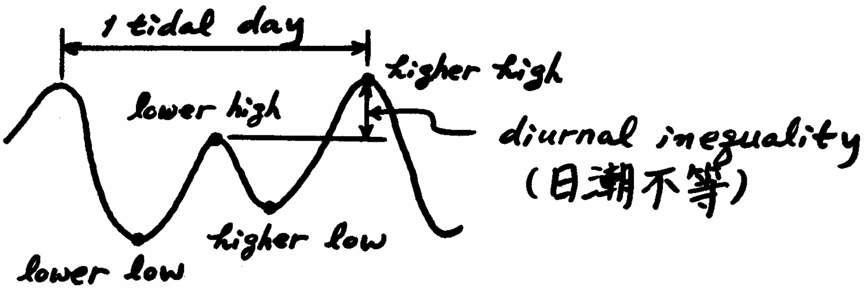
(a) semi-diurnal tide (半日周潮)



(b) diurnal tide (日周潮)



(c) mixed tide (混合潮)



Local effects on tide:

nearshore hydrography

bottom friction

Coriolis acceleration

resonant effects

⋮

5.3 Tidal Datums and Tide Prediction

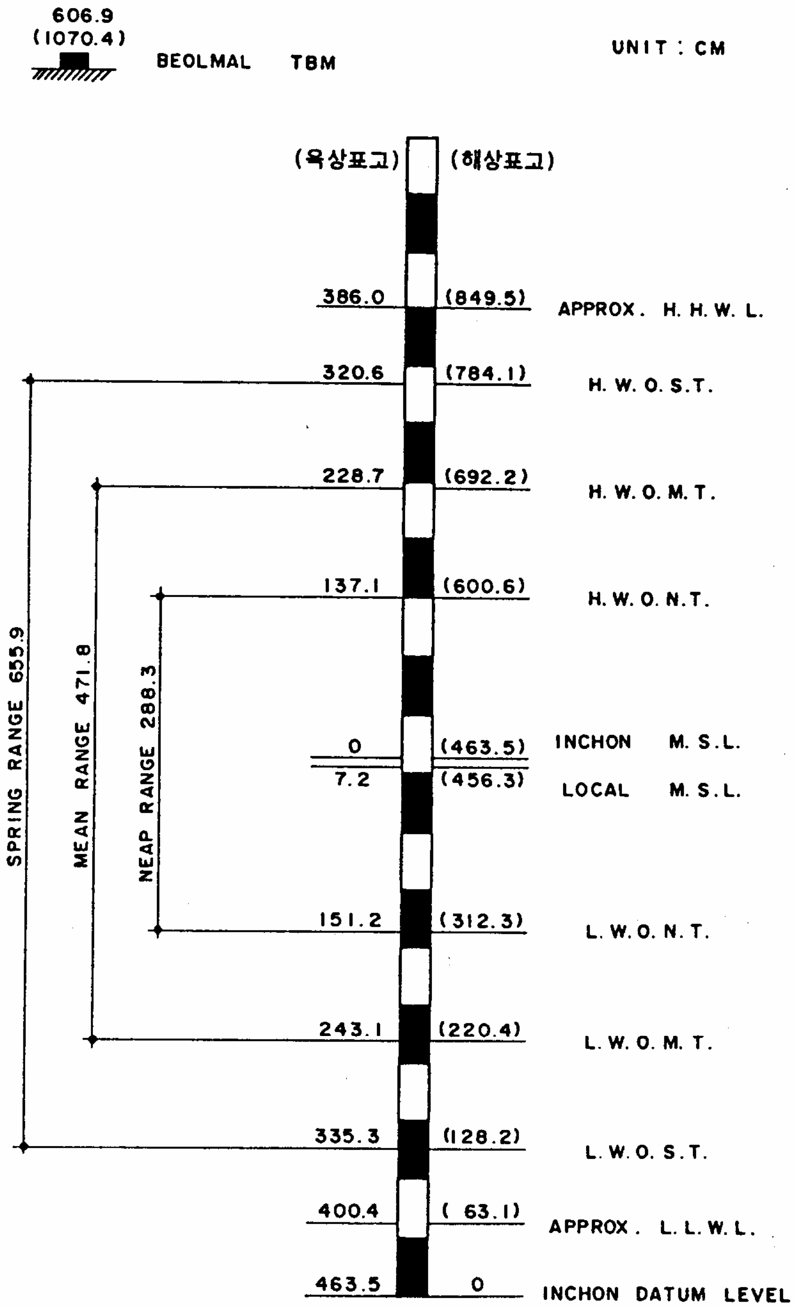
MSL (mean sea level; 평균해면): long-term averaged sea level

DL (datum level; 기본수준면): MSL $-\left(A_{M_2}+A_{S_2}+A_{K_1}+A_{O_1}\right)$

다음은 우리 나라에서 일반적으로 많이 쓰이는 해면에 대한 설명이다.

- *略最高高潮面(Approx. Higher High Water):*
  평균해면+ $(A_{M2}+A_{S2}+A_{O1}+A_{K1})$
- *大潮平均高潮面(High Water Ordinary Spring Tide):*
  평균해면+ $(A_{M2}+A_{S2})$
- *平均高潮面(High Water Ordinary Mean Tide):*
  평균해면+ A_{M2}
- *小潮平均高潮面(High Water Ordinary Neap Tide):*
  평균해면+ $(A_{M2}-A_{S2})$
- *(局地)平均海面((local) Mean Sea Level):*
  $A_{M2}+A_{S2}+A_{O1}+A_{K1}$
- *小潮平均低潮面(Low Water Ordinary Neap Tide):*
  평균해면− $(A_{M2}-A_{S2})$
- *平均低潮面(Low Water Ordinary Mean Tide):*
  평균해면− A_{M2}
- *大潮平均低潮面(Low Water Ordinary Spring Tide):*
  평균해면− $(A_{M2}+A_{S2})$
- *略最低低潮面(Approx. Lower Low Water) 또는 基本水準面(Datum Level):*
  평균해면− $(A_{M2}+A_{S2}+A_{O1}+A_{K1})$
- *既往高·低極潮位(Observed Highest High Water, Observed Lowest Low Water):*
  조화상수로 계산되는 이론적 해면이 아니라 관측기간 중 또는 그 이전에 출현된 최고조면 또는 최저조면

일반적으로 육상표고는 MSL을 기준으로 하고, 해도는 DL을 기준으로 함 (항해 시 안전 고려)



조위표 (충남 가로림만 벌말)

Tide prediction

$$\eta(t) = A + \sum_{i=1}^{N} A_i \cos\left(\frac{2\pi}{T_i} t + \Delta_i\right)$$

where η = elevation above DL

A = vertical distance from DL to MSL

N = number of tidal components (e.g. 4: M_2, S_2, K_1, O_1)

A_i, T_i, Δ_i = amplitude, period, phase angle of tidal components

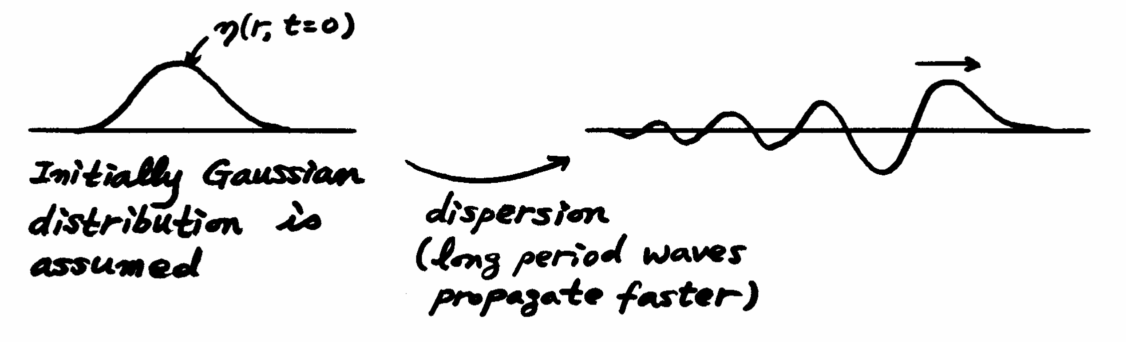
The amplitude, period, and phase angle of tidal components will be changed by local developments (e.g. land reclamation) but almost no change in deep oceans. To calculate tidal change due to local developments, solve the long wave equations in the interested area for given open boundary conditions, which are not influenced by local developments.

5.4 Tsunamis

Tsunamis are caused by underwater earthquake (most frequent), land slide, volcano, etc.

The wave height of a tsunami is small in the ocean (~ 1 m or less) but amplified in coastal areas by shoaling, refraction, and resonance.

A tsunami consists of a group of waves of T = 5 to 60 minutes, with the most common T = 20 to 30 minutes, which are long waves even in the ocean. A tsunami has a small H/L, but large L and C; see Example 5.4-1 of textbook.



Forecasting of arrival time, wave height, and runup:

Construction of refraction diagram

↓

$t_T = \sum \frac{\Delta S}{\sqrt{gd}}$ along a ray

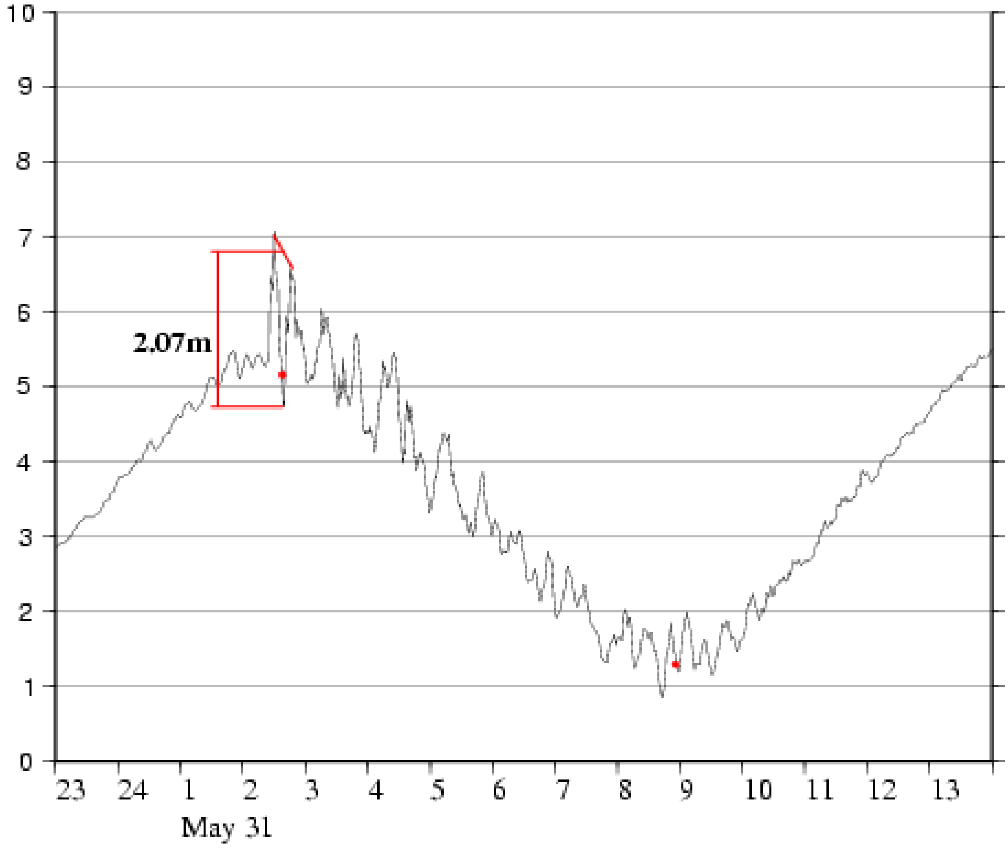
$H = H_0 K_s K_r$

$R \cong H$ or slightly higher than H

5.5 Basin Oscillation

Basin oscillation is a slow periodic rise and fall of water surface.

Exciting forces: long waves (tide, tsunami, see Fig. 5.5), wind stress, atmospheric pressure gradient, etc.



2007년 5월 31일 영광 검조소(계마항 내)에서의 수면 변동
(due to meteorological tsunami?)

Important for mooring system of large vessels (no response to short waves):

Mooring lines are broken
Fenders are damaged
Loading/unloading is delayed.

Analogy to pendulum (or swing):

free oscillation at natural frequency (depending on arm length ↔ length, depth of basin)

+

forcing at frequency close to natural frequency

↓

amplitude of oscillation increases with time (= resonance)

↓

but friction reduces the response amplitude

5.6 Resonant Motion in Two- and Three-Dimensional Basins

↑

standing wave system (antinode at wall)

shallow water waves ($d << L$)

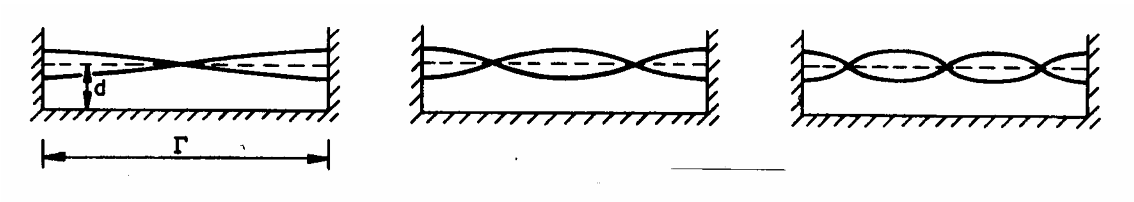
Need to predict: 1) resonance periods

2) locations of nodes and antinodes

3) amplitude and particle velocity

<u>2D basins</u>

closed basin:



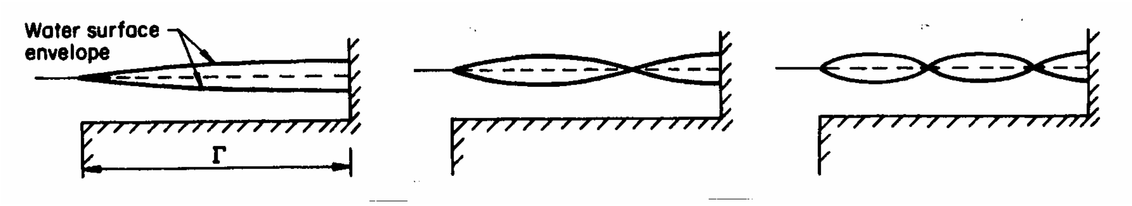
$$\Gamma = \frac{k+1}{2} L, \quad k = 0,1,2,\cdots$$

$$C = \sqrt{gd} = \frac{L}{T} \quad \rightarrow \quad L = \sqrt{gd}T$$

$$\Gamma = \frac{k+1}{2}\sqrt{gd}T \quad \rightarrow \quad T = \frac{2\Gamma}{(k+1)\sqrt{gd}}$$

fundamental mode (most dominant): $k = 0$, $T = \dfrac{2\Gamma}{\sqrt{gd}}$

open basin:



$$\Gamma = \frac{2k+1}{4} L = \frac{2k+1}{4}\sqrt{gd}T \quad k = 0,1,2,\cdots$$

$$T = \frac{4\Gamma}{(2k+1)\sqrt{gd}}$$

fundamental mode: $k = 0$, $T = \frac{4\Gamma}{\sqrt{gd}}$

For irregular bathymetry,

$$T = \frac{2}{k+1}\sum_{i=1}^{N}\frac{\Gamma_i}{\sqrt{gd_i}} \quad \text{for closed basin}$$

$$T = \frac{4}{2k+1}\sum_{i=1}^{N}\frac{\Gamma_i}{\sqrt{gd_i}} \quad \text{for open basin}$$

Particle velocity and excursion length:

$u_{\max}$ (at node) = $\frac{H}{2}\sqrt{\frac{g}{d}}$

$$u_{avg} = \frac{1}{T/4}\int_0^{T/4} u_{\max}\cos\sigma t dt = \frac{2}{\pi}u_{\max} = \frac{HL}{\pi dT}$$

horizontal excursion length, $X = u_{avg}\frac{T}{2} = \frac{HT}{2\pi}\sqrt{\frac{g}{d}}$

3D basins

Neglecting Coriolis force and bottom and surface shear stresses, linearized long wave equations in constant depth are

$$-g\frac{\partial \eta}{\partial x} = \frac{\partial U}{\partial t}: \quad x\text{-momentum equation}$$

$$-g\frac{\partial \eta}{\partial y} = \frac{\partial V}{\partial t}: \quad y\text{-momentum equation}$$

$$d\frac{\partial U}{\partial x} + d\frac{\partial V}{\partial y} + \frac{\partial \eta}{\partial t} = 0: \text{ continuity equation}$$

Differentiating the continuity equation with respect to time,

$$d\frac{\partial}{\partial x}\left(\frac{\partial U}{\partial t}\right) + d\frac{\partial}{\partial y}\left(\frac{\partial V}{\partial t}\right) + \frac{\partial^2 \eta}{\partial t^2} = 0$$

Substituting the momentum equations,

$$-gd\frac{\partial^2 \eta}{\partial x^2} - gd\frac{\partial^2 \eta}{\partial y^2} + \frac{\partial^2 \eta}{\partial t^2} = 0 \qquad \text{(5.12) of textbook}$$

A solution to (5.12) is

$$\eta = H\cos(k_x x)\cos(k_y y)\cos\sigma t$$

For a rectangular basin with lengths of Γ_x and Γ_y in x- and y-direction, respectively,

$$\eta = H\cos\left(\frac{\pi N}{\Gamma_x}x\right)\cos\left(\frac{\pi M}{\Gamma_y}y\right)\cos\left(\frac{2\pi}{T_{NM}}t\right)$$

where

$$N, M = 1, 2, 3, \cdots$$

$$T_{NM} = \frac{2}{\sqrt{gd}}\left[\left(\frac{N}{\Gamma_x}\right)^2 + \left(\frac{M}{\Gamma_y}\right)^2\right]^{-1/2}$$

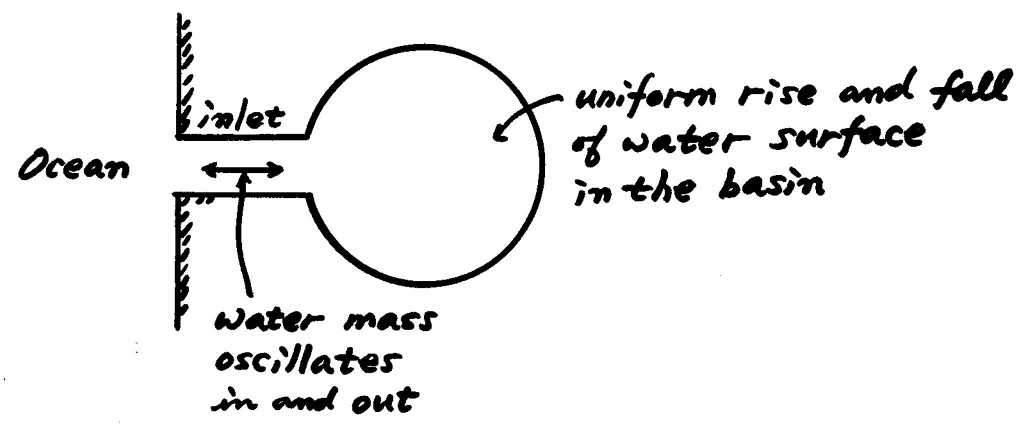
For nodal lines where $\eta = 0$ always,

$$\cos\left(\frac{\pi N}{\Gamma_x} x\right) = 0$$

$$\cos\left(\frac{\pi M}{\Gamma_y} y\right) = 0$$

Read Ex. 5.6-2 for oscillations in a square basin.

Helmholtz resonance



Example: harbor oscillation by tsunamis

5.7 Resonance Analysis for Complex Basins

• Hydraulic model tests:

undistorted model: $T_r = \sqrt{L_r}$ using Froude similitude

distorted model: $T_r = \dfrac{L_{rh}}{\sqrt{L_{rv}}}$

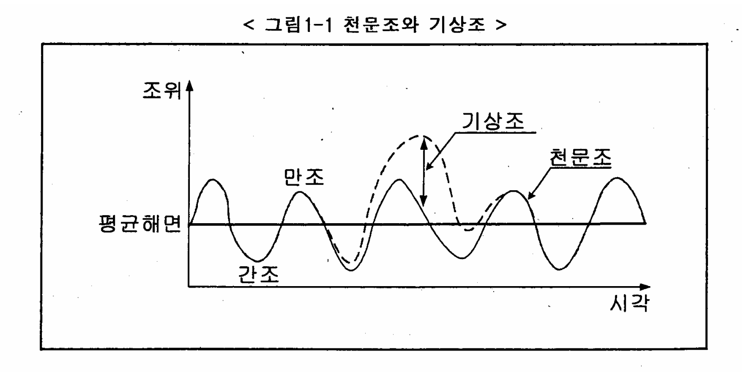
• Numerical model: Solve long wave equations with proper boundary conditions.

5.8 Storm Surge and Design Storms

↑

also called 'meteorological tide' (氣象潮) ↔ astronomical tide (天文潮).

local and temporal rise of MSL during storms → coastal flooding



Storm surge is influenced by

(1) surface wind stress

(2) bottom shear stress
(3) Coriolis acceleration
} related to wind - induced currents

(4) horizontal gradient of atmospheric pressure

(5) wave setup, precipitation, runoff, etc.

Wide continental shelf → large storm surge (e.g. southwest coast of Korea)

Storm surge + high tide → severe flooding in coastal areas

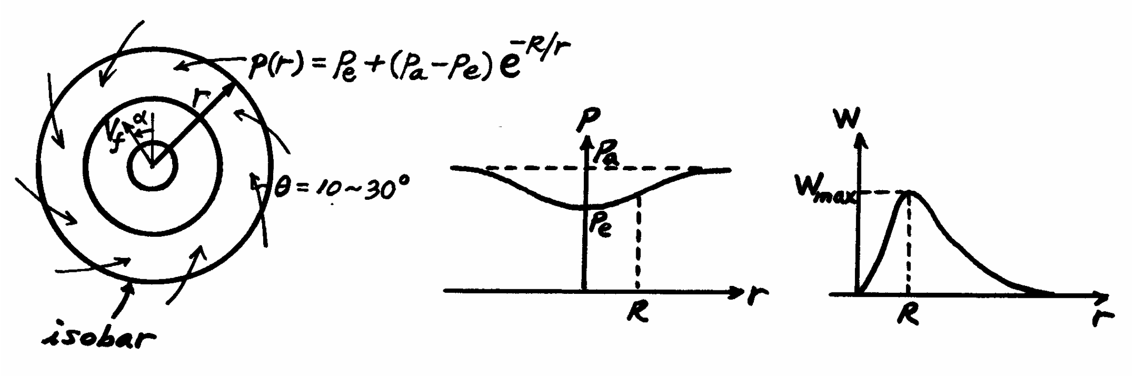
Design storm (typhoon) is specified by

p_e = central pressure
p_a = peripheral pressure (= standard atmospheric pressure)
R = radius to maximum wind speed

V_f = forward speed of typhoon

α = direction of V_f

θ = inflow angle of wind



$\Delta p = p_a - p_e$ = strength of typhoon
R = size of typhoon

5.9 Numerical Analysis of Storm Surge (read text)

Wind stress, $\tau_s = K_s \rho W^2$

where K_s = drag coefficient

ρ = density of sea water,

W = wind speed at 10 m above water level

Van Dorn (1953):

$$K_s = \begin{cases} 1.21\times 10^{-6}, & W \le 5.6\, m/s \\ 1.21\times 10^{-6} + 2.25\times 10^{-6}\left(1 - \dfrac{5.6}{W}\right)^2, & W > 5.6\, m/s \end{cases}$$

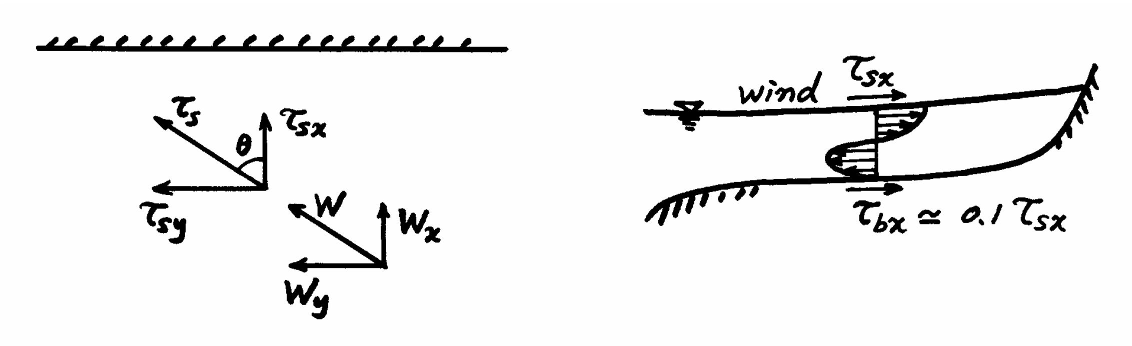
5.10 Simplified Analysis of Storm Surge

wind/bottom stress setup
atmospheric pressure gradient setup
Coriolis setup

} are calculated separately.

Static (not moving) storm is assumed → conservative estimation of setup

<u>Wind/bottom stress setup</u>

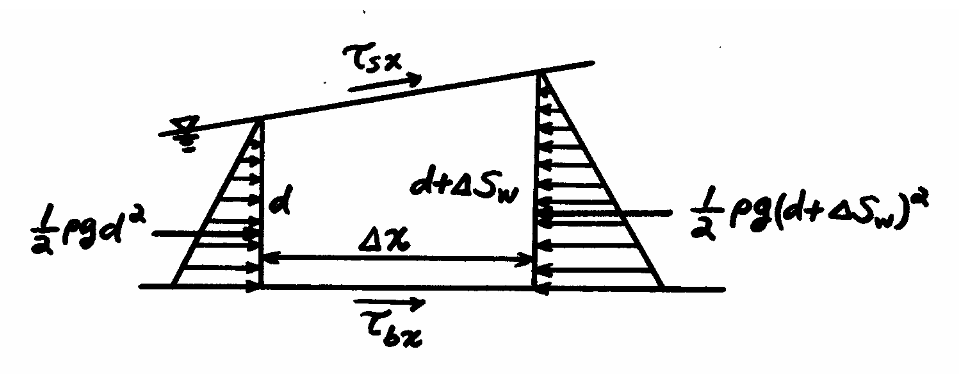


$$\tau_s = K_s \rho W^2$$

$$\tau_{sx} = K_s \rho W^2 \cos\theta = K_s \rho W W_x$$

$$\tau_{sy} = K_s \rho W^2 \sin\theta = K_s \rho W W_y$$

Assuming locally flat bottom, consider force balance in x-direction.



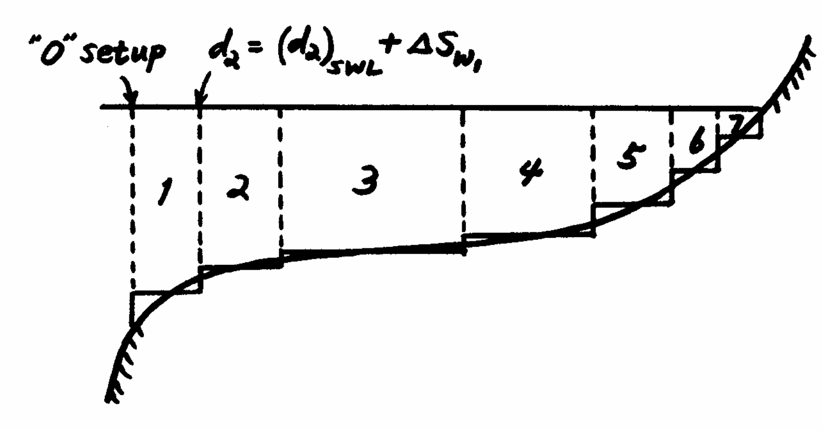
$$\left(\tau_{sx} + \tau_{bx}\right)\Delta x + \frac{1}{2}\rho g d^2 - \frac{1}{2}\rho g\left(d + \Delta S_w\right)^2 = 0$$

Using $\tau_{sx} + \tau_{bx} = 1.1 K_s \rho W W_x = K_{sb} \rho W W_x$,

$$K_{sb} \rho W W_x \Delta x + \frac{1}{2}\rho g d^2 - \frac{1}{2}\rho g d^2 - \rho g d \Delta S_w - \frac{1}{2}\rho g \Delta S_w^2 = 0$$

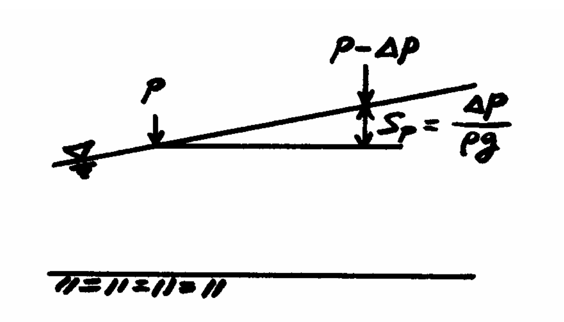
Solving the quadratic equation for ΔS_w,

$$\Delta S_w = d\left[\sqrt{\frac{2K_{sb} W W_x \Delta x}{g d^2} + 1} - 1\right]$$

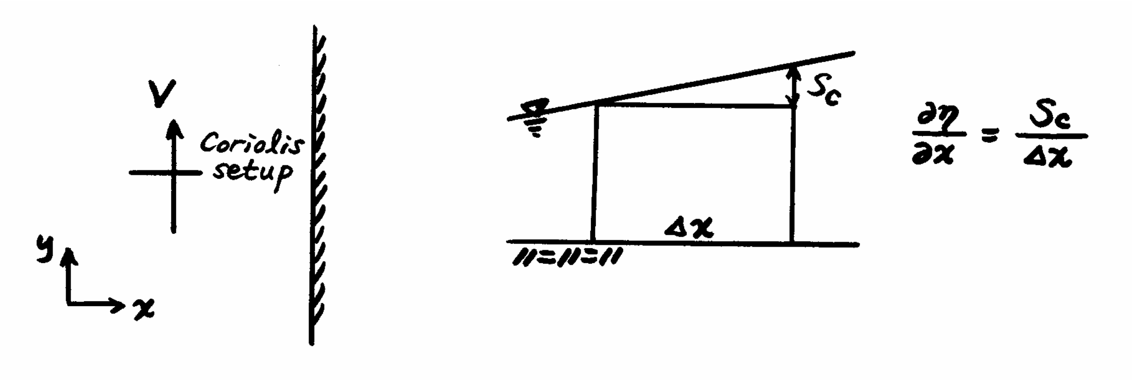


Calculate setup for each segment from offshore boundary toward shore. The water depth, d, at the beginning of each segment must include the setup at the previous segment.

<u>Atmospheric pressure gradient setup</u>



Coriolis setup



Linearized momentum equation in x - direction is

$$-g\frac{\partial \eta}{\partial x} + fV + \frac{1}{\rho(d+\eta)}\left(\tau_{sx} - \tau_{bx}\right) = \frac{\partial U}{\partial t}$$

Considering only Coriolis setup and assuming static storm,

$$-g\frac{S_C}{\Delta x} + 2\omega \sin\phi V = 0$$

$$S_C = \frac{2\omega}{g}\sin\phi V \Delta x \qquad (5.27)$$

V is given by Bretschneider (1967) as

$$V = Wd^{1/6}\sqrt{\frac{K_s}{14.6n^2}\sin\theta}$$

where n = Manning coefficient (~ 0.035)

5.11 Long-Term Sea Level Change

green house effect

↓

melting of polar ice, expansion of sea water

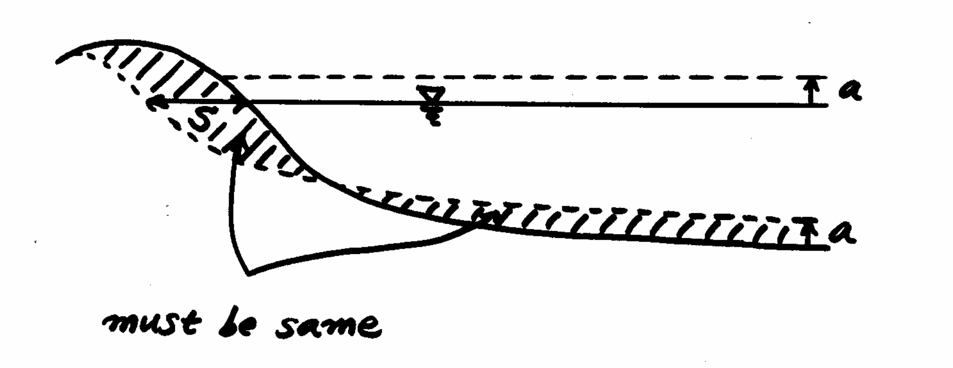
↓

rise of sea level

↓

coastal flooding, coastal erosion, salt water intrusion into ground water, etc

<u>Brunn's rule</u>



Brunn rule: $S = \dfrac{a}{\tan\beta}$

where β = average beach slope

Ex) $a = 10$ cm, $\tan\beta = 1/100$ $\rightarrow$ $S = 10$ m