Chapter 3 Dynamic Features and Methods of Analysis

3.1 Introduction

3.1.1 Fluid transport phenomena

= ability of fluids in motion to convey materials and properties from place to place

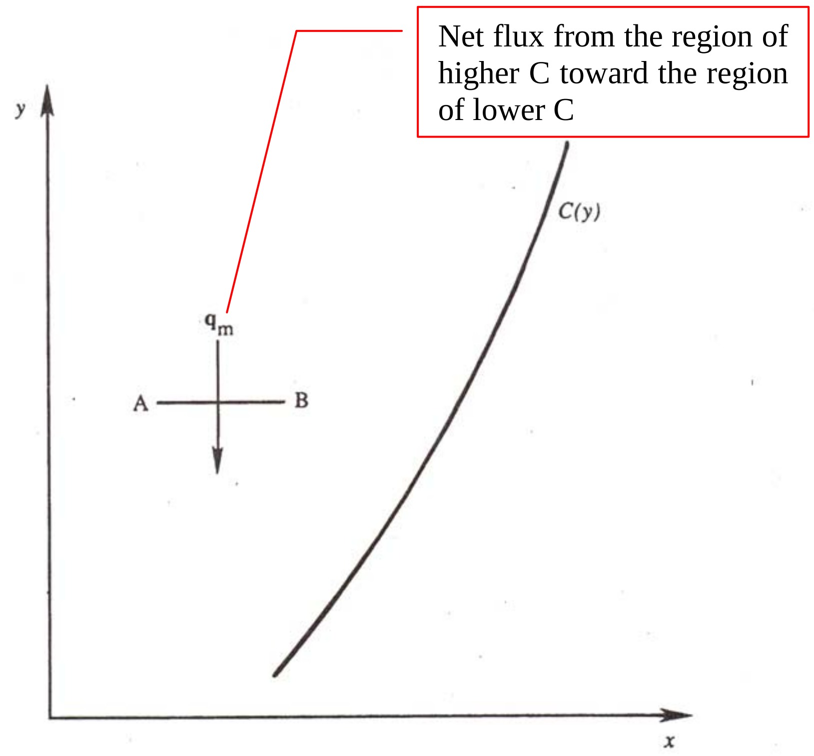
= mechanism by which materials and properties are diffused and transmitted through the fluid medium because of molecular motion

Transport of materials and properties in the direction of decreasing mass, temperature, momentum

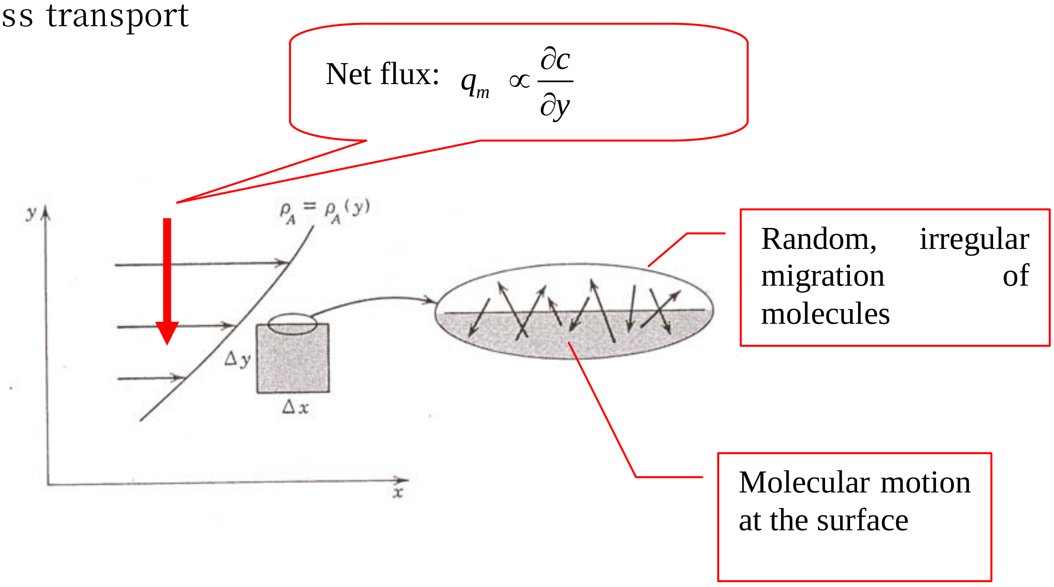
$$q = \left\{ \frac{dM/dt}{area} \right\} \propto \left\{ \frac{d(M/vol)}{ds} \right\}$$

Flux = quantity per unit time per area

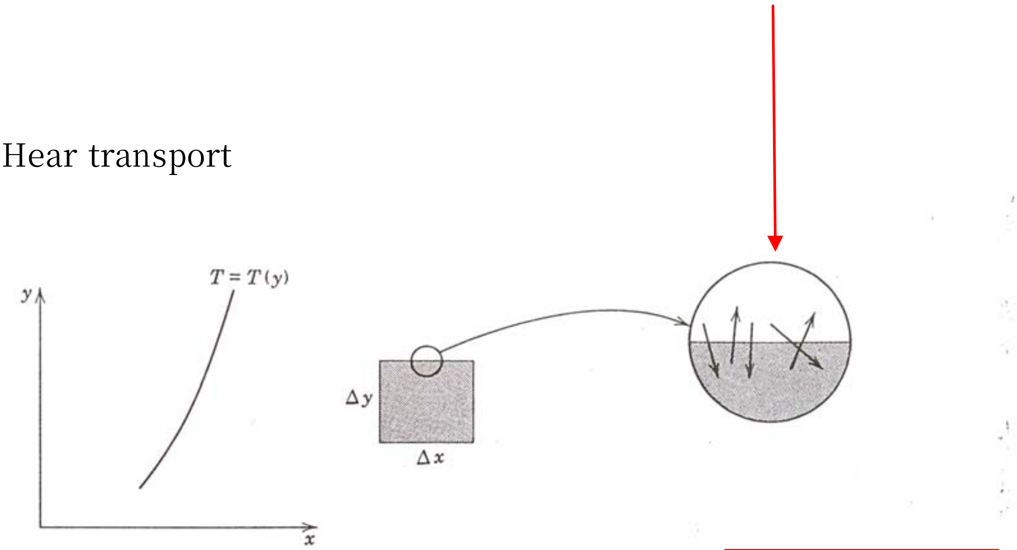
process	observational law
mass transport	conservation of matter
heat transport	conservation of energy (1st law of thermodynamics)
momentum transport	equation of motion (Newton's 2nd law)


Net flux from the region of higher C toward the region of lower C
y
C(y)
q_m
A
B
x

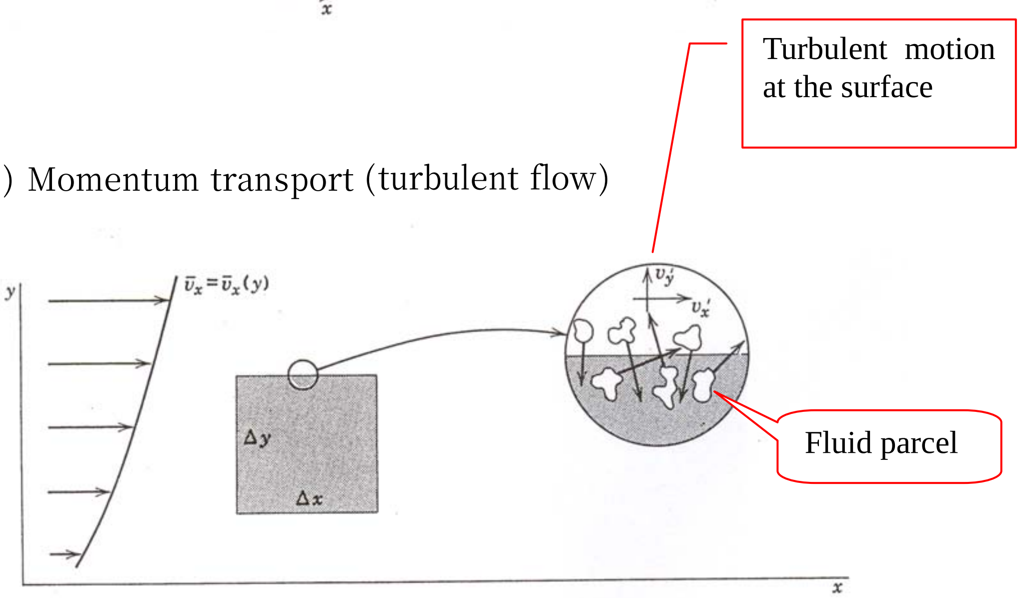
a) Mass transport



b) Hear transport



c) Momentum transport (turbulent flow)



3.1.2 Subsidiary Laws

→ Relations between fluxes and driving force (gradient)

→ Transport analogy

Flux	Driving force	Law	Relation
Mass flux q_m	concentration gradient $\frac{\partial c}{\partial x_j}$	Fick's law	$q_m = -D\frac{\partial c}{\partial x_j} = -D\nabla c$ gradient
Heat flux q_H	temperature gradient $\frac{\partial T}{\partial x_j}$	Fourier's law	$q_H = -k\frac{\partial T}{\partial x_j} = -k\nabla T$
Momentum Flux, q_{mo}	velocity gradient $\frac{\partial u_i}{\partial x_j}$	Newton's law of viscosity	$\tau = \mu\frac{\partial u_i}{\partial x_j}$

◈ momentum flux

$$q_{mo} = \frac{mu}{t(\Delta x\,\Delta y)} = \frac{m(u/t)}{A} = \frac{ma}{A} = \frac{F}{A} = stress = \tau \qquad (3.1)$$

Mass/heat transport:

c, T – scalar

q_m, q_H - vector

momentum transport:

u_i – vector

$q_{mo} = \tau$ - tensor

3.2 Mass Transport

All fluid motions must satisfy the principle of conservation of matter.

- homogeneous fluid — single phase
- non-homogeneous fluid
  - multi phase: air-liquid, liquid-solid
  - multi species: fresh water - salt water

Homogeneous fluid	Non-homogenous fluid
single phase	multi phase
single species	single phase & multi species
Continuity Equation [Ch. 4]	mass transport due to local velocity + mass transport due to diffusion → Advection-Diffusion Equation [Advanced Environmental Hydraulics]

Continuity equation: relation for temporal and spatial variation of velocity and density

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{q}\,) = 0$$

Advection-Diffusion Equation

= Advection + molecular diffusion

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}\left(uc - D\frac{\partial c}{\partial x} \right) = 0$$

$$u = \overline{u} + u'$$

Mean motion

fluctuation

$$q_d = \overline{u'c'} = D\frac{\partial c}{\partial x}$$

3.3 Heat Transport

thermodynamics ~ non-flow processes

equilibrium states of matter

fluid dynamics ~ transport of heat (scalar) by fluid motion

- Apply conservation of energy to flow process (= 1st law of thermodynamics)

~ relation between pressure, density, temperature, velocity, elevation, mechanical work, and heat input (or output).

~ since heat capacity of fluid is large compared to its kinetic energy, temperature and density remain constant even though large amounts of kinetic energy are dissipated by friction.

→ simplified energy equation

- Heat transfer in flow process

1) convection: due to velocity of the flow – advection

2) conduction: analogous to diffusion, tendency for heat to move in the direction of decreasing temperature

- Application

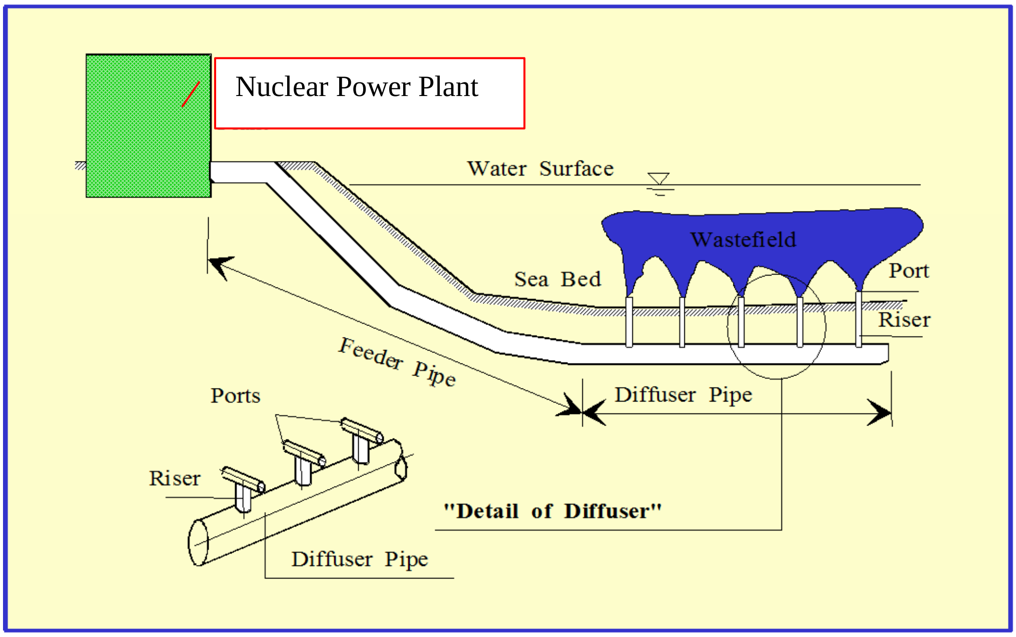
1) Fluid machine (compressors, pumps, turbines): energy transfer in flow processes

2) Heat pollution: discharge of cooling water for nuclear power plant

[Re] Thermal stratification

- Ocean, lake, reservoir

- density variations


Nuclear Power Plant
Water Surface
Wastefield
Sea Bed
Port
Riser
Feeder Pipe
Diffuser Pipe
Ports
Riser
"Detail of Diffuser"
Diffuser Pipe

3.4 Momentum Transport

3.4.1 Momentum transport phenomena

~ encompass the mechanisms of fluid resistance, boundary and internal shear stresses, and propulsion and forces on immersed bodies.

Momentum = mass · velocity vector = $m\vec{u}$

Adopt Newton's 2nd law

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{u}}{dt} = \frac{d}{dt}(m\vec{u}) \tag{3.2}$$

→ Equation of motion

- Effect of velocity gradient $\frac{\partial u}{\partial y}$

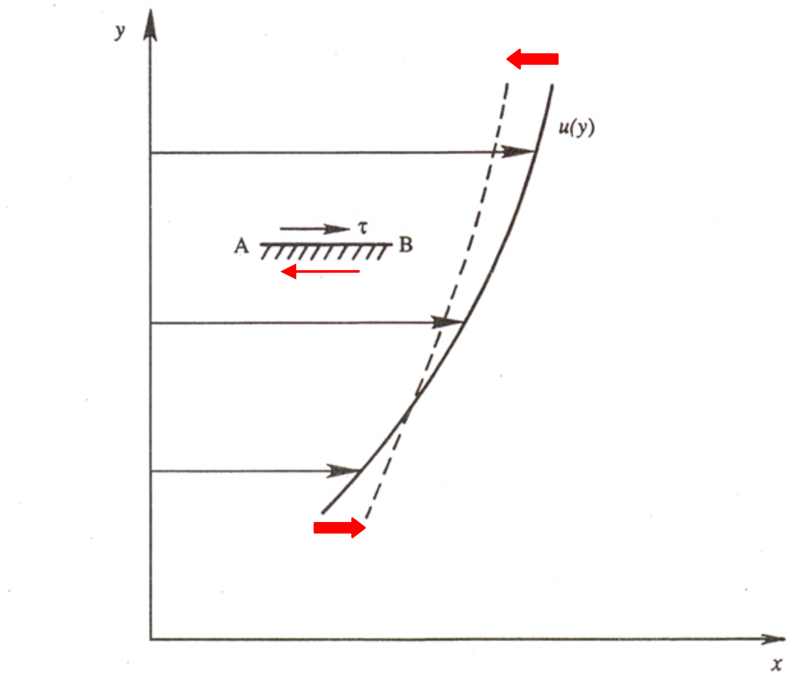
- macroscopic fluid velocity tends to become uniform due to the random motion of molecules because of intermolecular collisions and the consequent exchange of molecular momentum

→ the velocity distribution tends toward the dashed line

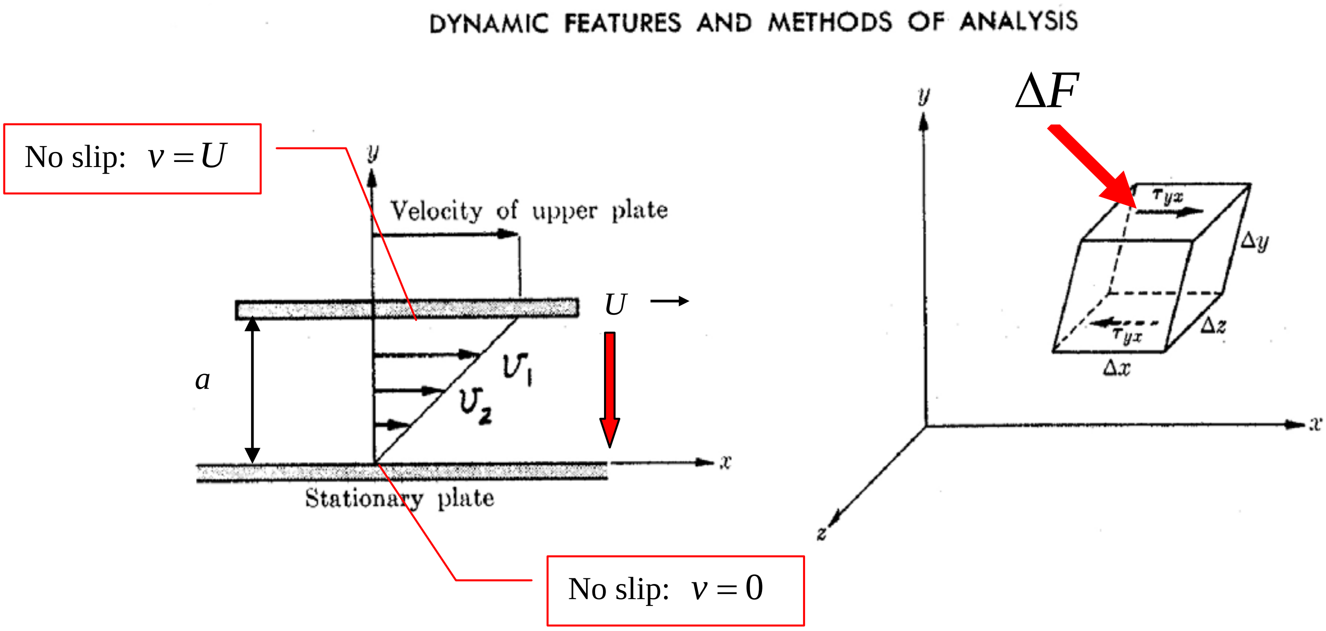
→ momentum flux is equivalent to the existence of the shear stress

$$\tau \propto \frac{\partial u}{\partial y}$$

$$\tau = \mu\frac{\partial u}{\partial y}$$ → Newton's law of friction (viscosity)


y
u(y)
A
τ
B
x

3.4.2 Momentum transport for Couette flow



Couette flow – laminar flow between two plates

transverse transport of longitudinal momentum (mv)

$\propto$ transverse gradient of longitudinal velocity $\left(\dfrac{dv}{dy}\right)$

in the direction of decreasing velocity (longitudinal momentum)

[Re] velocity gradient of Couette flow

- linear

$$\frac{dv}{dy} = \frac{U}{a}$$

3.5 Transport Analogies

(1) Transport

1) advection = transport by imposed current (velocity)

[cf] convection

2) diffusion = movement of mass or heat or momentum in the direction of decreasing concentration of mass, temperature, or momentum

[cf] conduction

(2) Driving force (= gradient)

mass, heat, momentum

$$flux = \left\{ \frac{dM/dt}{area} \right\} \propto \left\{ \frac{d(M/vol)}{ds} \right\}$$

Flux = Time rate of transport of M per unit area normal to transport direction

Gradient of M per unit volume of fluid in the transport direction

$$\frac{dM/dt}{A} = K\frac{d(M/vol)}{ds} \qquad (3.3)$$

where K = diffusivity constant (m^2/S) $[L^2/t]$

$K = f$ (modes of fluid motion, i.e., laminar and turbulent flow)

- molecular diffusivity for laminar flow
- turbulent diffusivity for turbulent flow

3.5.1 Momentum transport

Set $M = \text{momentum} = \Delta mu$

$$\therefore \quad \frac{d\left(\Delta mu\right)}{dt}\frac{1}{\Delta x \Delta z} = K\frac{d}{dy}\left(\frac{\Delta mu}{\Delta vol}\right)$$

Now, apply Newton's 2nd law to LHS

$$\frac{d}{dt}\left(mu\right) = m\frac{du}{dt} = ma = F$$

$$\therefore \quad \frac{d\left(\Delta mu\right)}{dt} = \Delta F_x$$

$$\therefore \quad LHS = \frac{\frac{d\left(\Delta mu\right)}{dt}}{\Delta x \Delta z} = \frac{\Delta F_x}{\Delta x \Delta z} = \tau_{yx}$$

$\tau_{yx} =$ shear stress parallel to the x-direction acting on a plane

whose normal is parallel to y-direction

RHS:

$$\frac{\Delta m}{\Delta vol} = \rho$$

$$\therefore \; RHS = K\frac{d}{dy}\left(\frac{\Delta mu}{\Delta vol}\right) = K\frac{d\left(\rho u\right)}{dy}$$

Combine (i) and (ii)

$$\tau_{yx} = K\frac{d\left(\rho u\right)}{dy} \tag{3.4}$$

If ρ = constant

$$\tau_{yx} = \rho K \frac{du}{dy} \tag{3.5}$$

K = molecular diffusivity constant $(\mathrm{m}^2/\mathrm{s})$

If $K \equiv \nu = \frac{\mu}{\rho}$ = kinematic viscosity

Then,

$$\tau_{yx} = \rho \nu \frac{du}{dy} = \mu \frac{du}{dy} \tag{3.6}$$

3.5.2 Heat transport

upper plate ~ high temperature

lower plate ~ low temperature

conduction of heat within fluid

Set

$$M = \text{heat} = Q = \Delta m C_p T \tag{3.7}$$

where C_p = specific heat at constant pressure

Then, Eq. (3.3) becomes

$$\frac{dQ}{dt}\frac{1}{\Delta x \Delta z} = q_{H_y} = -K \frac{d}{dy}\left[\frac{\Delta m C_p T}{\Delta vol}\right] \tag{3.8}$$

q_H = time rate of heat transfer per unit area normal

to the direction of transport $(j/\sec - \mathrm{m}^2)$

$K = \alpha =$ thermal diffusivity $\left(\mathrm{m}^2/\sec\right)$

If $\rho\,(=\dfrac{\Delta m}{\Delta vol})$ and C_p = const.

$$\therefore \quad q_{Hy} = -\rho C_p K \frac{dT}{dy} = -k \frac{dT}{dy} \tag{3.9}$$

where $k = \rho\, C_p\, K$ = thermal conductivity $\left(j/\sec - \mathrm{m} - K\right)$

3.5.3 Mass transport

Set

$$M = \text{dissolved mass of substancs} = \Delta m_f C_M \tag{3.10}$$

where $\Delta m_f =$ mass of fluid

$C_M =$ concentration

$\equiv$ mass of dissolved substance /unit mass of fluid

[Cf] $C_s = \dfrac{\Delta m_s}{\Delta vol_f}$ $(\mathrm{mg}/l, ppm)$

Then, Eq. (3.3) becomes

$$\frac{d\left(\Delta m_f C_M\right)}{dt}\frac{1}{\Delta x \Delta z} = j_{M_y} = -K\frac{d}{dy}\left[\frac{\Delta m_f C_M}{\Delta vol}\right] \tag{3.11}$$

$j_M =$ time rate or mass transfer per unit area normal to the direction

of transport $\quad \mathrm{kg/m^2 \cdot s}$

If $\quad \rho = \dfrac{\Delta m}{\Delta vol} = \text{const.} = \dfrac{\Delta m_f}{\Delta vol_f}$

$$j_{M_y} = -\rho K \frac{dC_M}{dy} \tag{3.12}$$

$$= -K \frac{dC_M \cdot \rho}{dy} = -K \frac{d\left(\dfrac{\Delta m_s}{\Delta m_f} \cdot \dfrac{\Delta m_f}{\Delta vol_f}\right)}{dy} = -K \frac{d\left(\dfrac{\Delta m_s}{\Delta vol_f}\right)}{dy} = -K \frac{dC_s}{dy}$$

Set $K = D =$ molecular diffusion coefficient $\left(\mathrm{m^2/sec}\right)$

$$j_{M_y} = -\rho K \frac{dC_M}{dy} = -D \frac{dC_s}{dy}$$

transport process	kinematic fluid property (m^2/s)
momentum	ν (kinematic viscosity)
heat	α (thermal diffusivity)
mass	D (diffusion coefficient)

3.6 Particle and Control-Volume Concepts

3.6.1 Infinitesimal elements and control volumes

- Eulerian equations of fluid mechanics

(1) Material method: particle approach

~ describe flow characteristics at a fixed point $(x,\ y,\ z)$ by observing the motion of a material particle of a infinitesimal mass

~ laws of conservation of mass, momentum, and energy can be stated in the differential form, applicable at a point.

~ Newton's 2nd law

$$d\vec{F} = dm\vec{a}$$

◦ If fluid is considered as a continuum, end result of either method is identical.

(2) Control volume method

① differential (infinitesimal) control volume – parallelepiped control volume

② finite control volume – arbitrary control volume

[Re] Control volume

- fixed volume which consists of the same fluid particles and whose bounding surface moves with the fluid

① Differential control volume method

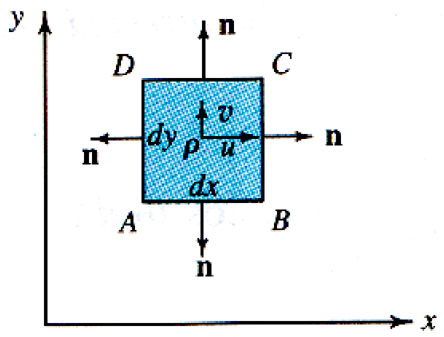
~ concerned with a fixed differential control volume $(= \Delta x \Delta y \Delta z)$ of fluid

~ 2–D or 3–D analysis, Ch. 6

$$\Delta \vec{F} = \frac{d}{dt}\left(\Delta m \vec{q}\right) = \frac{d}{dt}\left(\rho \Delta x \Delta y \Delta z \vec{q}\right)$$

~ $\Delta x, \Delta y, \Delta z$ become vanishingly small

→ point form of equations for conservation of mass, momentum, and energy



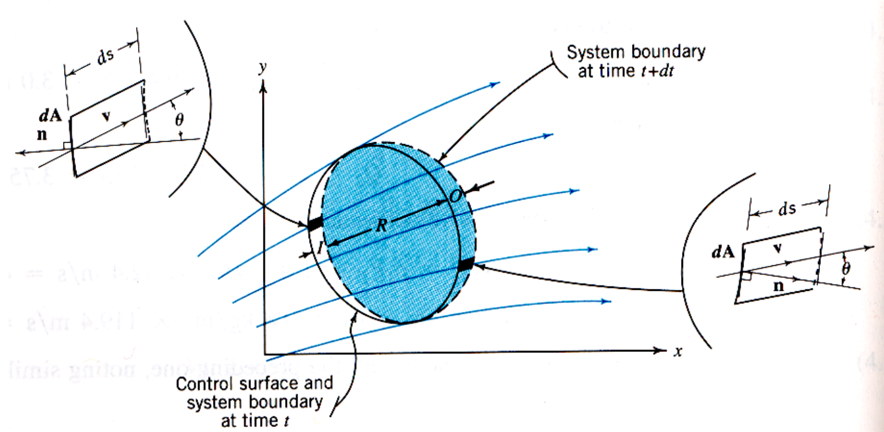
② Finite control volume method

→ frequently used for 1–D analysis, Ch. 4

~ gross descriptions of flow

~ analytical formulation is easier than differential control volume method

~ integral form of equations for conservation of mass, momentum, and energy


System boundary
at time t+dt
Control surface and
system boundary
at time t
dA
n
v
ds
θ
R
O
I
x
y